Estimating Ambiguity Aversion in a Portfolio Choice Experiment*

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Abstract

We report a laboratory experiment that enables us to estimate four prominent models of ambiguity aversion – Subjective Expected Utility (SEU), Maxmin Expected Utility (MEU), Recursive Expected Utility (REU), and α-Maxmin Expected Utility (α-MEU) – at the level of the individual subject. We employ graphical representations of three-dimensional budget sets over bundles of Arrow securities, one of which promises a unit payoff with a known probability and two with unknown (ambiguous) probabilities. The sample exhibits considerable heterogeneity in preferences, as captured through parameter estimates. Nonetheless, there exists a strong tendency to equate the demands for

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the securities that pay off in the ambiguous states. This feature is more easily accommodated by the α-MEU model than by the REU model.

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1 Introduction

The distinction between settings with risk, involving objective and known probabilities, and ambiguity, which involves subjective and unknown probabilities dates back to at least the work of Knight (1921). However, in Savage's (1954) celebrated theory of Subjective Expected Utility (SEU), an individual acts as if a single probability measure governs uncertainty over states of the world. Ellsberg (1961) countered this reduction of subjective uncertainty to risk with several thought experiments suggesting an aversion to ambiguity. Subsequently, a large experimental literature has confirmed Ellsberg's suggested choices, while a large theoretical literature has developed models to accommodate this behavior. On the other hand, with few exceptions, these literatures have developed in parallel with little contact with the other.

In this paper, we connect the experimental evidence with the theoretical models by estimating the parameters of four models of choice under ambiguity. The first is the SEU model; the second is the Maxmin Expected Utility (MEU); the third is Recursive Expected Utility (REU); and the fourth is α-Maxmin Expected Utility (α-MEU).[1] The SEU and MEU respectively capture probabilistic sophistication and extreme ambiguity aversion, while the REU and α-MEU models allow for heterogeneous degrees of ambiguity aversion.

In particular, the experimental test we study is a portfolio choice problem involving three assets. In our preferred interpretation, there are three *states of nature* denoted by $s = 1, 2, 3$ and, for each state s, there is an *Arrow security* that pays one dollar in state s and nothing in the other states. To distinguish the effects of risk (known probabilities) and ambiguity (unknown probabilities), we assume that state 2 has an objectively known probability, whereas the probabilities of states 1 and 3 are ambiguous. Specifically, state

[1] See, respectively, Savage (1954); Gilboa and Schmeidler (1989); Ergin and Gul (2004). Klibanoff, Marinacci, and Mukerji (2005), among others; and Ghirardato, Macherroni, and Marinacci (2004), and Olszewski (2006).

2 occurs with probability $\pi_2 = \frac{1}{3}$ and states 1 and 3 occur with unknown probabilities π_1 and π_3. Subjects were only informed that $\pi_1 + \pi_3 = \frac{2}{3}$.

We consider the problem of allocating an individual's wealth between the three securities. Let x_s denote the demand for the security that pays off in state s and let p_s denote its price. Without essential loss of generality, assume the individual's wealth is normalized to 1. The *budget set* is defined by the feasibility constraint $\mathbf{p} \cdot \mathbf{x} \leq 1$, where $\mathbf{x} = (x_1, x_2, x_3)$ and $\mathbf{p} = (p_1, p_2, p_3)$. The individual can choose any *portfolio* that satisfies this constraint. These budget sets are presented using a novel graphical interface that has proved useful in the (objective) risk experiment of Choi, Fisman, Gale and Kariv (2007a, 2007b) (henceforth CFGKa, CFGKb). The design's most useful feature is that it allows subjects to make numerous choices over a wide range of budget sets, and this yields a rich dataset that is well-suited to analysis at the level of the individual subject.

We begin the analysis of these experimental data by looking for general signs of ambiguity aversion in the aggregate data. In this choice problem, ambiguity aversion can realize itself behaviorally by a desire to hedge payoffs across ambiguous states. A subject can avoid ambiguity completely by demanding *equal* amounts of the securities that pay off in the ambiguous states $x_1 = x_3$. The resulting portfolio pays an amount x_2 with probability $\frac{1}{3}$ and an amount $x_1 = x_3$ with probability $\frac{2}{3}$, thus eliminating any ambiguity about the distribution of returns. Similarly, choosing x_1 close to x_3 will reduce exposure to ambiguity, without eliminating it altogether. The tendency to equate x_1 and x_3 could, of course, result from simple risk aversion, but this is where the unambiguous and risky state becomes useful: the tendency to equate demands is strikingly higher between the securities that pay off in the ambiguous states than between any other pair of securities.

The aggregate data tell us little about the particular portfolios chosen by individual subjects. In select cases, it is possible to readily identify subjects whose choices correspond to prototypical preferences simply from the scatterplots of their choices. We conduct case studies of several subjects where the regularities in the data are very clear. We also find many intermediate cases, but these are difficult to see directly on a scatterplot due to the fact that all prices shift in each observation. This is the purpose of our individual-level econometric analysis.

Before beginning this econometric analysis, we test whether the observed choices are consistent with utility maximization. Afriat (1967) shows that if finite choice data satisfy the Generalized Axiom of Revealed Preference (GARP), there exists a well-behaved utility function that the choices maximize. We replicate under ambiguity the conclusions of CFGKb for choice under risk: that although individual preferences are complex and highly heterogeneous, for most subjects, the violations are sufficiently minor that we can ignore them for the purposes of recovering preferences or constructing appropriate utility functions. We emphasize that the variation in budget sets (prices and incomes) is essential for a non-trivial test of consistency, and that while GARP implies rationality in the sense of a complete, transitive preference ordering, it does not imply the Savage axioms.

The centerpiece of the paper is the econometric estimation of four theories of ambiguity aversion/neutrality at the level of the individual subject. This exercise has two faces. First, the theory informs our view of the data. The estimated parameters serve as useful statistics to summarize the rich variation in individual-level choices generated by our design. Once this information is succinctly summarized, we can compare risk and ambiguity attitudes across subjects and get a broad picture of the heterogeneity of these attitudes. For any given model, the parameter estimates vary dramatically across subjects, implying that individual preferences are very heterogeneous, ranging from risk neutrality with ambiguity aversion to ambiguity neutrality with risk aversion to infinite risk aversion.

Second, the data informs our view of the theories. Most importantly, our experiment employs a broad range of budget sets that provide a serious test of the ability of each model to interpret the data. We did not expect to crown any model as a winner. Indeed, the absolute and relative ability of the models to fit observed individual data vary across subjects, suggesting that a variety of models may be needed to explain the different choices patterns in the population. One particular feature of the data is a widespread attraction by subjects to hedge ambiguity perfectly, i.e. select portfolios which equalize the demands for the securities that pay off in the ambiguous states. This pattern is much more easily accommodated by the α-MEU model than the REU model.

Finally, for each model, we compare the estimated parameters in our ambiguity experiment ($\pi_2 = \frac{1}{3}$ and $\pi_1 + \pi_3 = \frac{2}{3}$) with the analogous estimated parameters in an otherwise identical risk experiment, where the probabilities of all states are objectively known and equal ($\pi_1 = \pi_2 = \pi_3 = \frac{1}{3}$). While we find no difference in the distributions of the parameter interpreted as the coefficient of (absolute) risk aversion, the distributions of the second parameter (in the α-MEU and REU models), which measures ambiguity aversion, shift considerably to the right, especially in the REU model, when calculated using the ambiguity data as compared to the risk data.

The rest of the paper is organized as follows. Section 2 provides a discussion of closely related literature. Section 3 describes the experimental design and procedures. Section 4 summarizes some important features of the data, and Section 5 establishes the consistency of the data with utility maximization. Section 6 discusses the different models that are calibrated to the data. Section 7 provides the econometric analysis, and Section 8 contains some concluding remarks. Experimental instructions, technical details, and individual-level data are gathered in appendices.

2 Related Literature

We will not attempt to review the enormous experimental literature on ambiguity aversion.[2] Instead, we focus attention on some recent papers that are relevant to our study. Halevy (2007) cleverly designs an experiment to verify the connection between the reduction of objective compound lotteries and attitudes to ambiguity. Four different urns are used to elicit choices in the presence of risk, ambiguity, and two degrees of compound uncertainty. Different models of ambiguity with recursive structures generate different predictions about how the reservation values for these four urns will be ordered. The experiment can therefore classify each subject according to which model predicts his ordering of reservation values. We share Halevy's starting point that different models might be appropriate in describing different subjects' behaviors. But, we take this view of heterogeneity even further: not only is there variation in the appropriateness of different models, but also variation in the degree of ambiguity aversion within the subjects who do conform to a particular model.

Hayashi and Wada (2007) examine attitudes toward imprecise information, which incorporates objective restrictions on the distributions over states of nature as a consideration in decision making. This is obviously closely related to ambiguity aversion. They present subjects with different objective restrictions on the possible probabilities over states, and see how imprecision-free equivalents vary with these restrictions. Although the elicitation procedures are very different, our work shares with theirs the aim of estimating parameters at an individual level.

3 Experimental Design

The experimental procedures described below are identical to those used by CFGKb to study decisions under risk when there are two unambiguous

[2] See Camerer and Weber (1992) and Camerer (1995) for excellent, if now somewhat dated, surveys.

states and two associated securities. The experiment was conducted at the Experimental Social Science Laboratory (X-Lab) at UC Berkeley under the X-Lab Master Human Subjects Protocol. The subjects in the experiment were recruited from all undergraduate classes and staff at UC Berkeley and had no previous experience in experiments employing the graphical computer interface. After subjects read the instructions (reproduced in Appendix I), the instructions were read aloud by an experimenter. At the end of the instructional period subjects were asked if they had any questions or difficulties understanding the experiment. No subject reported difficulty understanding the procedures or using the computer interface. Each experimental session lasted about one and a half hours. Payoffs were calculated in terms of tokens and then converted into dollars, where each token was worth \$0.50. A \$5 participation fee and subsequent earnings, were paid in private at the end of the session.

Each experimental session consisted of 50 decision rounds. In each round, a subject allocates tokens between three accounts, labeled x, y and z. Each of these accounts corresponds to an axis in a three-dimensional graph. Each choice involved choosing a point on a budget set of possible token allocations. An example of one such budget set is illustrated in the experimental instructions reproduced in Appendix I. For each round, the computer selected a budget set randomly from the collection of those which intersect at least one of the axes at 50 or more tokens, but with no intercept exceeding 100 tokens. The budget sets selected for each subject in different decision problems were independent of each other and of the sets selected for any of the other subjects in their decision problems.

The axes of the graph were scaled from 0 to 100 tokens. The resolution compatibility of the budget sets was 0.2 tokens. At the beginning of each decision round, the experimental program dialog window went blank and the entire setup reappeared. The appearance and behavior of the pointer were set to the Windows mouse default and the pointer was automatically repositioned randomly on the budget constraint at the beginning of each decision round. Subjects could use the mouse or the keyboard arrows to move the pointer on the computer screen to the desired allocation. Choices were restricted to allocations on the budget constraint, so that subjects could not violate budget balancedness. Subjects could either left-click or press the Enter key to record their allocation. The process was repeated until all 50 rounds were completed.

The payoff in each decision round was determined by the number of tokens in each account. At the end of the round, the computer selected one of the accounts, x, y or z in a random manner such that account y was always selected with probability $\pi_y = \frac{1}{3}$ and accounts x and z were selected with unknown probabilities π_x and π_z. Subjects were only informed that $\pi_x+\pi_z = \frac{2}{3}$. In practice, π_x was drawn from the uniform distribution over $[0, \frac{2}{3}]$. As a counterpoint, we borrow data collected in a concurrent experiment which is identical to ours, except that $\pi_x = \pi_z = \frac{1}{3}$ and announced to the subjects, eliminating any ambiguity. We will refer to these data under objective risk (47 subjects) as belonging to the risk experiment, as opposed to the ambiguity experiment (76 subjects) described previously.

In each decision round, each subject received the number of tokens allocated to the account that was chosen. Subjects were not informed of the account that was actually selected at the end of each decision round. At the end of the experiment, the experimental program randomly selected one decision round from each participant to carry out for payoffs. Each round had an equal probability of being chosen, and the subject was paid the amount he had earned in that round. Note that the payoff method does not provide a randomization with which to hedge ambiguous portfolios across rounds since the probabilities π_x and π_z are unrelated across decision rounds.

4 Data description

In this section, we take an initial look at some broad features of the experimental data as a prelude to our estimation of parametric models of ambiguity aversion.

