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JOB MARKET SIGNALING AND JOB SEARCH

by

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Job Market Signaling and Job Search *

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Abstract

The high cost of searching for employers borne by prospective employees increases friction in the labor market and inhibits formation of efficient employer-employee relationships. It is conventionally agreed that mechanisms that reduce the search costs (e.g., internet portals for job search) lower unemployment and improve overall welfare. We demonstrate that a reduction of the search costs may have the converse effect. We consider a labor market in which workers can either establish a long-term relationship with an employer by being productive, or shirk and move from one employer to the next. In addition, the workers can signal to a potential employer their intention to be productive. We show that lower search costs lead to fewer employees willing to exert effort and, in a separating equilibrium, to more individuals opting to stay completely out of the job market and remain unemployed. Furthermore, we show that lower search costs not only deteriorate the market composition, but also impair efficiency by leading to more expensive signaling in a separating equilibrium.

Keywords: signaling; job market; job search; separating equilibrium; unemployment; moral hazard

JEL classification: D82; C72; C73; J64

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1 Introduction

Consider the situation where an employer wants to fill a qualified job position. It is not observable at the stage of job interviews whether a potential employee meets the requirements for this position (or, put differently, whether a potential employee would be willing to exert enough effort to cope with assigned tasks). Furthermore, an employee's qualifications become observable only after a certain period of time. Thus, in the initial period of employment, an employee can potentially work hard to demonstrate her abilities and to obtain tenure, or, alternatively, she can shirk and, when her real productivity becomes known, leave the job and search for a new vacancy at another firm.

In the described situation, job search costs play a crucial role. The more expensive it is to search for a new job, the less motivation an employee has to 'cheat' in the initial period of employment and move on to another employee. Therefore, an increase in the job search costs could be welfare-improving, since more employees would be willing to exert effort and to continue being employed by the same firm, and fewer individuals would be searching for a job. Furthermore, in a separating equilibrium of a signaling model, only 'productive' workers will be willing to enter the job market, while 'cheaters' will opt to stay completely out of the job market and remain unemployed [1] . As a result, an increase in search costs leads not only to greater employment stability, but also, counterintuitively, to lower unemployment rates. In addition, the signal separating 'productive' workers from 'cheaters' becomes less expensive. Conversely, technologies aimed at reducing search costs (e.g., internet job search sites) have a negative effect and, in the extreme case, when job search costs becomes less than the cost of effort, may lead to a collapse of the qualified labor market.

The early analysis of labor markets assumed a frictionless perfect market and, specifically, complete information and free movement within the market (Hicks, 1932). However, this was perceived as a simplified model, which

[1] Here by 'employment' we understand occupancy in the premium job market. An individual is 'unemployed' if he does not occupy a qualified job position which is potentially available for him.

should be amended and expanded in order to better reflect real labor markets (Kerr, 1950). The consequences of imperfect information and the constraints on flow of information within the market were first raised and studied by Stigler (1961), and expanded in the context of labor markets by Stigler (1962), followed by McCall (1970), Mortensen (1970a,b), and Gronau (1971). These pioneering papers were to a large degree motivated by the attempt to explain existing levels and durations of unemployment.[2]

The above-mentioned papers and their many successors[3] assume a sequential job search, in which a job seeker usually compares a current offer to the expected gain from a continued search. An offer is accepted (only) if it carries a higher value than the expected value from rejecting the offer and continuing to search. As McCall (1970) has pointed out, a potential employee whose unemployment benefits exceed her expected gains from a search will 'drop out' and forgo the job search altogether. Since an increase in search costs implies a decrease in the possible gains from a search, it would also increase unemployment in the market, even when there are jobs available. Additional unemployment will be observed in the labor market as there are always some employees who are in the process of searching.

In the following model, we will show that in an anonymous labor market, in which workers signal their willingness to exert effort, increasing search costs may have the converse effect: a reduction in unemployment and an increase in social welfare. To that end, we look at a simpler model, in which search costs are defined somewhat differently. The previous models focus on the uncertainty inherent in the market, so that the job search process is related to collecting information regarding specific wage offers.[4] As we are interested in other attributes of search costs, we shall assume a homogeneous employer-market, in which identical vacancies are always available, but there is a fixed cost for finding one. Thus, the main thrust of the previous findings remains, as

[2] In the introduction to their survey of the literature, Rogerson et al. (2005) identify eight questions addressed by the search theory, five of which relate to unemployment.

[3] For surveys of the job search literature, see Lippman and McCall (1976), Mortensen (1987), Mortensen and Pissarides (1999a,b), and Rogerson et al. (2005).

[4] Offers may include additional elements, such as employment duration (McCall, 1970).

some employees will still drop out of the job search, provided that their unemployment option carries a higher return than entering the market. As we shall see, the combination of search costs and the heterogeneity of the employees will result in additional entry costs and equilibrium unemployment.

