Partisan Politics and Aggregation Failure with Ignorant Voters[†]

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Abstract

We analyze a Downsian model of candidate competition with two modifications. First, some voters are uncertain about a candidate's policy choice and about the distribution of voter preference. Second, if both candidates offer the same policy, voters choose between them according to their "personality preferences." Equilibrium outcomes differ from standard Downsian predictions: the candidate with a personality advantage chooses partisan policies and gets elected. This departure from the Downsian prediction is most pronounced when candidates have a weak policy preference and care mostly about winning the election. In that case, uninformed voters' equilibrium strategy is to vote for the candidate with the preferred personality even if *on average* electing this candidate implies a lower payoff.

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1. Introduction

We study a Downsian model of candidate competition in which some voters cannot observe one candidate's policy choice. Our goal is to identify conditions under which voters' preferences over superficial, readily observable candidate characteristics leads to policy and election failures; that is, to election outcomes that put a candidate who has chosen a partisan policy in office at the expense of a candidate that has chosen the median-preferred policy.

Voters in our model care about policy but also have a "personality preference" for a particular candidate. This personality preference is small in the sense that when candidates choose different policies, the candidate with the favored policy is always preferred. All voters know which candidate's personality they prefer but some voters are ignorant of one candidate's policy position. We model voter ignorance as incomplete information regarding the policy choice.

There is empirical support for our voter ignorance assumption. Surveys routinely find that the American electorate is poorly informed.[1] Delli-Carpini and Keeter (1993) cite a 1990-91 National Election Study survey[2] indicating that only 57% of voters could correctly identify relative ideological positions of the Republican and Democratic parties[3] and only 45% of voters could correctly identify the parties' relative position on federal spending.[4] The same survey shows that voters are no better informed of the electorate than they are of party positions. For example, in the 1990-91 survey only 47% of voters correctly identified the party that holds the majority in the Senate.[5] (See Delli-Carpini and Keeter (1993), Table 2).

There is also evidence that ignorant voters react to superficial differences in candidates. For example, Redlawsk and Lau (2003) conduct mock election experiments that force subjects to choose between an unattractive candidate who has the right positions on the issues and a more personally attractive candidate who held many positions that

[1] For an early reference, see Berelson, Lazarsfeld and McPhee (1954)

[2] The National Election Study survey is conducted by the Survey Research Center, University of Michigan. The sample size was 449 and consisted of US citizen of voting age.

[3] 25% of the answers were "incorrect or incomplete" and 18% answered "don't know."

[4] 26% of the answers were "incorrect or incomplete" and 29% answered "don't know."

[5] 17% of answers were incorrect or incomplete and 36% answered "don't know."

the subject disagrees with. Their results show that voters often choose the candidate that has the more attractive personality and appearance but holds less desirable policy positions. However, voters with more expertise, measured by their level of interest, knowledge, and participation, where more likely the choose the unattractive candidate who held the preferred positions on the issues.

Our analysis combines these two assumptions (voter ignorance, personality preference) with the hypothesis that candidates have better information about the distribution of voter preferences than the voters themselves. Voters are unlikely to influence election outcomes and therefore have little incentive to get informed. By contrast, the candidates have much at stake when making a policy choice and hence invest significant resources to determine the distribution of voter preferences. Of course, some of this information may 'leak' to voters. Our model allows for the possibility that those voters know policies are also know the composition of the electorate. However, uninformed voters remain uncertain about the distribution of voter preferences.

Our main result demonstrates that the candidate with the personality advantage can get elected even if he chooses a partisan policy that is not median preferred. Such partisan policy outcomes will occur even if – in fact, especially if – the candidate with the personality advantage is an office seeker who cares relatively little about policy and more about getting elected. When uninformed voters confront office seekers, they will vote according to their personality preference. This behavior is optimal even though the candidate with the preferred personality may *on average* choose policies that lead to lower utility than the policies of his opponent. An observer who interprets voting behavior as non-strategic may therefore conclude that voters place an unreasonably large weight on personality or that voters have incorrect expectations about their favored candidate's policy choice.

The fraction of informed voters plays a central role in our analysis. When candidates are office seekers, increasing the fraction of informed voters increases the probability of median preferred outcomes. With few informed voters, candidates who have a personality advantage will rarely choose median preferred policies. This conclusion is in contrast to work that emphasizes the information aggregation properties of elections (See McKelvey and Ordeshook (1985, 1986) and Feddersen and Pesendorfer (1997)). Feddersen and Pesendorfer (1997) show that even if the fraction of informed voters is arbitrarily small, the outcome of a large election is as if all voters are fully informed. Feddersen and Pesendorfer (1997) assume that candidate's policy choices are exogenous, whereas in our model, one candidate chooses his policy strategically. Moreover, some voters are uncertain about policy choices and about the composition of the electorate. This multi-dimensionality uncertainty implies that elections cannot fully aggregate information.

We consider a very simple and stylized candidate competition model. There are two candidates; candidate A is committed to a fixed moderate policy m, while candidate B chooses between a partisan policy l and the moderate policy m. Voters prefer the moderate policy m to the partisan policy l. If both candidates choose the moderate policy, voters' personality preference leads them to prefer either A or B. We assume that personality preference is less important than policy preference; that is, no voter prefers candidate B if he chooses the partisan policy. In our model, candidate A chooses the median preferred policy and therefore offers the toughest possible competition for B. Nevertheless, we show that B is able to exploit voter ignorance to implement the partisan policy.

The standard model of party competition (Downs 1957) has two candidates who maximize the probability of getting elected. Both candidates choose policies before the election. Voters observe these policies and choose the candidate who offers the more attractive policy. The model predicts that the median preferred policy will be implemented. Candidate competition is similar to Bertrand competition: if a candidate chooses a policy other than the median voter's favorite, he will be "under-cut" by his opponent. As a result, the Downsian prediction of median preferred outcomes holds even when the candidates have policy preferences.

Our model differs from the standard Downsian model in two ways: first, we assume that some voters do not know one candidate's policy choice and the distribution of voter preferences. We incorporate policy ignorance into a strategic model by assuming that each voter observes the realized policy choice with a probability between 0 and 1. To model the voter's ignorance of the electorate, we assume that s, the probability that a random voter prefers A's personality, is uncertain and that voters do not observe the realized s. We call s the *state of the electorate*.

Second, unlike the voters, candidate B learns the state of the electorate before making his decision. We say that a candidate has a *personality advantage* if the probability that a voter prefers his personality is greater than $\frac{1}{2}$. Our assumption that candidates have better information about the state of the electorate is motivated by the fact that candidates often take (secret) opinion polls measuring how their personality is perceived by voters. These opinion polls may provide precise information.

Our focus is on large elections. Therefore, we study limit equilibria as the number of voters goes to infinity. We normalize candidate B's utility function so that his utility of winning the election with the partisan policy is 1, his utility of losing the election is 0, and his utility of winning with the moderate policy is $\mu \in (0, 1)$. Hence, μ close to 0 describes a candidate who derives utility from winning only if he can implement his favored policy while μ close to 1 describes a candidate who is motivated primarily by winning the election. If μ is close to 1, we refer to the candidate as an *office seeker*. Since we find this to be the more descriptive case, many of our results assume that candidate B is an office seeker.

We establish the following departures from the standard Downsian model if the candidate is an office seeker:

(1) *Non-median Election Outcomes:* The candidate who offers the median preferred policy may lose the election.

(2) *Personality Matters:* The candidate with the personality advantage wins the election. If B has a substantial personality advantage, he chooses the partisan policy l.

(3) *Information Matters:* The probability of the partisan outcome l is decreasing in the probability that a voter is informed. Hence, the voters' equilibrium payoff is lower if the electorate is more ignorant. If the electorate is sufficiently poorly informed, then B almost always chooses the partisan policy if he is elected. (Moreover, B wins if he has a personality advantage.)

(4) *Voting on Personality Preference:* Ignorant voters vote their personality preference. In particular, ignorant voters who prefer B's personality vote for B even if electing B implies, on average, a lower payoff.

Our results imply that small asymmetries between candidates can have large effects on election outcomes. Voters behave in this seemingly naive way because they condition on the event that their vote is pivotal. An office seeker (μ close to 1) will choose the moderate policy when he expects the election to be close. Anticipating this behavior, voters conclude that conditional on a vote being pivotal, the opportunistic candidate is likely to choose the moderate policy. As a result, uninformed voters who face an office seeker behave *as if* the opportunistic candidate always chooses the moderate policy and vote according to their personality preference.

A high probability of choosing the moderate policy when the election is close does not translate into a high unconditional probability of choosing the moderate policy. Because an office seeker receives a large share of the uninformed vote, choose the partisan policy without risking loosing the election whenever the state is sufficiently favorable. Conditioning on being pivotal creates a wedge between voting behavior and ex ante policy choices. An lower fraction of informed voters makes this wedge larger. As the fraction of informed voters goes to zero, the opportunistic office seeker will choose the partisan policy and win the election whenever the majority prefers his personality.

In section 4, we discuss robustness and extensions. In section 4.2, we argue that our main insights are survive a modeling change that gives candidate B more than two policy choices. We also argue (section 4.3) that a symmetric model with two strategic candidates would generate similar insights as our simple one-sided model. To isolate the effect of this asymmetric information, we consider two alternative models with symmetrically informed candidates and voters. In section 4.4, we assume that neither the candidates nor the voters observe the state of the electorate while in section 4.5, we assume that all agents know the state. In both cases, the Downsian prediction of median preferred outcomes is restored. Hence, asymmetric information (between the candidate and the voters) about the state of the electorate is crucial for our results.

Evidence of voter ignorance may be considered puzzling since we might expect political competition to force candidates to inform voters of their positions. In section 4.1, we investigate this hypothesis. We find that giving a candidate the opportunity to increase the proportion of informed voters has no effect if the candidate is an office seeker. Hence, permitting voluntary disclosure does not mitigate partisan politics or aggregation failure. This is true even though informing voters is costless. However, our analysis suggests that informing voters about the opponent's position, provided such information can be revealed credibly, may be an effective remedy for partisan politics and aggregation failure. Hence, we find a role for "negative campaigning."

1.1 Strategic Voters

We describe voting behavior as the equilibrium outcome of a voting game. Hence, we assume that voters are strategic. There is some experimental evidence that sheds light on the validity of strategic voting models. Battaglini, Morton and Palfrey (2005) conduct experiments based on a voting game with asymmetric information. They find that voters react to changes in the distribution of types of other voters in a way that is qualitatively consistent with the predictions of Nash equilibrium. In particular, Battaglini, Morton and Palfrey provide evidence that agents vote against their unconditionally preferred choice when this choice is inferior conditional on a vote being pivotal.

In their experiment, subjects play a voting game with uninformed, informed and partisan voters. Partisan voters always vote for alternative a. Informed and uninformed voters choose between a and b and prefer a in state A and b in state B. Uninformed voters have a prior probability over states while informed voters know the true state. Equilibrium predicts that – in the presence of a-partisans – uninformed voters will vote for b even if a is the better unconditional choice (i.e., a has a higher expected payoff when the uninformed voter uses his prior to weigh the states). Their experimental evidence is consistent with this equilibrium prediction.

