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Societal benefits of illiquid bonds

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Abstract

In this paper, I provide a possible explanation of why nominally risk-free bonds are essential in monetary economies. I argue that the role of nominal bonds is to enable agents to engage in intertemporal exchanges of money. I show that bonds can only serve this role if they are illiquid (costly to exchange for goods). Finally, I argue that in economies in which nominal bonds are essential, it is optimal for monetary policy to respond to changes in the distribution of liquidity needs.

JEL classification: E40; E51; D82

1. Introduction

In many societies, individuals trade both money and nominal bonds. Often, as with Treasury bills, the bonds are essentially risk-free in nominal terms, and so they provide little benefit over money itself in terms of risk sharing. Why then does a society find it beneficial to have both money and nominally risk-free bonds? In the language of monetary theory, why are both money and bonds *essential*?

In this paper, I provide an answer to this question. I show that in the equilibria of monetary economies, individuals may have different intertemporal marginal rates of substitution. Hence, individuals would like to engage in additional intertemporal trades of money. I argue that the role of nominal bonds is to allow households to make these trades. More precisely, I show that if nominal bonds exist, some

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households can give up bonds for money while others receive money for bonds. Later, the holders of the nominal bonds can exchange them for monetary interest rate payments. In this way, nominal bonds allow households to make intertemporal monetary exchanges.

In making this argument, I find that it is important to distinguish between *illiquid* and *liquid* bonds. By illiquid, I mean bonds that cannot, because of physical or informational reasons, be exchanged for goods. I show that illiquid bonds allow agents to make intertemporal exchanges of money, but liquid bonds do not. In this sense, I *endogenize* the illiquidity of bonds by showing that it is *efficient* for bonds to be illiquid rather than liquid.

I formalize these arguments in a model economy in which agents have unobservable differences in their marginal utilities of first-period consumption. I look at the consequences of adding three types of durable tokens—money, liquid bonds, and illiquid bonds—to this economy. I prove three results. The first is that, given the presence of money, liquid bonds are inessential; they can be eliminated with no loss in welfare. Intuitively, liquid bonds are equivalent to money, and there is never any reason to have both types of assets.

The second result concerns the essentiality of illiquid bonds. I show that the unobservable tastes lead to illiquid bonds' being essential. In particular, agents with a high temporary need for consumption sell bonds and buy money; agents with a low temporary need for consumption do the opposite. Intuitively, illiquid bonds provide a way for agents with relatively low current consumption needs to credibly commit within an asset market to forego consumption today in exchange for future consumption.

Finally, in environments in which illiquid bonds are essential, higher nominal interest rates may be associated with higher welfare. The last result is contrary to the analyses of optimal monetary policy in representative agent cash-in-advance settings. There, zero nominal interest rates are optimal, and there is an inverse relationship between welfare and interest rates. The intuition behind my result is simple. If illiquid bonds are essential, then adding a small amount of illiquid bonds improves welfare. But with an increased supply of bonds relative to money, the price of bonds has to fall. Thus, in any economy in which nominal bonds are essential, we should expect small increases in the supply of bonds (when the supply is in the neighborhood of zero) to lead simultaneously to higher interest rates and higher welfare.

I derive these three results in a particular model environment. However, I argue that they are robust. The inessentiality of liquid bonds is an arbitrage argument, and, as such, should be true in virtually any environment. Similarly, the positive association between welfare and interest rates when illiquid bonds are essential is a basic demand/supply story, and again should be true in most environments. The essentiality of illiquid bonds is more delicate: it hinges on the agents' having different nominal intertemporal marginal rates of substitution in a monetary equilibrium, and these differences being difficult to observe.

I also discuss the implications of my results for monetary policy. Macroeconomics typically couches discussions of optimal monetary policy in terms of responses to changes in aggregate variables. The social benefit of having both bonds and money is to allow agents to trade current liquidity for future liquidity among themselves. Hence, monetary policy needs to respond to variations in the benefits of liquidity exchange. These benefits are shaped by distributional, not aggregate, variables: namely, the cross-sectional distribution of liquidity holdings and liquidity needs.