Multitype labor markets and signaling were studied extensively since Spence (1973, 1974) first showed that productive workers can signal their type by sending a signal that carries a cost negatively correlated with quality.[5] We introduce a model in which quality is patience. In our model, an employer cannot condition a contract on the worker's future performance during the contract period, but can base his wage offers on beliefs about this future performance. These beliefs can be updated according to the worker's behavior in past periods. This gives rise to a repeated game, in which high productivity and high wages are possible to maintain in an equilibrium, as long as the players are patient enough (Friedman, 1971; Fudenberg and Maskin, 1986). Only a worker who obtains enough (discounted) utility from future contracts will exert effort in current periods, in order to receive high offers from the employer in the next round. Hence patience becomes quality, as only patient workers are productive.[6]

To study the role of search costs in this context, we consider a market with many workers and employers. Initially, workers and employers are matched randomly. Afterwards, in each contract period, the employer makes a wage offer, which the worker can accept, or reject and search for a new offer, at a fixed search cost.

Thus, a patient worker would like to signal her type to the potential employers, in order to receive a high wage offer and establish a continuing employment relationship. Previous models have studied repeated signaling games in which signals are sent in each period, either by new players coming into the market

[5] For a survey of the signaling literature see Riley (2001).

[6] For simplicity of exposition we assume that the types are differentiated only in patience (the discount factor). The results, however, will remain qualitatively the same if we allow the type differentiation not only in the discount factor, but also in other parameters, such as the productivity or the cost of effort (see the discussion in Section 7.5).

(Noldeke and Samuelson, 1997) or in a repeated interaction between the same players, in which the informed player distributes the signal over time (Kaya, 2007). Instead, we study a single committing public signal, which is sent by the worker at the beginning of the game and cannot be altered subsequently. We show that the repeated interaction structure described above enables the worker to send a reliable signal, even when no signal that carries a cost correlated with quality is available. We determine the unique cost of such a signal under assumptions of continuous worker types.

Let us first consider the market behavior in the absence of a separating signal. When the types are unknown to potential employers, patient workers will establish long-term employment relationships, while impatient workers will not. Thus, an impatient worker will shirk in each contract period. Following shirking, such a worker will face a reduced wage offer, and prefer to seek a new vacancy. This is repeated in each period, with the impatient worker moving from job to job and paying the search costs repeatedly in each period. In this view, the first beneficial attribute of search costs become apparent, since as search costs increase, the *shirk and switch* strategy becomes less appealing.

This difference in behavior between different worker types makes reliable signaling possible. Assume that firms only employ workers who signal. Workers who intend to establish lasting relationships stand to gain more from investing in a signal than workers who intend to shirk and switch, thus incurring repeated search costs. Thus, patient workers can signal their type by burning money, while impatient workers would prefer to remain unemployed. Increased search costs reduce the share of the workers who prefer the shirk and switch strategy, which implies reduced unemployment in the separating equilibrium.

To see the other beneficial attribute of search costs, in addition to reduced unemployment, consider that the cost of the separating signal is determined by weighing the possible gain from employment and unemployment, respectively, as well as by the search costs. As we saw, an increase in search costs implies reduced gains from the shirk and switch strategy, so that the signaling workers can invest less in signaling in order to separate themselves from the other workers; that is, signaling becomes cheaper.

The paper is organized as follows. Section 2 describes the moral hazard game.

Section 3 describes the population model of a multiplayer market with signaling. In Section 4 we define the different types of workers as they emerge from the game given a heterogeneity in patience levels. In Section 5 we prove the existence of a separating equilibrium and derive the cost of the separating signal. In Section 6 we discuss social welfare and the effects of search costs. Section 7 concludes.

2 The Moral Hazard Game

Consider a situation where an employer contracts a worker to do a job. The contract is for one period of work at a fixed wage, and can be renewed at the end of each period. The worker may either exert effort (resulting in high productivity) or shirk (resulting in low productivity). Since the worker's productivity is only revealed at the end of the period, the employer cannot condition the wage on the effort. However, as this interaction is repeated, the employer can base future wage offers on the current effort.

Formally, the game is described as follows. Before every period $t = 1, 2, \ldots$, the employer makes a wage offer w_t to the worker, contingent on the history of actions. The worker decides whether to accept it or not. If the offer is rejected, the worker receives the outside-option wage $\bar{w}$ in all future periods, and the game ends. If the offer is accepted, the worker decides to exert effort or to shirk, and receives in this period the payoff equal to $w_t - ce_t$, where $e_t \in \{0, 1\}$ is the level of effort and c is the cost of effort, and the game proceeds to the next period.

Denote by p and p_0 the worker's productivity when the worker, exerts effort or shirks, respectively, i.e., $p > p_0 \geq 0$. The outside-option wage $\bar{w}$ is the wage (net of the cost of effort) that the worker can guarantee elsewhere. It will be defined more precisely in the next section. To avoid trivialities, assume that $p_0 \leq \bar{w} < p - c$; that is, an outside-option pays less than working with (observed) high effort, but the worker can always guarantee the wage at least p_0.

