Discussion Papers
Department of Economics
University of Copenhagen

No. 98-11

Discrete Public Goods with Incomplete Information

Flavio M. Menezes, Paulo K. Monteiro,
and Akram Temimi

Studiestræde 6, DK-1455 Copenhagen K., Denmark
Tel.: +45 35 32 30 82 – Fax: +45 35 32 30 00
http://www.econ.ku.dk

ISSN: 1601-2461 (online) ISSN 0902-6452 (print)

Discrete Public Goods with Incomplete Information[¤]

Flavio M. Menezes
Department of Economics
Australian National University
Canberra ACT 0200
Australia

Paulo K. Monteiro
University of Copenhagen and IMPA

Akram Temimi
Department of Finance
University of Alabama
Tuscaloosa AL 35487
USA

June 1998

Abstract

We investigate a simultaneous discrete public good provision game with incomplete information. To use the terminology of Admati and Perry (1991), we consider both contribution and subscription games. In the former, contributions are not refunded if the project is not completed, while in the latter they are.

[¤]We would like to thank John Conley, Charles Kahn, Paul Pecorino and Harris Schlesinger for helpful discussions. Monteiro gratefully acknowledges the financial support of CNPq.

In the presence of complete information about individuals' valuations for the public good, the di®erence between the equilibrium outcomes of a subscription game and a contribution game is not signi¯cant. However, there is both casual evidence from the fund-raising literature and experimental evidence that subscription games are \superior ", i.e., a refund increases the chance of providing the good given that it is e±cient to do so. Our analysis shows that this is indeed the case in the presence of incomplete information.

We compute a symmetric equilibrium for the subscription game and show that it is not necessarily e±cient. This ine±ciency stems from the di±culties arising in coordinating to overcome the free-rider problem in the presence of incomplete information. Although it is well known that informational disparities impose limits on the e±ciency of outcomes, the novel feature of our analysis is to explicitly model the resulting trade-o® | when deciding how much to contribute towards the public good | between increasing the likelihood of provision and creating incentives for free-riding by the other player. Moreover, we show that for the contribution game, \contributing zero" is the only equilibrium for a given range of the ¯xed cost of provision and for a family of distributions.

Keywords: Public goods, incomplete information, continuous distribution.
JEL: D8, H4.

1 Introduction

The literature on private provision of public goods can be divided into two broad categories.[1] The ¯rst branch of the literature focuses on the provision of continuous public goods. Papers include Warr (1982, 1983), Bergstrom,

[1] In this paper, we focus on private provision. There is an extensive mechanism design literature that treats public provision. Papers in this literature develop e±cient mechanisms for the provision of public goods. These mechanisms are in general complex and require a central authority to implement them and hence would be best decsribed as mechanisms for public provision of public goods. Gradstein (1994) examines e±cient mechanisms for discrete public goods. Other papers that treat public provision include Maskin (1977), d'Aspremont and Varet (1979, 1982), Palfrey and Srivastava (1986). Cornelli (1996) examines an optimal mechanism for a monopolist which produces an excludable good that has large ¯xed costs.

Cornes and Sandler (1984), Blume and Varian (1986), Andreoni (1988), Gradstein, Nitzan and Slutsky (1994) and others. A standard result in this literature is that public goods are underprovided by voluntary contributions due to free riding behavior.[2] One might conjecture that the government can solve this underprovision problem by providing some of the good and financing it by imposing taxes on contributors. However, Warr (1982) and Roberts (1984), in two influential papers, show that government contributions result in a dollar to dollar reduction in private contributions if the tax on contributors does not change the set of contributing individuals. Bergstrom, Blume and Varian (1986) show that the crowding out effect is only partial if one allows for the taxes that pay for the government contribution to be also collected from non contributors. Hence the literature suggests that underprovision is a robust conclusion if the public good level is endogenously determined by voluntary contributions.[3]

The second strand of the literature focuses on discrete public goods where a fixed level of a public good is provided if enough contributions are collected to cover its cost c. Otherwise, the good is not provided. Typical examples include building a bridge, a library of a certain size, public radio fund raising to finance a certain program. In all of these examples, if enough money is raised to cover the cost of the public good, then the good is provided, otherwise the good is not provided.

