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Ex ante randomization in agency models

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We consider how a principal can use randomized strategies in designing optimal contracts in agency settings. We distinguish between ex post *randomization (over fee schedules following act selection by the agent) and* ex ante *randomization (over fee schedules before act selection). We show that* ex ante *randomization may be efficient in both full information and private information settings under certain assumptions regarding preferences and the production technology. It is particularly significant that additive separability and concavity of the agent's utility function as well as concavity of the production function do not rule out efficient* ex ante *randomized contracts in private information settings, although, as is known, they rule out* ex post *randomization.*

1. Introduction

■ Agency theory has provided considerable insights into the nature of optimal contracts in a variety of settings. The simplest agency model typically involves two individuals: a principal and an agent. The principal delegates responsibility and, perhaps, resources to the agent, who by act selection influences a stochastic process generating an outcome, usually monetary, which is shared by both actors. The agent has often been characterized as selecting an effort level that interacts with nature to determine the outcome. The agency problem arises because the interests of the agent and the principal do not coincide. The principal attempts to structure a sharing rule that both motivates the agent and shares risk.

Of course, the sharing arrangement is a function of, at most, jointly observable variables. If the principal can observe either the act or the state (along with the outcome), incentive problems can be avoided and optimal risk sharing can be achieved. (See Shavell (1979).) This is typically described as the *full information solution.* On the other hand, if the principal can observe only the outcome, optimal (first-best) contracts cannot be written (under reasonable conditions, such as agent risk aversion and constant probability support over outcomes), because of the confounding of effort and state. Inducing the agent to select a

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desirable action requires suboptimal risk sharing. This constrained optimum will be termed the *private information solution.*

The nature of the contract, or sharing rule, has been investigated under a variety of conditions. (See, for example, Harris and Raviv (1979) and Demski and Feltham (1978).) In the private information case, the contractual form requires some risk bearing by the agent (since the fee schedule depends on the outcome), even though the principal may be risk neutral. Thus, the agent is induced to select a desirable act. Holmstrom (1979) proves a very important result for this case. Suppose an additional signal (beyond outcome) is available for contracting purposes. Holmstrom shows that, where the agent's preferences over outcome and action are additively separable, the additional information yields a Pareto improvement if and only if the signal conveys some information about the act that is not contained in the outcome alone. In other words, a purely random signal cannot improve the contract structure. Conversely, all signals used in contracts in such circumstances are informative; they convey information regarding the agent's act. Holmstrom indicates that the sufficiency argument generalizes to nonseparable utility functions.

Holmstrom's results are based on randomization over fee schedules following act selection by the agent. We shall term such arrangements *ex post randomization.* Another possible contracting scheme available to the principal is randomization *before* act selection, which we term *ex ante randomization.* It is this set of schemes that we shall consider here. We demonstrate the following:

(1) *Ex ante* randomization may be desirable (Pareto efficient) in full information and private information settings under certain assumptions; in particular, *none* of the articles we examined eliminated the possibility of *ex ante* randomization.[1] Additive separability and concavity of the agent's preferences do not rule out such randomization in the private information setting, even where the production function is concave.
(2) In the private information case, contracts based on random signals are observationally equivalent to contracts based on "informative" signals. This is true if the observer (like the principal) cannot observe the agent's action.
(3) Over repeated observations, the random signals in contracts will be correlated (*ex post*) with outcomes. The signal will then appear to be "informative."

In Section 2 we review the agency model more formally and expand the characterization of *ex ante* and *ex post* randomization in light of related work. In Section 3 we identify conditions under which *ex ante* randomization will be desirable in a full information scenario. In Section 4 the private information case is considered; again *ex ante* randomization is shown to yield contracting improvements under some conditions. Section 5 contains concluding remarks.