Let $w_t \geq 0$ be the wage offered by the employer for period t, let $\mu_t \in \{0, 1\}$ be the worker's decision of accepting the wage offer, and let $e_t \in \{0, 1\}$ be the effort level of the worker. Next, let $h_t = \{(w_1, e_1), \ldots, (w_t, e_t)\}$ be the history of actions up to period t. Denote by H_t the set of all histories up to t, and let $H = \bigcup_{t=0}^{\infty} H_t$. A strategy of the worker is a pair (μ, e) of functions, $\mu : H \times \mathbb{R}_+ \to \{0,1\}$ and $e : H \times \mathbb{R}_+ \to \{0,1\}$, respectively, which associate with every history h_{t-1} and every offered wage w_t a decision $\mu_t = \mu(h_{t-1}, w_t)$ whether to accept the offer and (if the offer is accepted) a level of effort $e_t = e(h_{t-1}, w_t)$. A strategy of the employer $w : H \to [0,1]$ associates with every history h_{t-1} the wage $w_t = w(h_{t-1})$ that is offered to the worker for period t.

Given the employer's strategy w, the present-value payoff of the worker in period t is given by

$$u_t = u(\mu_t, e_t, h_{t-1}) = (1-\mu_t)\frac{\bar{w}}{1-\delta} + \mu_t \left[w(h_{t-1}) - ce_t + \delta u_{t+1}\right],$$

where δ is the worker's discount factor, $0 < \delta < 1$, and u_{t+1} is the next-period expected payoff conditional on $\mu_t = 1$. That is, if $\mu_t = 0$, the worker obtains wage $\bar{w}$ forever; if $\mu_t = 1$, in period t the worker obtains wage $w_t = w(h_{t-1})$ and, if he exerts effort, pays the cost of effort c.

Given the worker's strategy (μ, e), the employer's present-value payoff from hiring the worker in period t is given by

$$\pi_t = \pi(w_t, h_{t-1}) = \mu_t \left[e_t p + (1-e_t)p_0 - w_t + \delta^E \pi_{t+1}\right],$$

where δ^E is the employer's discount factor, $\mu_t = \mu(h_{t-1}, w_t)$, $e_t = e(h_{t-1}, w_t)$, and π_{t+1} is the next-period expected payoff which depends on the history up to t. That is, if $\mu_t = 1$, in period t the employer obtains p whenever the effort is exerted ($e_t = 1$) and otherwise p_0, and pays wage w_t; if $\mu_t = 0$, the worker has left the firm and does not contribute to its profit any more, and so the employer receives zero forever.

We assume that the job market is competitive among employers; thus in equilibrium the employer offers to the worker the wage equal to her (expected) productivity.[7] A strategy profile (w, μ, e) is a subgame-perfect equilibrium

[7] Explicit modeling of the wage-setting mechanism is not the issue that we pursue in this paper. Therefore, for simplicity, we follow Spence (1973, 1974) and others by assuming that the wage offered to an employee is equal to her expected productivity

(SPE) if in every period t

(i) (μ_t, e_t) maximizes the worker's present-value payoff,

$$(\mu_t, e_t) \in \operatorname{argmax} u(\mu_t, e_t, h_{t-1});$$

(ii) the offered wage is equal to the worker's actual productivity,

$$w_t = e_t p + (1 - e_t) p_0.$$

Definition 1. A worker is said to be *productive* if there exists an SPE where the worker always accepts the firm's wage offer and exerts effort; otherwise, the worker is said to be *non-productive*.

Denote by w^* the following *grim trigger strategy*: the employer pays wage $w_t^* = p$, as long as the worker always exerted effort in the past, $e_1 = \ldots = e_{t-1} = 1$, and otherwise $w_t^* = p_0$. Denote by (μ^*, e^*) the *productive strategy* of the worker: accept the offer and exert effort in every period t where $w_t \geq p$, and reject the offer otherwise.

Lemma 1. *A worker is productive if and only if strategy profile* (w^*, μ^*, e^*) *is a subgame-perfect equilibrium.*

Proof. The "if" part holds by definition. It remains to prove the "only if" part. Suppose that (w^*, μ^*, e^*) is not an SPE. The payoff of the worker with patience level δ from playing (μ^*, e^*) is $(p - c)/(1 - \delta)$, and her payoff from shirking in period 1 and subsequently rejecting the offer is $p + \delta\bar{w}/(1 - \delta)$. Since we assumed (w^*, μ^*, e^*) is not an SPE, it follows that $(p - c)/(1 - \delta) < p + \delta\bar{w}/(1 - \delta)$, or, after expanding the left-hand side and canceling out p on both sides, we obtain

$$-c + \delta(p - c) + \delta^2(p - c) + \ldots < \delta\bar{w}/(1 - \delta). \tag{1}$$

Suppose that there exists the employer's strategy w such that the productive strategy (μ^*, e^*) is the worker's best reply to w. In particular, the following inequality must hold:

$$(w_1 - c) + \delta(w_2 - c) + \delta^2(w_3 - c) + \ldots \geq w_1 + \delta\bar{w}/(1 - \delta).$$

(cf. Noldeke and Samuelson, 1997).

