GENERAL COMPETITIVE ANALYSIS

IN AN ECONOMY WITH PRIVATE INFORMATION*

by

Edward C. Prescott and Robert M. Townsend**

Discussion Paper No. 81 - 144, April 1981

---

* This paper is a revised version of "On Competitive Theory with Private Information" presented at the University of Chicago, Columbia University, Cornell University, Northwestern University, Yale University, the Summer Meetings of the Econometric Society in Montreal, and the NBER Conference-Seminar on the Theory of General Equilibrium at Berkeley. Helpful comments from Truman Bewley, Charles Wilson, and the participants of these seminars are gratefully acknowledged. We also thank the National Science Foundation and the Federal Reserve Bank of Minneapolis for financial support and accept full responsibility for any errors as well as the views expressed herein.

** The University of Minnesota and Carnegie-Mellon University, respectively.

Center for Economic Research

Department of Economics

University of Minnesota

Minneapolis, Minnesota 55455

Abstract

General competitive analysis is extended to cover a dynamic, pure-exchange economy with privately observed shocks to preferences. In the linear, infinite-dimensional space containing lotteries we establish the existence of optima, the existence of competitive equilibria, that every competitive equilibrium is an optimum, and, with some revealing qualifications, that every optimum can be supported as a competitive equilibrium. An example illustrates that rationing and securities with contrived risk have an equilibrium interpretation.

1. Introduction

The last decade has witnessed a virtual explosion in the economics of private information and moral hazard. We have for example a literature on optimal contracts, principal-agent relationships, and auctions which seeks to explain or evaluate observed arrangements and a literature on signaling and adverse selection in competitive insurance markets which uncovers various existence and welfare anomalies. [1/] There is no doubt now that private information has had and will continue to have important implications for positive and normative research.

Yet, despite these advances, or perhaps because of them, we believe more research is needed in relating the above-mentioned literatures to constructs with which economists are already familiar. In principal the gains from such research can be multiple. To the extent that a standard construct turns out to be inapplicable when private information is introduced into an otherwise standard environment, the economic nature of private information, and the difficulties caused by it, might be better understood. Such an outcome would be consistent with the above-mentioned literature which uncovers various existence and welfare anomalies. At the same time there remains the possibility that a standard construct might be modified with the introduction of private information in such a way as to create a construct which combines the explanatory power of the standard construct with the explanatory power of private information. Such a construct might also have important normative implications.

Some research along these lines has been undertaken. Independently, both Myerson [1979] and Harris and Townsend [1977][1981] have shown that standard

---

1/ No attempt can be made here to survey these literatures; interested readers are referred to Hirshliefer and Riley [1979].

concepts of feasibility and optimality are inapplicable in games or environments with private information. That is, following the Arrow [1953] and Debreu [1959] treatment of uncertainty, one may well index allocations by the realizations of random variables. But if these random variables are privately observed shocks or parameters, then not all shock-contingent allocations are achievable. It is as if shock-contingent allocations must be such that each agent has an incentive to correctly reveal his own privately-observed shock. It turns out, though, that the conditions which give each agent just such an incentive, which Hurwicz [1971] labeled the incentive compatibility conditions, are frequently both necessary and sufficient conditions for achievability. That is, in environments with private information, it may be enough to append these conditions onto the standard definitions of feasibility and optimality, and go on to characterize (private information) optimal allocations in the usual way. We would like to emphasize here the success of this and related techniques in various recent applications: Chari [1980], Green [1980] and Grossman and Hart [1980] explain underemployment; Chiang and Spatt [1980] explain observations in industrial organization; Morton [1980] explains strike duration; and Baron and Myerson [1979], Harris and Raviv [1979], [1980], and Myerson [1980] examine auction design and monopoly pricing schemes. [2/]

In addition to extending the analysis of private-information optima, the purpose of this paper is to begin an exploration of the applicability of general equilibrium competitive analysis, another standard construct, to economies with a large number of agents and private information. On the face of it, this undertaking would seem to be difficult; the incentive compatibility conditions increase more than proportionately with an increase in the number of agents.

[2/] Some of this recent literature postdates an earlier draft of this paper. We cite it here to make the case that standard constructs modified to allow for private information have indeed proved successful and in all likelihood will continue to be so.

Moreover such conditions can introduce nonconvexities, whereas in competitive analysis, convexity is usually assumed a priori. Here we handle the first problem by consideration of large economies in which the distribution of unobserved shocks in the population is the same as the probability distribution of shocks for each individual, and by treating agents with the same shock in the same way. We handle the second problem by following von Neumann and Mortenstern's [1947] seminal contribution, using lotteries to make spaces convex. Then, making use of some rather abstract theorems of Debreu, both the existence and the optimality of competitive equilibria in an environment with private information are established. These results illustrate the tremendous power of the work of Arrow, Debreu, McKenzie, and others in the theory of general economic equilibrium. And the tie-in indicates that there need be no existence and welfare anomalies for a large class of environments with private information. 3/

Of course the use of lotteries to make space convex and to establish existence in game theory has become standard. And lotteries have been used in Bayesian games which have private information and in social choice theory. 4/ But to our knowledge lotteries have not been used explicitly in general equilibrium competitive analysis. Moreover, we establish in this paper that lotteries sometimes have considerable power in overcoming the barriers to trade implicit in the incentive-compatibility conditions. That is, private-information optimal deterministic allocations, defined above, need not be optimal. We show by an

---

3/ In separate work we are establishing that externalities in the signaling environment of Spence [1974] and Riley [1979] and the insurance environment of Rothschild-Stiglitz [1976] and Wilson [1978] can generate nonexistence and nonoptimality.

4/ See Myerson [1979] and Fishburn [1972a], [1972b], Gibbard [1977], Intriligator [1979], Zeckhauser [1969],[1973] respectively. Other literature with lotteries is cited below.

example that there can exist a stochastic allocation which strictly dominates the best deterministic allocation [5/]. The example itself can be interpreted as a model of apparent disequilibrium phenomena (first-come first-serve, rationing, queues, etc.) and of securities with contrived risk. But the stochastic allocation of the example is actually both an optimal allocation in the relevant (*ex ante*) sense and a competitive equilibrium allocation in the linear space containing lotteries. The example thus gives a hint of the explanatory power of the constructs we develop in this paper, and of their welfare implications.

There is, however, one standard general equilibrium result which fails in this paper. In the linear space containing lotteries, the second fundamental welfare theorem, that optima can be supported as competitive equilibria, holds only with qualifications. We find these qualifications revealing of the difficulties of decentralization in environments with private information.

This paper proceeds as follows. Section 2 presents the example mentioned above and introduces many of the concepts which are developed in the rest of the paper. Section 3 makes explicit how lotteries overcome the barriers to trade and the nonconvexities associated with the incentive-compatibility conditions. Section 4 describes the underlying environment, a simple, pure-exchange, dynamic economy with period by period shocks to individual preferences, motivated by Lucas [1980]. Section 4 goes on to describe the linear space containing lotteries, the space of signed measures, and the linear product space L containing shock-contingent lotteries. Consumption sets and preferences are defined on the space L; certain incentive compatibility conditions are

---

5/ This is consistent with findings in the literature on optimal taxation, that stochastic taxation schemes can strictly dominate the best deterministic schemes. See Stiglitz [1976] and Weiss [1976].

loaded into the consumption set. Implementable allocations are then defined by certain resource constraints and by a prior self-selection constraint. A (private information) Pareto optimum is also defined. Section 5 establishes the existence of an optimum by consideration of a linear programming problem.

Section 6 introduces an aggregate production set in the space L and then defines attainable states as in Debreu [1954]. The production set and market clearing conditions are such that attainable states are equivalent with allocations satisfying the resource constraints in the pure exchange economy. A price system on L is also defined, a linear functional. Then a competitive equilibrium is defined in the usual way, following Debreu [1954]. The existence of a competitive equilibrium (in the linear space of signed measures) is established for various approximate economies, in which the underlying commodity space is finite, using a theorem of Debreu [1962] for Euclidean spaces. Then the existence of a competitive equilibrium for the unrestricted economy is established by taking a limit of the approximate economies, as suggested by Bewley [1972].

Section 7 considers the two fundamental theorems of contemporary welfare economics, following Debreu [1954] in linear spaces. That every competitive equilibrium is an optimum is virtually immediate. Also, every optimum can be supported as a competitive equilibrium with endowment selection. In the latter an agent chooses an endowment from a certain finite set, essentially by announcing his initial preference shock, and then trades from that endowment subject to the usual budget constraint and subject to a self-selection constraint which takes into account the preferences of others, and which depends on the announced type. Again, we find these qualifications revealing of the difficulties caused by private information.

2. A Model of Rationing and of Securities with Contrived Risk

Motivated by Lucas [1980] or Gale [1980], imagine an economy with a continuum of households. Each household is endowed initially with e units of the single consumption good of the model, and has preferences over consumption c described by the utility function $U(c, \theta)$. Here $U(\cdot, \theta)$ is continuous, strictly increasing, and concave. Parameter θ is interpreted as a shock to preferences at the beginning of the consumption period, known only to the household itself. In this sense there is private information. Parameter θ is viewed *a priori* as a random variable, taking on values θ' and θ'' with probabilities $\lambda(\theta')$ and $\lambda(\theta'')$, respectively. Suppose also that $\lambda(\theta)$ represents the fraction of households in the population in the consumption period with parameter draw θ; thus there is no aggregate uncertainty. [6/]

The introduction of shocks to preferences may be viewed as somewhat unsatisfactory. But the model can be given an alternative interpretation, with shocks to technology. Suppose there are actually two goods in the economy, a transferable good c which enters as an input into the household production function, and with which the household is endowed in amount e, and a nontransferable good q, the output of the household production process and over which the household has utility function $V(q)$. Imagine the household production technology f is subject to shocks θ, i.e., $q = f(c, \theta)$, where parameter θ is described above. As Lucas writes, think of a need for medical services, unanticipated to the household itself. With this set up, one induces an indirect utility function U over the input good c, namely $U(c, \theta) = V[f(c, \theta)]$.

Now as a special case of the above model, suppose that $U(c, \theta')$ is strictly

6/ In the planning period we suppose that each agent knows only what the distribution of the parameter in the population will be, and it is thus that *for each* the $\lambda(\theta)$ are regarded as probabilities. There are problems in the other direction, from independent and identically distributed random variables on the continuum to measurable (integrable) sample paths. Unlike Malinvaud [1973] we deal directly in the limit economy with a continuum of agents.

concave and continuously differentiable with $U'(0, \theta') = \infty$ and $U'(\infty, \theta') = 0$, and that $U(c, \theta'') = kc$. Equivalently, suppose the technology f is described by $q = c^{\theta}$ where $0 < \theta' < 1$ and $\theta'' = 1$, and that preferences over q are described by a linear function $V(q) = q$. Thus, either directly or indirectly, households of type θ' are *ex post* risk averse and household of type θ'' are *ex post* risk neutral. Admittedly this specification is somewhat extreme, but it will serve us well in making the points of this section. The crucial feature is that there be differences in curvatures *ex post*.