2. The basic agency model

■ We consider a setting in which the agent selects an action, denoted by a, which, combined with a state of nature, θ, yields an outcome, viewed as income or cash flows, denoted $x = x(a, \theta)$. The action may be viewed in a variety of ways (effort, leisure, perquisite consumption); here we view it as the amount of "effort" selected by the agent. As in Mirrlees (1974) and Holmstrom (1979), we suppress θ and view x as a random variable

[1] This includes, for example, Holmstrom (1979), Shavell (1979), Harris and Raviv (1979), Gjesdal (1982), Baiman and Demski (1980), and Grossman and Hart (1983). Holmstrom, however, in the working paper predecessor to Holmstrom (1979) notes (footnote 7) the possibility of a nonconcave Pareto frontier owing to the incentive compatibility condition in the private information case. Thus, potential efficient *ex ante* randomization is admitted. The footnote was deleted in the published version of the paper.

conditioned on a with the conditional probability density $f(x|a)$, where for $a_1 > a_2$, $f(x|a_1)$ dominates $f(x|a_2)$ in the sense of first-degree stochastic dominance.

Consider first the case in which the principal can observe both the outcome and effort. The basic problem for the principal is to find a sharing rule, s, to divide x in a manner that provides the agent incentives to expend effort and that shares risk in an optimal fashion. We denote the principal's preferences over income as $u(x - s)$. The agent's preferences are denoted $v(s, a)$. The following assumptions regarding these preferences are typical:[2]

$$u' > 0, \qquad u'' \leq 0$$

$$v_1 > 0, \qquad v_{11} < 0, \qquad v_2 < 0, \qquad v_{22} \leq 0.$$

Thus, the principal is risk neutral or risk averse, and the agent is risk averse in income, but risk averse or neutral in effort.[3] The principal's problem can be stated as:

$$\max_{s(x,a),a} \int u(x - s(x, a))f(x|a)dx \tag{1}$$

subject to

$$\int v(s(x, a), a)f(x|a)dx \geq \bar{v},$$

where $\bar{v}$ is the agent's minimum utility (opportunity cost or market wage) required to ensure contract acceptance. The solution to this problem is first best; there is no conflict between risk sharing and incentives.

If the principal can observe only the outcome, however, the sharing rule can no longer be set both to share risk optimally and to force the agent to select a first-best effort level. This private information problem is:

$$\max_{s(x),a} \int u(x - s(x))f(x|a)dx \tag{2}$$

subject to

$$\text{(i)} \qquad \int v(s(x), a)f(x|a)dx \geq \bar{v}$$

$$\text{(ii)} \qquad a \in \underset{a' \in A}{\operatorname{argmax}} \int v(s(x), a')f(x|a')dx,$$

where A is the set of feasible effort levels. The second constraint indicates that the principal accounts for the optimizing behavior of the agent for any fee schedule selected and ensures a Nash equilibrium.

The strategies of the principal in (1) and (2) have been represented as pure; however, randomization may be desirable. We distinguish between *ex post* and *ex ante* randomization as follows. Consider a setting in which a wheel is spun after the action is chosen by the agent. The random signal (y) thus generated is used to determine the fee paid to the agent. This postaction device is termed *ex post* randomization and is depicted in Figure 1.

Contrast this with *ex ante* randomization in which the wheel is spun before act selection

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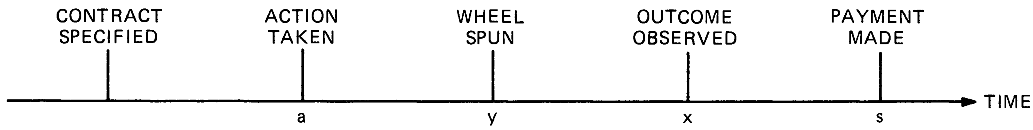
[2] We use the following convention in notation:

$$v_1 = \frac{\partial v}{\partial s}, \qquad v_{11} = \frac{\partial^2 v}{\partial s^2}, \qquad v_2 = \frac{\partial v}{\partial a}, \qquad v_{22} = \frac{\partial^2 v}{\partial a^2}, \qquad v_{12} = \frac{\partial^2 v}{\partial s \partial a}, \qquad \text{etc.}$$

[3] No assumption is made here regarding the concavity of the agent's utility function, since nonconcavity will prove important in some later results. Some definitions of multiattribute risk aversion do not rely on concavity (Richard, 1975), while others do (Kihlstrom and Mirman, 1974). Rarely, however, is concavity assumed in the agency literature. Of course, nonconcavities immediately suggest efficient randomization. As we show in Theorems 3 and 4, concavity is not sufficient to eliminate randomization in an agency setting.

FIGURE 1

EX POST RANDOMIZATION



by the agent. The initial formulation of the contract specifies two (or more) branches; the specific contract branch that is in effect depends on the random signal. See Figure 2.

