Participation Externalities and Asset Price Volatility

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This draft: June 2001

Abstract

I analyze how an exogenous cost of entry in a risky asset market a®ects two endogenous variables: the degree of market participation and the price volatility. I show that di®erent entry costs generate different participation equilibria and multiplicity of equilibria arises for some range of entry costs, but the new market entrants are always more risk-averse than the rest of the participants. Every participation equilibrium is associated with a certain volatility of the price of the asset. Most importantly, I show that increased market participation leads to increased asset price volatility, if the new entrants are su±ciently more risk-averse than the old participants. This is supported by empirical evidence. (JEL: G12, D40, C70)

Keywords: participation, volatility, risk-aversion, externalities

1 Motivation

Over the past years, the increase in the number and in the diversity of traders has generated changes in ¯nancial asset markets and their price volatility.

[¤] I am grateful to Pierpaolo Benigno, Jean-Pierre Benoit, Raquel Bernal, Alberto Bisin, Lorenzo Cappiello, George Comer, Diego Comin, Paolo Dudine, Jean-Baptiste Eslyn, Ester Faia, Douglas Gale, Boyan Jovanovic, Ron Miller, Marco Pagano, Paolo Pasquariello, Fabrizio Perri, Gideon Saar, Alex Shapiro, Paolo Siconol¯ and Bernard Yeung for helpful comments and discussions. Any errors in the paper are only mine. Comments are welcome.

Overall, stock market participation has increased consistently since after the war. In particular, the increase has been dramatic in the eighties and nineties.[1] Since 1995, a large variety of new investors started purchasing assets in ¯nancial markets worldwide especially through the Internet.[2] The improved accessibility and the signi¯cant decrease in transaction costs are the main reasons for the increasing popularity of Internet trading. Thanks to information technology and telecommunications improvements, the information on any given security can now be spread instantaneously and is immediately and directly available to every interested consumer virtually for free. As a result of this technological revolution, there is no doubt that the cost of acquiring information on assets declined dramatically in the past years.

Whereas in the past higher transaction and information costs kept certain types of investors out of risky asset markets, nowadays the easier access has driven new types of investors into ¯nancial markets. However, while there seems to be a consensus that the lowering of the pecuniary (brokerage) and non-pecuniary (information and setup) costs of participation has driven new investor-types into asset markets, whether the e®ect of these new market participants has increased or decreased the asset price volatility is a question that still needs theoretical and empirical exploration.

2 Agenda and Main Results

The theoretical goal of this paper is to study the following questions:

1. If a lower entry cost implies necessarily a higher market participation.

2. Under what conditions does increased market participation increase or decrease the asset price volatility.

[1] The number of shareowners in the United States increased by approximately 32 million from 1989 to 1998, when it reached approximately 84 million individuals. In particular, the fraction of families having direct ownership in publicly traded stocks rose to 19.2% in 1998 from 15.2% in 1995, according to the latest Survey of Consumer Finances (1998).

[2] From 1995 through mid-2000, investors opened 12.5 million on-line brokerage accounts, as Barber and Odean document (JEP 2001). This number is projected to grow to more than 42 million by 2003. By the end of the '90s, on-line trading accounted for nearly half of all retail (non-institutional) investor trades. Shareowners with on-line brokerage accounts trade more frequently than those without such accounts, they consequently account for a disproprtionately large share of total trading.

My aim is to correct the "conventional wisdom" according to which a higher market participation should decrease volatility, because the demand shocks of more participants average-out better. As Pagano (ReStud. 1989) shows, this dampening of the price volatility is caused by the cancellation of the independent demand shocks of the market participants due to a law of large numbers e®ect. Allen & Gale (AER 1994) also show an analogous result: that limited market participation generates excess price volatility through liquidity trading. As other participant-types with liquidity shocks uncorrelated with old participants' shocks enter the market, the price volatility is dampened due to this higher liquidity shock heterogeneity. In other words, these two models, which are benchmarks of the endogenous participation literature, both show that more market participation decreases price volatility. As I clarify in the next section, some empirical studies show the latter result is counterfactual.

The result of this paper is the opposite of the results of Allen&Gale and Pagano. I show that if the potential market participants have di®erent levels of risk-aversion, more participation can increase volatility. That is, the simple introduction of heterogeneity in risk-aversion is su±cient to invalidate the conventional wisdom, namely, the law of large numbers volatility-reduction result of Pagano and Allen&Gale. Note that, the more uncorrelated the idiosyncratic demand shocks of the potential investors are, the stronger the dampening of the price volatility as new participants enter the market. Even in the extreme case in which the shocks are independent and the dampening of the volatility is hence the highest, the volatility can increase due to a stronger opposing e®ect that arises from the heterogeneity in risk-aversion. My result of increased volatility with increased participation is missing (to the best of my knowledge) from the related limited participation theoretical literature. The main intuition behind this result is outlined in what follows.

I analyze the di®erent participation equilibria that arise as I lower the cost of entry in the asset market. Di®erent participation equilibria imply different price volatilities, depending on what types of investors enter the asset market. First, I ¯nd that the less risk-averse types enjoy a higher bene¯t than the more risk-averse types from participating in a risky asset market. The main reason for that is that the types with lower risk-aversion require a lower compensation than the higher risk-aversion types for bearing the risk of holding the asset, once they have entered the market. So, for any given such compensation the low risk-aversion types have a higher incentive to participate in the market. As a consequence, for a high cost of market participation, the types with low risk-aversion are the only types to enter the market. For a low entry cost, also the more risk-averse types enter the market. However for intermediate entry costs, there are multiple participation equilibria: depending on how the highly risk-averse types coordinate themselves, they may or may not decide to join the less risk-averse types in the market.

Second, I find that the less risk-averse types react more aggressively to any fluctuation in the price, by buying low or selling high larger amounts of the asset than the other types. Hence, if many types with low risk-aversion are participating in the market, there is a strong pressure on the asset price that pushes the price towards its benchmark value.[3] In other words, the low risk-aversion types tend to dampen the price fluctuations more than the other types, by aggressively buying and bidding up the price if the price is below the benchmark, and conversely. As market participation increases (lower entry cost) and the more risk-averse types join the less risk-averse types in the market, the less aggressive reaction to the price fluctuations of the new participants tends to increase the asset price volatility. This increasing effect is counterbalanced though by the, always present, mutual shock cancellation effect that tends to reduce the price volatility (law of large numbers effect). If the participant types differ sufficiently in risk-aversion levels, the law of large numbers effect is weaker than the risk-aversion effect and as a consequence the price volatility increases.

2.1 Empirical Evidence

If on the one hand, my result, as I said, is missing from the limited participation theoretical literature, on the other hand, my result is supported by empirical evidence. Indeed, the conventional wisdom that increased participation should decrease price volatility not only it is not robust to the introduction of heterogeneity in risk-aversion as I show, but does not reproduce some empirical evidence either. In the charts in the appendix, I take the monthly change in the price-earnings ratio of the S&P500 index and show that its fluctuations have increased over time in the past 25 years. I also show that the intra-day volatility of the S&P500 has increased in the nineties. Simultaneously, the stock market participation has consistently increased in

[3] In this model the benchmark price is equal to the expected return of the risky asset minus the risk-premium.

the last ¯ve decades and dramatically increased in the nineties. My model is consistent with this empirical evidence that documents this increase in price volatility.

Besides my suggestive empirical ¯ndings, the predictions of my model are consistent with signi¯cant econometric studies on stock return volatility for individual stocks. In a recent in°uential paper, Campbell, Lettau, Malkiel and Xu (JoF 2001) prove empirically that the "idiosyncratic" volatility of individual stocks has increased dramatically over the past three decades. That is, controlling for the volatility of the whole stock market and of the individual industrial sectors, the volatility of the stocks, taken individually, has increased in the last thirty years. The authors claim that the causes of this increased volatility remain unclear. My model o®ers a plausible explanation for this increase in volatility that Campbell et al. document. Indeed, the risky asset under consideration in my framework can be interpreted as a single stock rather than the stock market as a whole.[4]

My other important result that the new market entrants are more risk-averse than the old market participants is also suggested by the empirical literature which documents the di®erences between stock holders and non-stockholders. Most empirical papers on the determinants of stock ownership ¯nd that the probability of stockownership is increasing in wealth and self-reported risk tolerance (see for instance Mankiw and Zeldes (1991), Vissing-Jorgensen (1998 & 2000) and Polkovnichenko (2000)). In general, more wealth is associated with lower absolute risk-aversion, which is the measure of risk-aversion I adopted in my model, as it is customary in the asset-pricing literature. Therefore, the richer investors which are the ¯rst market participants according to every survey, are also the more risk-tolerant according to the absolute measure of risk-aversion. The less wealthy, which according to empirical evidence enter the market later (thanks to a lower entry cost), are indeed the more risk-averse in absolute terms, as my model predicts. The quantitative prediction of my model is that if the new market entrants are at least twice more risk-averse than the rest of the participants, then the price volatility increases with more participation, even if the demand shocks are completely uncorrelated and therefore their impact on the asset-price is maximally reduced with increased participation. Since I refer to the absolute (not the relative) risk-aversion coe±cient, this di®erence in risk-aversion of

[4]Moreover, in the 1990s the volatility of the high-turnover stocks rose to nearly double its highs from the previous three decades levels (Barber and Odean, JEP 2001).

the participant-types is not a strong heterogeneity requirement at all. For instance, assuming instead that the population is characterized by a constant relative risk-aversion, as often is done in macroeconomic models and as seems more realistic[5], it is su±cient for the a participant to have half the wealth to be twice more risk-averse in absolute terms, as my model needs. Indeed, the large relative di®erences in the US wealth distribution generate the (same) large relative di®erences in the absolute risk-tolerance of the population.

3 General Setup

I introduce a model of endogenous market participation. There are three asset:

1. Risky Asset X with uncertain return x (only traded in the market under consideration)
2. Cash w (or safe asset)
3. Private Asset e with uncertain return u_j (non-marketable and peculiar to the investor-j).

The private asset represents any return that an agent-j may have aside from cash and the risky asset, e.g. human capital, real estate, or other non-liquid assets.

My framework has three periods and entails two sequential decisions made by investors: in the ¯rst period, an entry decision in a given asset market, in the second period (only in the case of entry) a portfolio decision on how much of the risky asset to buy. In the third period all uncertain returns are realized.

Since the problem has two stages, I proceed solving backwards. First, I solve the second stage. I ¯nd every investor's demand for the asset and aggregate these demands taking as given the number of market participants, which is derived later when I solve the ¯rst stage problem. Equating aggregate demand to aggregate supply, I obtain the equilibrium price of the asset and the indirect utility from entry of every market participant as a function of the number of participants. Comparing the utility from entry minus the

[5]Empirical studies show that even the relative risk-aversion coe±cient is decreasing in wealth (Mankiw and Zeldes (1991), Brav, Constantinides and Geczy (1999)).

