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Dynamic Insurance with Private Information and Balanced Budgets

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This paper studies a dynamic insurance problem with bilateral asymmetric information and balanced budgets. There are two infinitely-lived agents in our model, both risk averse, and each has an i.i.d. random endowment stream which is unobservable to the other. In each period, each agent must have a non-negative consumption and together they must consume the entire aggregate endowment. Dynamic incentive compatibility in the Nash sense is defined. We give sufficient and necessary conditions for the existence of a constrained efficient contract. We show that a constrained efficient contract can be characterized in a Bellman equation. We demonstrate that the long-run distribution of expected utilities of each agent is not degenerate. We also develop an algorithm for computing the efficient contract and, in a numerical example, we find that the consumption processes of the agents form stationary Markov chains.

1. INTRODUCTION

This paper studies a dynamic insurance problem between two risk-averse agents with bilateral asymmetric information and balanced budgets. The two agents, both infinitely-lived in a pure exchange economy, face idiosyncratic risks in the endowments they receive. Specifically, at each date, they each draw independently a stochastic, privately observed endowment. The endowment is perishable, and there exist no opportunities for the two agents to borrow and lend with outside parties. The two agents hence are constrained to consume the aggregate endowment they receive at each date. Being risk averse, they would wish to pool their endowments together. But this is impeded by the private information about the endowments they receive. The problem that the two agents face, therefore, is to design a feasible trading mechanism which achieves Pareto efficiency subject to the constraint of incentive compatibility: they must both be given the incentives to truthfully reveal their endowments.

The problem we study here is closely related to the dynamic insurance literature led by Townsend (1982), Spear and Srivastava (1987), and Thomas and Worrall (1990), who model relationships between a principal and a single risk-averse agent; and Green (1987) and Phelan and Townsend (1990), who examine relationships between a principal and a continuum of risk-averse agents. In these models, as in standard principal-agent models such as Holmstrom (1979), Allen (1985), and Radner (1985), a key feature is that the principal, who typically has access to credit markets, can serve as a residual claimant to permit violation of the budget-balancing constraint. In a recent contribution, Atkeson and Lucas (1992) extend the literature by looking at a closed economy where a period-by-period aggregate resource constraint is imposed. In Atkeson and Lucas, the total consumption handed out by the principal each period to the population cannot exceed some constant endowment level.

This paper goes one step further in the direction started by Atkeson and Lucas. In the model studied in Atkeson and Lucas it is feasible for the constant aggregate endowment not to be consumed since the principal can always retain a positive residual. In our model, by contrast, the two agents are constrained to consume the entire (uncertain) aggregate endowment in each period. Therefore in our model the role played by the principal as residual claimant is completely discarded. Another important difference between our model and that of Atkeson and Lucas is that Atkeson and Lucas focus on multiplicative taste shocks rather than endowment shocks. Multiplicative taste shocks can sometimes be interpreted as endowment shocks, for example, when utility is exponential. In this case, however, the optimal contract may require the principal to take from an agent more than he claims to have. This feature is absent in our model. Instead, we focus directly on endowment shocks and we impose in a period-by-period feasibility constraint that the optimal contract does not take from an agent more than he claims to have received.

The model is presented in Section 2 where feasible and incentive compatible contracts are defined and the problem of constrained efficiency is formulated. In Section 3, we demonstrate that a constrained efficient contract that delivers ex ante expected utility V to agent 2 exists if and only if V is in some compact set we denote as Φ_V. Then, in Section 4, we show that a constrained efficient contract can be characterized in a Bellman equation. Our Bellman equation is quite different from those studied by earlier writers in the dynamic insurance literature. Among other things, a unique feature of our Bellman equation is that the value function enters into the incentive constraints.

A common important result in Green (1987), Atkeson and Lucas (1992), and Thomas and Worrall (1990) is, for efficient risk sharing, the expected utility of each agent converges to the minimum level in the set of possible expected utilities with probability one. This however is not the case here. In Section 5, we show that the expected utility of each agent converges to every level in the set of possible expected utilities with probability zero. We also show in this section that the constraint efficient contract is non-trivial and strictly dominates autarky.

As our model is not amenable to analytic solutions, an algorithm for numerical computation of an efficient contract is discussed in Section 6. Then in Section 7, an example, where utility is exponential and endowment takes on two values, is computed. Among other things, we find that in this example, each agent's consumption path forms a stationary Markov chain. Section 8 concludes the paper with several short remarks.

2. THE MODEL

Consider the following economy. Time is discrete and lasts forever: $t=1, 2, \ldots$. There are two infinitely-lived agents, indexed by $a=1, 2$. Both agents are risk averse and maximize their ex ante expected life-time utilities, and discount the future by the common discount factor $\beta \in (0, 1)$. There is one perishable good which the agents consume. The instantaneous utility function u: $\boldsymbol{R} \rightarrow \boldsymbol{R}$, shared by both agents, is assumed to satisfy the following conditions: $u'(c)>0$, $u''(c)<0$, for all $c \geqq 0$. At each date, each agent has a random endowment e_t^a drawn from a finite set $\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$, where $0<\theta_1<\theta_2<\cdots<\theta_n$. We assume that e_t^1 and e_t^2 are identically and independently distributed and Prob $\{e_t^a=\theta_i\}=\pi_i>0$, for all $t \geqq 1$, all $\theta_i \in \Theta$ and all a.

There exist no opportunities for the two agents to borrow or lend with outside parties. Self-imposed punishments, such as the bonfires discussed in Holmstrom (1982) and the consumption lotteries in Rasmusen (1987) are also infeasible: ex post it is inefficient for the two agents to commit to these strategies. Since endowments are perishable, the two agents are constrained to consume the entire aggregate endowment at each date. Being risk averse, they would wish to pool their endowments together. But this is impeded by a problem of information asymmetry. At each date, the history of realized endowments of each agent is his private information.

Given the information structure, any possible trades in our model are to be based solely upon what has been reported by the two agents. For all $t \geqq 1$, we denote by $g^{at}=(r_1^a, \ldots, r_t^a)$ agent a's reported history of endowments up to date t, where r_t^a is agent a's reported endowment at date t. Next, let the overall history of reported endowments up to date t be denoted by

$$g^t=(g^{1t}, g^{2t})=(r_1, \ldots, r_t) \in (\Theta \times \Theta)^t = H^t,$$

where $r_t=(r_t^1, r_t^2)$, and H^t is the set of possible histories up to date t. Let $H^0=\varnothing$.

Definition 1. A co-insurance contract σ is a sequence of functions $\{\sigma_t\}_{t=1}^{\infty}$ where σ_t: $H^t \to \boldsymbol{R}$. Call $\sigma_t(g^t)$ the amount of good transferred from agent 1 to agent 2 at date t, conditional on reported history g^t up to date t.

Therefore under contract σ, at date t, agent 1 will consume $-\sigma(g^t)+e_t^1$, and agent 2 $\sigma(g^t)+e_t^2$. Note that the above way of defining a contract automatically ensures that the two agents will consume exactly the entire aggregate endowment at each date. Also note that the case where $\sigma_t=0$ independent of date and history corresponds to autarky.

We now turn to define the feasibility of a contract. Let $\sigma_t(g^{t-1}, (\theta_i, \theta_j))$ be the date t net transfer of endowment from agent 1 to agent 2 if reported history up to date $t-1$ has been g^{t-1} and date t current reports by the two agents are θ_i and θ_j respectively.

Definition 2. A co-insurance contract σ is feasible if for all $t \geqq 1$ and $g^{t-1} \in H^{t-1}$,

$$-\theta_j \leqq \sigma_t(g^{t-1}, (\theta_i, \theta_j)) \leqq \theta_i, \quad \forall (\theta_i, \theta_j) \in \Theta^2. \tag{1}$$

Condition (1) simply requires that, at any date, the contract will not take from any agent more than he claims to have received. Therefore, suppose the two agents both report truthfully about their endowments at each date, as they will under the conditions of incentive compatibility to be given shortly, then both agents will consume a non-negative amount of the consumption good.

We proceed now to tackle the issue of incentive compatibility. Basically, a contract σ is said to be perfectly incentive compatible if, at any date, conditional on any history, the continuation profile of σ is such that truthful reporting strategies by both agents concerning all future endowments constitute a Nash equilibrium. By modifying the approach in Green (1987), and Spear and Srivastava (1987), this can be formulated in a recursive manner. The idea is to decompose the super-incentive problem that each agent faces at the beginning of each date into a sequence of one-step incentive problems, each associated with a single future date. Some additional notation is needed here. Denote by $U(\sigma | g^{t-1}, (\theta_i, \theta_j))$ the date t expected utility (discounted to date $t+1$) that the continuation profile (from date $t+1$ on) of σ will deliver to agent 1, conditional on reported history $(g^{t-1}, (\theta_i, \theta_j))$ up to date t and that both will report truthfully from date $t+1$ on. Define $V(\sigma | g^{t-1}, (\theta_i, \theta_j))$ analogously for agent 2.