Now consider the following resource allocation scheme. Prior to the realization of the shock θ some central planner (who could just as well be one of the households) instructs all households to surrender to him all endowed units of the consumption good e. Then, subsequent to the revelation of the shock θ, agents are asked to commit themselves to a choice of one of two distribution centers. In the first center the planner guarantees an allotment of c^* units of the consumption good. In the second, household are offered the possibility of $\bar{c}$ units of the consumption good, but there is no guarantee (Here $\bar{c} > c^*$). Households committed to the second center are imagined to arrive in a random fashion (independent of starting times) and to receive $\bar{c}$ on a first-come first-serve basis. Alternatively households might form a queue.

All households believe the guarantee in the first center and assess the probability of being served in the second center as α and of receiving zero as $1 - \alpha$. They then commit themselves to a center, and their beliefs turn out to be self-fulfilling. All who choose the first center receive c^*, and fraction α of those who choose the second center receive $\bar{c}$ while fraction $1 - \alpha$ go away empty-handed. Collusion among households has been ruled our *a priori*; households must respect their position in the queue. [7/]

[7/] We thank John Bryant for pointing out this implicit restriction.

Upon observing the number of unserved customers in the second center, a casual observer might find the above-described scheme somewhat unsatisfactory. Since some go away empty-handed, the "price" must be too low, that is, the potential allotment of $\bar{c}$ is too high. In fact, if the receipt were lowered to some c**, all could be served. This would of course be preferred ex post by those who go unserved. [8/]

The above-described scheme can be given a second interpretation. Prior to the realization of shock θ, each household agrees to surrender its endowment e in exchange for a security with two options. Under the first option the household receives c*. Under the second the household is to receive $\bar{c}$, but there is a possibility of default, assessed at probability α. That is, under the second option, the return is risky. But this risk is entirely man-made.

Situations with such contrived risk may seem somewhat unusual. Apart from the activity of risk-lovers (gamblers) we do not seem to see agents spinning wheels of fortune. But if nature provides a random variable with a continuous density unrelated to any households preferences, endowments, or technology, then lotteries can be effected by making the allocations contingent on the realizations of that random variable. Thus the lottery would not seem inconsistent with the usual state-contingent treatment of uncertainty. [9/] In fact a random device was implicit in the above-given model of first-come first-serve, which

---

[8/] Of course this is not the only model of apparent underpricing. In a provocative article Cheung [1977] argues that apparent underpricing of better seats in theaters, so that they fill up early on, is a way of reducing the costs of monitoring seat assignments. But the theory developed here has something in common with Cheung's, the use of apparent underpricing to discriminate among potential buyers with unobserved characteristics. Such discrimination also underlies the model of credit-rationing of Stiglitz and Weiss [1980], though they proceed in a different way and draw somewhat different conclusions than the analysis of this paper; see also Akerloff [1970], Stiglitz [1976], Wilson [1977].

[9/] We would like to thank Kenneth J. Arrow for pointing this out to us.

made use of random arrival times.

It is now argued that there is a specification of c*, c**, $\bar{c}$, $U(\cdot, \theta')$ and α such that the stochastic allocation of the above-described scheme is Pareto optimal. Following Arrow and Debreu, index consumption by the shock θ. That is, let $c(\theta)$ denote the allocation of each household of type θ. (We insist that each be treated identically as if each had no name). Suppose also that the assignment of $c(\theta)$ to households of type θ is possible, as if there were full information, even though θ is private to the household. Then consider the maximization of the expected utility of the (representative) household, prior to the realization of shock θ, by choice of consumption allocations $c(\theta)$, and subject to a resource constraint that economy-wide average consumption not exceed the economy-wide average endowment. That is

(2.1) $$\max_{c(\theta') \geq 0, c(\theta'') \geq 0} \lambda(\theta')U[c(\theta'),\theta'] + \lambda(\theta'')U[c(\theta''),\theta'']$$

subject to

(2.2) $$\lambda(\theta')c(\theta') + \lambda(\theta'')c(\theta'') \leq e.$$

Note here that the terms $\lambda(\theta)$ enter the objective function, the expected utility of the representative household prior to the parameter draw θ, as probabilities, while these terms enter the resource constraint as population proportions. Necessary and sufficient conditions for a solution to this problem are:

(2.3) $$U'[c(\theta'),\theta'] = U'[c(\theta''),\theta'']$$

(2.4) $$\lambda(\theta')c(\theta') + \lambda(\theta'')c(\theta'') = e.$$

This problem may be solved in two steps (see Figure 1): First let k be the slope of the linear utility function $U(\cdot,\theta'')$ and let $c^* > 0$ denote the unique solution to

$$U'[c(\theta'),\theta'] = k. \tag{2.5}$$

Then from (2.4), set

$$c^{**} = \frac{e-\lambda(\theta')c^*}{\lambda(\theta'')}$$

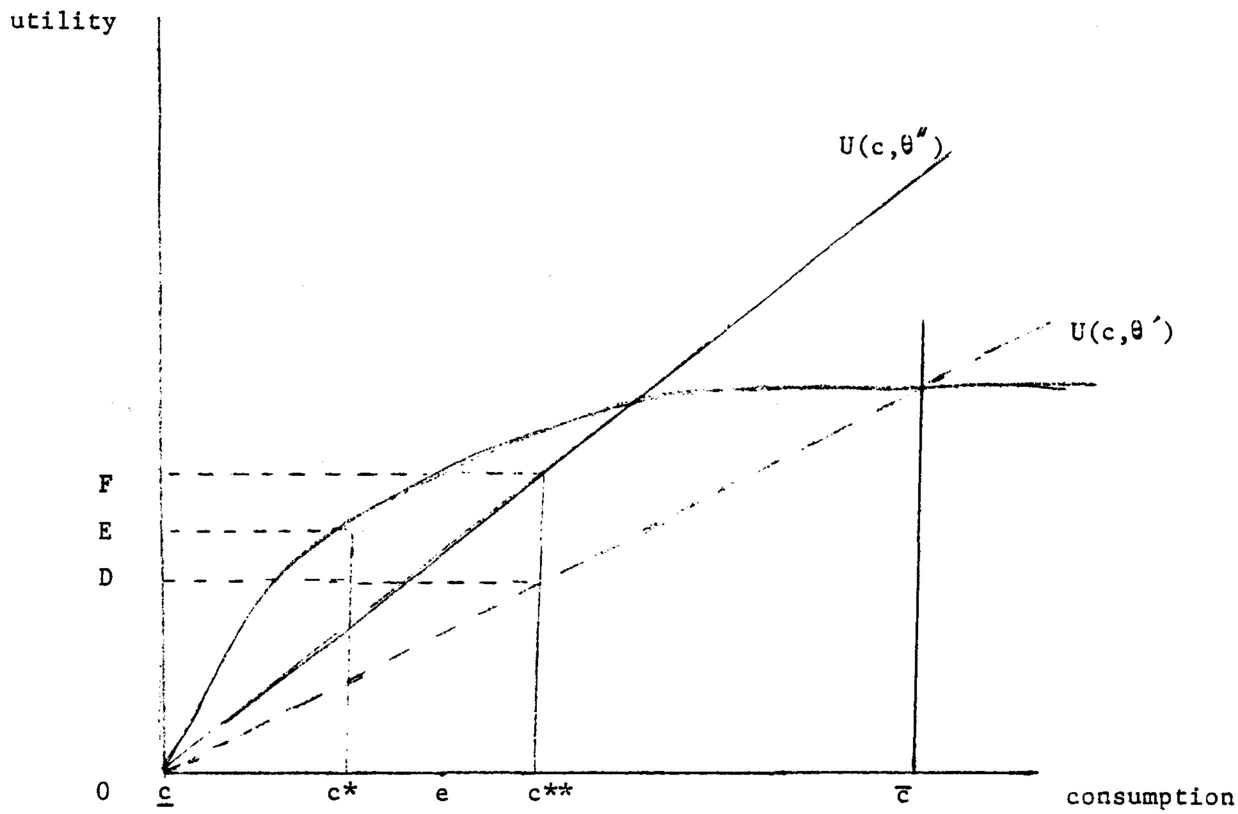
We shall assume for purposes of this section that $e > c^*$, as depicted in Figure 1.

Now returning to the actual private information economy, one notes an apparently severe implementation problem. The allocation $c(\theta') = c^*$, $c(\theta'') = c^{**}$ with $c^* < c^{**}$, is unattainable, at least in an announcement game with truth-telling. [10/] That is, all agents would announce θ'' and receive c^{**}, but of course this violates (2.2). But it is argued that the appropriate incentives can be induced by going to lotteries and exploiting differences in risk aversion. The allocation $c(\theta)$ described above is not attainable. But there is an allocation in lotteries which is attainable and which yields the same value for the objective function (2.1). In particular consider a lottery μ which is a random choice over two bundles 0 and $\bar{c}$ with probabilities $1 - \alpha$ and α respectively and has mean $E_\mu(c) \equiv (1-\alpha)0 + \alpha\bar{c} = c^{**}$. Figure 1 establishes that by setting $\bar{c} > c^{**}$ it <u>may</u> be possible to get the dispersion of the lottery μ large enough that a risk averse household prefers the sure thing,

---

[10/] Building on Harris and Townsend [1977][1981] and Myerson [1979], this can be shown to be without loss of generality.

Figure 1



OD = expected utility of gamble if θ = θ′

OE = utility of c* if θ = θ′

OF = utility for both c** and gamble if θ = θ″

c^*, to the lottery μ. Thus $U(c^*,\theta')$ is the utility to housholds of type θ'. Of course a risk neutral household would prefer the lottery μ as its mean consumption is higher, and would achieve the utility of the mean $U(c^{**},\theta'')$. Finally note that with the above scheme, and consequent choice of the agents, the resource constraint is

$$\lambda(\theta')c^* + \lambda(\theta'')[(1-\alpha)0 + \alpha\bar{c}] = e. \tag{2.6}$$

Here we interpret the lottery μ as a situation in which $1-\alpha$ is the fraction of those agents who choose the lottery who are assigned 0, and similarly for α and $\bar{c}$. Note also that (2.6) is satisfied since $E_\mu(c) = c^{**}$ and (2.4) is satisfied by construction. We have thus established that the above-described resource allocation scheme achieves the utility of a full-information optimum. It is therefore private-information optimal as well. 11/

As the above-described allocation in lotteries is optimal, it seems natural to ask whether such an allocation can be supported as a competitive equilibrium. We establish here that such a competitive equilibrium exists, making the point that the above-described apparent disequilibrium phenomena are in fact equilibrium phenomena, and that securities with contrived risk are consistent with exchange in competitive markets.

For this purpose, then, imagine that the underlying commodity space C is finite, i.e., c can take on only a finite number of values. The household is imagined to choose a probability measure $x(c,\theta)$, $c \in C$ on this finite space for each possible value of θ, namely θ' and θ''. (Here $0 \le x(c,\theta) \le 1$ and

11/ Note that we have not formally defined private-information optimal allocations. Such a definition naturally follows a more general treatment of the incentive compatibility conditions in section 3. In general, full-information optimality is an inappropriate welfare criterion; by altering the example, full-information optimal allocations can be made unachievable.