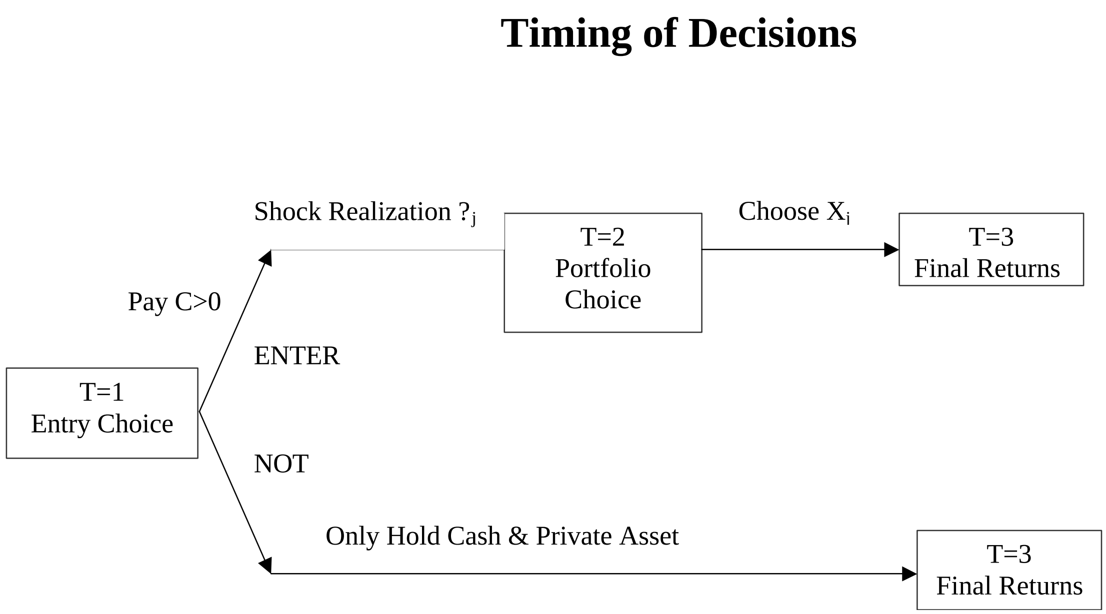


Figure 1:

entry cost to the utility of not entering, I obtain the entry condition which determines the ¯rst stage entry decision. Finally, I look for the Nash equilibria of the entry game and ¯nd the equilibrium number of participants. Di®erent equilibrium levels of participation imply di®erent volatilities of the asset price, which was previously derived in the second stage.

3.1 Demand of Market Participants

There are two types of investors. For every type-i there is a total number N_i of agents. The number of agents of type-i that participate in the asset market is $n_i \cdot N_i$. The total number of market participants $n = n_A + n_B$ is taken as given (exogenous) for now and is derived later as a function of the entry cost, when I solve backwards for the participation equilibrium. Ex-ante, that is, before making their participation decision, all potential participants face the same entry cost and may di®er only in their level of risk-aversion. I denote type-A as the aggressive (low risk-aversion) type and type-B as the backward (high risk-aversion) type.

A potential participant that decides to enter the risky asset market (market henceforth) chooses his demand schedule for the risky asset $X(p)$ and keeps the rest of his initial wealth in cash. If the investor decides not to enter the market, he can only hold cash.[6] The risky asset has a given distribution of the returns x. The distribution of x is common knowledge among agents:

$$x \sim \left(\mu_x; \sigma_x^2\right)$$

If the price of the risky asset is p and the initial wealth of the investor is w, the final wealth of the investor is:

$$w_f = (w - pX) + xX$$

I derive now the demand schedule for the market participant. If the investor enters the market, he then chooses X to maximize a linear mean-variance objective over final wealth:

$$U_i = \alpha_i E(w_f) - \beta_i Var(w_f)$$

The expectations are taken with respect to the distribution of final returns only: the price is observed at the moment the agent buys the asset, once he has entered the market. I assume $\alpha_i > 0$ and $\beta_i > 0$, to express how investors like a high expected return on their wealth and dislike a high volatility. I define the ratio $a_i \equiv \frac{\alpha_i}{\beta_i}$; which is the risk-tolerance (see Sharpe and Alexander 1990) and expresses the agent's trade-off between return and risk. For a given linear mean-variance objective, the bigger is this ratio the less risk-averse is the agent.[7] This implies that: $a_A > a_B$. The backward types-B are the more risk-averse and the aggressive types-A are still risk-averse but they are closer to risk-neutrality, because they have a higher trade-off ratio between return and risk than the types-B.

[6] Nothing would change in all the results of this paper if I introduced a safe asset with a return of $R > 1$; rather than simply cash.

[7] The objective U and the terms "risk-tolerance" and "risk-aversion" are used in a Mean-Variance framework (see Sharpe (1970) or Sharpe and Alexander (1990)). On the other hand, assuming multivariate normal return distribution and exponential vNM utility, the linear MV-objective U is consistent with the expected utility theorem and $2a^{-1}$ is the Arrow-Pratt measure of risk-aversion, which is constant, i.e. independent from the initial wealth w (CARA utility).

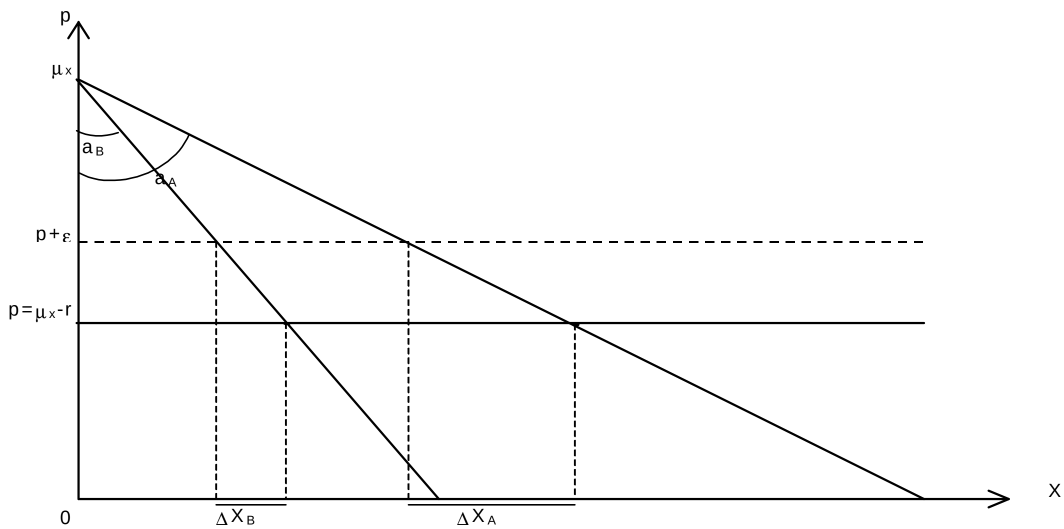


Figure 2: First, the B-types demand less of the risky asset for any given price, because they are more risk-averse, that is, they require a higher risk-premium. Second, the B-types react less aggressively to any price fluctuation having on aggregate a less dampening impact on the fluctuation than the less risk-averse A-types.

The FOCs yields a linear downward-sloping demand function[8]:

$$X_i(p) = \frac{\alpha_i}{\gamma_i} \frac{\mu_x - p}{2\sigma_x^2} \equiv a_i \frac{\mu_x - p}{2\sigma_x^2}$$

Note first that, the B-types demand less of the risky asset for any given price, because they are more risk-averse, that is, they require a higher risk-premium. Second, the B-types react less aggressively to any price fluctuation (aggregate shock) having on aggregate a less dampening impact on the fluctuation than the less risk-averse A-types.

[8]With this MV-objective the demand does not depend on the initial cash w or on the initial endowment of the asset X_0:

If borrowing constraints on cash are present, I assume that the investors have enouph initial cash to demand their optimal amount X of the asset.

3.2 Demand Shocks

I introduce now some heterogeneity in the agents' demands. All agents belonging to the same type-i are identical ex-ante, that is, before the time when the entry decision is made. Ex-post, that is, after the entry decision is made and before the portfolio decision, all agents differ in the realizations of their demand-shocks ρ_j as follows.

I assume that all investors have an endowment e of a non-marketable asset. Given that, the final wealth is:

$$w_f^j = (w - pX) + xX + u_j e;$$

where u_j is the return of this non-marketable asset for each agent-j. This non-marketable asset represents any private business or alternative return that an agent has aside from cash and from the risky asset. For any given agent j, the return of the non-marketable asset has a correlation ρ_j with the risky asset, that is in general different across agents. Before entering the market the agents do not know how their endowment of their private asset is correlated with the risky asset: they have some information about how these correlations are distributed (see below).[9] Since the market participants choose to purchase an amount X_i^j of the asset to maximize the mean-variance objective over the final wealth, the demand function is the following:

$$X_i^j(p) = a_i \frac{\mu_x - p}{2\sigma_x^2} - \frac{e\,Cov(x;u_j)}{\sigma_x^2}$$

where $Cov(x;u_j) \equiv \rho_j \sigma_u \sigma_x$ is the covariance between the personal asset return u_j with the risky asset return x. Agents have different demands for the same asset x; since they have different needs to diversify away the risk arising from their private asset. I assume that before entering the market ρ_j is itself a random variable (demand shock) with zero mean and assigned variance $\hat{\sigma}^2$:

$$\rho_j \overset{iid}{\sim} \left(0; \hat{\sigma}^2\right):$$

The prior distribution (not the ex-post realization value) of ρ_j is the information agents have before entering the market, i.e., when making their entry decision. The value of ρ_j for every participant is realized (learned) right before the agents make their portfolio decision, that is, after the every participant has payed the entry cost and learned how his personal asset is correlated with the risky asset he wants to purchase. The more positively correlated with the risky asset is the agent's personal asset, the less of the risky asset the agent demands. Everything else being equal, the higher the volatility of the returns of the asset σ_x^2, the less aggressively the agent's demand reacts to any given shock realization ρ_j, because all the agents are risk averse.

3.3 Equilibrium Price and Variance Analysis

The equilibrium price is obtained by aggregating the demands over all $n = n_A + n_B$ participants assumed to be the market. I assume a constant per-investor supply of the risky asset X_0[10]. The demand-supply equilibrium condition dividing by n both sides is:

$$\frac{\sum_{ij=1}^{n} X_i^j(p)}{n} = \bar{a}_n \frac{\mu_x - p}{2\sigma_x^2} - e\sigma_u \frac{\bar{\rho}_n}{\sigma_x} = X_0;$$

where: $\bar{a}_n \equiv \frac{a_A n_A + a_B n_B}{n}$ identifies the average risk-aversion that characterizes the asset market and $-e\sigma_u \frac{\bar{\rho}_n}{\sigma_x}$ denotes the demand shock averaged over the n participants, that conventionally has a negative sign in this setup. The sample mean of the shocks $\bar{\rho}_n$ is realized after the entry is made but before purchasing the asset.

[10]If I assume that a firm supplies the amount of the asset to maximize its value:

$$V = [nX_0(n)]\, E\,[p(n)];$$

then, the optimal per-investor supply X_0 of the asset is constant, that is, it does not depend on the number of participants n:

$$X_0^*(n) = a\frac{\mu_x}{4\sigma_x^2}$$

An elastic supply of the asset is custumary in the endogenous participation litterature, because with a fixed supply, entry by additional investors would permanently increase the market price and lower the expected rate of return. All the positive externalities form entry that endogenous market participation wants to analyze would be offset by this increase of the market price. For this reason, Pagano (ReStud. 1989) assumes an elastic supply of equities by introducing-value maximizing firms and Allen & Gale (AER 1994) assume a perfectly elasti supply ($p = 1$) of the asset in the first period.