Baiman (1982, p. 175) emphasizes the importance of the randomization issue:

> . . . the principal restricts his choice of payment schedules to the class of pure, nonrandomized, payment schedules. This seems intuitively reasonable since the agent and (usually) the principal are risk averse and therefore introducing any additional uncertainty by means of randomized payment schedules could only reduce utility. Accordingly, almost all of the agency research starts by implicitly restricting the payment schedule to be nonrandomized. However, if the Pareto efficient frontier is not concave then randomized payment schedules may be Pareto superior to nonrandomized ones.

Baiman points out that because of this restriction to nonrandomized strategies, many of the agency theory results may be based on suboptimal payment schedules. He notes, however, that Gjesdal (1981) has shown that additive separability of the agent's utility function in wealth and effort is a sufficient condition for the Pareto optimality of pure payment schedules. Consequently, Baiman concludes that the problem of randomized payment schedules is mitigated. As we show in the following sections, Gjesdal's demonstration applies to *ex post* randomization only.

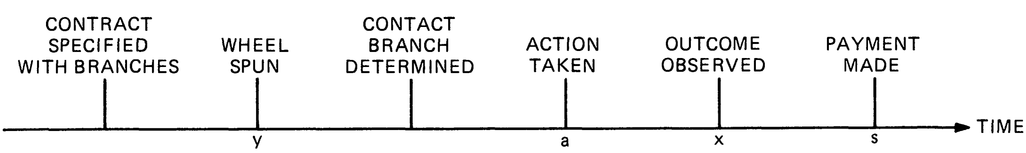
Holmstrom (1979) considers the case where $v(s(x), a)$ is additively separable: $v(s(x), a) = w(s(x)) - z(a)$.[4] Using this assumption, Holmstrom investigates the value of an additional signal, denoted y, in the private information case. He shows that y is valuable if and only if the following condition does *not* hold:

$$f(x, y|a) = g(x, y)h(x|a).$$ [5]

The construction of the proof implies that only *ex post* randomization is considered. The signal must follow the act, and only the fee is conditioned on y: $s(x, y)$. Thus, in this formulation, fee is the only choice variable over which randomization occurs.

FIGURE 2

EX ANTE RANDOMIZATION



[4] Keeney and Raiffa (1976, p. 232) state:

"The assumptions required for the justification of an additive utility function are rather restrictive. They allow for no interaction of the decision makers' preferences for various amounts of the two attributes. Often, we might expect the desirability of various amounts of one attribute to depend on the specified level of the other attribute. For instance, consider a farmer with preferences for various amounts of sunshine and rain because of the impact this will have on the season's crops. Here we might expect the farmer's preferences for various amounts of sunshine to be different depending on whether there had been only a little rain or much rain. Such an interaction of preferences cannot be expressed with the additive utility function."

[5] If Holmstrom's condition holds, the signal implies nothing about the action beyond what is already implicit in the outcome. The signal is, in such a case, not "informative." Holmstrom indicates that additive separability is unnecessary for the sufficiency part of this result.

Grossman and Hart (1983) construct the discrete analogue of Holmstrom's result for both additive and multiplicative separability of the agent's preferences. Again, only *ex post* randomization is considered.

Gjesdal (1982) also investigates circumstances under which *ex post* randomization is potentially desirable. He demonstrates that: (1) *ex post* randomization is never desirable if the agent's preferences are additively separable; and (2) where separability does not hold, *ex post* randomization can be beneficial if the incentive effect exceeds the risk effect. An example is provided where the agent's first-order condition is convex in s. Randomization then induces higher effort, and the benefits from the increase exceed the risk premium required by the agent.

Lambert (1981) identifies sufficient conditions for *ex ante* randomization to be inefficient in the full information setting where the agent's utility function is additively separable. Essentially, Lambert's result (consistent with our Theorem 1 if the agent is assumed to have a concave and additive separable utility function) is that stochastic increasing returns to scale must be present to make *ex ante* randomization efficient.

Thus, there has been considerable interest evidenced in the literature regarding the existence of efficient randomized contracting schemes. These schemes rely on nonconcavities in the Pareto surface which are generated by preference structures that are nonconcave or by production opportunities that exhibit increasing returns to scale. We formally characterize settings in which such nonconcavities can be exploited using (*ex ante*) randomized contracting strategies.