Palfrey and Rosenthal (1984) developed the first modern treatment of private provision of discrete public goods. More specifically they analyze contribution and subscription games — to use the terminology of Admati and Perry (1991) — for a discrete public good under complete information where players' strategies are restricted to either contribute zero or an exogenous positive amount. In a contribution game, contributions are not refunded if the sum of the contributions does not cover the cost of the public good c,

[2] The free riding problem becomes worse in the presence of incomplete information. Gradstein (1992) considers a dynamic model of private provision for a continuous public good with incomplete information with the restriction that players either contribute zero or an exogenous positive amount. He concludes that in addition to the standard underprovision results, inefficiency occurs because of a delay in contributions. This inefficiency does not disappear as the population becomes large.

[3] Andreoni (1997) analyzes the role of seed money in the presence of nonconvexities in the production of the public good. He shows that a small amount of seed money can generate a substantial amount of voluntary contributions.

while in a subscription game players get their money back if the project is not completed. An example of a subscription game can be given as follows. The Wisconsin Governor has recently pledged $27 million in state bonds to ¯nance a new $72 million basketball arena on the condition that the rest of the money be raised by private donations. (Andreoni, 1997). That is, the Governor will provide $27 million as long as the remaining $45 million is raised, otherwise the o®er is cancelled. Other examples can be found in the fund-raising literature. Examples of contribution games include public radio and TV fund-raising e®orts where contributors do not get their money back if the program is not provided. Other examples of contributions games are situations where contributions take the form of physical labor, in this case volunteers cannot recover their e®ort if the project is not completed.

In contrast with the standard underprovision result for continuous public goods, Palfrey and Rosenthal (1984) obtain the startling conclusion that e±cient provision of a discrete public good is a robust equilibrium of both contribution and subscription games. E±cient provision is rather intuitive since the coordination problem becomes easier when the level of public good is ¯xed and common knowledge to all the players. Bagnoli and Lipman (1989) extend Palfrey and Rosenthal model by allowing individuals to make continuous contributions. They show that the set of undominated perfect equilibrium outcomes of the subscription game is not only e±cient but coincides with the core of the economy. They also show that with a dynamic version of the subscription game, it is possible to obtain e±cient outcomes even if the level of the public good is not binary as long as the number of units of the public good is countable.[4] It follows that the general conclusion from the literature is that private provision of a discrete public good is in general e±cient in the presence of complete information. Moreover, e±cient provision is obtained for both subscription and contribution games. However, there is both casual evidence from the fund-raising literature of the superiority of subscription games and experimental evidence from Bagnoli and McKee (1991) and Cadsby and Maynes (1997) that a refund increases the chance of providing the good. More speci¯cally, Cadsby and Maynes

[4] Admati and Perry (1991) consider a dynamic private provision model for a discrete public good. They analyze contribution and subscription games in a Rubinstein type framework with complete information where players alternate in making contributions to the public good. They show that the equilibrium of the subscription game is e±cient while the equilibrium of the contribution game is ine±cient.

consider a discrete public goods experiment. They provide experimental evidence showing that provision is encouraged in a subscription game vis-a-vis a contribution game. They also provide evidence that a high c discourages provision in the contribution game but not in the subscription game. Our results will con¯rm these ¯ndings.

An important question that has not been addressed in the literature on private provision of discrete public goods is to what extent these results generalize to a model where players have incomplete information about other players' valuation for the public good. Moreover the important issue of how to measure departure from e±ciency in this incomplete information framework has not been explored.[5]

In this paper, we address these issues. More speci¯cally, we analyze contribution games and subscription games for a discrete public good in the presence of incomplete information about preferences. Our model also relaxes the binary contribution restriction imposed in the literature.[6] While there are important instances where binary contributions are relevant due to transaction costs, in general players can give any amount of money they desire (alumni donation, donations to a library, etc. .). Moreover, in a continuous contributions framework, individuals make contributions that best match their preferences as opposed to a discrete contribution model. Cadsby and Maynes (1997) provide experimental evidence showing that allowing continuous rather than binary \all-or-nothing " contributions facilitates provision.