4.1 Aggregate behavior

We begin with an overview of the basic features of the aggregate data. For any portfolio $\mathbf{x} = (x_1, x_2, x_3)$, we define the *token share* of the security that pays off in state s to be $x_s/(x_1 + x_2 + x_3)$, that is, the number of tokens payable in state s as a fraction of the sum of tokens payable in all three states. We also define the *expenditure share* of the security that pays off in state s to be the expenditure on tokens invested in this security as a fraction of total expenditure. Since prices are normalized so that total expenditure equals unity, the expenditure share is simply $p_s x_s$. Table 1 displays summary statistics and percentile values for the token share (Table 1A) and the expenditure share (Table 1B) of each security $s = 1, 2, 3$. Both distributions are very similar. On average, our subjects invested approximately 35 percent of the tokens in the security with unambiguous payoff, accounting also for 35 percent of total expenditure. This is only marginally higher than the 32 percent of the token and expenditure shares of the other securities.

[Table 1 here]

For any portfolio $\mathbf{x} = (x_1, x_2, x_3)$ and any pair of securities s and $s' \neq s$, we define the *relative demand* to be $x_s/(x_s + x_{s'})$, that is, the demand for the security that pays off in state s as a fraction of the sum of demands for securities that pay off in state s and s'. This ratio measures the extent to which the demand for securities s and s' are equalized. Table 1 above also displays summary statistics and percentile values for the three relative demands (Table 1C). In each case, we screen the data for portfolios that spend 10 percent or less of the total expenditure on tokens invested in securities s and s' (these portfolios account for only 2.8, 3.3 and 2.2 percent of the data, respectively). The three distributions are quite similar. Additionally, perhaps as expected, the distributions are nearly symmetric and concentrated around the midpoint of 0.5.

Interestingly, the mode around the midpoint is more pronounced in the relative demand for the securities that pay off in ambiguous state $x_1/(x_1 + x_3)$, which provides clear evidence of ambiguity aversion. In the presence of ambiguity, subjects can create a hedge against ambiguity by equalizing their portfolio holdings across the securities that pay off in the ambiguous states 1 and 3, making their payoffs less sensitive to their unknown probabilities π_1 and π_3. Figure 1 depicts kernel density estimation of $x_1/(x_1 + x_3)$ and compares it with $x_1/(x_1 + x_2)$, which measures the extent to which subjects equalize payoffs in two states, exactly one of which is ambiguous (the distribution of $x_3/(x_2 + x_3)$ is identical). For 45.5 percent of the portfolios, the the relative demand $x_1/(x_1 + x_3)$ is between 0.45 and 0.55. If we consider $x_1/(x_1 + x_2)$ and $x_3/(x_2 + x_3)$ this decreases to 34.2 and 35.0 percent, respectively.

[Figure 1 here]

4.2 Individual behavior

The aggregate distributions above tell us little about the portfolios chosen by particular subjects. To get some sense of the considerable heterogeneity of individual behavior, we discuss a few subjects whose choices can easily be explained in terms of some notion of ambiguity or risk aversion. For each subject, Figure 2 shows the portfolio choices in terms of token shares (left panel) and expenditure shares (right panel) for the three securities. The vertices of the unit simplex correspond to portfolios consisting of one of the three securities. Each point in the simplex represents a portfolio as a convex combination of the extreme points. The figures for the full set of subjects are available in Appendix II, where we also show, for each subject, the relationships between the log-price ratio $\ln(p_1/p_2)$ and the relative demand $x_1/(x_1+x_2)$ and between the log-price ratio $\ln(p_1/p_3)$ and the relative demand $x_1/(x_1+x_3)$. The scatter plots in these panels illustrate the sensitivity of portfolio decisions to changes in relative prices. We emphasize that for most subjects the data are much less regular and, for those subjects, it is more difficult to see these relationships in a scatterplot. Nevertheless, the portfolio choices for the full set of subjects reveal striking regularities *within* and marked heterogeneity *across* subjects.

[Figure 2 here]

Figure 2A depicts the choices of a subject (ID 129) who always chose nearly equal portfolio holdings $x_1 = x_2 = x_3$, suggesting infinite risk aversion. Figure 2B depicts a very different case, the choices of the only subject (ID 314) who, with a few exceptions, invested all his tokens in the cheapest security, indicating pure risk neutrality. Figure 2C depicts a subject (ID 339) who equalizes expenditure $p_1x_1 = p_2x_2 = p_3x_3$, rather than tokens, across the three securities. This behavior is consistent with a logarithmic von Neumann-Morgenstern utility function over tokens. A more interesting regularity is illustrated in Figure 2D, which depicts the decisions of a subject (ID 322) who, with very few exceptions, invested nearly equal amounts in the securities corresponding to ambiguous states 1 and 3, that is, $x_1 = x_3 \neq x_2$. A similar regularity, albeit implemented less precisely, is illustrated in Figure 2E, which depicts the decisions of another subject (ID 130). Finally, Figure 2F depicts a subject (ID 407) who did not equalize his payoffs in ambiguous states, but tended to choose payoffs x_1 and x_3 that were much closer to each other than to the value of x_2. The behaviors of these subjects (ID 322, 130 and 407) suggest ambiguity aversion, in the sense that they are trying to reduce the sensitivity of their payoffs to states with ambiguous probabilities.

5 Testing Rationality

As a matter of theory, ambiguity aversion may be perfectly rational, in the sense of being consistent with a complete and transitive preference ordering. In practice, however, one might suspect that individuals who exhibit ambiguity aversion are more likely to violate the axioms of rational behavior. For this reason, before considering alternative parametric utility functions, we investigate whether subjects' choices can be generated by maximizing *any* utility function.

Afriat's (1967) theorem tells us that there exists a utility function that rationalizes the observed portfolio choices if and only if the data satisfy the Generalized Axiom of Revealed Preference (GARP). Moreover, if GARP is satisfied the utility function can be chosen to be piecewise linear, continuous, increasing, and concave. Since GARP offers an exact test (either the data satisfy GARP or they do not), whereas in practice there will always be some small violations, we compute a measure of how nearly the data complies with GARP. Afriat's (1972) Critical Cost Efficiency Index (CCEI) measures the amount by which each budget constraint must be relaxed in order to remove all violations of GARP. The CCEI is bounded between zero and one. The closer it is to one, the smaller the perturbation of budget constraints required to remove all violations and thus the closer the data are to satisfying GARP. We refer the interested reader to CFGKa for a discussion of consistency tests.

The CCEI scores averaged 0.942 and 0.938 in the ambiguity and risk experiments, respectively, which is close enough to passing GARP to suggest that our subjects' choices are indeed consistent with utility maximization. Following Bronars (1987) we compare the actual CCEI scores with the scores of simulated subjects who randomize uniformly among all portfolios on each budget set. Figure 3 shows histograms of the distribution of CCEI scores for a random sample of 25,000 simulated subjects versus the scores for the actual subjects in the ambiguity (blue) and risk (red) experiments.

[Figure 3 here]

The histograms in Figure 3 make plain that the significant majority of our subjects came much nearer to consistency with utility maximization than random choosers would have done and that their CCEI scores were only slightly worse than the score of one of the perfect utility maximizers. The fact that choices are sufficiently consistent to be considered utility-generated is a striking result in its own right. The presence of ambiguity could cause not just a departure from expected utility, but a more fundamental departure from rationality. Our analysis suggests otherwise. At the very least, choices under ambiguity are at least as rationalizable as choices under risk.

6 Models of Ambiguity

In this section, we first provide an overview of the different models that will be estimated using individual-level data: Subjective Expected Utility (SEU), Maxmin Expected Utility (MEU), Recursive Expected Utility (REU), and α-Maxmin Expected Utility (α-MEU). We skip the models' development and analysis and instead focus on the identifying assumptions placed on these general models to generate specific parametric formulations amenable to analysis. We refer the interested reader to Appendix III for more details. Throughout, we assume that cardinal utility over tokens exhibits constant absolute risk aversion (CARA):

$$u(t) = -e^{\rho t},$$

where ρ is the coefficient of absolute risk aversion and t is the number of tokens. This specification has two advantages. First, it is independent of the (unobservable) initial wealth level of the subjects. Second, it can accommodate boundary portfolios, where $x_s = 0$ for some state s.

6.1 Models description

Subjective Expected Utility (SEU) As a benchmark we use the SEU model (Savage (1954)). The decision maker has a von Neumann-Morgenstern utility function that he integrates with respect to a subjective probability distribution. We proceed by making the identifying assumption that ambiguous states are equally probable, that is, the subjective probability distribution is $\pi = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. Letting $\exp\{t\} = e^t$, the utility function over portfolios $\mathbf{x} = (x_1, x_2, x_3)$ takes the form:

$$U_{\text{SEU}}(\mathbf{x}; \rho) = -\frac{1}{3}\exp\{-\rho x_1\} - \frac{1}{3}\exp\{-\rho x_2\} - \frac{1}{3}\exp\{-\rho x_3\}.$$

Maxmin Expected Utility (MEU) By contrast with the SEU model, a decision maker whose preferences are described by MEU (Gilboa and Schmeidler (1989)) evaluates a portfolio by its least expected utility over some set of subjective prior probabilities. This minimization reflects aversion to ambiguity. Here we make the identifying assumption that the subjective probabilities agree with the objective probabilities announced in the experiment. In other words, the set of admissible probability distributions over which the expected utility will be minimized is $\{\pi : \pi_2 = \frac{1}{3} \text{ and } \pi_1 + \pi_3 = \frac{2}{3}\}$. Then the utility function over portfolios $\mathbf{x} = (x_1, x_2, x_3)$ becomes:

$$U_{\text{MEU}}(\mathbf{x}; \rho) = \begin{cases} -\frac{2}{3}\exp\{-\rho x_1\} - \frac{1}{3}\exp\{-\rho x_2\} & \text{if} \quad x_1 \leq x_3 \\ \\ -\frac{2}{3}\exp\{-\rho x_3\} - \frac{1}{3}\exp\{-\rho x_2\} & \text{if} \quad x_1 > x_3 \end{cases}.$$

Recursive Expected Utility (REU) A recent view of ambiguity aversion (Ergin and Gul (2004), Klibanoff, Marinacci, and Mukerji (2005), Nau (2005), and Seo (2007); as well as related work by Ahn (2008), Giraud (2006), and Halevy and Feltkamp (2005)) assumes the decision maker has a subjective (second-order) prior over the possible (first-order) probabilities over states $\pi = (\pi_1, \pi_2, \pi_3)$. Unsure which of the possible first-order probabilities actually governs the states, the decision maker transforms the expected utilities for all distributions π using a concave function before integrating these utilities with respect to his second-order prior. This procedure is entirely analogous to the transformation of wealth into cardinal utility before computing objective expected utility. The concavity of this transformation captures ambiguity aversion. We follow Halevy (2007) in referring to this model as REU, owing to its recursive double expectation.

We parameterize this model by assuming that the second-order probability is uniformly distributed over the set $\{\pi : \pi_2 = \frac{1}{3} \text{ and } \pi_1 + \pi_3 = \frac{2}{3}\}$ consisting of all priors consistent with the objective information. In this case, the utility function over portfolios $\mathbf{x} = (x_1, x_2, x_3)$ takes the two-parameter form:

$$U_{\mathrm{REU}}(\mathbf{x}; \alpha_0, \rho) =$$
$$\frac{1}{\alpha_0} \int_0^{\frac{2}{3}} - \exp\left\{ -\alpha_0 \left(\begin{array}{c} -\pi_1 \exp\{-\rho x_1\} \\ -\frac{1}{3}\exp\{-\rho x_2\} - \left(\frac{1}{3} - \pi_1\right)\exp\{-\rho x_3\} \end{array} \right) \right\} d\pi_1,$$

where

$$\alpha = \frac{30\alpha_0}{1 - \exp\{30\alpha_0\}}$$

reflects the curvature of the aggregator and, hence, the decision maker's ambiguity aversion. The implicit equation for α in terms of α_0 normalizes the raw ambiguity coefficient to control for differences in risk aversion across decision makers (see Appendix III for precise details). It can be shown that the REU model reduces to SEU when $\alpha = 0$ and approaches MEU as $\alpha \to \infty$.

