Coordination in the Static and the Dynamic*

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Abstract

In this paper, we analyse three factors that are crucial in determining whether equilibrium is unique in coordination games with incomplete information. We show that a unique equilibrium exists if there is sufficiently (i) large uncertainty about the common component of agents' payoffs; (ii) small degree of strategic uncertainty; and (iii)large differences in agents' payoffs. We call these three factors **fundamental uncertainty, strategic uncertainty** and **heterogeneity**, respectively.

To show the trade-offs among the three factors, we construct a dynamic model where information is released gradually and decisions are made sequentially. The dynamic model demonstrates that gradual release of information combined with sequential choice facilitates unique equilibrium selection. The dynamic model allows the agents to make decision subject to a greater degree of fundamental uncertainty while reducing the strategic uncertainty. Hence unique equilubrium selection obtains in the dynamic model for a set of parameters which yields multiple equilibria if the model is formulated as a static one.

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1 Introduction

The difficulty of expectation formation in coordination games and its implication on the selection of equilibrium have been discussed frequently in the recent game theory literature. The multiplicity of equilibria is disturbing since when there are multiple equilibria, predicting which equilibrium is actually played is beyond the theory, rendering the theory obsolete.

The literature on global games (see Carlsson and van Damme (1994), Morris and Shin (2001)) addresses the issue of equilibrium selection relying on incomplete information. When agents have private information which is slightly distinct from each other, they can use the information as a coordination device and arrive at a unique equilibrium. As subsequent works in the area show, (see Hellwig (2001) and Morris and Shin (2002)) the equilibrium selection in global games depends on the interactions among a few components including how precise the private information is relative to the public information and how diverse the agents are in terms of the payoff from a particular action choice. The current paper attmepts to clarify the equilibrium selection mechanism focusing on these components and exploit the mechanism in a dynamic context to obtain unique equilibrium slection even if the combination of the same parameters in a static context implies the existence of multiple equilibria.

Consider a game where the agent's payoff depends on unknown state of nature as well as strategies chosen by all agents. Moreover the payoff from an action depends on how many agents coordinate on it while the payoff from the other action is fixed independent of the against the other. A simple intuition suggests that there is a unique equilibrium when individual-specific terms in agents' utility functions are more important (in some sense) than the term subject to the coordination.

This intuition is broadly correct, but misses much of the detail of the situation. We show that a unique equilibrium exists if there is sufficiently (i) large uncertainty about the common component of agents' payoffs; (ii) small degree of strategic uncertainty; and (iii)large differences in agents' payoffs. We call these three factors **fundamental uncertainty, strategic uncertainty** and **heterogeneity**, respectively. In order for multiple equilibria to exist, there must be a sufficiently large expected co-ordination effect between agents. It is easy to understand why heterogeneity decreases the expected co-ordination effect, since payoff heterogeneity makes agents behave idiosyncratically. When there is a large degree of fundamental uncertainty at the time of decision making, the effect of coordination success would be of secondary concern since most of the variation in the expected payoff comes from the fundamental uncetainty. In contast if the success of coordination has a dominant effect in the determination of the individual payoff, multiple equilibria more likely and we capture the payoff effect of the coordination as the strategic uncertainty.

Recently there have been many papers which address related issues. The literature on global games which have grown very rapidly relies on incomplete information to obtain unique equilibrium selection for coordination game. Carlsson and van Damme (1994) was taken further by Morris and Shin (1998) for the explanation of currency crisis. Morris and Shin (2001) provides an overview of the literature so far. This paper differs from the global game literature in that we do not rely on the asymmetric information among agents.

Burdzy, Frankel and Pauzner (2001), Frankel and Pauzner (2000), and Herrendorf, Valentinyi, and Waldman (2000) introduce heterogeneity among agents to have the same effect on the equilibrium selection. However they do not allow fundamental uncertainty on the underlying stochastic parameter, which is the crucial component in our model.

In the literature on industrial economics, the issue of path dependence has discussed for a long time. Indeed Farrell and Saloner (1985) construct a model where dynamic coordination takes place ina similar fashion to the present paper. They are mainly interested in producing inefficient "lock-in" and they do not allow gradual release of information. Their model can be regarded as a dynamic version of global game where agents have asymmetric information and move in a sequential fashion.

In section 2, a static model is developed that identifies the three factors and establish the necessary and sufficient condition for unique equilibrium selection. Section 3 extends the static analysis to two periods; the three factors are allowed to change over time (e.g., due to learning about an underlying state) in order to gain further understanding about their interaction. Section 4 analyses a particular property of the dynamic equilibrium, that is, the path dependence and the final section concludes.

2 Static Model

Let the state of the world be denoted $\theta \in \mathbf{R}$ which is not observed by agents; all agents have a common prior on θ which is normally distributed with mean μ_0 and variance σ_0^2. The agents receive the same noisy signal x of the true state, where $X = \theta + \epsilon$ and ϵ is assumed to be normally distributed with zero mean and variance $\sigma_\epsilon^2 > 0$.[1] The standard properties of normal distributions imply that

[1]We take the notational convention that a Roman alphabet denoting a random variable is written in upper-case and its realization is written in lower-case.

the agents' posterior is normally distributed with mean $\mu = (\sigma_\epsilon^2 \mu_0 + \sigma_0^2 x)/(\sigma_0^2 + \sigma_\epsilon^2)$ and variance $\sigma^2 = \sigma_0^2 \sigma_\epsilon^2/(\sigma_0^2 + \sigma_\epsilon^2)$.

There is a continuum of agents of total mass 1, represented by the unit interval $[0, 1]$. Agents must choose an action from a binary action space, denoted $\{0, 1\}$. Choosing action 0 guarantees the agent zero payoff. On the other hand, the utility from choosing action 1 consists of *i)* ζ, which is an idiosyncratic parameter in the agent's utility, *ii) the cost of choosing the action,* which is normalized to 1, and *iii)* γ, which depends on the occurrence of a particular event. The first component, ζ, represents the heterogeneity among agents and is assumed to be uniformly distributed on the interval $[0, \beta]$, $\beta > 0$, throughout the population of agents. The third component, $\gamma > 0$, represents the benefit of successful coordination: the agent receives it if the sum of the random state, θ, and the number of agents choosing action 1, α, is sufficiently large (greater than some parameter $D > 0$); otherwise he receives zero from this component. Hence choosing action 1 yields the utility given as:

$$U_\zeta(\theta, \alpha) = \begin{cases} \zeta - 1 + \gamma & \text{if} \quad \theta + \alpha \geq D, \\ \zeta - 1 & \text{if} \quad \theta + \alpha < D. \end{cases} \tag{1}$$

The additive separability in equation (1) and the uniform distribution of ζ are not substantive assumptions. The one important feature that is captured simply by this formulation is that there is heterogeneity in the population of agents.