$\Sigma_{c\in C} x(c,\theta) = 1$.) That is, the household is supposed to announce its actual shock θ, and receive c with probability $x(c,\theta)$.[12/] The household is effectively endowed with two such probability measures $\xi(c,\theta')$ and $\xi(c,\theta'')$, each putting mass one on the point e. Preferences of the household are described by expected utility over θ and over the chosen lotteries:

$$\sum_{\theta} \lambda(\theta) \sum_{c} x(c,\theta) U(c,\theta). \tag{2.7}$$

Imagine also that there is an intermediary or firm in the economy who can make commitments to buy and sell the consumption good from consumers of different types. A production choice $y(c,\theta)$, $c \in C$ specifies the number of units of the bundle with c units of the consumption good which the firm must deliver to consumers announcing they are of type θ. (If $y(c,\theta)$ is negative there is a commitment to take in resources). The production set Y is defined by

$$Y = \{y(c,\theta),\ c\in C,\ \theta = \theta',\ \theta'':\ \sum_{\theta} \lambda(\theta) \sum_{c} c y(c,\theta) \leq 0\}. \tag{2.9}$$

Thus the firm cannot distribute more than it takes in.

Finally the price system in this economy is an element of the same Euclidean space, denoted $p(c,\theta)$, $c\in C$, θ', θ''. We then have the obvious

<u>Definition</u>: A <u>competitive equilibrium</u> is a price system $\{p^*(c,\theta)\}$, a consumption allocation $\{x^*(c,\theta)\}$, and a production allocation $\{y^*(c,\theta)\}$ such that the $\{x^*(c,\theta)\}$ maximize objective function (2.7) subject to the budget constraint

$$\sum_{\theta} \sum_{c} p^*(c,\theta) x(c,\theta) \leq \sum_{\theta} \sum_{c} p^*(c,\theta) \xi(c,\theta); \tag{2.10}$$

[12/] In general constraints ensuring this outcome will have to be imposed explicitly. Here they are not needed.

the $\{y^*(c,\theta)\}$ maximize profits

$$\sum_\theta \sum_c p^*(c,\theta)y(c,\theta) \tag{2.11}$$

constrained by the production set (2.9); and markets clear

$$x^*(c,\theta) = y^*(c,\theta) + \xi(c,\theta) \qquad c \in C,\ \theta = \theta',\theta''. \tag{2.12}$$

Now for an equilibrium specification let the price system be $p^*(c,\theta) = \lambda(\theta)c$, let the consumption allocation $x^*(c,\theta)$ be

$$x^*(c^*,\theta') = 1 \qquad x^*(\bar{c},\theta'') = \alpha \qquad x^*(0,\theta'') = 1-\alpha$$

and let $y^*(c,\theta)$ be determined by (2.12).

With price system $p^*(c,\theta)$ the problem facing the consumer is

$$\text{Max} \sum_\theta \lambda(\theta) \sum_c x(c,\theta)\ U\ (c,\theta)$$

subject to $\quad \sum_\theta \sum_c \lambda(\theta)c\ \xi(c,\theta) = e.$

This is just the stochastic version of program (2.1)-(2.2). The allocation $x^*(c,\theta)$ satisfies the constraint and yields value equal to the optimal solution of the deterministic program. If some allocations $x^{**}(c,\theta)$ yielded greater value than $x^*(c,\theta)$ for the stochastic system, the deterministic allocation with the same means would be feasible and would yield greater value for the deterministic problem that that problem's optimal solution. This is impossible, establishing $x^*(c,\theta)$ is optimal for the stochastic program. By construction of $y^*(c,\theta)$, market clearing condition (2.12) is satisfied. In addition under $p^*(c,\theta)$ the value of any $y \in Y$ is

$$\sum_\theta \sum_c \lambda(\theta)\ c\ y(c,\theta)$$

which is nonpositive by definition of Y. As the budget constraint is binding and $y^*(c,\theta) = x^*(c,\theta) - \xi(c,\theta)$ the value of $y^*(c,\theta)$ is zero. Thus $y^*(c,\theta)$ maximizes profits. Thus the existence of a competitive equilibrium supporting the optimal allocation has been established.

3. The Use of Lotteries to Overcome Barriers to Trade and Nonconvexities

Thus far there has been no formal treatment of the incentive-compatibility conditions, though these implicitly motivated the use of lotteries in the previous section. So, returning to deterministic allocations for a moment, consider a set of shock-contingent consumptions $c(\theta)$, $\theta = \theta', \theta''$. Under a direct revelation mechanism with truth-telling, there can be an assignment of $c(\theta)$ to a θ-type agent if and only if

$$U[c(\theta'),\theta'] \geq U[c(\theta''),\theta']$$

(3.1)
$$U[c(\theta''),\theta''] \geq U[c(\theta'),\theta''].$$

These are the appropriate incentive-compatibility conditions in deterministic allocations for the simple economy of the previous section as well as for economies in which the consumption set is a subset of R^{ℓ}_{+}, so that $c(\theta)$ is an ℓ-dimensional vector.

We should emphasize here that each agent is assumed to know <u>all</u> aspects of the environment other than the particular parameter draws of other agents. For example, the utility <u>function</u> $U(\cdot,\theta)$ of others is known up to the parameter draw θ. Thus conditions like (3.1) or their stochastic analogues are supposed to capture completely all the incentive (or disincentive) effects of private information in any well-defined game or resource allocation scheme for our economy. Hereafter we shall cease to make reference to mechanisms and take conditions like (3.1) to be natural restrictions in the space of parameter-contingent allocations. They are for us a given of the analysis. Thus private information Pareto optima are defined relative to such restrictions.

There are two difficulties associated with constraints like (3.1). First these constraints impose rather severe restrictions on mutually beneficial exchange. Second the space of parameter-contingent allocations restricted by (3.1) is generally not convex.

To illustrate the first difficulty we consider the single-commodity economy of the previous section. Then, with more preferred to less, conditions (3.1) imply that $c(\theta') \geq c(\theta'')$ and $c(\theta'') \geq c(\theta')$. Thus the only implementable allocations are $c(\theta') = c(\theta'')$, so there can be no gains from trade. We have shown in the previous section that lotteries sometimes can overcome such barriers to trade.

The second difficulty is that the space of parameter-contingent allocations restricted by (3.1) is generally not convex. To illustrate this consider a two-commodity economy with preferences described by the utility function

$$U(c,\theta) = \theta\ u(c_1) + (1-\theta)\ u(c_2)$$

where $u(\cdot)$ is strictly increasing and strictly concave and where $0 < \theta' < \theta'' < 1$. Then the nature of condition (3.1) is illustrated in Figure 2a. The preferences of the agent depend on the parameter draw θ: Essentially, there are two sets of indifference curves, the flatter set corresponding to the parameter draw θ'. Thus in Figure 2a if $c(\theta')$ is the allocation to an agent of type θ', then the allocation $c(\theta'')$ to an agent of type θ'', which satisfies (3.1), must lie in the shaded region. Now in Figure (2b) the pairs $c_A = (c(\theta')_A, c(\theta'')_A)$, $c_B = (c(\theta')_B, c(\theta'')_B)$ both satisfy (3.1) with $c(\theta')_A = c(\theta')_B$ and with $c(\theta'')_A$ and $c(\theta'')_B$ distinct but both on the upper boundary of the shaded region. Now from a convex combination of these two pairs, $c(t) = tc_A + (1-t)c_B$, $0 < t < 1$. Then $tc(\theta'')_A + (1-t)c(\theta'')_B$ cannot lie in the

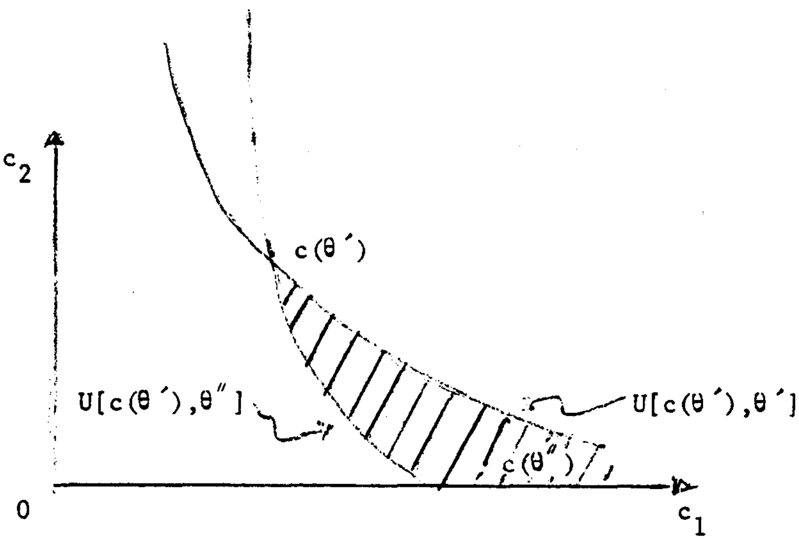


Figure 2a

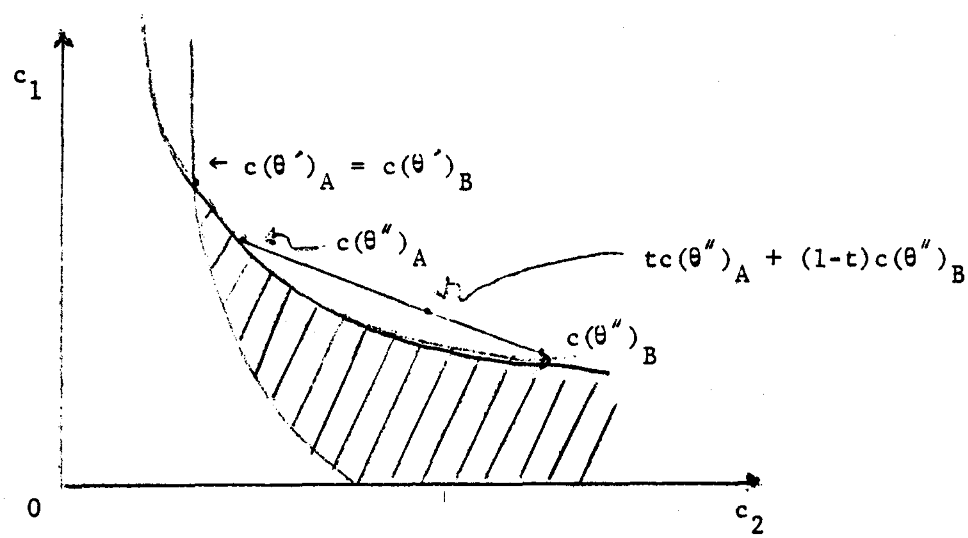


Figure 2b

shaded region, and hence c(t) does not satisfy (3.1). See Figure 2.