After canceling out w_1 on both sides, we obtain

$$-c + \delta(w_2 - c) + \delta^2(w_3 - c) + \ldots \geq \delta\bar{w}/(1-\delta). \tag{2}$$

Since $w_t = e_t p + (1 - e_t)p_0 \leq p$ for all t, after combining (1) and (2) we obtain

$$\begin{aligned}\delta\bar{w}/(1-\delta) &\leq -c + \delta(w_2 - c) + \delta^2(w_3 - c) + \ldots \leq \\ &\leq -c + \delta(p - c) + \delta^2(p - c) + \ldots < \delta\bar{w}/(1-\delta).\end{aligned}$$

A contradiction. **End of Proof**.

The following corollary follows from Lemma 1 and inequality (1).

Corollary 1. *A worker is productive if and only if she is patient enough, that is,*

$$\delta \geq \frac{c}{p - \bar{w}}.$$

3 Population Model with Signaling

Consider now a job market with a large (infinite) population of firms $\mathcal{N}$ and workers $\mathcal{L}$. The population of firms is homogeneous, while the population of workers consists of heterogeneous individuals. Every individual i's type is characterized by a patience level δ_i; otherwise all individuals are identical.[8] Let $G : (0,1) \to [0,1]$ be a continuous distribution function that associates with every $x \in (0,1)$ the measure of workers in the population with a patience level less than x, or, in other words, the probability that a worker chosen at random has type $\delta \leq x$,

$$G(x) = \Pr[\delta \leq x].$$

A worker in $\mathcal{L}$ is matched with a firm in $\mathcal{N}$ chosen at random, and the two of them play the moral hazard game described above, over periods $t = 1, 2, \ldots$, until the worker decides to reject the offer and leave the firm. Once the worker leaves the firm, she immediately (i.e., in the same period) returns to the job market and is randomly rematched with a new firm.[9] We assume that a job

[8] See Footnote 6.

[9] One may argue that it is unrealistic to say that a new job is found immediately, as a job search usually takes time. In fact, this assumption is only for convenience:

search is costly: a worker needs to pay a fee $s > 0$ to participate in the job market. However, participation in the job market is voluntary; that is, before each round, the job seeker decides whether he wants to participate in the job market or to stay out and receive unemployment benefits[10] u_0 in the current period. To simplify the analysis, we assume that u_0 satisfies $p_0 \leq u_0 < p - c$; that is, the outside-option payoff is at least as high as the payoff of a low-effort task (the wage for a low-effort job) and it does not exceed the net payoff of a high-effort task.

We assume that a worker's type is unobservable to firms. A firm observes only past actions of its employees. Thus, if a firm has repeatedly interacted with a worker for a few consecutive periods, it may infer her type by observing her past actions. However, when a new worker is hired, her type, as well as her past actions during employment with other firms, are completely unknown[11] to the firm.

In order to communicate the type, before period 1 each worker is allowed to send a public signal. We assume that the signal costs $F \geq 0$ to a sender and (presumably) communicates that the sender is 'productive.' The signal may be sent only once (it is public and irreversible), it is observed by everyone, and the firms can condition their actions on the workers' signals.

Fix a worker in $\mathcal{L}$. In every period $t = 1, 2, \ldots$, the following three-stage game is played.

Entry decision. The worker (if she is seeking a job) has a choice to 'enter' (i.e., participate in the *job market*) or to 'stay out.' If the worker enters, she is called an *entrant* and pays a participation fee s, $0 < s < p - u_0$; if she stays out, she obtains the stage payoff u_0 and the game proceeds to the next period.

Matching. An entrant is matched with a firm drawn at random[12] (anony-

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all job search-related costs are aggregated in the variable s.

[10] We take this to be the outside-option payoff, which includes any monetary and non-monetary utility to be gained from unemployment including, for example, leisure time.

[11] In the infinite population, the probability be randomly matched with the same individual more than once is zero.

[12] Alternatively, the firms may use the signal as a prerequisite to hiring an employee;

mously) from $\mathcal{N}$.

Moral hazard game. The pair of matched players plays the moral hazard game described above and receives the stage payoffs of this game.

In the following period $t+1$, if the worker has accepted the firm's offer, she plays the moral hazard game with the same firm; if she has rejected the offer and left the firm, she returns to the job market and makes the 'entry decision.'

Note that the outside-option wage $\bar{w}$ of the worker in the moral hazard game is equal to the highest payoff that she can guarantee after leaving the firm.