Given α, write the expected utility of an agent who chooses action 1 on receiving a signal X as

$$\mathbb{E}[U_\zeta(\theta, \alpha)|X] = \zeta - 1 + V(X; \alpha, \sigma) \tag{2}$$

where

$$V(X; \alpha, \sigma) \equiv \mathbb{E}[\gamma \cdot \mathbf{1}_{\{\theta+\alpha \geq D | X \geq 0\}}], \tag{3}$$

where $\mathbf{1}_{\{.\}}$ is the indicator function. It is easy to see that $V(X; \alpha, \sigma)$ is increasing in X and α. Note that the function $V(X; \alpha, \sigma)$ is also parameterized by the standard deviation of the posterior (equivalently its variance), σ, which is determined by the variance of the signal. This notation facilitates our analysis where the fundamental uncertainty, represented by the posterior variance, affects the decision making. Indeed we shall show that the variance of the posterior distribution plays a crucial role for the unique equilibrium selection.

The timing of the game is that all agents receive the same random signal X at $t = 0$, and then simultaneously choose an action at $t = 1$.

One story supporting this model is as follows. A firm operates with an existing debt of D which has to be serviced at the end out of the firm's profit. If the profit is less than debt service requirement, then the firm goes bankrupt. Profit is earned from selling to consumers; in addition, random shocks affect the firms' profit. Each consumer has unit demand, and gains additional utility if the firm is not bankrupt at the end (this utility may come from e.g., continued availability of parts after purchase).[2] A unit mass of consumers decides whether to purchase the firm's good. With a fraction α buying, total sales are α. The firm charges a fixed price, normalized to 1. Hence the firm's realized profit, before servicing debt, is $\theta + \alpha$.

A similar story can be told of A *pure strategy* for an agent is a mapping from its type ζ and the signal X to the binary action space $\{0, 1\}$; i.e., $p : [0, \beta] \times \mathbf{R} \to \{0, 1\}$. Agents form a posterior based on the prior and the signal according to Bayes' rule. A Bayesian equilibrium (in pure strategies) is a set of pure strategies such that each agent's strategy maximizes its expected utility given the strategies of all other agents and its Bayesian posterior.

The equilibrium of the model is defined by the optimizing behavior of the agents, which is represented by the strategy choice, together with consistency condition on their beliefs, which is represented by the expected proportion of agents who choose action 1.

Definition 1 *The agents' choices $p(\zeta, \alpha)$ and the beliefs on the state θ constitute a Bayesian equilibrium of the game if*

1. *for $\zeta \in [0, \beta]$, $p(\zeta, \alpha) = \mathbf{1}_{\{\mathbb{E}[U_\zeta(\theta,\alpha)|X] \geq 0\}}$,[3] and*
2. *$\alpha(X) = \int_0^\beta p(\zeta, X) \frac{1}{\beta}\, d\zeta$.*

To determine the equilibrium $\alpha(X)$, note that for given x and α, all agents with $\beta \geq \zeta \geq -V(x; \alpha, \sigma)$ have a non-negative expected utility and so choose action 1. This observation implies that $\alpha(X)$ is determined as

$$\alpha(X) = \begin{cases} 0 & \text{if } \; -V(X; o, \sigma) \geq \beta, \\ 1 & \text{if } \; -V(X; 1, \sigma) \leq 0, \\ \frac{\beta + V(X;\alpha,\sigma)}{\beta} \in (0, 1) & \text{otherwise} \end{cases} \tag{4}$$

[2] The model can be applied to various settings other than the current case of durable foods. For instance, a factor supplier might be concerned about the financial viability of a firm and willing to invest in the relationship only if the firm is likely to survive. The concern indicates the possibility of coordination difficulty as modeled here.

[3] This condition follows from the fact that optimization requires choosing action 1 if and only if its payoff is positive.

From the observation it is immediate that there is unique equilibrium for all realization of X if $\alpha(X)$ is monotonically increasing. The following proposition provides the necessary and sufficient condition under which there is a unique equilibrium.

Proposition 1 *There is a unique equilibrium if and only if*

$$\frac{\partial V(X;\alpha,\sigma)}{\partial \alpha} \leq \beta \tag{5}$$

for all $X \in \mathbf{R},\ \alpha \in [0,1]$, *and* $\sigma > 0$.

Proof. Totally differentiating the last line of the implicit equilibrium relation (4), we get

$$\frac{d\alpha(X)}{dX} = \frac{\partial V(X;\alpha,\sigma)/\partial X}{\beta - \partial V(X;\alpha,\sigma)/\partial \alpha}. \tag{6}$$

Note that

$$\begin{aligned}\frac{\partial V(X;\alpha,\sigma)}{\partial X} &= \frac{\partial \mathbb{E}[\gamma \cdot \mathbf{1}_{\{\theta+\alpha \geq D|X \geq 0\}}]}{\partial X} \\ &= \gamma \cdot \frac{\partial \Pr[\theta+\alpha \geq D|X]}{\partial X}\end{aligned}$$

Since θ is normally distributed, we can further manipulate the term having the probability on the last line as:

$$\begin{aligned}\frac{\partial \Pr[\theta+\alpha \geq D|X]}{\partial X} &= \frac{\partial}{\partial X}\int_{\frac{D-\alpha-\mu(X)}{\sigma}}^{\infty} \phi(\theta)d\theta \\ &= \frac{\partial \mu(X)}{\partial X}\cdot\frac{1}{\sigma}\cdot\phi\left(\frac{D-\alpha-\mu(X)}{\sigma}\right)\end{aligned}$$

where $\mu(X)$ is the mean of the state conditional on the signal X and $\phi(\cdot)$ is standard normal density.