The Battaglini, Morton and Palfrey experiment uses a different setup than the model presented here. However, in both models, conditioning on being pivotal creates a wedge between the unconditionally optimal choice and equilibrium behavior. Battaglini, Morton and Palfrey provide evidence that voters recognize this wedge and react in a way consistent with equilibrium predictions.

1.2 Related Literature

Several authors have examined the robustness of Downs' results by introducing policy motivated candidates and uncertainty about median voter preferences. For example, Wittman (1977) and Calvert (1985) consider a model with two candidates, uncertain distribution of voter preferences but no asymmetric information. In Chan (2001) and Bernhard, Duggan and Squintani (2003), candidates have asymmetric information. In all these models, candidates typically choose distinct policy positions. Because the median's policy preference is not known, candidates trade-off the probability of losing against winning with a less desired policy. However, if candidates are office seekers and mostly care about winning, their policy positions converge. In contrast, an office seeker in our model is more likely to choose a partisan position than a candidate who has a strong policy preference.

Ansolabehere and Snyder (2000), Groseclose (2001), Aragones and Palfrey (2002, 2004) examine Downsian competition models with uncertain median preferences where one candidate has a "valence" advantage formally equivalent to the personality advantage analyzed here. Aragones and Palfrey (2002) show that equilibrium entails mixed strategies and hence candidates typically choose distinct policy positions.

In all related studies of Downsian competition, aggregation failure cannot occur because voters know candidates' policy choices. Candidates hope that their partisan positions will match the realized median preference. Hence, distinct policy positions benefit the median voter in some states of the world. In our model, candidate B chooses a partisan position even though he knows the median prefers the moderate policy in all states of the world. Candidate B benefits from this behavior because uninformed voters cannot detect the partisan choice.

Feddersen and Pesendorfer (1996, 1997) study models with asymmetrically informed voters. The Feddersen and Pesendorfer papers show that large elections effectively aggregate information if policy positions are fixed and voters are uncertain about the "quality" of the candidates' policies. Our model has both asymmetrically informed voters and candidate competition. In our context, the Feddersen and Pesendorfer result would correspond to a situation where the opportunistic candidate's policy is exogenously (and randomly) chosen. The difference here is that the candidate's policy choice is a strategic variable.

McKelvey and Ordeshook (1985, 1986) argue that even if voters are ignorant of policy choices they may still infer which candidate offers the preferred policy from polling data, endorsements, and other public information. In other words, McKelvey and Ordeshook argue that ignorance about policy choices alone may not lead to non-median outcomes. In our model, voters are uncertain about the policy choice and about the state of the electorate. Therefore, voters cannot infer policy choices and non-median outcomes ensue. We show in section 4 that when voters know the state, the election yields Downsian outcomes even if an arbitrarily large fraction of voters are ignorant of policy. Hence, as suggested by the McKelvey-Ordeshook argument, Downsian outcomes are attained whenever voters can infer policy from public information.

2. The Model

Two candidates stand for election. Candidate A is committed to the moderate policy m while candidate B chooses between the partisan policy l and the moderate policy m. Candidate B's payoff is 1 if he is elected and implements l, $\mu \in (0,1)$ if he is elected and implements m, and 0 if he is not elected. The parameter μ quantifies how B trades off getting elected and implementing his preferred policy. If μ is close to one, then the candidate cares mostly about winning the election, while if μ is close to zero the candidate cares mostly about implementing his preferred policy.

The $2n+1$ voters care about the implemented policy and about who wins the election. There are three possible election outcomes, denoted l, m_a, m_b, where m_a stands for 'A wins and implements m' and m_b stands for 'B wins and implements m.' We let $o \in \{l, m_a, m_b\}$ denote the election outcome.

Every voter prefers the moderate policy m to the partisan policy l irrespective of who gets elected. Each voter also has a *personality preference* that determines his ranking of the candidates if both choose m. Hence, a voter's preference type is some $j \in \{a, b\}$ where $j = a$ prefers A's personality and $j = b$ prefers B's personality. A voter type j has payoff $u(j, o)$, where

$$u(j,o) = \begin{cases} 1+\epsilon & \text{if } o = m_j \\ 1 & \text{if } o = m_i, i \neq j \\ 0 & \text{if } o = l \end{cases}$$

We assume that $\epsilon > 0$.

Voters are assigned a type prior to the election. With probability $s \in [0,1]$, a voter is type a and with probability $1-s$ a voter is type b. We refer to s as the *state of the electorate*; it specifies which candidate has a personality advantage. If $s > \frac{1}{2}$, then a-types are more likely than b-types and hence candidate A has a personality advantage. Conversely, B has a personality advantage if $s < \frac{1}{2}$. The probability distribution G describes how s is chosen. We assume that G has support $[0,1]$ and admits a continuous, strictly positive, and continuously differentiable density g.

We analyze the following model of political competition.

(i) Nature draws s according to G and independently assigns each voter type a with probability s and type b with probability $1-s$. Voters learn their preference types but not the preference types of other voters.

(ii) Candidate B observes s and chooses a policy.

(iii) Each voter is independently informed of B's realized policy choice with probability $\delta \in (0,1)$. Voters do not observe the realized state s.

(iv) Each voter casts a vote for A or B.

(v) The candidate who receives the most votes ($n+1$ or more) wins the election and implements his policy.

We analyze *symmetric Nash equilibria in weakly undominated strategies.* We refer to such equilibria as *voting equilibria.* Symmetry requires that all voters with the same strategy preferences and information have the same strategy. Note that voters and candidates may use mixed strategies.

Informed and uninformed a-types have a simple dominant strategy: always vote for A. Informed b-types also have a simple dominant strategy: vote for B if and only if B chooses m. Hence, the only agents with a non-trivial decision are *uninformed* b-types. Let $x \in [0,1]$ be the probability that a b-type votes for B. Below, we will suppress the behavior of informed and a-type voters and refer to x as the voters' strategy.

A strategy for candidate B is a *cutoff-strategy* if there is $y \in [0,1]$ such that B chooses l if $s < y$ and m if $s > y$. Since the probability of any single state s is 0, the number x suffices to describe candidate B's behavior. A voting equilibrium in which candidate B uses a cutoff strategy is a *cutoff equilibrium.*[6]

Proposition 1: *There exists a voting equilibrium and every voting equilibrium is a cutoff equilibrium.*

Proof: See Appendix. □

Given a voter strategy $x \in [0,1]$, let $\pi^p(x,s)$ be the probability that a randomly selected voter casts a vote for B in state s conditional on B choosing policy $p \in \{l,m\}$. If B chooses l, then only uninformed b-types vote for him; if B chooses m, then informed b-types also vote for him. Hence, we have

[6] In a cutoff equilibrium, voters may use a mixed strategy. An earlier version of this paper has voters differentiated according to the strength of their personality preference. That model has only pure strategy equilibria.

$$
\begin{aligned}
\pi^{l}(x,s) &= (1-s)(1-\delta)x \\
\pi^{m}(x,s) &= (1-s)\left[(1-\delta)x+\delta\right]
\end{aligned}
\tag{1}
$$

Let $B_n(z)$ be the probability of at least $n+1$ successes out of $2n+1$ trials when the probability of success in each trial is z. Hence,

$$
B_n(z) = \sum_{k=n+1}^{2n+1} \binom{2n+1}{k} z^k (1-z)^{2n+1-k} \tag{2}
$$

Candidate B wins with probability $B_n(\pi^p(x,s))$ in state s if he chooses p. Therefore, B chooses l if

$$
B_n(\pi^l(x,s)) > \mu B_n(\pi^m(x,s))
$$

and m if this inequality is reversed. To prove that B's best response is a cutoff strategy we show that $\frac{B_n(\pi^l(x,s))}{B_n(\pi^m(x,s))}$ is strictly decreasing in s. It follows that if m is optimal at s, then it is the only optimal action at $s' > s$. Hence, the best response must be a cutoff strategy. We use a fixed-point argument to establish the existence of a cutoff equilibrium.

Our model of policy choice is highly stylized: we assume that one candidate is committed to a fixed policy while the other candidate has a binary choice. By assuming that A is committed to m – the policy preferred by all voters – we create the toughest possible competition for B. Our main result demonstrates that electoral competition cannot prevent B from getting elected with the partisan policy *even if* he faces an idealized opponent who is committed to the median preferred policy. In section 4.3, we discuss how our analysis changes if both candidates have a non-trivial policy choice. The assumption that B has a binary policy choice simplifies our analysis but is not essential for our results. In section 4.2, we discuss how our results extend to a model in which B chooses among multiple policies.

Uninformed voters learn their type but remain uncertain about B's policy choice and about the state of the electorate.[7] The assumption that candidates are better informed than voters about the state of the electorate is essential for our results. In sections 4.4

[7] Note, however, that a voters' own preference type is informative about the state of the electorate. In particular, a's beliefs about the state of the electorate put more weight on higher states than b's beliefs about the state of the electorate in the sense of first order stochastic dominance.

and 4.5, we describe how the analysis changes if there is symmetric information between voters and candidates.

Candidates are likely to be better informed than voters because information is more valuable for them. Hence, candidates have a much greater incentive to collect information than voters. Candidates are reported to spend substantial resources on polling and focus groups prior to selecting their policies. Of course, some of this information may become public and accessible to voters. However, not all voters pay attention to this information. In particular, voters who are uninformed of the policy choices are likely to be uninformed of the state of the electorate as well.

For any voting equilibrium (x, y), let $\phi^o(s)$ be the probability that the outcome in state s is $o \in \{l, m_a, m_b\}$. Hence,

$$\begin{aligned}
\phi^l(s) &= \begin{cases} B_n(\pi^l(x,s)) & \text{if } s < y \\ 0 & \text{otherwise.} \end{cases} \\
\phi^{m_b}(s) &= \begin{cases} B_n(\pi^m(x,s)) & \text{if } s > y \\ 0 & \text{otherwise.} \end{cases} \\
\phi^{m_a}(s) &= 1 - \phi^{m_b}(s) - \phi^l(s)
\end{aligned} \tag{3}$$

Let $\Phi_n(x, y)$ denote the outcome of the voting equilibrium (x, y) as defined by equation (3). A *limit equilibrium* is a pair (x, y) such that for every voting game with $2n+1$ voters, there exists a sequence (x_n, y_n) of voting equilibria converging to (x, y). For any limit equilibrium (x, y), $\Phi(x, y)$ denotes the set of limit outcomes associated with (x, y). That is, $\phi \in \Phi(x, y)$ if there exists voting equilibria (x_n, y_n) for all $2n+1$ voter games such that (x_n, y_n) converges to (x, y) and $\phi_n(x_n, y_n)$ converges to ϕ.[8]

An outcome ϕ is Downsian if $\phi^{m_b}(s) = 1$ for $s < \frac{1}{2}$, and $\phi^{m_a}(s) = 1$ for $s > \frac{1}{2}$. Hence, an outcome is Downsian if and only if the candidate with a personality advantage wins and implements the moderate policy. Two different Downsian outcomes agree at every $s \neq \frac{1}{2}$. Our second proposition serves as a benchmark. It shows that if the fraction of informed voters is greater than $\frac{1}{2}$, then any limit is Downsian.