Definition 3. A contract σ is incentive compatible if, for all $t \geqq 1$, $g^{t-1} \in H^{t-1}$, and for all $(\theta_i, \theta_j) \in \Theta^2$ and $\theta_k \in \Theta$,

$$u(-\sigma_t(g^{t-1}, (\theta_i, \theta_j)) + \theta_i) + \beta U(\sigma | g^{t-1}, (\theta_i, \theta_j))$$
$$\geqq u(-\sigma_t(g^{t-1}, (\theta_k, \theta_j)) + \theta_i) + \beta U(\sigma | g^{t-1}, (\theta_k, \theta_j)), \tag{2}$$

and, for all $(\theta_i, \theta_j) \in \Theta^2$ and $\theta_l \in \Theta$,

$$u(\sigma_t(g^{t-1}, (\theta_i, \theta_j)) + \theta_j) + \beta V(\sigma | g^{t-1}, (\theta_i, \theta_j))$$
$$\geqq u(\sigma_t(g^{t-1}, (\theta_i, \theta_l)) + \theta_j) + \beta V(\sigma | g^{t-1}, (\theta_i, \theta_l)). \tag{3}$$

In constraint (2), given that θ_i and θ_j are the endowments the two agents receive at date t, and reported history up to date $t-1$ has been g^{t-1}, on the left-hand side of the inequality, $-\sigma_t(g^{t-1}, (\theta_i, \theta_j)) + \theta_i$ is agent 1's current consumption and $U(\sigma | g^{t-1}, (\theta_i, \theta_j))$ his future utility if he and agent 2 both report truthfully, today and from tomorrow on. On the right-hand side of the inequality, $-\sigma_t(g^{t-1}, (\theta_k, \theta_j)) + \theta_i$ is agent 1's current consumption and $U(\sigma | g^{t-1}, (\theta_k, \theta_j))$ his future utility, if agent 1 cheats by reporting θ_k rather than θ_i, given that agent 2 reports honestly θ_j and that they both will report truthfully from tomorrow on. Hence by Definition 3, incentive compatibility means that, at any date, given any reported history, if one agent chooses to adopt the truthful reporting strategy from that date on, then the other cannot benefit from any one-period misrepresentation at that date. Note that although feasibility will guarantee that the "truth reporting" current consumptions $-\sigma_t(g^{t-1}, (\theta_i, \theta_j)) + \theta_i$ and $\sigma_t(g^{t-1}, (i, j)) - \theta_j$ are non-negative, it may still be the case that the "deviating"—off the equilibrium path—current consumptions $-\sigma_t(g^{t-1}, (\theta_k, \theta_j)) + \theta_i$ and $\sigma_t(g^{t-1}, (\theta_i, \theta_l)) + \theta_j$ take on negative values. This is why we required for mathematical convenience that the utility function u be defined on the whole real line.

We are now in a position to define constrained efficiency. We say that a feasible and incentive compatible contract σ is efficient if it maximizes the ex ante expected utility of agent 1, denoted by $U(\sigma)$, subject to delivering a given ex ante expected utility, denoted $V(\sigma)$, to agent 2. Formally,

Definition 4. A co-insurance contract σ is constrained efficient at V if it maximizes $U(\sigma)$ subject to constraints (1), (2), (3), and

$$V(\sigma) = V. \tag{4}$$

To close this section, we note that following the literature, we only look at contracts which implement truthful reporting strategies. We also note that here, as in Green, and Atkeson and Lucas, we do not impose that each agent be entitled an expected utility that is at least as high as the autarkic expected utility at any ex post date. We leave this type of enforcement issue aside to focus on the issues of private information and budget balancing.

3. EXISTENCE OF THE EFFICIENT CONTRACT

Obviously, the efficient contract defined in Definition 4 does not exist for all values of V. In this section, we solve the problem of existence by establishing that an efficient contract at V exists if and only if V is in some compact set we call Φ_V. We also develop notations and a fixed point argument that will prove useful in the later sections.

Let Φ be the set of feasible and incentive compatible expected utilities:

$$\Phi \equiv \{(U(\sigma), V(\sigma)) \in \boldsymbol{R}^2 | \sigma \text{ s.t. (1), (2), and (3)}\}.$$

Set Φ is non-empty because the autarkic contract is always feasible and incentive compatible. Φ is also bounded. We shall show that Φ is compact and is a fixed point of a one-step operator. To this end, we borrow from Abreu, Pearce and Stacchetti (1990) the concept of "self-generation" as our major mathematical tool.

Let Ψ be any non-empty and bounded set in $\boldsymbol{R}^2$. Let $\mathscr{C} \equiv [\sigma(\theta_i, \theta_j)]_{(\theta_i,\theta_j)\in\Theta^2}$. Call $\mathscr{U}: \Theta^2 \to \boldsymbol{R}^2$ a continuation value function with respect to Ψ if $\mathscr{U}(\theta_i, \theta_j) = (U(\theta_i, \theta_j), V(\theta_i, \theta_j)) \in \Psi$, for all $(\theta_i, \theta_j) \in \Theta^2$.

Definition 5. Given Ψ, a pair $(\mathscr{C}, \mathscr{U})$ is said to be admissible to Ψ if $\mathscr{U}$ is a continuation value function with respect to Ψ, and the following conditions are satisfied:

$$-\theta_j \leqq \sigma(\theta_i, \theta_j) \leqq \theta_i, \quad \forall (\theta_i, \theta_j) \in \Theta^2; \tag{5}$$

$\forall(\theta_i, \theta_j) \in \Theta^2$ and $\forall \theta_k \in \Theta$,

$$u(-\sigma(\theta_i, \theta_j) + \theta_i) + \beta U(\theta_i, \theta_j) \geqq u(-\sigma(\theta_k, \theta_j) + \theta_i) + \beta U(\theta_k, \theta_j); \tag{6}$$

and, $\forall(\theta_i, \theta_j) \in \Theta^2$ and $\forall \theta_l \in \Theta$,

$$u(\sigma(\theta_i, \theta_j) + \theta_j) + \beta V(\theta_i, \theta_j) \geqq u(\sigma(\theta_i, \theta_l) + \theta_j) + \beta V(\theta_i, \theta_l). \tag{7}$$

Note that the constraints in the above definition are the one-step analogues of constraints (1), (2), and (3) in the previous section. Specifically, (5) is feasibility, and (6) and (7) are incentive compatibility. Now for any given $(\mathscr{C}, \mathscr{U})$, which is admissible to Ψ, define the one-step expected utilities generated by this pair by

$$\mathscr{E}(\mathscr{C}, \mathscr{U}) = (\mathscr{E}_1(\mathscr{C}, \mathscr{U}), \mathscr{E}_2(\mathscr{C}, \mathscr{U})),$$

where

$$\mathscr{E}_1(\mathscr{C}, \mathscr{U}) = \sum \pi_i \pi_j [u(-\sigma(\theta_i, \theta_j) + \theta_i) + \beta U(\theta_i, \theta_j)],$$

$$\mathscr{E}_2(\mathscr{C}, \mathscr{U}) = \sum \pi_i \pi_j [u(\sigma(\theta_i, \theta_j) + \theta_j) + \beta V(\theta_i, \theta_j)].$$

If we let $\boldsymbol{L}^\infty(\Theta^2; \boldsymbol{R}^2)$ denote the space of bounded functions mapping from Θ^2 to $\boldsymbol{R}^2$, then function $\mathscr{E}: \boldsymbol{R}^{\Theta^2} \times \boldsymbol{L}^\infty(\Theta^2; \boldsymbol{R}^2) \to \boldsymbol{R}^2$ which is defined above is continuous when the product space $\boldsymbol{R}^{\Theta^2} \times \boldsymbol{L}^\infty(\Theta^2; \boldsymbol{R}^2)$ is endowed with the proper product topology. Further, define operator B stepwise in the following way:

$$B(\Psi) \equiv \{\mathscr{E}(\mathscr{C}, \mathscr{U}) | (\mathscr{C}, \mathscr{U}) \text{ admissible to } \Psi\}.$$

Note that the operator B which maps from the collection of all non-empty and bounded sets in $\boldsymbol{R}^2$ to $\boldsymbol{R}^2$, is non-empty, bounded valued, and monotone in the sense that $\Psi_1 \subseteq \Psi_2$ implies $B(\Psi_1) \subseteq B(\Psi_2)$.

Following Abreu, Pearce and Stacchetti (1990), Ψ is called self-generating if $\Psi \subseteq B(\Psi)$, i.e. if its image under operator B contains Ψ itself. In the appendix we show in Lemma 1 that if Ψ is self-generating, then $B(\Psi) \subseteq \Phi$. We also show in Lemma 2 that Φ itself is self-generating. With these lemmas, we can then establish:

Proposition 1. (i) Φ *is compact.* (ii) $\Phi = B(\Phi)$.

Corollary 1. (i) $\Phi_V = \{V \in \boldsymbol{R} |$ *there exists U such that* $(U, V) \in \Phi\}$ *is compact.* (ii) *For all* $V \in \Phi_V$, $\Phi(V) = \{U \in \boldsymbol{R} | (U, V) \in \Phi\}$ *is compact.*

The proof of Proposition 1 is in the Appendix. The proof of Corollary 1 is a straightforward exercise using the first part of Proposition 1. What Corollary 1 tells us is that an efficient contract exists for any V in a compact set Φ_V. Given that there is not a feasible and incentive-compatible contract for any V outside the set Φ_V, we can then conclude that an efficient contract at V exists if and only if $V \in \Phi_V$. Finally, the second part of Proposition 1, which states that Φ is a fixed point of B, is a useful result for analysis in Section 6 when an algorithm for computing Φ is developed.

4. A BELLMAN EQUATION

Given Corollary 1, we can usefully define function $U^*: \Phi_V \to \boldsymbol{R}$ in the following way:

$$U^*(V) \equiv \max_{U \in \Phi(V)} U, \quad \forall V \in \Phi_V.$$

That is, given that agent 2 receives an expected utility V, $U^*(V)$ is the maximum expected utility of agent 1 that can be achieved by a feasible and incentive-compatible contract. Our aim is to show that U^* is a fixed point of a mapping that we now seek to define.