It is important not to confuse the goal of this paper with that of a related stream of literature. There are many papers in monetary theory that ask the question: why do individuals simultaneously hold money and bonds, even though bonds appear to generate the same flow of payments at a lower price? This question about *individual* behavior is fundamentally different from my question about *societal* behavior. My paper instead asks the question: given a choice between bonds that are costly to use in goods transactions, and bonds that are not, why do societies find it efficient to use the former? My paper *endogenizes* transaction costs: it shows why it is socially optimal for interest-bearing bonds to be costlier to use in goods markets than money is.

In the model, societies face a choice between liquid and illiquid bonds. Arguably, this choice is similar to the United States government's choosing whether to issue its debt as bearer bonds or as registered bonds. With a bearer bond, possession establishes ownership. This means that it is just as easy to exchange bearer bonds for goods as it is to exchange them for cash in an asset market. At least along this dimension, bearer bonds look like liquid bonds.

With registered bonds, ownership is recorded in an account. In the United States, the appropriate records are kept either by a depository institution or by the Treasury. These same entities can broker asset market transactions. The bundling of record keeping and brokering implies that it is not costly to transfer ownership of a registered bond in the process of making an asset market trade. In contrast, if an individual tries to buy apples with a registered bond, transferring ownership of that bond requires a lot of resources.

The asymmetry of trading costs between the goods and asset markets means that registered bonds resemble illiquid bonds. It is worth emphasizing that all United States government debt takes the form of registered bonds.

Of course, one could give other rationalizations for why United States government bonds are relatively hard to exchange for goods. I do not want to choose a particular type of illiquidity: The goal of this paper is to show that illiquidity, whatever its source, has previously unappreciated societal benefits.

I want to distinguish my paper from those that rationalize the co-existence of money and credit (among others, Corbae and Ritter [2] and Townsend [8]). In these papers, a barren token termed money is essential because it can be used for transactions among strangers; simultaneously, though, it is not needed to generate trade within enduring relationships. In a similar vein, Kocherlakota and Wallace [6] show that in worlds with imperfect centralized recordkeeping systems, money can be essential because it helps fill the "holes" in the centralized recordkeeping system. In contrast, my paper provides a rationalization of why it is essential to have two different types of intrinsically useless tokens.

There is one other paper to my knowledge that addresses the societal benefits of having multiple nominally denominated assets. Aiyagari et al. [1] argue that if bonds and money are both indivisible, then their co-existence provides societal benefits. I think of my approach as being a useful complement to theirs: while indivisibility may well be an important factor in some situations, I doubt that it is the whole story in all settings. In particular, in modern economies, the indivisibility of money does not appear to be an important societal constraint.

2. Environment

I begin by describing a simple islands economy. The environment is essentially that used by many authors to motivate a cash-in-advance monetary economy.

2.1. Physical structure

There is a unit measure of households in the environment. Each household has two members, a consumer and a producer. The households are characterized by a vector $(i,j) \in \{1,2\} \times \{H,L\}$; there are equal measures of the four types of households.

There are two types of perishable goods in the household's setting. The first component of this vector describes the types of goods produced and consumed by the household: type 1(2) households produce type 1(2) goods and consume type 2(1) goods. The latter two components are preference parameters. Specifically, a type $(1,j)$ household has preferences representable by

$$\theta_j \ln(c_1) - \alpha y_1 + \sum_{t=2}^{\infty} \beta^{t-1}\{\ln(c_t) - \alpha y_t\}, \tag{1}$$

where c_t is consumption of type 2 goods and y_t is production of type 1 goods. I assume that $0 \leqslant y_t \leqslant 1$, and $\theta_H > 1 > \beta$, and $\theta_L = 1$. Type $(2,j)$ households have similar preferences with the obvious changes in goods produced and consumed.

There are three islands in the world, labelled 1, 2, and 3. At the beginning of each period, all households are located on island 3 together. This island has no endowment of goods or means of production. Then, the type 2(1) producers go to island 2(1) and the type 1(2) consumers go to island 2(1). On these islands, production and consumption take place. All goods are perishable.