In the full information regime, efficient randomization is ruled out if preferences and opportunities are restricted to be concave. Somewhat surprisingly, these same restrictions imposed in a private information setting do not necessarily eliminate the possibility of efficient *ex ante* randomization. They do, however, render any *ex post* randomization inefficient, as shown by Holmstrom. We formally characterize private information situations in which *ex ante* randomization is efficient.

3. Full information case

■ Consider the problem described by (1), the full information scenario. If the principal is risk neutral (as we shall assume throughout), the solution is characterized by a fixed fee (not conditional on outcome) for the agent, so that the principal bears all risk. Randomization can dominate this pure strategy only if the Pareto frontier is nonconcave around the optimal (s^*, a^*) utility levels. We identify in Theorems 1 and 2 when this can occur.

We consider only the simplest randomization device: a signal, y, that takes on values y_1 or y_2, each with probability ½.[6] The signal is assumed independent of x. In addition, we consider a risk-neutral principal. If no arguments are shown for derivatives, it is assumed that they are evaluated at the first-best solution under consideration. All proofs are in the Appendix.

Theorem 1. Let (s^*, a^*) be a first-best solution such that a^* is an interior point. Assume $v_1 > 0$ and $v_{11} < 0$. If

$$\int xf_{aa}(x|a)dx > 1/v_1\left[\frac{(v_{12})^2}{v_{11}} - v_{22}\right] \tag{3}$$

holds at (s^*, a^*), then the solution can be improved by *ex ante* randomization.

[6] In fact, the *optimal* randomization device will consist of a signal that takes on only two values spanning the nonconcave portion of the Pareto frontier. The optimal device, however, will generally not arbitrarily assign equal probabilities to each of the signals. We do not attempt here to characterize optimal contracts, which will generally depend on the bounds on action or fee or on the nonconcave region of the Pareto frontier.

Remark. The theorem can easily be interpreted. Condition (3) requires stochastic increasing returns to effort $\left(\frac{\partial^2 Ex}{\partial a^2} > 0\right)$ *or* nonconcavity of the agent's utility function. Assuming that Ex is characterized by constant or decreasing returns to effort (Ex is concave), then the left-hand side of (3) is zero or negative. Since $v_1 > 0$, the right-hand side will be negative only if the term in brackets is negative, which can easily be seen to imply nonconcavity. If Ex is concave in a, an additively separable utility function renders *ex ante* randomization inefficient since $v_{11} < 0$, $v_{22} \leqq 0$ by assumption. Other functions, which display interactions between s and a, such as those which are multiplicatively separable, need not be concave (depending on the degree to which s and a act as substitutes or complements).

Next we provide a simple two-outcome example that illustrates the theorem.

Example 1. Assume there are two outcomes with associated probabilities:

x	$f(x\|a)$
100	$\frac{50 - a}{100}$
200	$\frac{50 + a}{100}$,

where $0 \leqq a \leqq 50$. Let $v = 2\sqrt{s}(150 - a)$ and $\bar{v} = 2048$. Under the above assumptions, the solution to (1) is $s^* = 64$, $a^* = 22$, with $Eu^* = 108$ and $Ev^* = 2048$.

Consider the randomized contract:[7]

$$\begin{cases} s = 41.53, & a = 43 \quad \text{if } y_1 \text{ is observed;} \\ s = 83.53, & a = 1 \quad \text{if } y_2 \text{ is observed.} \end{cases}$$

Then we obtain:

$$Eu = 109.47 \quad \text{and} \quad Ev = 2051.3301,$$

and *ex ante* randomization is valuable. This example is constructed such that Ex displays constant returns to effort $\left(\frac{\partial^2 Ex}{\partial a^2} = 0\right)$ and the agent's utility function is nonconcave $(v_{12}^2/v_{11} - v_{22} = -\sqrt{s}(2/(150 - a)) < 0)$ at (s^*, a^*).