Let $C(V) = [\sigma(\theta_i, \theta_j)(V), V(\theta_i, \theta_j)(V)]_{(\theta_i, \theta_j) \in \Theta^2}$, for all $V \in \Phi_V$. Let $\bar{U}: \Phi_V \to \boldsymbol{R}$ be any bounded function. Given $\bar{U}$, for all $V \in \Phi_V$, let

$$\xi(C(V), \bar{U}) = \sum \pi_i \pi_j [u(-\sigma(\theta_i, \theta_j) + \theta_i) + \beta \bar{U}(V(\theta_i, \theta_j))],$$

for all $C(V)$ such that the following constraints are satisfied: $\forall (\theta_i, \theta_j) \in \Theta^2$,

$$-\theta_j \leqq \sigma(\theta_i, \theta_j)(V) \leqq \theta_i, \; V(\theta_i, \theta_j)(V) \in \Phi_V; \tag{8}$$

$\forall (\theta_i, \theta_j) \in \Theta^2$ and $\forall \theta_k \in \Theta$,

$$u(-\sigma(\theta_i, \theta_j)(V) + \theta_i) + \beta \bar{U}(V(\theta_i, \theta_j)(V)) \geqq u(-\sigma(\theta_k, \theta_j)(V) + \theta_i) + \beta \bar{U}(V(\theta_k, \theta_j)(V)); \tag{9}$$

$\forall (\theta_i, \theta_j) \in \Theta^2$ and $\forall \theta_l \in \Theta$,

$$u(\sigma(\theta_i, \theta_j)(V) + \theta_j) + \beta V(\theta_i, \theta_j)(V) \geqq u(\sigma(\theta_i, \theta_l)(V) + \theta_j) + \beta V(\theta_i, \theta_l)(V); \tag{10}$$

and

$$\sum_{(\theta_i, \theta_j) \in \Theta^2} \pi_i \pi_j [u(\sigma(\theta_i, \theta_j)(V) + \theta_j) + \beta V(\theta_i, \theta_j)(V)] = V. \tag{11}$$

Condition (8) requires that $C(V)$ be feasible. Constraints (9) and (10) require that $C(V)$ be incentive compatible, given $\bar{U}$. Constraint (11) is the one-step analogue of (4) which promises that expected utility V be delivered to agent 2. Now define an operator T, which maps from bounded functions to bounded functions, as follows. Given function $\bar{U}$, let

$$T(\bar{U})(V) = \sup_{C(V)} \xi(C(V), \bar{U}), \quad \forall V \in \Phi_V,$$

where $C(V)$ satisfies constraints (8) through (11).

Proposition 2. $T(U^*)(V) = U^*(V)$, *for all* $V \in \Phi_V$.

A proof of Proposition 2 is in the Appendix. The following lemma states that the "sup" in the Bellman equation is actually attained.

Lemma 3. $T(U^*)(V) = \max_{C(V)} \xi(C(V), U^*)$, *for all* $V \in \Phi_V$.

A proof of Lemma 3 is in the appendix. Now let $C^*(V)=[\sigma^*(\theta_i,\theta_j)(V), V^*(\theta_i,\theta_j)(V)]_{(\theta_i,\theta_j)\in\Theta^2}$. Call $C^*=\{C^*(V): C^*(V)\in\text{argmax}_{C(V)}\,\xi(C(V),U^*), V\in\Phi_V\}$ an efficient allocation rule,[1] where $\{\sigma^*(\theta_i,\theta_j)(V): (\theta_i,\theta_j)\in\Theta^2, V\in\Phi_V\}$ is the efficient trading scheme and $\{V^*(\theta_i,\theta_j)(V): (\theta_i,\theta_j)\in\Theta^2, V\in\Phi_V\}$ is the optimal law of motion of the state variable. We say that an efficient allocation rule C^* can generate a contract σ, (or σ can be generated by C^*), if for all $t\geqq 1$, $h^{t-1}\in H^{t-1}$, and $(\theta_i,\theta_j)\in\Theta^2$,

$$\sigma_t(h^{t-1},(\theta_i,\theta_j))=\sigma^*(\theta_i,\theta_j)(V(\sigma|h^{t-1})),$$
$$V(\sigma|h^{t-1},(\theta_i,\theta_j))=V^*(\theta_i,\theta_j)(V(\sigma|h^{t-1})),$$
$$U(\sigma|h^{t-1},(\theta_i,\theta_j))=U^*(V^*(\theta_i,\theta_j)(V(\sigma|h^{t-1}))).$$

The following lemma establishes in some loose sense an equivalence relationship between efficient allocation rules and efficient contracts.

Lemma 4. (i) *Let σ be an efficient contract. Then there exists an efficient allocation rule C^* that generates σ.* (ii) *Let C^* be an efficient allocation rule. Then for all $V\in\Phi_V$, an efficient contract σ can be generated by C^* such that $V(\sigma)=V$.*

A formal proof of Lemma 4 is left for the reader. Due to (ii) of Lemma 4 then, to solve for an efficient contract, it is sufficient to solve for an efficient allocation rule which in turn amounts to solving the Bellman equation. For illustrative purposes, we now describe briefly how the contract σ that is generated by C^* in (ii) of Lemma 4 works. Let V_0 be the ex ante expected utility of agent 2. Suppose, at date 1, θ_i and θ_j are reported respectively by the two agents. Then the contract says that $\sigma^*(\theta_i,\theta_j)(V_0)$ amount of the consumption good is to be transferred from agent 1 to agent 2. In the meantime, the contract also determines that, from date 2 on, agent 2 is entitled to an expected utility $V_1=V^*(\theta_i,\theta_j)(V_0)$. Now as the two agents move to date 2, suppose $\theta_{i'}$ and $\theta_{j'}$ are reported, then $\sigma^*(\theta_{i'},\theta_{j'})(V_1)$ will be transferred from agent 1 to agent 2, and $V_2=V^*(\theta_{i'},\theta_{j'})(V_1)$ will be promised to agent 2 as his expected utility from date 3 on. In this way the contract rolls forward date by date. Notice that here the expected utility of agent 2, V_t, is acting as a state variable to summarize history. At the beginning of each date t, nothing but V_{t-1} matters, for today and for the future.

We now go on to derive a characterization of an efficient allocation rule. The following proposition says that no matter what agent 1 reports, agent 2 should receive a smaller transfer of current endowment from agent 1, and be entitled to a higher expected utility from tomorrow on, if he reports a higher endowment. Similarly, if he reports a lower endowment, then he should receive a larger transfer from agent 1 and be entitled to a lower expected utility from tomorrow on.

Proposition 3. *Let C^* be an efficient allocation rule. Then for all $V\in\Phi_V$, if $\theta_k\leqq\theta_i$ and $\theta_l\leqq\theta_j$, then*

$$\sigma^*(\theta_k,\theta_j)(V)\leqq\sigma^*(\theta_i,\theta_j)(V)\leqq\sigma^*(\theta_i,\theta_l)(V),$$
$$V^*(\theta_i,\theta_l)(V)\leqq V^*(\theta_i,\theta_j)(V)\leqq V^*(\theta_k,\theta_j)(V),$$
$$U^*(V^*(\theta_k,\theta_j)(V))\leqq U^*(V^*(\theta_i,\theta_j)(V))\leqq U^*(V^*(\theta_i,\theta_l)(V)).$$

1. This terminology is borrowed from Atkeson and Lucas (1992) but used here in a slightly different sense.

A proof of Proposition 3 is in the appendix. To fully characterize an efficient allocation rule, it is desirable that the Bellman equation in Proposition 2 be solved analytically. Green (1987), Thomas and Worrall (1990), and Atkeson and Lucas (1992) have shown that, for some special forms of the utility function, (exponential utility functions in particular), it is possible to derive closed-form solutions to their Bellman equations. This, however, is difficult here. There are two reasons, each can be viewed as a unique feature of our Bellman equation, compared to those in related models. First, in the Bellman equation here, there are explicit upper and lower boundaries for the net transfers, $\sigma(\theta_i, \theta_j)(V)$, whereas in the Bellman equations of Green (1987), Thomas and Worrall (1990), and Atkeson and Lucas (1992) for the exponential utility case, there is not a boundary on an individual agent's consumption. Second, and more important, in our Bellman equation, the value function $U^*(\cdot)$ enters into not only the objective but also both sides of the incentive constraints.

5. FURTHER CHARACTERIZATIONS OF THE EFFICIENT CONTRACT: THE CASE OF TWO ENDOWMENT VALUES

In this section, we present two propositions to further characterize the efficient contract without solving for it analytically. For tractability, we focus on the case where the endowment takes on only two values, i.e. $n=2$ and $\Theta=\{\theta_1, \theta_2\}$. First, we show in Proposition 4 that a contract where agent 1 transfers a constant amount of the endowment to agent 2 in every period cannot be efficient. A corollary of Proposition 4 hence is that the autarkic contract is dominated by an efficient contract.

Proposition 4. *The contract σ^c where $\sigma^c_t = c$, c being a constant, for all t is not efficient.*

A constructive proof of Proposition 4 is in the appendix. The rest of this section is devoted to looking at the long-run behaviour of the two agents' expected utilities. To motivate our result, note that a central proposition in Green (1987) is that the long-run distribution of expected utilities across agents is degenerate: for each individual agent in the population, his expected utility converges to negative infinity with probability one. Green assumes that his agents have an exponential utility function. Thomas and Worrall (1990) in their single-agent model show that for a family of utility functions which are not bounded from below, the agent's expected utility also converges to negative infinity with probability one. Atkeson and Lucas (1992) show that in cases in which the utility function of the agents takes either the logarithmic form, the CRRA form, or the CARA form, the expected utility of any individual agent converges to the minimum level in the set of possible expected utilities with probability one. This type of result, however, does not apply here. First, here by assuming that a feasible contract never takes away from any agent more than he receives, the two agents in our model will never consume a negative amount of endowment. This implies that their expected utilities are essentially bounded from below, although the two agents here may have the same unbounded utility function as the agents in, for example, Green (1987) have. This certainly rules out possibilities for the expected utilities of the two agents to converge to minus infinity. Further, given that in our model we have identical agents and they are constrained to consume the entire aggregate endowment each date, it is also unikely that their expected utilities will converge to the minimum in the expected utility possibilities with probability one. Our aim in the rest of the section is to show that the expected utility of each agent actually converges to every expected utility, including the minimum, in Φ_V with probability zero. This will guarantee that the long-run distributions of expected utilities of the two agents are not degenerate.

Let $V_0 \in \Phi_V$ be any arbitrary ex ante expected utility that the constrained efficient contract would promise to agent 2. Let $V_t (t \geqq 1)$ be the random variable representing the expected utility to which agent 2 is entitled at the end of date t.

