# Discussion Papers Department of Economics University of Copenhagen 

No. 98-02

## Wage Determination And Employment In Traditional Agriculture

James Mcl ntosh

Studiestræde 6, DK-1455 Copenhagen K., Denmark Tel.: +4535323082-Fax: +45353230 00
http://www.econ.ku.dk

ISSN: 1601-2461 (online) ISSN 0902-6452 (print)

# Wage Determination A nd Employment In Traditional A griculture 

J ames M cl ntosh<br>M arch 11, 1998<br>Department of Economics<br>Concordia University<br>1455 de M aisonneuve West<br>M ontreal, Quebec H3G 1M 8<br>Canada<br>Tel. (514) 8483910<br>Fax (514) 8484536<br>E mail: jamesm@vax2.concordia.ca<br>and<br>Institute of Economics<br>University of Copenhagen<br>Studiestraede 6<br>1455 Copenhagen, Denmark ${ }^{1}$


#### Abstract

Wages in traditional agrarian societies are often observed to be above reseravtion wages even in the slack season when markets are in a state of excess labour supply. Models of noncooperative wage setting by landlords which explicitly take account of the costs of supervising hired labour and emphasize worker heterogeneity are developed and analysed. B oth symmetric and asymmetric information cases are considered. Conditions are given for the existence of competitive equilibria and their relationship to Nash equilibria. Nash equilibria are shown to be more likely to exist. Nash equilibria exhibit wage dispersion and involuntary unemployment or underemployment with identical workers earning di ®erent wage rates.


K ey W ords: Wages, Involuntary unemployment, Underemployment, Supervision costs.

[^0]
## 1 Introduction

One of the features of third world rural labour markets is the presence of high unemployment rates in the o®peak seasons. In addition, as a number of authors have noted, wage rates for casual or landless labour tend to be less than fully ${ }^{\circ}$ exible in response to changing seasonal demands for labour. Consequently, unemployment is involuntary and the unemployed can not improve their position by o®ering to work for less or by looking harder for work. The plight of landless labour is, in fact, one of the most distressing problems facing less developed countries. ${ }^{2}$ While there seems to be little disagreement among scholars who study these markets that they are characterized by substantial periods of involuntary unemployment there is considerable variety in terms of potential explanations of the causes of such phenomena.

This paper examines the theory of agricultural labour markets. The next section brie ${ }^{\circ} y$ surveys the literature in this area. ${ }^{3}$ There are a number of papers which attempt to explain wage rigidity and involuntary unemployment as products of the behaviour of rational optimizing agents. Two such classes of model are discussed here. These are variations on $\backslash \mathrm{e} \pm$ ciency wage" models and labour bargaining models. This discussion reveals certain limitations in the established theory of agricultural labour markets and the need for a new class of model to be developed. This is done in Section (3) in a model which explicitly considers supervision costs and permits workers to be paid wages which re ect the observable characteristics that determine their productivity. ${ }^{4}$ Section (4) contains a summary and a discussion of the main results.

[^1]
## 2 Theoretical Wage M odels

A mong endogenous explanations of involuntary unemployment $e \pm$ ciency wage models are perhaps the most well known. Although, there are quite a large number of papers on this topic only a small sub-sample will be considered here. There are two versions, one based on nutritional requirements, the other on employer inability to determine individual worker productivity. ${ }^{5}$ In the nutrition based version worker productivity is determined by consumption which is in turn determined by the worker's wage. Workers with very low wages do not have physical strength and stamina to main high eßort or productivity levels. Raising wages does alleviate this problem, however, and consequently there is a link between wages and productivity. Under certain not unreasonable assumptions there exists a lower bound on the wage rate, the e $\pm$ ciency wage. If farmers pay wages below this rate their pro ${ }^{-}$ts actually fall because worker e $\pm$ciency falls and reduces revenue more than wage costs. Even when there is an excess supply of labour at this wage farmers prefer to pay the e $\pm$ ciency wage rather than let the wage fall to clear the market. This is what Dasgupta and Ray (1986, p. 1024) describe as a competitive non-Walrasian equilibrium in which there is involuntary unemployment.

The same outcome is obtained in the asymmetric model, but the mechanism is quite di ®erent. When farmers can not observe the characteristics which determine the productivity of farm labour they can not discriminate on this basis and thus must pay a common wage to everyone they employ. If farms are identical then all farms must pay the same wage rate. Individuals di ®er not only with respect to their ability but have di ®erent reservation wages which usually are assumed to be positively correlated with their ability. Since workers enter the labour market only if their reservation wage is at least as great as the market wage average productivity increases with the wage rate.

An e $\pm$ ciency wage can arise in this case and will be supported by farmers if reductions in the wage by driving the highest productivity workers out of the market reduce average productivity more than wage costs. As already mentioned, the causal mechanism is based on the presence asymmetric information and while both versions of the model are characterized by involuntary unemployment this version is immune to the criticism leveled against the nutrition based version that this was a longer term relationship and that individual farmers had no incentive to pay the e $\pm$ ciency wage since they would not likely reap the bene- ts. However, the validity of this type of model really does depend on there being asymmetries in the distribution of information about worker characteristics.

What farmers know about the labour which they employ is an empirical question. There may be situations where there is less than perfect information about worker characteristics in which case this version of the model is relevant. ${ }^{6}$

[^2]A radically di Rerent approach is adopted by Mukherjee and Ray (1992) who see the determination of wages as the outcome of con ${ }^{\circ}$ ict or bargaining between land-owning farmers and landless labourers. There are two periods representing the slack and peak seasons. There is surplus labour in the slack season and wages are low. Workers have opinions as to what a \fair wage" is and when the peak season arrives they remember which farmers paid fair wages and those which did not. In the peak season those farmers which paid unfair wages in the slack season experience higher recruitment costs due to refusals by workers who thought their slack season wages were unfair. To avoid this costly situation farmers have an incentive to pay more than reservation wages in the slack season. The authors propose a Nash equilibrium which farms of di ®erent size pay possibly di ®erent wages which and these minimize total expected two period costs by paying more than reservation wages in the slack season to reduce expected recruitment costs in the peak season.

Now there are two problems with this model. The ${ }^{-}$rst the is credibility of the behaviour of landless peasants. For workers who must survive a large number of two season years is it reasonable to assume that they would threaten a landlord by refusing to work for him in the peak season because he paid low wages in the previous slack season and risk the possible retaliation of this landlord in the following slack season? This possibility is assumed away because there are only two periods but it is a problem nonetheless. Secondly, the emphasis on intertemporal considerations gives the model a seasonal interdependence that runs counter to what many observers report.