I make the following three assumptions about enforcement and information. The first two are standard ways to generate a role for money: I assume that there is no recordkeeping technology (other than the durable tokens that I describe later), and I assume that society cannot impose any penalties on households. This means, among other things, that type 2 producers cannot be required to produce for type 1 consumers, because there is no way to force them to do so. Finally, I assume that, for a type (i,j) household, i is publicly observable, but j is not.

2.2. Tokens

I investigate economies that are distinguished by the types of tokens that are available. All of the economies have a durable and divisible token called money. Money can be costlessly created by society (but individual households cannot do so).

The economies differ, though, in the availability of bonds. A bond is a token which physically lasts for only one period: it physically disappears after the period 2 meeting of all households on island 3. All of the bonds are nominal: immediately before a unit of bonds completely depreciates, the holder can exchange it for a unit of newly created money. Thus, societies can commit to convert nominal bonds into money, even though individuals cannot commit to repay loans. This commitment power on the part of society plays an important role in making bonds essential.

There are two types of bonds: liquid and illiquid. A liquid bond is a bond which can be traded on all islands at all dates. An illiquid bond can be traded only on island 3. These differences in bond type can be physically interpreted as saying that bonds are either portable or not.[1]

In what follows, I measure holdings of bonds in terms of their nominal payoff at time of maturity. Thus, B units of one-period bonds refers to the payoff of B units of money that the holder receives in period 2. In a *no-bond* economy, all households begin life with M units of money. In a *liquid-bond* economy, all households begin life with M units of money and B units of liquid bonds. Finally, in an *illiquid-bond* economy, all households begin life with M units of money and B units of one-period illiquid bonds.

The rules of exchange in all economies are the same. On island 3, agents trade money, liquid bonds, and illiquid bonds in a competitive market.[2] I denote the relative price of bonds in terms of money in this market by q. On islands 1 and 2, the agents located on those islands trade money, liquid bonds, and goods in a competitive market. I denote the period t relative price of goods in terms of money in this market by p_t. Thus, all of the economies are essentially cash-in-advance economies, where cash includes both money and liquid bonds.

3. Results

In this section, I describe and prove the three main results in the paper. Note first, though, that because of the enforcement and recordkeeping limitations, money itself is essential in all economies.

[1] Following the discussion of registered bonds in the introduction, illiquid bonds need not be durable tokens. Alternatively, there could be a record on the central island 3. Each person begins life with a credit of B units in this record; if they enter period 2 with B' units in the record, they receive B' units of money. (Obviously, this requires that people are distinguishable.)
For this record to function just like illiquid bonds, we need two key restrictions. First, agents cannot go negative in terms of their credit record. The idea here is that while agents can credibly establish their identities, they can disappear so that the planner cannot find them to collect. Second, the record is not accessible from islands 1 and 2.
I thank the associate editor for this (more realistic) re-interpretation.

[2] The restriction to competitive trade is not innocuous; see [9].

3.1. Inessentiality of liquid bonds

The first result is that any equilibrium allocation in the liquid-bond economy is an equilibrium allocation in the no-bond economy. This is easily demonstrated through arbitrage. In the liquid-bond economy, to avoid arbitrage opportunities, the relative price of money and bonds must be one in both goods and asset markets in periods 1 and 2. Suppose this were not so. In the period 1 goods market on either island 1 or island 2, there is a positive supply of bonds and money. If the relative price of bonds in terms of money is less (more) than one, then all households will hold only bonds (money). Hence, the relative price must be one. But this kind of argument can be unraveled backwards to prove that the relative prices must be always be one. Given this is true, we can replace all bonds with an equivalent amount of money, and the liquid-bond equilibrium will then be a no-bond equilibrium.

The intuition behind this argument is clear. If bonds are as liquid as money, then people will only hold money if nominal interest rates are zero. But then the bonds can just be replaced by money: there is no difference between the two instruments at all.

Despite (because of?) its simplicity, the implications of the result are strong: adding nominal bonds to a monetary economy only improves welfare if those bonds are at least somewhat less liquid than money. Any essentiality of nominal bonds can be traced directly to their (relative) illiquidity.