α-Maxmin Expected Utility (α-MEU) An alternative approach (Ghirardato, Macherroni, and Marinacci (2004), and Olszewski (2006)) considers a decision maker who evaluates each portfolio by a convex combination of its worst and best expected utilities over some fixed set of subjective probabilities over states, using the weights α and $1 - \alpha$ for the best and worst expected utilities, respectively. The parameter α can be interpreted as a measure of ambiguity aversion. Assuming that the set of subjective probabilities is consistent with the objective information, the utility function over portfolios $\mathbf{x} = (x_1, x_2, x_3)$ takes the two-parameter form:

$$U_{\alpha\text{-MEU}}(\mathbf{x}; \alpha, \rho) = \begin{cases} \begin{array}{c} -\frac{2}{3}\alpha \exp\{-\rho x_1\} - \frac{1}{3}\exp\{-\rho x_2\} \\ -\frac{2}{3}(1-\alpha)\exp\{-\rho x_3\} \end{array} & \text{if} \quad x_1 \leq x_3 \\ \begin{array}{c} -\frac{2}{3}\alpha \exp\{-\rho x_3\} - \frac{1}{3}\exp\{-\rho x_2\} \\ -\frac{2}{3}(1-\alpha)\exp\{-\rho x_1\} \end{array} & \text{if} \quad x_1 > x_3 \end{cases} .$$

As with REU, the α-MEU model reduces to SEU when $\alpha = \frac{1}{2}$ and MEU when $\alpha = 1$.[3]

6.2 Properties of demand

The models described above can all be characterized by axioms over preferences. Since we are interested in estimating the models using our design, what matters most is the implications of the models for portfolio choice. To better understand the properties of each model, we have simulated the demand for securities as a function of prices. Figure 4 below illustrates the relationships between the log-price ratio $\ln(p_1/p_3)$ and the optimal relative demand $x_1^*/(x_1^* + x_3^*)$ (left panels) and between $\ln(p_1/p_2)$ and $x_1^*/(x_1^* + x_2^*)$ (right panels), for each model, using a range of parameter values.

Figure 4A illustrates the case of SEU. Note that an increase in the level of absolute risk aversion ρ makes the curves flatter. For high values of ρ, the decision maker is extremely risk averse and always chooses nearly safe portfolios $x_1^* = x_2^* = x_3^*$. The relationships between $\ln(p_1/p_3)$ and $x_1^*/(x_1^* + x_3^*)$ and between $\ln(p_1/p_2)$ and $x_1^*/(x_1^* + x_2^*)$, which illustrate the tradeoffs that the decision maker makes between the payoffs in ambiguous states and between the payoffs in an ambiguous state and the unambiguous state, are identical.

The case of the MEU model is illustrated in Figure 4B. By contrast with the case of SEU, the optimal relative demand for securities paying off in ambiguous states, $x_1^*/(x_1^* + x_3^*)$, is insensitive to the level of risk aversion. This is because MEU embodies an extreme form of ambiguity aversion that implies the optimal portfolio choice satisfies $x_1^* = x_3^*$. On the other hand, the relationship between $\ln(p_1/p_2)$ and $x_1^*/(x_1^* + x_2^*)$ is qualitatively very similar, whether preferences satisfy MEU or SEU.

Figure 4C depicts the same relationships in the case of REU, for different values of α and ρ (each panel assumes a different value for α). We note

[3] We can also use a change of variables to interpret α-MEU as a model of imprecise information, in the sense of Hayashi, Gajdos, Tallon, and Vergnaud (forthcoming), as explained in Appendix III.

that the two optimal relative demand curves are smooth for all price ratios. The ambiguity aversion parameter α flattens the $x_1^*/(x_1^*+x_3^*)$ curves in a manner qualitatively similar to having increased risk aversion, whereas the $x_1^*/(x_1^*+x_2^*)$ curves remain very similar to those in the case of SEU with the corresponding level of risk aversion ρ.

Finally, Figure 4D illustrates the case of α-MEU. If the prices of the securities that pay off in the ambiguous states, p_1 and p_3, are similar ($\ln(p_1/p_3)$ is close to zero), then the optimum portfolio choice satisfies $x_1^* = x_3^*$ and is insensitive to ambiguity. The only effect of increasing the level of ambiguity aversion α is to make this intermediate range of price ratios larger. For more extreme price ratios (the absolute value of $\ln(p_1/p_3)$ is large), the relationship between $\ln(p_1/p_3)$ and $x_1^*/(x_1^*+x_3^*)$ is the same as in the SEU model (the same curve "stretched" apart at the point 0.5). The key feature of the α-MEU model is this flat range in which the portfolio satisfies $x_1^* = x_3^*$ and is insensitive to ambiguity. By contrast, the choice of a portfolio without ambiguity is a knife-edge case in the REU model.

[Figure 4 here]

7 Estimation

7.1 Specification

The data generated by an individual's choices are denoted by $\left\{\left(\mathbf{x}^i, \mathbf{p}^i\right)\right\}_{i=1}^{50}$, where $\mathbf{x}^i = \left(x_1^i, x_2^i, x_3^i\right)$ is the actual portfolio chosen by the subject and $\mathbf{p}^i = (p_1^i, p_2^i, p_3^i)$ denotes the vector of the security prices. For each subject n and for each model, we generate estimates of the parameters for ambiguity aversion $\hat{\alpha}_n$ (for the REU and α-MEU models) and the parameter for risk aversion $\hat{\rho}_n$ using nonlinear least squares (NLLS). These estimates are chosen to minimize

$$\sum_{i=1}^{50} \left\|\mathbf{x}^i - \mathbf{x}^*(\mathbf{p}^i; \alpha_n, \rho_n)\right\|, \tag{1}$$

where $\|\cdot\|$ denotes the three-dimensional Euclidean norm.

We note two observations about the econometric specification:

- First, very large coefficients of risk aversion ρ are computationally unidentified. For high values of ρ, all models predict perfect hedging by choosing the safe portfolio $x_1 = x_2 = x_3$. We thus impose the restriction $\rho \leq 2$ on the estimations in order to avoid this identification problem. Similarly, we also impose the restriction $\alpha \leq 2$ in the REU model. Moreover, when ρ is large, ambiguity aversion cannot be separately identified, since all of the hedging between the securities that pay off in the ambiguous states can be attributed to the extreme risk aversion. Therefore, when $\rho = 2$, we set $\alpha = 0$ in the REU model and $\alpha = \frac{1}{2}$ in the α-MEU model, the respective parameter values which correspond to ambiguity neutrality in the two models.

- Secondly, according to Afriat's (1967) theorem, if choices are made from convex budget sets there is no loss of generality in assuming that utility functions are well-behaved (piecewise linear, continuous, increasing, and concave). In particular, price and quantity data do not allow us to distinguish between risk- or ambiguity-loving behavior, on the one hand, and risk- or ambiguity-neutral behavior, on the other. We therefore restrict the parameters to the range of risk and ambiguity averse values: $\rho \geq 0$ in all models, and $\alpha \geq 0$ in the REU model and $\alpha \geq 1/2$ in the α-MEU model.

7.2 Econometric results

Appendix IV presents, subject by subject, the estimated $\hat{\alpha}_n$ (for the REU and α-MEU models) and $\hat{\rho}_n$ for each model. The additional column lists the CCEI scores. Table 2 below displays summary statistics and percentile values. An interesting feature of the estimates is the considerable heterogeneity in the parameters $\hat{\alpha}_n$ and $\hat{\rho}_n$, in both the ambiguity and risk experiments. Despite this heterogeneity, the distributions of risk aversion across the ambiguity and risk experiments, as reflected in the percentile values for $\hat{\rho}_n$, are very similar, which is comforting since there should be no difference in risk attitudes across the ambiguity and risk experiments (the differences in means and standard deviations are driven by a few subjects with very large coefficients in the risk experiment). On the other hand, the distributions of the ambiguity parameter $\hat{\alpha}_n$ in the α-MEU and REU models in the two experiments are very different. Precisely, the distributions of $\hat{\alpha}_n$ calculated using the ambiguity data lies considerably to the right (especially in the REU model) compared to distributions of $\hat{\alpha}_n$ calculated using the risk data.

[Table 2 here]

Figure 5 presents the data from Appendix IV graphically in the form of scatterplots of the estimates, and illustrates the heterogeneity of preferences that we find in all models. Figure 5A shows a scatterplot of $\hat{\rho}_n$ in the SEU and MEU models with the sample split by ambiguity (blue) and risk (red) experiments. To facilitate presentation of the data, we omit 11 subjects in the ambiguity experiment and eight subjects in the risk experiment whose $\hat{\rho}_n$ value is higher than 0.2 in either model. Perhaps most interestingly, the estimates are concentrated around the 45 degree line indicating a strong similarity *within* subjects between the risk estimates in the SEU and MEU models. Figure 5B shows a scatterplot of $\hat{\alpha}_n$ and $\hat{\rho}_n$, in the REU model with the sample again split by ambiguity (blue) and risk (red) experiments. Again, to facilitate presentation of the data, we omit 19 subjects in the ambiguity experiment and 11 subjects in the risk experiment whose $\hat{\alpha}_n$ or $\hat{\rho}_n$ value is higher than 0.2. Finally, Figure 5C shows the same scatterplot for the α-MEU model after omitting 13 of the ambiguity subjects and ten of the risk subjects, whose $\hat{\alpha}_n$ value is higher than 0.7 or whose $\hat{\rho}_n$ value is higher than 0.2.

[Figure 5 here]

Finally, Figure 6 shows the relationship between log-price ratio $\ln(p_1/p_3)$ and the actual relative demand $x_1/(x_1 + x_3)$ (blue) and estimated relative demand $\hat{x}_1/(\hat{x}_1+\hat{x}_3)$ (red) in the two-parameter models REU (left panel) and α-MEU (right panel) for the group of subjects that we followed in the non-parametric analysis (ID 129, 130, 314, 322, 339, 407). Note that $\hat{x}_1/(\hat{x}_1 + \hat{x}_3)$ is calculated using the individual-level estimates, $\hat{\alpha}_n$ and $\hat{\rho}_n$. Through these examples, we hope to reinforce the message that different subjects have qualitatively different responses to ambiguity and that neither model obviously "wins" and dominates the other in its ability to fit the behavior of all subjects. The figures for the full set of subjects and for all models are available in Appendix V, which also depicts the relationship between the log-price ratio $\ln(p_1/p_2)$ and the actual relative demand $x_1/(x_1 + x_2)$ (blue) and estimated relative demand $\hat{x}_1/(\hat{x}_1+\hat{x}_2)$, as well as the actual and estimated portfolios in terms of token and expenditure shares represented as points in a simplex.

[Figure 6 here]

Figure 6A shows the relationship between $\ln(p_1/p_3)$ and the estimated relative demand $x_1/(x_1+x_3)$ for a subject (ID 129) with $\hat{\rho}_{\text{REU}} = \hat{\rho}_{\alpha\text{-MEU}} = 2$. This subject very precisely implemented infinite risk aversion preferences. The ambiguity aversion parameter of this subject is therefore unidentified and set equal to the neutral value, $\hat{\alpha}_{\text{REU}} = 0$ and $\hat{\alpha}_{\alpha\text{-MEU}} = 0.5$. The fit is nearly perfect in both REU and α-MEU. Figure 6B shows the subject (ID 314) who most closely approximated risk neutral preferences with

$(\hat{\alpha}, \hat{\rho})_{\mathrm{REU}} = (0.01, 0.00)$ and $(\hat{\alpha}, \hat{\rho})_{\alpha\text{-MEU}} = (0.52, 0.00)$. Both the REU and α-MEU models suggest a nontrivial degree of ambiguity aversion, which is driven by a few exceptional choices where this subject chose nearly unambiguous portfolios $x_1 = x_3$. Notice that both models do a very good job of predicting his boundary portfolios, but perform less well in predicting the "outliers." This subject appears to be very close to risk neutrality, but perhaps reveals some degree of ambiguity aversion through his outlying portfolios.

Figure 6C shows the subject (ID 339) who precisely implemented logarithmic preferences with $(\hat{\alpha}, \hat{\rho})_{\mathrm{REU}} = (0.00, 0.08)$ and $(\hat{\alpha}, \hat{\rho})_{\alpha\text{-MEU}} = (0.50, 0.08)$. Nonetheless, the exponential form performs quite well in terms of fit. More interestingly, Figure 6D shows the relationship for a subject (ID 322) with $(\hat{\alpha}, \hat{\rho})_{\mathrm{REU}} = (2.00, 0.42)$ and $(\hat{\alpha}, \hat{\rho})_{\alpha\text{-MEU}} = (0.71, 0.43)$, who quite precisely chose unambiguous portfolios $x_1 = x_3$. The estimated ambiguity parameters for both the REU and α-MEU models are among the highest in the sample. A visual inspection of the estimation results and the observed data reveals the fit to be quite good for both the REU and α-MEU models. However, both REU and α-MEU provide substantial improvement in fit over SEU, but little improvement over MEU. This subject thus closely approximates MEU preferences.