To illustrate how this nonconvexity is overcome in the space of probability measures, again suppose for simplicity that the underlying commodity space is finite, i.e., c can be one of a finite number of possible bundles in C. Then let $x(\theta)$ be a random assignment to each agent of type θ, where $x(c,\theta)$ is the probability of bundle c. Then a parameter-contingent random allocation $(x(\theta'),x(\theta''))$ can be achieved in a direct-revelation mechanism with truth-telling if and only if

$$\text{(3.2a)} \qquad \sum_{c\in C} U(c,\theta')x(c,\theta') \geq \sum_{c\in C} U(c,\theta')x(c,\theta'')$$

$$\text{(3.2b)} \qquad \sum_{c\in C} U(c,\theta'')x(c,\theta'') \geq \sum_{c\in C} U(c,\theta'')x(c,\theta') .$$

These conditions are the random analogues of (3.1). These conditions are linear in the $x(c,\theta)$ and therefore constitute convex constraints.

In the previous section no use was made of convexity in establishing the existence of a competitive equilibrium or its optimality. But in general the incentive-compatibility conditions must be imposed explicitly, as constraints, and convexity will be needed.

4. The Formal Securities Model

Consider now a three-period model with a continuum of agents and ℓ commodities. Each of the agents has an endowment vector $e_t \gg 0$ in each period t, t = 0,1,2. Letting c_t denote the consumption vector in period t, each agent has preferences over consumption sequences $\{c_t\}_{t=0}^{2}$ as described by the utility function

$$E\ \Sigma_{t=0}^{2} U(c_t,\theta_t).$$

Here E is an expectations operator (the random variables will be described momentarily). Also consumption is bounded, $0 \leq c_t \leq b$. Each single-period utility function $U(\cdot,\theta_t)$ is continuous, concave, and strictly increasing with $U(0,\theta_t) \geq 0$. The parameter θ_t is interpreted as a shock to individual preferences at the beginning of period t, observed only by the individual agent. For simplicity parameter θ_t is assumed to take on only a finite number of values; that is, for each t, $\theta_t \in \Theta = \{1,2,\ldots,n\}$. Fraction $\lambda(\theta_t)$ of agents in the population have the parameter draw θ_t at time t, where

$0 < \lambda(\theta_t) < 1$, $\Sigma_{\theta_t \in \Theta}\lambda(\theta_t) = 1$. From the point of view of the individual agent at the beginning of time 0, θ_0 is known, and $\lambda(\theta_t)$ represents the probability of the parameter draw θ_t at time t, t = 1,2. Notationally it will be convenient in what follows to convert the parameter θ_0 to the parameter i, and thus we may refer to agents of type i, i = 1,2,...,n classified by their initial parameter draw.

We have deliberately kept our model simple, rather than attempting great generality. Some obvious extentions are possible. First, one may easily increase the number of periods to any finite horizon. Three periods were the smallest number necessary to illustrate the nature of the incentive compatibility constraints. Second, utility functions may be supposed to depend on the entire history of individual shocks. Third, there can be statistical dependence in the θ_t, $t \geq 1$, as long as there is independence from the initial parameter $\theta_0 = i$. Observable heterogeneous characteristics and nontrivial production could be introduced. We did not do so in order to focus on private information.

This section now makes precise the notion of a lottery on the underlying space of possible consumptions. The space of lotteries is shown to be a subset of a linear space. Individual consumption sets, preferences, and endowments are defined on this linear space. Implementable allocations and Pareto optimal allocations are also defined.

First, denote the underlying commodity space by $C = \{c \in R^{\ell} : 0 \le c \le b\}$. We then begin with the space S of all finite, real-valued, countable-additive set functions on the Borel sets of C, denoted by $\mathcal{B}(C)$, i.e., functions mapping such Borel sets into the reals. The operations of addition and scalar multiplication are defined as follows:

(i). Given any two elements μ and η of S, a third element $\mu + \eta$ is S called the <u>sum</u> is determined by the condition

$$(\mu + \eta)(B) = \mu(B) + \eta(B) \qquad B \in \mathcal{B}(C).$$

(ii). Given any real number α and any element μ of S, a second element $\alpha\mu$ in S called the <u>scalar product</u> is determined by the condition

$$(\alpha\mu)(B) = \alpha \cdot \mu(B) \qquad B \in \mathcal{B}(C).$$

With these definitions the axioms defining a linear space are satisfied. 13/ Finally integration of measurable functions is well defined. 14/

13/ See Kolmogorov and Fomin, [1970], p. 118. The zero element of S assigns the number zero to every Borel set and the negative element $-\mu$ of an element μ is defined by $(-\mu)(B) = -\mu(B)$ for every $B \in \mathcal{B}(C)$. Note that the space of probability measures on C is not a linear space, since if $\mu(C) = 1$, $(\alpha\mu)(C) < 1$ for $\alpha < 1$.

14/ Note that here and below the integral is Lebesgue; see for example Ash [1972] pp. 36-37. Note that typically, and in Ash, integration is defined relative to <u>measures</u>, i.e., <u>nonnegative</u> real-valued, countably-additive set functions. By the Jordan-Hahn decomposition theorem, however, any countably-additive, real-valued set function μ on the σ-field $\mathcal{B}(C)$ may be expressed as the difference of two measures μ^+ and μ^-, i.e., $\mu = \mu^+ - \mu^-$. Hence for any Borel measurable function h, define $\int h d(\mu^+ - \mu^-) = \int h d\mu^+ - \int h d\mu^-$, where the two terms on the right-hand side are defined in the usual way. For this last equality we are also using the fact that for any two measures μ and η and any two scalars α and β, and for any Borel measurable function h, $\int h d(\alpha\mu + \beta\eta) = \alpha\int h d\mu + \beta \int h d\eta$. With the above-given definition of integration relative to general countably-additive set functions, this linearity continues to hold.

Motivated by the previous discussion one suspects that "consumption" in period one should be indexed by θ_1 and "consumption" in period two should be indexed by θ_1 and θ_2. This leads us to consider the space L with typical element $\mu = [\mu_0, \{\mu_1(\theta_1)\}, \{\mu_2(\theta_1,\theta_2)\}]$ where the components, μ_0, the $\mu_1(\theta_1)$ and the $\mu_2(\theta_1,\theta_2)$ are each elements of S. Addition and scalar multiplication on the space L is defined in the obvious way -- termwise. Then it is easily verified that since S is a linear space, so also is L. Consumption sets, preferences, and endowments are all to be defined relative to the linear space L. Note L is the $1 + n + n^2$ cross product space of S.

The consumption sets and preferences are defined first. Returning to the space S, recall that a probability measure p is a real-valued, countably-additive, nonnegative set function with $p(C) = 1$. Thus a probability measure $p \in S$ is our desired notion of a lottery. The one-period expected utility of an agent under such a probability measure p, given the parameter draw θ, is

$$\int_C U(c,\theta)p(dc).$$

Note here since $U(\cdot,\theta)$ is continuous on compact set C it follows that $U(\cdot,\theta)$ is Borel measurable and bounded. Thus expected utility is well defined. Now let

$$P \equiv \{\mu \in L\text{: } \mu_0\text{, the } \mu_1(\theta_1)\text{, and the } \mu_2(\theta_1,\theta_2) \text{ are all probability measures of } S\}.$$

Then given any $\mu \in P$, impose the further requirement that

$$(4.1) \quad \int U(c,\theta_2)\mu_2(dc, \theta_1,\theta_2) \geq \int U(c,\theta_2)\mu_2(dc,\theta_1,\theta_2') \; \forall \; \theta_1 \in \Theta,\ \theta_2,\ \theta_2' \in \Theta.$$

Condition (4.1) is a period $t = 2$ incentive compatibility requirement. Its analogue in section 3 is (3.2). If restricted in period $t = 2$ to choosing a member of $\{\mu_2(\theta_1,\theta_2)\}$ with some θ_1 fixed in advance, the representative agent would weakly prefer $\mu_2(\theta_1,\theta_2)$ if his parameter draw is θ_2. Given (4.1), the period t = 1 incentive compatibility requirement is

$$\text{(4.2)} \qquad \int U(c,\theta_1)\mu_1(dc,\theta_1) + \sum_{\theta_2} \lambda(\theta_2) \int U(c,\theta_2)\mu_2(dc,\theta_1,\theta_2)$$

$$\geq \int U(c,\theta_1)\mu_1(dc,\theta_1') + \sum_{\theta_2} \lambda(\theta_2) \int U(c,\theta_2)\mu_2(dc,\theta_1',\theta_2) \ \forall\ \theta_1,\theta_1' \in \Theta.$$

If asked in period t = 1 to choose a member of $\{\mu_1(\theta_1),\{\mu_2(\theta_1,\theta_2)\}\}$ the representative agent would weakly prefer the pair $(\mu_1(\theta_1),\{\mu_2(\theta_1,\theta_2)\})$ if his parameter draw is actually θ_1.

Finally let

$$X \equiv \{\mu \in P\colon \mu \text{ satisfies (4.1) and (4.2)}\}.$$

The space $X \subset L$ is the <u>consumption set</u> of the representative agent. Given any $x \in X$, let preferences be given by

$$\text{(4.3)} \quad W(x,i) \equiv \int U(c,i)x_0(dc) + \sum_{\theta_1} \lambda(\theta_1) \int U(c,\theta_1)x_1(dc,\theta_1)$$

$$+ \sum_{\theta_1} \lambda(\theta_1) \sum_{\theta_2} \lambda(\theta_2) \int U(c,\theta_2)x_2(dc,\theta_1,\theta_2) \ .$$

A point $x^o \in X$ is a satiation point in X for agent i if $W(x,i) \leq W(x^o,i)$ for all $x \in X$.

The endowment of agent i in each period t is a ℓ-dimensional vector $e_t \gg 0$, $e_t \in C$. So let ξ be that element of P such that ξ_0 puts all mass on e_0, $\xi_1(\theta_1)$ puts all mass on e_1 for each $\theta_1 \in \Theta$, and $\xi_2(\theta_1,\theta_2)$ puts all mass on e_2 for $\theta_1,\theta_2 \in \Theta$.

We now have a pure exchange economy defined by the population fractions $\lambda(i)$, $i \in \Theta = \{1,2,\ldots,n\}$, the linear space L, the common consumption set $X \subset L$, the common endowment $\xi \in L$, and preferences $W(\cdot,i)$ defined on X for every agent of type i, $i \in \Theta$.

An implementable allocation for this economy is an n-tuple (x_i) with $x_i \in X$ for every i which satisfies the resource constraints in each period t, t = 0,1,2, [15]

(4.4) $$\sum_i \lambda(i) \int c\, x_{i0}(dc) \le e_0$$

(4.5) $$\sum_i \lambda(i) \sum_{\theta_1} \lambda(\theta_1) \int c\, x_{i0}(dc,\theta_1) \le e_1$$

(4.6) $$\sum_i \lambda(i) \sum_{\theta_1} \lambda(\theta_1) \sum_{\theta_2} \lambda(\theta_2) \int c\, x_{i2}(dc,\theta_1,\theta_2) \le e_2$$

and which satisfies a prior self-selection constraint

(4.7) $$W(x_i,i) \ge W(x_j,i) \qquad \forall\ i,j \in \Theta.$$

The three resource constraints (4.4) - (4.6) are the analogues of (2.6) in the example of section 2. Thus we assume that fraction $x_{i0}(B)$ of the agents of type i, those who have chosen the lottery x_{i0}, is assigned an allocation in $B \in \mathcal{B}(C)$ in period zero, and similarly for $x_{i1}(B,\theta_1)$, $x_{i2}(B,\theta_1,\theta_2)$.