3.2. Essentiality of illiquid bonds

The second main result concerns the essentiality of illiquid bonds. I construct equilibria in the no-bond economy and the illiquid-bond economy. I demonstrate that welfare is higher in the latter equilibrium. Throughout, I assume that $\beta > \alpha$. As we shall see, this assumption serves to eliminate the inflationary distortion on household labor supply, which makes welfare comparisons more straightforward.

3.2.1. No-bond economy

In the no-bond economy, if $\{p_t\}_{t=1}^{\infty}$ is the sequence of price levels, a type (i,j) household faces the following decision problem:

$$\max_{\{c_{jt}, y_{jt}, M_{jt}\}_{t=1}^{\infty}} \theta_j \ln(c_{j1}) - \alpha y_{j1} + \sum_{t=2}^{\infty} \beta^{t-1}\{\ln(c_{jt}) - \alpha y_{jt}\}$$

$$\begin{aligned} \text{s.t.} \quad & p_t c_{jt} \leqslant M_{j,t-1}, \\ & p_t c_{jt} + M_{jt} \leqslant M_{j,t-1} + p_t y_{jt}, \\ & M_{jt}, \ c_{jt} \geqslant 0, \\ & 0 \leqslant y_{jt} \leqslant 1, \\ & M_{j0} = M. \end{aligned}$$

Here, c_{jt} is consumption in period t, y_{jt} is production in period t, and M_{jt} is moneyholdings in period t. Because of the spatial separation in the environment, consumers in household j have to use previously accumulated money balances to buy goods in period t. This "cash-in-advance" restriction is embedded in the set of constraints in the household's problem.

It is then straightforward to show that the following is an equilibrium:

$$p_t = M \text{ for all } t,$$

$$c_{jt} = 1 \text{ for all } t, j,$$

$$y_{jt} = 1 \text{ for all } t, j.$$

(My choice of numeraire rules out the equilibrium in which money is not valued at all.) In the appendix, I prove that this is the unique equilibrium for any θ_H. I also allow for the possibility that in the no-bond economy, the government injects additional money into the economy at the beginning of every period via equal transfers for all households.[3] I show that for any weakly increasing sequence of money supplies $\{M_t\}_{t=1}^{\infty}$, and any θ_H, the equilibrium allocation of resources is given by

$$c_{jt} = y_{jt} = \min(\alpha^{-1}\beta M_t/M_{t+1}, 1) \text{ for all } t \text{ and } j = H, L \tag{2}$$

which provides no more utility to the households than the equilibrium obtained for the constant-money case.[4]

Because they are liquidity constrained, households both consume the same amount of consumption in period 1. Note that this is not consistent with a first-best allocation, in which θ_H households would consume more in period 1 than the other households.

The unobservability of the households' tastes plays an important role here. If tastes were observable, the social planner could give $(\theta_H - 1)M$ additional units of money to the type H households before trading begins. In the resulting monetary equilibrium, type H households consume $2\theta_H/(1+\theta_H)$ in period 1, and consume 1 in all following periods. This consumption allocation is Pareto optimal (it solves a planner's problem in which the planner weights all agents equally).

3.2.2. *Illiquid-bond economy*

In the illiquid-bond economy, if q is the relative price between bonds and money in period 1, and $\{p_t\}_{t=1}^{\infty}$ is the sequence of price levels, type (i,j) households face the

[3] Lump-sum taxes of money are impossible because society cannot impose any penalties on households. (If such penalties could be imposed, then there is clearly no role for money, because producers could just be forced to produce for households.)
Even without these penalties, it might be possible to impose proportional taxes on money holdings (if money is a depreciable token or if money produced at different dates is distinguishable). But such proportional taxes are neutral.