The equilibrium price is:

$$p = \left(\mu_x - 2X_0 \frac{\sigma_x^2}{\bar{a}_n}\right) - 2\frac{\sigma_x}{\bar{a}_n}\rho_n;$$

I call the expected price the benchmark price:

$$E(p) = \mu_x - 2X_0 \frac{\sigma_x^2}{\bar{a}_n}$$

The benchmark price is an increasing function of the expected return and a decreasing function of the variance of returns. The term $-2\frac{X_0}{\bar{a}_n}\sigma_x^2$ expresses the risk-premium, i.e., the compensation in terms of price that the market participants require for the risk they are bearing when purchasing the risky asset. The bigger this variance σ_x^2 the less the agents will demand of the risky asset. The reduction in demand leads to a higher compensation and a lower equilibrium price. As intuition suggests, the decline in price is stronger the higher the average risk-aversion of the agents that participate in the market (the smaller the coefficient $\bar{a}_n$).

For a fixed per-agent supply X_0, the demand-shock term alone is responsible for the price volatility. The variance of the price is the product of four factors:

$$Var(p) = \frac{4}{n}\frac{1}{\bar{a}_n^2}\sigma^2\sigma_x^2$$

The first factor expresses the standard law of large numbers result: with independent shocks, as more agents enter the market the variance of the price is reduced due to mutual shock cancellation. The second factor expresses the negative dependence of the variance of the price on the average risk-tolerance $\bar{a}_n$ of the market: A higher average $\bar{a}_n$ means that there are relatively many type-A agents in the market. These investors with low risk-aversion react more aggressively to the variations in price due to idiosyncratic or aggregate shocks, because they have a more elastic demand. Hence they have on aggregate a stronger stabilizing effect on the equilibrium price and therefore reduce its variance. This can be seen easily if you imagine one agent that has no demand shocks ($\rho_j = 0$ always), so that he only contributes to reduce the price volatility by pushing the price back to target, without contributing to increment it with his shock. This push is stronger the lower the risk-aversion. In the limit if the agent is extremely aggressive, i.e. risk-neutral

($a_i = \infty$), his demand is perfectly elastic, the price is pinned to the expected return ($p = \mu_x$) and the price volatility is reduced to zero as well as the risk-premium. In this extreme case, with no borrowing or short sales constraints, the presence of a single risk-neutral participant is sufficient to eliminate all uncertainty in the price. If the shock is present, by looking at the individual demands you can see that its magnitude does not depend on the level of risk-aversion, so all types make the same positive contribution to the price volatility.[11] The third factor says that the variance of the demand shocks of the agents that enter the market is passed on to the variance of the price. This happens because more volatile individual demands make the aggregate demand more volatile and the asset supply is constant in this model. Lastly, not surprisingly the price volatility depends positively on the variance of the returns σ_x^2: A high variance σ_x^2 translates into a high covariance σ_{xu} of the returns of the risky asset x with the returns of the private asset u: Hence, since this covariance is the source of the demand shocks of the participants, a high σ_x^2 generates significant fluctuations in the aggregate demand, that translate into significant fluctuations in the equilibrium price since the supply is constant.

The amount of the risky asset demanded in equilibrium by every participant is:

$$X_i^j = \frac{a_i}{\bar{a}_n} X_0 + e\sigma_u \left(\frac{a_i}{\bar{a}_n} \frac{\bar{\rho}_n}{\sigma_x} - \frac{\rho_j}{\sigma_x} \right)$$

As figure 2 shows, the more risk-tolerant A-types demand on average more than the per agent supply and push more aggressively than the types-B in the direction opposite to the aggregate shock, mitigating more the price volatility. The contribution of a single agent to the price volatility $-e\sigma_u \frac{\rho_j}{\sigma_x}$ is the same across types though.

Now that it is clear what is the relation between participation $(n_A; n_B)$ and volatility $Var(p)$, it is left to find the relation between entry cost C and participation, which is done in the following sections. It is clear that to some extent a lower entry cost implies a higher market participation. But to determine the volatility implications of an increased market participation, it is crucial to know who are the new market entrants, i.e., how they are distributed across risk-aversion levels with respect to the rest of the market participants.

3.4 Utility from Market Participation

All agents of a given type are identical ex-ante, namely, before entering the market. The heterogeneity within a given type arises only ex-post, namely before the portfolio choice and after the participation decision, when the i.i.d. distributed correlations ρ_j of the non-marketable assets with the risky asset are realized.

The utility for a given type depends on how many agents of both types enter the market, that is:

$$E(U_i)(n_A;n_B) - [\alpha_i w - \beta_i \mathrm{Var}(\tilde{e}_j)] = -\beta_i \left[\left(\frac{a_i}{\bar{a}_n}\right)^2 X_0^2\sigma_x^2 + (e\sigma_u)^2 E\left(\frac{a_i}{\bar{a}_n}\bar{\rho}_n - \rho_j\right)^2\right]$$

The intuition for the last expectation term is simple if you assume no heterogeneity in risk-aversion ($\frac{a_i}{\bar{a}_n} = 1$). The more distant the agent-j is from the sample mean shock the better-off he is, since he can better exploit the market through speculation. The "diversity" of one investor with respect to the rest of the market generates his possibilities of speculation. More importantly, the speculative value of the asset market increases with the number of participants n: Suppose that the investor-j has a high realization of ρ_j; in a thin market this realization increases significantly the entire sample mean $\bar{\rho}_n$. If the market is not thin, i.e. there are many independent investors in the market, one shock realization does not influence significantly the sample mean. In terms of equilibrium price the same concept can also be stated in the following way. In a thin market the demand shock of a single agent generates a significant price movement. This price movement is adverse to the agent, that is, it reduces his utility from speculation. In a market with many participants the price is not influenced significantly by a single agent's demand, so there are no adverse price movements that reduce speculative profits.

Recalling that the shocks have zero mean, the RHS becomes:

$$-\beta_i\left[\left(\frac{a_i}{\bar{a}_n}\right)^2 X_0^2\sigma_x^2 + (e\sigma_u)^2\left(\mathrm{Var}\left(\rho_j\right) + \left(\frac{a_i}{\bar{a}_n}\right)^2 \mathrm{Var}(\bar{\rho}_n) - 2\frac{a_i}{\bar{a}_n}\mathrm{Cov}\left(\rho_j;\bar{\rho}_n\right)\right)\right]$$

The four terms in this expression have an important economic interpretation, which I give when I study the participation condition. The assumption that the demand shocks are i.i.d., $\rho_j \overset{iid}{\sim} \left(0; \sigma_\rho^2\right)$; makes the potential participants differ ex-ante only in the level of their risk-aversion and allows to analyze the effects on participation and volatility of risk-aversion heterogeneity alone. With this assumption the RHS takes the form:

$$\gamma_i \left[\left(\frac{a_i}{\bar{a}_n}\right)^2 X_0^2 \sigma_x^2 + \left((e\sigma_u)^2 \sigma_\rho^2 + \left(\frac{a_i}{\bar{a}_n}\right)^2 \frac{\sigma_\rho^2}{n} - 2\frac{a_i}{\bar{a}_n}\frac{\sigma_\rho^2}{n} \right) \right]$$

To simplify notation, for the rest of the paper I define: $\sigma \equiv e\sigma_u \sigma_\rho$.

4 Participation Analysis

Now that the second stage of the problem is solved, in the remaining sections I solve the first stage problem. I find the (Nash-)equilibrium level of market participation $(n_A; n_B)$ for any given entry cost. To each equilibrium corresponds a certain level of the variance of the price of the asset.

4.1 Participation Condition

If an agent decides to stay out of the market, he cannot buy the risky asset: $X_i^j = 0$, so he can only keep in his portfolio his initial cash (or the safe asset) and his personal asset. Hence his final wealth is:

$$w_f^o = w + u_j e$$

His expected utility from staying out is:

$$E\left(U_i^o\right) = \left[\alpha_i w - \beta_i \mathrm{Var}\left(u_j e\right)\right]$$

Every agent faces a utility cost C^0 when entering the market. Agent-i participates in the market if:

$$E\left(U_i\right)\left(n_A; n_B\right) - C^0 > E\left(U_i^o\right)$$

The above inequality is the participation condition, which becomes:

$$\begin{aligned} V_i\left(n_A; n_B\right) &\equiv \left[\left(\frac{a_i}{\bar{a}_n}\right)^2 X_0^2 \sigma_x^2 + \left(\sigma^2 + \frac{a_i^2}{4\sigma_x^2} \mathrm{Var}(p) - 2\frac{a_i}{\bar{a}_n}\frac{\sigma^2}{n} \right) \right] = \\ &= \left[\left(\frac{a_i}{\bar{a}_n}\right)^2 X_0^2 \sigma_x^2 + \left(\sigma^2 + \left(\frac{a_i}{\bar{a}_n}\right)^2 \frac{\sigma^2}{n} - 2\frac{a_i}{\bar{a}_n}\frac{\sigma^2}{n} \right) \right] > \frac{C^0}{\gamma_i} \equiv C \end{aligned}$$

$V_i(n_A, n_B)$ depends on the risk-aversion ratios $a = \frac{\alpha}{\beta}$. The fact that the RHS above does depend on the level of β_i is just a matter of scaling. If we double α and β the a-ratio along with the LHS remains constant. However, the mean-variance objective function doubles, or equivalently, the utility cost of participation is halved relative to the benefit of participation. To make a fair comparison among the utilities of types with different risk-aversion ratios a_i, I normalize: $\beta_i = 1$ for every type, or equivalently consider the quantity: $C \equiv \frac{C'}{\beta_i}$ to be constant across types. This guarantees that, everything else being equal, agents with the same risk-aversion coefficient a_i face the same participation condition.[12] Allowing the quantity $\frac{C'}{\beta_i}$ to vary across types is equivalent in this model, de facto, to allowing heterogeneity of entry costs.

The following gives the economic intuition behind the terms in $V_i(n_A, n_B)$:

The term proportional to σ_x^2 represents the risk-premium compensation in utility terms. The more risk-averse the market is ($\bar{a}_n$ low), the higher the compensation for every agent-type. In this sense, agents with high risk-aversion are always welcome into a market with lower average risk-aversion, because they exercise a positive externality on all participants, by increasing the average risk-aversion and the risk-premium. An aggressive type-A, i.e., a low risk-aversion agent (a_i big), requires a lower risk-premium compensation than the more risk-averse agents. Hence, an A-type is better-off than a B-type in that respect, i.e., for a given compensation: $\frac{1}{\bar{a}_n} X_0^2 \sigma_x^2$, the type-A agents feel more rewarded than the type-B agents.[13]

The higher the expected "diversity" σ^2 of the agent, the higher his chances of finding the asset at a market price that is very convenient to him, taking into account his idiosyncratic risk-hedging needs. Obviously, this benefit is absent if only one agent is in the market, because the adverse movement of the price offsets one to one his demand shock.

The variance of the equilibrium price always enters positively in the utility from entry. This is the "speculative value" of market participation: a high price volatility raises welfare because it increases the chances of buying low or (short-)selling high. Everything else being equal, a high price volatility encourages market participation. For a given price volatility, the more risk-loving type-A agents (a_i big) react more aggressively to every fluctuation

[13] Note that this last result does not rely at all on the fact that this risk-premium term depends positively on the variance of the returns σ_x^2, which is a peculiar feature of the linear MV-objective adopted. This crucial result would be unchanged if the risk-premium term depended negatively on σ_x^2.

of the price, because their demand function is more price-elastic than the types-B. This results in a higher benefit for the types-A from participating in this market. Price volatility is endogenous here though. Since for now the demand shocks are assumed to be i.i.d., the price volatility declines as more agents enter the market, reducing the speculative value of the market and discouraging participation.