Figure 6E shows the fitted relationships for a subject (ID 130) with $(\hat{\alpha}, \hat{\rho})_{\mathrm{REU}} = (0.00, 0.06)$ and $(\hat{\alpha}, \hat{\rho})_{\alpha\text{-MEU}} = (0.57, 0.05)$, who with few exceptions chose unambiguous portfolios ($x_1 = x_3$). This subject's departures from unambiguous portfolios are precipitated by extreme log-price ratios $\ln(p_1/p_3)$. This is entirely consistent with α-MEU. Furthermore, the α-MEU model provides an improved fit over MEU, especially in picking up the few extremely ambiguous portfolios. On the other hand, REU fails to pick up this subject's aversion to ambiguity. Clearly, it cannot accommodate both the large interval of intermediate log-price ratios $\ln(p_1/p_3)$ in which unambiguous portfolios $x_1 = x_3$ are chosen and the extremely ambiguous portfolios $x_1 = 0$ or $x_3 = 0$ chosen for low and high price ratios, respectively. The estimated relationship for REU makes clear that the model gives up the former to improve its fit for the latter. A review of the full set of subjects shows, in many cases, a substantial number of unambiguous portfolios ($x_1 = x_3$), which can only be accommodated by the MEU or α-MEU models.

Finally, Figure 6F shows the fitted relationship for a subject (ID 407) with $(\hat{\alpha}, \hat{\rho})_{\mathrm{REU}} = (1.99, 0.08)$ and $(\hat{\alpha}, \hat{\rho})_{\alpha\text{-MEU}} = (0.87, 0.08)$. This subject chose portfolios with smaller differences between x_1 and x_3 than between any other pair of securities, but did not usually move to the extreme of choosing unambiguous portfolios ($x_1 = x_3$). Although this subject is averse to ambiguity, the fit of the α-MEU model barely improves on the MEU model. On the other hand, the REU model improves the fit considerably over either MEU or SEU. This is especially true for extreme log-price ratios $\ln(p_1/p_3)$, where MEU predicts values for x_1 far too close to x_3, while SEU predicts values far too distant.

8 Conclusion

In this paper, we have estimated four models of choice under uncertainty – SEU, MEU, REU and α-MEU. In undertaking this estimation exercise, we had two main objectives in mind. In the first place, we were interested in the parameter estimates as measures of individual attitudes toward risk and ambiguity. In the second place, we wanted to explore the ability of the models to capture important features of the data. The experimental method enables us to collect many observations per subject, and we can therefore estimate the models at the individual level. Most importantly, the broad range of budget sets that our experiment employs provides a serious test of the ability of each theory, and a structural econometric model based on the theory, to interpret the data.

The parameter estimates reveal a large amount of individual heterogeneity. Nevertheless, there is a strong tendency to equate the demands for the securities that pay off in the ambiguous states, $x_1 = x_3$. This feature is more easily explained by the α-MEU model than by the REU model. We do not regard the estimation exercise as a competition to determine a general winning model. Obviously, all theoretical models have their limitations and a model that works well in the present setting may not work so well in others. Further, we made specific parametric assumptions in order to implement the different models of ambiguity preferences, which may have affected the performance of the different models. We do believe, however, that some of the features of the data generated by this experiment, particularly the observation of so many hedged portfolios satisfying $x_1 = x_3$, are robust and need to be taken into account in future work.

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Table 1. Aggregate statistics

A. Token shares

	$s=1$	$s=2$	$s=3$
Obs.	3800	3800	3800
Mean	0.326	0.351	0.323
SD	0.203	0.222	0.204
5	0.001	0.001	0.002
25	0.224	0.233	0.217
50	0.333	0.333	0.330
75	0.404	0.445	0.393
95	0.705	0.804	0.706

B. Expenditure shares

	$s=1$	$s=2$	$s=3$
Obs.	3800	3800	3800
Mean	0.323	0.353	0.322
SD	0.191	0.208	0.194
5	0.002	0.002	0.004
25	0.228	0.235	0.218
50	0.318	0.338	0.312
75	0.406	0.456	0.406
95	0.650	0.726	0.642

C. Relative demands

	$x_1/(x_1+x_2)$	$x_1/(x_1+x_3)$	$x_3/(x_2+x_3)$
Obs.	3694	3676	3716
Mean	0.488	0.503	0.486
SD	0.255	0.241	0.255
5	0.003	0.003	0.004
25	0.368	0.442	0.364
50	0.500	0.499	0.499
75	0.599	0.582	0.585
95	0.994	0.993	0.997

Table 2. Summary of estimation results

A. SEU

	ρ	
Exp.	Ambiguity	Risk
Mean	0.136	0.242
SD	0.291	0.559
5	0.007	0.011
25	0.027	0.018
50	0.045	0.043
75	0.113	0.079
95	0.607	2.000

B. MEU

	ρ	
Exp.	Ambiguity	Risk
Mean	0.127	0.252
SD	0.288	0.571
5	0.013	0.015
25	0.031	0.026
50	0.042	0.043
75	0.096	0.070
95	0.463	1.999

C. REU

	ρ		α	
Exp.	Ambiguity	Risk	Ambiguity	Risk
Mean	0.115	0.252	0.247	0.125
SD	0.257	0.603	0.516	0.364
5	0.007	0.010	0.000	0.000
25	0.022	0.017	0.007	0.000
50	0.038	0.038	0.042	0.021
75	0.092	0.068	0.153	0.058
95	0.464	2.043	1.985	0.506

D. α-MEU

	ρ		α	
Exp.	Ambiguity	Risk	Ambiguity	Risk
Mean	0.119	0.223	0.565	0.552
SD	0.274	0.532	0.089	0.114
5	0.006	0.010	0.500	0.500
25	0.022	0.018	0.500	0.500
50	0.041	0.041	0.529	0.503
75	0.088	0.070	0.604	0.541
95	0.428	1.999	0.789	0.881

Figure 1: The distribution of relative demands $x_1/(x_1+x_3)$ and $x_1/(x_1+x_2)$

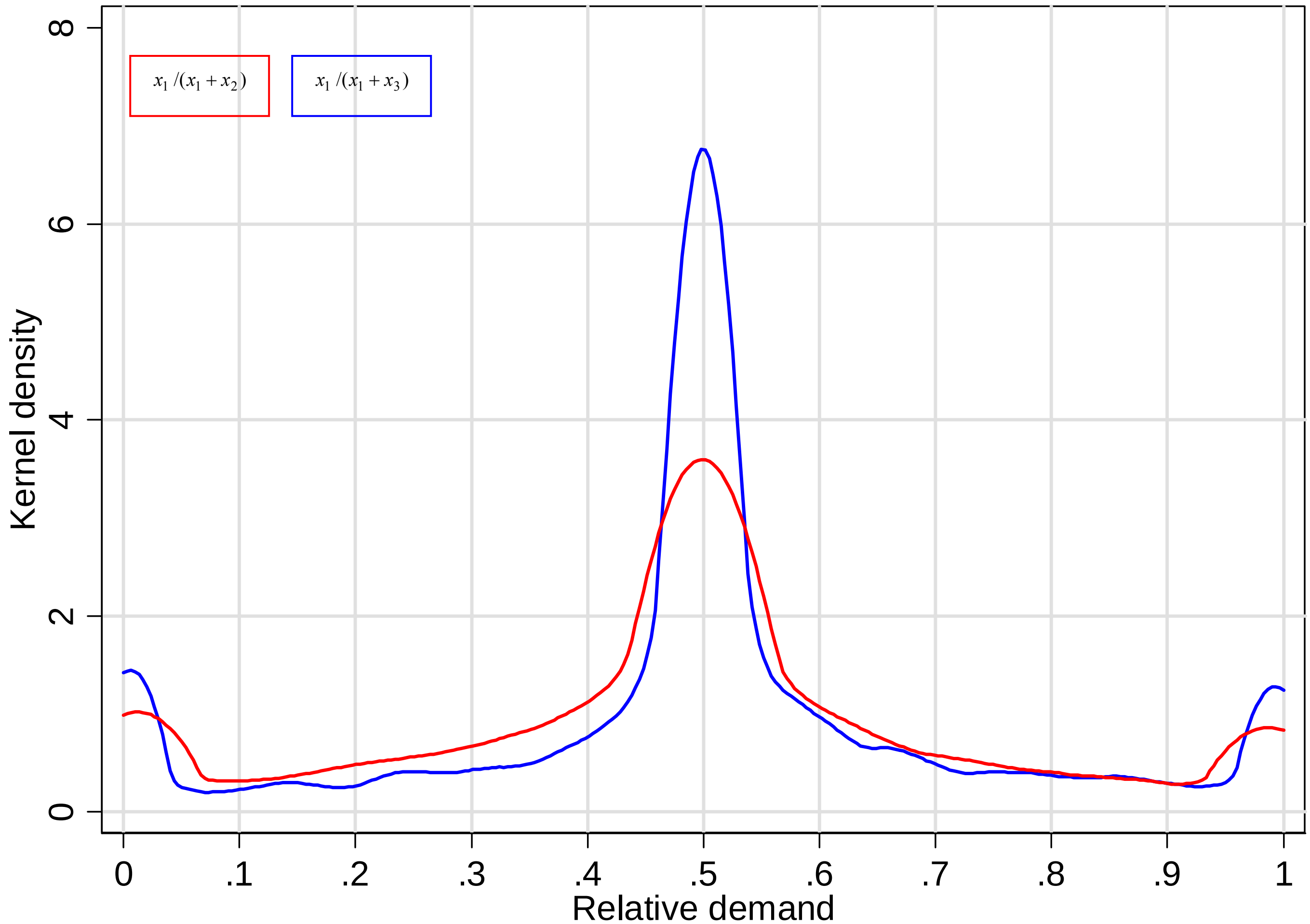
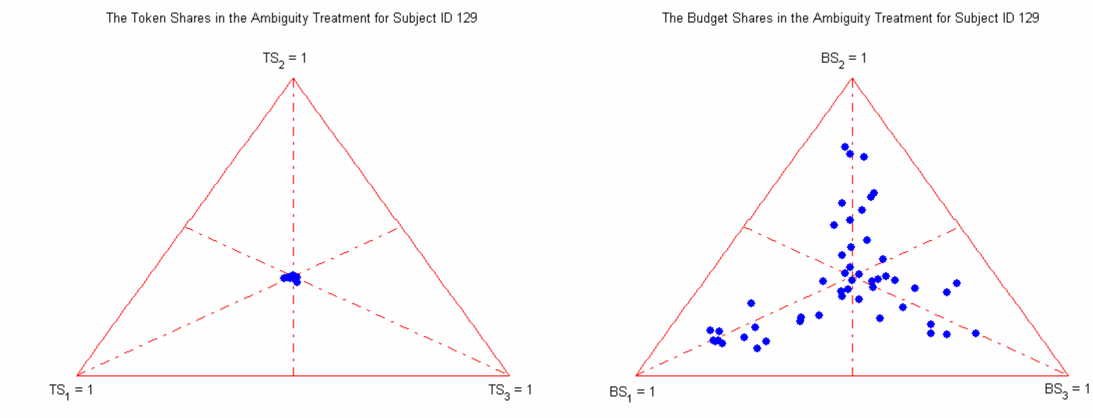