4 Productive vs. Non-productive Workers

Consider a subgame after the worker has been matched with a firm. A worker is productive if there exists an equilibrium where she prefers a long-term productive relationship with a firm to shirking and then seeking another job. Since non-productive workers will shirk in *any* equilibrium, firms prefer that only productive workers enter the job market, and non-productive ones stay out.

Consider a worker with patience level δ_i. As follows from Lemma 1, the worker is *productive* if and only if, when the firm plays the grim trigger strategy w^*, the worker's payoff from accepting the wage offer and exerting effort in every period is higher than the payoff for a one-time deviation, namely,

$$\frac{p-c}{1-\delta_i} \geq p + \delta_i \bar{w},$$

where $\bar{w} = -s + (p-c) / (1-\delta_i)$, since, after leaving the firm, the worker can enter the job market at cost s and receive $(p-c)$ in all subsequent periods. Plugging $\bar{w}$ into the above inequality, we obtain, after simplification,

$$\delta_i \geq c/s. \tag{3}$$

Thus, a worker with patience level δ_i is productive if and only if (3) holds. The value of the discount factor $\delta^* = c/s$ is the threshold level of patience that separates productive from non-productive workers.

thus only those who send a signal participate in the matching.

5 Separating Equilibrium

We focus on two models of worker behavior. A worker is said to be a *participant* if (i) she participates in the job market (whenever unemployed), and (ii) she exerts effort in every period $t = 1, 2, \dots$ and accepts the firm's wage offer w_t for the following period, as long as $w_t \geq p$ (otherwise she rejects the offer and leaves the firm). The worker is said to be a *dropout* if (i) she stays out of the job market (whenever unemployed), and (ii) she shirks and rejects the firm's next-period wage offer (whenever employed).

Suppose that $s > c$ and suppose that population $\mathcal{L}$ contains positive measures of both productive and non-productive workers, i.e., $0 < G(\delta^*) < 1$, where $\delta^* = c/s$ is the threshold level of patience.

Definition 2. A (perfect Bayesian) equilibrium is said to be *separating* if there exists a signal cost F such that every productive worker sends the signal and becomes a participant, while every non-productive worker does not send the signal and becomes a dropout.

Note that the separation is on both signaling behavior and further play. A separation on signaling only is not sufficient to induce different play: there is a trivial equilibrium in which every productive worker sends a costless signal, while every non-productive worker does not (thus the signal is effective in communication of the types); but in the subgame after the signaling stage, everyone stays out of the job market and receives the stage payoff of u_0 in all periods.

Next, note that existence of a separating equilibrium requires costly communication of the type. Indeed, if the cost of the signal is small, then a non-productive worker can mimic the productive one's signal and enter the job market. Since she will receive a high wage p (as only productive workers are supposed to enter the market in a separating equilibrium, a firm's best reply is $w_t = p$ in all periods), and since by assumption $s < p - u_0$, her stage payoff is $-F - s + p > u_0$ for small enough F.

Theorem 1. *A separating equilibrium always exists. The cost F of the signal* *in every separating equilibrium satisfies*

$$F = \frac{(p - u_0) - s}{1 - c/s}. \tag{4}$$

Before proving the theorem, let us examine condition (4) on the cost of the signal. This condition can be written as a system of the following inequalities:

$$-F + \frac{p - s}{1 - \delta} \leq \frac{u_0}{1 - \delta} \quad \text{for every } \delta < c/s \tag{5}$$

and

$$-F + \frac{p - s}{1 - \delta} \geq \frac{u_0}{1 - \delta} \quad \text{for every } \delta \geq c/s. \tag{6}$$

Since by sending the signal and entering the job market a non-productive worker receives

$$-F + (-s + p) + \delta(-s + p) + \ldots = -F + \frac{p - s}{1 - \delta},$$

and by staying out she receives $u_0/(1 - \delta)$, inequality (5) is a non-productive worker's incentive compatibility constraint; it requires that a non-productive worker have no incentive to mimic a productive worker. Next, since a productive worker's payoff in a separating equilibrium is $-F - s + \frac{p-c}{1-\delta}$, the inequality

$$-F - s + \frac{p - c}{1 - \delta} \geq \frac{u_0}{1 - \delta}, \quad \delta \geq c/s \tag{7}$$

is a productive worker's participation constraint. By the definition of 'productive worker', $-s + \frac{p-c}{1-\delta} \leq \frac{p-s}{1-\delta}$, and for $\delta = c/s$,

$$-s + \frac{p - c}{1 - c/s} = \frac{-s + c + p - c}{1 - c/s} = \frac{p - s}{1 - c/s}, \tag{8}$$

and hence (6) yields precisely (7). The cost of effort F given by (4) is the unique number satisfying both constraints.

Proof of Theorem 1. Suppose that every firm plays the grim trigger strategy w^* whenever it is matched with a worker who sent a signal, and otherwise plays $w_t = p_0$ (i.e., a worker who has not sent a signal is always offered wage p_0).