---

[15] The integration below is coordinate wise. Thus in (4.4) for example,

$$\int c_j\, x_{i0}(dc) = \int \pi_j(c) x_{i0}(dc)$$

where $\pi_j(c)$ is the projection of c onto the jth coordinate axis.

The prior self-selection constraint captures the idea that an allocation (x_i) can be actually implemented only if each agent of type i reveals his true type by the choice of the bundle x_i from among the n-tuple (x_i).

An implementable allocation (x_i) is said to be a <u>Pareto optimum</u> if there does not exist an implementable allocation (x_i') such that

$$W(x_i',i) \geq W(x_i,i) \qquad i = 1,2,\ldots,n \tag{4.8}$$

with a strict inequality for some i.

5. Existence of a Pareto Optimum

To establish the existence of a Pareto optimum for our economy it is enough to establish the existence of a solution to the following problem.

<u>Problem (1)</u>: Maximize a weighted average of the utilities of the agent types

$$\sum_i \omega(i)\, W(x_i,i) \tag{5.1}$$

where

$$0 < \omega(i) < 1, \ \sum_i \omega(i) = 1$$

by choice of the n-tuple (x_i), $x_i \in X$, subject to the resource constraints (4.4) - (4.6) and the prior self-selection constraint (4.7). We wish to make use of the theorem that continuous real-valued functions on nonempty, compact sets have a maximum.

We need to introduce a topology on the space of probability measures so notions of continuity and compactness may be well defined. Let P* denote the space of probability measures with common sigma algebra $\mathcal{B}(C)$, the Borel sets of the underlying commodity space C. Let the topology on P* be defined by integrals of (bounded) continuous, real-valued functions on C. 16/ That is, let sets of the form

$$V_{\mu^o} = \{\mu \in P^*: |\int f_j(c)\, \mu(dc) - \int f_j(c)\mu^o(dc)| < \epsilon_j \quad j = 1,2,\ldots,J\}$$

define the base for our topology, where μ^o is an arbitrary probability measure in P^*, f_j is an arbitrary (bounded) continuous function on C, ϵ_j is an arbitrary positive number, and J is an arbitrary positive integer. With this topology a sequence of measures μ^m in P^* converges to a measure μ in P^* if and only if

$$\lim_{m \to \infty} \int f(c)\mu^m(dc) = \int f(c)\mu(dc)$$

for every (bounded) continuous function f on C. Notationally we write $\mu^m \overset{w}{\to} \mu$, i.e., weak convergence of measures.

The underlying commodity space C is a subset of R^ℓ, and so is a separable metric space. It follows that the space of probability measures on C, P^*, with the above topology, is metrizable, i.e., there exists a metric on P^* which induces the same open sets. (See Parthasarathy [1967], Theorem 6.2, Chapter 2.) Moreover, since C is compact, P^* is a compact (metric) space (Parthasarathy, Theorem 6.4, Chapter 2). Since P, to which the x_i belong, is the $(1+n+n^2)$ product space of P^* and we are concerned with the n-tuple (x_i), let P^n be the n product space of P and therefore the $n(1+n+n^2)$ product of P^*. Let the topology on P^n be the product topology. (See Royden [1968] Theorem 19, p. 166). Since P^* is metrizable, so also in P^n (Royden [1965], p. 151). Hence P^n is a compact metric space. Convergence of measures in P^n is equivalent with

16/ We similarly introduce a topology on S and the associated $1 + n + n^2$ product topology on L.

weak convergence of measures coordinate-wise.

The objective function in Problem (1) is (5.1). From (4.3) we have

$$(5.2)\qquad W(x_i,i) \equiv \int U(c,i)x_{i0}(dc) + \sum_{\theta_1} \lambda(\theta_1) \int U(c,\theta_1)x_{i1}(dc,\theta_1)$$

$$+ \sum_{\theta_1} \lambda(\theta_1) \sum_{\theta_2} \lambda(\theta_2) \int U(c,\theta_2)x_{i2}(dc,\theta_1,\theta_2) .$$

To establish continuity of (5.1) it is enough to show that for every sequence $(x_i^m) \overset{w}{\rightarrow} (x_i)$,

$$\lim_{m\to\infty} \sum_{i=1}^{n} \omega(i)W(x_i^m,i) = \sum_{i=1}^{n} \omega(i)W(x_i,i).$$

So it is enough to show that

$$(5.3)\qquad \lim_{m\to\infty} W(x_i^m,i) = W(x_i,i) \qquad \forall i \in \Theta.$$

Since the $U(\cdot,\theta_t)$ are (bounded) continuous functions on C, the continuity of $W(\cdot,i)$ with respect to x_i is immediate.

Now consider the domain of the choice elements in Problem (1), space P^n restricted by the resource constraints (4.3) - (4.6), the prior self-selection constraint (4.7) and the incentive compatibility constraints (4.1) and (4.2) for each agent type. Call this space T. This restricted space T is nonempty since it contains the endowment n-tuple, ξ^n. As closed subsets of compact spaces are compact (Kolmogorov and Fomin [1970], Theorem 2, p. 93), we need only establish T is closed. So it is enough to establish that given any sequence $(x_i^m) \overset{w}{\rightarrow} (x_i)$ with $(x_i^m) \in T$, that $(x_i) \in T$. Now if $(x_i^m) \in T$, then in (4.1) for example

$$\int U(c,\theta_2)x^m_{i2}(dc,\theta_1,\theta_2) \geq \int U(c,\theta_2)x^m_{i2}(dc,\theta_1,\theta_2') \quad \forall i,\theta_1,\theta_2',\theta_2 \in \Theta.$$

Taking the limit of this inequality as $m\to\infty$, and using the fact that $U(\cdot,\theta_2)$ is a (bounded) continuous function, one obtains

$$\int U(c,\theta_2)x_{i2}(dc,\theta_1,\theta_2) \geq \int U(c,\theta_2)x_{i2}(dc,\theta_1,\theta_2') \quad \forall i,\theta_1,\theta_2,\theta_2' \in \Theta.$$

A similar argument applied termwise establishes the desired property for (4.2). The same type of argument is used to establish the desired property for (4.7), and also for the resource constraints (4.4) - (4.6), where coordinate-wise the integrand is a (bounded) continuous function.

We conclude by noting (again) that continuous real-valued functions on compact topological spaces achieve a maximum. (Royden [1968], Proposition 9, p. 161). Hence the existence of a Pareto optimum is established.

The above argument relies heavily on the compactness of C. In fact this assumption is crucial. By modifying the first example of section 2 where C is not compact we have produced an environment in which one can get arbitrarily close to but not attain the utility of a full-information optimum; thus for this environment a Pareto optimum does not exist.

6. Existence of a Competitive Equilibrium

In this section we establish that our economy can be decentralized with a price system, that is, that there exists a competitive equilibrium. We accomplish this task by introducing a firm into the analysis, with a judiciously chosen (aggregate) production set. We then follow the spirit of a method developed by Bewley [1972] for establishing the existence of a competitive equilibrium with a continuum of commodities. Various approximate economies are considered, with a finite number of commodities. Existence of a competitive equilibrium for these economies is established with a theorem of Debreu [1962]. One then takes an appropriate limit.

Let there be one firm in our economy with production set $Y \subset L$, where

$$Y = \{y \in L \ : \ (6.1), (6.2), \text{ and } (6.3) \text{ below are satisfied}\}:$$

(6.1) $\int c y_0(dc) \leq 0$

(6.2) $\Sigma_{\theta_1} \lambda(\theta_1) \int c y_1(dc, \theta_1) \leq 0$

(6.3) $\Sigma_{\theta_1} \lambda(\theta_1) \Sigma_{\theta_2} \lambda(\theta_2) \int c y_2(dc, \theta_1, \theta_2) \leq 0.$

To be noted here is that the components of some $y \in Y$ are elements of S, and thus each is a way of adding. A negative weight corresponds to a commitment to take in resources and positive weight corresponds to a commitment to distribute resources. Thus in (6.1), for example, the term $\int c_j y_0(dc)$ should be interpreted as the net trade (sale) of the jth consumption good in period zero. Inequality (6.1) states that as a clearing house or intermediary, the firm cannot supply more of the consumption good than it acquires. When indexed by the parameter θ, a component of y should be interpreted as a commitment to agents who announce they are of type θ. The production set Y, it should be noted contains the zero element of L and also displays constant returns to scale.

Following Debreu [1954] we define a <u>state</u> of our economy as an (n+1)-tuple $[(x_i), y]$ of elements of L. A state $[(x_i), y]$ is said to be <u>attainable</u> if $x_i \in X$ for every $i \in \Theta$, $y \in Y$, and $\Sigma_{i=1}^{n} \lambda(i) x_i - y = \xi$. Now suppose a state $[(x_i), y]$ is attainable. Then setting $y = \Sigma_i \lambda(i) x_i - \xi$ in (6.1) - (6.3), one obtains the resource constraints (4.4) - (4.6). Similarly, given any n-tuple (x_i), $x_i \in X$, satisfying the resource constraints (4.4) - (4.6), define y by

$y = \Sigma_i \lambda(i)x_i - \xi$, and then $y \in Y$. Thus there is a one-to-one correspondence between attainable states in the economy with production and allocations in the pure exchange economy satisfying the resource constraints. An attainable state $[(x_i),y]$ is said to be a Pareto optimum if the n-tuple (x_i) satisfies (4.7) and there does not exist an attainable state $[(x_i'),y']$ which satisfies (4.7) and Pareto dominates, i.e., satisfies (4.8). Again there is a one-to-one correspondence between optimal states and optimal allocations.

A price system for our economy is some real-valued linear functional on L, that is, some mapping $v : L \to \mathbb{R}$. More will be said about price systems v in what follows, but we may note here that v will have $(1+n+n^2)$ components each of which is a continuous linear functional on S relative to the weak topology. That is, given some $\mu \in L$, then

$$v(\mu) = \int f_0(c)\mu_0(dc) + \sum_{\theta_1} \int f_1(c,\theta_1)\mu_1(dc,\theta_1) + \sum_{\theta_1}\sum_{\theta_2} \int f_2(c,\theta_1,\theta_2)\mu_2(dc,\theta_1,\theta_2)$$

where the functions $f_0(\cdot)$, $f_1(\cdot,\theta_1)$, $f_2(\cdot,\theta_1,\theta_2)$ are (bounded) continuous functions on C. (See Dunford and Schwartz, [1957], Theorem 9, p. 421). We now make the following

Definition: A competitive equilibrium is a state $[(x_i^*),y^*]$ and a price system v^* such that

(i) for every i, x_i^* maximizes $W(x_i,i)$ subject to $x_i \in X$ and $v^*(x_i) \le v^*(\xi)$;

(ii) y^* maximizes $v^*(y)$ subject to $y \in Y$; and

(iii) $\Sigma_{i=1}^n \lambda(i)x_i^* - y^* = \xi$.