Proposition 5. Prob $\{\lim_{t\to\infty} V_t = V\} = 0$, *for all* $V \in \Phi_V$.

Proof. Let $\{v_t\}_{t=1}^{\infty}$ [2] be any time-series (or path) of agent 2's expected utilities that has the following property

$$\lim_{t\to\infty} v_t = V. \tag{12}$$

For each v_t, let $\{\sigma^*(\theta_i, \theta_j)(v_t), V^*(\theta_i, \theta_j)(v_t)\}_{(\theta_i,\theta_j)\in\Theta^2}$ be the one-step profile of the constrained efficient contract at the state $V = v_t$. We show for the first step of the proof that either of the following two inequalities must hold:

$$\lim_{t\to\infty} V^*(\theta_1, \theta_n)(v_t) \neq V, \tag{13}$$

$$\lim_{t\to\infty} V^*(\theta_n, \theta_1)(v_t) \neq V. \tag{14}$$

Suppose not and $\lim_{t\to\infty} V^*(\theta_1, \theta_n)(v_t) = \lim_{t\to\infty} V^*(\theta_n, \theta_1)(v_t) = V$. Then, since

$$V^*(\theta_n, \theta_1)(v_t) \leqq V^*(\theta_i, \theta_j)(v_t) \leqq V^*(\theta_1, \theta_n)(v_t), \quad \forall (\theta_i, \theta_j) \in \Theta^2,$$

for all v_t, it is immediate that

$$\lim_{t\to\infty} V^*(\theta_i, \theta_j)(v_t) = V, \quad \forall (\theta_i, \theta_j) \in \Theta^2. \tag{15}$$

Apply this to the incentive constraints for agent 2 in the Bellman equation to yield

$$\lim_{t\to\infty} [\sigma^*(\theta_i, \theta_j)(v_t) - \sigma^*(\theta_{i'}, \theta_{j'})(v_t)] = 0, \quad \forall (\theta_i, \theta_j), (\theta_{i'}, \theta_{j'}) \in \Theta^2.$$

Since for each $(\theta_i, \theta_j) \in \Theta^2$, the sequence $\{\sigma^*(\theta_i, \theta_j)(v_t)\}_{t=1}^{\infty}$ is bounded and hence contains a convergent sub-sequence. For convenience we assume that this sub-sequence is the sequence itself. We therefore can write:

$$\lim_{t\to\infty} \sigma^*(\theta_i, \theta_j)(v_t) = c, \quad \forall (\theta_i, \theta_j) \in \Theta^2, \tag{16}$$

where c is some constant in $[-\theta_1, \theta_1]$. Now notice that for each v_t,

$$v_t = \textstyle\sum_{(\theta_i,\theta_j)\in\Theta^2} \pi_i \pi_j [u(\sigma^*(\theta_i, \theta_j)(v_t) + \theta_j) + \beta V^*(\theta_i, \theta_j)(v_t)].$$

Let $t \to \infty$ and due to (15) and (16), the above will yield:

$$V = \frac{1}{1-\beta} \textstyle\sum_{j\in\Theta} \pi_j u(c + \theta_j).$$

This implies that the contract σ^c where $\sigma_t^c = c$ for all t is efficient, contradicting Proposition 4. Therefore, either (13) or (14) must be true.

2. Note that as is standard we use the upper case for the random variable and the lower case for its realization.

We now proceed with the second step of the proof. Suppose (13) is true. Define two sub-sequences $\{x_q\}$ and $\{y_q\}$ of $\{v_t\}$ be such that

$$x_q = V^*(\theta_1, \theta_n)(y_q), \quad \forall q. \tag{17}$$

We show that $\{x_q\}$ can not contain infinitely many elements. Suppose the contrary, then due to (12),

$$\lim_{q\to\infty} x_q = \lim_{q\to\infty} y_q = V.$$

However, $\lim_{q\to\infty} y_q = V$ and (17) together would imply $\lim_{q\to\infty} x_q \neq V$, due to (13).[3] This is a contradiction. Therefore $\{x_q\}$ can have at most finitely many elements and hence the path $\{v_t\}$ allows only finitely many (θ_1, θ_n) to occur. Such paths have a measure zero. Supposing (14) is true will lead us to the same conclusion. ||

6. COMPUTING THE EFFICIENT CONTRACT

As our discussion in Section 4 indicates, the complex nature of our Bellman equation makes it difficult to solve analytically for an efficient contract. In order to obtain greater insight into the structure of an efficient contract, in this and the next section, we turn to pursue a computational approach. As a first step, we explore in this section two related algorithms for numerical computation of an efficient contract.

Following our analysis in Section 4, the key to solving for an efficient contract is to solve for Φ, the set of admissible expected utility pairs. Once Φ is obtained, then the set of admissible states, i.e. the set of admissible expected utilities of agent 2, Φ_V, and the value function of the Bellman equation, $U^*(V)$, $V\in\Phi_V$, are readily computed. Finally, solving the Bellman equation given Φ_V and $U^*(V)$ will yield the efficient trading scheme $\{\sigma^*(\theta_i, \theta_j)(V)\}$ and the optimal law of motion of the state variable $\{V^*(\theta_i, \theta_j)(V)\}$.

The following lemma, which is in the spirit of Abreu–Pearce–Stacchetti (1990), provides an algorithm for solving for Φ. Basically, starting with a set $W_0 \subseteq \boldsymbol{R}^2$ which is large enough, and operating on it iteratively using the operator B, we will then obtain a monotone sequence of sets converging to Φ.

Lemma 5. *Let W_0 be the space on which (U, V), the pair of expected utilities of the two agents, are allowed to take values, and assume $B(W_0)\subseteq W_0$. Let $W_{t+1} = B(W_t)$, $\forall t \geqq 0$. Then $\{W_t\}$ is monotone decreasing and* $\lim_{t\to\infty} W_t = W_\infty = \Phi$.

Here a natural candidate for W_0 is $[\underline{a}, \bar{a}]\times[\underline{a}, \bar{a}]$, where $\underline{a}$ is the expected life-time utility of the agent if he consumes zero units of the consumption good every period, and $\bar{a}$ is the expected life-time utility of the agent if he consumes the aggregate endowment every period. Of course any set in $\mathbf{R}^2$ that contains $[\underline{a}, \bar{a}]\times[\underline{a}, \bar{a}]$ will also do the job.

To numerically implement the above algorithm, W_0 is not allowed to take on continuous values. We can assume that W_0 contains N^2 grid points uniformly distributed over the space $[\underline{a}, \bar{a}]\times[\underline{a}, \bar{a}]$. That is, $W_0 = \{(U_p, V_q), p, q = 1, 2, \ldots, N\}$, where $U_p = V_p = \underline{a} + (\bar{a} - \underline{a})(p-1)/(N-1), p = 1, 2, \ldots, N$. To obtain $W_1 = B(W_0)$, we are essentially searching over W_0 for all the (U_p, V_q)s where there exist $[(U(\theta_i, \theta_j), V(\theta_i, \theta_j)) \in W_0]_{(\theta_i,\theta_j)\in\Theta^2}$, and $[\sigma(\theta_i, \theta_j)]_{(\theta_i,\theta_j)\in\Theta^2}$, such that they satisfy conditions (5) through (7) and $U_p = \sum \pi_i\pi_j[u(-\sigma(\theta_i, \theta_j)+\theta_i)+\beta U(\theta_i, \theta_j)]$, $V_q = \sum \pi_i\pi_j[u(\sigma(\theta_i, \theta_j)+\theta_j)+\beta V(\theta_i, \theta_j)]$.

3. Remember that in (13) $\{v_t\}$ is any arbitrary sequence converging to $V_{\min}$.

Since now we are dealing with a finite space of possible expected utilities, convergence of the sequence $\{W_t\}$ will occur after a finite number of iterations.

A deficiency of the above algorithm however is that the amount of computation that it requires to reach the solution can be large. At any $(t+1)$th iteration, given the complex nature of incentive compatibility, and that the space W_t over which we search for admissible expected utilities is two-dimensional, a large number of nonlinear programming problems must be solved. To reduce computation, we now proceed to develop an alternative algorithm by modifying the one in Lemma 5. The idea here is to compute the value function of the Bellman equation directly, without having to keep track of the whole set of admissible expected utilities Φ, as we do in Lemma 5.

Instead of covering the whole space on which the pair of expected utilities (U, V) takes on values with a grid, we now assume that only V is restricted to take values on a discrete space containing N grid points $\{V(K): K=1, 2, \ldots, N\}$, which are distributed over the interval $[\underline{\alpha}, \bar{\alpha}]$. But for each V, U is allowed to take continuous values on the closed interval $[\underline{\alpha}, \bar{\alpha}]$. The following proposition lays out an algorithm for computing an efficient contract under these assumptions.