A nother type of bargaining model is considered by Osmani (1991, p. 6) who believes \} ...that it is the workers rather than employers who resist the wage rate from being pushed down to the competitive level." He sees the modelling of casual labour markets as a repeated non-cooperative game between a group of homogenous peasant workers and a group of passive landlords. Given this representation of the casual labour market it is possible to show that all slack season workers get a wage above their common reservation wage and this is sustained as a sub-game perfect Nash equilibrium with each worker following a \trigger strategy".

W hile there in nothing wrong with the logic of the model the plausibility of the assumptions is another matter. As has already been noted, there are di ßerentials across individuals that a rect both productivity and the probability of employment. W hile these can be accommodated in a more general \trigger strategy" equilibria when farmers have no role to play in wage determination, the situation is radically di ®erent when they do. As the models developed in the next section show, worker heterogeneity and the presence of supervision costs or not being able to observe ability gives farmers an incentive to be selective with respect to their labour force and to set wage rates accordingly.

The importance of supervision costs has long been appreciated by economists interested in the study of traditional third world agriculture. It is fairly obvious that farmers expose themselves to moral hazard problems if they do not monitor or direct hired labour because of the incentives that wage earners have to either overstate their production in a piece-rate system or shirk when they are being paid by the day. Frisvold (1994) in his study of Indian farms provides convincing evidence that supervision improves the productivity of agricultural
hired workers as well as brief survey of the theoretical literature on supervision costs. For further discussion of this issue see Foster and Rosenzweig (1993, p. 763).

However, the implications of costly supervision have yet to be fully worked out. When workers di ®er by ability or productivity the presence of supervision costs as a function of farm employment causes a number of things happen. First, the substitutability between di ®erent levels of ability is no Ionger solely determined by the characteristics of the individuals but depends on the farms that could potentially employ them. It is well known that in competitive markets workers with di ßerent but observable productivities are paid wages which are equal to these productivities. In many cases the competitive pricing of labour makes perfect substitutes out of di ßerent quality labour by making them equally costly to the employer. ${ }^{7}$ Secondly, supervision costs are not part of the compensation package paid to labour although they must be paid by the employer. This means that when there are supervision costs farms can improve pro ${ }^{-}$tability by hiring proportionately more high ability labour because the higher wage costs are o Bset by lower supervision costs. As will be seen in the models developed in the next section this trade-0®can be accompanied by a subset of the farms specializing in high ability labour, the subset being determined by the shape of the supervision cost function, and paying a premium not only above reservation wages but above that justi ed by the di ßerence in individual abilities.

There are also implications for employment. Higher wages generally lead to lower employment levels. This will turn out to be true for models based on supervision costs and like some of the models referred to earlier in this section there will be involuntary underemployment or unemployment and because the wage rates are chosen by farms those looking for work will be unable to obtain it by bidding down the wage.

## 3 A M odel of Slack Season E mployment

Production in traditional agriculture is a continuous process throughout the crop cycle. It culminates with the harvest; but in order for this to happen tasks have to be performed at various times prior to harvesting. These include plowing, weeding, applying fertilizer and pesticides, preparing irrigation ditches etc. The amount of work that has to be done at each point in the crop cycle is assumed to be determined by the maximization of an intertemporal objective function. Without specifying the precise nature of this procedure we assume that farmer i has a number of tasks, q; that have to be performed. Here time subscripts are suppressed to simplify the notation but it should be remembered that these requirements vary over time.

Starting with workers, we assume that they are characterized by their ability level, $\mu$ Workers can either work for wages if they can get acceptable employment with farmers in their

[^3]locality or they can work, possibly with other members of their family, in a house-hold mode of production. A worker of type $\mu$ has a utility function
\[

$$
\begin{equation*}
u(` ; \mu)=y(\mu) ; c(` ; \mu) \tag{1}
\end{equation*}
$$

\]

In equation (1) ` is the amount of e®ort expended by the worker and \(y(\mu)\) is the income earned. For house-hold production this is generated by a concave house-hold production function, \(g\left({ }^{`} \mu\right)\); for a wage earner it is his wage, $w(\mu)$, which is also assumed to depend on the worker's type. Here ability has a very simple interpretation. $\mu$ is a measure of e $\pm$ ciency and converts labour time into e $\pm$ ciency units multiplicatively. The nominal value of utility is the di ®erence between income and the nominal disutility of working, $\left.d^{`} ; \mu\right)$. The following sign pattern for the partial derivatives of $c$ is assumed. $c \gg 0, c^{\prime}>0, c_{\mu}<0, c_{\mu}<0$, and $\mathrm{C}_{\mu \mu}>0 .{ }^{8}$

Individuals who are engaged in house-hold production choose an e®ort level which maximizes their utility and generates the following value function

$$
\begin{equation*}
V(\mu)=\max \left[g\left(\mu^{\prime}\right) i \quad c(` ; \mu)\right] \tag{2}
\end{equation*}
$$

Since farms have to compete with this alternative when they attempt to hire labour for wages they must pay at least the reservation wage for a type $\mu$ which is

$$
\begin{equation*}
r(\mu)=V(\mu)+c(1 ; \mu) \tag{3}
\end{equation*}
$$

In the de ${ }^{-}$nition of $r(\mu)$ it is assumed that workers involved in wage employment always provide one unit of labour. R outine calculations using the fact that $\mathrm{g}^{\infty}<0$ and the assumed sign pattern of the partial derivatives of $c$ reveal that both $V(\mu)$ and $r(\mu)$ are increasing concave functions of $\mu$

To simplify the analysis assume that there are just two types of worker: those with low ability and those with high ability. It is assumed that workers are paid on a piece-rate basis and that the number of pieces produced or the number of times a task is performed per day by a type i worker is ${ }^{1}\left(\mu_{\boldsymbol{k}}\right)$. For simplicity it will be assumed that ${ }^{1}\left(\mu_{\mathrm{H}}\right)=\mu_{\mathrm{H}}$ so that in the present context ability is synonymous with productivity. Let values of ability or productivity be $\mu_{L}<\mu_{H}$ and let $n_{i L}$ and $n_{i}$ be the number of low and high ability labour hired at piece

[^4]rates $\left(p_{L} ; p\right) .{ }^{9}$ Assuming that farms have only labour costs, a farm with a total piece-work requirement of $q$ has labour costs of $p_{L} \mu_{L} n_{i L}+p_{H} n_{i}+s\left(n_{i L}+n_{i}\right)$ where $n_{i L} \mu_{L}+n_{i} \mu_{H}=q$. In the expression for labour costs, $s$ is the cost of supervising the labour force. This depends only on total employment and does not depend on ability levels. Clearly, $s^{0}, 0$; although, the second derivative of s may be of either sign depending on the particular type of work involved.