The negative term $-2\frac{a_i}{\bar{a}_n}\frac{\sigma^2}{n}$ comes directly from the correlation of the single demand shock with the average shock $\bar{\rho}_n$. This represents the adverse price movement caused by the demand of the agent under consideration. Since ceteris paribus the more risk-loving type opposes $\bar{\rho}_n$ more aggressively than any agent of the other type, he is responsible for a stronger price movement in that direction (opposing $\bar{\rho}_n$), which more often than not is an adverse direction for him because his shock ρ_i is a component of the average shock $\bar{\rho}_n$. This effect discourages entrance but vanishes for n large: since the demand shocks are independent, this negative term declines in a thicker market due to the law of large numbers shock-cancellation. If there are only agents of one type in the market, the comparison between the two effects of points 3 and 4 is straightforward: the negative effect of the adverse price movement is stronger than the positive effect of price speculation. Since these terms are decreasing in n at the same rate, market participation depends positively on n:

$$V_i \equiv \left[X_0^2\sigma_x^2 + \sigma^2 - \frac{\sigma^2}{n} \right]$$

In the particular case in which there is only one type of agents in the market, entrance by an additional agent generates a positive externality an all participants by making the market less thin, that is, by reducing the adverse price movement that each individual demand generates. In this case, it is true that "the more, the merrier": the more investors enter the market the higher the benefit of all the market participants.

4.2 Participation Equilibria

In this section I find the Nash equilibria of this entry game with $(N_A + N_B)$ players, i.e. ,all the potential market participants. For a given entry cost C; $(n_A; n_B) \in (0; N_A) \times (0; N_B)$ is an (interior) equilibrium if and only if:

$$\begin{aligned} V_A(n_A; n_B) &\geq C \geq V_A(n_A + 1; n_B) \\ V_B(n_A; n_B) &\geq C \geq V_B(n_A; n_B + 1); \end{aligned}$$

so that investors already in the market weakly prefer to stay in and an additional investor of every type weakly prefers not to enter. This implies that a necessary condition to have an interior equilibrium is that the functions $V_i(n_i; \cdot)$ are both non-increasing for some common range of values. Total participation of agents of type-B $(n_A; N_B)$ is an equilibrium only if:

$$V_B(n_A; N_B) \geq C$$

No participation of type-2 of agents: $(n_A; 0)$ is an equilibrium only if:

$$V_B(n_A; 1) \leq C$$

and similarly for type-A agents. In the picture below, I show how different participation equilibria arise for a given entry cost C: In the picture I assume, for mere illustrative purposes, that there is only one type of agents: I denote by $n \leq N$ the number of investors that participate in the market and by N the total number of investors. It is clear from the picture that for any entry cost C, an interior equilibrium, such as Eq.2, exists only if the value function $V(n)$ is decreasing for some range of values of n: If the value function is strictly increasing, then for any entry cost, the only two possible participation equilibria are the no-participation equilibrium (Eq.1) and the full participation equilibrium (Eq.3).

Since in the model, there are two types of agents finding the participation equilibria involves studying the value function of both types simultaneously. Before finding the equilibria of this game, I assume that N_A and N_B are large, that is, the total number of agents of any type is large. This means that if all the agents of any given type decide to enter the market, then there are many investors in the market, i.e. the market is not thin.[14]

Lemma 1 For every n_A; $V_B(n_A; n_B)$ is strictly increasing in n_B:

Proof. See appendix. ■

The above lemma extends "the more, the merrier" result to the backward types: no matter how many type-A agents are in the market, if a type-B

[14] Of course, if many agents of a given type decide not to participate ($n_i < N_i$), the market can be thin. However as it turns out, a number of participants n_i; such that $0 < n_i < N_i$, never corresponds to a participation equilibrium in this model.

For the purpose of this model N_i need not be too large. It is sufficient N_i large enough, so that in the V_i functions the terms proportional to $\frac{\sigma^2}{n}$ are negligible with respect to the other terms (e.g. $N_i > 10$).

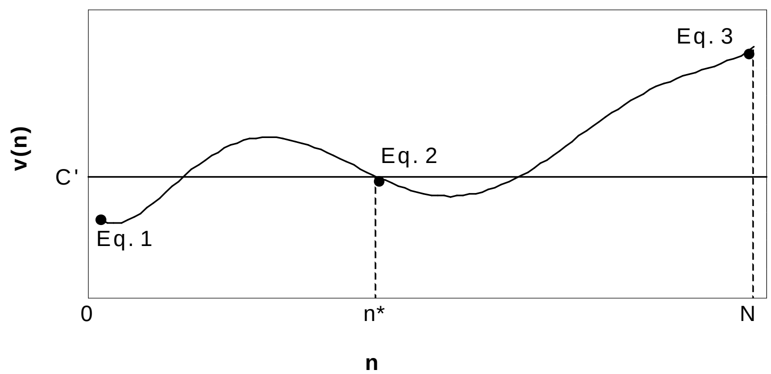


Figure 3:

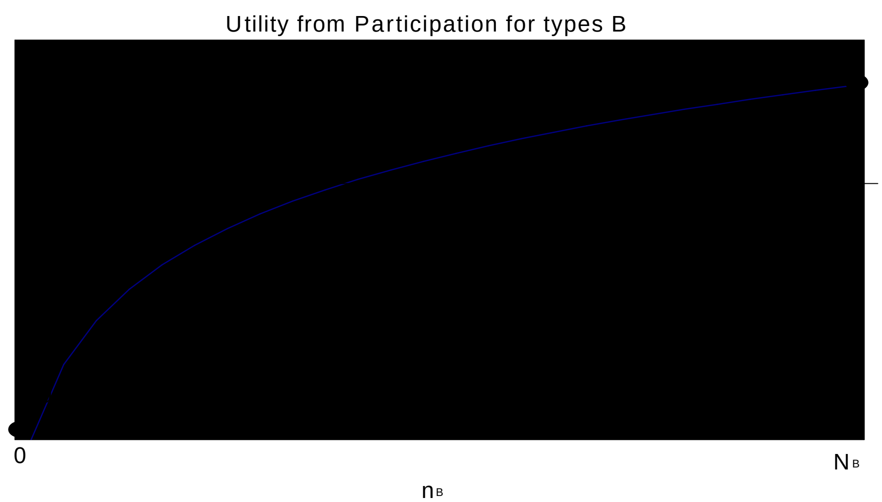


Figure 4:

investors decides to participate, all the type-B participants are better-off. Indeed, the entrance of a B-type is a positive externality for all the other participants regardless of their type. Since the value function for the B-types is always increasing, in equilibrium the agents of type-B enter the market only as a whole group. Since N_B is large, the terms proportional to $\frac{1}{n}$ become negligible with respect to the other terms. Hence:

$$V_B(n_A; N_B) = \max_{n_B} V_B(n_A; n_B) \overset{N_B \text{ large}}{\approx} \left[\left(\frac{a_B}{\bar{a}_n} \right)^2 X_0^2 \sigma_x^2 + \sigma^2 \right] :$$

Lemma 2 For every $(n_A; n_B)$ such that n is large enough, $V_A(n_A + 1; n_B) > V_B(n_A; n_B)$

Proof. See appendix. ■

Proposition 3 For any entry cost C, there are only three possible participation equilibria: no participation $(n_A; n_B) = (0; 0)$, partial participation $(n_A; n_B) = (N_A; 0)$ and total participation $(n_A; n_B) = (N_A; N_B)$.

Proof. See appendix. ■

All possible (that is, not considering the level of the entry cost) participation Nash-Equilibria are represented as the black dots on the graph below. The important result is that no equilibrium lies inside the rectangle, namely, agent types enter the market only as a whole group.

The vertical arrows represent the fact that for any number of A-participants, the entrance of a B-type makes all the B-participants better-off. Hence, the types with higher risk-aversion enter in blocks (this is true for any number of types). This positive participation externality arising from the entry of B-types has two origins. First, the entry of an additional B-investor it generates a thicker market, improving the participants' welfare. Namely, the demand of every single participant has a smaller effect on the equilibrium price, that is, the adverse price movements generated by the demand of any participant is reduced. Second, it increases the average risk-aversion of the market participants, generating a higher risk-premium and a lower price. For the same reason, the entrance of an A-type generates a negative externality on all participants by decreasing the average risk-aversion and a positive externality by making the market thicker. However, if there are only agents of one type in the market this risk-premium externality is absent because the

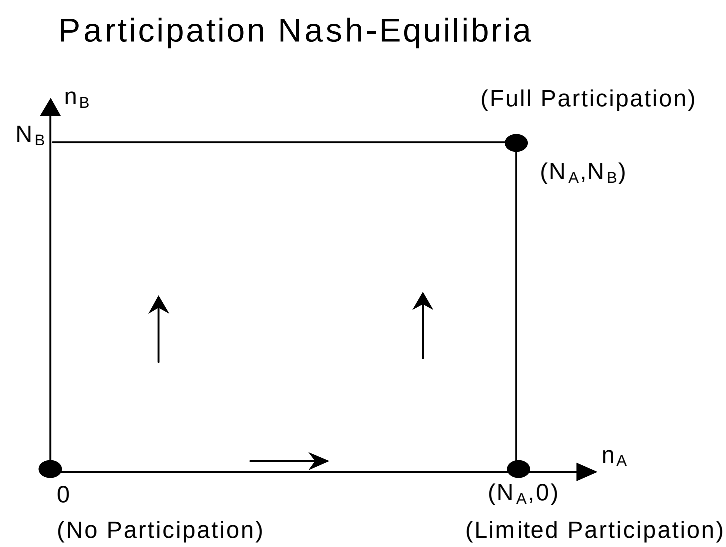


Figure 5:

average risk-aversion remains unchanged as an additional agent enters. In this case the positive the thicker market externality becomes predominant. The horizontal arrow represents the fact that if there are only A-types in the market, the entry of an additional A-type generates a positive externality on all participants by making the market less thin. Hence, if there are only A-types in the market, then an additional A-type increases the utility of all participants.

For any given entry cost C, some of the three possible Nash-equilibria exist. The following proposition give the necessary and sufficient conditions for the existence of each of the three possible participation equilibria.

Proposition 4 No participation is an equilibrium if and only if the entry cost is above a minimum level: $C \geq X_0^2\sigma_x^2$. Full participation is an equilibrium if and only if the entry cost is below a maximum level: $C \leq \left(\frac{a_B}{a_N}\right)^2 X_0^2\sigma_x^2 + \sigma^2$. Partial participation is an equilibrium if and only if the entry cost is in the interval: $\left(\frac{a_B}{a_A}\right)^2 X_0^2\sigma_x^2 + \sigma^2 \leq C \leq X_0^2\sigma_x^2 + \sigma^2$.