Proposition 2: *Let $\delta > \frac{1}{2}$. Then, $(x, y) = (1, 0)$ is the unique limit equilibrium and every limit outcome is Downsian.*

[8] We say that the function ϕ^n converges ϕ if $\phi^n(s)$ converges to $\phi(s)$ at every continuity point of ϕ.

Proof: See Appendix.

If $\delta > \frac{1}{2}$, the electorate is sufficiently informed to ensure that the election outcome (in a large electorate) is as if all voters observe the policy choice.

3. Poorly Informed Electorates

In this section, we analyze voting equilibria when the probability that a voter is informed is less than $\frac{1}{2}$. Henceforth, we assume $\delta \in (0, \frac{1}{2})$. This assumption ensures that in a large electorate more than $\frac{1}{2}$ of the voters are uninformed and therefore uninformed voters can be decisive.

The states in which half of the electorate is expected to vote for either candidate, given the strategy x, play an important role in limit equilibria. We refer to those states as *marginal states*. Hence, $s^p(x)$, policy p's marginal state at x is the state at which a randomly drawn voter chooses B with probability $\frac{1}{2}$ if B chooses policy p. That is, $s^p(x)$ is the unique solution to

$$\pi^p(x, s) = \tfrac{1}{2}$$

Define,

$$\underline{x} := \frac{1}{2(1-\delta)}$$

and note that $\pi^l(\underline{x}, 0) = \frac{1}{2}$ and therefore l's marginal state at $\underline{x}$ is zero. For $x < \underline{x}$, l's marginal state at x is not well defined. For $x \in [\underline{x}, 1]$, we have:

$$\begin{aligned} s^l(x) &= 1 - \frac{1}{2(1-\delta)x} \\ s^m(x) &= 1 - \frac{1}{2((1-\delta)x + \delta)} \end{aligned} \tag{4}$$

Clearly, both s^l and s^m are increasing functions of x. Moreover,

$$s^l(x) < s^m(x) \leq \tfrac{1}{2}$$

Proposition 3 below establishes that the limit equilibrium cutoff for B is equal to l's marginal state $s^l(x)$.

Proposition 3: *If (x, y) is a limit equilibrium and $\phi \in \Phi(x, y)$, then $x \in [\underline{x}, 1], y = s^l(x)$ and*

$$\begin{aligned} \phi^l(s) = 1 &\qquad \text{if } s < s^l(x) \\ \phi^{m_b}(s) = 1 &\qquad \text{if } s^l(x) < s < s^m(x) \\ \phi^{m_a}(s) = 1 &\qquad \text{if } s^m(x) < s \end{aligned}$$

Proof: see Appendix

For $s < s^l(x)$, Proposition 3 establishes that candidate B chooses the partisan policy l and wins the election. The probability that a randomly selected voter will vote for candidate B if B chooses l is greater than $\frac{1}{2}$ at any state $s < s^l(x)$. Hence, in a large electorate, B wins with probability (close to) 1 when he chooses l, and since he strictly prefers l to m, he chooses l. Therefore, $\phi^l(s) = 1$ at states $s < s^l(x)$.

Proposition 3 also asserts that candidate B chooses the moderate policy at state $s > s^l(x)$. At such states, B loses the election with probability close to 1 if he chooses l. In that case, B is better off if he chooses m and thereby secures the vote of informed agents who prefer his personality. At $s \in (s^l(x), s^m(x))$, informed agents are decisive and candidate B wins the election. At $s > s^m(x)$, candidate B chooses m and loses the election. The following figure summarizes Proposition 3.

–Insert figure 1 here–

Proposition 3 implies that the probability of outcome m_b is small whenever the probability that a voter is informed is small. To see this, note that m_b is the outcome in states $s \in (s^l(x), s^m(x))$ and therefore is chosen with probability $G(s^m(x)) - G(s^l(x))$. From the definition of s^l and s^m (equation (4)), it follows that $s^m(x) - s^l(x)$ converges to 0 uniformly in $x \in [\underline{x}, 1]$ as δ goes to 0. Therefore, the probability of outcome m_b is arbitrarily small when δ is arbitrarily small.

For $x \in [\underline{x}, 1]$, we define $\ell(x)$ as follows:

$$\ell(x) := \frac{g(s^l(x))}{g(s^m(x))} \cdot \left(\frac{(1-\delta)x + \delta}{(1-\delta)x} \right)^2 \tag{5}$$

Proposition 4 characterizes the voters' equilibrium strategy. We say that $x \in [\underline{x}, 1]$ is a critical point if

$$(1-\mu)\ell(x) \begin{cases} \leq \epsilon & \text{for } x = 1 \\ = \epsilon & \text{for } x \in (\underline{x}, 1) \\ \geq \epsilon & \text{for } x = \underline{x} \end{cases} \tag{6}$$

A critical point $x \in (\underline{x}, 1)$ is *regular* if $l'(x) \neq 0$. (Note that $l(x)$ is differentiable since g is differentiable). A critical point $x \notin (\underline{x}, 1)$ is *regular* if (6) holds with strict inequality.

Proposition 4: *If $(x, s^l(x))$ is a limit equilibrium, then x is a critical point. Conversely, if x is a regular critical point, then $(x, s^l(x))$ is a limit equilibrium.*

Proof: see Appendix

To get some intuition for Proposition 4, assume for the moment that there are only two states of the electorate s_l, s_m with $s_l < s_m$ and prior probabilities g_l, g_m respectively. Assume also that B chooses l in state s_l and m in state s_m. Furthermore, assume that the voter's strategy x is such that s^l and s^m are marginal states, i.e., a randomly chosen voter votes for B with probability $\frac{1}{2}$ in both states. (Hence, s^l, s^m, x satisfy Equation (4) above.)

Let θ be the probability of s_l conditional on a b-type's information and conditional on being pivotal. Note that the probability that a given b-type voter is pivotal is the same in either state s_l or s_m. Moreover, the probability of being a b-type voter in state s is equal to $1 - s$. Therefore,

$$\theta = \frac{g_l(1-s_l)\binom{2n}{n}2^{-2n}}{g_l(1-s_l)\binom{2n}{n}2^{-2n} + g_m(1-s_m)\binom{2n}{n}2^{-2n}} = \frac{g_l(1-s_l)}{g_l(1-s_l) + g_m(1-s_m)} \tag{7}$$

If $x \in (0, 1)$ is optimal, then a b-type voter must be indifferent between A and B. Hence,

$$1 = (1-\theta)(1+\epsilon) \tag{8}$$

where the left hand side is the expected utility of voting for A and the right hand side the expected utility of voting for B (conditional on a vote being pivotal and on a b-type's information). Equations (4), (7), and (8) imply

$$\begin{aligned} \frac{\theta}{1-\theta} &= \frac{g_l}{g_m} \cdot \frac{1-s^l}{1-s^m} \\ &= \frac{g_l}{g_m} \cdot \frac{(1-\delta)x + \delta}{(1-\delta)x} \end{aligned} \tag{9}$$

Then, equations (8) and (9) yield

$$\frac{g_l}{g_m} \cdot \frac{(1-\delta)x+\delta}{(1-\delta)x} = \epsilon \tag{10}$$

In our model, there are many states. However, as the number of voters becomes large, the probability of being pivotal is concentrated around the marginal states, $s^l(x)$ and $s^m(x)$. Hence, conditional on being pivotal and on his information, a b-type voter knows that the state is in one of two small "critical intervals" around the marginal states. Therefore, the relative likelihood g_l/g_m in Equation (10) has to be replaced by the relative likelihood of the events

$$E_l := [s : \pi^l(s,x) \text{ is close to } 1/2 \text{ and } B \text{ chooses } l]$$

and

$$E_m := [s : \pi^m(s,x) \text{ is close to } 1/2 \text{ and } B \text{ chooses } m]$$

Consider candidate B's incentives when $\pi^l(s,x)$ is close to $1/2$. If B chooses m he wins with probability close to one because the added vote of the informed voters imply a vote share greater than $1/2$. Therefore, if $B_n(\pi^l(s,x)) < \mu$, then B strictly prefers the moderate policy m. We conclude that the interval E_l is truncated at the state where the probability of winning drops below μ. Hence, the closer the parameter μ is to 1, the smaller is the interval E_l. The key step in the proof is showing that the relative likelihood of E_l and E_m converges to

$$(1-\mu)\frac{g(s^l(x))}{g(s^m(x))}\frac{(1-\delta)x+\delta}{(1-\delta)x}$$

at the number of voters becomes large. The $(1-\mu)$ term arises from the truncation described above and the term $\frac{(1-\delta)x+\delta}{(1-\delta)x}$ results from the change of variables from π to s. Substituting the relative likelihood of E_l and E_m for g^l/g^m in equation (10) yields

$$(1-\mu)\cdot\frac{g(s^l(x))}{g(s^m(x))}\cdot\left(\frac{(1-\delta)x+\delta}{(1-\delta)x}\right)^2 = (1-\mu)\ell(x) = \epsilon$$

and therefore x is a critical point as defined in (6).

3.1 Limit Equilibria when the Candidate is an Office Seeker

In this section, we consider an office seeking candidate B who cares mostly about winning the election; that is μ is close to one. Note, however, that even an office seeker has a *strict* preference for the partisan policy (i.e., $\mu < 1$). Proposition 5 below establishes that if B is an office seeker, then all uninformed b-types will vote for him.

Proposition 5: *There is $\bar{\mu} < 1$ (independent of δ) such that for $\mu > \bar{\mu}$, the unique limit equilibrium is $(1, \frac{1-2\delta}{2-2\delta})$ and the corresponding limit outcome ϕ satisfies*

$$\phi^o(s) = \begin{cases} 1 & \text{if } o = l, \quad s \in \left[0, \frac{1-2\delta}{2-2\delta}\right) \\ 1 & \text{if } o = m_b, s \in \left(\frac{1-2\delta}{2-2\delta}, \frac{1}{2}\right) \\ 1 & \text{if } o = m_a, s \in \left(\frac{1}{2}, 1\right] \end{cases}$$

.

Proof: Note that $\ell(x)$ is uniformly bounded for all $\delta \in [0, \frac{1}{2}]$ and all $x \in [\underline{x}, 1]$. Therefore, Proposition 4 ensures the existence of $\bar{\mu}$ such that $\mu > \bar{\mu}$ implies $x = 1$. The corresponding ϕ follows from Proposition 3. $\square$

The intuition for Proposition 5 is straightforward: an office seeker will choose the moderate policy whenever the election is close and therefore, an uninformed voter believes that conditional on his vote being pivotal, it is very likely that B will choose m. But then, all uninformed b-types vote for B.

Although we have assumed that voters are strategic, equilibrium behavior – as characterized in Proposition 5 – seems very naive: uninformed voters simply vote according to their personality preference. This behavior is optimal because voters expect candidate B to choose the moderate policy whenever the election is close.