Given piece rates daily wage rates for each type of worker are $w_{L}=p_{L} \mu_{L}$ and $w=p_{H}$. In what follows the analysis will by carried out in terms of the daily wage rates and $q$ will be referred to as farm i's demand. Furthermore, piece-rates will be allowed to vary across types so that $p_{\mathrm{L}}$ will be allowed to be di ®erent from p . This may be more general than is required to represent the payments systems that are actually used in agrarian labour markets. However, it is important to see whether optimizing farms would gain from such a choice.

The problems that concern us in this paper are those involving wage determination when there is \surplus labour" under varying scenarios concerning information availability and market structure. The model developed in this section assumes that farms can observe an individual's ability; that is, every worker's value of $\mu$ is observable to all employers. ${ }^{10}$ The market structure is oligopsonistic; with a - nite number of farms bidding for labour. Farms compete with each other in setting wages and employment levels. Labour is passive; workers act as individuals and their strategies consist of either accepting or rejecting wage contracts. This is a game of imperfect and complete information; play is simultaneous and there is no uncertainty, all players are fully aware of other players characteristics and payo@s, and this fact is known to all players.

We start by examining competitive equilibria when there are $K$ farms with demands $f q_{1}>$ $q_{\mathrm{z}}:::>q_{k}$. As stated earlier, workers will work as paid labourers if the wage rate for their type is at least as great as their reservation wage. Let $N_{\mathrm{L}}$ and N be the number of low ability and high ability workers respectively, that are available for potential employment in the market. A ssume that $N<{ }_{i=1}^{k} q \neq H$ so that an excess demand exists for high ability workers if all farms wanted to employ only high ability workers. In all of what follows $N_{L}$ will be assumed to be large in the sense that any farm will always be able to hire as many low ability workers at $r\left(\mu_{L}\right)=r_{L}$ as it wishes. This is what is meant by $\backslash$ surplus labour" in this model.

In a competitive market farms minimize costs, taking the wage rate for high ability workers, w , as a parameter.

Let $n_{i}(q ; w)$ minimize

$$
\begin{equation*}
C_{i}\left(n_{i} ; w\right)=r_{L}\left(q i \mu H n_{i}\right) \neq L+w n_{i}+s\left[\left(q i \mu H n_{i}\right) \neq \mu+n_{i}\right] \tag{4}
\end{equation*}
$$

[^5]subject to the constraint
\[

$$
\begin{equation*}
0 \cdot n_{i} \cdot q \neq \#_{H}: \tag{5}
\end{equation*}
$$

\]

For future reference note that when $n_{i}(q ; w)$ is an interior solution to the minimization problem above $@_{i}(q ; w)=@>0$ and $@_{i}(q ; w)=@ v<0$.

## De- nition 1

A competitive equilibrium has the following properties. The equilibrium wage rate, $\mathrm{w}^{\text {axa }}$, satis ${ }^{-}$es the equation

$$
\begin{equation*}
{ }_{i=1}^{k} n_{i}\left(q ; w^{\text {wax }}\right)=N \tag{6}
\end{equation*}
$$

and $n_{i}\left(q_{;} ; w^{\text {max }}\right)=q \neq H_{H}$ if $\varrho_{i}\left(q \neq H_{H} ; w^{\text {max }}\right)=@_{i}<0,0<n_{i}\left(q ; w^{\text {max }}\right)<q_{\# H}$, if $@_{i}\left(n_{i} ; w^{\text {max }}\right)=@_{i}=$ 0 , and $n_{i}\left(q ; w^{\text {axa }}\right)=0$ if $\bigodot_{i}\left(0 ; w^{\text {axa }}\right)=@_{i}>0$.

The characteristics of the supervision cost function turn out to be important both for the existence of a competitive equilibrium and the properties of $N$ ash equilibria. Consequently, two cases will be considered; one where s is convex and the other where s is concave. The case when $s$ is convex is considered ${ }^{-r} r$ st.

## Theorem 1

Let $j, 2^{11}$ If $s^{\infty}>0$ and $j$ is de ned by ${ }^{P}{ }_{j i=1}^{i=1} q \neq H<N<{ }^{P}{ }_{i=1} q \neq H$ a competitive equilibrium exists and $n_{i}\left(q ; w^{2 x a}\right)=q \neq H$ if $i<j, 0<n_{j}\left(q ; w^{2 x a}\right)<q \neq H$, and $n_{i}\left(q ; w^{\text {max }}\right)=0$ if $\mathrm{i}>\mathrm{j}$.

Proof

Under the proposed equilibrium equation (7) can be written as $n_{j}\left(q ; w^{\text {max }}\right)=N_{j}$ where $N_{j}=N_{i}{ }^{P}{ }_{i=1}^{i=1}(q \neq H)$. De $e^{-}$ne $W_{i}$ be the largest wage rate at which farm $i$ can employ high ability labour. Clearly, when $s^{\infty}, 0$

$$
\begin{equation*}
\left.W_{i}=r_{L} H_{H} \neq L i s q q \neq \mu\right)\left(1 ; \mu_{H} \neq L\right) \tag{7}
\end{equation*}
$$

[^6]and is the wage rate that makes marginal cost equal to zero when $n_{i}=0$. C ost when each type is paid its reservation wage is $\mathrm{C}_{\mathrm{i}}\left(\mathrm{n}_{\mathrm{i}} ; \mathrm{r}_{\mathrm{H}}\right)$. Now it follows from relatively simple calculations that
\[

$$
\begin{equation*}
@_{i}\left(n_{i} ; r_{H}\right)=@_{i}=\mu_{H}\left(r_{H} \neq H ; r_{L} \neq L\right)+s^{q}\left(1_{i} \mu_{H} \neq \mu\right) \tag{8}
\end{equation*}
$$

\]

is negative. This follows from the concavity of $r(\mu)$ which makes ( $r_{H} \neq H$ i $\left.r_{L} \neq \mu\right)<0$. Consequently, $n_{j}\left(q ; r_{H}\right)=q \neq H$. The function $C_{j}\left(n_{j} ; w\right)$ is continuous and convex since $s^{\infty}$, 0 . Thus, $n_{j}(q ; w)$ maps the interval $\left[r_{H} ; w_{j}\right]$ continuously onto the interval $[0 ; q \neq H]$. Since $0<N_{j}<q \neq H$ the continuity of $n_{j}$ guarantees that there exists $w^{\text {vad }}$ for which $n_{j}\left(q ; w^{\text {vad }}\right)=N_{j}$ and $w^{\text {axa }}<w_{j}$. And, of course, $œ_{j}\left(n_{j}\left(q ; w^{\text {axa }}\right) ; w^{\text {axa }}\right)=œ_{j}=0$. However, there is only one farm with an interior solution. To show this suppose in order to obtain a contradiction that farms $j$ and $k$ have interior solutions. The consequence of this is that $\left.s q\left(q ; \mu_{H} n_{j}\right) \neq \mathrm{L}+n_{j}\right]=$ $\left.s q\left(q_{k} i \mu_{H} n_{k}\right) \neq k+n_{k}\right]$. But this can not happen generically unless $q=q_{k}$ since $n_{i}$ is a non-linear function of q .