Within this framework, we ¯rst show that we no longer obtain e±ciency when we introduce incomplete information. This ine±ciency stems from the di±culties arising in coordinating to overcome the free-rider problem in the presence of incomplete information. Although this type of ine±ciency is well known in economic theory, we go beyond that by explicitly characterizing the trade-o® involved when information is incomplete. We explicitly show the

[5]It is important to note that there are several papers that introduce incomplete information in one form or another but do not address the above questions. Palfrey and Rosenthal (1988) analyze the provision of a discrete public good when individuals have incomplete information about the degree of altruism of other players under the restriction that players are only allowed to make discrete contributions. Nitzan and Romano (1990) show that when the cost of the discrete public good is uncertain to the players, then e±ciency is no longer obtained.

[6]Bagnoli and Lipman (1989) also considers continuous contributions. However their model is with complete information.

trade-off between free riding behavior and the probability of provision of the public good. Note that the individual problem of an individual deciding how much to pledge is to forecast the lowest pledge one can make, given that it is below one's value, and still have the good being provided. This is similar but not the same as the problem of an individual deciding how much to bid in a first-price auction who faces a trade-off between profits conditional on winning and the probability of winning.

Moreover, we measure how inefficient the provision is and we are able to do that since we are able to explicitly solve for the contribution functions of the players. We also show the significant difference in the equilibrium outcome between the contribution game and the subscription game in contrast to the similarity obtained in the absence of incomplete information. This reconciles the theory with the evidence from the experimental literature of the superiority of subscription games over contribution games. In contrast to the mechanism design literature, our approach of characterizing equilibrium behavior in existing mechanisms enables us to provide results which can be tested in a laboratory environment or by using empirical data.

2 The Model

Before we present the general model, consider the following simple example with discrete distributions. Two players, 1 and 2, have the following valuations for a threshold public good:

$$v_i = \begin{cases} 0; & \text{with probability } \frac{1}{2} \\ 1; & \text{with probability } \frac{1}{2} \end{cases} ; i = 1; 2$$

Valuations are private information. That is, each individual knows her own valuation but only the distribution of her opponent's valuation. The public good will be provided if the threshold $1 < c < 2$ is met through private contributions. The money contributed by both players is returned in the event that the sum of the pledges is less than c. If the sum of contributions is greater or equal than c, the good is provided but any resources above c are not returned to individuals.[7] This game has a symmetric Bayesian Nash

[7] This assumption is standard in the literature on discrete public goods. One can think of the the excess as accruing as "profits" to the provider of the public good (Nitzan and

equilibrium where an individual pledges \$0 if her value is 0 and \$$\frac{c}{2}$ if her value is 1.[8] Note that in this equilibrium, the sum of the pledges never exceeds c, individuals never pledge more than their values, and all equilibria are efficient, as the good is always provided whenever it is socially efficient to do so.

Consider now a contribution game where the contributions are not returned to individuals when the threshold c is not met. Recall that $1 < c < 2$: We examine this game from the perspective of Player 1. (As the game is symmetric, it does not really matter who we choose.) Player 1 will of course contribute zero if her value is zero. The question is what will she contribute if her value is equal to one. We can readily verify that the equilibrium of the subscription game where each player contributes $\frac{c}{2}$ if her value is equal to one and zero if her value is equal to zero does not emerge in this game. If Player 2 follows this strategy, player 1 will make negative profits by following it as well as her profits equal $\frac{1}{2} - \frac{c}{2} < 0$ in this case. It follows (by symmetry) that a player cannot contribute $\frac{c}{2}$ or more and make positive profits. (The maximum any player could possibly contribute and make nonnegative profits is clearly $\frac{1}{2} \le \frac{c}{2}$.) Therefore, the only pure strategy equilibrium of this game is for both players to contribute zero no matter what their values are. That is, the good is never provided in equilibrium.[9] In this paper, we investigate to what extent the intuition emerging from the above example generalizes to

Romano 1990).