An outline of our proof for the existence of a competitive equilibrium for our economy is as follows. First the underlying commodity space C is restricted to a finite number of points, the nodes of a mesh or grid on C. In this restricted economy a countably-additive, real-valued set function is completely defined by an element of a Euclidean space, with dimension equal to the dimension of the restricted C. The linear space of these restricted economies is the $1 + n + n^2$ cross product of this Euclidean space. Consumption sets, preferences, endowments, and a production set may be defined on this space in the obvious way. The existence of a competitive equilibrium for the restricted economy is established using a theorem of Debreu [1962]. Then letting the grid get finer and finer, one can construct a sequence competitive equilibria for the economies which are less and less restricted. A subsequence of these competitive allocations and prices converges and the limiting allocations and prices are shown to be a competitive equilibrium for the unrestricted economy. We now give a more detailed argument.

The first restricted economy may be constructed in an essentially arbitrary way by subdividing each of the ℓ coordinate axes of the commodity space C into intervals, subject to the following restrictions. First, each endowment point e_t, t = 0,1,2, must be one of the nodes of the consequent grid. Second, letting

$$c_0^* > \max_i \left[\frac{e_0}{\lambda(i)}\right], \quad c_t^* > \max \left[\frac{e_t}{\lambda(\theta_t)}\right] \text{ for } t = 1,2, \tag{6.4}$$

each point c_t^*, t = 0,1,2 must be one of these nodes. (We thus suppose that the upper bound b of C is such that $0 < c_t^* \leq b$). Third, the element zero must be an element of the consequent grid. The first of these restrictions will mean the endowment points lie in each of the restricted consumption sets, and the second will mean that no agent is ever satiated in his <u>attainable</u> consumption sets. (See condition b.1 of the theorem below).

The second restricted economy is obtained from the first by equal subdivision of the original intervals of the ℓ coordinate axes. The third is obtained by equal subdivision of the second, and so on. In what follows we let the subscript k be the index number of the sequence of restricted economies. Note that the length of each of the intervals goes to zero as $k\to\infty$, so that these grids are finer and finer.

For the kth restricted economy let C^k be the restricted underlying commodity space and L^k be the finite dimensional subspace of L for which the support of each of the $n^2 + n + 1$ measures is C^k. That is, let $x_0(c)$, the $x_1(c,\theta_1)$ and the $x_2(c, \theta_1, \theta_2)$ for $c \in C^k$ each be the measure of $\{c\}$, the set containing the single point c. Then the space L^k is finite dimensional and a point is characterized by the vector $\{x_0(c),\ x_1(c,\theta_1),$ and $x_2(c,\theta_1,\theta_2)\}$ $c \in C^k$, $\theta_1,\theta_2 \in \Theta$. Note that the integral of an integrable function f: $C \to R$ with respect to a measure x on C^k is

(6.5) $$\int_C f(c)x(dc) = \sum_{c \in C^k} f(c)x(c).$$

The consumption and production possibility sets for the k^{th} restricted economy are $X^k = X \cap L^k$ and $Y^k = Y \cap L^k$ respectively. By result (6.5), the integrals used in the definition of X, Y and W, namely in (4.1)-(4.2), (6.1)-(6.3) and (4.3) respectively, have representations as finite sums over the elements of C^k. As e_0, e_1 and e_2 belong to C^k, the endowment for economy k is $\xi^k = \xi \in L^k$.

As our linear space for the kth restricted economy is a subset of Euclidean space, the price system is also an element of this Euclidean space. Thus we may define a price system $p^k = \{(p_0^k(c)),\ (p_1^k(c,\theta_1)),\ (p_2^k(c,\theta_1,\theta_2))\}$,

$c \in C^k$, $\theta_1, \theta_2 \in \Theta$, where each component is an element of R.

Now let m be the least common denominator of the $\lambda(i)$, $i = 1,2,\ldots,n$ and consider the kth restricted *finite* economy containing number $\lambda(i)m$ agents of type i and production set mY^k. [17/] Now restrict attention to an m-agent economy in which all agents of any given type i must be treated identically. Then following Debreu [1962] we have the following

Definition: a quasi-equilibrium of the kth restricted finite economy is a state $[x_i^{k*}, y^{k*}]$ and a price system p^{k*} such that

(α) for every i, x_i^{k*} is a greatest element $\{x_i \in X^k\colon p^{k*} \cdot x_i \leq p^{k*} \cdot \xi^k\}$ under $W(\cdot,i)$ and/or $p^{k*} \cdot x_i^{k*} = p^{k*} \cdot \xi^k = \text{Min } p^{k*} \cdot X^k$;

(β) $p^{k*} \cdot my^{k*} = \text{Max } p^{k*} \cdot mY^k$

(γ) $\sum_i m\lambda(i)x_i^{k*} - my^k = m\xi^k$

(δ) $p^{k*} \neq 0$.

A quasi-equilibrium is a competitive equilibrium if the first part of condition (α) holds. In what follows we shall establish the existence of a quasi-equilibrium using a theorem of Debreu [1962], and then establish directly that it is also a competitive equilibrium. It is immediate that a competitive equilibrium for the kth restricted finite economy is also a competitive equilibrium for the original kth restricted economy with a continuum of agents (m cancels out of conditions (β) and (γ)).

We make use of the following theorem, as a special case of Debreu [1962],

---

17/ We are assuming that each $\lambda(i)$ is rational. An extension to arbitrary real $\lambda(i)$'s would entail a limiting argument.

<u>Theorem</u> (Debreu): The kth restricted finite economy has a quasi-equilibrium if

(a.1) $A(m\ X^k) \cap (-A(m\ X^k)) = \{0\}$,

(a.2) X^k is closed and convex;

for every i,

(b.1) for every consumption x_i in $\hat{X}_i^k$, there is a consumption in X^k preferred to x_i,

(b.2) for every x_i' in X^k, the sets

$\{x_i \in X^k : W(x_i, i) \geq W(x_i', i)\}$

$\{x_i \in X^k : W(x_i, i) \leq W(x_i', i)\}$ are closed in X^k,

(b.3) for every x_i' in X^k, the set $\{x_i \in X^k : W(x_i, i) \geq W(x_i', i)\}$ is convex,

(c.1) $(\{m\xi^k\} + m\ Y^k) \cap m\ X^k \neq \emptyset$,

(c.2) $(\{\xi^k\} + A(m\ Y^k)) \cap X^k \neq \emptyset$,

(d.1) $0 \in m\ Y^k$,

(d.2) $A(m\ X^k) \cap A(m\ Y^k) = \{0\}$,

where A(H) is the asymptotic cone of set H, $mH = \{s : s = mh,\ h \in H\}$, and $\hat{X}_i^k$ is the attainable consumption set for the i^{th} type consumer in k^{th} restricted economy.

Each of these conditions holds for our restricted finite economy, as we indicate in the appendix. Thus the existence of a quasi-equilibrium is established. We now verify that the first part of condition (α) must hold. In a quasi-equilibrium condition (β) holds, i.e., there exists a maximizing element in Y^k given p^{k*}. It follows that no component of p^{k*} can be negative. Also from condition (δ) not all components can be zero. Therefore at least one component of p^{k*} is positive. Maximizing $p^{k*} \cdot y$ with respect to y in Y^k one obtains

$$\text{(6.6a)} \qquad p_0^{k*}(c) - \psi_0^k \cdot c = 0 \qquad \forall c \in C^k$$

$$\text{(6.6b)} \qquad p_1^{k*}(c,\theta_1) - \lambda(\theta_1)\psi_1^k \cdot c = 0 \qquad \forall c \in C^k, \forall \theta_1 \in \Theta$$

$$\text{(6.6c)} \qquad p_2^{k*}(c,\theta_1,\theta_2) - \lambda(\theta_1)\lambda(\theta_2)\psi_2^k \cdot c = 0 \qquad \forall c \in C^k, \forall \theta_1,\theta_2 \in \Theta$$

where the ψ_t^k, t = 0,1,2 are nonnegative ℓ-dimensional vectors of Lagrange multipliers. By virtue of the existence of a maximum and the existence of at least one positive price, one of these Lagrange multipliers is positive. Thus

$$p^{k*} \cdot \xi^k = \psi_0^k \cdot e_0 + \psi_1^k \cdot e_1 + \psi_2^k \cdot e_2 > 0$$

since $e_t > 0$, t = 0,1,2. But the measure which puts mass one on the zero element of the underlying commodity space for all possible parameter draws has valuation zero under p^{k*}. Thus $p^{k*} \cdot \xi^k > \text{Min}\ p^{k*} \cdot X^k$ and the second part of condition (α) cannot hold.

Now x_i^{k*} denotes the maximizing element for the ith agent type in a competitive equilibrium of the kth restricted economy. For any i, $\{x_i^{k*}\}_{k=0}^{\infty}$ is a sequence in the space of $1 + n + n^2$ dimensional vectors of probability measures on the underlying consumption set C. This metric space is compact, so there exists a convergent subsequence. Since there are a finite number of agent types, it is thus possible to construct a subsequence of the sequence allocations $\{(x_i^{k*})\}$ which converges to some allocation (x_i^{∞}). It may be guessed that this limit, (x_i^{∞}), will constitute part of an equilibrium specification for the unrestricted economy.

For every restricted economy k, the price system is (6.6). Moreover, the price system may be normalized by dividing through by the sum of all the Lagrange multipliers so that in fact each Lagrange multiplier may be taken to be between zero and one. Thus one may again find a further subsequence of sequence of vectors $\{\psi_t^k\}$ which converges to some number $\{\psi_t^{\infty}\}$ with components between zero and one. Moreover the Lagrange multipliers in $\{\psi_t^{\infty}\}$ must sum to 1. In what follows then we restrict attention to the subsequence of economies, indexed by h, such that for every i, $x_i^{h*} \overset{w}{\rightarrow} x_i^{\infty}$ and for every t, $\psi_t^h \rightarrow \psi_t^{\infty}$.

For each economy h the equilibrium price system is a linear functional v^h defined by

$$(6.7) \qquad v^h(x) = \sum_{c \in C^k} p_0^{h*}(c)x_0(c) + \sum_{\theta_1} \sum_{c \in C^k} p_1^{h*}(c,\theta_1)x_1(c,\theta_1)$$

$$+ \sum_{\theta_1} \sum_{\theta_2} \sum_{c \in C^k} p_2^{h*}(c,\theta_1,\theta_2)x_2(c,\theta_1,\theta_2)$$

$$= \sum_{c \in C^k} \psi_0^h \cdot c\, x_0(c) + \sum_{\theta_1} \lambda(\theta_1) \sum_{c \in C^k} \psi_1^h \cdot c\, x_1(c,\theta_1)$$

$$+ \sum_{\theta_1} \lambda(\theta_1) \sum_{\theta_2} \lambda(\theta_2) \sum_{c \in C^k} \psi_2^h \cdot c\, x_2(c,\theta_1,\theta_2) .$$

Thus it may be guessed that an equilibrium price system v^{∞} for the unrestricted economy will be

$$(6.8) \qquad v^{\infty}(x) = \psi_0^{\infty} \cdot \int c\, x_0(dc) + \sum_{\theta_1} \lambda(\theta_1)\psi_1^{\infty} \cdot \int c\, x_1(dc,\theta_1)$$

$$+ \sum_{\theta_1} \lambda(\theta_1) \sum_{\theta_2} \lambda(\theta_2)\psi_2^{\infty} \cdot \int c \, x_2(dc,\theta_1,\theta_2).$$

Note that since the sum of the Lagrange multipliers is strictly positive, $v^{\infty}(\xi) > 0$.