Since there can only be one farm with an interior solution, the marginal cost of farm ( $\mathrm{j}-1$ ) must be either strictly positive or strictly negative for all values of $n_{j i 1}$. Suppose that it is
 $s^{9}\left(q \neq \mu_{l}\right)\left(1_{i} \mu_{H} \neq \mu_{l}\right)$. However these inequalities imply that $q_{i_{1}}<q$ contrary to assumption. Hence the marginal cost of farm ( $j-1$ ) must be negative making $n_{j i}=q_{i 1} \neq H$ the desired level of employment. Since $@_{0} C_{i}\left(q \neq{ }_{H} ; w^{\text {axa }}\right)=@_{i} @_{q}<0$, farm i with $i<(j-1)$ also has a negative marginal cost. Furthermore, $w^{\text {ada }}<w_{i}$ when $i<j$ since $d w_{i}=d q=i s^{a}\left(1_{i} \mu_{H} \neq \mu\right)>0$ so that all farms with $\mathrm{i}<\mathrm{j}$ can arord to pay $\mathrm{w}^{\text {nad. }}$. Finally, it is possible to show using the same type of argumentation that farm i for $\mathrm{i}>\mathrm{j}$ has a positive marginal cost and prefers not to hire high ability labour. C onsequently, the proposed equilibrium satis ${ }^{-}$es the conditions of the de- nition and is, therefore, a competitive equilibrium.

Having de- ned a competitive equilibrium and demonstrated its existence for this model we now examine $N$ ash equilibria for wage setting behaviour in this model and establish conditions when the two equilibria are the same. For expositional purposes it is simpler to derive these results when there are only two farms.

At reservation wages, costs are minimized by employing only high ability workers, that is when $n_{i}=q \neq{ }_{H}$. However, this is sustainable as a Nash equilibrium only if the total quantity of high ability workers is greater than the demand by both farms. W hen this condition does not hold and there is not enough high ability labour for both farms, the equilibrium wage rate paid by at least one farm for high ability workers will have to be set above the reservation wage, $r\left(\mu_{H}\right)=r_{H}$.

The strategy spaces for each farm are the sets $S_{i}=f\left(n_{i} ; w_{i}\right) 2[0 ; q \neq H]-\left[r_{H} ; W_{i}\right] g$. Costs for each farm are de- ned in the following way using equation (4). Let $\mathfrak{a} g$ be the indicator function for the event contained in $\mathrm{fg}^{12}$. For $\mathrm{w}_{1}<\mathrm{w}_{2} \mathrm{de}^{-}$ne

[^7]\[

$$
\begin{equation*}
C_{2}\left(n_{1} ; w_{1} ; n_{2} ; w_{2}\right)=C_{2}\left(n_{2} ; w_{2}\right) \tag{9}
\end{equation*}
$$

\]

$$
\begin{equation*}
C_{1}\left(n_{1} ; w_{1} ; n_{2} ; w_{2}\right)=C_{1}\left(n_{1} ; w_{1}\right) \text { wa } n_{1}: n_{1} \cdot \min \left[q_{1} \neq H ; N ; n_{2}\left(q_{2} ; w_{2}\right)\right] g \tag{10}
\end{equation*}
$$

Likewise, for $w_{2}<w_{1}$

$$
\begin{equation*}
C_{1}\left(n_{1} ; w_{1} ; n_{2} ; w_{2}\right)=C_{1}\left(n_{1} ; w_{1}\right) \tag{11}
\end{equation*}
$$

$$
\begin{equation*}
C_{2}\left(n_{1} ; w_{1} ; n_{2} ; w_{2}\right)=C_{2}\left(n_{2} ; w_{2}\right) \operatorname{ma}_{2}: n_{2} \cdot \min \left[q_{2} \neq H ; N ; n_{1}\left(q_{1} ; w_{1}\right)\right] g \tag{12}
\end{equation*}
$$

And, - nally, for the case $\mathrm{w}_{1}=\mathrm{w}_{2}$

$$
\begin{equation*}
C_{1}\left(n_{1} ; w_{1} ; n_{2} ; w_{2}\right)=C_{1}\left(n_{1} ; w_{1}\right) \not \alpha_{1}-f n_{1}: n_{1} \cdot \min \left[q_{1} \neq H ; N=2\right] g \tag{13}
\end{equation*}
$$

$$
\begin{equation*}
C_{2}\left(n_{1} ; w_{1} ; n_{2} ; w_{2}\right)=C_{2}\left(n_{2} ; w_{2}\right) \propto x_{1}-n_{2}: n_{2} \cdot \min \left[q_{2} \neq H ; N=2\right] g \tag{14}
\end{equation*}
$$

In these de- nitions two points should be noted. First, it is assumed that the farm paying the higher wage has an unlimited choice over the entire labour market. W hereas, the farm paying the lower wage is constrained to hiring its labour from the pool of workers not wanted by the high wage farm. The indicator function in equation (10) will be used to impose this restriction in the de ${ }^{-}$nition of the $N$ ash equilibrium below. Secondly, when both farms attempt to pay a common wage it is assumed that workers allocate themselves randomly across the two farms making the supply to each farm equal to $\mathrm{N}=2$.