[8] "Always pledge zero" is another symmetric equilibrium. This equilibrium is known in the literature as the "strong free-riding equilibrium". However this equilibrium involves weakly dominated strategies. As with the game of complete information, this game has infinitely many asymmetric equilibria both efficient and inefficient. In what follows we will focus on symmetric equilibria since the game is completely symmetric.

[9] In the presence of complete information about individual's valuations for the public good, the difference between the set of equilibria in a contribution game and the set of equilibria in a subscription game is not significant. Consider a two-player provision game, where both individuals value for the public good are equal to one. The public good will be provided if the threshold $1 < c < 2$ is met through private contributions — any contributions above c are not returned to individuals. The subscription game has a continuum of equilibria, which includes the pairs $(0;0)$, $(\epsilon;\delta)$ — where ϵ and δ are such that $c - \epsilon > 1$ and $c - \delta > 1$ — and all pairs of pledges $(b_1;b_2)$ such that $b_1 + b_2 = c$; $b_1 \le 1$ and $b_2 \le 1$: Note that the set of Nash equilibria of the contribution game coincides with the set of Nash equilibria of the subscription games with the exception of the pairs of pledges $(\epsilon;\delta)$ as described above. However the $(\epsilon;\delta)$ are weak in the sense that a player's deviation from the equilibrium does not affect any player's payoff.

a situation where each bidder believes her opponent's value can be any of a continuum.

We now present the formal model. We consider an independent-private values model where each individual i, $i = 1; 2$, knows her value for a certain discrete public good but only the distribution of her opponent's value. That is, each bidder i knows that her opponent has a value v_j; $j \neq i$; that is drawn from a distribution $F(:)$ with positive density $f(:)$ on the support $[0; 1]$. We denote the cost of providing the public good by c. Whatever the method to elicit donations is, the good is only provided if at least \$c are donated. If more than \$c are donated, the additional money is not returned to the contributors. We consider the case when c belongs to the interval [1,2], that is, when a single player cannot provide the good by herself.

2.1 The Subscription Game

First we consider a subscription game where both players make pledges that are only materialized if the threshold is met. Player 1's expected profits given that she has a value v_1, makes a pledge b_1 and that Player 2 follows a "pledging" strategy $b_2(v_2)$ is

$$\pi_1(v_1; b_1; b_2(v_2)) = (v_1 - b_1)\chi_{b_1 + b_2(v_2) \geq c}: \tag{1}$$

For the moment we will assume $b_2(v_2)$ is nondecreasing and differentiable in the relevant interval. Taking the inverse, we can rewrite the above expression as

$$\pi_1(v_1; b_1; b_2(v_2)) = (v_1 - b_1) \Pr\left(v_2 \geq b_2^{-1}(c - b_1)\right): \tag{2}$$

Player 1 chooses b_1 in order to maximize her expected profits. The first-order condition for an interior solution is given by

$$(v_1 - b_1)\left(f\left(b_2^{-1}(c - b_1)\right)\left(b_2^{-1}\right)'(c - b_1)\right) = 1 - F\left(b_2^{-1}(c - b_1)\right): \tag{3}$$

We can now use the fact that we are looking for a symmetric equilibrium where $b_1(:) = b_2(:) = b(:)$ yielding the following differential equation

$$(v - b(v))\left(f\left(b^{-1}(c - b(v))\right)\left(b^{-1}\right)'(c - b(v))\right) = 1 - F\left(b^{-1}(c - b(v))\right)$$

This differential equation does not admit an explicit solution for general distribution functions. For the case of a uniform distribution on the interval [0,1], the solution is:

$$b(v) = \frac{2c - 1}{6} + \frac{v}{2} \tag{4}$$

Note, however, that in equilibrium a player may not follow $b(v)$ for any v in [0,1] as this may lead to pledging more than her value or more than the cost of the public good. Hence we need to impose the following boundary conditions

$$b(v) \leq c \tag{b1}$$

$$b(v) \leq v \tag{b2}$$

$$b(v) \geq 0 \tag{b3}$$

It turns out that (b1) and (b3) are not binding since $c > 1$. Condition (b2) is binding as $b(v) > v$ for $v < \frac{2c-1}{3}$.