Proof. See appendix. ■

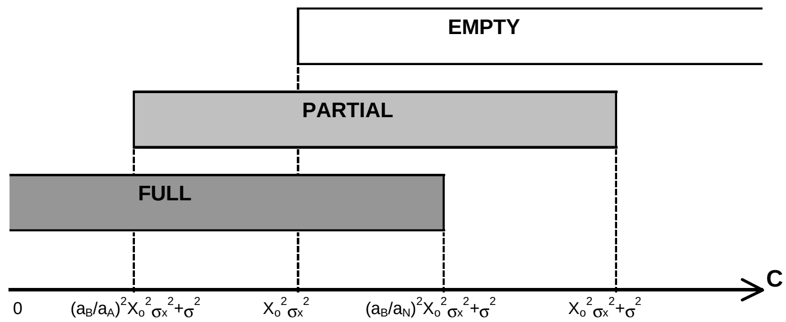


Figure 6:

4.3 Multiple Equilibria

For given parameter values some of the three participation equilibria coexist, that is, multiple equilibria arise for some range of entry costs, as this figure shows:

I call the no-participation equilibrium the trivial equilibrium. For some range of entry costs, the trivial equilibrium can coexist with either (see footnote) or both the partial and the full participation equilibria. When there are multiple equilibria, if the trivial equilibrium is selected, then it means that a potential asset market is missing due to coordination failure.[15] The following result expresses what the picture illustrates.

Proposition 5 For a sufficiently low cost $C \le \left(\frac{a_B}{a_A}\right)^2 X_0^2\sigma_X^2 + \sigma^2$ full participation is the unique equilibrium. For $\left(\frac{a_B}{a_A}\right)^2 X_0^2\sigma_X^2 + \sigma^2 \le C \le \left(\frac{a_B}{a_N}\right)^2 X_0^2\sigma_X^2 +$

[15] In the picture above the location of the no-participation equilibrium relative to the other two equilibria depends on the values of the parameters of the model. For this picture I select $\left(\frac{a_B}{a_A}\right)^2 k^2\sigma_X^2 + \sigma^2 < k^2\sigma_X^2 < \left(\frac{a_B}{a_N}\right)^2 k^2\sigma_X^2 + \sigma^2$, but the lower bound for the no-participation equilibrium ($k^2\sigma_X^2$) can also be located below the lower bound for the partial participation equilibrium (if $k^2\sigma_X^2 < \left(\frac{a_B}{a_A}\right)^2 k^2\sigma_X^2 + \sigma^2$) or above the upper bound for the full participation equilibrium (if $k^2\sigma_X^2 > \left(\frac{a_B}{a_N}\right)^2 k^2\sigma_X^2 + \sigma^2$). The location of the partial participation equilibrium relative to the full participation equilibrium is independent though from the parameter values.

σ^2 full and partial participation are both equilibria. For $\left(\frac{a_B}{a_N}\right)^2 X_0^2\sigma_x^2 + \sigma^2 < C \leq X_0^2\sigma_x^2 + \sigma^2$ partial participation is the unique (non-trivial) equilibrium. For $C > X_0^2\sigma_x^2 + \sigma^2$ nobody participates in the market (trivial equilibrium).

In the case of multiple equilibria, the different participation equilibria can always be Pareto-ranked. The partial participation equilibrium is Pareto-superior to the no-participation equilibrium, because the B-types have a higher welfare from participating than not. The full participation equilibrium is Pareto-superior to the other two equilibria for the following reason. On the one hand, in a full participation equilibrium the B-types have a higher welfare from participating then if they were the only types in the market, since they receive a higher risk-premium compensation due to the participation of the A-types. The A-types also have a higher welfare in a full participation equilibrium than in a no-participation equilibrium. On the other hand, the B-types have a higher welfare from participating (full participation equilibrium) than not entering (partial participation equilibrium or no-participation equilibrium). Indeed in the case of multiplicity of equilibria, the failure to enter the market of one type of investors (as a whole) is a coordination failure that locks the market into a lower participation-lower welfare equilibrium.

All these results on market participation can be generalized to more types/levels of risk-aversion $(A, B, C, \ldots, Z)$. Assuming to rank them in order of increasing risk-aversion, the following result holds.

Proposition 6 All the possible participation equilibria for many types are of the form: $(0, 0, 0, \ldots, 0)$; $(N_A, 0, 0, \ldots, 0)$; $(N_A, N_B, 0, \ldots, 0)$; $(N_A, N_B, N_C, \ldots, 0)$:etc.

So types inter in blocks. The intuition is that the types (say Z-types) with the highest risk-aversion enter in blocks because their utility is strictly increasing. For a given entry cost, if the Z-types enter, then all the other less risk-averse types want to enter as well, because they have a higher benefit in terms of risk-premium. If Z-types do not enter, then the above argument applies to the second more risk-averse Y-types, which, in turn, now become the more risk-averse potential participants. The process unravels this way by induction. The multiplicity of equilibria, their dependance on the entry cost and the Pareto ranking are qualitatively the same as in the two types case.

4.4 Equilibrium Volatility

The volatility of the price of the asset depends on the particular participation equilibrium selected. An important result of this paper is that the price volatility can increase as the types-B agents enter the market and the market moves from a partial participation to a full participation equilibrium. That is a thicker market can be more volatile than a thinner market. In the partial participation equilibrium the price volatility is:

$$Var_p(p) = \frac{1}{[a_A]^2}\left(\frac{1}{N_A}\right)\left[4\sigma^2\sigma_x^2\right]$$

In the full participation equilibrium the price volatility is:

$$Var_f(p) = \frac{1}{[(1-z)a_A + za_B]^2}\left(\frac{1}{N_A + N_B}\right)\left[4\sigma^2\sigma_x^2\right]$$

where z is the proportion of new participants in the market:

$$z = \frac{N_B}{N_A + N_B}$$

The ratio of the price-variances after and before the entry of new participants is:

$$\frac{Var_f(p)}{Var_p(p)} = \frac{1-z}{[1-(1-\delta)z]^2}$$

where:

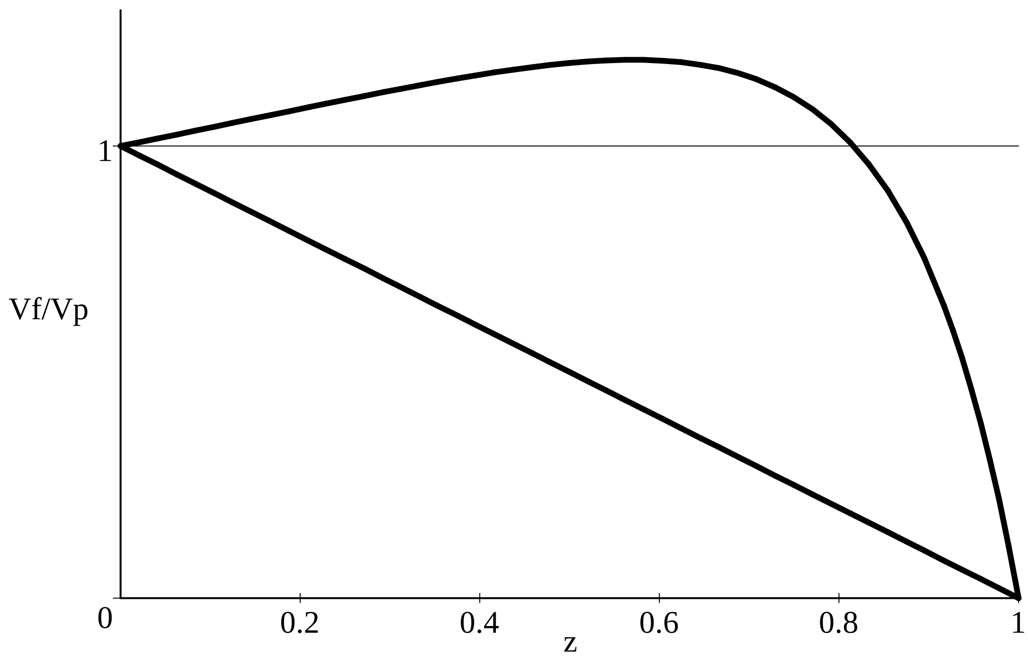
$$\delta = \frac{a_B}{a_A} < 1$$

is the ratio of the risk-tolerance, or its inverse δ^{-1} is the ratio of the risk-aversion of the two participant-types.

Proposition 7 Volatility increases with more participation only if the new participants are at least twice more risk-averse than the rest:

$$a_B < \frac{a_A}{2}$$

Proof. The variance-ratio is grater than 1 for every z in the interval: $0 < z < z_1 \equiv \frac{1-2\delta}{(1-\delta)^2}$. This interval is non-empty only for $z_1 > 0$, that is: $\delta < \frac{1}{2}$. ■



The continuous line shows the effect on volatility for $\delta = 0.3$.
The dotted line shows the steady decrease in volatility due to the Law of Large Numbers effect only, that is, $\delta = 1$ (no risk-aversion heterogeneity).

As the market shifts from a partial participation to a full participation equilibrium and the type-B agents enter the market, two effects push the price volatility in opposite directions. One effect is due to the law of large numbers: additional market participants reduce the price volatility because their demand shocks tend to average-out better. The cross-line in the figure shows the steady decrease in volatility due to this effect alone, that is, without risk-aversion heterogeneity. The other effect is the risk-aversion effect that tends to increase volatility: the highly risk-averse B-agents react less aggressively to any price movement, contributing less than the more risk-loving A-types to the dampening of the price fluctuations. Which of the two effects dominates depends on how different the two participant-types are in their risk-aversion levels and on the proportion z of new participants. If the new entrants are enough more risk-averse than the other market participants, then the effect of more participation in an asset market is an increase in the price volatility. However, if the relative number of new participants is very large the variance-ratio decreases and drops below one, no matter how risk-averse the new participants are. This is intuitive. Eventually, if the new participants dominate the market, that is, the old participants are negligible with respect to the new participants (high z), then the average risk-aversion of the market is almost constant. Since the law of large numbers effect is the only effect in a market with homogenous risk-aversion, volatility decreases sharply for high values of z.

It should be clear that the assumption of independent shocks is an extreme case. In general, the demand shocks are not completely independent across agents, that is, there is some degree of correlation among the demand-shocks. A systematic component in the agent's shocks is equivalent to an aggregate shock. In the presence of aggregate shocks, the mutual shock cancellation is less pronounced. If the correlation among demand shocks is $\gamma > 0$, then the volatility becomes:

$$\mathrm{Var}(p) = \frac{4\sigma_x^2}{\bar{a}_n^2}\left[\frac{\sigma^2 - \gamma}{n} + \gamma\right]$$

The term in square parentesis, which is the LLN cancellation term, decreases at a slower pace and does not coverge to zero anymore, but to $\gamma > 0$. With full correlation ($\gamma = \sigma^2$) the shock cancellation is totally absent. As a consequence of this only partial shock cancellation, a smaller difference in risk-aversion among the market participants is sufficient to increase the price volatility.

5 Extension: OLG Model

In this section I consider a dynamic version of the original static model and show that my results still hold. I assume that on every period one cohort of participants enters the market purchasing the risky asset inelastically supplied by the previous cohort that entered the market one period before. So, the last period in the static model now becomes the period at which the market participants sell inelastically (and then exit the market) all their risky asset holdings to a new generation of participants that enters the market in that period. As a consequence, a unit of the asset now has the exogenous risky return x as before, and, if it was purchased at a price p_t, it also has a return p_{t+1} from its sale price. The relevance of this extension comes from the fact that in the static model investors liked the price volatility because it merely had a speculative value, namely, it enhanced their chances of speculation. In the dynamic extension the buyers of today are sellers tomorrow, so they like volatility today but dislike it tomorrow, since it generates variability in their final wealth. I analyze only steady-state participation Nash-equilibria of this model. I assume that the total per agent supply of the asset in the market is constant X_0: The final wealth of investor-j is:

$$w_f^j = R\left(w - p_t X\right) + \left(p_{t+1} + x\right) X + u_j e;$$

where R is the return of the safe asset (e.g. cash if $R = 1$).