## De- nition 2

A $N$ ash equilibrium ( $n_{1}^{\frac{\alpha}{2}} ; w_{1}^{\alpha} ; n_{2}^{\frac{\alpha}{\alpha}} ; w_{2}^{\frac{\alpha}{2}}$ ) satis ${ }^{-}$es the following inequalities:

$$
\begin{array}{ll}
\biguplus_{1}\left(n_{1}^{\alpha} ; w_{1}^{\alpha} ; n_{2}^{\alpha} ; w_{2}^{\alpha}\right) \cdot e_{1}\left(n_{1} ; w_{1} ; n_{2}^{\alpha} ; w_{2}^{\alpha}\right) & 8\left(n_{1} ; w_{1}\right) 2 S_{1} \\
e_{2}\left(n_{1}^{\alpha} ; w_{1}^{\alpha} ; n_{2}^{\alpha} ; w_{2}^{\alpha}\right) \cdot e_{2}\left(n_{1}^{\alpha} ; w_{1}^{\alpha} ; n_{2} ; w_{2}\right) & 8\left(n_{2} ; w_{2}\right) 2 S_{2} \tag{16}
\end{array}
$$

In order to insure that the appropriate constraints are imposed in the minimization problem © $=C+\left(1 ;{ }^{a}\right) K$ is used where $K$ is a very large positive number.

A formal proof of existence of Nash equilibria does not appear to befeasible since $C_{i}\left(n_{i} ; w_{i} ; n_{j} ; w_{j}\right)$ is not necessarily convex in $\left(n_{i} ; w_{i}\right)$. However, these equilibria do exist; inequalities (15) and (16) do have solutions and they are generically unique, although they usually involve corner solutions. As will be shown later, they often generate the same labour allocation as the corresponding competitive equilibrium but wage rates that are usually lower.

Discussion continues within the framework of the two farm model. When $q_{q} \neq \psi_{H}<N$ and there is not enough labour for the largest farm the only possible competitive equilibrium has $@_{1}\left(n_{1} ; w^{\text {ax }}\right)=@_{1}=0$ and $n_{1}\left(q_{1} ; w^{\text {ax }}\right)=N$ together with $@_{2}\left(n_{2} ; w^{\text {ax }}\right)=@_{2}>0$, and $\mathrm{n}_{2}\left(\mathrm{o}_{\mathrm{R}} ; \mathrm{w}^{\text {max }}\right)=0$.

However, this is not a Nash equilibrium. Since $w^{\text {axa }}>W_{2}$ farm 1 only has to pay $\mathcal{W}_{2}$ to attract the N high ability workers; consequently, $\mathrm{W}_{2}$ is the N ash equilibrium wage rate.

Suppose now that there is more high ability labour available and that $\mathrm{a}_{\mathrm{L}} \neq \mathrm{H}_{\mathrm{H}}<\mathrm{N}<$ $\left(q_{1}+q_{R}\right) \neq \#_{H}$. A competitive equilibrium exists with $n_{1}\left(q_{1} ; w^{\text {axa }}\right)=q_{q_{1}} \neq H_{H}$ and $0<n_{2}\left(q_{2} ; w^{\text {ax }}\right)=$ N i $\mathrm{q}_{\mathrm{H}} \neq{ }_{H}<\mathrm{q}_{2} \neq \mathrm{H}$. Like the ${ }^{-}$rst example this competitive equilibrium is not sustainable as a N ash equilibrium either. $\mathrm{De}^{-} \mathrm{ne} \mathfrak{W}_{2}$ by the following condition:

$$
\begin{equation*}
C_{2}\left(n_{2}\left(o_{2} ; w_{2}\right) ; \hat{w}_{2}\right)=C_{2}\left(N ; q_{1} \neq H_{H} ; r_{H}\right) \tag{17}
\end{equation*}
$$

Equation (17) $\mathrm{de}^{-}$nes the wage rate at which farm 2 is indi ®erent between employing $n_{2}\left(q_{2} ; w_{2}\right)<q_{2} \neq H$ high ability workers at $w_{2}$ or employing the $N_{i} q_{1} \neq H_{H}$ residual high ability workers which farm 1 does not want to employ together with some low ability workers both at their respective reservation wage rates. Now $w_{2}$ is a Nash equilibrium. Here, the existence of a $N$ ash equilibrium is a relatively simple matter to demonstrate since the existence of $W_{2}$ is guaranteed by the continuity of $\mathrm{C}_{2}\left(\mathrm{n}_{2}\left(\mathrm{o}_{2} ; w\right) ; \mathrm{w}\right)$ ) on the interval $\left[\mathrm{r}_{\mathrm{H}} ; \mathrm{w}_{2}\right]$ and that is all that is required.

If $C_{1}\left(q_{1} \neq H_{H} ; \hat{W}_{2}\right)<C_{1}\left[N ; n_{2}\left(q_{2} ; \hat{W}_{2}\right) ; w_{2}\right]$ the high ability labour allocation is the same as it is in the competitive equilibrium, with farm 2 paying $r_{H}$ and farm 1 paying $w_{2}$ to high ability workers. This case is shown in Figures $1 A$ and $1 B$. If, on the other hand, $C_{1}\left(q_{1} \neq q_{H} ; W_{2}\right)>$ $\mathrm{C}_{1}\left[\mathrm{~N} ; \mathrm{n}_{2}\left(\mathrm{o}_{2} ; \hat{w}_{2}\right) ; \hat{w}_{2}\right]$ then farm 2 employs $\mathrm{n}_{2}\left(\mathrm{o}_{2} ; w_{2}\right)$ high ability workers at $w_{2}$ and farm 1 employs the residual, $\mathrm{N}_{\mathrm{i}} \mathrm{n}_{2}\left(\mathrm{o}_{\mathrm{R}} ; \hat{W}_{2}\right)$, at $\mathrm{r}_{\mathrm{H}}$.

Both farms have lower total labour costs than they did under $w^{\text {ax }}$ and neither farm has any incentive to change its wage rate. For the labour allocation indicated in Figure 1 farm 2 has no incentive to increase its employment of high quality workers by setting a wage above $r_{H}$ since that would raise its costs and paying a wage below $w_{2}$ would not attract high ability workers away from farm 1. Likewise, farm 1 has no incentive to deviate from $w_{2}$ since this is the lowest wage at which it only hires high ability workers and if it paid less than $\mathrm{w}_{2}$ it would end up with higher total costs employing N ; $\mathrm{n}_{2}\left(\mathrm{o}_{\mathrm{Z}} ; \mathrm{w}_{2}\right)$ high ability workers at $\mathrm{r}_{\mathrm{H}}$.

What is, perhaps, surprising is that high ability workers on one of the farms are forced to accept their reservation wages when identical workers on the other farm are earning $w_{2}$. However, they have no bargaining power since an o ®er of working for a wage less than whould be refused by farm 1 for the reasons outlined above. In this example the Nash equilibrium is characterized by involuntary underemployment. It should be noted, moreover, that the presence of involuntary underemployment is not restricted to the two farm example. Adding a third farm to this example still leaves some high ability workers underemployed but they get $\mathfrak{w}_{3}$ instead of $\mathrm{r}_{\mathrm{H}}$.