Note that as in the informed electorate benchmark analyzed in Proposition 2, candidate B is elected whenever $s < \frac{1}{2}$, i.e., whenever he has a personality advantage. However, unlike the informed electorate benchmark, Proposition 5 shows that there is a strictly positive probability that the outcome is l, the partisan policy. Recall that *all* voters strictly prefer m to l irrespective of who implements m. Therefore, the outcome l represents *aggregation failure*, i.e., a failure of to choose the median preferred alternative.

For $o \in \{l, m_a, m_b\}$, let $\bar{\phi}^o = E[\phi^o]$ denote the ex ante probability of outcome o. Proposition 5 shows that if B is an office seeker, the ex ante probability of outcome l is

$$\bar{\phi}^l = G\left(\tfrac{1-2\delta}{2-2\delta}\right) \tag{11}$$

and the probability of outcome m_b is

$$\bar{\phi}^{m_b} = G\left(\tfrac{1}{2}\right) - G\left(\tfrac{1-2\delta}{2-2\delta}\right) \tag{12}$$

The ex ante probability that B wins the election is the sum of $\bar{\phi}^l$ and $\bar{\phi}^{m_b}$ and therefore is equal to $G(\frac{1}{2})$. The ex ante probability of outcome m_a is

$$\bar{\phi}^{m_a} = 1 - G\left(\tfrac{1}{2}\right)$$

Equation (11) implies that $\bar{\phi}^l$ is decreasing in δ and $\lim_{\delta \to 0} \bar{\phi}^l = G(\frac{1}{2})$. Equation (12) implies that $\bar{\phi}^{m_b}$ is increasing in δ and $\lim_{\delta \to 0} \bar{\phi}^{m_b} = 0$. Hence, when δ is small, the probability of m_b is close to zero and the probability of l is close to its maximum probability $G(\frac{1}{2})$. Conversely, we have $\lim_{\delta \to \frac{1}{2}} \bar{\phi}^l = 0, \lim_{\delta \to \frac{1}{2}} \bar{\phi}^{m_b} = G(\frac{1}{2})$. Hence, as the proportion of informed voters approaches $\frac{1}{2}$, the election outcome approaches the Proposition 2 benchmark: the informed voters are almost always pivotal and candidate B chooses m to get elected when he has the personality advantage.

Let u_b denote the expected utility of a $b-$type in a limit equilibrium. Hence,

$$u_b = \bar{\phi}^{m_a} + (1+\epsilon)\bar{\phi}^{m_b}$$

Note that if candidate A wins the election ($\bar{\phi}^{m_a} = 1$) then $u_b = 1$. Substituting for $\bar{\phi}^{m_a}$ and $\bar{\phi}^{m_b}$ yields

$$u_b = 1 - G\left(\tfrac{1-2\delta}{2-2\delta}\right) + \epsilon\left(G\left(\tfrac{1}{2}\right) - G\left(\tfrac{1-2\delta}{2-2\delta}\right)\right) \tag{13}$$

Proposition 6 shows that for small ϵ, the equilibrium payoff of b-types is less than 1, that is, less than the payoff when A is elected.

Proposition 6: *There is $\bar{\mu} < 1$ such that $\epsilon < \frac{G\left(\frac{1-2\delta}{2-2\delta}\right)}{G\left(\frac{1}{2}\right) - G\left(\frac{1-2\delta}{2-2\delta}\right)}$ and $\mu > \bar{\mu}$ implies $u_b < 1$.*

Proof: Proposition 6 follows from Equation (13) and Proposition 5. □

When ϵ is small and candidate B is an office seeker, uninformed b-types vote for B despite the fact that electing B implies, on average, a lower payoff than electing A. An observer who interprets the behavior of uninformed b-types as non-strategic may conclude that b-types vote "against their interests" and put an unreasonably large weight on the candidates' personalities. However, this analysis misses the point that average payoffs are not the correct criterion for strategic voters.

3.2 Limit Equilibria when the Candidate is a Partisan

Next, we analyze the opposite case in which B has a strong partisan preference. Proposition 7 shows that if μ is sufficiently small, then candidate B always chooses the moderate policy and wins only if he has a significant personality advantage.

Proposition 7: *There are $\underline{\mu} > 0$ and $\underline{\epsilon} > 0$ (independent of δ) such that for $\mu < \underline{\mu}, \epsilon < \underline{\epsilon}$, the unique limit equilibrium is $(\underline{x}, 0)$ and the corresponding limit outcome ϕ satisfies*

$$\phi^o(s) = \begin{cases} 0 & \text{if } o = l, \quad \forall s \in [0,1] \\ 1 & \text{if } o = m_b, s \in \left(0, \frac{2\delta}{1+2\delta}\right) \\ 1 & \text{if } o = m_a, s \in \left(\frac{2\delta}{1+2\delta}, 1\right] \end{cases}$$

.

Proof: Note that ℓ is uniformly bounded for all $\delta \in (0,1)$ and $x \in [\underline{x}, 1]$. Therefore, Proposition 4 implies that there are $\underline{\mu}$ and $\underline{\epsilon} > 0$ such that for $\mu > \underline{\mu}$, $\epsilon < \underline{\epsilon}$, $x = \underline{x}$ in any limit equilibrium. The corresponding ϕ follows from Proposition 3. □

When B is a partisan, voters expect him to choose l even if the election is reasonably close. As a result, uninformed voters are reluctant to vote for him. This, in turn, forces B to choose m, since otherwise he is almost sure to lose the election. Proposition 7 may seem paradoxical because candidate B never chooses l yet voters assume that conditional on a vote being pivotal there is a significant probability that he chooses l. Note that Proposition 7 describes the limit of a sequence of equilibria (x_n, y_n) such that y_n converges to 0. Along the sequence, B chooses l if the state s is close to 0. Hence, for any finite electorate, the probability that B chooses l is strictly positive and conditional on a vote being pivotal the probability that B chooses l stays bounded away from 0 for all n.

The probability that a partisan wins the election is equal to $\bar{\phi}^{m_b}$, the probability of outcome m_b. Proposition 7 shows that

$$\bar{\phi}^{m_b} = G\left(\tfrac{2\delta}{1+2\delta}\right)$$

Hence, the ex ante probability that a partisan wins the election is smaller than in the benchmark Downsian case. Moreover, that probability is increasing in the fraction of informed voters. We have

$$\lim_{\delta \to \frac{1}{2}} \bar{\phi}^{m_b} = G\left(\tfrac{1}{2}\right), \lim_{\delta \to 0} \bar{\phi}^{m_b} = 0$$

Hence, when the proportion of informed voters is small, a partisan almost never wins the election. As the proportion of informed voters approaches $\frac{1}{2}$, we converge to the Proposition 2 benchmark.

Proposition 7 demonstrates a second type of aggregation failure. In states s such that $\frac{2\delta}{1+2\delta} < s < \frac{1}{2}$, candidate B does not win the election even though he has chosen the moderate policy and has a personality advantage. Hence, in those states, B is the median preferred alternative yet loses the election.

3.3 A Uniform Example

When G is uniform, Proposition 4 yields a simple characterization of limit equilibria. In the uniform case,

$$\ell(x) = \left(\frac{(1-\delta)x + \delta}{(1-\delta)x}\right)^2$$

Assume that $\mu = \frac{7}{9}$ and $\epsilon = \frac{1}{2}$. Depending on δ, we can distinguish three cases.

Case (i) $\delta < \frac{1}{4}$: Uninformed b-types vote for B with probability 1 in the unique limit equilibrium (see section 3.1). Hence, the equilibrium strategies are

$$x = 1, y = \frac{1-2\delta}{2-2\delta}$$

The equilibrium payoff of a b-type is

$$u_b = \frac{2+\delta}{4(1-\delta)}$$

and therefore $u_b < 1$, for $\delta < \frac{1}{4}$. Note that u_b is increasing in δ and $u_b = 1/2$ for $\delta = 0$. In this equilibrium, candidate B is elected whenever he has a personality advantage (i.e., with probability $\frac{1}{2}$). Type b voters' expected payoff is below 1. Hence, for all voters, election B is worse *on average* than electing A.

Case (ii) $\delta > \frac{1}{3}$: The unique limit equilibrium corresponds to the case analyzed in section 3.2:

$$x = \underline{x}, y = 0$$

Hence, B chooses the moderate policy in all states and b-types choose the strategy $\underline{x}$. For $s \in [0, \frac{2\delta}{1+2\delta})$, the election outcome is m_b and for $s \in (\frac{2\delta}{1+2\delta}, 1]$, it is m_a. In this case,

$$u_b = 1 + \frac{\delta}{1 + 2\delta}$$

Note that u_b is increasing in δ and strictly greater than 1. Hence, b-types are better off on average if B is elected.

Case (iii) $\frac{1}{4} < \delta < \frac{1}{3}$: There are three limit equilibria; the two extreme equilibria identified in cases (i) and (ii) above and the interior equilibrium

$$x = \frac{2\delta}{1 - \delta}, y = \frac{4\delta - 1}{4\delta}$$

In this equilibrium, B chooses the partisan policy when

$$s < \frac{4\delta - 1}{4\delta} \tag{14}$$

Note that the right hand side of (14) is *increasing* in δ and hence a higher fraction of informed voters makes B more likely to choose the partisan policy (within the range given for this case). The utility of a b-type is

$$u_b = \frac{3 - 6\delta}{8\delta} + \frac{1}{2}$$

which is *decreasing* in δ.

The interior equilibrium is like a mixed equilibrium in a coordination game. Fixing the strategies of players, the larger the fraction of informed voters, the more attractive candidate A becomes. This follows because $\ell(x)$ is increasing in δ. To keep uninformed voters indifferent despite the higher fraction of informed voters, more uninformed b-types must vote for B, i.e., x must increase. (Recall that $\ell(x)$ is decreasing in x). More uninformed b-types voting for B, in turn, implies more partisan outcomes (a higher y). Hence, increasing the fraction of informed voters increases the probability of partisan outcomes and decreases b-types' payoff along the interior equilibrium trajectory.

The uniform example demonstrates that multiplicity of limit equilibria can occur. The example also reveals an indirect effect of the parameter δ on election outcomes. For a fixed μ, ϵ, a large δ implies that only the partisan equilibrium, $(\underline{x}, 0)$, exists while a small δ implies that only the office seeker equilibrium, $(1, \frac{1-2\delta}{2-2\delta})$, exists. Intermediate δ values create the possibility of an interior equilibrium with counter-intuitive comparative statics.

4. Robustness and Extensions

In this section we examine how changing our model affects the results of the previous section.

4.1 Control of Information

So far, we have assumed that candidates cannot affect voters' information. Here, we briefly analyze an extension in which B chooses the fraction of informed voters. Since a voter is unlikely to be pivotal, he has little incentive to acquire information. Hence, there is a tendency for voters to remain ignorant. Our objective is to investigate if candidates have incentives to combat this tendency.