Proposition 6. *Let* $\Phi_V^0=\{V(K): K=1, 2, \ldots, N\}$. *Let* $U_{\min}^0(V)=\underline{\alpha}$, *and* $U_{\max}^0(V)=\bar{\alpha}, \forall V\in\Phi_V^0$. *Let* $\Phi_0=\{(U, V) \mid U\in[U_{\min}^0(V), U_{\max}^0(V)], V\in\Phi_V^0\}$. *For* $t\geqq 0$ *and* $V\in\Phi_V^t$, *let*

$$S_{t+1}(V)=\{\textstyle\sum \pi_i\pi_j[u(-\sigma(\theta_i, \theta_j)+\theta_i)+\beta U(\theta_i, \theta_j)]\},$$

where $[\sigma(\theta_i, \theta_j), U(\theta_i, \theta_j)]_{(\theta_i,\theta_j)\in\Theta^2}$ *is such that there exists* $[V(\theta_i, \theta_j)]_{(\theta_i,\theta_j)\in\Theta^2}$ *so that* $[\sigma(\theta_i, \theta_j), U(\theta_i, \theta_j), V(\theta_i, \theta_j)]_{(\theta_i,\theta_j)\in\Theta^2}$ *is admissible to* Φ_t *and satisfies the following*:

$$\textstyle\sum \pi_i\pi_j[u(\sigma(\theta_i, \theta_j)+\theta_j)+\beta V(\theta_i, \theta_j)]=V.$$

Let

$$\Phi_{t+1}=\bar{B}(\Phi_t)=\{(U, V)\in\Phi_t \mid V\in\Phi_V^t, S_{t+1}(V)\neq\varnothing, U\in[U_{\min}^{t+1}(V), U_{\max}^{t+1}(V)]\},$$

where $U_{\max}^{t+1}(V)$ *and* $U_{\min}^{t+1}(V)$ *respectively are the maximum and minimum values in the set* $S_{t+1}(V)$. *Then, let*

$$\Phi_V^{t+1}=\{V\in\Phi_V^t: \quad \exists U \text{ s.t. } (U, V)\in\Phi_{t+1}\}.$$

Let $\Phi_V^\infty=\lim_{t\to\infty}\Phi_V^t$, *and* $U_{\max}^\infty(V)=\lim_{t\to\infty}U_{\max}^t(V), \forall V\in\Phi_V^\infty$. *If* $T(U_{\max}^\infty)(V)=U_{\max}^\infty(V), \forall V\in\Phi_V^\infty$, *then* $\Phi_V=\Phi_V^\infty$, *and* $U^*(V)=U_{\max}^\infty(V), \forall V\in\Phi_V$.

Proof. We begin the proof by noticing several simple facts. First, $\bar{B}$ is monotonic and $\bar{B}(\Phi_0)\subseteq\Phi_0$. Second, $B(\Psi)\subseteq\bar{B}(\Psi), \forall\Psi\in\mathbf{R}^2$. Third, the two sequences $\{\Phi_t\}$ and $\{\Phi_V^t\}$ are monotone decreasing. Fourth, for all $V\in\Phi_V^\infty$, $\{U_{\max}^t(V)\}$ is monotone decreasing but $\{U_{\min}^t(V)\}$ is monotone increasing.

Given the conditions, we have: $\{(U_{\max}^\infty(V), V): V\in\Phi_V^\infty\}\subseteq\Phi$, and it then follows immediately that $\Phi_V^\infty\subseteq\Phi_V$, and $U_{\max}^\infty(V)\leqq U^*(V), \forall V\in\Phi_V^\infty$.

Now let $\{W_t\}$ be the monotone sequence of sets generated by operating B (rather than $\bar{B}$) iteratively on Φ_0. Then

$$\Phi\subseteq B(\Phi)\subseteq B(\Phi_0)=W_1\subseteq\bar{B}(\Phi_0)=\Phi_1.$$

By induction it can be shown that in general,

$$\Phi \subseteq W_t \subseteq \Phi_t, \quad \forall t \geqq 0.$$

But this implies that $\Phi_V \subseteq \Phi_V^\infty$ and $U^*(V) \leqq U_{\max}^\infty(V)$, $\forall V \in \Phi_V$. ||

In Proposition 6, by allowing U to take continuous values leads to a large reduction in the amount of search over the grid points. At any $(t+1)$th iteration, we only search for admissible expected utilities of agent 2 along a single dimensional space Φ_V^t, whereas in Lemma 5, we were searching over a two-dimensional space W_t for admissible expected utility pairs. However, as is commonplace in dynamic programming problems where the mapping that defines the Bellman equation is not a contraction and uniqueness of a fixed point of this mapping is not guaranteed, Proposition 6 as an algorithm only works when a certain requirement is satisfied. Here the key requirement is that the function $U_{\max}^\infty$ must in fact be a fixed point of operator T. Obviously, a sufficient condition for this is that the operator B preserves convexity, in the sense that for all t and V, $(U_1, V) \in W_t$ and $(U_2, V) \in W_t$ together will imply $(\alpha U_1 + (1-\alpha)U_2, V) \in W_t$, $\forall \alpha \in (0, 1)$. In this case, B and $\bar{B}$ will essentially be equivalent. Finally, we note that although the condition that $U_{\max}^\infty$ is a fixed point of T may be hard to verify analytically, it is straightforward to check computationally after the convergence occurs.

7. COMPUTING THE EFFICIENT CONTRACT: AN EXAMPLE

For illustrative purposes, in this section we solve numerically a parameterized example of our model using the algorithm provided by Proposition 6. Assume utility is exponential, i.e. $u(c) = -\exp(-c)$. Assume $\beta = 0{\cdot}96$. Assume the endowment can be either low or high: $\theta_1 = 0{\cdot}2$ and $\theta_2 = 0{\cdot}4$. The low and high endowments are received by each agent with equal probabilities: $\pi_1 = \pi_2 = 0{\cdot}5$. Assume that the expected utility of agent 2 can only take values on a finite set that contains one hundred grid points, $\{V(1), \ldots, V(100)\}$, which are uniformly distributed over the interval $[\underline{\alpha}, \bar{\alpha}]$, where $\underline{\alpha}$ and $\bar{\alpha}$ are as defined in the previous section.

We find, for the efficient contract, $\Phi_V = \{V(22), \ldots, V(84)\}$. That is, any expected utility which is below $V(22)$ or above $V(84)$ is not achievable. The value function $U^*(V(K))$ is found to be concave and monotone decreasing. The efficient trading scheme is depicted in Figure 1. Notice that $\sigma^*(\theta_1, \theta_2)(V(K)) < \sigma^*(\theta_i, \theta_i)(V(K)) < \sigma^*(\theta_2, \theta_1)(V(K))$, $i = 1, 2$. Remember that this is the property we prove analytically in Proposition 3, which describes the impact of current endowments on current trades. Also notice that $\sigma^*(\theta_i, \theta_j)(V(K))$ are monotone increasing in K, which means that $\sigma^*(\theta_i, \theta_j)(V(K))$, and hence $\sigma^*(\theta_i, \theta_j)(V(K)) + \theta_j$, which is the current consumption of agent 2, tends to increase as K increases. That is, agent 2 will receive more transfer of the consumption good from agent 1 and hence consume more currently, as his wealth accumulates. Similarly, agent 1's current consumption $-\sigma^*(\theta_i, \theta_j)(V(K)) + \theta_i$ decreases as K increases. Note that this is how history affects current consumption.

The optimal law of motion of the state variable is as follows:

$$V^*(\theta_1, \theta_1)(V(K)) = V(K), \; K = 22, \ldots, 84.$$

$$V^*(\theta_1, \theta_2)(V(K)) = V(K+1), \; K = 22, \ldots, 83; \; V^*(\theta_1, \theta_2)(V(84)) = V(84).$$

$$V^*(\theta_2, \theta_1)(V(K)) = V(K-1), \; K = 23, \ldots, 84; \; V^*(\theta_2, \theta_1)(V(22)) = V(22).$$

$$V^*(\theta_2, \theta_2)(V(K)) = V(K), \; K = 22, \ldots, 84.$$

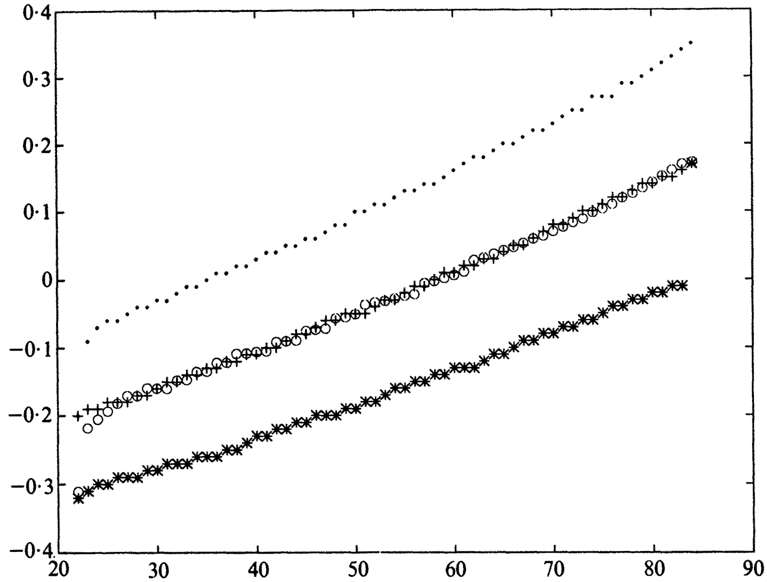

FIGURE 1

Net Transfer from agent 1 to agent 2. '+' $=\sigma^*(\theta_1, \theta_1)(V(K))$, '*' $=\sigma^*(\theta_1, \theta_2)(V(K))$, '·' $=\sigma^*(\theta_2, \theta_1)(V(K))$, 'O' $=\sigma^*(\theta_2, \theta_2)(V(K))$.

Notice that for all (θ_i, θ_j), $V^*(\theta_i, \theta_j)(V(K))$ is monotone increasing in K. Note that this is how history affects future wealth: for given current endowment realizations, the agent will be in a better wealth position tomorrow if he is in a better wealth position today.

The above law of motion of the state variable indicates that the expected utilities of each agent form a stationary Markov chain. This in turn implies that the consumption process of each agent also forms a stationary Markov chain. To illustrate graphically, Figure 2 plots an example of the expected utility paths of the two agents who start with almost the same ex ante expected life-time utilities over a period of 400 dates. Notice that although the two agents have ergodic long-run distributions in expected utilities, their wealth positions may still fan out temporarily. Finally, Figure 3 plots the associated consumption paths of the two agents. Notice the persistence in consumption that shows up in this figure.

8. CONCLUDING REMARKS

This paper studies a model of dynamic insurance under private information in a pure exchange economy. There are two infinitely-lived agents in our model, both risk-averse and each having an i.i.d. stochastic endowment stream which is unobservable to the other. We give sufficient and necessary conditions for the existence of a constrained efficient contract. We show that a constrained efficient contract can be characterized in a Bellman equation. An algorithm for numerical computation of an efficient contract is discussed and an example with exponential utility is computed.