Figure 2: Individual-level data

A: ID 129



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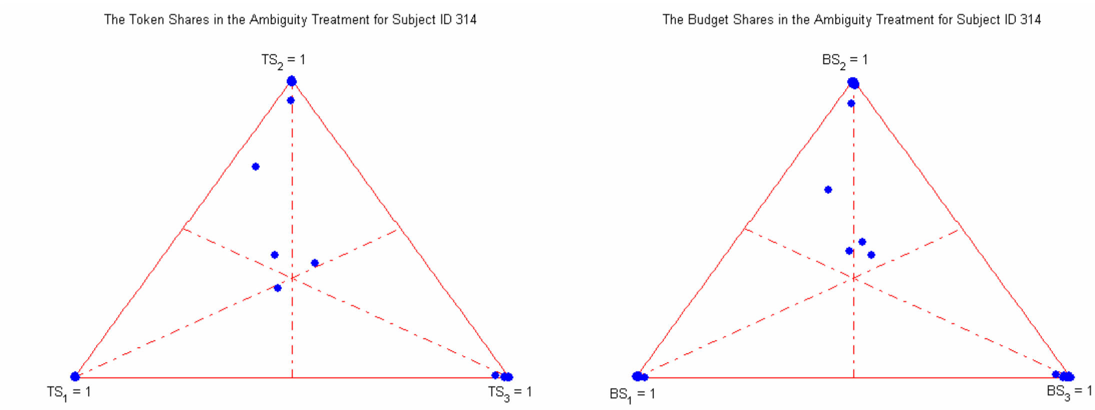
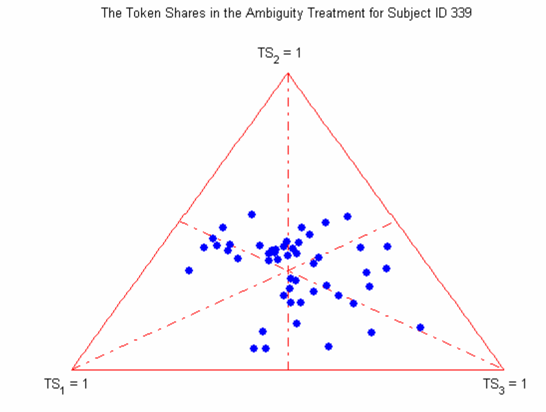
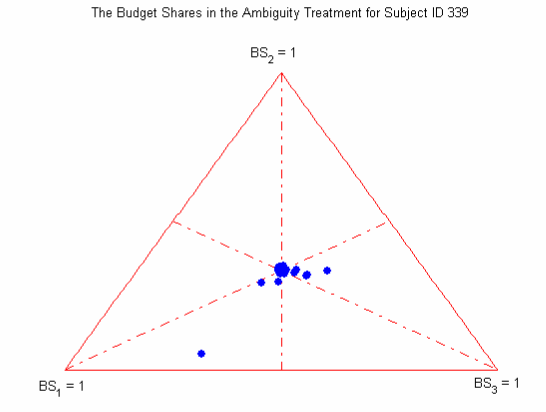
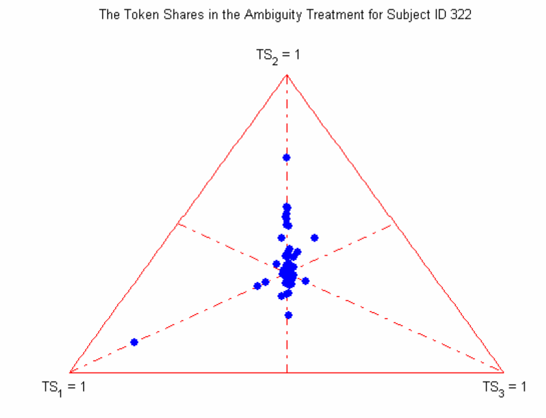


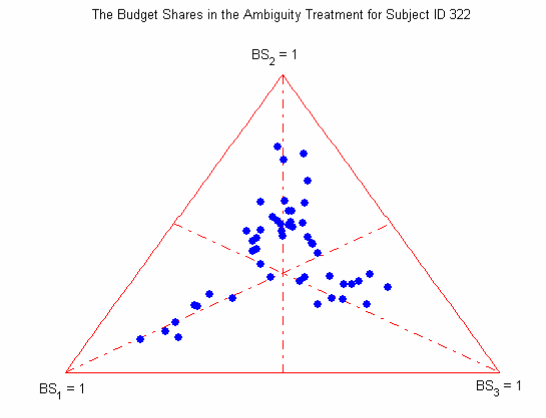
C: ID 339





D: ID 322





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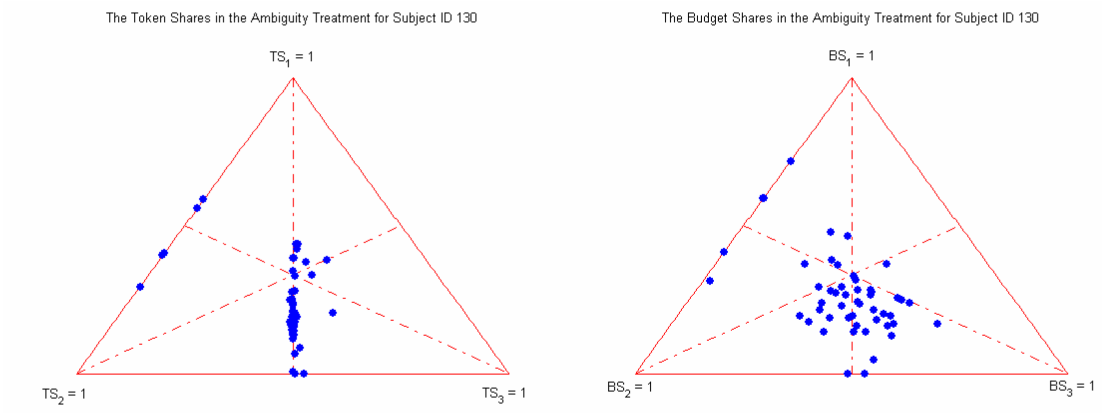


F: ID 407

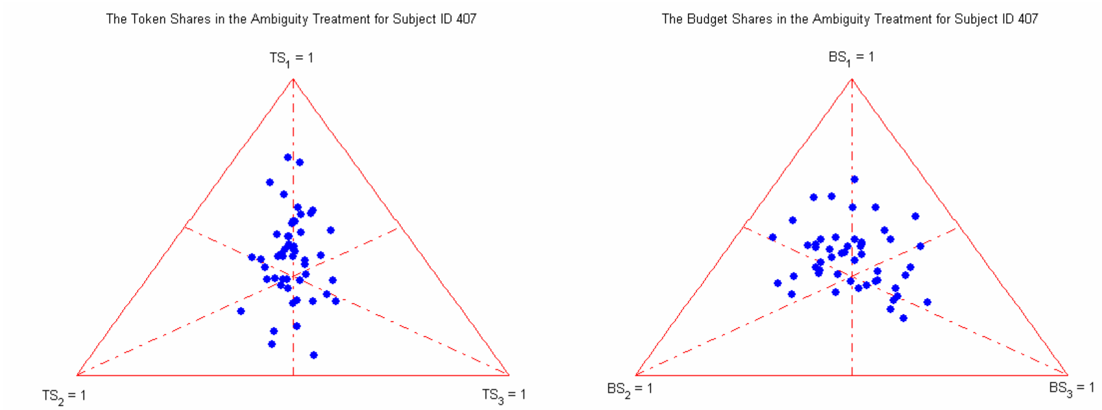


Figure 3: The distribution of CCEI scores

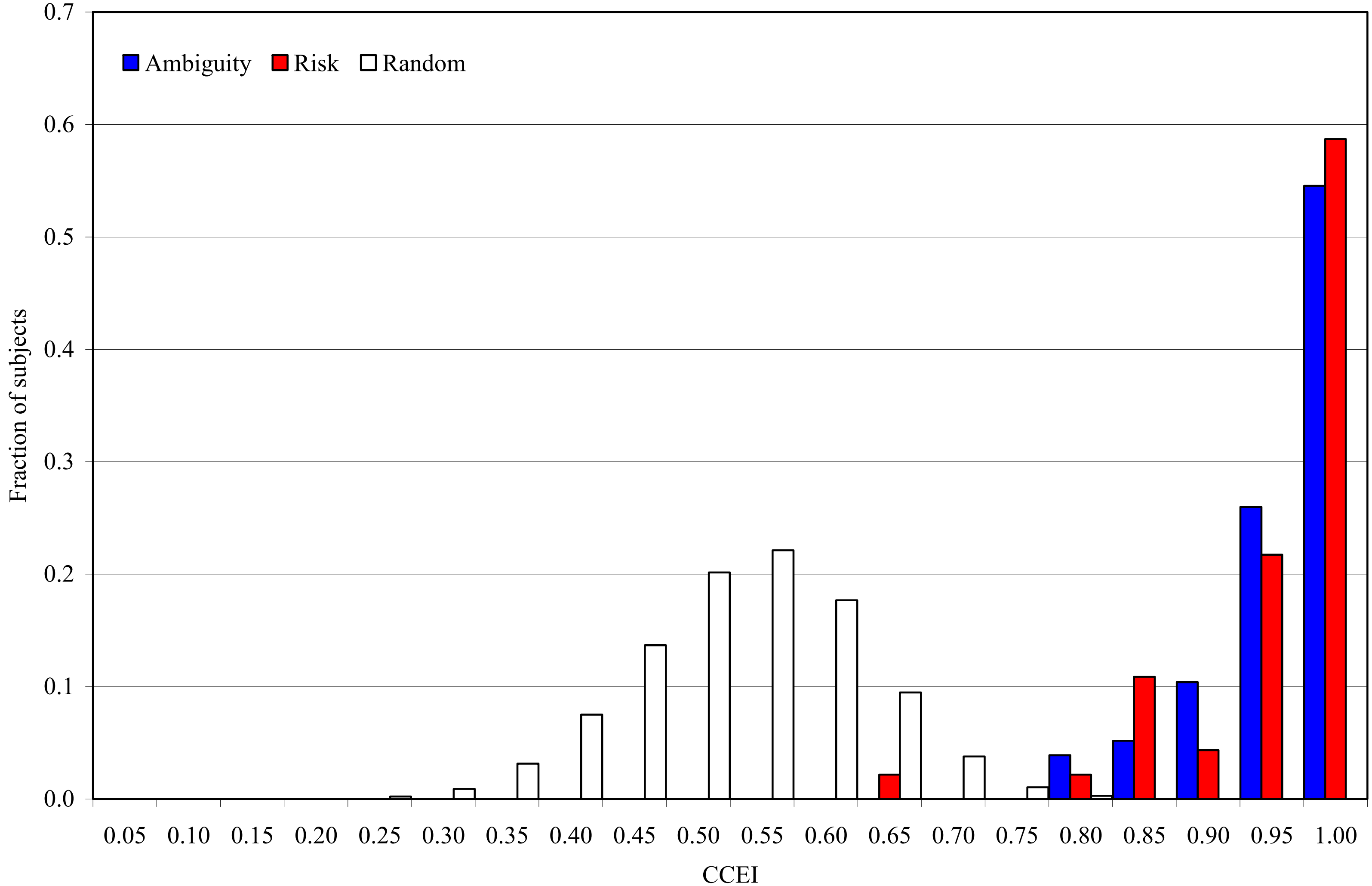
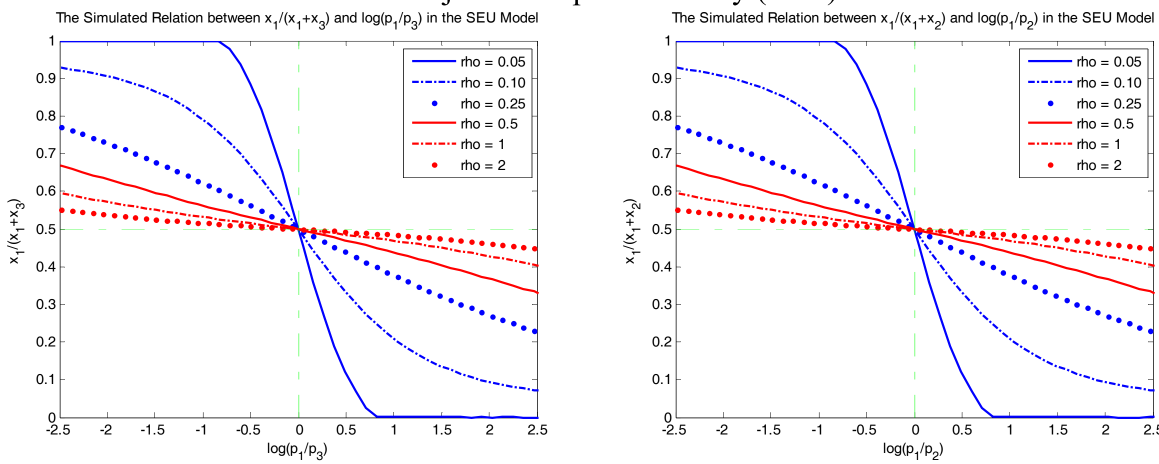
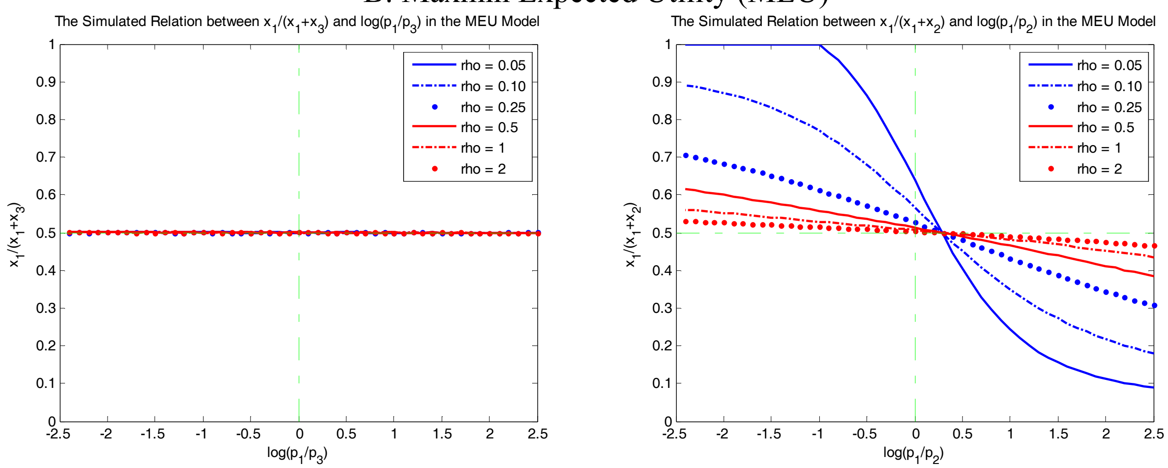