Since $\frac{\partial \mu(X)}{\partial X} > 0$ and the standard normal density function has positive vale everyhwfere, the deirvative of $V(X;\alpha,\sigma)$ with respect to X is positive as desired. It follows that $\alpha(X)$ is monotonically increasing if $\beta - \partial V(X;\alpha,\sigma)/\partial\alpha \geq 0$. and the sufficient part follows.

To prove the necessary part, suppose that there is X for which

$$\frac{\partial V(X;\alpha,\sigma)}{\partial \alpha} > \beta.$$

There are multiple equilibria if the equilibrium $\alpha(X)$ is *backward bending.* Consider its inverse function $X(\alpha)$. If $X(\alpha)$ is increasing and then decreasing, then $\alpha(X)$ is backward bending. Rewriting (6), we get

$$\frac{dX(\alpha)}{d\alpha} = \frac{\beta - \partial V(X;\alpha,\sigma)/\partial\alpha}{\partial V(X;\alpha,\sigma)/\partial X}.$$

If $\frac{\partial V(X;\alpha,\sigma)}{\partial\alpha} \geq \beta$ for some X, then there is a range of X over which the numerator is smaller than 0 and thus α is backward-bending. It follows that there are multiple equilibria if $\frac{\partial V(X;\alpha,\sigma)}{\partial\alpha} \geq \beta$. The proof is complete. ■

We can interpret the necessary and sufficient condition in Proposition 1 in terms of the three components. To do this we first demonstrate a trade-off between the fundamental uncertainty and the strategic uncertainty in the following lemma.

Lemma 1 *If* $\sigma \leq \sigma'$, *then*

$$\frac{\partial V(X;\alpha,\sigma)}{\partial\alpha} \geq \frac{\partial V(X;\alpha,\sigma')}{\partial\alpha}. \tag{7}$$

Proof. Note that

$$\begin{aligned}\frac{\partial V(X;\alpha,\sigma)}{\partial\alpha} &= \gamma\frac{\partial}{\partial\alpha}\int_{\frac{D-\alpha-\mu(X)}{\sigma}}^{\infty}\phi(\theta)d\theta \\ &= \gamma\cdot\frac{1}{\sigma}\cdot\phi\left(\frac{D-\alpha-\mu(X)}{\sigma}\right)\end{aligned}$$

Suppose $D-\alpha-\mu(X) \geq 0$. If $\sigma \leq \sigma'$, then $\frac{D-\alpha-\mu(X)}{\sigma} \geq \frac{D-\alpha-\mu(X)}{\sigma'}$. Since the standard normal density function is decreasing to the right of zero,

$$\gamma\cdot\frac{1}{\sigma}\cdot\phi\left(\frac{D-\alpha-\mu(X)}{\sigma}\right) \leq \gamma\cdot\frac{1}{\sigma}\cdot\phi\left(\frac{D-\alpha-\mu(X)}{\sigma'}\right)$$

The case where $D-\alpha-\mu(X) \leq 0$ can be proved similarly except that the standard normal density function is increasing to the left of zero. The two cases together prove the claim. ■

The necessary and sufficient condition in Proposition 1 crucially depends on the merginal contribution of the proportion of agents who coordinate on action 1. The lemma implies that the marginal contribution gets smaller when the agents have noisier information about the state. This is the trade-off we exploit to obtain unique equilibrium selection in the dynamic model. The intuition behind the lemma is made explicit in the following corollary where we obtain a characterization of the conditions for unique equilibrium selection. We first provide a more precise definition of the following three factors

The necessary and sufficient condition (5) allows a characterization of the equilibrium uniqueness in terms of three factors in the game:

1. **Heterogeneity** measured by β.

2. **Fundamental uncertainty** measured by σ.

3. **Strategic uncertainty** measured by γ.

The heterogeneity is captured by β in the model. Burdzy, Frankel, and Pauzner (2000) and Herendorf, Valentinyi, and Waldman (2000) exploited this form of heterogeneity to obtain unique equilibrium selection in a model of endogenous growth. It measures how different preference agents may have. The fundamental uncertainty is measured by the standard deviation of the posterior; it captures how much uncertainty the agents have after the receipt of signals. If this uncetainty vanishes, the game is a pure coordination game. Finally the strategic uncetainty is represented by γ which is the utility effect of coordinatin success. We observe that the size of the population is another component of the model which can potentially determine the strategic uncertainty. If there are more agents who may buy the product of the firm, the firm's profit may fluctuate more dueto the coordination among the agents. In our model we take the population size as normalized quantity and any effect due to the significance of the market size relative the debt is capture by the utility effect of coordinatin success.

Corollary 1 *(i) For any given γ and σ, there exists $\beta^* > 0$ such that equilibrium is unique if $\beta \geq \beta^*$.*

(ii) For any given β and γ, there exists $\sigma^* \geq 0$ such that equilibrium is unique if $\sigma \geq \sigma^*$.

(iii) For any given β and σ, there exists $\gamma^* > 0$ such that equilibrium is unique if $\gamma \leq \gamma^*$.

Proof. From the intermediate step in the calculation in the proof of Lemma 1, we have

$$\frac{\partial V(X;\alpha,\sigma)}{\partial \alpha} = \gamma \cdot \frac{1}{\sigma} \cdot \phi\left(\frac{D-\alpha-\mu(X)}{\sigma}\right)$$

It follows that depending on the ralization of X,

$$0 \leq \frac{\partial V(X;\alpha,\sigma)}{\partial \alpha} \leq \gamma \cdot \frac{1}{\sigma} \cdot \frac{1}{\sqrt{2\pi}} \tag{8}$$

since the standard normal density function has values bewteen 0 and $\frac{1}{\sqrt{2\pi}}$.

We can rewrite the necessary and sufficient condition in Proposition 1 as

$$\beta \geq \gamma \cdot \frac{1}{\sigma} \cdot \frac{1}{\sqrt{2\pi}}. \tag{9}$$

For a given values of γ and σ, let $\beta^* = \gamma \cdot \frac{1}{\sigma} \cdot \frac{1}{\sqrt{2\pi}}$. Then for any $\beta \geq \beta^*$, the necessary and sufficient condition is satisfied and thus there exists a unique equilibrium, which proves part i). Part ii) and iii) are proved similarly. We omit the detail. ■

3 Dynamic Model

We construct a dynamic model based on the static one to demonstrate how the three factors—heterogeneity, fundamental uncertainty and strategic uncertainty—interact over time to resolve the difficulty of coordination. We analyse the interaction of the factors in a two period model in which information is received over time.