We now provide the intuition for equation (4). Recall that in a symmetric equilibrium of a first-price sealed-bid auction — where the object is awarded to the individual with the highest bid — an individual bids in such a way to outbid the opponent with the highest value. That is, conditional on her value being the highest, her bid is equal to the expected value of the first-order statistics of her opponents.

In the subscription game, however, the good is provided to both players if their contributions add up to the cost of provision. Thus, the problem becomes one of forecasting the lowest pledge one can make, given that it is below one's value, and still have the good being provided. Thus, (4) states that Player 1, for example, pledges the equivalent to the expected value of player 2 being lower than her own, conditional on the interval $[\frac{2c-1}{3}; 1]$, that is, on the interval where pledges are less than or equal to the values and add up to c.

Notice the distinction between the solution of the subscription game and the solution of the first-price auction. In the latter, if a bidder's value is not the highest, then in any symmetric equilibrium with increasing bids she will lose the object and therefore, in equilibrium, she does not have to consider what she would do if her value is not the highest one. In the subscription game this is not the case. If her value is the lowest of the two, she may still obtain the object and, thus, following (4) guarantees that this is the minimum pledge so that the object is provided and the players are sharing the cost in such way as to equalize their marginal contributions. This property of the equilibrium pledging strategies is very distinct from the result for first-price auctions and it captures the nature of the trade-off between the probability of the public good being provided and the free-riding behavior.

The next proposition characterizes a symmetric Bayesian equilibrium of the subscription game.

Proposition 1 The following are symmetric equilibrium pledging strategies for the subscription game with two players whose values are uniformly distributed on $[0;1]$ and $c \geq 1$:

$$b^*(v) = \begin{cases} \frac{2c-1}{6} + \frac{v}{2}; & \text{if } \frac{2c-1}{3} \leq v \leq 1 \\ 0; & \text{otherwise} \end{cases}$$

Proof: Given that Player 2 is following the proposed equilibrium pledging strategy, we have to find the best response of player 1. We first show that for $\frac{2c-1}{3} \leq v \leq 1$; $b_1(v) = \frac{2c-1}{6} + \frac{v}{2}$ is a best response to $b^*(v_2)$. Player 1's expected profit, if the bid is b; is given by

$$\phi(b) = (v - b)\Pr(b + b^*(v_2) \geq c):$$

To find the maximum of ϕ first note that if $0 \leq c - b < b^*\left(\frac{2c-1}{3}\right) = \frac{2c-1}{3}$ then $\phi(b) = (v-b)\left(1 - \frac{2c-1}{3}\right) \leq \left(v - \frac{c+1}{3}\right)\left(1 - \frac{2c-1}{3}\right) = \phi\left(\frac{c+1}{3}\right):$ Note that if $c - b \geq b^*(1)$ then $\phi(b) = 0$: Let us consider now $c - b^*(1) < b < c - b^*\left(\frac{2c-1}{3}\right):$ Then we have

$$\begin{aligned}\phi(b) &= (v-b)\Pr\left(\left\{v_2 \geq \frac{2c-1}{3}; v_2 \geq \frac{4c+1}{3} - 2b\right\}\right) = \\ &\quad (v-b)\left(1 - \max\left\{\frac{2c-1}{3}; \frac{4c+1}{3} - 2b\right\}\right):\end{aligned}$$

There are two cases to consider:

a) $\frac{2c-1}{3} > \frac{4c+1}{3} - 2b$

In this case $\phi(b) = (v-b)\left(1 - \frac{2c-1}{3}\right) < \phi\left(\frac{c+1}{3}\right):$

b) $\frac{2c-1}{3} \le \frac{4c+1}{3} - 2b$

In this case $\phi(b) = (v-b)\left(1 - \frac{4c+1}{3} + 2b\right)$. This quadratic function has a unique maximum at $b^* = \frac{2c-1}{6} + \frac{v}{2}$. Thus b^* is the optimal bid if $b^* \in \left[c - b(1), c - b\left(\frac{2c-1}{3}\right)\right]$ and $\frac{2c-1}{3} \le \frac{4c+1}{3} - 2b^*$. The last inequality is valid for all $v \in [0,1]$. The first inequality is valid if $v \in [\frac{2c-1}{3}, 1]$.