Avoiding all the derivations (see appendix), the investors of type-i enter the market if the quantity below is bigger than C :

$$\left\{ \left(\frac{a_i}{\bar{a}_n}\right)^2 X_0^2 \left[\mathrm{Var}(p) + \sigma_x^2\right] + \frac{\sigma_x^2}{\mathrm{Var}(p) + \sigma_x^2} \left[\left(\frac{a_i}{\bar{a}_n}\right)^2 \frac{\sigma^2}{n} - 2\left(\frac{a_i}{\bar{a}_n}\right)\frac{\sigma^2}{n} + \sigma^2 \right] \right\}$$

The variance of the price of the asset is the same as in the static version of the model:

$$\mathrm{Var}(p) = \frac{4}{n}\frac{1}{\bar{a}_n^2}\sigma^2\sigma_x^2$$

The variance of the future price is now liked because it generates a higher risk-premium (first term) and disliked because it affects positively the volatility of the final wealth when selling the asset (second term). The variance of the current price is accounted for in the term proportional to $\left(\frac{a_i}{\bar{a}_n}\right)^2 \frac{\sigma^2}{n}$: It enters positively in the utility from entry because it enhances the chances of price speculation when purchasing the asset.

The participation equilibria of the dynamic model are the same as in static model and the same pattern of multiple equilibria arises as the entry cost changes (see appendix). Since the expression of the price volatility is the same as in the basic model, the volatilities that correspond to the different participation levels are also the same as in the static model. So the dynamic extension adds nothing new to the basic model, but allows another interpretation of the results on volatility, as follows.

5.1 Volatility of Returns

The dynamic extension is important because, the results on the price volatility easily translate into return volatility. Returns are so defined:

$$r_{t+1} = \log(p_{t+1} + x_{t+1}) - \log p_t$$

Since in my model, the price is i.i.d. distributed in every period t, a higher range of fluctuation in the price implies a higher range of fluctuation in the returns. If, more precisely, I take the variance as the measure of volatility instead of the range of fluctuation, I can show that.

Proposition 8 A higher variance of the price implies a higher variance of the returns of the asset.

Proof. See appendix ■

Summarizing, when I say that more participation increases volatility, I refer to both price and return volatility, the latter is what matters more and is studied in the empirical literature.

6 Summary

I have answered the two questions I outlined in the introduction.

First, this model replicates the stylized fact that the lowering of the entry costs drives more agent-types into the asset markets in the following way. As the entry cost is lowered, the market moves from a participation equilibrium in which only the more risk-tolerant types-A are in the market, to a multiple equilibrium situation in which also the more risk-averse types-B can participate in the market. As the entry cost is lowered further all types necessarily enter the market regardless of their risk-aversion. The backward types-B require a higher compensation than the aggressive types-A for the risk they are bearing when purchasing the risky asset. Hence, for any given number of participants $(n_A; n_B)$, the types-A perceive a higher risk-premium compensation in utility terms than the types-B. This is a reason why the aggressive types enjoy a higher bene¯t from participating and have a higher incentive to enter the market than the backward types.[16] As a consequence for some range of entry costs, the A-types are better-o® participating in the market and the B-types are better-o® staying out of the market.

Second, the more risk-tolerant agents react more aggressively to any price °uctuation, by buying less (or more) of the asset in larger amounts than the other types in response to any given price shock. Hence, as market participation increases and the more risk-averse types join the less risk-averse types in the market, the less aggressive reaction to the price °uctuations of the new participants tends to increase the asset price volatility. The increase in volatility is more likely the more risk-averse the new participants are and it is more pronounced in the presence of aggregate price shocks, that is, if the demand shocks of the participants are correlated.

[16] This becomes the compelling reason when the market is thick, that is, when the terms other than the risk-premium (i.e. the terms proportional to $\frac{1}{n}$) in the entry condition become negligible with respect to the risk-premium compensation term.

7 Appendix

7.1 Proofs

Proof. 1 The term $\left(\frac{a_B}{\bar{a}_n}\right)^2 X_0^2\sigma_x^2$ is increasing in n_B: Since: $\frac{a_B}{\bar{a}_n} \leq 1$; the term $\left[\left(\frac{a_B}{\bar{a}_n}\right)^2 - 2\frac{a_B}{\bar{a}_n}\right]\frac{\sigma^2}{n}$ is always negative, vanishes for big n and its first derivative with respect to n_B is always positive. Hence, $V_B(n_A; n_B)$ is always strictly increasing in n_B. ∎

Proof. 2 Since the terms proportional to $1/n$ are negligible with respect to the rest, I need to show that:

$$\left[\left(\frac{a_A}{\bar{a}_{n+1}}\right)^2 X_0^2\sigma_x^2 + \sigma^2\right] > \left[\left(\frac{a_B}{\bar{a}_n}\right)^2 X_0^2\sigma_x^2 + \sigma^2\right]$$

Simple algebra shows that the above inequality is equivalent to the inequality:

$$\frac{a_B}{a_A}n + \frac{a_B}{\bar{a}_n} < n + 1;$$

which is trivially satisfied. ∎

Proof. 3 Since $V_B(n_A; n_B)$ is strictly increasing in n_B; the only possible values of n_B compatible with equilibria are $n_B = 0$ and $n_B = N_B$: First, I look for the possible equilibria with $n_B = 0$. Trivially, $V_A(n_A; 0) = X_0^2\sigma_x^2 + \sigma^2 - \frac{\sigma^2}{n_A}$ is strictly increasing in n_A (if only agents of one type enter the market "the more, the merrier" result holds). Hence, the only equilibria with $n_B = 0$ are $(n_A; n_B) = (0; 0)$ and $(n_A; n_B) = (N_A; 0)$: Second, I look for the possible equilibria with $n_B = N_B$: In order for $(n_A^*; N_B)$ to be an equilibrium for any given n_A^*, it must be the case that: $V_B(n_A^*; N_B) \geq C$: Since N_B is large enough, Lemma 2 implies that for every n_A^*:

$$V_A(n_A^* + 1; N_B) \overset{N_B \text{ large}}{>} V_B(n_A^*; N_B) \geq C$$

In other words, the additional agent of type-A strictly prefers to participate at the entry cost C, meaning that $(n_A^*; N_B)$ with $n_A^* < N_A$ cannot be an equilibrium. Hence, the only possible equilibrium with $n_B = N_B$ is $(n_A; n_B) = (N_A; N_B)$:

Proof. 4 Since $V_A(1; 0) = V_B(0; 1) = X_0^2\sigma_x^2$, every agent prefers not to enter the empty market if and only if $X_0^2\sigma_x^2 \leq C$: Since $V_A(N_A; N_B) >$

$V_B(N_A;N_B) \simeq \left(\frac{a_B}{a_N}\right)^2 \left[X_0^2\sigma_x^2+\sigma^2\right]$; every agent prefers not to exit the full market if and only if $\left(\frac{a_B}{a_N}\right)^2 \left[X_0^2\sigma_x^2+\sigma^2\right] \geq C$: Since $V_A(N_A;0) \overset{N_A\ \text{large}}{\simeq} X_0^2\sigma_x^2+\sigma^2$ and $V_B(N_A;1) \overset{N_A\ \text{large}}{\simeq} \left(\frac{a_B}{a_A}\right)^2 X_0^2\sigma_x^2+\sigma^2$; given that all agents of type-A are participating in the market and no agent of type-B is participating $(n_A;n_B)=(N_A;0)$, a type-B agents prefers to stay out of the market if and only if $C \geq \left(\frac{a_B}{a_A}\right)^2 X_0^2\sigma_x^2+\sigma^2$ and type-A agent prefers to stay in the market if and only if $C \leq X_0^2\sigma_x^2+\sigma^2$: ■ ■

Proof. 8 By taking a first order approximation of the log around the mean[17] and recalling that the random variable return is independent from the random variable price, I obtain:

$$\mathrm{Var}\left[\log\left(p_{t+1}+x_{t+1}\right)\right] \simeq \left(\frac{1}{E(p)+\mu_x}\right)^2\left[\mathrm{Var}(p)+\sigma_x^2\right]$$

$$\mathrm{Var}\left[\log p_t\right] \simeq \left(\frac{1}{E(p)}\right)^2 \mathrm{Var}(p)$$

The benchmark price E(p) has decreased with more participation because of the higher risk-premium, required by the more risk-averse new entrants. Hence, moving from a partial participation equilibrium to a full participation equilibrium, the variance of the log-price increases if the variance of the price increases. Remembering that the covariance of functions of stochastically independent random variables (prices) is zero, I obtain:

$$\mathrm{Var}(r) \simeq \left(\frac{1}{E(p)+\mu_x}\right)^2\left[\mathrm{Var}(p)+\sigma_x^2\right]+\left(\frac{1}{E(p)}\right)^2 \mathrm{Var}(p)$$

So the variance of the returns increases with the variance of the price. ■

[17] This approximation is accurate if the price (and dividend x) fluctuations are small relative to the mean price level: $\frac{\Delta p}{\overline{p}} \ll 1$. This is typically the case, since it is the same approximation that in first place allows to define the returns as the difference of the log-prices:

$$r = \frac{\Delta p}{p} \simeq \Delta \log p$$

$$\log p \simeq \log \overline{p} + \frac{\Delta p}{\overline{p}} - \frac{1}{2}\left(\frac{\Delta p}{\overline{p}}\right)^2 + O\left(\frac{\Delta p}{\overline{p}}\right)^3$$

7.2 OLG Model Calculations

I present here all the derivations for the extended model. The derivations for the static model can be obtained from the extended model by setting $p_{t+1} = 0$.

I restate now the following definitions.