First, we show that if F satisfies (4), no worker is willing to deviate. As noted above, a non-productive worker has no incentive to mimic a productive worker; a productive worker does not benefit by staying out of the job market. Also, by the definition of 'productive worker,' one cannot benefit by shirking. It is straightforward to verify that any other deviation is either equivalent or inferior to the above possibilities.

To prove the uniqueness of the signal cost, we shall show that when the cost of signal is $F' \neq F$, either a productive or a non-productive worker is willing to deviate.

Assume there is a separating equilibrium when $F' > F$. Let δ' be such that $F' = \frac{p-p_0-s}{1-\delta'}$. Solving for δ', we obtain $\delta' > c/s$, since

$$\delta' = 1 - \frac{p - p_0 - s}{F'} > 1 - \frac{p - p_0 - s}{F} = 1 - (1 - c/s) = c/s.$$

By assumption, G is continuous and $0 < G(c/s) < 1$; hence there is a positive measure of productive workers with types in $[c/s, \delta')$. Consider a productive worker with type $\delta \in [c/s, \delta')$. By the definition of 'productive worker,' $-s + \frac{p-c}{1-\delta} \leq \frac{p-s}{1-\delta}$, and since $\delta' > \delta$, we have

$$-s + \frac{p - p_0 - c}{1 - \delta} \leq \frac{p - p_0 - s}{1 - \delta} < \frac{p - p_0 - s}{1 - \delta'} = F'.$$

By playing productively, this player receives $-F' - s + \frac{p-c}{1-\delta} < \frac{p_0}{1-\delta}$, and so she would be better off sending no signal and staying out of the job market.

The proof for the case of $F' < F$ is symmetric. **End of Proof**.

6 Welfare Analysis and Job Search Costs

The *payoff* for an individual, U_δ, is defined as a discounted sum of her stage payoffs in all periods, where the discount factor is the individual's 'patience level' δ. In a separating equilibrium, every dropout receives u_0 in each period; hence her individual payoff is

$$U_\delta(s) = \frac{u_0}{1 - \delta}, \quad \delta < c/s.$$

Every participant pays $(F + s)$ once and receives $(p - c)$ in each period; hence her individual payoff is

$$U_\delta(s) = -F - s + \frac{p - c}{1 - \delta}, \quad \delta \geq c/s.$$

By Theorem 1, $F = F(s) = \frac{p-u_0-s}{1-c/s}$, and by (8) $F(s) = -s + \frac{p-u_0-c}{1-c/s}$. Hence,

$$U_\delta(s) = -\frac{p-u_0-c}{1-c/s} + \frac{p-c}{1-\delta}, \quad \delta \geq c/s.$$

The *welfare* of an individual is also defined as a discounted sum of her stage payoffs in all periods, but the discount factor is some parameter, called a *social discount factor*, which is common for all individuals (e.g. an inflation rate or a rate-of-return on savings). Denote the social discount factor by γ, $0 < \gamma < 1$. In a separating equilibrium, each dropout's individual welfare is

$$W_\delta(s) = \frac{u_0}{1-\gamma}, \quad \delta < c/s.$$

and every participant's individual welfare is

$$W_\delta(s) = -F - s + \frac{p-c}{1-\gamma} = -\frac{p-u_0-c}{1-c/s} + \frac{p-c}{1-\gamma}, \quad \delta \geq c/s.$$

The *total welfare* is the sum of all players' individual welfare.[13] Since in equilibrium the firms pay the wage equal to the expected productivity, their profit is zero. Thus only payoffs for workers matter when the welfare is evaluated. Since the measure of dropouts is $G(c/s)$, the total welfare in the separating equilibrium is equal to

$$\begin{aligned}\bar{W}(s) &= \frac{u_0}{1-\gamma}G(c/s) + \left[-\frac{p-u_0-c}{1-c/s} + \frac{p-c}{1-\gamma}\right](1-G(c/s)) \\ &= \left[\frac{p-c}{1-\gamma} - \frac{p-u_0-c}{1-c/s}\right] - G(c/s)\left[\frac{p-u_0-c}{1-\gamma} - \frac{p-u_0-c}{1-c/s}\right]\end{aligned}$$

Let us examine the effects of an increase in job search costs s on the described labor market. Assume that following constraint is satisfied;

$$\gamma \geq c/s \tag{9}$$

(that is, the social discount factor is not too low).[14] Then we obtain the

[13] The standard way to define the total welfare as the sum of payoffs of all players seems to be inappropriate in this case, since the players have different discount factors, and thus their payoffs are incomparable (see also the discussion in Section 7.4).

[14] See discussion in Section 7.3.

following results. As s increases:

(a) The threshold $\delta^*(s) = c/s$ separating productive and non-productive workers shifts down; therefore more workers are willing to exert effort in long-term relationships.

(b) The signal becomes less expensive; i.e., $F(s)$ strictly decreases.

(c) Every productive worker remains a participant, and her individual payoff and her individual welfare strictly increase.