[4] In [7], the socially optimal allocation of resources is an equilibrium if the money supply grows sufficiently fast. This is not true in this setting. Intuitively, even though the low-shock households are more patient than the high-shock households, the low-shock households are still impatient (in the sense that, at their endowments, their shadow real interest rates are positive). It is therefore not possible to separate the two types by using inflationary policy as in [7].

following decision problem:

$$\max_{\substack{\{c_{jt}, y_{jt}, M_{jt}\} \\ M'_{j0}, B_{j1}}} \theta_j \ln(c_{j1}) - \alpha y_{j1} + \sum_{t=2}^{\infty} \beta^{t-1}\{\ln(c_{jt}) - \alpha y_{jt}\}$$

$$\begin{aligned} \text{s.t.} \quad & M'_{j0} + qB_{j1} \leqslant M + qB, \\ & p_1 c_{j1} \leqslant M'_{j0}, \\ & M_{j1} \leqslant M'_{j0} - p_1 c_{j1} + p_1 y_{j1} + B_{j1}, \\ & p_t c_{jt} \leqslant M_{j,t-1}, t \geqslant 2, \\ & M_{jt} \leqslant M_{j,t-1} - p_t c_{jt} + p_t y_{jt}, t \geqslant 2, \\ & M_{jt}, c_{jt}, y_{jt}, M'_{j0}, B_{j1} \geqslant 0, \\ & 0 \leqslant y_{jt} \leqslant 1. \end{aligned}$$

This decision problem represents the sequence of markets that households face. In period 1, households can trade money and bonds at price q. Then, consumers take the available money (bonds are illiquid) and buy goods. In period 2, households can use the money balances and the payments from bonds to buy goods. From period 3 on, the decision problem is the same as in the no-bond economy.

Given this decision problem, it is straightforward to show that the following is an equilibrium for sufficiently small B:

$$y_{jt} = 1 \text{ for all } j, t, \tag{3}$$

$$q = \beta M/(M + \beta B + 2B), \tag{4}$$

$$p_1 = M, \tag{5}$$

$$p_t = (M + B), t > 1, \tag{6}$$

$$c_{h1} = (M + Bq)/M, \tag{7}$$

$$c_{l1} = (M - Bq)/M, \tag{8}$$

$$c_{h2} = M/(M + B), \tag{9}$$

$$c_{l2} = (M + 2B)/(M + B), \tag{10}$$

$$c_{j2} = 1, t > 2. \tag{11}$$

Let me explain how this equilibrium works. In period 1, on island 3, type H agents sell all of their illiquid bonds at price q to the type L agents. Because they have more cash, the type H agents can consume more in period 1 than type L households (note that the type L households end up carrying both bonds and money into period 2). All households work the full amount possible, because they are at a corner; hence, they all carry the same amount of money into the next period.

In period 2, on island 3, the type L agents receive the payoff from their illiquid bonds. Hence, they can consume more in period 2 than the type H agents. Again, in period 2, all agents work the same amount. This ensures in all future periods, all agents consume the same amount.

It is straightforward to show that all agents are made better off in this equilibrium than in the no-bond equilibrium allocation. In particular, it is budget-feasible for the agents to spend M in period 1, and then spend $(M + B)$ in period 2. This means that the no-bond equilibrium allocation is budget feasible in the illiquid-bond economy, but agents strictly prefer not to choose it.

Thus, in this model environment, all agents are made better off by the introduction of illiquid bonds. Both illiquid bonds and money are essential.[5]

The economics underlying this result are simple. At their endowments, agents have two different shadow real interest rates. It follows that they want to engage in an intertemporal consumption trade. To do so, they must trade an asset that pays a rate of return which is intermediate to their shadow real interest rates. Money does not work: its real rate of return is no more than zero, and all agents' shadow interest rates are positive. The agents need another asset with a positive return. It follows that there is a role in this economy for illiquid bonds.

The result is reminiscent of Woodford [10]. Woodford shows that in an economy with borrowing-constrained agents, efficient outcomes require issuing a positive amount of public debt. The important innovation here is that I show explicitly that the relevant bonds must be illiquid to offer a welfare benefit over what agents can do with money alone.

3.3. Welfare benefits of high nominal interest rates

Now consider increasing the supply B of bonds in the above illiquid-bond economy. If B increases slightly, then, since:

$$q = \beta M/((2 + \beta)B + M)$$

q falls as B rises. Moreover, household welfare is increasing in B (at least) for small values of B. It follows that as M/B varies, interest rates and household welfare move in the same direction.