Figure 4: An illustration of the relationships between log-price ratio $\ln(p_1 / p_3)$ and optimal token share $x^*_1 /(x^*_1 + x^*_3)$

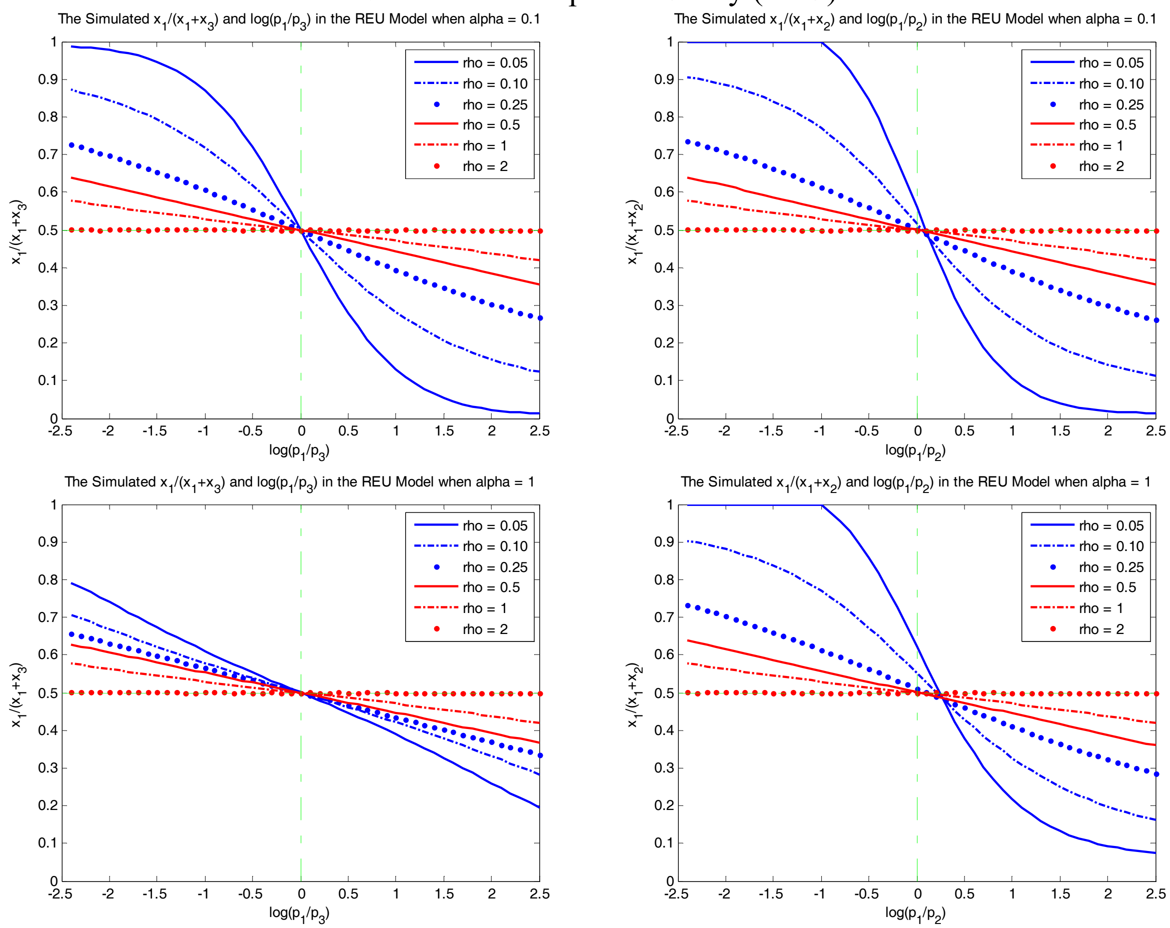
A: Subjective Expected Utility (SEU)

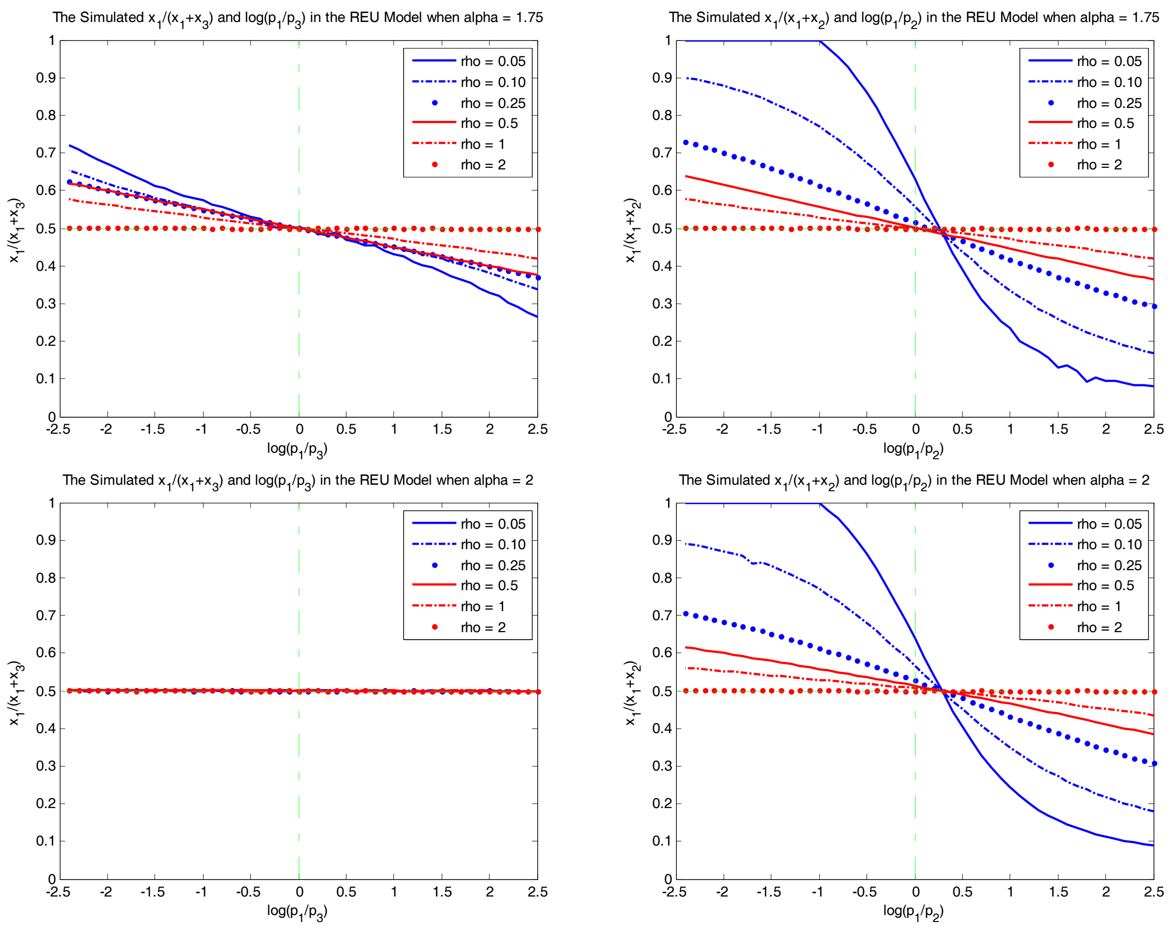


B: Maxmin Expected Utility (MEU)



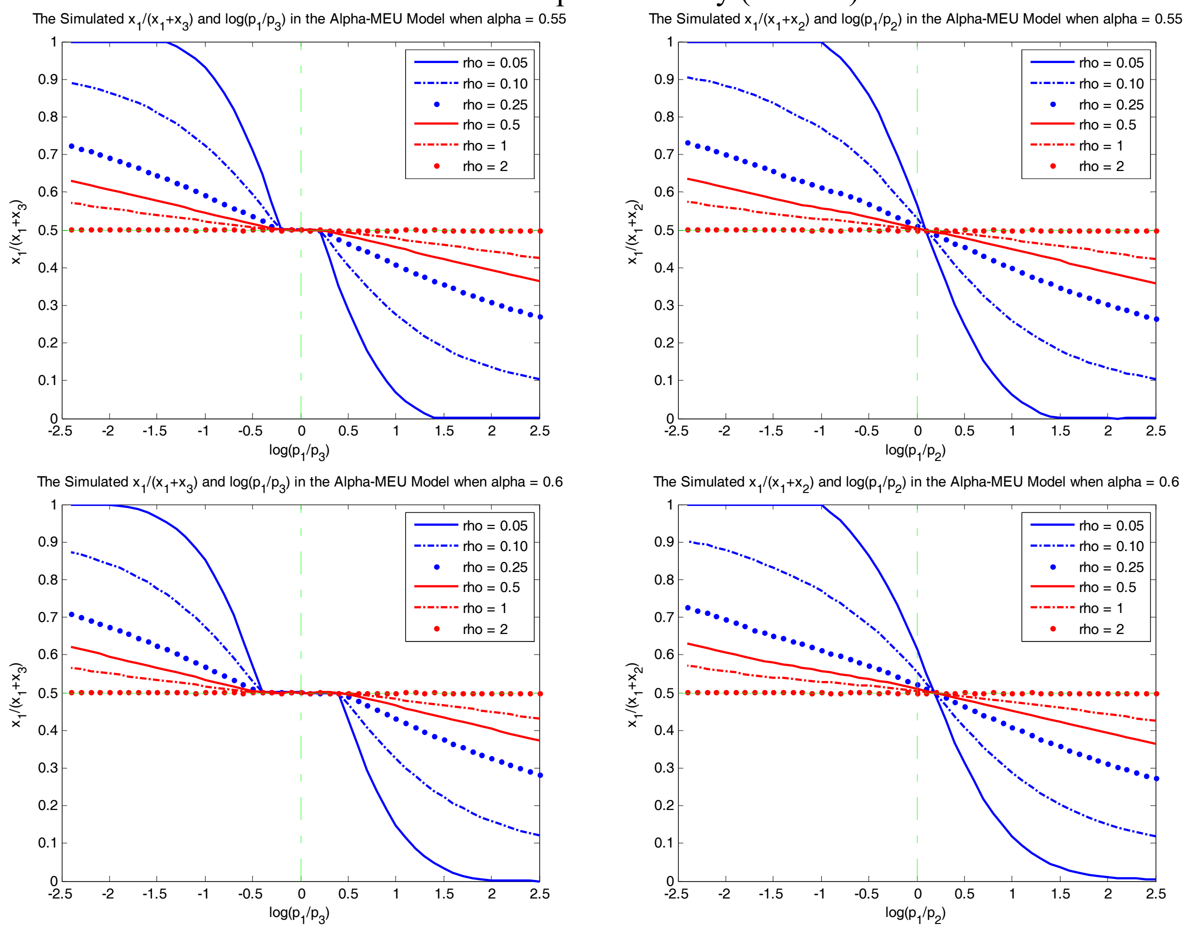
C: Recursive Expected Utility (REU)