As in the previous section, we assume that B chooses a policy $p \in \{l, m\}$. Candidate B also chooses the fraction of voters $\delta^* \in \{\delta, \Delta\}$, where $0 < \delta < \Delta < \frac{1}{2}$, who will be informed of his policy choice. We assume that voters cannot observe δ^*.

One interpretation of this model is the following. Suppose B must decide how many informative campaign commercials to run. The more commercials are run, the more likely it is that a voter observes the policy choice.

Since all voters strictly prefer m to l and voters never use weakly dominated strategies, B will choose $\delta^* = \Delta$ whenever he chooses the moderate policy m and $\delta^* = \delta$ whenever he chooses the partisan policy.[9]

The following proposition establishes that when μ is close to 1, the equilibrium outcome is *as if* δ^* is fixed at δ. That is, Proposition 8 identifies the same limit equilibrium as Proposition 5. In other words, B's ability to disclose additional information (choose $\delta^* = \Delta$) has no effect if he is an office seeker. Limit equilibria and limit outcomes are defined as in the previous section.

Proposition 8: *There is $\bar{\mu} < 1$ (independent of δ) such that for $\mu > \bar{\mu}$, $(1, \frac{1-2\delta}{2-2\delta})$ is the unique limit equilibrium and the corresponding limit outcome ϕ satisfies*

$$\phi^o(s) = \begin{cases} 1 & \textit{if } o = l, \quad s \in \left[0, \frac{1-2\delta}{2-2\delta}\right) \\ 1 & \textit{if } o = m_b, s \in \left(\frac{1-2\delta}{2-2\delta}, \frac{1}{2}\right) \\ 1 & \textit{if } o = m_a, s \in \left(\frac{1}{2}, 1\right] \end{cases}$$

.

Proof: See Appendix.

For μ close to 1, the fact that B informs voters whenever he chooses m has no effect on the equilibrium outcome even though voters know that they are more likely to be uninformed whenever B chooses l. Proposition 8 shows that if δ is close to 0 – that is, if voters remain ignorant unless B voluntarily discloses information – equilibrium is *as if* voters are uninformed. Candidate B chooses l and wins the election whenever he has a personality advantage. All uninformed voters who prefer B's personality vote for him. Hence, when candidate B has control over information, the effect described in section 3 is exacerbated.

4.2 Multiple Policies

Suppose candidate B chooses among three policies $p \in \{l_1, l_2, m\}$ and he prefers l_1 to l_2 and l_2 to m. If all voter types prefer m_a to l_2 and l_1, then our analysis remains

[9] Choosing (l, Δ) could be optimal if $x = 0$. But, if all uninformed voters vote for A, then B's unique best response is to choose (m, Δ). Hence, B cannot choose (l, Δ) in equilibrium.

unchanged. Candidate B will choose either policy l_1 or policy m. He will never choose the intermediate policy l_2 because choosing l_2 does not win any informed voters.

The many policy extension is more interesting if some b-type voters have more intense personality preferences than others. Suppose weak b-types prefer B only if he chooses m and strong b-types prefer B if he chooses l_2 or m. Then, in any the state, B will choose the most partisan policy that guarantees a vote share greater than $\frac{1}{2}$. Hence, candidate B may choose all three policies. Such a model is more difficult to analyze because the inference conditional on a vote being pivotal is more complicated. However, if B is an office seeker, he will switch to a less partisan policy whenever the probability of losing exceeds some small threshold. Hence, as in our two-policy model, conditional on a vote being pivotal, B is unlikely to choose l_1 or l_2 and hence uninformed voters will believe that B chooses m with probability close to one. This implies that all uninformed b-types (strong and weak) will vote for B. We conclude that the analysis of section 4 is valid even if candidate B has multiple policy options.

4.3 Both Candidates have Policy Choice

In our model, candidate A is committed to the policy that all voters prefer. One could extend our model so that A chooses between m and a partisan policy r. As noted in section 4.2, the inference conditional on a vote being pivotal is more difficult with more than two critical states. However, any candidate who is an office seekers will switch to the moderate policy when the probability of losing the election with the partisan policy exceeds some small threshold. As a result, conditional on a vote being pivotal, voters expect both candidates to choose the moderate policy. Hence, we conjecture that a symmetric version of our model would yield a result analogous to Proposition 5.

4.4 Uninformed Candidates

The assumption that candidates have better information about voter preferences plays a central role in our analysis. To illustrate the importance of this assumption, we modify the election game so that candidate B cannot observe the parameter s. For simplicity, we assume that G is uniform on $[0, 1]$. Thus, neither candidate B nor the voters know the distribution of preferences. This means that B's policy choice cannot depend on s and is simply his probability of choosing m. Voters strategies are the same as in section 3. Limit equilibria (x, z) and the corresponding limit outcomes $\Phi^s(x, z)$ are defined as in section 3.

Proposition 9 below shows that if μ is above $\frac{1-2\delta}{1-\delta}$, then $x = 1, z = 1$ is the unique equilibrium. That is, the game where neither voters nor candidate B knows the distribution of preferences yields again the Proposition 2 benchmark: there is no aggregation failure and no partisan politics.

Proposition 9: *If candidate B is uninformed, then $(1, 1)$ is the unique limit equilibrium and every limit outcome is Downsian.*

Proof: See Appendix.

There is an $\varepsilon > 0$ such that given any voter strategy x, the probability that B wins if he chooses m (without knowing the state) is at least ε higher than if he chooses l. Therefore, an office seeker will always choose m and all b-types will voter for him. Hence, partisan politics and aggregation failure cannot occur if B is an uninformed office seeker. This model re-produces the standard Downsian prediction of median preferred policy outcomes.

4.5 Informed Voters

In this section, we assume that both voters and candidates observe the state s before they choose their actions. A strategy profile is a pair (X, Z), where $X(s)$ is the probability that an uninformed b-type votes for B given state s and $Z(s)$ is the probability with which candidate B chooses m in state s. When $X \equiv x$ or $Z \equiv z$ are constant functions, we write (x, z) rather than (X, Z). The definitions of π and of ϕ are unchanged. Proposition 10 shows that median preferred outcomes result even if δ is small.

Limit equilibria and the corresponding limit outcomes are defined as above. Proposition 10 states that if the voter know the distribution of preferences, then the moderate policy m is implemented with probability 1. Hence, the fact that most voters do not know B's policy choice has virtually no impact on the election outcome. Moreover, candidate B is elected if and only if he has a personality advantage. That is, personality affects who gets elected but does not influence the policy outcome.

Proposition 10: *If voters and candidates both observe the state, then $(1, 1)$ is the only limit equilibrium and every limit outcome is Downsian.*

Proof: See Appendix.

Together, Propositions 9 and 10 show that voter ignorance by itself cannot yield partisan politics and aggregation failure; non-Downsian outcomes can occur only if candidates have better information regarding the state of the electorate than the typical voter.

We can contrast Proposition 10 with the Feddersen and Pesendorfer (1997) information aggregation results. Suppose that B's strategy $z \in (0,1)$ is fixed and $s < \frac{1}{2}$. If m is realized, then most voters prefer B (when n is large) whereas if l is realized, the majority prefers A. The Feddersen and Pesendorfer result implies that for a *fixed* strategy z and large n, candidate B is elected with probability close to 1 if the realized action is m and candidate A is elected with probability close to 1 if the realized action is l.

In our model, the strategy z is not fixed but endogenous. Proposition 10 pins down both voter and candidate behavior. For $s < \frac{1}{2}$ and n large, B chooses l with positive probability and conditional on choosing l wins with positive probability. Nevertheless, information is aggregated because the probability that B chooses l converges to 0.

Next, we provide intuition for Proposition 10. Recall that π^o denotes the probability that a randomly chosen voter votes for B if B chooses policy $o \in \{l, m\}$. If π^l is less than $\frac{1}{2}$ and bounded away from $\frac{1}{2}$, then B strictly prefers m to l when n is large. This is clear if B's vote share is greater than $\frac{1}{2}$ conditional on m, which would mean that he wins for sure with m and loses for sure with l. If his vote share is less than $\frac{1}{2}$ in both cases, then his probability of winning goes to 0 with either policy, but it goes to 0 much faster if he chooses l than if he chooses m. Hence, in both cases, B strictly prefers m to l.

The second step is to note that for large n, candidate B must mix in equilibrium. If B were to choose l for sure, then $\pi^l = 0$. Then, by the argument above, he strictly prefers m. If B were to choose m for sure, then the uninformed voters would guarantee victory for B no matter which policy he chooses, i.e., $\pi^l > \frac{1}{2}$. Then, B strictly prefers l.

The third step is to observe B's indifference between l and m, implies that π^l converges to $\frac{1}{2}$. If π^l stays bounded above $\frac{1}{2}$, then B wins for sure with l and hence would never choose m. If π^l stays bounded below $\frac{1}{2}$, then the rate of convergence argument above establishes that B strictly prefers m, contradicting the fact that he must be indifferent between l and m.

Finally, since the probability of winning with l converges to $\frac{1}{2}$ (and therefore the probability of winning with m converges to 1), conditional on a vote being pivotal, it is much more likely that B has chosen l than m. Therefore, to maintain the uninformed voters' incentives, B must choose l with vanishing probability as n goes to infinity. Hence, in large electorates, B will choose m almost all the time and almost always wins when he has a personality advantage.

5. Conclusion

We have analyzed how candidate competition is altered when most voters do not know the policy choices. We show that when a candidate is an office seeker with a weak partisan preference, voter ignorance will enable him to implement partisan policies without suffering a reduced probability of winning the election. Uninformed voters behave as if the office seeker always chooses the median preferred policy.

One consequence of this effect is that candidates have little incentive to spend resources on informing voters. Providing an office seeker with the opportunity to inform voters costlessly has does not affect the equilibrium outcome. As long as voters are convinced that a candidate will "do what it takes" to get elected, voter ignorance does not harm his chance of getting elected. At the same time, a poorly informed electorate allows the candidate to choose policies that closer match his policy preference.

To simplify the exposition, we have considered a one-sided model in which only candidate B has a non-trivial policy choice and assumed that candidate A is committed to the median preferred policy. Hence, A provides the stiffest possible competition for the opportunistic candidate B. As our main result shows, even in this case, the median preferred moderate policy may not be implemented.