Our model here is simple and restricted. For example, there are only two agents in our model. One natural extension is to allow for multiple agents, and it is clear that the technical approach here is able to be modified to confront this situation. Specifically, in the case of N agents, an efficient contract can be defined as one which maximizes the

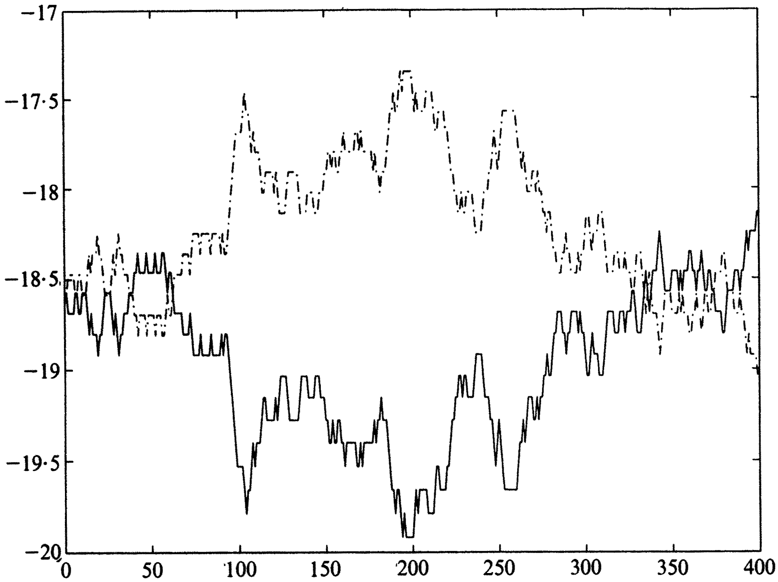

FIGURE 2
Expected utility paths of the two agents: '-.' = expected utility of agent 1, '-' = expected utility of agent 2.

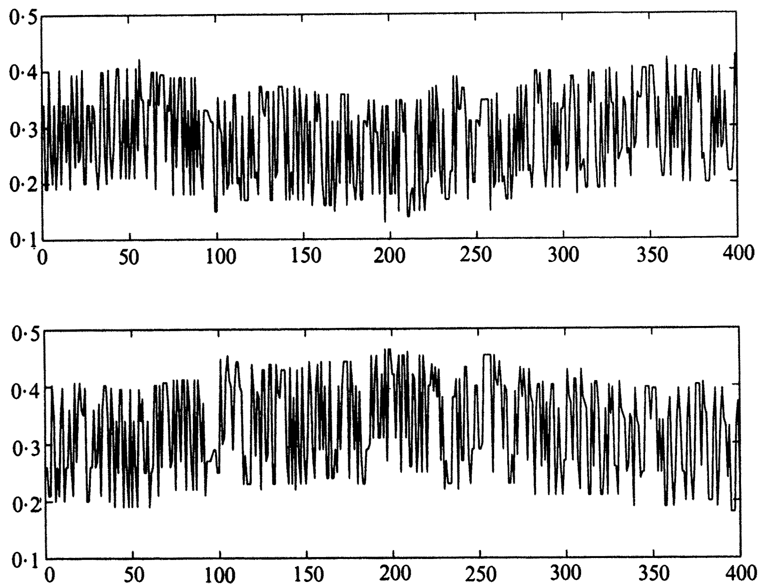

FIGURE 3
Consumption paths of the two agents. Top panel: consumption path of agent 1. Bottom panel: consumption path of agent 2.

expected utility of the Nth agent, subject to delivering a given vector of expected utilities to the rest $N-1$ agents. For the Bellman equation, $N-1$ state variables, each corresponding to the expected utilities of the $N-1$ agents, will need to be defined.

Other extensions of the model are also possible. For example, the only consumption good here is perishable, it will be interesting to see how savings can be determined in a bilateral trading context by allowing for storage in our model. Of course it is also important to understand to what extent the efficient allocations in our model can be achieved in a decentralized environment with price-taking traders.

APPENDIX

Lemma 1. *If Ψ is self-generating, then $\Psi \subseteq \Phi$.*

Proof. Let Ψ be self-generating and let $\psi(h^0)=(\psi^1(h^0), \psi^2(h^0)) \in B(\Psi)$. We need to show that $\psi(h^0) \in \Phi$, i.e., there exists a feasible and incentive-compatible co-insurance contract $\sigma(\psi(h^0))$ such that $(U, V)(\sigma(\psi(h^0)))=\psi(h^0)$.

We start by constructing the contract $\sigma(\psi(h^0))$. By the definition of $B(\Psi)$, there exists a pair $(\mathscr{C}(\psi(h^0)), \mathscr{U}(\psi(h^0)))$, where $\mathscr{C}(\psi(h^0))=[\sigma(\psi(h^0))(\theta_i, \theta_j)]_{(i,j)\in\Theta^2}$, admissible with respect to Ψ such that $\mathscr{E}(\mathscr{C}(\psi(h^0)), \mathscr{U}(\psi(h^0)))=\psi(h^0)$. Define, for all $(\theta_i, \theta_j)\in\Theta^2$, that $\sigma_1(h^0, (\theta_i, \theta_j))=\sigma(\psi(h^0))(\theta_i, \theta_j)$. Then for any date 1 reported realization of endowments, say (θ_i, θ_j), let

$$\psi(h^1)=\psi(h^0, (\theta_i, \theta_j))=\mathscr{U}(\psi(h^0))(\theta_i, \theta_j)\in\Psi\subseteq B(\Psi).$$

Where the "$\in$" is due to the fact that $\mathscr{U}(\psi(h^0))$ is a selection from Ψ and the "$\subseteq$" is due to the fact that Ψ is self-generating.

Now for $\psi(h^1)\in B(\Psi)$ instead of $\psi(h^0)\in B(\Psi)$, follow the above procedure to obtain $\sigma_2(h^2)$ and $\psi(h^2)$. Repeat this for all t to obtain:

$$\sigma(\psi(h^0))=\{\sigma_1(h^1), \sigma_2(h^2), \ldots, \sigma_t(h^t), \ldots\},$$
$$S(\psi(h^0))=\{\psi(h^0), \psi(h^1), \ldots, \psi(h^t), \ldots\}.$$

We now demonstrate that $S(\psi(h^0))$ is the sequence of expected utility vectors that the contract $\sigma(\psi(h^0))$ will generate for the two agents. Precisely,

$$(U, V)(\sigma(\psi(h^0))|h^t)=\psi(h^t), \qquad t=0, 1, 2, \ldots. \tag{18}$$

Note that if the above equation is indeed true, then it is easy to perceive that the co-insurance contract $\sigma(\psi(h^0))$ is feasible, incentive-compatible and gives the two agents the expected utility vector $\psi(h^0)$, as is desired. To show that (18) is true, observe that a simple fact from the above recursive construction of $\sigma(\psi(h^0))$ is:

$$\sigma(\psi(h^0))|h^t=\sigma(\psi(h^t)), \qquad t=0, 1, 2, \ldots.$$

Use this relationship to write:

$$\begin{aligned}U(\sigma(\psi(h^0))&=\sum \pi_i\pi_j[u(-\sigma_1(h^0, (\theta_i, \theta_j))+\theta_i)+\beta U(\sigma(\psi(h^0))|h^0, (\theta_i, \theta_j))]\\&=\sum \pi_i\pi_j[u(-\sigma_1(h^0, (\theta_i, \theta_j))+\theta_i)+\beta U(\sigma(\psi(h^0, (\theta_i, \theta_j))))].\end{aligned}$$

On the other hand, by the construction of $\psi(h^0)$, we have:

$$\begin{aligned}\psi^1(h^0)&=\sum \pi_i\pi_j[u(-\sigma(\psi(h^0))(\theta_i, \theta_j)+\theta_i)+\beta U(\psi(h^0))(\theta_i, \theta_j))]\\&=\sum \pi_i\pi_j[u(-\sigma_1(h^0, (\theta_i, \theta_j))+\theta_i)+\beta\psi^1(h^0, (\theta_i, \theta_j))].\end{aligned}$$

Therefore,

$$\begin{aligned}|\psi^1(h^0)-U(\sigma(\psi(h^0))| &\leqq \beta\sum \pi_i\pi_j|\psi^1(h^0, (\theta_i, \theta_j))-U(\sigma(\psi(h^0, (\theta_i, \theta_j))))|\\&\leqq \beta \sup_{(\theta_i,\theta_j)\in\Theta^2}|\psi^1(h^0, (\theta_i, \theta_j))-U(\sigma(\psi(h^0, (\theta_i, \theta_j))))|\\&\cdots\\&\leqq \beta^t \sup_{(\theta_i,\theta_j)\in\Theta^2}|\psi^1(h^{t-1}, (\theta_i, \theta_j))-U(\sigma(\psi(h^{t-1}, (\theta_i, \theta_j))))|.\end{aligned}$$

Note that the above is true for all $t\geqq 1$ and all $h^{t-1}\in H^{t-1}$. Now let $t\to\infty$. Since $0<\beta<1$ and utilities are bounded, it is immediate that $\psi^t(h^0)=U(\sigma(\psi)(h^t))$, $\forall t\geqq 0$. Therefore half of (18) is proven. In the same way we can show the other half to be true. ||

Lemma 2. *Φ is self-generating.*

Proof. Let $\phi=(\phi^1, \phi^2)\in\Phi$. We need to show that $\phi\in B(\Phi)$. By definition of Φ, there exists a co-insurance contract $\sigma(\phi)$ such that $(U, V)(\sigma(\phi))=\phi$, or equally,

$$\phi^1=\sum \pi_i\pi_j[u(-\sigma_1(\phi)(h^0, (\theta_i, \theta_j)+\theta_i)+\beta U(\sigma(\phi)|h^0, (\theta_i, \theta_j))],$$

$$\phi^2=\sum \pi_i\pi_j[u(\sigma_1(\phi)(h^0, (\theta_i, \theta_j)+\theta_j)+\beta V(\sigma(\phi)|h^0, (\theta_i, \theta_j))].$$