Theorem 2. Assume that $v_2 < 0$, $v_{11} < 0$, $\int xf_a(x|a)dx > 0$ and

$$\frac{\int xf_{aa}(x|a)dx}{\int xf_a(x|a)dx} > \frac{1}{v_2}\left[v_{22} - \frac{(v_{12})^2}{v_{11}}\right] \tag{4}$$

at the first-best solution (s^*, a^*). Then *ex ante* randomization improves the solution.[8]

Remark. As in Theorem 1, the most reasonable circumstance in which Theorem 2 holds is where the agent's utility is nonconcave. If Ex is concave, the left-hand side ≤ 0; since $v_2 < 0$ by assumption, the term in brackets is again critical, and only nonconcavity will satisfy the inequality. Again, additive separability resolves the issue (assuming $v_{22} < 0$) if decreasing or constant returns to effort characterize the problem.

[7] This contract is obtained by taking $t = 21$ and $\rho = 1/300$ in the proof (see the Appendix).

[8] The proof of this theorem is similar to that of Theorem 1 and is available from the authors upon request.

Thus, the most reasonable circumstance in which *ex ante* randomization is efficient in the full information scenario is nonconcavity of the agent's utility function. There is, however, no reason *a priori* to eliminate interactions between effort and fee, which could yield such nonconcavity. As we have observed, the assumption of additive separability resolves the issue in the full-information case. Unfortunately, such a simple device is not sufficient in the private information setting.

4. Private information case

■ Consider the problem described by (2), the private information scenario. In this setting, optimal risk sharing has been shown to be impossible; the fee must depend on the outcome to induce the agent to expend effort. We use the following notational conveniences in Theorems 3 and 4:

$$B = \frac{\partial^2 Ev}{\partial a^2} = \int (v_{22} f + 2v_2 f_a + v f_{aa})$$

$$C = \frac{\partial^2 Ev}{\partial a \partial s} = \int (v_{12} f + v_1 f_a)$$

$$D = \frac{\partial^2 Ev}{\partial s^2} = \int v_{11} f.$$

Theorem 3. Let $s^*(x)$ be a private information solution where the agent's optimal action, a^*, is an interior point. Assume that the following two inequalities hold at $(s^*(x), a^*)$:

$$\int (x - s(x)) f_{aa} dx \geqq 0 \tag{5a}$$

$$C^2 \geqq BD \tag{5b}$$

with at least one inequality being strict. Then *ex ante* randomization improves the nonrandomized solution.

The conditions in Theorem 3 depend on the concavity of *expected* utilities of the principal and agent at the solution $(s^*(x), a^*)$. Unfortunately, concavity of v does not imply concavity of Ev because of the interactions of effort on both utility and probabilities. Thus, additive separability of v is *not* sufficient to eliminate the possibility of *ex ante* randomization, as the next example illustrates.

Example 2.

	x	$f(x \mid a)$
x_1	10	$\frac{3-a}{6}$
x_2	$\frac{638}{17}$	$\frac{3+a}{6}$,

where $0 \leqq a \leqq 3$. Let $v(s, a) = -\frac{1}{s} - \frac{a^2}{170}$ and let $\bar{v} = -9/85. = -.1058824$. It can be shown that $s^*(x_1) = 5$, $s^*(x_2) = 17$, and $a^* = 2$, with $Eu^* = 17.941176$ and $Ev^* = -.1058824$. Consider the randomized contract:[9]

[9] This contract is obtained by taking $t = 1.3$ in the proof (see the Appendix).

$$\begin{cases} s(x_1) = 6.3, & s(x_2) = 18.3 \quad \text{if } y_1 \text{ is observed;} \\ s(x_1) = 3.7, & s(x_2) = 15.7 \quad \text{if } y_2 \text{ is observed.} \end{cases}$$

Using (A5) in the proof, we obtain $a^*|y_1 = 1.4745425$ and $a^*|y_2 = 2.9264934$. Thus, we have $Eu = 18.460164$ and $Ev = -.1052507$.

Several observations can be made regarding this example. First, as in Example 1, constant returns to effort are assumed so that condition (5a) in Theorem 3 is an equality. Second, additive separability is assumed for the agent. But the *expected* utility function of the agent is constructed to be nonconcave at $(s^*(x), a^*)$, as some tedious computations show. (The construction resulted in the precise nature of the parameters in the utility function.) Note, however, that the utility function is concave, so that it is the interaction between the density function and utility function that gives rise to the nonconcavity.[10]

In addition, Example 2 confirms observations (b) and (c) in the Introduction. Unless one can observe the act, which we assume is impossible for the principal in the private information case, the randomized contract cannot be distinguished from one conditioned on a signal that is informative in the sense of Holmstrom. In each case, we observe a contract, $s(x, y)$, that depends on both the outcome and the signal. If the agent's act occurs before y is observed, then by Holmstrom's and Gjesdal's results, the signal must be informative (if the agent's preferences are additively separable) or preferences must be such that *ex post* randomization is efficient. If the agent's act occurs after y is observed, then clearly y is not necessarily informative. The observer does not know whether the act occurs before or after the signal, which is the crucial distinction between *ex ante* and *ex post* randomization.