6. Appendix

Lemma 1: $B_n(z) = \frac{\int_0^z \theta^n(1-\theta)^n d\theta}{\int_0^1 \theta^n(1-\theta)^n d\theta}$

Proof: (i) The binomial theorem implies that

$$\int_0^z \theta^n(1-\theta)^n d\theta = \int_0^z \sum_{k=0}^{n} \binom{n}{k} (-1)^k \theta^{n+k} d\theta = \sum_{k=0}^{n} \binom{n}{k} \frac{z^{n+k+1}}{n+k+1} (-1)^k \qquad (A1)$$

Next, we show that

$$B_n(z) = \frac{(2n+1)!}{n!n!} \sum_{k=0}^{n} \binom{n}{k} \frac{z^{n+k+1}}{n+k+1} (-1)^k \qquad (A2)$$

The binomial theorem yields

$$\begin{aligned} B_n(z) &= \sum_{k=n+1}^{2n+1} \binom{2n+1}{k} z^k (1-z)^{2n+1-k} \\ &= \sum_{k=n+1}^{2n+1} \sum_{m=0}^{2n+1-k} z^{k+m} (-1)^m \binom{2n+1-k}{m} \binom{2n+1}{k} \end{aligned}$$

Hence, letting $t = m + k$, $r = t - (n+1)$ and rearranging terms yields

$$\begin{aligned} B_n(z) &= \sum_{k=n+1}^{2n+1} \sum_{t=k}^{2n+1} z^t (-1)^{t-k} \binom{2n+1-k}{t-k} \binom{2n+1}{k} \\ &= \sum_{t=n+1}^{2n+1} z^t \sum_{k=n+1}^{t} \binom{2n+1-k}{t-k} (-1)^{t-k} \binom{2n+1}{k} \\ &= \sum_{t=n+1}^{2n+1} \frac{(2n+1)!\, z^t}{(2n+1-t)!} \sum_{r=0}^{t-(n+1)} \frac{(-1)^{t-r-(n+1)}}{(t-r-(n+1))!(r+n+1)!} \end{aligned}$$

Feller (1967) pg 65 provides the following identity:

$$\binom{a}{b} - \binom{a}{b-1} + \binom{a}{b-2} \cdots \binom{a}{0} = \binom{a-1}{b}$$

Hence, the last equation implies

$$\begin{aligned}
B_n(z) &= \sum_{t=n+1}^{2n+1} \frac{(2n+1)!\, z^t}{(2n+1-t)!} \frac{(-1)^{t-(n+1)}}{t!} \frac{(t-1)!}{(t-(n+1))!n!} \\
&= \frac{(2n+1)!}{n!n!} \sum_{t=n+1}^{2n+1} \frac{(-1)^{t-(n+1)} n!\, z^t}{(2n+1-t)!(t-(n+1))!\, t} \\
&= \frac{(2n+1)}{n!n!} \sum_{k=0}^{n} \binom{n}{k} \frac{z^{n+k+1}}{n+k+1} (-1)^k
\end{aligned}$$

We conclude from $(A1)$ and $(A2)$ that

$$A \cdot \frac{\int_0^z \theta^n (1-\theta)^n d\theta}{\int_0^1 \theta^n (1-\theta)^n d\theta} = B_n(z)$$

for some constant $A > 0$. Clearly, $A = 1$ since

$$1 = B_n(1) = A \cdot \frac{\int_0^1 \theta^n (1-\theta)^n d\theta}{\int_0^1 \theta^n (1-\theta)^n d\theta} = A$$

which proves part (i). $\square$

Lemma 2: *Let $\bar{z} > z, s \in [0,1)$. Then, $\frac{B_n((1-s)z)}{B_n((1-s)\bar{z})}$ is strictly decreasing in s.*

Proof: Clearly it suffices to show that $\ln B_n((1-s)z) - \ln B_n((1-s)\bar{z}))$ is strictly decreasing in s. Hence, we must show that

$$\frac{d}{dy} \frac{y B_n'(y)}{B_n(y)} < 0 \tag{A6}$$

By Lemma 1, $(A6)$ is equivalent to

$$\frac{((n+1)(1-y) - ny) \int_0^y \zeta^n (1-\zeta)^n d\zeta - y^{n+1}(1-y)^{n+1}}{(\int_0^y \zeta^n (1-\zeta)^n d\zeta)^2} < 0 \tag{A8}$$

Substituting $v = \zeta - \zeta^2$ yields $\int_0^y \zeta^n (1-\zeta)^n (1-2\zeta) d\zeta = \int_0^{y-y^2} v^n dv = \frac{y^{n+1}(1-y)^{n+1}}{n+1}$. Hence, proving $(A8)$ is equivalent to proving

$$h(y) := \int_0^y \zeta^n (1-\zeta)^n \left(2\zeta - \frac{2n+1}{n+1} y\right) d\zeta < 0$$

Since $h(0) = 0$, to conclude the proof, it suffices to show that $h'(y) < 0$ for $y \in (0, 1)$.

$$\begin{aligned}(n+1)\frac{\partial}{\partial y}\hat{h}(y) &= y^{n+1}(1-y)^n - (2n+1)\int_0^y \zeta^n(1-\zeta)^n d\zeta \\ &= y^{n+1}(1-y)^n - \frac{2n+1}{n+1}y^{n+1}(1-y)^n - \frac{n(2n+1)}{n+1}\int_0^y \zeta^{n+1}(1-\zeta)^{n-1}d\zeta \\ &= -\frac{n}{n+1}y^{n+1}(1-y)^n - \frac{n(2n+1)}{n+1}\int_0^y \zeta^{n+1}(1-\zeta)^{n-1}d\zeta\end{aligned}$$

Hence, $h'(y) < 0$ as desired. □

6.1 Proof of Proposition 1

We will show that B's best response to voter strategy x is a cutoff strategy. A best response requires B to choose m whenever

$$\mu \cdot B_n(\pi^l(x,s)) < B_n(\pi^m(x,s))$$

and l if this inequality is reversed. To show that this yields a cutoff strategy, it suffices to show that $\frac{B_n(\pi^l(x,s))}{B_n(\pi^m(x,s))}$ is strictly decreasing in s. Recall that $\pi^l(x,s) = (1-s)(1-\delta)x$ and $\pi^m(x,s) = (1-s)[(1-\delta)x + \delta]$. Therefore, we can apply Lemma 2 to conclude that $\frac{B_n(\pi^l(x,s))}{B_n(\pi^m(x,s))}$ is strictly decreasing in s.

To prove that equilibrium exists, define

$$\tau_n(x,y) := \frac{\int_0^y \pi^l(x,s)^n(1-\pi^l(x,s))^n(1-s)g(s)ds}{\int_y^1 \pi^m(x,s)^n(1-\pi^m(x,s))^n(1-s)g(s)ds} \tag{A9}$$

Let $\tau_n(x,y) = \infty$ if the denominator in ($A9$) is 0. Note that $\tau_n(x,y)$ is the ratio of the probability that B chooses l to the probability that B chooses m for b-type voter given that he is pivotal; that is, $\tau_n(x,y) = \frac{\theta}{1-\theta}$. An uninformed $b-$type voter will (weakly) prefer candidate B if and only if

$$\tau_n(x,y) \leq \epsilon$$

Define

$$\rho_n(x,y) := \min[\max[x + \epsilon - \tau_n(x,y), 0], 1]$$

It follows that x is a best response to (x,y) if and only if $\rho_n(x,y) = x$.

Let $\sigma_n : [0,1] \to [0,1]$ be defined as follows:

$$\sigma_n(x) := \begin{cases} 1 & \text{if } \frac{B_n(\pi^l(x,1))}{B_n(\pi^m(x,1))} < \frac{1}{\mu} \\ 0 & \text{if } \frac{B_n(\pi^l(x,0))}{B_n(\pi^m(x,0))} > \frac{1}{\mu} \\ y \ \text{ such that } \frac{B_n(\pi^l(x,y))}{B_n(\pi^m(x,y))} = \frac{1}{\mu} & \text{otherwise.} \end{cases}$$

Note that σ_n is well-defined since $\frac{B_n(\pi^l(x,s))}{B_n(\pi^m(x,s))}$ is strictly decreasing and continuous in s. The cutoff $\sigma_n(x)$ is B's best response to x. Let $\nu_n(x,y) = (\rho_n(x,y), \sigma_n(x))$ for all $(x,y) \in [0,1] \times [0,1]$. The lemma below follows from the above observations:

Lemma 3: *The strategy profile is (x,y) is a cut-off equilibrium if and only if it is a fixed-point of ν_n.*

To conclude the proof of Proposition 1, note that $\rho_n : [0,1] \times [0,1] \to [0,1]$ is continuous. Since $\frac{B_n(\pi^l(x,s))}{B_n(\pi^m(x,s))}$ is continuous in (x,s), σ_n is also continuous. We conclude that a fixed-point of $\nu_n : [0,1] \times [0,1] \to [0,1] \times [0,1]$ exists. □

6.2 Proof of Proposition 2

If $\delta > \frac{1}{2}$, then $\pi^l(x,s) < \frac{1}{2}$ for all $x, s \in [0,1]$. Therefore, $B_n(\pi^l(x_n,0)) \to 0$ for any sequence of voting equilibria (x_n, y_n). Since $\pi^m(x,0) - \pi^l(x,0) = \delta > 0$, this implies that $B_n(\pi^l(x_n,0))/B_n(\pi^m(x_n,0)) \to 0$. Therefore, $B_n(\pi^l(x_n,0)) < \mu B_n(\pi^m(x_n,0)$ for large n. It follows $y_n = 0$ for large n. But $y_n = 0$ implies that $x_n = 1$ since B always chooses m. The characterization of outcomes follows from a straightforward application of the law of large numbers. □

6.3 Proof of Propositions 3 and 4

The proofs below will use the functions τ_n, ρ_n, σ_n as defined in the proof of Proposition 1. Recall that τ_n is the relative likelihood that candidate B will choose l as opposed to m, conditional on any voter i being pivotal and of type b. We extend the function $s^l : [\underline{x}, 1] \to [0,1]$ continuously to $[0,1]$ by setting $s^l(x) = 0$ for all $x < \underline{x}$.

Lemma 4: *The sequence σ_n converges uniformly to s^l.*

Proof: If not, there exists $\varepsilon > 0$ and a sequence x_n, such that $\sigma_n(x_n) - s^l(x_n) > \varepsilon$ for all n. Since the sequences $x_n, \sigma_n(x_n)$ both lie in compact sets, we can assume that they converge to some $x, y \in [0,1]$ respectively such that $|y - s^l(x)| \geq \varepsilon$. If $y - s^l(x) \geq \varepsilon$, then choose s', s'' such that $[s', s''] \subset (s^l(x), y)$ and note that there exist N such that for all $n \geq N$ candidate B chooses m in any state $s \in [s', s'']$ and loses with probability arbitrarily close to 1 even though he would have won with probability arbitrarily close to 1 had he chosen l, contradicting the fact σ_n is his best response. Similarly, if $s^l(x) - y \leq \varepsilon$, then choose s', s'' such that $[s', s''] \subset (s^l(x), \min\{y, s^m(x)\})$ and note that there exist N such that for all $n \geq N$, candidate B chooses l in any state $s \in [s', s'']$ and loses with probability arbitrarily close to 1 even though he would have won with probability arbitrarily close to 1 by choosing policy m, again, a contradiction. $\square$

Lemma 5: *Let (x_n, y_n) be a convergent sequence of equilibria converging to (x, y). Then, (i) $x \geq \underline{x}$ and $y < \frac{1}{2}$; (ii) $y_n > 0$ for n sufficiently large; (iii) $0 < y_n$ for all n implies $\lim B_n(\pi^l(x_n, y_n)) = \frac{1}{\mu}$.*

Proof: (i) If $x < \underline{x}$, then, $\pi^l(x_n, y_n) \leq \frac{1}{2} - \eta$ for some $\eta > 0$ and for n sufficiently large and therefore $\tau_n(x_n, y_n) \to 0$. Note that $\rho_n(x_n, y_n) > x_n$ whenever $x_n < 1$. But Lemma 3 ensures that $\rho_n(x_n, y_n) = x_n$, a contradiction.