(d) For every initially non-productive worker who remains non-productive after the change, both the individual payoff and the individual welfare remain unchanged.

(e) For every initially non-productive worker who becomes productive after the change (and thus becomes a participant), the individual payoff increases. That is, suppose that the search cost has increased from s' to s, $s' < s$. Then for every δ such that $c/s \leq \delta < c/s'$,

$$U_\delta(s) - U_\delta(s') = \left[-\frac{p-u_0-c}{1-c/s} + \frac{p-c}{1-\delta}\right] - \frac{u_0}{1-\delta}$$
$$= (p-u_0-c)\left[\frac{1}{1-\delta} - \frac{1}{1-c/s}\right] \geq 0,$$

where the inequality holds since $\delta \geq c/s$. Also, the individual welfare increases, since

$$W_\delta(s) - W_\delta(s') = \left[-\frac{p-u_0-c}{1-c/s} + \frac{p-c}{1-\gamma}\right] - \frac{u_0}{1-\gamma}$$
$$= (p-u_0-c)\left[\frac{1}{1-\gamma} - \frac{1}{1-c/s}\right] \geq 0,$$

where the inequality holds by condition (9).

(f) The total welfare $\bar{W}(s)$ strictly increases.

Note that the minimum that the total welfare can possibly attain under any equilibrium behavior is at least $u_0/(1-\gamma)$, since every worker can guarantee the stage payoff of u_0 in every period. In the separating equilibrium, the total welfare attains its minimum, $\bar{W}(s) = u_0/(1-\gamma)$, when the search cost s is equal to the least value that satisfies condition (9), $s = c/\gamma$.

To sum up, the effect of an increase of the search cost is rather strict: it makes every individual worker, whether productive or non-productive, better off from both her own and society's perspective. Furthermore, productive workers are strictly better off, and so the total welfare strictly goes up. This positive effect of search costs increase rises due to the fact that higher search costs entail lower benefits from 'cheating' on the job, that is, shirking and then moving on to another employer. In other words, some workers may *prefer* to have higher search costs in order to remove the incentives to 'cheat' and to let 'exerting high effort' become an equilibrium behavior.

7 Discussion

7.1 Fixed Costs

In his seminal paper in which he introduced the topic of job market signaling, Spence (1973) writes:

> *It is not difficult to see that a signal will not effectively distinguish one applicant from another, unless the costs of signaling are negatively correlated with productive capability.*

The bulk of the signaling literature reflects this notion of differential signal costs as essential for reliable signaling (e.g., Riley, 2001).[15] However, a signal may still be reliable even if its cost is uncorrelated with type[16] (*fixed-costs signal*). Austen-Smith and Banks (2000) have analyzed the conditions under which an informed player can use costly signaling in the form of burned money in order to reliably transmit her type. Specifically, they show that an asymmetry in payoffs across types can lead to fixed-costs signaling (Austen-Smith and Banks, 2000, footnote 5). Milgrom and Roberts (1986), following Nelson (1970, 1974, 1978), have shown how differential potential gains from advertising allow for reliable signaling through fixed-costs advertising. In their model, they assume an 'experience good,' which means that the product's quality can be assessed only through use. For an experience good, the benefits of adver-

[15] The same principle is prevalent in the biological literature, where it is known as the *Handicap Principle* (Zahavi, 1975).

[16] Provided the type is one-dimensional (see the discussion on multi-dimentional types in Section 7.5).

tising are larger for a high-quality good, which will attract repeated purchases following an initial sale, than for a low-quality good, which will not. Thus advertising through burned money is reliable, as long as the cost of the (advertising) signal is higher than the benefit of an initial sale of a low-quality product. The model presented in this paper carries some resemblance to the advertising model, as the quality of a worker can be determined by the firm only through employment, and repeated interactions are key in allowing reliable fixed-costs signaling.

Note, however, that for our analysis of the effects of search costs on individual and social utilities, the signaling model can be replaced with a differential-costs signal in the tradition of Spence (1973). Nonetheless, the availability of fixed-costs signaling that we prove here carries significant weight, as we can assume that such a signal is readily available to workers, while a differential-costs signal may or may not be. For instance, a certificate of a higher education is traditionally considered a signal that is less costly for more talented individuals. However, in some post-Soviet countries, in particular Ukraine, it is not a secret now that in institutions of higher education (including top universities), bribery in exchange for admission or high grades is quite commonplace (see, e.g., Osipian, 2007). Acquiring an undergraduate or graduate degree is more a matter of money than personal effort; hence the cost of such a signal is uncorrelated with an individual's abilities.

7.2 Pooling Equilibrium

An equilibrium is said to be *pooling* if the signal is uninformative; i.e., the firm's behavior disregards the signal. In a pooling equilibrium the firm does not know the worker's type in the first match with a worker and has no means to discriminate between productive and non-productive workers. Thus the firm sets the wage according to the worker's expected productivity, $xp + (1 - x)p_0$, where x is the fraction of productive workers among all job applicants, and later sets the wage according to p if the worker has exerted effort in all previous periods, and p_0 otherwise. Then a productive worker will always exert effort and stay with the same firm; a non-productive worker will shirk and switch to another firm in every period.