Perhaps more important and more confounding from an empirical perspective, the (purely random) signal and outcome are correlated. This is straightforward since the probability distribution over outcomes is parameterized by the act, which is influenced by the signal. In Example 2, if y_1 is observed, the outcome, x, is on average 30.5, whereas if y_2 is observed, the average outcome is 37.2. Thus the signal, which is purely random, has the *appearance* of being influenced by the action of the agent, when, in fact, the opposite is occurring.

In no way do our results disagree with Holmstrom's. Rather, the direction of causality is different in the models, and as a result, there is observational equivalence between the two signals considered. In both cases the only observables are s, x, and y. In Holmstrom's case, s depends on y because the distribution of y is influenced by a, and thus y is informative. In our case a is influenced by y through s. But unless we know the timing of the act relative to the signal, we cannot distinguish between these situations.

The problem of inferential error naturally arises in such a setting. Holmstrom's result implies that if a signal is utilized in a contract, then the signal must be associated with the agent's act. It may be, however, that the signal-generating system is being used merely as a randomizing device.[11]

[10] This result creates difficulties for some solution techniques found in the literature. Grossman and Hart (1983), for example, consider a technique whereby the principal solves the agency problem in a two-stage process: first, the least-cost method of implementing a particular act is determined; then, the optimal act is found. If randomization is efficient, such a two-stage process is inappropriate, since no fixed act is optimal. The technique is valuable, however, inasmuch as the optimal randomization scheme involves randomizing between two points on the Pareto frontier, each of which is a nonrandomized solution.

[11] Many contracts that depend in part on general market conditions may contain at least a component of *ex ante* randomization. For example, stock appreciation rights and stock options provided to management are partially dependent on market movements that occur before investment decisions. Similarly, the evaluation of management based on the performance of the firm as measured by accounting income may contain a random component if demand for the firm's products depends on general economic conditions. These apparently random components could be easily abstracted out of remuneration plans if desirable.

Theorem 3 is related to Theorem 1 in the sense that nonconcavities are exploited by randomization. Similarly, Theorem 4 is the private information analogue of Theorem 2.

Theorem 4. Let $s^*(x)$ be a private information solution with an interior optimal action a^*. If there exists a number ρ such that:

$$2\rho\left[1+\frac{C}{B}\int(x-s)f_a\right]<\frac{C^2}{B^2}\int(x-s)f_{aa} \tag{6a}$$

$$2\rho\left[\int v_1 f-\frac{C}{B}\int(vf_a+v_2 f)\right]>\frac{C^2}{B}-D \tag{6b}$$

hold at $(s^*(x), a^*)$, then *ex ante* randomization can improve $s^*(x)$.[12]

5. Conclusions

■ *Ex ante* randomization is potentially desirable in an agency setting. When all variables are observable (the full information case), interactions exhibited in the agent's preferences (creating nonconcavities in the agent's utility function) or stochastic increasing returns to scale can give rise to profitable randomized contracts. In the private information case, we require binding commitments on the part of both the principal and agent so that, in particular, the agent cannot leave the firm following observation of an adverse (random) signal. The signal might specify a branch of the contract which does not provide the agent with his reservation expected utility. The agent must precommit not to leave the firm should this occur. Otherwise, the benefits of randomization cannot be exploited. This is similar to the case investigated by Harris and Raviv (1979) in which the agent observes the state before the act selection, and the agent cannot "renege" and accept alternative employment in the case of adverse state occurrence.

Observed contracts may contain randomization. Unless the act itself can be observed, it will not be possible to distinguish random signals from "informative" signals. More problematic is the fact that random signals will appear to be informative since they will be correlated with the outcome. There does not seem to be any easy solution to the problem.

Appendix

■ Proofs of Theorems 1 and 3 follow.