(ii) Note that $s^l(x) < \frac{1}{2}$ for all x and therefore $y < \frac{1}{2}$ follows from Lemma 4. Suppose there is a subsequence $y_{n_j} = 0$ for all n_j. Then, $\tau_{n_j}(x_{n_j}, y_{n_j}) = 0$ and hence $x_{n_j} = 1$ for all n_j. Hence, Lemma 4 implies $\lim y_{n_j} = \lim s_l(x_{n_j}) = s_l(1) > 0$, a contradiction.

(iii) Part (ii) implies $0 < y_n < \frac{1}{2}$. Then, $\mu = \frac{B_n(\pi^l(x_n, y_n))}{B_n(\pi^m(x_n, y_n))}$ and Lemma 4 ensures that $\pi^l(x_n, y_n)$ converges to $\frac{1}{2}$. Therefore $B_n(\pi^m(x_n, y_n))$ converges to 1 and hence, $B_n(\pi^l(x_n, y_n))$ converges to $\frac{1}{\mu}$. $\square$

Lemma 6: *Assume (i) $\lim a_n = \frac{1}{2}$, $\lim \alpha_n = \alpha \in (\frac{1}{2}, 1]$, $\lim b_n = b \in [0, \frac{1}{2})$, and $\lim \beta_n = \beta \in (\frac{1}{2}, 1]$. (ii) $\{f_1, h_1, f_2, h_2, \ldots\}$ are equicontinuous functions on $[0,1]$ such that for some $c, C \in I\!R_+$ $c \leq f_n \leq C$, $c \leq h_n \leq C$ for all n, and (iii) $\lim f_n(\frac{1}{2})$, $\lim h_n(\frac{1}{2})$, $\gamma := \lim \frac{\int_{a_n}^1 z^n(1-z)^n dz}{\int_0^1 z^n(1-z)^n dz}$ exist. Then,*

$$\lim \frac{\int_{a_n}^{\alpha_n} z^n(1-z)^n f_n(z) dz}{\int_{b_n}^{\beta_n} z^n(1-z)^n h_n(z) dz} = \gamma \cdot \lim \frac{f_n(\frac{1}{2})}{h_n(\frac{1}{2})}$$

Proof: Define

$$q_n(z) = z^n(1-z)^n$$
$$Z_n(r,t) = \int_r^t q_n(z)dx$$

Step 1: $\lim r_n = r < t = \lim t_n$ and $\frac{1}{2} \notin [r,t]$ implies

$$\lim \frac{Z_n(r_n,t_n)}{Z_n(0,1)} = 0$$

Assume that $\frac{1}{2} < r$ and choose $y, y' \in (\frac{1}{2}, r)$ such that $y > y'$. (The proof $t < \frac{1}{2}$ is symmetric and omitted.) Note that q_n is a strictly quasiconcave function and attains its unique maximum at $\frac{1}{2}$. Hence, $q_n(x) \leq q_n(y) < q_n(y')$ for all $x \geq y$ and $q_n(x) \geq q_n(y')$ for all $x \in [1-y', y']$. Therefore, for n sufficiently large

$$\frac{Z_n(r_n,t_n)}{Z_n(0,1)} \leq \frac{(1-y)q_n(y)}{(2y'-1)q_n(y')} = \frac{(1-y)}{(2y'-1)} \cdot \left(\frac{q_1(y)}{q_1(y')}\right)^n$$

Since $\left(\frac{q_1(y)}{q_1(y')}\right) < 1$, step 1 follows.

Step 2: $\lim r_n = r < t = \lim t_n$ and $\frac{1}{2} \in (r,t)$ implies

$$\lim \frac{Z_n(r_n,t_n)}{Z_n(0,1)} = 1$$

Choose $\eta \in (0, \min\{\frac{1}{2} - r, t - \frac{1}{2}\}$. Then, for n large enough

$$1 \geq \lim \frac{Z_n(r_n,t_n)}{Z_n(0,1)} \geq \frac{Z_n(\frac{1}{2}-\eta, \frac{1}{2}+\eta)}{Z_n(0,1)} = 1 - \frac{Z_n(0, \frac{1}{2}-\eta)}{Z_n(0,1)} - \frac{Z_n(\frac{1}{2}+\eta, 1)}{Z_n(0,1)}$$

By step 1, the second and third terms on the right go to 0 as n goes to infinity, proving step 2.

Let

$$N_n = \int_{a_n}^{\alpha_n} q_n(z)f_n(z)dx$$
$$D_n = \int_{b_n}^{\beta_n} q_n(z)h_n(z)dx$$
$$T_n = \frac{N_n}{D_n}$$

Step 3: $\lim T_n = \gamma \cdot \frac{\lim f_n(\frac{1}{2})}{\lim h_n(\frac{1}{2})}$.

The equicontinuity of f_n, h_n ensures that for any $\eta > 0$, there exists $\eta' > 0$ such that for n large enough

$$[f_n(\tfrac{1}{2}) - \eta]Z_n(a_n, \tfrac{1}{2} + \eta') \leq N_n \leq [f_n(\tfrac{1}{2}) + \eta]Z_n(a_n, \tfrac{1}{2} + \eta') + CZ_n(\tfrac{1}{2} + \eta', 1)$$

$$[h_n(\tfrac{1}{2}) - \eta]Z_n(\tfrac{1}{2} - \eta', \tfrac{1}{2} + \eta') \leq D_n \leq [h_n(\tfrac{1}{2}) + \eta]Z_n(\tfrac{1}{2} - \eta', \tfrac{1}{2} + \eta') + CZ_n(0, \tfrac{1}{2} - \eta') + CZ_n(\tfrac{1}{2} + \eta', 1)$$

Using the expressions above to bound $\frac{N_n}{D_n}$, then dividing terms by $Z_n(0,1)$, letting n go to infinity and applying steps 1 and 2 yields

$$\frac{\lim f_n(\frac{1}{2}) - \eta}{\lim h_n(\frac{1}{2}) + \eta} \cdot \lim \frac{Z_n(a_n, 1)}{Z_n(0,1)} \leq \lim T_n \leq \frac{\lim f_n(\frac{1}{2}) + \eta}{\lim h_n(\frac{1}{2}) - \eta} \cdot \lim \frac{Z_n(a_n, 1)}{Z_n(0,1)}$$

Since the equation above holds for any η, we conclude that

$$\lim T_n = \frac{\lim f_n(\frac{1}{2})}{\lim h_n(\frac{1}{2})} \cdot \lim \frac{Z_n(a_n, 1)}{Z_n(0,1)} = \gamma \cdot \frac{\lim f_n(\frac{1}{2})}{\lim h_n(\frac{1}{2})}$$

as desired. □

Lemma 7: *Suppose (x_n, y_n) is a voting equilibrium for the game with $2n+1$ voters and (x_n, y_n) converges to (x, y). Then, (i) $\underline{x} < x$ implies $\lim \tau_n(x_n, y_n) = (1-\mu)\ell(x)$; (ii) $\underline{x} = x$ and $\eta > 0$ implies $\tau_n(x_n, y_n) \leq (1-\mu)\ell(x) + \eta$ for all n sufficiently large.*

Proof: Let $a_n := \pi^l(x_n, y_n), \alpha_n := \pi^l(x_n, 0), b_n := \pi^m(x_n, 1), \beta_n := \pi^m(x_n, y_n)$. Since (x_n, y_n) is convergent, it follows that $(a_n, \alpha_n, b_n, \beta_n)$ converges to some (a, α, b, β). Note that $a = \frac{1}{2}$ and since $\delta > 0$, it follows that $\beta > \frac{1}{2}$. Since $x > \underline{x}$, we have $\alpha > \frac{1}{2}$. Since $\pi_n(x, 1) = 0$, we have $b < \frac{1}{2}$.

Let $q_n(r) := r^n(1-r)^n$. A change of variables yields

$$\tau_n(x_n, y_n) = \frac{\int_{a_n}^{\alpha_n} q_n(r)h_n^l(r)dr}{\int_{b_n}^{\beta_n} q_n(r)h_n^m(r)dr}$$

where

$$h_n^l(r) = \frac{g(z_n^l(r))(1 - z_n^l(r))}{(1-\delta)x_n}$$

$$h_n^m(r) = \frac{g(z_n^m(r))(1 - z_n^m(r))}{(1-\delta)x_n + \delta}$$

and $z_n^o(r)$ is the unique solution to $\pi^o(x_n, z_n^o) = r$ for $o \in \{l, m\}$.

Since $x_n \to x$, the functions $\{h_n^l, h_n^m\}$ for $n = 1, \ldots$ are equicontinuous. Also, by Lemma 4, $y = a = s^l(x)$ and therefore

$$\lim h^l(\tfrac{1}{2}) = \frac{g(s^l(x))(1 - s^l(x))}{(1-\delta)x}$$

Similarly,

$$\lim h^m(\tfrac{1}{2}) = \frac{g(s^m(x))(1 - s^m(x))}{(1-\delta)x + \delta}$$

Then, by Lemma 6,

$$\begin{aligned}
\lim \tau_n(x_n, y_n) &= \frac{\lim h^l(\frac{1}{2})}{\lim h^m(\frac{1}{2})} \cdot \lim \frac{\int_{a_n}^{\alpha_n} q_n(r)dr}{\int_{b_n}^{\beta_n} q_n(r)dr} \\
&= \frac{g(s^l(x))}{g(s^m(x))} \frac{1 - s^l(x)}{1 - s^m(x)} \frac{(1-\delta)x + \delta}{(1-\delta)x} \lim \frac{\int_{a_n}^{\alpha_n} q_n(r)dr}{\int_{b_n}^{\beta_n} q_n(r)dr} \qquad (A10) \\
&= \frac{g(s^l(x))}{g(s^m(x))} \left(\frac{(1-\delta)x + \delta}{(1-\delta)x}\right)^2 \lim \frac{\int_{a_n}^{\alpha_n} q_n(r)dr}{\int_{b_n}^{\beta_n} q_n(r)dr}
\end{aligned}$$

But by steps 1 and 2 of Lemma 6 and Lemma 1,

$$\lim \frac{\int_{a_n}^{\alpha_n} q_n(r)dr}{\int_{b_n}^{\beta_n} q_n(r)dr} = \lim \frac{\int_{a_n}^{1} q_n(r)dr}{\int_{0}^{1} q_n(r)dr} = 1 - \lim \frac{\int_{0}^{a_n} q_n(r)dr}{\int_{0}^{1} q_n(r)dr} = 1 - \lim B_n(a_n)$$

and since $a_n = \pi^l(x, y)$, Lemma 5(iii) yields $B_n(a_n) = \frac{1}{\mu}$. Then, $(A10)$ establishes that $\lim \tau_n = (1 - \mu)\ell(x)$ as desired.