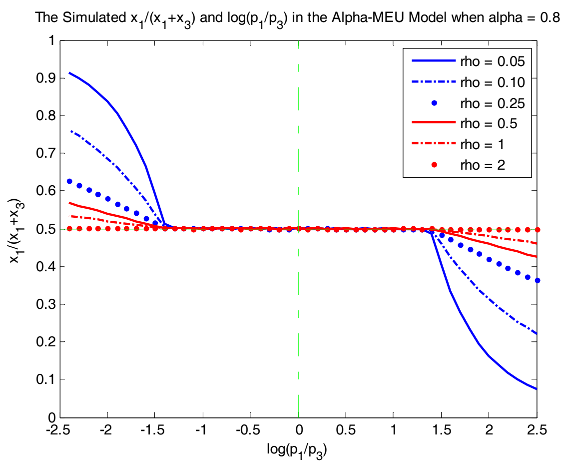


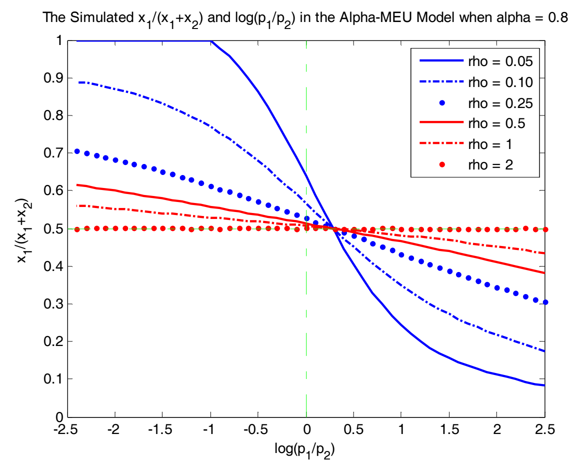


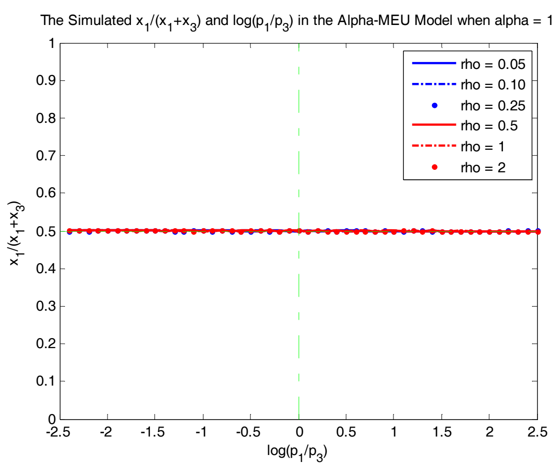
The Simulated x1/(x1+x3) and log(p1/p3) in the REU Model when alpha = 1.75
The Simulated x1/(x1+x2) and log(p1/p2) in the REU Model when alpha = 1.75
The Simulated x1/(x1+x3) and log(p1/p3) in the REU Model when alpha = 2
The Simulated x1/(x1+x2) and log(p1/p2) in the REU Model when alpha = 2
rho = 0.05
rho = 0.10
rho = 0.25
rho = 0.5
rho = 1
rho = 2
x1/(x1+x3)
x1/(x1+x2)
log(p1/p3)
log(p1/p2)

D: α-Maxmin Expected Utility (α-MEU)




The Simulated x1/(x1+x3) and log(p1/p3) in the Alpha-MEU Model when alpha = 0.8
rho = 0.05
rho = 0.10
rho = 0.25
rho = 0.5
rho = 1
rho = 2
x1/(x1+x3)
log(p1/p3)


The Simulated x1/(x1+x2) and log(p1/p2) in the Alpha-MEU Model when alpha = 0.8
rho = 0.05
rho = 0.10
rho = 0.25
rho = 0.5
rho = 1
rho = 2
x1/(x1+x2)
log(p1/p2)


The Simulated x1/(x1+x3) and log(p1/p3) in the Alpha-MEU Model when alpha = 1
rho = 0.05
rho = 0.10
rho = 0.25
rho = 0.5
rho = 1
rho = 2
x1/(x1+x3)
log(p1/p3)

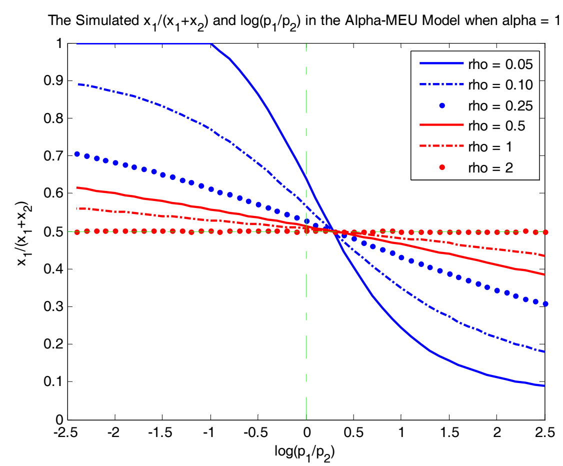

The Simulated x1/(x1+x2) and log(p1/p2) in the Alpha-MEU Model when alpha = 1
rho = 0.05
rho = 0.10
rho = 0.25
rho = 0.5
rho = 1
rho = 2
x1/(x1+x2)
log(p1/p2)