It is first established that x_i^{∞} solves

$$\max_{x \in X} \; W(x,i) \qquad \text{s.t.}$$

(6.9) $$v^{\infty}(x) \le v^{\infty}(\xi) \; .$$

Note that in the competitive equilibrium of the h^{th} restricted finite economy, x_i^{h*} solves

$$\max_{x \in X^h} \; W(x,i) \qquad \text{s.t.}$$

(6.10) $$v^h(x) \le v^h(\xi^h).$$

So from (6.21) and the definition of v^h in (6.7)

(6.11) $$\psi_0^h \cdot \int c \, x_{i0}^{h*}(dc) + \sum_{\theta_1} \lambda(\theta_1)\psi_1^h \cdot \int c \, x_{i1}^{h*}(dc,\theta_1)$$

$$+ \sum_{\theta_1} \lambda(\theta_1) \sum_{\theta_2} \lambda(\theta_2)\psi_2^h \cdot \int c \, x_{i2}^{h*}(dc,\theta_1,\theta_2)$$

$$\le \psi_0^h \cdot e_0 + \psi_1^h \cdot e_1 + \psi_2^h \cdot e_2 \; .$$

Recalling that $x_i^{h*} \xrightarrow{w} x_i^{\infty}$, and noting that in (6.11) we are integrating (bounded) continuous functions on C, we may take the limit of both sides of (6.11) as

$h \to \infty$, and obtain that x_i^∞ satisfies (6.9). Thus x_i^∞ is a feasible solution. Now suppose x_i^∞ is not a maximizing element, so that there exists some $\hat{x}_i \in X$ satisfying (6.9) with

$$W(\hat{x}_i, i) > W(x_i^\infty, i).$$

Then it is possible to construct some $\hat{x}_i^h$ such that $W(\hat{x}_i^h, i) > W(x_i^{h*}, i)$ and $v^h(\hat{x}_i^h) < v^h(\xi^h)$. This will contradict x_i^{h*} as maximizing in the h^{th} restricted economy. 18/

Now define $y^\infty = \sum_i \lambda(i) x_i^\infty - \xi$. We want to show y^∞ solves

$$\underset{y \in Y}{\text{Max}}\ v^\infty(y).$$

Then we will have both profit maximization and market clearing, the remaining conditions of a competitive equilibrium to be verified. First note that the budget constraint (6.9) may be assumed to hold as an equality under x_i^∞. Then inserting the functional v^∞ from (6.8), multiplying through by $\lambda(i)$ and summing over i

$$(6.12) \quad \psi_0^\infty \cdot \{\sum_i \lambda(i) \int c\, x_{i0}^\infty(dc) - e_0\} + \sum_{\theta_1} \lambda(\theta_1)\psi_1^\infty \cdot \{\sum_i \lambda(i) \int c\, x_{i1}^\infty(dc, \theta_1) - e_1\}$$

$$+ \sum_{\theta_1} \lambda(\theta_1) \sum_{\theta_2} \lambda(\theta_2)\psi_2^\infty \cdot \{\sum_i \lambda(i) \int c\, x_{i2}^\infty(dc, \theta_1, \theta_2) - e_2\} = 0.$$

We shall make use of (6.12) below. Now in each restricted economy h, y^{h*} is the maximizing production vector in Y^h. Thus from the market clearing condition and the definition of Y^h

18/ For the details of this argument see Prescott and Townsend [1979] .

(6.13) $$\sum_i \lambda(i) \int c\, x^{h*}_{i0}(dc) \le e_0$$

(6.14) $$\sum_{\theta_1} \lambda(\theta_1) \sum_i \lambda(i) \int c\, x^{h*}_{i1}(dc,\theta_1) \le e_1$$

(6.15) $$\sum_{\theta_1} \lambda(\theta_1) \sum_{\theta_2} \lambda(\theta_2) \sum_i \lambda(i) \int c\, x^{h*}_{i2}(dc,\theta_1,\theta_2) \le e_2.$$

Taking the limit as $h\to\infty$, recalling that $x_i^{h*} \overset{w}{\to} x_i^{\infty}$, and noting that we are integrating over (bounded) continuous functions

(6.16) $$\sum_i \lambda(i) \int c\, x^{\infty}_{i0}(dc) \le e_0$$

(6.17) $$\sum_{\theta_1} \lambda(\theta_1) \sum_i \lambda(i) \int c\, x^{\infty}_{i1}(dc,\theta_1) \le e_1$$

(6.18) $$\sum_{\theta_1} \lambda(\theta_1) \sum_{\theta_2} \lambda(\theta_2) \sum_i \lambda(i) \int_c c\, x^{\infty}_{i2}(dc,\theta_1,\theta_2) \le e_2.$$

So from the construction of y^∞, $y^\infty \in Y$. Now under the price system v^∞, the problem of the firm is

$$\text{Max } \psi_0^\infty \cdot \int c\, y_0(dc) + \psi_1^\infty \cdot \{\sum_{\theta_1} \lambda(\theta_1) \int c\, y_1(dc,\theta_1)\}$$

$$+ \psi_2^\infty \cdot \{\sum_{\theta_1} \lambda(\theta_1) \sum_{\theta_2} \lambda(\theta_2) \int c\, y_2(dc,\theta_1,\theta_2)\}$$

subject to (6.1)-(6.3). Thus profits are nonpositive. Moreover at y^∞ profits are zero, using (6.12). Hence, y^∞ is profit maximizing. This completes the proof of the existence of a competitive equilibrium for the limit economy.

It is readily verified that for one-period economy (with period zero only) there need be no randomness in a competitive equilibrium. Agents are risk averse, and the incentive-compatibility conditions need not be imposed explicitly. In this sense the work developed here reduces to standard competitive analysis when the information structure is private but not sequential.

7. The Welfare Theorems

We now turn to the two fundamental theorems of contemporary welfare economics and ask whether any competitive equilibrium allocation is optimal and whether any optimum can be supported in a competitive equilibrium. Both questions may be answered in the affirmative, but the second affirmative answer has some revealing qualifications.

In the context of private information we rely heavily on Debreu's [1954] treatment in general linear spaces. To establish that any competitive equilibrium is an optimum, just two properties are sufficient:

(I) X is convex.

(II) $\forall\ x', x'' \in X$ and $\forall\ i \in \Theta$,

$W(x',i) < W(x'',i)$ implies $W(x',i) < W(x^{\alpha},i)$

where $x^{\alpha} = (1 - \alpha)x' + \alpha x''$, $0 < \alpha < 1$.

For property I, note that a linear combination of two probability measures is again a probability measure, and that constraints (4.1) and (4.2) hold under convex combinations, as indicated in the discussion in Section 3. For property II, it is readily verified that

$$W(x^{\alpha},i) = (1 - \alpha)\ W(x',i) + \alpha W(x'',i).$$

That is, the objective function is linear in probability measures, a natural consequence of the expected-utility hypothesis. In summary we have

<u>Theorem 1</u>: Every competitive equilibrium with state (x^*,y^*) and price system v^* is an optimum.

<u>Proof</u>: The proof follows Debreu [1954] quite closely. For details see Prescott and Townsend [1979].

To establish that any optimum can be supported as a competitive equilibrium three more properties are sufficient:

(III) $\forall$ $x, x', x'' \in X$ and $\forall$ $i \in \Theta$, the set $\{\alpha \in [0,1]: W(x^{\alpha},i) \leq W(x,i)\}$ is closed where $x^{\alpha} = (1-\alpha)x' + \alpha x''$.

(IV) Y is convex.

(V) Y has an interior point.

Property III follows immediately from the linearity of the objective function. Property IV follows from the linearily of L and from the fact that constraints (6.1)-(6.3) hold under convex combinations. For property V pick a degenerate element of L such that (6.1)-(6.3) hold as strict inequalities. [19/] There now follows

<u>Theorem 2</u>: Every optimum $[(x_i^*), y^*]$ for which the set $N \equiv \{(x_i): x_i \in X_i^{\geq}(x_i^*), x_k \in X_k^{>}(x_k^*)$ for at least one k, (x_i) satisfies (3.7)$\}$ is nonempty, is associated with a nontrivial continuous linear functional v^* on L such that

(i) x_i^* solves

$$\underset{x_i \in X_i^{\geq}(x_i^*)}{\text{Min}} \quad v^*(x_i)$$

19/ Here the interior point is relative to the product topology on L; see footnote 16.

subject to

(7.1) $$W(x_i,j) \leq W(x^*_j,j) \qquad \forall\ j \neq i,$$

(ii) y^* solves

$$\max_{y \in Y} v^*(y)$$

where

$$X^{\geq}_i(x^*_i) = \{x_i \in X:\ W(x_i,i) \geq W(x^*_i,i)\}$$

$$X^{>}_k(x^*_k) = \{x_k \in X:\ W(x_k,k) > W(x^*_k,k)\}$$

Proof: Again the proof follows Debreu [1954] with suitable modifications. For details see Prescott and Townsend [1979].

Here of course x^*_i is a minimizer of expenditure on the weak upper contour set relative to x^*_i restricted by (7.1). Relative to this, Debreu makes the

Remark: Suppose that for every $i \in \Theta$ there exists an x_i' satisfying (7.1) with $v^*(x_i') < v^*(x^*_i)$. Then x^*_i solves

Problem (2):

$$\max_{x_i \in X} W(x_i,i)$$

subject to

$$v^*(x_i) \leq v^*(x^*_i)$$

(7.1) $W(x_i, j) \leq W(x_j^*, j) \qquad \forall\ j \neq i.$

Proof: Again the proof follows Debreu [1954]. See Prescott and Townsend [1979] for details.

Thus, under the conditions of the Remark, [20] an optimum $[(x_i^*), y^*]$ can be supported as a kind of competitive equilibrium, relative to a price system v^*. But note first that the problem confronting each agent of type i (problem 2) is not that which appears in the definition of a competitive equilibrium, even with ξ replaced by x_i^*. In particular constraint (7.1) has been imposed. Thus, unlike the standard decentralization result, each agent type i must know not only his own endowment and preferences (and prices), but also the preferences and assignment of other agents. Second, no agent of type i can be forced to solve problem (2); on an a priori basis each agent's type is not known, yet problem 2 is defined relative to the parameter i. We circumvent these difficulties by modifying the definition of a competitive equilibrium to allow for endowment selection.