The pooling equilibrium may sometimes yield a higher welfare than the separating equilibrium. This is the case when the measure of non-productive workers in the population is low enough, so that it is too expensive to make all productive workers pay the signal cost.

In the pooling equilibrium, the net effect of an increase in search costs is less clear, since *all* individuals pay the search costs in period 1, while the users of the shirk and switch strategy continue doing so thereafter; hence a negative effect of a search cost increase (though to a lesser extent for productive workers than for non-productive workers). However, our main message remains intact: higher search costs decrease the benefits of the shirk and switch strategy, thus causing more workers to exert effort and creating a positive effect on welfare.

7.3 Social Discount Factor

Constraint (9) requires that the social discount factor be close enough to 1, in other words, that the social discounting rate $r = 1/(1-\gamma)$ be sufficiently low. This assumption is justified for two reasons. First, if this constraint does not hold, then a dropout's individual welfare is greater than a participant's:

$$\frac{u_0}{1-\gamma} > -F(s) - s + \frac{p-c}{1-\gamma};$$

that is, dropping out is more desirable from society's point of view. In this situation, signaling the type is too costly; that is, *separating signaling is socially undesirable* (see the discussion about pooling equilibrium).

Second, many economic and ethical arguments (e.g., that one generation should not be favored over another generation just because of its temporal placement) have been put forward in favor of low or zero social discounting rates, often incorporating different methods of discounting, such as decreasing social discounting rates (cf. Hepburn, 2007; Pearce et al., 2003, and references within). Although the debate usually revolves around intergenerational issues as distinct from our narrower time frame, many of those arguments still hold in our case.

7.4 Social Welfare

Our analysis shows that increasing search costs lead to a Pareto improvement of the market in equilibrium. Some unemployed workers will be productively employed, while all workers previously employed under lower search costs will incur reduced signaling costs. It follows trivially that the sum of utilities increases. Nevertheless, an argument might be put forward that in the context of a longitudinal model with discounting agents, social welfare should encompass more than total utility, since a Pareto improvement from the agents' point of view may reduce welfare from society's point of view. Indeed, in the basic moral hazard game, the interests of non-productive workers are in opposition to those of a hypothetical social planner who is interested in increasing productivity.[17]

We show that, assuming that condition (9) holds in a labor market that has a significant share of productive workers, social welfare from a social planner's point of view is strictly increasing with search costs, in line with total utility as seen from the workers' point of view. We can safely conclude that in a long-term labor market with heterogeneous workers and signaling, search costs have the surprising attribute of reducing unemployment and increasing social welfare. This conclusion is opposed to results obtained under different assumptions (McCall, 1970; Mortensen, 1970a,b; Gronau, 1971), and should be regarded as significant for institutional design.

[17] This point can be illustrated using Rubinstein's (2006) example: Adam likes apples and is rather impatient; he is always willing to exchange up to 2 apples tomorrow for 1 apple today. Suppose that Adam lives no more than 120 years. If Adam "is endowed with a stream of 1 apple per day starting on day 18 for the rest of his life, ... he should be willing to exchange his endowment for a single apple right away!" (Rubinstein, 2006, pp. 868–869). Thus, allowing an option of 'a single apple right away' will be a Pareto improvement for Adam. It can be argued, however, that it will hurt the social welfare: assuming that the society is composed of many 'Adams' of various age, the sum of their current utilities would be higher if every 'Adam' were left with his initial endowment.

7.5 Multi-Dimensional Types

We assumed that the employees' types are differentiated only in the discount factor δ. If we had instead assumed the type differentiation in the cost of effort c, the analysis would have required no changes and the results would have been qualitatively the same. However, allowing for multi-dimensional types would present additional complexities, since the existence of a separating equilibrium would require existence of a signal correlated with type. For instance, suppose that each worker i is described by a pair (c_i, δ_i). While some workers are productive due to low discounting, as in the basic model, some workers are productive due to low costs. Imagine a worker who is very impatient (δ_i close to zero), but very skilled, so that she can be effortlessly productive ($c_i = 0$). Such a worker would be productive by definition, but stands to gain very little from working, and can thus afford no more than a signal cost of p. At the same time, there are non-productive workers who are patient, but have high costs, who would gain more than p from sending the signal and shirking and switching. Hence a separating signal cannot have a fixed cost, but has to be one that is more costly for unskilled workers, such as the education signal in Spence's (1973) original model. The cost of the separating signal given in (4) should then be replaced by the following, which depends on the type:

$$F_i = F(c_i) = \frac{p - u_0 - s}{1 - c_i/s}.$$

If such a signal exists, so does the separating equilibrium. Apart from that, the statement of Theorem 1 and all conclusions regarding the effects of search costs remain the same.

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