The characterization of Nash equilibrium when there are $K$ farms turns out to be quite complicated since there are a very large number of possible labour allocations. However, they all appear to have the properties of the example above.

Also, there is some wage dispersion with a $\backslash$ pivotal farm" paying a wage rate to high ability workers that is lower than the wage paid by farms specializing in high ability workers but above $\mathrm{r}_{\mathrm{H}}$.

So far the analysis has dealt with the case wheres is a convex function of total employment. $W$ hen $s$ is concave the results are quite di ®erent. First, there are no competitive equilibria. The concavity of $s$ makes the cost function concave. Consequently, minimizing it leads to a corner solution and the demand function for high ability labour for farm $\mathrm{i}, \mathrm{n}_{\mathrm{i}}(\mathrm{q} ; \mathrm{w})$ is a discontinuous step function of $w$ taking on the values 0 or $q \neq H$. Without the continuity of demand functions there is little hope for the existence of a market clearing wage rate. On the other hand Nash equilibria can exist and their characterization is a relatively simple task when $K=2$. Complexity grows exponentially with $K$ so the analysis is restricted to the case where there are only two farms.
$W$ hen $s$ is concave $W_{i}$ needs to be rede ${ }^{-}$ned. It now must satisfy

$$
\begin{equation*}
r_{L} q \neq \mu+s(q \neq L)=w_{i} q \neq H H+s(q \neq H) \tag{18}
\end{equation*}
$$

The meaning of $W_{i}$ is still the same: it is the highest wage at which farm i can employ high ability labour. It is also the case that under reasonable assumptions $d W_{i}=d q$ is negative. ${ }^{13}$ To

[^8]describe $N$ ash equilibria another de $^{-}$nition is needed.
\[

$$
\begin{equation*}
r_{L} q \neq \mu+s(q \neq L)=r_{L}\left(q ; M \mu_{H}\right) \neq \mu+w_{i}(M) M+s\left[\left(q ; M \mu_{H}\right) \neq L+M\right] \tag{19}
\end{equation*}
$$

\]

for any $M 2[0 ; q \neq H]$. Notice that $\mathcal{W}_{i}(M)$ has the following properties: $\mathcal{W}_{i}(M)$ is increasing in $M, W_{i}(M)<W_{i}, w_{i}\left(q \neq W_{H}\right)=W_{i}$, and $d W_{i}(M)=d q<0$

Even when there are two farms the characteristics of the equilibrium depend on the relationship between N and fqg . There are three possible cases to be considered.

Case 1: $0<N<q_{Z} \neq H_{H}<q_{1} \neq H_{H}<\left(q_{1}+q_{R}\right) \neq H_{H}$. Here there is only enough high ability labour for the small farm so the only possible Nash equilibrium is $n_{2}=N, w_{2}=w_{1}(N)$ and $\mathrm{n}_{1}=0 . \mathrm{w}_{1}(\mathrm{~N})$ is the wage rate that makes farm 1 indi ®erent between employing N high ability workers at this wage or employing no high ability labour at all. $\mathfrak{W}_{1}(N)<W_{2}(N)$ so farm 2 can a ®ord to pay this.

Case 2: $0<q_{R} \neq \#_{H}<N<q_{1} \neq H_{H}<\left(q_{1}+q_{R}\right) \neq H_{H}$. With more than enough high ability labour for farm 2 farm 1 can become a player. There are two possibilities. If $\sigma_{1}(N)<$ $w_{2}\left(q_{2} \neq H\right), n_{2}=q_{2} \neq H$ and $w_{2}=w_{1}(N)$ but $n_{1}=N_{i} \quad q_{R} \neq H$ and $w_{1}=r_{H}$. Farm 1 would like to employ all $N$ workers but it is prevented from doing this by farm 2 paying $W_{1}(N)$. It is left with the residual, $\mathrm{N}_{\mathrm{i}} \mathrm{q}_{\mathrm{Z}} \neq \#_{H}$, which it pays $\mathrm{r}_{\mathrm{H}}$. On the other hand, if $\mathrm{w}_{1}(\mathrm{~N})>\mathrm{w}_{2}\left(q_{2} \neq H\right)$, then $n_{2}=0 ; w_{2}=r_{H}, n_{1}=N$; and $w_{1}=W_{2}\left(q_{2} \neq H\right)$ : Farm 2 will be unable to prevent farm 1 from employing all of the high quality labour.

Case 3: $0<q_{2} \neq H_{H}<q_{1} \neq H_{H}<N<\left(q_{1}+q_{2}\right) \neq H_{H}$. Like case two there are two possibilities. If $W_{2}\left(q_{2} \neq H\right)<W_{1}\left(q_{1} \neq H\right), n_{1}=q_{1} \neq H, w_{1}=w_{2}\left(q_{2} \neq H\right), n_{2}=N i \quad q_{1} \neq H$, and $w_{2}=r_{H}$. Conversely, if $w_{2}\left(q_{2} \neq H\right)>w_{1}\left(q_{1} \neq H\right)$, then $n_{1}=N$ i $\left.q_{2} \neq H, w_{1}=r_{H}\right), n_{2}=q_{2} \neq H$, and $w_{2}=W_{1}\left(q_{1} \neq \psi_{1}\right)$

Like the previous example in which the supervision cost function was convex high ability workers on one of the farms are always involuntarily underemployed.

## 4 Comments and Discussion

This section starts with a brief summary of the properties of supervision cost models. While both competitive and $N$ ash equilibria were discussed it is the latter which deserve our attention. While there may be a fairly large number of employers bidding for labour even in the slack season it is quite clear that it is the farmers themselves who actually set wages. Since Nash behaviour is a description of this process it seems to be more relevant than an abstract unspeci- ed market equilibrating process. The Nash equilibria display some wage dispersion, although most of it is due to di ®erences in ability. There are only three possible wages rates: reservation wages
for low ability workers, premium wages for some high ability and a wage rate below that, but possibly above $r_{H}$, for the rest of the high ability workers. Premium wages have an interesting property; they are higher than $r_{L} \mu_{H} \neq \mathrm{L}$, which is what high ability workers would be paid if farmers priced high ability labour purely on arbitrage principles. However, numbers matter and competition between farmers causes the wage to rise above this level. The model also makes predictions about who works where, but that depends on the shape of the supervision cost function. However, there is one property that the model does not have and that is there is no involuntary unemployment.