In each period t, a signal X_t is drawn and observed by all agents. The signal X_t in period t about the state θ is determined by $X_t = \theta + \epsilon_t$; $\{\epsilon_t\}_{t\in\{1,2\}}$ are drawn independently from the same normal distribution with zero mean and variance σ_ϵ^2. The common prior over θ is normally distributed with mean μ_0 and variance σ_0^2. Hence initially $\{X_t\}_{t\in\{1,2\}}$ are distributed normally with mean μ_0 and the variance $\sigma^2 = \sigma_0^2 + \sigma_\epsilon^2$. The information sets Ω_t for all agents at time t are, therefore, $\Omega_1 = \{X_1\}$ and $\Omega_2 = \{X_1, X_2, \alpha_1\}$. Notice that α_2 and X_2 are all random variables at the beginning of period 1 (after the signal X_1 is observed). In period 2, the signal, X_1, and fraction of agents choosing 1 in the first period, α_1, have been observed, and the signal X_2 received.

In the dynamic model with two periods, choosing action 1 yields the utility given as:

$$U_\zeta(\theta,\alpha_1,\alpha_2) = \begin{cases} \zeta - 1 + \gamma & \text{if} \quad \theta + \alpha_1 + \alpha_2 \geq D, \\ \zeta - 1 & \text{if} \quad \theta + \alpha_1 + \alpha_2 < D. \end{cases} \tag{10}$$

A pure strategy for an agent in the first period is a mapping from its type ζ and the signal X_1 to the binary action space $\{0, 1\}$; i.e., $s : [0, \beta] \times \mathbf{R} \to \{0, 1\}$. A pure strategy for an agent in the second period is a mapping from its type ζ, the signals X_1, X_2 and the fraction of agents who purchased in the first period, to the binary action space $\{0, 1\}$; i.e., $s : [0, \beta] \times \mathbf{R}^2 \times [0, 1] \to \{0, 1\}$.

A Bayesian perfect equilibrium (in pure strategies) is a set of pure strategies and set of beliefs such that (i) beliefs are determined by Bayes' rule and the equilibrium strategies; (ii) each agent's strategy maximizes its expected utility given the subsequent strategies of all other agents and its beliefs. It can be obtained with minor modification from the Bayesian equilibrium defined in the previous section.

In the following we take 3 different move orders / information structures to highlight the interaction of the factors.

3.1 Simultaneous Information and Choice

With the sequential set-up explained, we first analyse the benchmark case where two signals are received simultaneously and a mass 2 of agents with idiosyncratic valuations ζ distributed on $[0, \beta]$ choose simultaneously whether to purchase or not. Hence $\Omega_1 = \Omega_2 = \{X_1, X_2\}$. This case is identical to the static model explained in the previous section except for additional structure on the agent's decision making procedure.

Given two independently drawn signals x_1 and x_2, agents' common posterior on the state is normally distributed with mean $\mu_2 = (\sigma^2\mu_0 + \sigma_0^2(x_1 + x_2))/(\sigma^2 + 2\sigma_0^2)$ and variance $\sigma_2^2 = \sigma_0^2\sigma^2/(\sigma^2 + 2\sigma_0^2)$. Expected utility of a type-ζ agent from choosing action 1 is therefore

$$\mathbb{E}[U_\zeta(\theta, \alpha)] = \zeta - 1 + \gamma\mathbb{P}[\theta + 2\alpha \geq D | X_1, X_2],$$

(For comparison with later calculations, α is the fraction of a unit mass of agents choosing 1, so that in total a mass 2α chooses 1.) Since θ is normally distributed with mean μ_2 and variance σ_2^2,

$$\mathbb{E}[U_\zeta(\theta, \alpha)] = \zeta - 1 + \gamma\left(1 - \Phi\left(\frac{D - 2\alpha - \mu_2}{\sigma_2}\right)\right),$$

where $\Phi(\cdot)$ is the distribution function of the standard normal. Comparing with equation (3), let

$$V(\mu_2, \alpha; \sigma_2) \equiv \gamma\left(1 - \Phi\left(\frac{D - 2\alpha - \mu_2}{\sigma_2}\right)\right).$$

Note that

$$\sup_{x,\alpha,\sigma} \gamma \left(1 - \Phi\left(\frac{D - 2\alpha - \mu_2}{\sigma_2}\right)\right) = \gamma$$

and so the parameter γ is the measure of strategic uncertainty identified in section 2.

Agents choose 1 if and only if the expected net utility from doing so is greater than zero. In equilibrium, therefore, α is determined as

$$\alpha = \begin{cases} 0 & \text{if } 1 - \gamma\left(1 - \Phi\left(\frac{D-\mu_2}{\sigma_2}\right)\right) \geq \beta, \\ 1 & \text{if } 1 - \gamma\left(1 - \Phi\left(\frac{D-2-\mu_2}{\sigma_2}\right)\right) \leq 0, \\ \frac{\beta - 1 + \gamma\left(1-\Phi\left(\frac{D-2\alpha-\mu_2}{\sigma_2}\right)\right)}{\beta} \in (0,1) & \text{otherwise.} \end{cases} \tag{11}$$

Proposition 2 *There is a unique equilibrium in the simultaneous choice / information case if and only if* $\frac{\beta}{\gamma} \geq \sqrt{\frac{2}{\pi}}\frac{1}{\sigma_2}$.

Proof. α is a monotonically increasing function of μ_2 if

$$\beta > \frac{\partial V(x; \alpha, \sigma_2)}{\partial \alpha} = \frac{2\gamma}{\sigma_2}\phi\left(\frac{D - 2\alpha - \mu_2}{\sigma_2}\right) \tag{12}$$

where $\phi(\cdot)$ is the density function of the standard normal distribution, and α is determined by equation (11). Since $\phi(\cdot) \leq \frac{1}{\sqrt{2\pi}}$, this gives the sufficient condition that $\frac{\beta}{\gamma} \geq \sqrt{\frac{2}{\pi}}\frac{1}{\sigma_2}$.