The final wealth of investor-j is:

$$w_f^j = R(w - p_t X) + (p_{t+1} + x) X + u_j e;$$

where:

$$x \sim \left(\mu_x; \sigma_x^2\right)$$

is the risky asset's return (e.g. a dividend for a stock) which is distributed with known mean and variance, whereas R is the return of the safe asset (e.g. cash if R = 1). The returns of the private assets are i.i.d. distributed across the investors:

$$u_j \overset{iid}{\sim} \left(\mu_u; \sigma_u^2\right)$$

This private asset denotes any return that an investor may have aside from the risky asset x under consideration and the safe asset. Before entering the market the every potential participant knows how these assets are distributed but does not know how his private asset u_j is correlated with the risky asset x:

$$Cov(x; u_j) \equiv \sigma_x \sigma_u \rho_j \quad \text{with} \quad \rho_j \overset{iid}{\sim} \left(0; \bar{\sigma}^2\right)$$

that is, before entering the market the potential participant only knows that this correlation is zero on average but could be positive or negative with a variance of $\bar{\sigma}^2$. To enter the market investor-j pays an entry cost and becomes informed of this correlation, that is, he obtains a realization of the random variable ρ_j. So, after entering the market every participant knows how his private asset is correlated with the risky asset and based on this information he formulates his optimal demand for the latter. To obtain the optimal demand, I must maximize the mean-variance objective, so I evaluate the mean and the variance of the final wealth:

$$\begin{aligned}
E_t\left(w_f^j\right) &= R(w - p_t X) + (E_t(p_{t+1}) + \mu_x) X + \mu_u e \\
Var_t\left(w_f^j\right) &= X^2 Var_t(p_{t+1} + x) + e^2\sigma_u^2 + 2eX\,Cov_t(p_{t+1} + x; u_j)
\end{aligned}$$

By differentiating the objective $U_i^j \equiv \alpha_i E_t\left(w_f^j\right) - \beta_i Var_t\left(w_f^j\right)$; with respect to X, I obtain the FOC:

$$\alpha_i\left[E_t(p_{t+1}) + \mu_x - Rp_t\right] - \beta_i\left[2X\,Var_t(p_{t+1}+x) + 2e\,Cov_t(p_{t+1}+x; u_j)\right] = 0$$

Demand at time t:

$$\begin{aligned}
X_i^j(p_t) &= a_i \frac{E_t(p_{t+1}) + \mu_x - Rp_t}{2Var_t(p_{t+1}+x)} - \frac{e\,Cov_t(p_{t+1}+x; u_j)}{Var_t(p_{t+1}+x)} \\
&= a_i \frac{E_t(p_{t+1}) + \mu_x - Rp_t}{2\left(Var_t(p_{t+1}) + \sigma_x^2\right)} - e\frac{Cov_t(p_{t+1}; u_j) + Cov_t(x; u_j)}{Var_t(p_{t+1}) + \sigma_x^2} \\
&= a_i \frac{E_t(p_{t+1}) + \mu_x - Rp_t}{2\left(Var_t(p_{t+1}) + \sigma_x^2\right)} - e\frac{Cov_t(x; u_j)}{Var_t(p_{t+1}) + \sigma_x^2} \\
&\equiv a_i \frac{E_t(p_{t+1}) + \mu_x - Rp_t}{2\left(Var_t(p_{t+1}) + \sigma_x^2\right)} - e\frac{\sigma_x \sigma_u \rho_j}{Var_t(p_{t+1}) + \sigma_x^2}
\end{aligned}$$

The last steps have the following justification. Since p_{t+1} is the price at which the next cohort of investors is buying the asset from the current cohort which is selling all the assets inelastically, p_{t+1} depends on the demand shocks of the next cohort only. That is, p_{t+1} is not correlated with the returns x and u_j of the current generation's agents:

$$\begin{aligned}
Cov_t(p_{t+1}; u_j) &= 0 \\
Cov_t(p_{t+1}; x) &= 0
\end{aligned}$$

Given that the total supply is $(n_A + n_B)X_0$, the demand-supply equilibrium is:

$$X_0 = \bar{a}_n \frac{E_t(p_{t+1}) + \mu_x - Rp_t}{2\left(Var_t(p_{t+1}) + \sigma_x^2\right)} - e\frac{\sigma_x \sigma_u \bar{\rho}_{n;t}}{Var_t(p_{t+1}) + \sigma_x^2};$$

where $\bar{\rho}_{n;t}$ is the sample average correlation of the time t-participants's private asset return with the risky asset return. The equilibrium price at time t becomes:

$$p_t = \frac{1}{R}\left\{E_t(p_{t+1}) + \mu_x - \frac{2X_0}{\bar{a}_n}\left(Var_t(p_{t+1}) + \sigma_x^2\right) - \frac{2e}{\bar{a}_n}\sigma_x \sigma_u \bar{\rho}_{n;t}\right\}$$

Leading one period, the equilibrium price for the following cohort is:

$$\begin{aligned}
p_{t+1} &= \frac{1}{R}\left[E_{t+1}(p_{t+2}) + \mu_x - \frac{2X_0}{\bar{a}_n}\left(Var_{t+1}(p_{t+2}) + \sigma_x^2\right)\right] + \\
&\quad - \frac{1}{R}\frac{2e}{\bar{a}_n}\sigma_x\sigma_u\rho_{n,t+1} \\
E_t(p_{t+1}) &= \frac{1}{R}\left[E_{t+1}(p_{t+2}) + \mu_x - \frac{2X_0}{\bar{a}_n}\left(Var_{t+1}(p_{t+2}) + \sigma_x^2\right)\right] \\
Var_t(p_{t+1}) &= \frac{4e^2}{\bar{a}_n^2}\sigma_x^2\sigma_u^2\frac{\sigma^2}{n}
\end{aligned}$$

Hence, the variance of the price is constant over time and it is the same as the one found in the two period static model. Since on every period the world looks the same in expectations, then for every t:

$$E(p) \equiv E_t(p_{t+1}) = E_{t+1}(p_{t+2}) = \frac{1}{R-1}\left[\mu_x - \frac{2X_0}{\bar{a}_n}\left(Var_{t+1}(p_{t+2}) + \sigma_x^2\right)\right]$$

So, from now on I can omit the time subscripts on the expectation and the variance of the price, since these quantities do not depend on time but only on the number of participants of each type, that is on the particular participation equilibrium. The equilibrium price is:

$$\begin{aligned}
p_t &= \frac{1}{R}\left\{\left(\frac{1}{R-1} + 1\right)\left[\mu_x - \frac{2X_0}{\bar{a}_n}\left(Var(p) + \sigma_x^2\right)\right] - \frac{2e}{\bar{a}_n}\sigma_x\sigma_u\rho_{n,t}\right\} \\
p_t &= \frac{1}{R-1}\left[\mu_x - \frac{2X_0}{\bar{a}_n}\left(Var(p) + \sigma_x^2\right)\right] - \frac{1}{R}\frac{2e}{\bar{a}_n}\sigma_x\sigma_u\rho_{n,t}
\end{aligned}$$

I derive here some quantities useful for later:

$$\begin{aligned}
Rp_t &= \frac{R}{R-1}\left[\mu_x - \frac{2X_0}{\bar{a}_n}\left(Var(p) + \sigma_x^2\right)\right] - \frac{2e}{\bar{a}_n}\sigma_x\sigma_u\rho_{n,t} \\
(E(p) + \mu_x - Rp_t) &= \frac{1}{R-1}\left[\mu_x - \frac{2X_0}{\bar{a}_n}\left(Var_{t+1}(p_{t+2}) + \sigma_x^2\right)\right] + \mu_x + \\
&\quad - \frac{R}{R-1}\left[\mu_x - \frac{2X_0}{\bar{a}_n}\left(Var(p) + \sigma_x^2\right)\right] + \frac{2e}{\bar{a}_n}\sigma_x\sigma_u\rho_{n,t} \\
&= \frac{2X_0}{\bar{a}_n}\left(Var_{t+1}(p_{t+2}) + \sigma_x^2\right) + \frac{2e}{\bar{a}_n}\sigma_x\sigma_u\rho_{n,t}
\end{aligned}$$

Substituting the equilibrium price in the equilibrium demand, I obtain the equilibrium demand:

$$
\begin{aligned}
X_i^j &= a_i \frac{E(p) + \mu_x - Rp_t}{2\left(Var(p) + \sigma_x^2\right)} - e\frac{\sigma_x \sigma_u \rho_j}{Var(p) + \sigma_x^2} \\
X_i^j &= a_i \frac{\frac{2X_0}{\bar{a}_n}\left(Var_{t+1}(p_{t+2}) + \sigma_x^2\right) + \frac{2e}{\bar{a}_n}\sigma_x\sigma_u\bar{\rho}_{n;t}}{2\left(Var(p) + \sigma_x^2\right)} - e\frac{\sigma_x \sigma_u \rho_j}{Var(p) + \sigma_x^2} \\
X_i^j &= \frac{a_i}{\bar{a}_n}X_0 + \left[\frac{a_i}{\bar{a}_n}\bar{\rho}_{n;t} - \rho_j\right] e\frac{\sigma_x \sigma_u}{Var(p) + \sigma_x^2}
\end{aligned}
$$

Having calculated the equilibrium demand of every participant, I can now calculate the benefit from entering the market, by substituting this demand into the objective:

$$
\begin{aligned}
U_i^j \quad &\equiv \alpha_i E_t\left(w_f^j\right) - \beta_i Var_t\left(w_f^j\right) \\
&= \alpha_i \left\{ Rw + \left(E(p) + \mu_x - Rp_t\right) X_i^j + \mu_u e \right\} + \\
&\quad - \beta_i \left\{ \left(X_i^j\right)^2 \left(Var(p) + \sigma_x^2\right) + e^2\sigma_u^2 + 2eX_i^j\sigma_x\sigma_u\rho_j \right\} \\
&= \left[\alpha_i\left(Rw + \mu_u e\right) - \beta_i e^2 \sigma_u^2\right] + \\
&\quad + \alpha_i\left[E(p) + \mu_x - Rp_t\right] X_i^j - \beta_i \left\{ \left(X_i^j\right)^2 \left(Var(p) + \sigma_x^2\right) + 2eX_i^j\sigma_x\sigma_u\rho_j \right\} .
\end{aligned}
$$

The quantity $U_i^0 \equiv \left[\alpha_i\left(Rw + \mu_u e\right) - \beta_i e^2\sigma_u^2\right]$ is the utility from not entering the market, so I calculate the quantity $\left(U_i^j - U_i^0\right)$, the expectation of which is a crucial magnitude. I can substitute the demand schedule:

$$
X_i^j(p_t) = a_i \frac{E(p) + \mu_x - Rp_t}{2\left(Var(p) + \sigma_x^2\right)} - e\frac{\sigma_x \sigma_u \rho_j}{Var(p) + \sigma_x^2}
$$

into this quantity $\left(U_i^j - U_i^0\right)$:

$$
\begin{aligned}
&\alpha_i\left[E(p) + \mu_x - Rp_t\right] X_i^j - \beta_i \left[\left(X_i^j\right)^2 \left(Var(p) + \sigma_x^2\right) + 2eX_i^j\sigma_x\sigma_u\rho_j \right] \\
&\frac{\alpha_i}{a_i}\left[2\left(Var(p) + \sigma_x^2\right) X_i^j + 2e\sigma_x\sigma_u\rho_j\right] X_i^j + \\
&- \beta_i \left\{ \left(X_i^j\right)^2 \left(Var(p) + \sigma_x^2\right) + 2eX_i^j\sigma_x\sigma_u\rho_j \right\} \\
&\beta_i \left(Var(p) + \sigma_x^2\right) \left(X_i^j\right)^2
\end{aligned}
$$

So taking expectations, and normalizing $\beta_i = 1$, I define the following quantity:

$$V_i\left(n_A; n_B\right) \equiv \left[E\left(U_i^j\right) - U_i^0\right] = \left(\mathrm{Var}\left(p\right) + \sigma_x^2\right) E\left[\left(X_i^j\right)^2\right];$$

so that the entry condition becomes:

$$V_i - C > 0$$

Recall that the variance of the price is the following function of the market participants $(n_A; n_B)$:

$$\mathrm{Var}\left(p\right) = \frac{4e^2}{\bar{a}_n^2}\sigma_x^2\sigma_u^2\frac{\sigma^2}{n}$$

