DISCUSSION PAPERS
Department of Economics
University of Copenhagen

00-07

On the Welfare Economic Foundations
of Health Status Measures

Jens Leth Hougaard
Hans Keiding

ON THE WELFARE ECONOMIC FOUNDATIONS
OF HEALTH STATUS MEASURES

Jens Leth Hougaard
Copenhagen Business School

Hans Keiding
University of Copenhagen*

January 2000

Abstract

Measures of health status, such as e.g. the QALY (Quality Adjusted Life Years) measure, have been proposed as a tool in the economic assessment of new medical technologies, and its possible foundations in individual utility theory has been discussed in the literature. However, the problems of aggregation or interpersonal utility comparison inherent in the application of such measures has went largely unnoticed. In the present work, we consider a general equilibrium model of a society, where different aspects of health are identified as Lancasterian characteristics. In this model, we consider the welfare theoretical basis for evaluation of changes of allocation from a Pareto optimum, and in particular, we investigate conditions on the economy under which the individuals will have the same marginal rates of substitution between characteristics, which is a precondition for a meaningful measurement of health status. It turns out that this equality will obtain only under very restrictive conditions of separability in characteristics production.

Keywords: QALYs, consumers' characteristics, general equilibrium.

JEL classification: D6, I0.

---

* Communication with: Hans Keiding, Institute of Economics, University of Copenhagen, Studiestraede 6, DK-1455 Copenhagen K, Denmark.

E-mail: Hans.Keiding@econ.ku.dk

1. Introduction

In recent years, the growth of expenditure on health in most countries has led to an increased concern for setting the right priorities and for methods of evaluation of new medical technologies. Unfortunately, most of the output produced in the health care sector of society is not directly marketed, and moreover, the final goal of the sector's production, health, largely evades precise measurement, thus making investment analysis along traditional lines quite complicated. It is not surprising, therefore, that attempts to measure health status or changes in health status has attracted much attention.

There is by now a very extensive literature on health status measurements (see e.g. Sintonen (1981), Mooney (1986)) and in particular on QALYs (Quality Adjusted Life Years) (see e.g. Broome (1993), Baldwin, Godfrey, Propper (1990), and the survey in Mooney (1994)). The basic idea behind such measurements is as follows: A list of fundamental aspects or characteristics of health is indentified (Sintonen (1986), works with a total of 11 such characteristics – ability to sleep, walk, see, hear etc.) and different states of perfection within these characteristics are ranked by a group of individuals. Afterwards, the characteristics are ranked according to the importance assigned to them by the group, and positions for each characteristic as well as the weights between characteristics may now be used to assign numbers to all conceivable states of health.

Other measurements, as e.g. the QALY measurements, introduced in the 1970s, cf. e.g. Torrance (1986), employ particular techniques to set up scales and assign values (as the so-called 'time trade-off' and 'standard-gamble' methods), but the goal is largely the same, constructing a utility scale for health states. Several authors (cf.e.g. Torrance and Feeny (1989), Loomes and McKenzie (1989), Bleichrodt, Wakker and Johannesson (1997), Bleichrodt and Quiggin (1997)) discuss foundational problems of QALY measurement, where the QALY is viewed as a representation of an individual preference relation defined on lotteries over health states. We shall not be concerned with these aspects of QALY measurement on the individual level. What concerns us the application of QALY measures in aggregate welfare considerations.

Measuring health status or quality of life is important for evaluating society's gain by using new medical technologies. A change in treatment involves a change in ressource use and a change in outcome. If the outcome can be measured in e.g. QALYs, then alternative technologies may be compared by their QALY scores, that is the amount of dollars given up per QALY gained. To be able to perform analyses of this type was the main reason for introducing QALYs, and is is currently being implemented to the extent that tables exist for QALY gain per dollar in a large number of new technologies.