(ii) For $x = \underline{x}$, note that $\lim \alpha_n = \frac{1}{2}$ and hence, we cannot apply Lemma 6. Define, $\hat{h}^l$ as follows: $\hat{h}_n^l(r) = h_n^l(r)$ for $r \leq \alpha_n$ and $h_n^l(r) = h_n^l(\alpha_n)$ for $r > \alpha$. Then,

$$\tau_n(x_n, y_n) = \frac{\int_{a_n}^{\alpha_n} q_n(r)\hat{h}_n^l(r)dr}{\int_{b_n}^{\beta_n} q_n(r)h_n^m(r)dr} \leq \frac{\int_{a_n}^{1} q_n(r)\hat{h}_n^l(r)dr}{\int_{b_n}^{\beta_n} q_n(r)h_n^m(r)dr} \qquad (A11)$$

Repeating the argument above for the last term of $(A11)$ yields the desired bound. $\square$

Proof of Proposition 3: Lemma 4 implies that $y = s^l(x)$ for any limit equilibrium (x, y). Lemma 5 (i) implies that $x \in [\underline{x}, 1]$. Note that at $s < y$, $\pi^l(x, s) > \frac{1}{2}$; at $s \in [y, s^m(x))$,

$\pi^m(x,s) > \frac{1}{2}$; and at $s > s^m(x)$, $\pi^m(x,s) < \frac{1}{2}$. Hence, the characterization of ϕ follows the law of large numbers. $\square$

Proof of Proposition 4: Let (x,y) be a limit equilibrium. Then, there exists equilibria (x_n,y_n) such that (x_n,y_n) converges to (x,y). By Lemma 4, $y = s^l(x)$ and by Lemma 3, $x_n > 0$ implies $\tau_n(x_n,y_n) \leq \epsilon$ (with equality if $x < 1$). By Lemma 7(i), $\tau_n(x_n,y_n)$ converges to $(1-\mu)\ell(x)$ if $x > \underline{x}$. Hence, $x > \underline{x}$ implies $(1-\mu)\ell(x) \leq \epsilon$ and $(1-\mu)\ell(x) = \epsilon$ if $\underline{x} < x < 1$. If $x = \underline{x}$, then Lemma 3 implies $\epsilon = \tau_n(x_n,y_n)$ and Lemma 7(ii) implies that for any $\eta > 0$, $\tau_n(x_n,y_n) \leq (1-\mu)\ell(x) + \eta$ for all n sufficiently large. Therefore, $\epsilon \leq (1-\mu)\ell(\underline{x})$ if $x = \underline{x}$.

Let x be a regular critical point. We will show that for $\eta > 0$, there is N such that for $n > N$ there is a voting equilibrium (x_n,y_n) with $|(x_n,y_n) - (x, s^l(x))| < \eta$.

First, consider $x \in (\underline{x}, 1)$ and let $\eta > 0$. Since x is regular, there is x', x'' such that $x' < x < x''$ and $x - x' < \eta, x'' - x < \eta$ and $(1-\mu)\ell(x') < \epsilon = (1-\mu)\ell(x) < (1-\mu)\ell(x'')$. Moreover, since s^l is continuous, we may choose x', x'' such that $|s^l(\hat{x}) - s^l(x)| < \eta$ for all $\hat{x} \in [x', x'']$. Let $y'_n = \sigma_n(x'), y''_n = \sigma_n(x'')$. By Lemma 4, there is N' such that for $n > N'$, $|s^l(\hat{x}) - \sigma_n(\hat{x})| < \eta$ for all $\hat{x}$. By Lemma 7(i), there is N'' such that for $n > N'$, $\tau_n(x', y'_n) < \epsilon$ and $\tau_n(x'', y''_n) > \epsilon$. Let $n > N = \max\{N', N''\}$. Continuity of τ_n and σ_n imply that there is $x^* \in [x', x'']$ such that $\tau_n(x^*, \sigma_n(x^*)) = \epsilon$. By Lemma 3, $(x^*, \sigma_n(x^*))$ is an equilibrium. Note that $|(x^*, \sigma_n(x^*)) - (x, s^l(x))| < 3\eta$. Since η is arbitrary the desired result follows.

Next, let $x = \underline{x}$. Since x is regular, there is $x > \underline{x}$ such that $x - \underline{x} < \eta$ and $\tau_n(x, \sigma_n(x)) > \epsilon$ for n sufficiently large. Note that $\sigma_n(\underline{x} - \eta) = 0$ for n sufficiently large. Now, we can repeat the argument above to prove the existence of an equilibrium $(x^*, \sigma_n(x^*))$ such that $|(x^*, \sigma_n(x^*)) - (\underline{x}, s^l(\underline{x}))| < 3\eta$.

Finally, let $x = 1$. Since x is regular, we conclude that $\tau_n(1, \sigma_n(1)) < \epsilon$ for n sufficiently large. Hence, by Lemma 3, $(1, \sigma_n(1))$ is an equilibrium. By Lemma 4, $|\sigma_n(1) - s^l(1)| < \eta$ for n large. This completes the proof of Proposition 4. $\square$

7. Proof of Proposition 8

Adapting the analysis of the previous section to this new game is straightforward. Marginal states are redefined as follows:

$$\hat{s}^l(x) = 1 - \frac{1}{2(1-\delta)x}$$
$$\hat{s}^m(x) = 1 - \frac{1}{2((1-\Delta)x + \Delta)}$$

Let $\underline{\hat{x}}$ be such that $(1-\delta)\underline{\hat{x}} = \frac{1}{2}$. Define

$$\hat{\ell}(x) = \frac{g(\hat{s}^m(x))}{g(\hat{s}^l(x))} \cdot \left(\frac{(1-\delta)x + \Delta}{(1-\Delta)x}\right)^2 \qquad (A12)$$

With these modified definitions, Proposition 4 holds for the new game. Now, we can repeat the argument for Proposition 5 to prove Proposition 8. □

8. Proof of Proposition 9

First assume $x < \underline{x}$. Then, as n becomes arbitrarily large, $\frac{B_n(\pi^l(s,x))}{B_n(\pi^m(s,x))}$ gets arbitrarily close to 0. Hence, $\mu \cdot B_n(\pi^l(s,x)) < B_n(\pi^m(s,x))$ for large enough n. Therefore, $z = 1$ is the only optimal strategy for B, implying $x = 1$.

Next, assume that $x \geq \underline{x}$. Recall that in any limit equilibrium, at any state $s < s^l(x)$, candidate B wins no matter what policy he chooses; wins if he choose m at states $s < s^m(x)$; and he loses at any state $s > s^m(x)$ no matter what policy he chooses. Hence, (since G is uniform), the probability that B wins if he chooses m is $s^m(x)$, while the probability that B wins if he chooses l is $s^l(x)$. Therefore, B chooses $z = 1$ if

$$\mu > \frac{s^l(x)}{s^m(x)}$$

Substituting for s^l, s^m we can rewrite this equation as

$$1 - \mu < \frac{\delta}{(1-\delta)x(2x(1-\delta) + 2\delta - 1)}$$

Since $x \leq 1$, the above inequality will hold if

$$\mu > \frac{1-2\delta}{1-\delta}$$

Then, $z = 1$ which in turn implies $x = 1$ as desired. □

9. Proof of Proposition 10

For $s > \frac{1}{2}$, the law of large numbers ensures that B loses in any limit equilibrium no matter which strategy he chooses. However, his probability of winning goes to 0 much faster if he choose m than if he chooses l. So, $Z(s) = 1$ and therefore $Z(s) =$ as desired. Next, assume $s < \frac{1}{2}$ and the sequence of equilibria (X_n, Z_n) converging pointwise to the limit equilibrium (X, Z). Since s is fixed, we omit the argument s in π^p and let $x_n = X_n(s)$, $z_n = Z_n(s)$.

If $z_n = 0$, then voter optimality implies $x_n = 0$ and therefore deviating to $z_n = 1$ strictly increases candidate B's expected payoff. Hence, for all n, $z_n > 0$. Next, note that if $z_n = 1$, then $\pi^m = 1 - s > \frac{1}{2}$ and the result again follows from the law of large numbers. Hence, only the case where (along some subsequence) $z_n \in (0, 1)$ for all n is left to consider.

Since B chooses both policies with strictly positive probability he must be indifferent between them. Hence,

$$\mu B_n(\pi^m(x_n)) = B_n(\pi^m(x_n)) \tag{A13}$$

Next, we show that (8) implies $\lim \pi^l(x_n) = \frac{1}{2}$. If $\pi^l(x_n) \geq \frac{1}{2} + \eta$ along any (sub)sequence x_{n_j}, then $\lim \frac{B_{n_j}(\pi^l(x_{n_j}))}{B_{n_j}(\pi^m(x_{n_j}))} = 1$, violating (8). Similarly, $\pi^l(x_{n_j}) \leq \frac{1}{2} - \eta$ for all n implies $\frac{B_{n_j}(\pi^l(x_{n_j}))}{B_{n_j}(\pi^m(x_{n_j}))} = 0$, again violating ($A13$). Since $\pi^l(x_n)$ converges to $\frac{1}{2}$, $\lim x_n > 0$. Then, voter optimality requires that $\frac{\theta}{1-\theta} \leq \epsilon$, where θ is the conditional probability that candidate B has chosen l given that the voter is pivotal and is a b-type. Hence,

$$\theta = \frac{z_n \binom{2n+1}{n+1} \pi^m(x_n)^n (1 - \pi^m(x_n))^n}{z_n \binom{2n+1}{n+1} \pi^m(x_n)^n (1 - \pi^m(x_n))^n + z_n \binom{2n+1}{n+1} \pi^l(x_n)^n (1 - \pi^l(x_n))^n} \tag{A14}$$

Some simplification of ($A14$) yields

$$\frac{1 - z_n}{z_n} \cdot \frac{\alpha_n^l}{\alpha_n^m} \leq \epsilon$$

where $\alpha_n^p = \pi^p(x)^n (1 - \pi^p(x))^n$ for $p \in \{m, l\}$. Note that $\frac{\alpha_n^l}{\alpha_n^m}$ converges to infinity since $\pi^l(x_n)$ converges to $\frac{1}{2}$ and $\pi^m(x_n)$ is bounded away from $\frac{1}{2}$. Therefore, $\lim x_n > 0$ implies $\lim z_n = 1$ and the probability of B winning conditional on choosing m converges to 1, as desired. □

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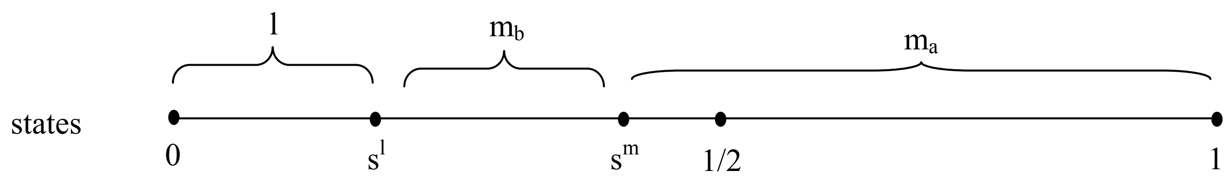

Figure 1