Thus $b(v) = \frac{2c-1}{6} + \frac{v}{2}$ is the best response to $b^*(v_2)$ if $v \in [\frac{2c-1}{3}, 1]$. To finish let us find the best response for $v \in [0, \frac{2c-1}{3})$. It is clear from the reasoning in (b) above that the maximum of ϕ is not interior. Thus we need only to compare $\phi(c - b^*(1)) = \phi\left(\frac{2c-1}{3}\right) = \left(v - \frac{2c-1}{3}\right)\left(1 - \frac{4c+1}{3} + 2\frac{2c-1}{3}\right) = 0$ and $\phi\left(c - b^*\left(\frac{2c-1}{3}\right)\right) = \phi\left(\frac{c+1}{3}\right) = \left(v - \frac{c+1}{3}\right)\left(1 - \frac{2c-1}{3}\right) < 0$. Thus if $v \in [0, \frac{2c-1}{3})$ the maximum expected utility is zero. Hence since bidding zero and bidding $\frac{2c-1}{3}$ gives the same expected profit we finished the proof that $b(\cdot)$ is an equilibrium. ■

Note that this proposition shows an equilibrium bidding strategy that is increasing and differentiable in the relevant range and, therefore, our previous analysis is justified.

We now provide a simple example to illustrate the analysis above.

Example 2 For simplicity, assume that the realizations of v_1 and v_2 are both equal to $\frac{1}{2}$ and that $c = 1$. The predicted symmetric equilibrium according to Proposition 1 is for both players to pledge $b^*(v) = \frac{1}{6} + \frac{v}{2} = \frac{5}{12}$. Since $c = 1 \ge \frac{5}{12} + \frac{5}{12}$, the good is not provided in this equilibrium, although it is efficient to do so once we know individuals' valuations.

Let us consider the game from Player 1's perspective and find what is his best response to Player 2 playing the proposed equilibrium strategy

$$b^*(v_2) = \begin{cases} \frac{1}{6} + \frac{v_2}{2}; & \text{if } \frac{1}{3} \le v_2 \le 1 \\ 0; & \text{otherwise} \end{cases}.$$

Player 1's expected profit is given by:

$$\pi_1\left(\frac{1}{2}; b_1; b^*(v_2)\right) = \left(\frac{1}{2} - b_1\right) \chi_{b_1 + b^*(v_2) \ge 1}$$

Maximizing with respect to b_1 yields $b_1 = \frac{5}{12}$ as expected.

The above example illustrates that a subscription game may not be ex-post efficient, i.e., after we learn individuals' valuations. This is not really surprising given some of the results listed in Section 2. In any case, perhaps ex-post efficiency is too strong a requirement. An alternative measure of efficiency is given below. It indicates the probability that the good will be provided whenever is efficient to do so.

Proposition 3 $\Pr\left(b^*(v_1) + b^*(v_2) \geq c \mid v_1 + v_2 \geq c\right) = \frac{2}{3}$

Proof. We need to compute

$$\frac{\Pr\left(\{(v_1; v_2) ; b^*(v_1) + b^*(v_2) \geq c \text{ and } v_1 + v_2 \geq c\}\right)}{\Pr\left(\{(v_1; v_2) ; v_1 + v_2 \geq c\}\right)}$$

Note that

$$\Pr\left(\{(v_1; v_2) ; b^*(v_1) + b^*(v_2) \geq c \text{ and } v_1 + v_2 \geq c\}\right) =$$
$$\frac{1}{2}\left(\frac{4-2c}{3}\right)^2 + \left(\frac{2c-1}{3} - (c-1)\right)\left(1 - \frac{c+1}{3}\right) = \frac{4}{3} - \frac{4}{3}c + \frac{1}{3}c^2 = \frac{1}{3}(c-2)^2 :$$

Since $\Pr\left(\{(v_1; v_2) ; v_1 + v_2 \geq c\}\right) = \frac{1}{2}(2-c)^2$ we finish the proof. ■

Notice that an increase in c affects this probability by two opposing effects. An increase in c causes $b(:)$ to increase but the interval for which $b(:)$ is different from zero shrinks. The random variable $v_1 + v_2$ has a triangular distribution (as the sum of two random variables that are uniformly distributed). Its density peaks at $c = 1$. Beyond this point, these two opposing effects completely offset each other as shown above. Therefore, a grant towards reducing the cost of provision has no effect on the probability of provision as it is perfectly offset by individual's behavior in equilibrium.