To prove the necessary part, assume that $\frac{\beta}{\gamma} < \sqrt{\frac{2}{\pi}}\frac{1}{\sigma_2}$ It suffices to find signal draws for which multiple equilibria exist. Consider signal draws such that

$$\mu_2 = D - \frac{2(\beta - 1) + \gamma}{\beta}.$$

Then α, determined by the equilibrium condition (11), is $(2(\beta - 1) + \gamma)/2\beta$ and $D - 2\alpha - \mu_2 = 0$. For such signal draws,

$$\frac{\partial V(x; \alpha, \sigma_2)}{\partial \alpha} = \frac{2\gamma}{\sigma_2}\phi\left(\frac{D - 2\alpha - \mu_2}{\sigma_2}\right) = \sqrt{\frac{2}{\pi}}\frac{\gamma}{\sigma_2}$$

since $\phi(0) = \frac{1}{\sqrt{2\pi}}$. Hence the necessary condition for unique equilibrium in Lemma 1

$$\beta \geq \frac{\partial V(x; \alpha, \sigma_2)}{\partial \alpha}$$

is violated. ■

The necessary and sufficient condition illustrates the balance between heterogeneity β, fundamental uncertainty, measured here by σ_2 and strategic uncertainty γ, identified in section 2.

3.2 Simultaneous Information and Sequential Choice

The next benchmark examined, before turning to the 'fully' sequential problem, is that in which a unit mass of agents choose in each of the two periods, with the same information: signals X_1 and X_2 drawn at the beginning of period 1 and observed by both sets of agents. Hence $\Omega_1 = \{X_1, X_2\}$ and $\Omega_2 = \{X_1, X_2, \alpha_1\}$.

As in the previous section, the common posterior on the state is normally distributed with mean μ_2 and variance σ_2^2.

Proposition 3 *If $\frac{\beta}{\gamma} \geq \sqrt{\frac{2}{\pi}\frac{1}{\sigma_2}}$, then there is a unique equilibrium in the sequential choice / simultaneous information case.*

Proof. Consider the decision of agents in the second period. Expected utility of a type-ζ agent is

$$\begin{aligned} \mathbb{E}[U_\zeta(\theta, \alpha_2)] &= \zeta - 1 + \gamma \mathbb{P}[\theta + \bar{\alpha}_1 + \alpha_2 \geq D | x_1, x_2], \\ &= \zeta - 1 + \gamma \left(1 - \Phi\left(\frac{D - \bar{\alpha}_1 - \alpha_2 - \mu_2}{\sigma_2}\right)\right). \end{aligned}$$

(The notation $\bar{\alpha}_1$ emphasizes that α_1 is parametric to agents in the second period.)

The equivalent of previous calculations for necessary and sufficient condition for unique equilibrium in the second period shows that α_2 is a monotonically increasing function of μ_2 if and only if

$$\beta \geq \frac{\gamma}{\sigma_2}\phi\left(\frac{D - \bar{\alpha}_1 - \alpha_2 - \mu_2}{\sigma_2}\right) \tag{13}$$

when α_2 is determined by

$$\alpha_2 = \frac{\beta - 1 + \gamma\left(1 - \Phi\left(\frac{D - \bar{\alpha}_1 - \alpha_2 - \mu_2}{\sigma_2}\right)\right)}{\beta}. \tag{14}$$

Hence the necessary and sufficient condition for unique equilibrium in the second period is $\frac{\beta}{\gamma} \geq$

$\sqrt{\frac{2}{\pi}}\frac{1}{\sigma_2}$.

Similarly expected utility of a type-ζ agent in the first period is

$$\mathbb{E}[U_\zeta(\theta,\alpha_1)] = \zeta + \gamma\left(1-\Phi\left(\frac{D-\alpha_1-\alpha_2-\mu_2}{\sigma_2}\right)\right) - 1,$$

where

$$\alpha_2 = \begin{cases} 0 & \text{if } 1-\gamma\left(1-\Phi\left(\frac{D-\alpha_1-\mu_2}{\sigma_2}\right)\right) \geq \beta, \\ 1 & \text{if } 1-\gamma\left(1-\Phi\left(\frac{D-\alpha_1-1-\mu_2}{\sigma_2}\right)\right) \leq 0, \\ \frac{\beta-1+\gamma\left(1-\Phi\left(\frac{D-\overline{\alpha}_1-\alpha_2-\mu_2}{\sigma_2}\right)\right)}{\beta} & \text{otherwise.} \end{cases} \tag{15}$$

α_1 is given by the implicit equation

$$\alpha_1 = \frac{\beta-1+\gamma\left(1-\Phi\left(\frac{D-\alpha_1-\alpha_2(\alpha_1)-\mu_2}{\sigma_2}\right)\right)}{\beta}.$$

Hence α_1 is an increasing function of μ_2 if and only if

$$\beta \geq \frac{\gamma}{\sigma_2}\left(1+\frac{\partial\alpha_2}{\partial\alpha_1}\right)\,\phi\left(\frac{D-\alpha_1-\alpha_2(\alpha_1)-\mu_2}{\sigma_2}\right). \tag{16}$$

$\frac{\partial\alpha_2}{\partial\alpha_1}$ in equation (16) can be obtained from total differentiation of equation (14):

$$\frac{\partial\alpha_2}{\partial\alpha_1} = \begin{cases} 0, \\ \frac{\frac{\gamma}{\sigma_2}\phi\left(\frac{D-\alpha_1-\alpha_2-\mu_2}{\sigma_2}\right)}{\beta-\frac{\gamma}{\sigma_2}\phi\left(\frac{D-\alpha_1-\alpha_2-\mu_2}{\sigma_2}\right)}, \end{cases}$$

depending on the value of α_2. Note that we need α_2 is increasing in α_1 to make equation (16) well-defined. It is easy to see that α_2 is increasing in α_1 if and only if $\sigma_2 \geq \frac{\gamma}{\sqrt{2\pi}\beta}$.