4. Discussion

In this section, I discuss three aspects of the results: the intuition underlying them, their implications for monetary policy, and their robustness to adding other forms of wealth, like claims to physical capital.

[5] Note that it would make agents even better off to allow them to trade one-period illiquid bonds on island 3 every period, not just in period 1.

4.1. Intuition

Here, I provide intuition for the three results. The first result is that liquid bonds are inessential. The intuition behind this result is that if bonds are liquid, then their price must always be the same as money, or bond/money markets cannot clear. Hence, money and bonds are exactly equivalent instruments—there is no need to distinguish between the two. This arbitrage-based result seems likely to be highly robust.

The second result is that illiquid bonds are essential. This result is more delicate, because it depends on three frictions. The first is that enforcement be limited. If there are no enforcement frictions, agents can borrow and lend among themselves without the outside supply of illiquid bonds. We know from Kocherlakota [5] and Huggett and Krasa [3], though, that without some enforcement frictions, money itself is inessential.

The second friction is that recordkeeping must be limited. Otherwise, private credit could be supported as an equilibrium outcome in a gift-giving game [5]. Again, as Kocherlakota [5] emphasizes, without recordkeeping limitations of some kind, money itself is inessential.

Finally, the households' shadow interest rates (at their endowments) must be positive and different. The households would like to eliminate this difference. They cannot use money to do so (because their shadow interest rates are positive). More strongly, because of the above enforcement/recordkeeping limitations, they can only do so by using bonds.

There is a counterintuitive element to the essentiality of illiquid bonds: why are households better off using a less flexible asset? The reason is simple. Households wish to engage in an intertemporal exchange of consumption. Because of spatial separation, they can only engage in these trades in the asset market. So, in the asset market, a type L household must guarantee that it is giving up first-period consumption in exchange for more second period consumption. The only way that they can do so is by buying bonds that are not as liquid as money.

Again, the intuition behind this result seems fairly robust. In particular, I conjecture that it will survive other ways of introducing differences in liquidity demands across model entities. (For example, the result should be true in a setting with firms that need money to hire inputs into production, and that differ in terms of productivity shocks.)

The final result is that in the illiquid-bond economy, for B/M small, larger bond supplies are associated with higher period 1 nominal interest rates and with higher levels of household welfare. Again, there is a simple (and robust) intuition behind this result. In an economy in which illiquid bonds are essential, adding a small amount of illiquid bonds makes agents better off (by definition). But increasing the supply of bonds will typically drive their price downwards, and increase the nominal interest rate. Hence, if illiquid bonds are essential, then higher nominal interest rates should be expected to be associated with higher levels of welfare.

4.2. Policy implications

Understanding why bonds and money are both essential has important consequences for our understanding of monetary policy.[6] Monetary policy in the above economy is about adjusting the relative supply of money and bonds. If B/M is small, the type H agents are borrowing-constrained. By increasing B/M, the planner/government essentially endows the agents with claims to future consumption. Hence, increases in B/M loosens the borrowing constraint on type H agents.

Once a sufficient amount of bonds is injected, the borrowing constraint is implicitly eliminated. In equilibrium, the agents' marginal rates of substitution are equated, and the allocation is Pareto optimal. There is then no effect from injecting any more bonds (until B/M is so large that the resulting inflation distorts labor supply).[7]

This thinking has major positive and normative consequences for monetary policy. In the islands economy studied above, prices are fully flexible. Moreover, inflation is basically a lump-sum tax when α is small, and so bond supply increases are Ricardian. Nonetheless, monetary policy can affect the real interest rate. Shrinking the money supply (or expanding the supply of bonds) loosens the borrowing constraint. Looser borrowing constraints means that there is more demand for loans, which drives up real interest rates.

Normatively, macroeconomists typically think about how monetary policy should respond to *aggregate* variables.[8] The above analysis indicates that it is optimal for open market operations to respond to *distributional* changes. Consider an extension of the above economy, in which the planner can condition B/M on the realization of

[6] As above, I assume in this subsection that agents are initially endowed with bonds. However, the qualitative effects of monetary policy are the same if the monetary authority injects bonds by pegging a nominal interest rate instead.