Define a pair $(\mathscr{C}, \mathscr{U})(\phi)$ such that for all $(\theta_i, \theta_j)\in\Theta^2$, $\sigma(\phi)(\theta_i, \theta_j)=\sigma_1(\phi)(h^0, (\theta_i, \theta_j))$, $U(\phi)(\theta_i, \theta_j)=U(\sigma(\phi)|h^0, (\theta_i, \theta_j))$, and $V(\phi)(\theta_i, \theta_j)=V(\sigma(\phi)|h^0, (\theta_i, \theta_j))$. It is then obvious that $(\mathscr{C}, \mathscr{U})(\phi)$ thus constructed is feasible and incentive compatible in the one-step sense defined by constraints (5), (6) and (7), and also gives the vector of expected utilities (ϕ^1, ϕ^2) to the two agents. ||

Proof of Proposition 1. $\Phi=B(\Phi)$ is an immediate consequence of Lemma 1 and Lemma 2. The proof for the compactness of Φ takes two steps. We first show that if Ψ is closed, then $B(\Psi)$ is closed, too. In other words, B preserves closedness. Let Ψ be closed and let sequence $\{\psi_n\}\subseteq B(\Psi)$ be such that $\psi_n\to\psi$, as $n\to\infty$. By the definition of $B(\Psi)$, there exists a sequence $\{(\mathscr{C}_n, \mathscr{U}_n)\}$ with each element admissible with respect to Ψ such that $\mathscr{E}(\mathscr{C}_n, \mathscr{U}_n)=\psi_n$, $\forall n$. Since the space of all admissible pairs with respect to Ψ is bounded, $\{(\mathscr{C}_n, \mathscr{U}_n)\}$ has a convergent subsequence $(\mathscr{C}_{n_q}, \mathscr{U}_{n_q})\to(\mathscr{C}, \mathscr{U})$, as $q\to\infty$. But $\mathscr{E}(\mathscr{C}, \mathscr{U})$ is continuous in $(\mathscr{C}, \mathscr{U})$, we have $\mathscr{E}(\mathscr{C}, \mathscr{U})=\lim_{q\to\infty}\mathscr{E}(\mathscr{C}_{n_q}, \mathscr{U}_{n_q})=\lim_{n\to\infty}\psi_n=\psi$. Left to be shown are: *(a)* $\mathscr{U}$ is a continuation value function with respect to Ψ; and *(b)* $(\mathscr{C}, \mathscr{U})$ satisfies equations (5) through (7). To show *(a)*, simply notice that since $\mathscr{U}_{n_q}(\theta_i, \theta_j)\in\Psi$, $\forall(\theta_i, \theta_j)\in\Theta^2$, and Ψ is closed, we have $\mathscr{U}(\theta_i, \theta_j)=\lim_{q\to\infty}\mathscr{U}_{n_q}(\theta_i, \theta_j)$, $\forall(\theta_i, \theta_j)\in\Theta^2$. *(b)* is obvious. Therefore we have shown that $\psi\in B(\Psi)$, and hence $B(\Psi)$ is closed.

Now we can proceed with the second step of the proof. We need only show that Φ is closed since it is certainly bounded. Let $\bar{\Phi}$ be the closure of Φ. By definition, $\Phi\subseteq\bar{\Phi}$. Since the operator B is monotone increasing, we thus have $B(\Phi)\subseteq B(\bar{\Phi})$. But $B(\Phi)=\Phi$, therfore $\Phi\subseteq B(\bar{\Phi})$. Now since $\bar{\Phi}$ is closed, by the result of the first step of the proof then, $B(\bar{\Phi})$ is also closed. However, since $\bar{\Phi}$ is the smallest closed set containing Φ, it must be the case that $\bar{\Phi}\subseteq B(\bar{\Phi})$, that is, $\bar{\Phi}$ is self-generating. Therefore by Lemma 1, $B(\bar{\Phi})\subseteq\Phi$, implying $\bar{\Phi}\subseteq\Phi$. Hence we have shown that $\bar{\Phi}=\Phi$, or Φ is closed. ||

Proof of Proposition 2. Fix V. Let $C(V)$ be such that $(C(V), U^*)$ meets (8), (10), (11), and

$$u(-\sigma(\theta_i, \theta_j)(V)+\theta_i)+\beta U^*(V(\theta_i, \theta_j)(V))\geqq u(-\sigma(\theta_i, \theta_j)(V)+\theta_j)+\beta U^*(V(\theta_i, \theta_j)(V)).$$

To show $T(U^*)(V)\leqq U^*(V)$, we need only show that there exists a contract σ which is feasible and incentive compatible and is such that $V(\sigma)=V$, and $U(\sigma)=T(U^*)(V)$. Now for each $(\theta_i, \theta_j)\in\Theta^2$, since $(U^*(V(\theta_i, \theta_j)(V)), V(\theta_i, \theta_j)(V))\in\Phi$, there exists a feasible and incentive-compatible contract σ_{ij} such that

$$U(\sigma_{ij})=U^*(V(\theta_i, \theta_j)(V)), \qquad V(\sigma_{ij})=V(\theta_i, \theta_j)(V), \quad \forall(\theta_i, \theta_j)\in\Theta^2.$$

We can then let the contract $\sigma=\{\sigma_t(h^t)\}$ be constructed in the following way:

$$\sigma_1(h^0, (\theta_i, \theta_j))=\sigma(\theta_i, \theta_j)(V), \qquad \sigma|h^0, (\theta_i, \theta_j)=\sigma_{ij}, \quad \forall(\theta_i, \theta_j)\in\Theta^2.$$

We now proceed to show that $U^*(V)\leqq T(U^*)(V)$. For all $U(\sigma)\in\Phi(V)$, we have

$$U(\sigma)=\sum \pi_i\pi_j[u(-\sigma_1(h^0, (\theta_i, \theta_j))+\theta_i)+\beta U(\sigma|h^0, (\theta_i, \theta_j))]$$

and

$$V=\sum \pi_i\pi_j[u(\sigma_1(h^0, (\theta_i, \theta_j))+\theta_j)+\beta V(\sigma|h^0, (\theta_i, \theta_j))],$$

where $\{\sigma_1(h^0, (\theta_i, \theta_j)), U(\sigma|h^0, (\theta_i, \theta_j)), V(\sigma|h^0, (\theta_i, \theta_j))\}_{(\theta_i, \theta_j)\in\Theta^2}$ satisfies (1), (2), and (3). But by definition of $U^*(V)$,

$$U(\sigma|h^0, (\theta_i, \theta_j))\leqq U^*(V(\sigma|h^0, (\theta_i, \theta_j))).$$

Therefore for all $U(\sigma)\in\Phi(V)$,

$$\begin{aligned} U(\sigma) &\leqq \sum \pi_i\pi_j[u(-\sigma_1(h^0, (\theta_i, \theta_j))+u_i)+\beta U^*(V(\sigma|h^0, (\theta_i, \theta_j)))] \\ &\leqq \sup_{C(V)} \xi(C(V), U^*) \\ &= T(U^*)(V). \end{aligned}$$

Thus taking the maximum across $U(\sigma)$ yields $U^*(V)=\max_{U(\sigma)\in\Phi(V)} U(\sigma)\leqq T(U^*)(V)$. ||

Proof of Lemma 3. Fix V. Since $\Phi(V)$ is compact, there exists a contract σ^* such that $U(\sigma^*)=U^*(V)$. Then $[\sigma_1^*(h^0,(\theta_i,\theta_j)),\ V(\sigma^*|h^0,(\theta_i,\theta_j))]_{(\theta_i,\theta_j)\in\Theta^2}\in \text{argmax}_{C(V)}\ \xi(C(V),U^*)$. ||

Proof of Proposition 3. Fix V. Fix θ_l and θ_j. Let $\theta_l<\theta_j$. Manipulate agent 2's incentive-compatibility constraints to get

$$u(\sigma^*(\theta_i,\theta_j)(V)+\theta_j)-u(\sigma^*(\theta_i,\theta_l)(V)+\theta_j)\geqq\beta[V^*(\theta_i,\theta_l)(V)-V^*(\theta_i,\theta_j)(V)],$$

$$u(\sigma^*(\theta_i,\theta_l)(V)+\theta_l)-u(\sigma^*(\theta_i,\theta_j)(V)+\theta_l)\geqq\beta[V^*(\theta_i,\theta_j)(V)-V^*(\theta_i,\theta_l)(V)].$$

Adding these two inequalities yields:

$$u(\sigma^*(\theta_i,\theta_j)(V)+\theta_j)-u(\sigma^*(\theta_i,\theta_l)(V)+\theta_j)\geqq u(\sigma^*(\theta_i,\theta_j)(V)+\theta_l)-u(\sigma^*(\theta_i,\theta_l)(V)+\theta_l).$$

Define function f: $\boldsymbol{R}_+\to\boldsymbol{R}$ as follows:

$$f(\theta)=u(\sigma^*(\theta_i,\theta_j)(V)+\theta)-u(\sigma^*(\theta_i,\theta_l)(V)+\theta).$$

Then we have: $f(\theta_j)\geqq f(\theta_l)$. Suppose, by way of contradiction, that $\sigma^*(\theta_i,\theta_j)(V)>\sigma^*(\theta_i,\theta_l)(V)$, then

$$f'(\theta)=u'(\sigma^*(\theta_i,\theta_j)(V)+\theta)-u'(\sigma^*(\theta_i,\theta_l)(V)+\theta)<0.$$

Since $\theta_l<\theta_j$, $f(\theta_l)>f(\theta_j)$, we have a contradiction. Therefore $\sigma^*(\theta_i,\theta_j)(V)\leqq\sigma^*(\theta_i,\theta_l)(V)$ must be the case. Applying this result to the first inequality in this proof, it is immediate that $V^*(\theta_i,\theta_l)(V)\leqq V^*(\theta_i,\theta_j)(V)$. In almost the same way the remaining parts of the lemma can be shown to be true. ||

Proof of Proposition 4. We prove the proposition by constructing a feasible and perfectly incentive compatible contract which strictly improves upon contract σ^c in the Pareto sense. Without losing generality assume $c\geqq 0$. Let $\delta\in[0,\theta_2-c]$ and let $\Delta\in[0,c]$. Construct a contract called $\sigma(\delta,\Delta)$ in the following way.