Suppose in what follows that (x_i^*) is an optimal allocation and v^* is the price system in Theorem 2. Let each agent choose one component of (x_i^*) as his endowment, and then maximize (solve problem 2). That is, suppose agent type i chooses x_k^*, $k \neq i$ as his endowment. Then under v^* his problem would be

$$\max_{x \in X} W(x, i)$$

subject to

---

[20] Recall from section 6 the equilibrium price system puts value zero on the vector of probability measures putting all mass on the zero element of the underlying commodity space. So the conditions of the remark are frequently satisfied.

$$v^*(x) \le v^*(x_k^*)$$

$$W(x,j) \le W(x_j^*,j) \qquad \forall\ j \ne k.$$

Of course any solution to this problem, say $\hat{x}_i^k$, must satisfy the constraints. In particular, setting j = i.

$$W(\hat{x}_i^k,i) \le W(x_i^*,i).$$

That is, agent type i can do no better than x_i^* by "pretending" to be some type $k \ne i$. Alternatively, if agent type i chooses the endowment x_i^*, his problem would be problem 2, and we know x_i^* solves that problem. It follows that x_i^* is a maximizing endowment choice. In summary the (allocation-type) tuple (x_i^*,i) solves

<u>Problem 3</u>:

$$\max_{x \in X,\, k \in \Theta} W(x,i)$$

subject to

$$v^*(x) \le v^*(x_k^*)$$

$$W(x,j) \le W(x_j^*,j) \qquad \forall\ j \ne k.$$

This gives us the following

<u>Definition</u>: <u>A competitive equilibrium with endowment selection</u> is a state $[(x_i^*),y^*]$ and a price system v^* such that

(i) $\forall$ i, (x_i^*,i) solves problem 3;

(ii) y^* solves

$$\max_{y \in Y} v^*(y);$$

(iii) $$\sum_i \lambda(i) x_i^* = y^* - \xi.$$

We have thus shown that under the conditions of the Remark, any optimum can be supported as a competitive equilibrium with endowment selection.

Appendix

Section 5 -- Verifying the conditions of the Theorem (Debreu)

As for (a.1) note that the asymptotic cone of mX^k, denoted $A(mX^k)$ equals the singleton $\{0\}$. (See Debreu [1959] for a definition of the asymptotic cone.) Thus (a.1) follows immediately. Also, any asymptotic cone must contain zero, so (d.2) is immediate. Condition (a.2) may be verified directly by using the definition of X^k and taking a limit of elements of X^k for closure and a convex combination for convexity.

For condition (b.1), $\hat{X}_i^k$ is the attainable consumption set of any agent of type i, the set of all consumption allocations x_i for the agents of type i consistent with consumption allocations (x_i) for all agents satisfying the resource constraints (4.4)-(4.6), restricted to C^k. Let $\hat{X}_i$ denote the attainable consumption set when unrestricted to C^k. Now pick any consumption x_i in $\hat{X}_i^k$. In the unrestricted economy x_i is weakly dominated under preferences (4.3) by a consumption which puts probability one on the mean consumption under x_i, denoted

$$E(c_i) = \{E(c_{i0}), (E(c_{i1}(\theta_1))), (E(c_{i2}(\theta_1,\theta_2)))\}. \tag{1}$$

This mean consumption $E(c_i)$ is consistent with (4.4)-(4.6) since x_i is. Now consider the consumption $\bar{c}_i$ defined by

$$\bar{c}_{i0} = E(c_{i0}) \qquad \text{at } t = 0$$

$$\bar{c}_{i1} = \sum_{\varphi_1} \lambda(\varphi_1)E(c_{i1}(\varphi_1)) \qquad \text{at } t = 1, \text{ for all } \varphi_1$$

$$\bar{c}_{i2} = \sum_{\varphi_1} \lambda(\varphi_1) \sum_{\varphi_2} \lambda(\varphi_2)E(c_{i2}(\varphi_1,\varphi_2)) \qquad \text{at } t = 2, \text{ for all } \varphi_1,\varphi_2.$$

The consumption $\bar{c}$ must weakly dominate under (3.3) the consumption $E(c_i)$, is consistent also with (4.4)-(4.6) and satisfies the incentive compatibility constraints (4.1) and (4.2) since it is parameter independent. Thus the consumption $\bar{c}_i$ must be in $\hat{X}_i$. But then

$$\bar{c}_{i0} < c_0^* \quad \bar{c}_{i1} < c_i^* \quad \bar{c}_{i2} < c_2^*$$

by the construction of c_t^*, t = 0,1,2 in condition (6.4). So $c^* \in X^k$ strictly dominates $\bar{c}$ which weakly dominates x_i.

For (b.2) one may note that W(x,i) is linear in x and consider the limit of convergent sequences. For (b.3) one may take convex combinations. For (d.1) $0 \in mY^k$ from the definition of Y^k. For (c.1) and (c.2) note that $0 \in mY^k$ and $0 \in A(mY^k)$, and also that $\xi^k \in X^k$ and $m\xi^k \in mX^k$.

References

Akerlof, G., "The Market for 'Lemons': Qualitative Uncertainty and the Market Mechanism," Quarterly Journal of Economics, 84 (August 1970). 488-500.

Arrow, K. J., "Le rôle des valeurs boursieres pour la répartition la meilleure des risques," Econometrie, Paris, Centra National de la Recherche Scientifique, (1953), 41-48.

Ash, R. B., Real Analysis and Probability (New York: Academic Press, 1972).

Baron, D. P. and R. Myerson, "Regulating a Monopoly with Unknown Costs," Center for Math Studies, Discussion Paper No. 412, Northwestern University, 1979.

Bewley, T. F., "Existence of Equilibria in Economies with Infinitely Many Commodities," Journal of Economic Theory 4 (1972), pp. 514-540.

Chari, V. V., "Involuntary Unemployment and Implicit Contracts," manuscript, Carnegie-Mellon University, August 1980.

Cheung, S. H., "Why Are Better Seats "Underpriced"? Economic Inquiry, vol XV, October 1977.

Chiang, R. and C. Spatt, "Imperfect Price Discrimination and Welfare," manuscript, November 1979.

Debreu, G., "New Concepts and Techniques for Equilibrium Analysis," International Economic Review 3 (1962), pp. 257-273.

Debreu, G., Theory of Value: An Axiomatic Analysis of Economic Equilibrium (New Haven: Cowles Foundation, 1959).

Debreu, G., "Valuation Equilibrium and Pareto Optimum," Proceedings of the National Academy of Science 40 (1954), pp. 588-592.

Dunford, N. and J. Schwartz, Linear Operators Part I: General Theory (New York: Interscience, 1964).

Fishburn, P. C., "Even-Chance Lotteries in Social Choice Theory" Theory and Decision, 3, 1972a, 18-40.

Fishburn, P. C., "Lotteries and Social Choices" Journal of Economic Theory, 5, 1972, 189-207.

Gale, D., "Money, Information and Equilibrium in Large Economies, Journal of Economic Theory 23, 28-65, 1980.

Gibbard, A., "Manipulation of Schemes that Mix Voting with Chance," Econometrica, 45-3, April 1977.

Green, J., "Wage-Employment Contracts," manuscript, September 1980.

Grossman, S. and O. Hart, "Implicit Contracts, Moral Hazard, and Unemployment" CARESS Working Paper #80-20, University of Pennsylvania, August 1980.

Harris, M. and A. Raviv, "A Theory of Monopoly Pricing Schemes with Demand Uncertainty," GSIA working paper #15-79-80, Carnegie-Mellon, revised August 1980.

Harris, M. and A. Raviv, "Allocation Mechanisms and the Design of Auctions," GSIA working paper #5-78-79, Carnegie-Mellon, revised November 1979.

Harris, M. and R. M. Townsend, "Allocation Mechanisms for Asymmetrically Informed Agents," Carnegie-Mellon Working Paper No. 35-76-77, March 1977.

Harris, M. and R. M. Townsend, "Resource Allocation Under Asymmetric Informa-rion," Carnegie-Mellon Working Paper No. 43-77-78, revised October 1979, published in Econometrica, January 1981.

Hirshliefer, J. and J. Riley, "The Analytics of Uncertainty and Information - An Expository Survey, Journal of Economic Literature, December 1979.

Hurwicz, L., "On Informationally Decentralized Systems," in Decision and Organization edited by R. Radner and B. McGuire (Amsterdam, North Holland, 1971).

Intriligator, M. D., "Income Redistribution: A Probabilistic Approach, American Economic Review, March 1979, p. 97-105.

Kolmogorov, A. N. and S. V. Fomin, Introductory Real Analysis, translated by Richard A. Silverman (New York: Dover, 1970).

Lucas, R. E., Jr., "Equilibrium in a Pure Currency Economy" in Models of Monetary Economies, J. Kareken and N. Wallace, eds., Federal Reserve Bank of Minneapolis, 1980.

Malinvaud, E., "Markets for an Exchange Economy with Individual Risks" Econometrica, (May 1973).

McKenzie, L., "On the Existence of General Equilibrium for a Competitive Market," Econometrica, 27 (1959),54-71.

Morton, S., "The Optimality of Strikes in Labor Negotiations", manuscript, Carnegie-Mellon University, August 1980.

Myerson, R. B., "Incentive Compatibility and the Bargaining Problem", Econometrica, 47,1 January 1979, 61-74.

Myerson, R., "Optimal Auction Design," forthcoming in Mathematics of Operations Research.

Neumann, J. von, and O. Morgenstern, Theory of Games and Economic Behavior, Princeton University Press, 1947.

Parthasarathy, K.R., Probability Measures on Metric Spaces, Academic Press, New York, 1967.

Prescott, E. C. and R. M. Townsend, "On Competitive Theory with Private Information," manuscript, June 1979.

Riley, J. G., "Informational Equilibrium," Econometrica, 47, 2, March, 1979, 331-360.

Rothschild, M. and J. Stiglitz, "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Impsrfect Information," Quarterly Journal of Economics, 90, (November, 1976) pp. 629-649.

Royden, H. L., Real Analysis, 2nd ed., The MacMillan Company, New York, 1968.

Spence, A. M., Market Signaling: Informational Transfer in Hiring and Related Screening Processes, (Cambridge: Harvard University Press, 1974).

Stiglitz, J. E., "Prices and Queues as Screening Devices in Competitive Markets," Stanford University, IMSS Technical Report, 212, August 1976.

Stiglitz, J. E., and A. Weiss, "Credit Rationing in Markets with Imperfect Information, Part II: Constraints as Incentive Devices," Research Memorandum No. 268, Princeton, August 1980.

Weiss, L. "The Desirability of Cheating Incentives and Randomness in the Optimal Income Tax," Journal of Political Economy, 99, 1343-52, 1976.

Wilson, C., "A Model of Insurance Markets with Incomplete Information," Journal of Economic Theory, 19, (1978).

Wilson, C., "The Nature of Equilibrium in Markets with Adverse Selection," Social System Research Institute, University of Wisconsin-Madison Report 7715, (November 1977).

Zeckhauser, R., "Majority Rule with Lotteries on Alternatives," Quarterly Journal of Economics, 83 (1969) 696-703.

Zeckhauser, R., "Voting Systems, Honest Preferences, and Pareto Optimality," American Political Science Review, 67 (1973) 934-946.