[7] In particular, there is a cutoff value $(\theta_H - 1)/(2\beta + 2)$. If B/M is less than this cutoff, then the type H agents are borrowing-constrained, and the equilibrium consumption allocation is determined as in (7)–(11). On the other hand, if B/M is at least as large as the cutoff:

$$c_{H1} = 2(1+\beta)/(2+\beta+\beta/\theta_H),$$

$$c_{H2} = \beta\theta_H^{-1}(1+r)c_{H1},$$

$$c_{Ht} = 1, t > 2,$$

$$c_{Lt} = 2 - c_{Ht} \text{ for all } t,$$

where $r = (2\theta_H\beta^{-1} - 2\beta)/(1 + \theta_H + 2\beta)$ is the real interest rate between periods 1 and 2. (These formulae assume that B/M is above the cutoff value, but is not sufficiently large so as to create an inflation distortion in labor supply.)
When B/M exceeds the cutoff value, the equilibrium allocation is Pareto optimal. However, it solves a planner's problem which puts less weight on the type H agents than the type L agents. In fact, for values of B/M near to, but smaller than, the cutoff value, the type H agents' welfare is decreasing in B/M.

[8] Assuming, as is true in this economy, that the Friedman Rule is not attainable because the money supply cannot be reduced through lump-sum transfers.

a random θ_H. Then, if θ_H is large, the planner needs a high value of B/M to achieve Pareto optimal allocations.

More generally, this explanation for the co-existence of money and bonds implies that monetary policy's goal is to correct mismatches between liquidity holdings and liquidity needs. If the mismatch between holdings and needs is large, then the monetary authority should ease borrowing constraints by *shrinking* the money supply and *expanding* the amount of bonds.

4.3. Other forms of wealth

In actual economies, money and bonds are not the only forms of wealth. People also trade claims to the rental income from physical capital and land. To what extent does the above analysis survive in the presence of these other claims?

The answer to this question depends crucially on the additional claims' liquidity and risk characteristics. In the above model economy, illiquid bonds are nominally riskless, are costlessly tradeable in asset markets, and are costly to trade in goods markets. If there are financial assets which share these same characteristics, then there is no additional role for illiquid bonds.

In the world, there is reason to suspect that other financial assets are not as liquid as Treasury bills (or other United States government debt). Other financial assets tend to have much higher risk-adjusted returns than Treasury bills. These extra returns are often attributed to Treasury bills' being more liquid (see [4]).

5. Conclusions

The purpose of this paper is to provide a rationalization of the co-existence of illiquid bonds and money. The key ingredients in this rationalization are twofold. First, enforcement is limited, and recordkeeping is impossible except through durable tokens. Second, households have unobservable differences in tastes that lead to a difference in their willingness to substitute money over time. Together, these frictions give rise to a need for an auxiliary illiquid durable token.

The paper shows that in economies in which illiquid bonds are essential, standard welfare analyses of monetary policy must be revised. Adjustments in the supply of money relative to the supply of bonds have important allocational effects. Specifically, an important consequence of shrinking the money supply is that individual borrowing constraints are loosened. This drives up real interest rates (even without sticky prices or inflation tax effects) and (as long as inflationary effects on output are small) makes agents better off.

This paper represents a first step toward understanding the social benefits of having assets that differ in their liquidity properties. It would be useful to extend the analysis to account for finer gradations of liquidity. I plan to address this question in future work.

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Appendix

In this appendix, I consider an extended version of the no-bonds economy presented in the text. I allow for the possibility that at the beginning of every period, households receive equal non-negative transfers of new money. I show that for any weakly increasing sequence of money supplies, there is a unique monetary equilibrium in the no-bond islands economy.

The proof works as follows. I first show that there is a unique equilibrium in the case in which $\theta_H = 1$. I then build on this lemma to prove that there is a unique equilibrium for all θ_H.

Lemma 1. *Let $\{M_t\}_{t=0}^{\infty}$ be a weakly increasing sequence of money supplies. If $\theta_H = 1$, there is a unique equilibrium in which $c_{jt} = y_{jt} = \min(\alpha^{-1}\beta M_t/M_{t+1}, 1)$, and $p_t = M_t/c_{jt}$ for all t.*

Proof. It is straightforward to show that any equilibrium must be symmetric in that all households have the same consumption-labor choices.