Below we outline briefly the role of health measures in cost-benefit analyses as applied to medical treatments. Assume that society's preferences with respect to consumption of commodities and health is described by a (differentiable) social welfare function

$$S(u_1(h_1, x_1), \ldots, u_m(h_m, x_m))$$

depending on the consumers' utility from consuming marketed commodities $x_1, \ldots, x_l$ as well as their health, which is described by several (say L) parameters or characteristics; these characteristics cannot be bought directly in the market but may be achievable by

individual and collective effort. In a very simple setup, the characteristics may be physical and mental health (as in SF-36, cf. Ware (1995)), or it may be a much more detailed description of definite aspects of health (moving, hearing, speaking, seeing, working etc., cf. Sintonen (1981)).

As usually, society's gain from a small displacement in health and commodity consumption can be assessed by

$$dS = \sum_{i=1}^{m} S_i' \sum_{j=1}^{L} u_{ij}' dh_{ij} + \sum_{i=1}^{m} S_i' \sum_{j=1}^{l} u_{ij}' dx_{ij};$$

Assuming that
- the commodities are bought at prices p_j,
- the income distribution in society is optimal for given health,

the second term reduces to $Kp \cdot \sum_{i=1}^{m} dx_i$ (this is obtained by substituting from the consumers' first order conditions $\lambda_i p_j = u_{ij}'$ (for utility maximzation at given prices and income) and using that the Lagrangian multiplier λ_i is consumer i's marginal utility of income; the assumption of optimal income distribution then gives us that $S_i' \lambda_i$ is independent of i, so that it equals some $K > 0$).

If we want to perform a similar reduction of the health part of the expression similarly, then we need similar assumptions; since there is no market for health, we cannot state them in the same way, but we can of course state what we need, that is
- the vectors of marginal utilities of each health parameter are all proportional, i.e.
$$(u_{ih}')_{h=1}^{L} = \mu_i v, \text{ all } i$$
for some vector v, and
- society's initial distribution of any health parameter is optimal (so that $S_{ih}' u_{ih}' = S_{i'h}' u_{i'h}'$ for all consumers i and i', and for $h = 1, \ldots, L$).

On this assumptions, the criterion for desirability of the displacement becomes

$$dS = Mv \cdot \sum_{i=1}^{m} dh_i + Kp \cdot \sum_{i=1}^{m} dx_i,$$

meaning that the evaluation reduces to assessing pairs $(v \cdot \sum_{i=1}^{m} dh_i, p \cdot \sum_{i=1}^{m} dx_i)$ according to a linear criterion with unknown weights M and K; comparison of such pairs may then be performed using the ratio of the two coordinates, the celebrated cost-effectiveness ratio of health economic appraisals. Thus, we have a theoretical foundation of cost-effectiveness analysis in medicine (cf. Gold, 1996) , and as a by-product, we get a QALY-like measure Q of health status displacements, namely by $Q(dh) = v \cdot h$.

However, this project clearly hinges on its assumptions; those pertaining to social optimality of distribution of income and health are to some extent inherent in the approach; if welfare could be improved by redistribution already, there are conceptual difficulties in measuring society's gain from some new technology. But the assumption of proportional

vectors of marginal utility of health is an assumption which is open to debate; the present paper investigates whether it can reasonably be expected to be fulfilled.

In order to access this problem, we consider a general equilibrium model with health characteristics. We take as a point of departure that health states can be measured, even in an objective way, at least when the description of the health state is sufficiently precise (thus, a health characteristic should not be thought of as a particular aspect of health). However, health is inherently individual and cannot be transferred. Consumers buy commodities, which can be transferred, and use them to produce health. This approach to health consumption using Lancaster's (1971) consumer characteristics has already been used in health economics, e.g. in the intertemporal health consumption model of Grossman (1972), and the connection between QALYs and Lancaster's characteristics was noticed by Williams (1985) and Culyer (1990).

In the present paper we consider the general problem of whether the agents in an economy where consumers buy goods which they transform to characteristics, will agree on the marginal rates of substitutions, not of goods but of characteristics. In terms of health status measures, we look for conditions on the economy which will give us that in equilibrium, when each consumer has bought the best commodity bundle with a view to its transformation to consumption *and* health, the marginal rates of substitution between different types of health will be the same for all consumers. If this is not the case, there is no a priori meaningful measure of overall health.