Proof of Theorem 1. Choose ρ such that

$$-\frac{1}{2}\int xf_{aa}(x|a)dx<\rho<\frac{1}{2v_1}\left[v_{22}-\frac{(v_{12})^2}{v_{11}}\right]. \tag{A1}$$

Given a small t, set

$$\epsilon=-\frac{v_{12}}{v_{11}}t. \tag{A2}$$

Consider the offer

$$\begin{cases}(s^*-\rho t^2+\epsilon,\ a^*+t)\text{ if }y_1\text{ is observed;}\\(s^*-\rho t^2-\epsilon,\ a^*-t)\text{ if }y_2\text{ is observed.}\end{cases}$$

Consider, in turn, the expected utility of the principal and agent with this new contract. First, the principal's expected utility is

$$\begin{aligned}Eu(t)&=\tfrac{1}{2}Eu|y_1+\tfrac{1}{2}Eu|y_2\\&=\frac{1}{2}\int x[f(x|a^*+t)+f(x|a^*-t)]dx-(s^*-\rho t^2).\end{aligned}$$

[12] The proof of Theorem 4 is similar to that of Theorem 3 and is available from the authors upon request.

Using a Taylor series expansion around $t = 0$, we have

$$Eu(t) = Eu(0) + Eu'(0)t + Eu''(0)\frac{t^2}{2} + \dots,$$

where

$$Eu(0) = Eu^*$$

$$Eu'(t) = \frac{1}{2}\int x\{f_a(x|a^* + t) - f_a(x|a^* - t)\}dx + 2\rho t$$

$$tEu'(0) = 0$$

$$Eu''(t) = \frac{1}{2}\int x\{f_{aa}(x|a^* + t) + f_{aa}(x|a^* - t)\}dx + 2\rho$$

$$\frac{t^2}{2}Eu''(0) = t^2\left\{\frac{1}{2}\int xf_{aa} + \rho\right\}$$

$$Eu(t) - Eu^* = t^2\left\{\frac{1}{2}\int xf_{aa}dx + \rho\right\} + \text{higher order terms } (t^3, \text{etc.})$$

$$\frac{Eu(t) - Eu^*}{t^2} = \left\{\frac{1}{2}\int xf_{aa}dx + \rho\right\} + \text{higher order terms}$$

$$\lim_{t \to 0}\frac{Eu(t) - Eu^*}{t^2} = \frac{1}{2}\int xf_{aa}dx + \rho > 0^{13} \tag{A3}$$

(since higher order terms $\to$ 0).

A similar procedure is used to examine the agent's expected utility under the new randomized contract. Since

$$\epsilon = -\frac{v_{12}}{v_{11}}t \qquad \left(\text{where } \frac{v_{12}}{v_{11}} \text{ is evaluated at } (s^*, a^*)\right)$$

$$Ev(t) = \frac{1}{2}\left\{v\left[\left(s^* - \rho t^2 - \frac{v_{12}}{v_{11}}t\right), a^* + t\right]\right\} + \frac{1}{2}\left\{v\left[s^* - \rho t^2 + \frac{v_{12}}{v_{11}}t, a^* - t\right]\right\}.$$

A Taylor series expansion about $t = 0$ yields:

$$Ev(t) = Ev(0) + Ev'(0)t + Ev''(0)\frac{t^2}{2} + \dots;$$

$$Ev(0) = Ev^*;$$

$$Ev'(t) = \frac{1}{2}\left\{v_1\left(-2\rho t - \frac{v_{12}}{v_{11}}\right) + v_2 + v_1\left(-2\rho t + \frac{v_{12}}{v_{11}}\right) - v_2\right\};$$

$$tEv'(0) = 0;$$

$$Ev''(t) = \frac{1}{2}\left\{v_{11}\left(-2\rho t - \frac{v_{12}}{v_{11}}\right)^2 + v_{12}\left(-2\rho t - \frac{v_{12}}{v_{11}}\right) - 2\rho v_1 + v_{12}\left(-2\rho t - \frac{v_{12}}{v_{11}}\right) + v_{22}\right.$$
$$\left. + v_{11}\left(-2\rho t + \frac{v_{12}}{v_{11}}\right)^2 - v_{12}\left(-2\rho t + \frac{v_{12}}{v_{11}}\right) - 2\rho v_1 - v_{12}\left(-2\rho t + \frac{v_{12}}{v_{11}}\right) + v_{22}\right\};$$

$$\frac{t^2}{2}Ev''(0) = \frac{t^2}{4}\left\{2\left(v_{22} - \frac{v_{12}^2}{v_{11}}\right) - 4\rho v_1\right\}$$