2.2 The Contribution Game

In this subsection we consider a game where both players make contributions that are not refunded if the threshold is not met. Let's write Player 1's expected profits given that she has a value v_1, makes a pledge b_1 and that Player 2 follows a "pledging" strategy $b_2(v_2)$

$$\pi_1(v_1; b_1; b_2(v_2)) = v_1 \chi_{b_1 + b_2(v_2) \geq c} - b_1 \tag{5}$$

For the moment we will assume $b_2(v_2)$ is nondecreasing and differentiable in the relevant interval. Taking the inverse we can rewrite the above expression as

$$\pi_1(v_1; b_1; b_2(v_2)) = v_1 \Pr\left(v_2 \geq b_2^{-1}(c - b_1)\right) - b_1 \tag{6}$$

For the uniform [0,1] distribution, (6) becomes

$$v_1 \left(1 - b_2^{-1}(c - b_1)\right) - b_1 \tag{7}$$

Player 1 chooses b_1 in order to maximize her expected profits. The first-order condition for an interior solution is given by

$$v_1 \left(b_2^{-1}\right)'(c - b_1) = 1 \tag{8}$$

That is,

$$\frac{1}{b_2'\left(b_2^{-1}(c - b_1)\right)} = \frac{1}{v_1} \tag{9}$$

As we are looking for symmetric equilibrium, we set $b_1(.) = b_2(.) = b(.)$.

$$b'\left(b^{-1}(c - b(v))\right) = v \tag{10}$$

Contrarily to the subscription game case, we do not know whether this differential equation has a solution. Even if the differential equation has a solution, it may not be a solution to the problem given the relevant boundary conditions. However, the next proposition shows that the strategy "contributing zero" for both players is the only equilibrium for a family of distributions and for $1 < c < 2$.

Proposition 3 Suppose $1 < c < 2$. If the distribution function satisfy either $F(x) \geq x$; $x \in (0; 1)$ or is concave then the best response to $b_2(v_2)$ such that $b_2(v_2) \leq v_2$ is $b_1(v_1) = 0$.

Proof. Define $\pi(b) = vP(b_2(v_2) \ge c - b) - b$. If $b > v$ then $\pi(b) \le v - b < 0$. Thus to maximize π necessarily $b \le v$. Suppose $b > 0$. If $b < c - 1$ then $\pi(b) = v \cdot 0 - b = -b < 0$. Suppose now $c - 1 \le b \le v$. We have $\pi(c - 1) = -b$ and $\pi(v) \le 0$. Now:

$$\pi(b) \le vP(v_2 \ge c - b) - b = v(1 - F(c - b)) - b =: g(b).$$

First suppose F is concave. Then g is convex. Thus we conclude that

$$\max_{c-1 \le b \le v} g(b) = \max\{g(c-1), g(v)\} < 0.$$

If $F(x) \ge x$ the reasoning is even simpler:

$$g(b) \le v(1 - (c - b)) - b = v - c + (v - 1)b.$$

■

Note that the above family of distributions includes many known distributions such as the uniform and the exponential distributions. Proposition 3 extends the intuition given in the example from the beginning of this section to continuous distributions. That is, the coordination problem is so severe in a contribution game that "contributing zero" is the only equilibrium.

3 Conclusion

A standard result in the literature on discrete public goods with complete and perfect information is that private provision is efficient. In this paper we tested the robustness of this result to the information structure. We developed a model where individuals have incomplete information about other individuals' valuations for the public good. Within this framework, we showed that the standard efficiency result in the literature on voluntary provision of discrete public goods no longer holds. Moreover, we explicitly analyzed the trade-off between free riding behavior and the probability of provision of the public good. We measure how inefficient the provision is and we are able to do that since we are able to explicitly solve for the contribution functions of the players.