Substituting for $\frac{\partial\alpha_2}{\partial\alpha_1}$, equation (16) can be written as:

$$\beta \geq \frac{\gamma}{\sigma_2}\left(\frac{\beta}{\beta-\frac{\gamma}{\sigma_2}\phi(\cdot)}\right)\,\phi\left(\frac{D-\alpha_1-\alpha_2(\alpha_1)-\mu_2}{\sigma_2}\right). \tag{17}$$

Equation (17) can be manipulated as $\frac{\beta}{\gamma} \geq \sqrt{\frac{2}{\pi}}\frac{1}{\sigma_2}$ because $\phi(\cdot) \leq \frac{1}{\sqrt{2\pi}}$. Since this condition implies the monotonicity of α_2 in μ_2 as well as α_1, there is a unique equilibrium for the second period as well as the first period if and only of this condition is satisfied. The proof is complete. ■

The proposition reveals an interesting aspect of the decision making under coordination difficulty. The necessary and sufficient condition for sequential choice/ simultaneous information case is exactly the same as that for simultaneous choice / information case. Hence simply uncoupling the timing of decision for different agents does not alter the nature of coordination difficulty. Those agents who move later may use the decision of the early-movers to remove strategic uncertainty as indicated in the intermediate step in the proof. However the early movers who take account of the late movers face the same coordination difficulty since their decision is subsequently amplified by those who decide in the later stage; the early mover's decision effectively has a bigger impact on the strategic uncertainty. It follows that early movers require the same necessary and sufficient condition on the three components since the smaller mass of agents who move together in the early stage is exactly offset by the amplification of the strategic uncertainty.

3.3 The Sequential Information and Choice

Finally, we can analyse the case in which choices are made and signals received sequentially. The decision of agents in the second period is the same as for the previous case (sequential choice and simultaneous information). The first period problem is different, however. Previously, the fraction α_2 of agents choosing 1 in the second period was certain (even if indeterminate due to multiplicity). Now, α_2 is a random variable from the perspective of period-1 agents. Hence $\Omega_1 = \{X_1\}$ and $\Omega_2 = \{X_1, X_2, \alpha_1\}$.

In the second period, the agents' common posterior on θ is given identical to the one in the previous subsection: it is normally distributed with mean $\mu_2 = (\sigma^2\mu_0 + \sigma_0^2(x_1 + x_2))/(\sigma^2 + 2\sigma_0^2)$ and variance $\sigma_2^2 = \sigma_0^2\sigma^2/(\sigma^2 + 2\sigma_0^2)$. In the first period, agents' common posterior on θ is normally distributed with mean $\mu_1 = (\sigma^2\mu_0 + \sigma_0^2 x_1)/(\sigma^2 + \sigma_0^2)$ and variance $\sigma_1^2 = \sigma_0^2\sigma^2/(\sigma^2 + \sigma_0^2)$.

Proposition 4 *If $\frac{\beta}{\gamma} \geq (\frac{1}{\sigma_1} + \frac{1}{\sigma_2})\frac{1}{\sqrt{2\pi}}$, then there is a unique equilibrium in the sequential choice / information case.*

Proof. We start with the second period problem for the sequential case, which is identical to that for the sequential choice/simultaneous information case considered in the previous subsection.

We collect the main results here for reference: α_2 is a monotonically increasing function of μ_2 if and only if

$$\beta \geq \frac{\gamma}{\sigma_2}\phi\left(\frac{D - \bar{\alpha}_1 - \alpha_2 - \mu_2}{\sigma_2}\right) \tag{18}$$

when α_2 is determined by

$$\alpha_2 = \frac{\beta - 1 + \gamma\left(1 - \Phi\left(\frac{D - \bar{\alpha}_1 - \alpha_2 - \mu_2}{\sigma_2}\right)\right)}{\beta}. \tag{19}$$

Hence the necessary and sufficient condition for unique equilibrium in the second period is $\sigma_2 \geq \frac{\gamma}{\sqrt{2\pi}\beta}$.

Expected utility of a type-ζ agent in the first period is

$$\zeta - 1 + \gamma \mathbb{P}[\theta + \alpha_1 + \tilde{\alpha}_2 \geq D | x_1],$$

where the notation $\tilde{\alpha}_2$ emphasizes that it is a random variable.

Those agents who make decision in the first period must compute the probability of the sum of two random variables, θ and α_2; since the second period's decision is made conditional on X_2, α_2 is a function of the random variable X_2.

To apply the necessary and sufficient condition in Lemma 1, we assess

$$\frac{d\mathbb{P}[\theta + \tilde{\alpha}_2 \geq D - \alpha_1 | x_1]}{d\alpha_1}$$

where the left hand side of the inequality inside the probability contains only random variables while the right hand side contains only parameters.

First observe that θ and $X_2 = \theta + \epsilon_2$ are bivariate-normally distributed random variables; conditional on the observation of x_1, θ and X_2 have the same mean μ_1 and they have variances σ_1^2 and $\sigma_1^2 + \sigma_\epsilon^2$. Their covariance is given by σ_1^2 so that the correlation coefficient $\rho = \frac{\sigma_1^2}{\sigma_1 \sqrt{\sigma_1^2 + \sigma_\epsilon^2}}$.

Hence write

$$\begin{aligned} \mathbb{P}[\theta + \tilde{\alpha}_2(X_2 | \alpha_1) \geq D - \alpha_1 | x_1] &= \int_{-\infty}^{\infty} \int_{D - \alpha_1 - \tilde{\alpha}_2(X_2 | \alpha_1)}^{\infty} \phi_{x_1}(\theta, X_2)\, d\theta\, dX_2 \\ &= \int_{-\infty}^{\infty} \int_{\frac{D - \alpha_1 - \tilde{\alpha}_2(X_2 | \alpha_1) - \mu_1}{\sigma_1}}^{\infty} \phi(\hat{\theta}, \hat{X}_2)\, d\hat{\theta}\, d\hat{X}_2 \end{aligned} \tag{20}$$

where $\phi_{x_1}(\theta, X_2)$ on the first line is the bivariate normal distribution of θ and X_2 conditional on the observation of x_1, while $\hat{\theta} = \frac{\theta - \mu_1}{\sigma_1}$ and $\hat{X}_2 = \frac{X_2 - \mu_1}{\sqrt{\sigma_1^2 + \sigma_\epsilon^2}}$ so that $\phi(\hat{\theta}, \hat{X}_2)$ on the second line is the standard bivariate normal distribution.