But the generalization of the model mentioned in footnote 9 will permit this. One of the simplifying assumptions of the model was that individual piece-rate productivities depended only on individual ability and were not in ${ }^{\circ}$ uenced by the rate, $p$, paid per piece by farm $i$. W hen this assumption is relaxed not only does the model become more realistic; it also has equilibria that exhibit involuntary unemployment. To see the implications of this extension suppose, to make the argument as simple as possible, that there is only one type of labour. If, as before, farm i has $q$ tasks that have to be done and the number of tasks per day that a worker performs is ${ }^{1}\left(\mu ; p_{i}\right)$ which is assumed to be increasing in $p_{i}$ farm's $i$ objective is to minimize

$$
\begin{equation*}
\left.p_{i} q+s\left(q=1 \mu_{i} p_{i}\right)\right) \tag{20}
\end{equation*}
$$

subject to

$$
\begin{equation*}
p_{i}^{1}\left(\mu ; p_{i}\right), r \tag{21}
\end{equation*}
$$

where $q=1\left(\mu ; p_{i}\right)$ is the number of workers employed and $r$ is their common daily reservation wage rate. W hen there is surplus labour and farm i can select its piecerate it may want to pay a piecerate which gives workers a daily wage rate above the reservation wage rate. Workers who are not employed can not bid the piece-rate down because farmers would have to pay higher supervision costs and the piece-rate that they have chosen produces the right balance between wage and supervision costs.

Notice, however, in the absence of supervision costs the optimal strategy is to pay the equivalent of reservation wages by selecting $p_{i}$ which makes (28) hold with equality.

W hen there are two levels of ability and both types of worker produce more at higher piece-rates the piece-rate for low ability workers will be higher than that which generates the reservation daily wage rate leading to involuntary unemployment. Piecerates for high ability workers which are already higher than reservation piece-rates may be further increased because of this erect.

On this issue there is, however, an important qualifying restriction. It has been assumed that farms set piece rates but not the number of tasks that workers perform. If farmers have
control over both of these then all workers will be forced to accept their reservation wages and there will be no involuntary unemployment.

It was pointed out in the introduction that one of the one of the crucial issues in the determination of employment decisions by farmers was the amount of information they had on the workers that were available for hire. In this model farmers are assumed to know all the characteristics of all workers. This may not be a particularly appropriate assumption in all circumstances since the amount of information that farmers have concerning the abilities of individuals in the work force is one of the more important answered question in the - eld. W hile it is not clear what farmers actually know it is possible to model their actions when they have varying amounts of information on their workers and to confront the predictions about wages and employment that arise from these models with what is actually observed.

Consider what happens when ability can not be observed by farmers. W hen employers know the distribution of ability the consequences of assuming the unobservability of $\mu$ have been well understood since the classic Weiss (1980) contribution. Suppose, in contrast with the model of the previous section, that ability is continuously distributed over the interval [ $\mu_{\mathrm{H}} ; \mu_{H}$ ] with a probability density function $f(\mu)$. If there is no way to distinguish between individuals each farm must pay all of its workers the same wage. If $w_{i}$ is the wage o®ered by farm $i$, the average ability of the workforce for farm i is

$$
\begin{equation*}
\mu^{1}\left(w_{i}\right)=Z_{\mu L}^{Z_{r^{1}}^{1}\left(w_{i}\right)} \mu f(\mu) d \mu \tag{22}
\end{equation*}
$$

and is generated by a random sample drawn from those workers whose reservation wage is no greater than $w_{i}$. Farms choose $w_{i}$ by minimizing ${ }^{14}$

$$
\begin{equation*}
w_{i} q \neq{ }^{1}\left(w_{i}\right)+s\left(q \neq{ }^{1}\left(w_{i}\right)\right) \tag{23}
\end{equation*}
$$

W hen there are K farms there are K di ®erent wage rates. This is, in fact, a dominant strategy Nash equilibrium since each farm can select its cost minimizing wage rate independently of what other farms do. Farm i selects a random sample of workers from the set of workers who are willing to work for $w_{i}$. Since this sampling is random the distribution of ability is unchanged by farm ios action and all of the other farms are free to do what they wish. Like the previous case workers who do not get jobs can not bid the wage rate down so there is involuntary unemployment. In this case, however, supervision costs are not needed to obtain the result.

There are three features of this model which are di $\pm$ cult to reconcile with what is actually observed in terms of traditional wage payments systems. First, while there may be some

[^9]instances where wages are farm speci ${ }^{-}$c this appears not the norm in most casual labour markets. Secondly, there are no wage premiums paid to the high ability workers and it may be the case that the wage is too low to attract the highest quality workers. Finally, the model makes no prediction as to who gets work since the employer has no way to determine who the best workers are.

There is one feature of the model that might appear restrictive to some readers and that is the inelasticity of demand for labour. In all versions of the model $q$ is a constant and, in particular, it does not depend on the wage or piece-rate that farm i has to pay. This assumption is designed to capture the reality of the situation. In the slack season certain tasks have to be performed at certain times otherwise there will be no crop at harvest time. Farmers have very little ${ }^{\circ}$ exibility and the constancy of q re ${ }^{\circ}$ ects that.

Finally, the models of the previous section allowed individuals with di ®erent ability levels to earn di ®erent piece-rates. If piece-rate were constrained to be equal across ability levels then this would require $w_{H} \neq N_{L}=\mu_{H} \neq L$. Since $w_{L}=r_{L}$ this means that all farms pay the same wage rate to high ability workers. This is certainly not a property of any of the equilibria discussed above. When farmers have to pay supervision costs they have an incentive to pay premium wages to better workers and would pay di ßerential piecerates provided this was acceptable within the norms of village society.