Our analysis showed that there is a significant difference in the equilibrium outcome between subscription and contribution games. This is in contrast with the literature with complete information where the two games lead to essentially the same outcomes and it reconciles the theory with the evidence from the experimental literature of the superiority of subscription games over contribution games.

There are several issues that deserve additional research. Perhaps the two most important issues are the investigation of individual behavior when valuations for the public good are correlated and whether or not it is possible to establish an optimal mechanism as one that maximizes some notion of ex-post e±ciency like the one we use in this paper.

References

[1] Admati, A. R. and M. Perry, 1991, \Joint Projects without Commitment," Review of Economic Studies 58, 259-276.

[2] Andreoni, J., 1997, \Toward a Theory of Charitable Fundraising," Forthcoming in Journal of Political Economy.

[3] Andreoni, J., 1988, \Privately Provided Public Goods in a Large Economy: The Limits of Altruism," Journal of Public Economics 35, 57-73.

[4] Bagnoli, M and M. McKee, 1991, \Voluntary Contribution Game: E±-cient Private Provision of Public goods," Economic Inquiry 29, 351-356.

[5] Bagnoli, M. and B. L. Lipman, 1989, \Provision of Public Goods: Fully Implementing the Core through Private Contributions," Review of Economic Studies 56, 583-602.

[6] Bergstrom, T, L. Blume and H. Varian, 1986, \On the Private Provision of Public Goods," Journal of Public Economics 29, 25-49.

[7] Cadsby, C. B. and E. Maynes, 1997, \Voluntary Provision of Public Goods with Continuous Contributions: Experimental Evidence," Forthcoming in Journal of Public Economics.

[8] Cornes, R. and T. Sandler, 1984, \Easy Riders, Joint Production, and Public Goods," Economic Journal 94, 580-598.

[9] Cornelli, F., 1996, \Optimal Selling Procedures with Fixed Costs," Journal of Economic Theory 71, 1-30.

[10] d'Aspremont, C. and L. A. Gerard-Varet, 1982, \Bayesian Incentive Compatible Beliefs," Journal of Mathematical Economics 10(1), 83-103.

[11] d'Aspremont, C. and L. A. Gerard-Varet, 1979, \Incentives and Incomplete Information," Journal of Public Economics 11(1), 25-45.

[12] Gradstein, M., 1994, \E±cient Provision of a Discrete Public Good," International Economic Review 35(4), 877-897.

[13] Gradstein, M., 1992, \Time Dynamics and Incomplete Information in the Private Provision of Public Goods," Journal of Political Economy 100(3), 1236-1244.

[14] Gradstein, M., S. Nitzan and S. Slutsky, 1994,\Neutrality and the Private Provision of Public Goods with Incomplete Information," Economics Letters 46, 69-75.

[15] Harsanyi, J., 1973, \Oddness of the Number of Equilibrium Points: A New Proof," International Journal of Game Theory 2, 235-250.

[16] Nitzan, S. and R. Romano, 1990, \Private Provision of a Discrete Public Good with Uncertain Cost," Journal of Public Economics 42, 357-370.

[17] Palfrey, T. and H. Rosenthal, 1988, \Private Incentives in Social Dilemmas: The E®ects of Incomplete Information and Altruism," Journal of Public Economics 35, 309-332.

[18] Palfrey, T. and H. Rosenthal, 1984, \Participation and the Provision of Discrete Public Goods: A Strategic Analysis," Journal of Public Economics 24, 171-193.

[19] Palfrey, T. and S. Srivastava., (1986), \Nash Implementation Using Undominated Strategies," Mimeo. Carnegie Mellon University.

[20] Roberts, R., (1984). \A Positive Model of Private Charity and Wealth Transfers," Journal of Political Economy 92, 136-148.

[21] Warr, P. G., 1983, \Pareto Optimal Redistribution and Private Charity," Journal of Public Economics 19, 131-138.

[22] Warr, P. G., 1982, \The Private Provision of a Public Good Independent of the Distribution of Income," Economics Letters 13, 207-211.