Using equation (20), the derivative of the probability with respect to α can be computed as follows:

$$\frac{d}{d\alpha_1}\mathbb{P}[\theta + \tilde{\alpha}_2(X_2|\ \alpha_1) \geq D - \alpha_1|x_1] = \int_{-\infty}^{\infty} \frac{d}{d\alpha_1} \int_{\frac{D-\alpha_1-\tilde{\alpha}_2(X_2)}{\sigma_1}}^{\infty} \phi(\hat{\theta}, \hat{X}_2)\ d\hat{\theta}\ d\hat{X}_2 \tag{21}$$

Applying Leibniz's rule, the derivative inside the outer integral is computed.

$$\frac{d}{d\alpha_1} \int_{\frac{D-\alpha_1-\tilde{\alpha}_2(X_2|\alpha_1)}{\sigma_1}}^{\infty} \phi(\hat{\theta}, \hat{X}_2)\ d\hat{\theta}\ d\hat{X}_2 = \left(\frac{1}{\sigma_1}(1 + \frac{d\tilde{\alpha}_2(X_2|\alpha_1)}{d\alpha_1})\right) \phi(\frac{D - \alpha_1 - \tilde{\alpha}_2(X_2)}{\sigma_1}, X_2)$$

Since α_2 is determined from the implicit equation (19), we can totally differentiate it to obtain

$$\frac{d\tilde{\alpha}_2(X_2|\alpha_1)}{d\alpha_1} = \frac{\gamma\phi(.)}{\beta\sigma_2 - \gamma\phi(.)} \leq \frac{\gamma\frac{1}{\sqrt{2\pi}}}{\beta\sigma_2 - \gamma\frac{1}{\sqrt{2\pi}}}$$

where the inequality follows from the fact that $\phi(.) \leq \frac{1}{\sqrt{2\pi}}$.

Collecting these results and substituting them into (21) yields,

$$\frac{d}{d\alpha_1}\mathbb{P}\left[\theta + \tilde{\alpha}_2(X_2|\ \alpha_1) \geq D-\alpha_1|x_1\right] \leq \left(\frac{1}{\sigma_1}\frac{\beta\sigma_2}{\beta\sigma_2 - \gamma\frac{1}{\sqrt{2\pi}}}\right) \int_{-\infty}^{\infty} \phi\left(\frac{D-\alpha_1-\tilde{\alpha}_2(X_2)}{\sigma_1}, X_2\right) dX_2. \tag{22}$$

The integral on the right hand side of equation (22) is bounded above by $\frac{1}{\sqrt{2\pi}}$ since

$$\int_{-\infty}^{\infty} \phi\left(\frac{D - \alpha_1 - \tilde{\alpha}_2(X_2)}{\sigma_1}, X_2\right) dX_2 \leq \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \exp[-\frac{X_2^2}{2}]\ dX_2 = \frac{1}{\sqrt{2\pi}}.$$

It follows that $\frac{d}{d\alpha_1}\mathbb{P}[\theta + \tilde{\alpha}_2(X_2|\ \alpha_1) \geq D - \alpha_1|x_1] \leq (\frac{1}{\sigma_1}\frac{\beta\sigma_2}{\beta\sigma_2 - \gamma\frac{1}{\sqrt{2\pi}}})\frac{1}{\sqrt{2\pi}}$.

We now apply the necessary and sufficient condition for the unique equilibrium from Lemma 1: there is unique equilibrium if and only if

$$\beta \geq \gamma\frac{d}{d\alpha_1}\mathbb{P}[\theta + \tilde{\alpha}_2(X_2|\ \alpha_1) \geq D - \alpha_1|x_1] \tag{23}$$

Substituting equation (22) into (23) we obtain the following sufficient condition for uniqueness: there is unique equilibrium if

$$\beta \geq \gamma\left(\frac{1}{\sigma_1}\frac{\beta\sigma_2}{(\beta\sigma_2 - \gamma\frac{1}{\sqrt{2\pi}})}\right)\frac{1}{\sqrt{2\pi}} \tag{24}$$

The condition can be rewritten as

$$\frac{\beta}{\gamma} \geq (\frac{1}{\sigma_1} + \frac{1}{\sigma_2})\frac{1}{\sqrt{2\pi}}.$$

Observe that under this condition, the necessary and sufficient condition for uniqueness in the second period is also satisfied and the proof is complete. ■

The proposition indicates that there is a unique equilibrium in a model with sequential choice and information for a milder condition on the three factors than the simultaneous choice /information model or sequential choice /simultaneous information. The following corollary formalizes this observation.

Corollary 2 *Given an arbitrary set of parameters on the fundamental uncertainty, there is a set of parameters on the strategic uncertainty and heterogeneity for which there is a unique equilibrium under sequential choice /information while there are multiple equilibria under simultaneous choice /information or sequential choice /simultaneous information. The parameters satisfy*

$$\frac{2}{\sigma_2}\frac{1}{\sqrt{2\pi}} \geq \frac{\beta}{\gamma} \geq (\frac{1}{\sigma_1} + \frac{1}{\sigma_2})\frac{1}{\sqrt{2\pi}}$$

.

Proof. Notice that $\frac{2}{\sigma_2} > \frac{1}{\sigma_1} + \frac{1}{\sigma_2}$ since $\sigma_2 < \sigma_1$. Hence we can always find $\frac{\beta}{\gamma}$ that satisfies the condition in the corollary. The proof follows by combining the conditions from Propositions 2, 3, and 4. ■

The intuition behind Corollary 2 can be explained by the interaction among the three factors which together determine the difficulty of equilibrium selection. In the simultaneous choice / information model the requirement on the three factors is exactly identical to the one in the static model. In the sequential choice / simultaneous information model, strategic uncertainty unravels gradually and thus it appears that the requirement for uniqueness might be milder. However those agents who move in the first period fully anticipate the consequence of their choice in terms of the reduction in strategic uncertainty. When the choice of the agents who move in the second period is correctly anticipated, the reduced fundamental uncertainty due to full revelation of information implies the same degree of difficulty in coordination. In contrast the sequential choice / information model suggests that gradual revelation of information is critical in removing the coordination difficulty. Those agents who move in the first period fully anticipate the response of the agents who move in the future. However they do not have as precise information as to the fundamental uncertainty so that they can make a unique choice conditional on the information available. In the second period, additional information as to the fundamental uncertainty is available. However part of the strategic uncertainty is resolved since half of the population already made their choice in the first period so that unique equilibrium exists in the second period as well.