The quantity left to calculate is $E\left[\left(X_i^j\right)^2\right]$, which yields:

$$E\left[\left(\frac{a_i}{\bar{a}_n}X_0^2 + e\frac{\sigma_x\sigma_u}{\mathrm{Var}\left(p\right) + \sigma_x^2}\left[\frac{a_i}{\bar{a}_n}\bar{\rho}_{n;t} - \rho_j\right]\right)^2\right]$$

$$E\left[\left(\frac{a_i}{\bar{a}_n}\right)^2 X_0^2 + \left(e\frac{\sigma_x\sigma_u}{\mathrm{Var}\left(p\right) + \sigma_x^2}\right)^2\left[\frac{a_i}{\bar{a}_n}\bar{\rho}_{n;t} - \rho_j\right]^2\right]$$

$$\left\{\left(\frac{a_i}{\bar{a}_n}\right)^2 X_0^2 + \left(e\frac{\sigma_x\sigma_u}{\mathrm{Var}\left(p\right) + \sigma_x^2}\right)^2\left[\left(\frac{a_i}{\bar{a}_n}\right)^2\frac{\bar{\rho}^2}{n} - 2\left(\frac{a_i}{\bar{a}_n}\right)\frac{\bar{\rho}^2}{n} + \bar{\rho}^2\right]\right\}$$

I define the variable: $\sigma^2 \equiv e^2\sigma_u^2\bar{\rho}^2$ to simplify the notation. The investors of type-i enter the market if the quantity below is bigger than C^0:

$$\left(\mathrm{Var}\left(p\right) + \sigma_x^2\right)\left\{\left(\frac{a_i}{\bar{a}_n}\right)^2 X_0^2 + \left(\frac{\sigma_x}{\mathrm{Var}\left(p\right) + \sigma_x^2}\right)^2\left[\left(\frac{a_i}{\bar{a}_n}\right)^2\frac{\sigma^2}{n} - 2\left(\frac{a_i}{\bar{a}_n}\right)\frac{\sigma^2}{n} + \sigma^2\right]\right\}$$

$$\left\{\left(\frac{a_i}{\bar{a}_n}\right)^2 X_0^2\left(\mathrm{Var}\left(p\right) + \sigma_x^2\right) + \frac{\sigma_x^2}{\mathrm{Var}\left(p\right) + \sigma_x^2}\left[\left(\frac{a_i}{\bar{a}_n}\right)^2\frac{\sigma^2}{n} - 2\left(\frac{a_i}{\bar{a}_n}\right)\frac{\sigma^2}{n} + \sigma^2\right]\right\}$$

Variance of the price depends on the distribution of participants on the following period:

$$\mathrm{Var}\left(p\right) = \frac{4}{\bar{a}_m^2}\sigma_x^2\frac{\sigma^2}{m}$$

The variance of the future price is now liked because it generates a higher risk-premium (first term) and disliked because it affects positively the volatility of the final wealth when selling the asset (second term). The variance of the current price is accounted for in the term proportional to $\sigma_x^2 \left(\frac{a_i}{a_n}\right)^2 \frac{\sigma^2}{n}$: It enters positively in the utility from entry because it enhances the chances of price speculation when purchasing the asset.

7.3 Participation Nash-Equilibria in Steady-State

In any period there is a participation game among the potential participants of that period. In every period there is a participation Nash-equilibrium. I look for a steady-state participation equilibrium, that is, a sequence of Nash-equilibria, where the same equilibrium distribution of participants enter in every period. This task seems hard, because the number (distribution) of participants in a given period influence the utility from entering of the participants in the previous period, since the purchasing price of the asset for one cohort is the selling price of the asset for the previous cohort. Note the following. The market participants in any period take as given exogenously the variance of the returns to their investment: $\sigma_r^2 \equiv Var(p) + \sigma_x^2$; so the Nash-equilibria for participation in any given period are the same as in the static model. In mathematical terms, the only difference is that I am calling σ_r^2 the variance of the returns instead of simply σ_x^2. Therefore as in the static model, the only possible equilibria are no participation, full participation and partial participation. There is still block-entry of participants in classes of increasing risk-aversion. There are only three possible steady-states: the no participation steady-state, the partial participation and the full participation. The (possibly simultaneous) existence of these equilibria depends on the entry cost C:

Proposition 9 No participation is a steady-state equilibrium if and only if the entry cost is high enough: $C \geq X_0^2 \sigma_x^2$

Proposition 10 Full participation is a steady-state equilibrium if and only if the entry cost is low enough:

Proof.

$$V_A(N_A;N_B) > V_B(N_A;N_B) \equiv \left\{ \left(\frac{a_B}{\bar{a}_N}\right)^2 X_0^2 \left(\mathrm{Var}_f(p) + \sigma_x^2\right) + \frac{\sigma_x^2}{\mathrm{Var}_f(p) + \sigma_x^2}\sigma^2 \right\} \geq C$$

$$\mathrm{Var}_f(p) = \frac{4\sigma_x^2}{\bar{a}_N^2}\frac{\sigma^2}{N_A + N_B}$$

■

Proposition 11 Partial participation is a steady-state equilibrium if and only if the cost lies in an intermediate range: $V_B(N_A;1) \leq C \leq V_A(N_A;0)$:

Proof.

$$V_A(N_A;0) \equiv X_0^2 \left(\mathrm{Var}_p(p) + \sigma_x^2\right) + \frac{\sigma_x^2}{\mathrm{Var}_p(p) + \sigma_x^2}\sigma^2$$

$$V_B(N_A;1) \equiv \left(\frac{a_B}{a_A}\right)^2 X_0^2 \left(\mathrm{Var}_p(p) + \sigma_x^2\right) + \frac{\sigma_x^2}{\mathrm{Var}_p(p) + \sigma_x^2}\sigma^2$$

$$\mathrm{Var}_p(p) = \frac{4\sigma_x^2}{[a_A]^2}\frac{\sigma^2}{N_A}$$

■

7.4 Multiple Steady-State Equilibria

Full participation and partial participation coexist for every entry cost that satisfies simultaneously the two conditions below:

$$C \leq \left\{ \left(\frac{a_B}{\bar{a}_N}\right)^2 X_0^2 \left(\mathrm{Var}_f(p) + \sigma_x^2\right) + \frac{\sigma_x^2}{\mathrm{Var}_f(p) + \sigma}\right.$$

$$\left(\frac{a_B}{a_A}\right)^2 X_0^2 \left(\mathrm{Var}_p(p) + \sigma_x^2\right) + \frac{\sigma_x^2}{\mathrm{Var}_p(p) + \sigma_x^2}\sigma^2 \leq C \leq X_0^2 \left(\mathrm{Var}_p(p) + \sigma_x^2\right) + \frac{\sigma_x^2}{\mathrm{Var}_p(p) + \sigma_x^2}\sigma^2$$

Summarizing, nothing changes qualitatively in the dynamic extension of the model compared to the static version.

$$\left\{ \left(\frac{a_i}{\bar{a}_n}\right)^2 X_0^2 \left(\mathrm{Var}(p) + \sigma_x^2\right) + \frac{\sigma_x^2}{\mathrm{Var}(p) + \sigma_x^2} \left[\left(\frac{a_i}{\bar{a}_n}\right)^2 \frac{\sigma^2}{n} - 2\left(\frac{a_i}{\bar{a}_n}\right)\frac{\sigma^2}{n} + \sigma^2 \right] \right\}$$

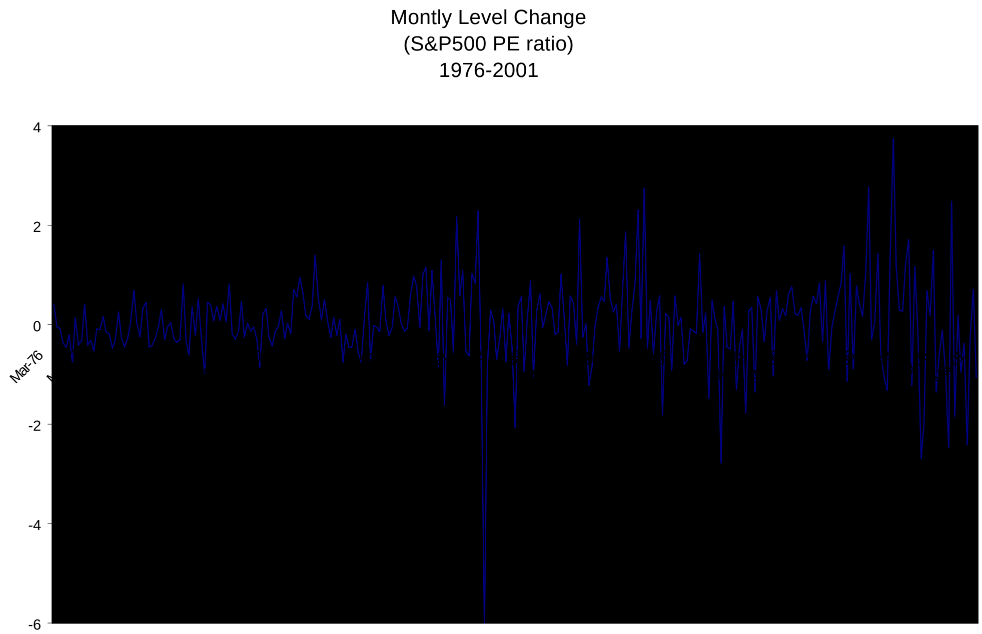


Figure 7: The P-E Ratio of the S&P500 shows increasing dispersion in the past 25 year period.

8 Charts

Here are the charts I refer to in the introduction:

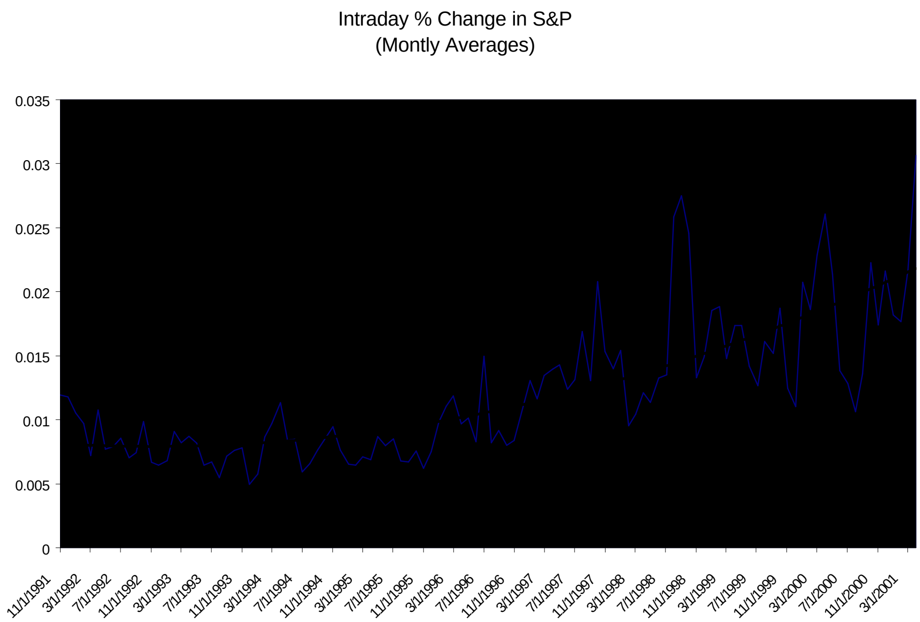


Figure 8: The intra-day volatility, i.e. the maximum daily °uctuation of the S&P500 trends upwards in the nineties.

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