## R eferences

[1] Basu, K. (1994) \E $\pm$ ciency Wage Theory with M onopolistic Landlords", Ch. 8 in Capital, I nvestment, and Development, Edited by K aushik Basu, M ukul M ajumdar, and Tapan Mitra, Blackwell, Oxford, U.K.
[2] Bliss, C. J. and Stern, N.H. (1978), \Productivity, Wages, and Nutrition I: The Theory". J ournal of Development Economics, Vol. 5, p330-362.
[3] Dasgupta, P., Ray, D. (1986), \Inequality As A Determinant of Malnutrition and Unemployment: Theory", Economic J ournal, Vol. 96, p. 1011-1034.
[4] Dr\&ze, J. P and Mukherjee, A. (1989), \Labour Contracts in Rural India: Theories and Evidence" Ch. 10 in Vol. 3 of The Balance Between Industry and Agriculture in Economic Development, edited by S. Chahravarty for the international Economic A ssociation. M acMillan Press, B asingstoke England.
[5] Dr\&ze, J. P and Sen, A.K. (1989), Hunger and Public A ction, Clarendon Press, Oxford, U.K.
[6] Foster, D., R osenzweig, M. (1993), \Information, Learning, and Wage Rates in Low-Income Rural Areas", J ournal of Human Resources, Vol. 28, p759-90.
[7] Foster, D., R osenzweig, M. (1996), \Comparative Advantage, information and The Allocation of Workers To Tasks: Evidence From An Agricultural Labour M arket", R eview of Economic Studies, Vol. 63, p 347-374.
[8] Frisvold, G. (1994), \Does Supervision Matter? Some Hypothesis Tests Using Indian Farm-Level Data", J ournal of Development Economics, Vol. 43, p 217-238.
[9] M as-Colell, A. Whinston, M.D. and Green, J.R. (1995), M icroeconomic Theory, Oxford University Press, Oxford England.
[10] M clntosh, J. (1984), \An Oligopsonistic M odel of Wage Determination in Agrarian Societies", Economic J ournal, Vol. 94, p 569-579.
[11] M ukherjee, A. R ay, D. (1992), I Wages and involuntary Unemployment in T he Slack Season of A Village Economy", J ournal of Development Economics, Vol. 37, p 227-264.
[12] Osmani, S.R. (1991), I Wage Determination in Rural Labour Markets", J ournal of Development Economics, Vol. 34, p 3-23.
[13] Stiglitz, J. E. (1976), \The E $\pm$ ciency Wage Hypothesis, Surplus Labour, and The Distribution of income in L.D.C.'s", Oxford Economic Papers. Vol. 28, p 185-207.
[14] Walker, W., Ryan, J. (1990), V illage and Household Economies in India's Semiarid Tropics, The J ohns Hopkins University Press, B altimore, USA.
[15] Weiss, A. (1980), \J ob Queues and Layo®s in Labor M arkets W ith F lexible Wages" , J ournal of Political Economy, Vol. 88, p 526-538.


[^0]:    ${ }^{1}$ From August 1, 1997 to December 31, 1997. An earlier version of this paper was presented at The E conomic Policy Research Unit of the University of Copenhagen Business School. Helpful comments of the participants are gratefully acknowledged. This research was supported by Concordia University and the Institute of Economics, University of Copenhagen. To them go my sincere thanks.

[^1]:    ${ }^{2}$ Dr $\ddagger$ ze and Sen (1989, p. 5) describe it in the following way: \People who possess no means of production excepting their own labour power, which they try to sell for a wage in order to an adequate income to buy enough food, are particularly vulnerable to changes in labour market conditions. A decline in wages vis- $\boldsymbol{a}$-vis food prices, or an increase in unemployment, can spell disaster for this class. .... The class of landless wage labourers has indeed recurrently produced famine victims in modern times. For example, in the Indian subcontinent, the majority of famine victims in this century and the last has come from this group.
    ${ }^{3} \mathrm{Dr}$ ize and Mukherjee (1989) provide an excellent survey of research on labour markets in the Indian subcontinent. They list a number of $\backslash$ stylized facts" for casual labour markets (page 246) the -rst four of which are: (1) Casual labour is the most important type of labour contract. It is generally hired on a day to day basis. (2) Search on the casual labour market is usually carried out on the previous night by employers who usually 'call' on the evening preceding the execution of the work. (3) The village labour market is largely closed; labour hiring across nieghbouring villages is rare. (4) Involuntary unemployment is common, particularly during the slack agricultural season. Less productive workers are especially vulnerable to forced leisure."
    ${ }^{4}$ These are important features of casual agricultural labour markets as the following description of the hiring process for daily rate labour by Walker and Ryan (1992, p. 110) shows. \In Sholapur, employers look for more $\mathrm{e} \pm$ cient and reliable workers ${ }^{-}$rst and o ®er premiums. Workers who have to approach prospective employers generally accept discounted wages. ...E mployees are willing to work for almost all employers and they regularly change their employer throughout the year." See Foster and Rosenzwieg (1993, Table 7 p. 777) for empirical support for this proposition. A paper that considers the problems that arise with worker heterogeneity is Mclntosh (1984). Unfortunately the model developed there does not consider the possibility of involuntaury unemployment or underemployment.

[^2]:    ${ }^{5}$ For the ${ }^{-}$rst type see Stiglitz (1976), Bliss and Stern (1978) and Dasgupta and Ray (1986) and Basu (1994). The asymmetric information model which has the same structural properties as the nutrition based models was - rst developed by Weiss (1980) and is discussed in some detail in chapter 13 of M as-Colell et al (1995).
    ${ }^{6}$ Foster and Rosenzwieg (1996, p. 368) in their study of P hilippine labour markets show that less than half the variation in worker productivity is explained by what farmer employers know.

[^3]:    ${ }^{7}$ See M clntosh (1984, p. 572) for examples.

[^4]:    ${ }^{8}$ This representation of consumer behaviour is widely used in the literature on incentives. See, for example, Mas-Colell et al (1995, p. 450), which is the source of much of the notation employed in this paper.

[^5]:    ${ }^{9}$ For the moment it will be assumed that these piece-rate productivities do not involve incentive erects; that is, they do not depend on the piece-rate paid to workers. De ${ }^{-}$ning the piece-rate productivities as ${ }^{1}$ ( $\mu ; p_{i}$ ) raises interesting possibilities since it is not unreasonable to assume that workers would produce at a greater rate if the rate per unit was higher. The implications of this are discussed in the next section.
    ${ }^{10}$ In section (4) the properties of this model are compared to those of model in which $\mu$ is unobservable.

[^6]:    ${ }^{11}$ The proof for the case when $N<q_{1} \neq H$ is straightforward but requires di Berent notation.

[^7]:    ${ }^{12}$ a ( $\mathrm{x}: \mathrm{x}, 0$ ) takes the value 1 if $\mathrm{x}, 0$ and the value 0 otherwise.

[^8]:    ${ }^{13}$ If $3 / 4$ ) is the elasticity of $s$ with respect to total employment then $3 / 4$ ) constant or a decreasing function is su $\pm$ cient to generate this result.

[^9]:    ${ }^{14} \mathrm{As}$ W iess (1980 p. 530) notes, this is an approximation since no account is taken of the fact that farm $\mathrm{i}^{0} \mathrm{~s}$ average productivity di ${ }^{\text {®ers }}$ from ${ }_{\mu}^{1}\left(w_{i}\right)$ by a sampling error.