4 Path Dependence

The intuition behind the unique equilibrium selection in the sequential choice /information model has an interesting implication on the dynamic behavior of the model. The agents who move in the first period have to make their choice with less information than those in the second period. On the other hand the decision made by the former is regarded as a commitment to a particular strategy by the agents who move in the second period. Part of the difficulty in the equilibrium selection stems from the fact that agents face too much strategic uncertainty compared to the fundamental uncertainty. The commitment made by the first period agents helps reducing the strategic uncertainty in the second period when more information is available. This observation implies that the first period agents effectively select one of the possible equilibria. This observation in turn is evidenced by path dependence in sense that the different sequence of signals alter the overall selection of equilibrium. Due to the commitment power of the agents who make decision in the first period, the signals which arrive in the first period may have a bigger effect on the determination of equilibrium path. This is confirmed along the equilibrium path, which we call path dependence.

Casual observation of various economics time series indicates that the fluctuation depends not only on the information content of economic signals driving it, but also on the order of the information arrival. The dynamic equilibrium selection in the sequential information and choice generates such a feature.

Proposition 5 *The distribution of α_2 conditional on α_1 is ranked in the sense of first order stochastic dominance.*

Proof. We show that if $\alpha_1 \geq \alpha_1'$, then $\mathbb{P}\{\alpha_2 \geq a | \alpha_1\} \geq \mathbb{P}\{\alpha_2 \geq a | \alpha_1'\}$ for $a \in [0,1]$. First note that α_2 is a function of α_1 and X_2: $\alpha_2(\alpha_1, X_2)$. Hence the distribution of α_2 is derived from the distribution of X_2 which is normally distributed. Moreover given α_1, there is unique x for which $X_2 \geq x$ if and only $\alpha_2(\alpha_1, X_2) \geq a$ because under the unique equilibrium for sequential choice /

information model, α_2 is uniquely determined once X_2 is. Denote such x as $x(\alpha_2|\alpha_1)$ and let $\psi(\alpha_2)$ be the density function of α_2. Write

$$\mathbb{P}\{\alpha_2 \geq a|\ \alpha_1\} = \int_a^\infty \psi(\alpha_2|\alpha_1)d\alpha_2 \tag{25}$$

$$= \int_{x(a|\alpha_1)}^\infty \phi(X_2)dX_2 \tag{26}$$

where $\phi(.)$ is normal density function.

To show that $\mathbb{P}\{\alpha_2 \geq a|\ \alpha_1\} \geq \mathbb{P}\{\alpha_2 \geq a|\ \alpha_1'\}$ for $a \in [0,1]$, it suffices to prove that $x(a|\alpha_1)$ is decreasing in α_1. Since α_2 is determined from the implicit equilibrium condition,

$$\alpha_2 = \frac{\beta - 1 + (1 - \Phi(\frac{D-\alpha_1-\alpha_2-\mu(x_2)}{\sigma_2})}{\beta},$$

total differentiation gives

$$\frac{dx_2}{d\alpha_1} = -\frac{\frac{\gamma}{\beta\sigma_2}\phi(.)}{\frac{\gamma\sigma_2^2}{\beta\sigma^2\sigma_2}\phi(.)} < 0.$$

It follows that $x(a|\alpha_1) < x(a|\alpha_1')$ if $\alpha_1 \geq \alpha_1'$. ■

The stochastic dominance demonstrated in the proposition holds even after controlling for the informational effect of first period signal. Since the signals are drawn conditional on the realization of the fundamental uncertainty, positive signal in the first period implies that there is a bigger probability that the state is good. The following example shows that the relationship of stochastic dominance remains even if the agents in the second period may entertain the same belief about the state of nature. In particular it implies that the resolution of strategic uncertainty has a substantial effect on the equilibrium path in addition to that of fundamental uncertainty.

Example 1 *Consider two realizations of signals of the same total information content but reversed order,* $x = (x_1, x_2)$ *and* $x' = (x_2, x_1)$ *where* $x_1 > x_2$*. Then* $\alpha_1|_x > \alpha_1|_{x'}$ *and* $\alpha_2|_x > \alpha_2|_{x'}$*.*

The example can be proved as follows. Given that $x_1 > x_2$, the first period agents' choice satisfies $\alpha_1|_{x_1} > \alpha_1|_{x_2}$ since the posterior distribution in the first period has a higher mean conditional on x_1 than x_2.

Recall that the second period equilibrium is determined from

$$\alpha_2 = \frac{\beta - 1 - \gamma\Phi(\frac{D-\alpha_1-\alpha_2-\mu_2}{\sigma_2})}{\beta}.$$

Notice that both sequences of signals produce the same mean and the same variances for θ in the second period: μ_2 and σ_2^2 are identical for both $x = (x_1, x_2)$ and $x' = (x_2, x_1)$. Moreover $\alpha_1|_{x_1} > \alpha_1|_{x_2}$ from above. We know that α_2 as a function of α_1 is increasing in α_1. Since α_2 depends on μ_2, σ_2, and α_1 where the first two are identical for both sequences and only α_1 differs in the two sequences, $\alpha_2|_x > \alpha_2|_{x'}$ as stated in the example.

5 Conclusion

We considered a model of coordination game under incomplete information which may have multiple equilibria depending on the realization of information. The coordination problem arises because the payoff effect due to coordination dominates that due to correct statistical decision and heterogeneity. We constructed a dynamic model where gradual information release together with distributed decision timing facilitates the unique equilibrium selection.

Our result implies that empirical test of economic phenomena which are subject to coordination problem should pay attention to the dynamic nature of the economics environment. For instance, ignoring the dynamic information release may lead the economist to conclude that the economy was subject to multiple equilibria although actual agents who lived the time did not have difficulties due to multiple equilibria.

The path dependence result seems to indicate the relevance of techinical analysis for inverstment decision. Modern finance theory assumes that the price process follows random walk hypothesis so that the past price path carries no additional information over and above the current price. According to the path dependence, paying attention to the price path may provide more information about the future direction of the movement.

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