[13] A critical requirement of this formulation is that the limit (as $t \to 0$) of the sum of terms in the Taylor series expansion of order greater than 2 is zero. That this will be true can be easily seen by restating the sum in remainder form:

$$R_N(t) = \frac{Eu^{(N)}(\delta)}{N!}t^N,$$

where δ is some point in the interval $(0, t)$. Then $\frac{Eu^{(N)}(\delta)}{N!}$ is bounded and $\lim_{t \to 0}\frac{t^N}{t^2} = 0$. (See Hildebrand (1976, p. 122)).

$$= \left\{\frac{1}{2}\left(v_{22} - \frac{(v_{12})^2}{v_{11}}\right) - \rho v_1\right\}t^2;$$

$$Ev(t) - Ev^* = \left\{\frac{1}{2}\left(v_{22} - \frac{(v_{12})^2}{v_{11}}\right) - \rho v_1\right\}t^2 + \text{terms of higher order } (t^3, \text{ etc.});$$

$$\lim_{t\to 0} \frac{Ev(t) - Ev^*}{t^2} = \frac{1}{2}\left(v_{22} - \frac{(v_{12})^2}{v_{11}}\right) - \rho v_1 > 0 \tag{A4}$$

(since higher order terms go to zero).

Thus, we have the conclusion by (A3) and (A4). *Q.E.D.*

Proof of Theorem 3. For a small t, consider the contract

$$\begin{cases} s^* + t \text{ if } y_1 \text{ is observed;} \\ s^* - t \text{ if } y_2 \text{ is observed.} \end{cases}$$

If y_1 is observed, the agent then solves:

$$\int v_2(s^* + t, a)f(x|a)dx + \int v(s^* + t, a)f_a(x|a)dx = 0. \tag{A5}$$

(We assume an interior solution.) Using a Taylor series expansion, note that

$$a(t) = a(0) + a'(0)t + a''(0)\frac{t^2}{2} + \dots,$$

where $a' = \frac{da}{dt}$. Implicitly differentiate (A5) with respect to t:

$$\int \left(v_{12} + v_{22}\frac{da}{dt}\right)f + \int v_2 f_a \frac{da}{dt} + \int \left(v_1 + v_2\frac{da}{dt}\right)f_a + \int v f_{aa}\frac{da}{dt} = 0$$

$$\frac{da}{dt} = -\frac{\int (v_{12}f + v_1 f_a)}{\int (v_{22}f + 2v_2 f_a + v f_{aa})}.$$

Since $a(0) = a^*$, we have

$$a^*|y_1 = a^* - \frac{\int (v_{12}f + v_1 f_a)}{\int (v_{22}f + 2v_2 f_a + v f_{aa})}t + (\text{terms of degree} \geqq 2). \tag{A6}$$

$a^*|y_2$ can be found in a similar manner:

$$a^*|y_2 = a^* + \frac{\int (v_{12}f + v_1 f_a)}{\int (v_{22}f + 2v_2 f_a + v f_{aa})}t + (\text{terms of degree} \geqq 2). \tag{A7}$$

Using the same techniques as in Theorem 1, some calculations then yield

$$\lim_{t\to 0}\frac{Eu(t) - Eu^*}{t^2} = \frac{1}{2}\left\{\frac{\int (v_{12}f + v_1 f_a)}{\int (v_{22}f + 2v_2 f_a + v f_{aa})}\right\}^2 \int (x - s(x))f_{aa}$$

$$\geqq 0 \text{ by (5a);}$$

$$\lim_{t\to 0}\frac{Ev(t) - Ev^*}{t^2} = \frac{1}{2}\left\{\int v_{11}f - \frac{\left[\int (v_{12}f + v_1 f_a)\right]^2}{\int (v_{22}f + 2v_2 f_a + v f_{aa})}\right\}$$

$$\geqq 0 \text{ by (5b).}$$

Since at least one of the above inequalities is strict by assumption, this completes the proof. *Q.E.D.*

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