Figure 5A: Scatterplot of the SEU and MEU estimates

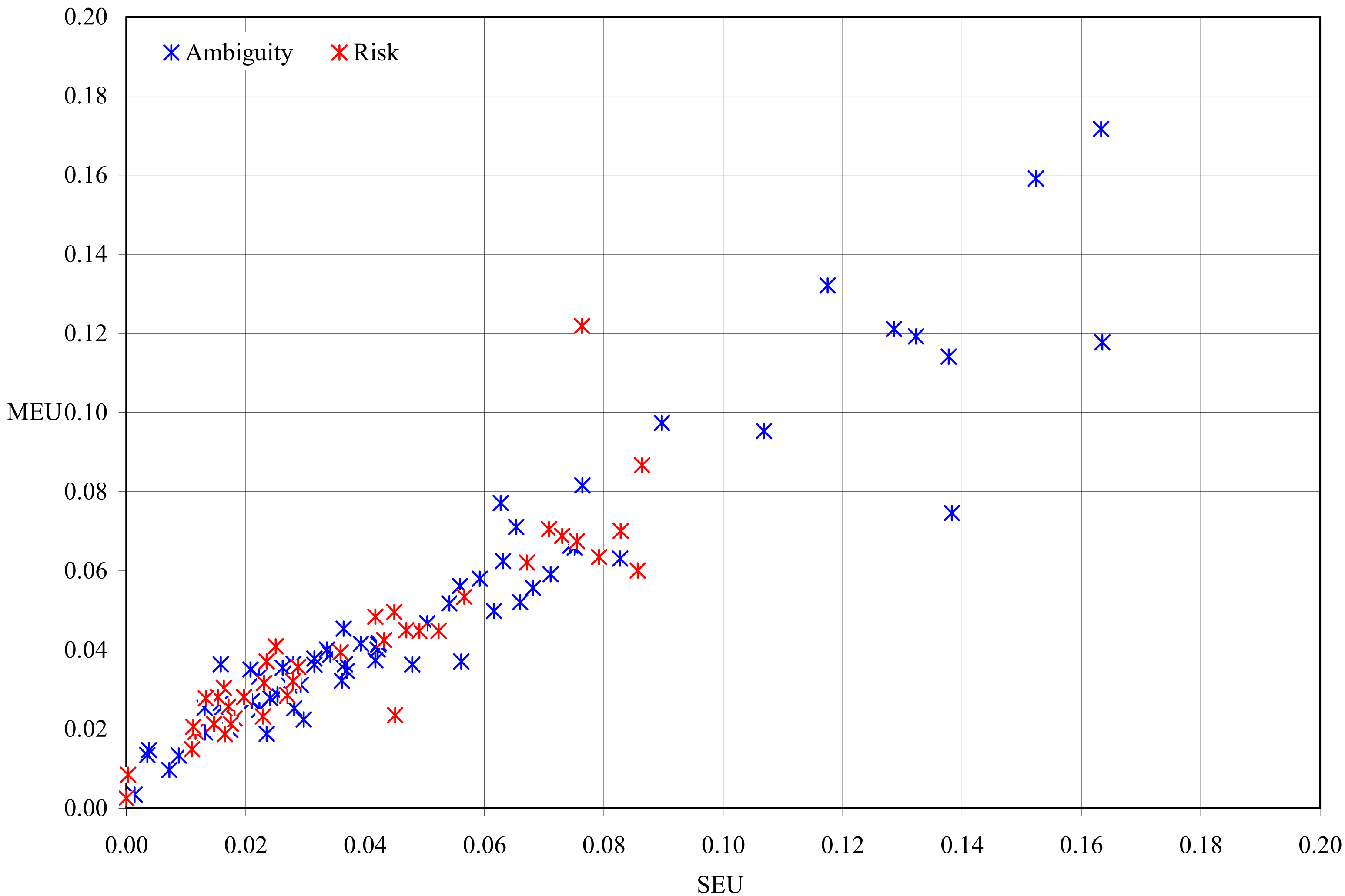


Figure 5B: Scatterplot of the REU estimates, α and ρ

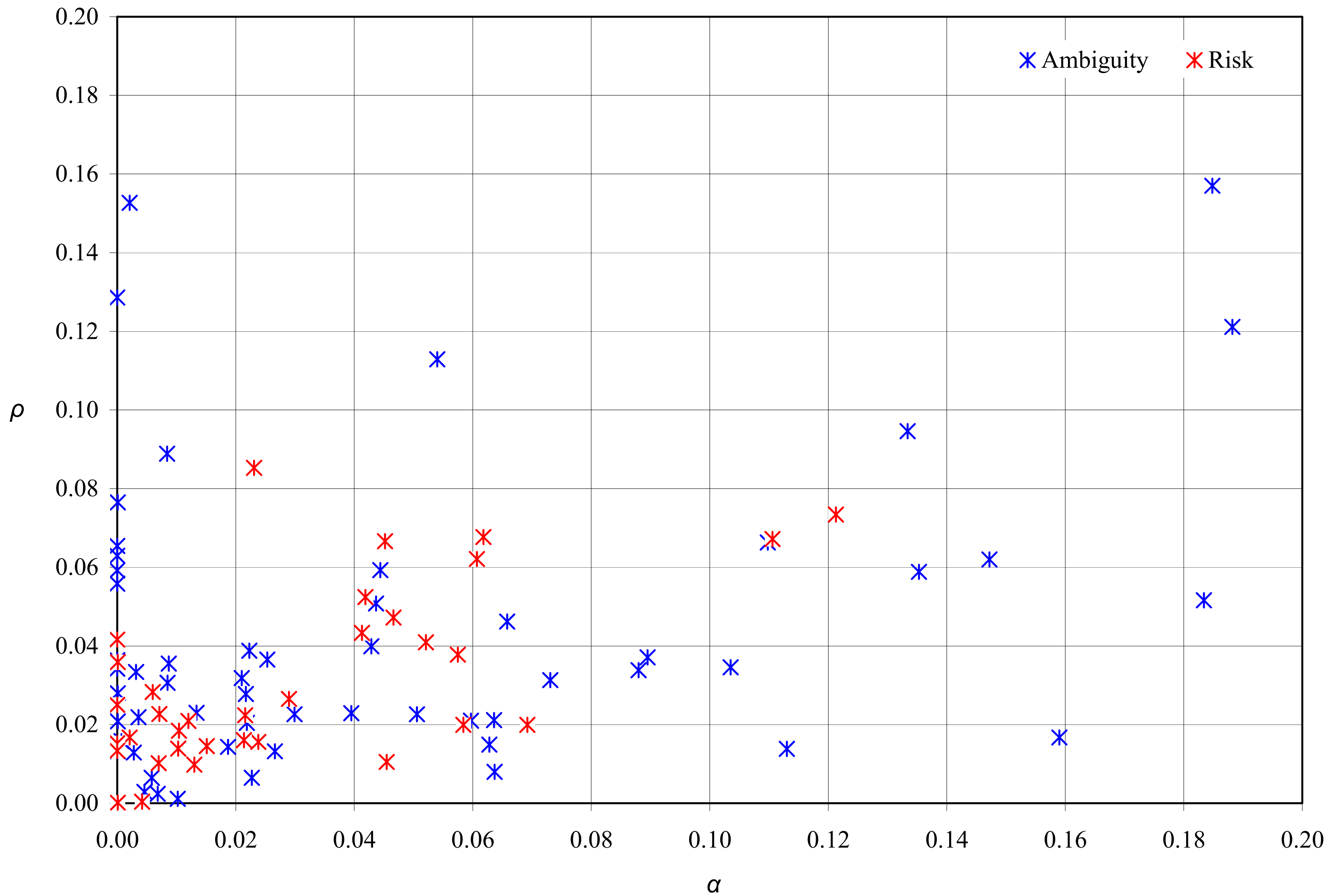


Figure 5C: Scatterplot of the α-MEU estimates, α and ρ

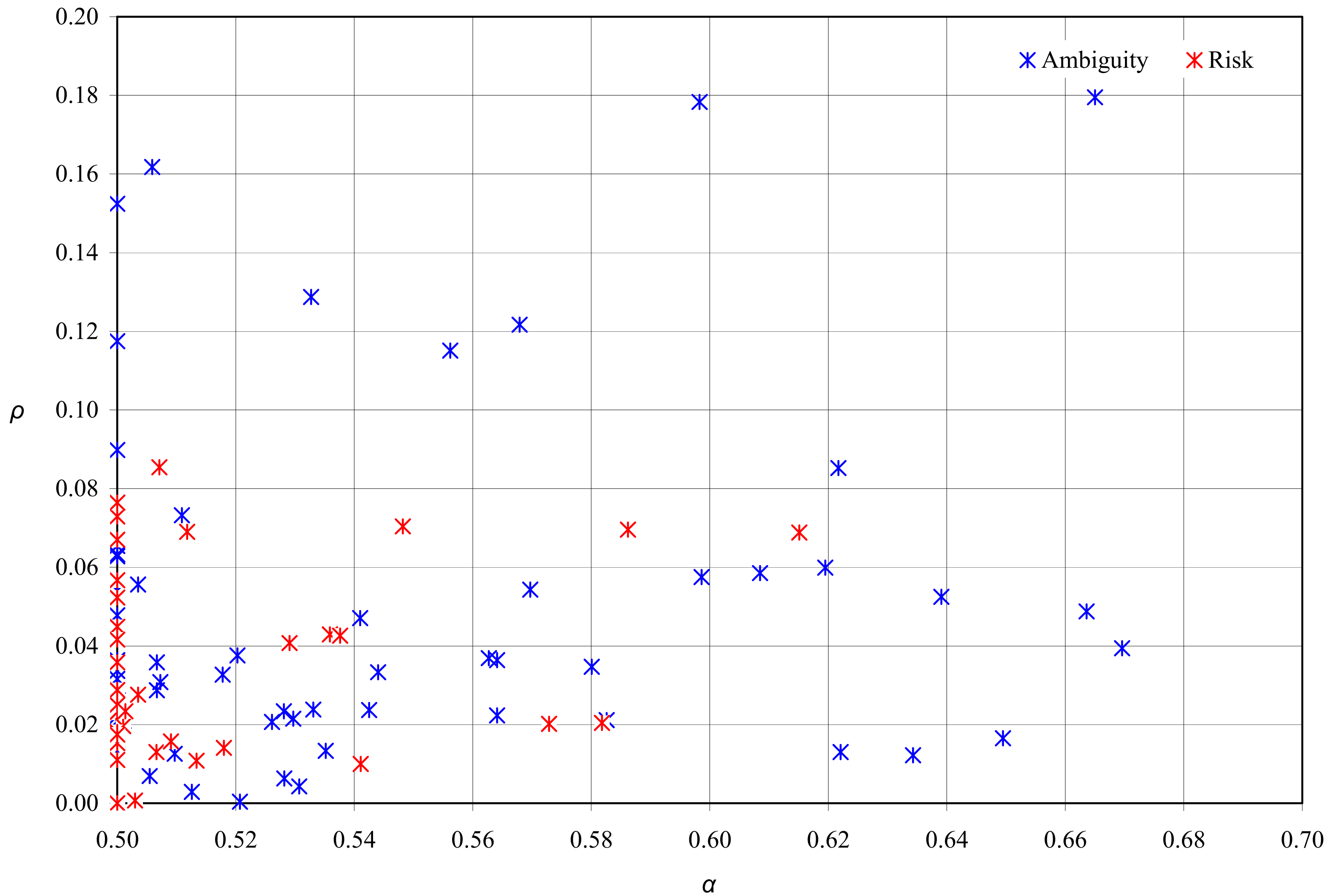
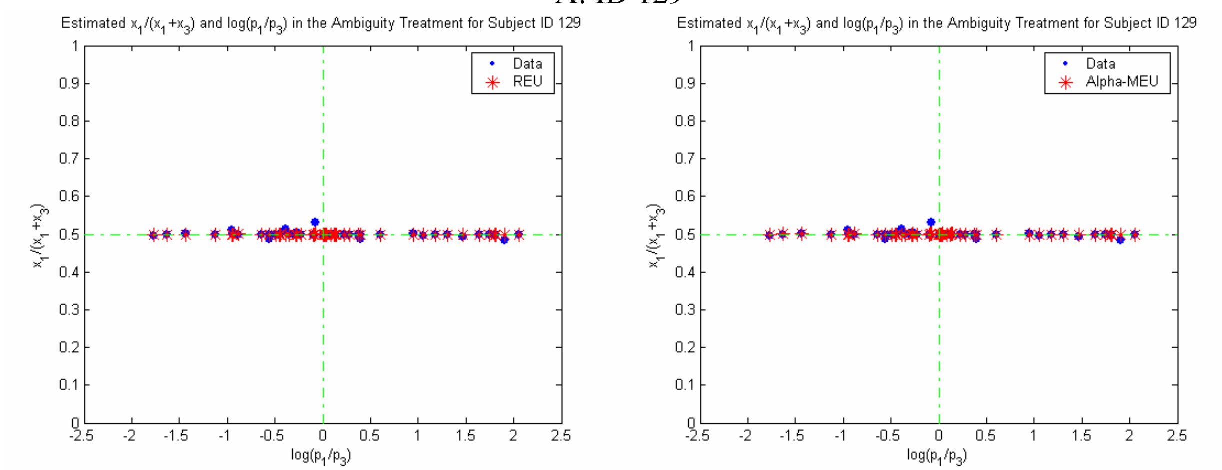
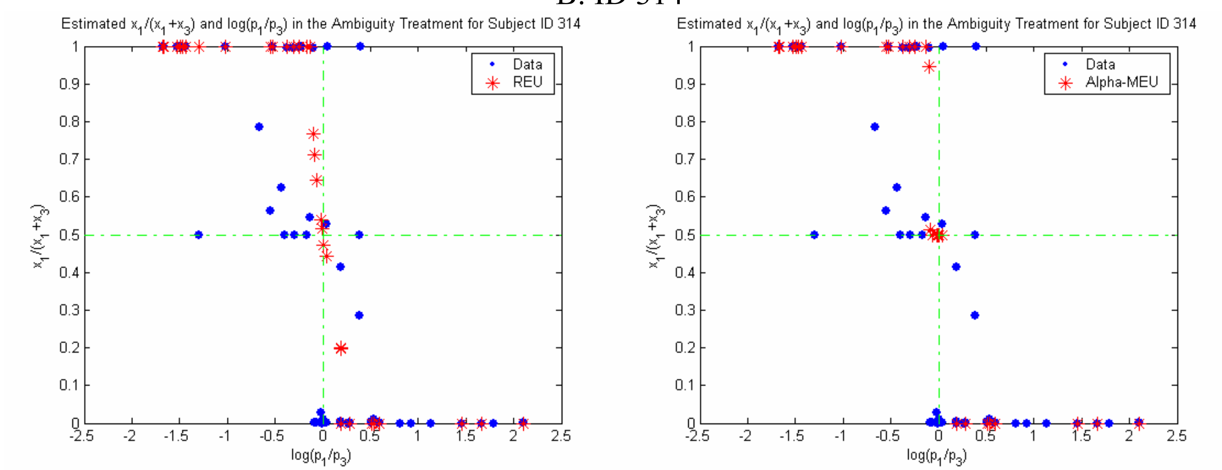


Figure 6: The relationship between log-price ratio $\ln(p_1 / p_3)$ and estimated token share $\hat{x}_1 / (\hat{x}_1 + \hat{x}_3)$

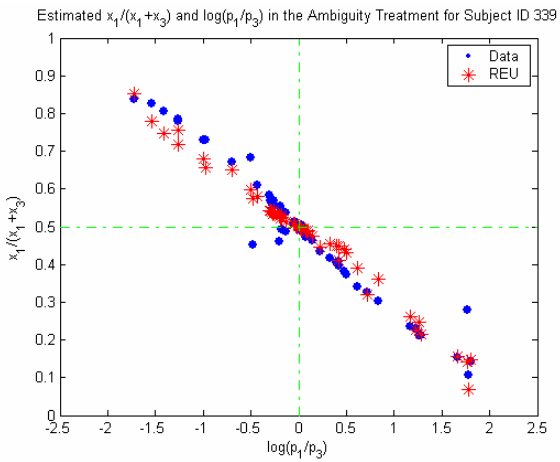
A: ID 129

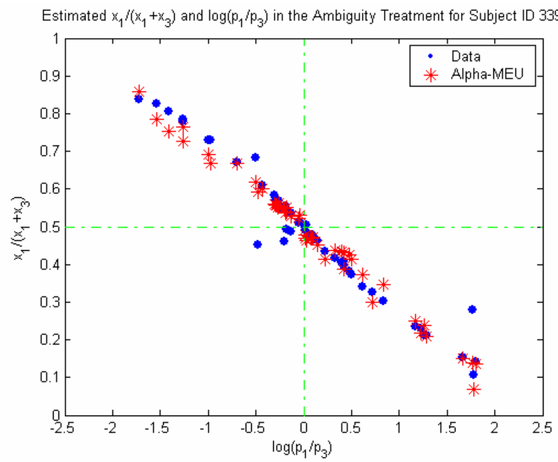


B: ID 314

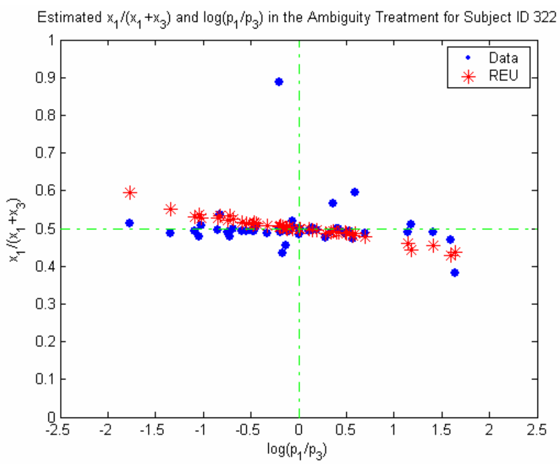


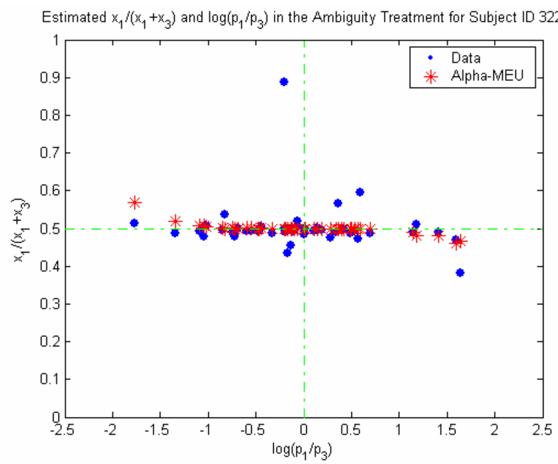
C: ID 339



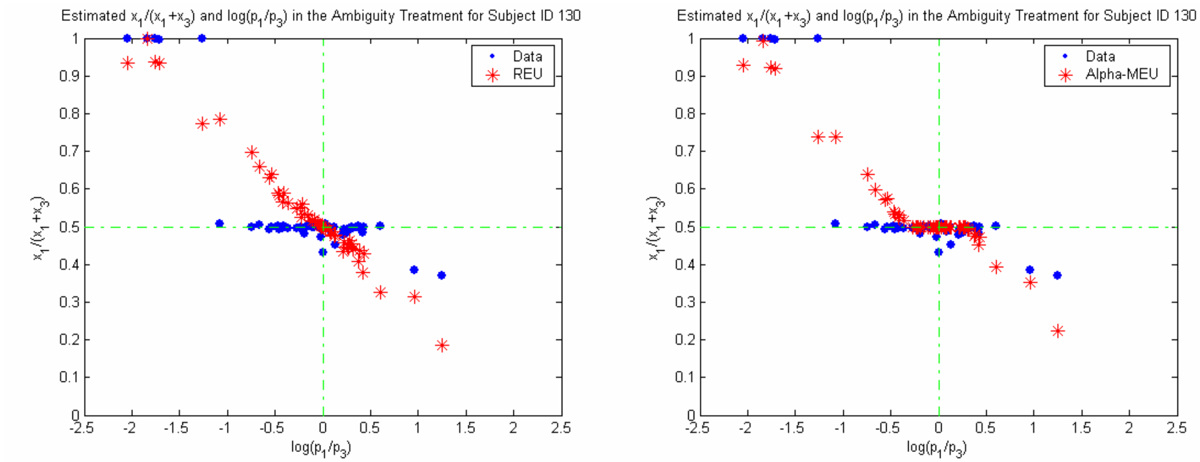


D: ID 322





E: ID 130



F: ID 407