For $t=1$, let $\sigma_1(\delta,\Delta)(g^0,(\theta_2,\theta_1))=c+\delta$, and $\sigma_1(\delta,\Delta)(g^0,(\theta_i,\theta_j))=c$, for $(\theta_i,\theta_j)\neq(\theta_2,\theta_1)$. For $t=2$, let $\sigma_2(\delta,\Delta)(g^2)=c-\Delta$, if $g^1=(g^0,(\theta_2,\theta_1))$; otherwise $\sigma_2(\delta,\Delta)(g^2)=c$. Finally, for $t\geqq 3$, let $\sigma_t(\delta,\Delta)(g^t)=c$, for all $g^t\in H^t$. Obviously, then, $\sigma(0,0)$ is just σ^c. Note that for all δ and Δ the contract $\sigma(\delta,\Delta)$ thus constructed is certainly feasible, and it is also temporarily incentive compatible at all the dates $t\geqq 2$. We now proceed to show that by choosing the magnitudes of d and Δ properly, $\sigma(\delta,\Delta)$ can be made temporarily incentive compatible at date 1 as well and satisfy the desired Pareto dominance requirement.

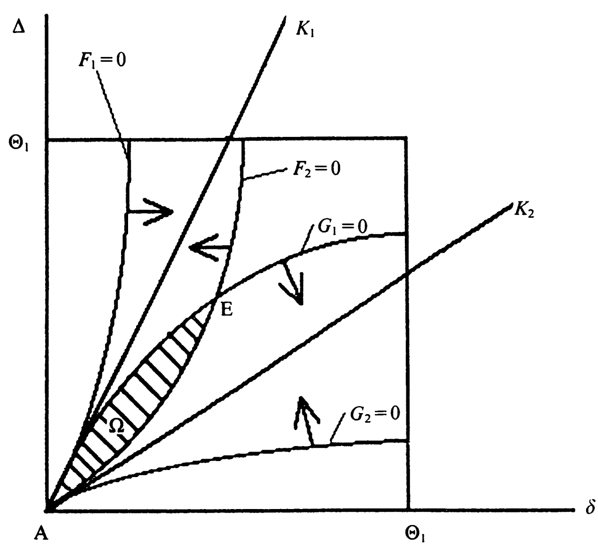

FIGURE 4

It is straightforward to show that $\sigma(\delta, \Delta)$ is temporarily incentive compatible at date 1 if and only if the following inequalities hold:

$$F_1(\delta, \Delta) = u(-c-\delta+\theta_1) + \beta[\pi_1 u(-c+\Delta+\theta_1) + \pi_2 u(-c+\Delta+\theta_2)] \\ -u(-c+\theta_1) - \beta[\pi_1 u(-c+\theta_1) + \pi_2 u(-c+\theta_2)] \leqq 0, \tag{19}$$

$$F_2(\delta, \Delta) = u(-c-\delta+\theta_2) + \beta[\pi_1 u(-c+\Delta+\theta_1) + \pi_2 u(-c+\Delta+\theta_2)] \\ -u(-c+\theta_2) - \beta[\pi_1 u(-c+\theta_1) + \pi_2 u(-c+\theta_2)] \geqq 0, \tag{20}$$

$$G_1(\delta, \Delta) = u(c+\delta+\theta_1) + \beta[\pi_1 u(c-\Delta+\theta_1) + \pi_2 u(c-\Delta+\theta_2)] \\ -u(c+\theta_1) - \beta[\pi_1 u(c+\theta_1) + \pi_2 u(c+\theta_2)] \leqq 0, \tag{21}$$

$$G_2(\delta, \Delta) = u(c+\delta+\theta_2) + \beta[\pi_1 u(c-\Delta+\theta_1) + \pi_2 u(c-\Delta+\theta_2)] \\ -u(c+\theta_2) - \beta[\pi_1 u(c+\theta_1) + \pi_2 u(c+\theta_2)] \geqq 0. \tag{22}$$

And, $\sigma(\delta, \Delta)$ strictly dominates autarky if and only if either (20) or (21) holds in strict inequality. let $\Omega = \{(\delta, \Delta) \in [0, \theta_1]^2 | (\delta, \Delta) \text{ s.t. } (19), (20), (21), (22)\}$. Notice that since $F_i(0, 0) = G_i(0, 0) = 0$, $i = 1, 2$, we have $(0, 0) \in \Omega$. In the following we will show that Ω contains a point at which either (20) or (21) holds with strict inequality. To this end, we find that Ω is characterized by the following facts:

$$\left.\frac{d\Delta}{d\delta}\right|_{F_i(\delta,\Delta)=0} > 0, \quad \left.\frac{d\Delta}{d\delta}\right|_{G_i(\delta,\Delta)=0} > 0, \quad i = 1, 2;$$

$$\left.\frac{d^2\Delta}{d\delta^2}\right|_{F_i(\delta,\Delta)=0} > 0, \quad \left.\frac{d^2\Delta}{d\delta^2}\right|_{G_i(\delta,\Delta)=0} < 0, \quad i = 1, 2;$$

$$\left.\frac{d\Delta}{d\delta}\right|_{F_1(0,0)=0} = \left.\frac{d\Delta}{d\delta}\right|_{G_1(0,0)=0} = K_1 > K_2 = \left.\frac{d\Delta}{d\delta}\right|_{F_2(0,0)=0} = \left.\frac{d\Delta}{a\delta}\right|_{G_2(0,0)=0}.$$

[4]With these facts in hand, the set Ω is depicted graphically in Figure 4.[5] Obviously then, for all $(\delta, \Delta) \in \Omega - \{A, E\}$, $\bar{\sigma}(\delta, \Delta)$ strictly dominates autarky. ||

4. To show these facts, define functions α and γ that map from $\boldsymbol{R}$ to $\boldsymbol{R}$ as follows:

$$\alpha(x) = (1/\beta)[\pi_1 u'(x-c+\theta_1) + \pi_2 u'(x-c+\theta_2)]^{-1},$$
$$\gamma(x) = (1/\beta)[\pi_1 u'(x+c+\theta_1) + \pi_2 u'(x+c+\theta_2)]^{-1}.$$

Then for $i = 1, 2$, we have:

$$\left.\frac{d\Delta}{d\delta}\right|_{F_i(\delta,\Delta)=0} = \alpha(\Delta) u'(c-\delta+\theta_i) > 0,$$

$$\left.\frac{d\Delta}{d\delta}\right|_{G_i(\delta,\Delta)=0} = \gamma(-\Delta) u'(c+\delta+\theta_i) > 0,$$

$$\left.\frac{d^2\Delta}{d\delta^2}\right|_{F_i(\delta,\Delta)=0} = -\alpha(\Delta) u''(-c-\delta+\theta_i) - \alpha(\Delta)^3 \beta u'(-c-\delta+\theta_i)^2 \\ \times [\pi_1 u''(-c+\Delta+\theta_1) + \pi_2 u''(-c+\Delta+\theta_2)] > 0,$$

$$\left.\frac{d^2\Delta}{d\delta^2}\right|_{G_i(\delta,\Delta)=0} = \gamma(-\Delta) u''(c+\delta+\theta_i) + \gamma(-\Delta)^3 \beta u'(c+\delta+\theta_i)^2 \\ \times [\pi_1 u''(c-\Delta+\theta_1) + \pi_2 u''(c-\Delta+\theta_2)] < 0,$$

$$\left.\frac{d\Delta}{d\delta}\right|_{F_i(0,0)=0} = \alpha(0) u'(-c+\theta_i),$$

$$\left.\frac{d\Delta}{d\delta}\right|_{G_i(0,0)=0} = \gamma(0) u'(c+\theta_i).$$

It is straightforward to verify that $\alpha(0)u'(-c+\theta_i) > \gamma(0)u'(c+\theta_i)$.

5. Note that the arrows in the figure point to the directions that are consistent with the inequalities from (19) to (22).

Proof of Lemma 5. $\{W_t\}$ is monotone decreasing because B is mototonic and $B(W_0)\subseteq W_0$. To prove the limiting result, first, we show that the sequence $\{W_t\}$ converges. It is obvious that $B(W_0)\subseteq W_0$. Now operate B repeatedly on both sides of this expression to yield: $W_{t+1}=B(W_t)\subseteq W_t$, for all $t\geqq 0$, as the operator B is monotone increasing. Therefore $\{W_t\}$ is a bounded and monotone decreasing sequence. It converges and in fact $W_\infty=\lim_{t\to\infty} Wt=\bigcap_{t=0}^{\infty} W_t$. Second, we show that $\Phi\subseteq W_\infty$. Obviously, $\Phi\subseteq W_0$. Monotonicity of B implies $B(\Phi)\subseteq B(W_0)$. But $B(\Phi)=\Phi$ by Proposition 1 and $B(W_0)=W_1$ by construction, we thus have: $\Phi\subseteq W_1$. Iterate the above procedure to obtain: $\Phi\subseteq W_t$, for all $t\geqq 0$. Therefore $\Phi\subseteq W_\infty$. Third, we show that $W_\infty\subseteq\Phi$. By the construction and convergence of $\{W_t\}$, $B(W_\infty)=W_\infty$. Therefore W_∞ is self-generating. By Lemma 1 then, $W_\infty=B(W_\infty)\subseteq\Phi$. ||

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