The first part of the proof is that in equilibrium, if the multiplier on the household's cash-in-advance constraint is zero in any period t, then it must be zero in all future periods. Let λ_t be the multiplier on the household's period t flow wealth constraint, and μ_t be the multiplier on its period t cash-in-advance constraint. Suppose $\mu_\tau = 0$ for $s > \tau \geqslant t$ and $\mu_s > 0$. Then

$$\beta^t/(p_t c_t) = \lambda_t = \lambda_s + \mu_s = \beta^s/(p_s c_s) = \beta^s/M_s,$$

which implies that

$$1 < \beta^{t-s} = p_t c_t/M_s \leqslant M_t/M_s,$$

which contradicts the specification that the money supply is weakly increasing.

The second part of the proof is that if $\mu_t = 0$ for all $t \geqslant T$, then the household's transversality condition is violated, because

$$\lim_{s\to\infty} \beta^s/\{p_s c_s\} = \beta^T/(p_T c_T) > 0.$$

The first two parts of the proof combine to tell us that $\mu_t > 0$ for all t. It follows that

$$p_t c_t = M_t$$

for all t, and so all households consume the same amount in every period. Moreover, the households' labor supply first-order condition looks like

$$\begin{aligned} &-\alpha + \beta p_t/(p_{t+1}c_{t+1}) \\ &\quad = -\alpha + \beta M_t/(M_{t+1}c_t). \end{aligned}$$

If $\alpha \leqslant \beta M_t/(M_{t+1})$, then $c_t = 1$. If $\alpha > \beta M_t/M_{t+1}$, then $c_t = \alpha^{-1}\beta M_t/M_{t+1}$. Hence, in equilibrium, $y_t = \min(\alpha^{-1}\beta M_t/M_{t+1}, 1)$ for all t. □

The lemma (and its proof) can be used to prove the proposition that for any θ_H, there is a unique monetary equilibrium, and in that equilibrium, the cash-in-advance constraint always binds.

Proposition 1. *Let $\{M_t\}_{t=0}^{\infty}$ be a weakly increasing sequence of money supplies. Then, for all θ_H, there is a unique equilibrium in which for all j, t:*

$$c_{jt} = y_{jt} = \min(\alpha^{-1}\beta M_t/M_{t+1}, 1),$$

$$p_t = M_t/c_{jt}.$$

Proof. Consider first an equilibrium in which, in period 1, the cash-in-advance constraint binds type L households. Then, the type L household's solution to its choice problem satisfies the first order conditions of the type H household's problem, and therefore also solves the type H household's problem. It follows that the cash-in-advance constraint binds type H households in period 1. From Lemma 1, we know that in any such equilibrium, the cash-in-advance constraint binds both types of households in all periods (because they begin period 2 with equal amounts of money). Hence, we know that the only equilibrium in which the cash-in-advance constraint binds type L households in period 1 has $c_{jt} = y_{jt} = \min(\alpha^{-1}\beta M_t/M_{t+1}, 1)$.

Now, I prove that there is no equilibrium in which the cash-in-advance constraint fails to bind type L households in period 1. Consider such an equilibrium. From the logic in the proof of Lemma 1, the cash-in-advance constraint must eventually bind type L households (or their transversality conditions are not satisfied). Suppose the cash-in-advance constraint binds first in period s. Then, again using logic from the proof of Lemma 1, we know that:

$$1/(p_1 c_{L1}) = \lambda_{L1} = \lambda_{Ls} + \mu_{Ls} = \beta^{s-1}/(p_s c_{Ls}) = \beta^{s-1}/M_{Ls}.$$

Moreover, because of their preferences, type L households spend no more in period 1 than type H households, and therefore $M_{Hs} \leqslant M_{Ls}$. It follows that $M_{Ls} > M_0$. But this implies that $M_0 < p_1 c_{L1}$, which violates the cash-in-advance constraint. It follows that there is no such equilibrium. □

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