It might be noticed at this point that it is not a priori excluded that equality in equilibrium of individual marginal rates of substitution for characteristics might hold even without trade in characteristics. Indeed, the factor price equalization theorem of international trade theory provides a similar case, where such an equality does obtain; and in the paper we show that under conditions which are in a certain sense analogous, we get a similar result. However, the main result is negative, since the conditions under which equality of marginal rates of substitution between characteristics must be considered as too restrictive to be widely satisfied in practice.

The paper is structured as follows: In section 2, we introduce the general model to be used in the sequel. In section 3, we present some simple versions of the economies of 2, and we show that in these models, the individual marginal rates of substitutions between characteristics are different, not only in one or in some Pareto optimal allocation, but essentially in all of them. Also, some results of a more general character are presented here. In section 4, we turn the problem around and show that a rather strong condition which is clearly sufficient for equality of individual marginal rates of substitution no matter what the preference relations of the consumers would be, is also necessary. This condition essentially amounts to separate production of characteristics - each characteristic is obtained from commodity inputs in a way which is completely independent of what is produced of the other characteristics. The concluding section 5 contains some discussion of the results; in particular, we return to the initial problem of health measurement and interpret our results in this context. The proof of Theorem 2 in section 2, which is rather long, has been put in an appendix.

2. A general model with consumer characteristics

We consider an economy $\mathcal{E}$ with l commodities. Consumers $i \in \{1, \ldots, m\}$ buy commodity bundles $x_i \in \mathbf{R}_+^l$ in the market and transform them into bundles ξ_i of L different consumers' characteristics which are used for final consumption.

There are m consumers, each having the set $\mathbf{R}_+^L$ as feasible consumption set. The consumer i has a preference relation $\succsim_i$ on $\mathbf{R}_+^L$. of consumption bundles; to obtain the characteristics bundle, the consumer uses a household technology $T_i \subset \mathbf{R}_+^l \times \mathbf{R}_+^L$ transforming commodity bundles to characteristics bundles; a household production is a pair (x_i, ξ_i) with x_i a commodity bundle and ξ_i a characteristics bundle.

We shall treat commodity production (as distinct from household production of commodities) in a very summary way: There is a given set $Y \subset \mathbf{R}_+^l$ of feasible aggregate commodity bundles in the economy $\mathcal{E}$.

We collect some assumptions on the economy $\mathcal{E}$:

Assumption 1. *The economy $\mathcal{E}$ satisfies the following assumptions:*
  (i) *For each i, $\succsim_i$ is continuous (the sets $\{\xi_i' \mid \xi_i' \succsim_i \xi_i\}$ and $\{\xi_i' \mid \xi_i \succsim_i \xi_i'\}$ are closed for each ξ_i), weakly monotonic ($\xi_i' > \xi_i$ implies that $\xi_i' \succ_i \xi_i$), and convex (for each ξ_i, the set $\{\xi_i' \mid \xi_i' \succsim_i \xi_i\}$ is convex).*
  (ii) *For each i, T_i is closed, convex, monotonic in the sense that for all $(x_i, \xi_i) \in T_i$, if $x_i' > x_i$ ($x_i' \geq x_i$, $x_i' \neq x_i$), then there is $\xi_i' > \xi_i$ ($\xi_i' \geq \xi_i$, $\xi_i' \neq \xi_i$) with $(x_i', \xi_i') \in T_i$, and relevant: if $(x_i, \xi_i) \in T_i$ and $\xi_i > 0$ ($\xi_i \geq 0$, $\xi_i \neq 0$), then $x_i > 0$ ($x_i \geq 0$, $x_i \neq 0$). Finally, T_i satisfies free disposal: if $(x_i, \xi_i) \in T_i$, and if $x_i' \geq x_i$, $\xi_i' \leq \xi_i$, then $(x_i', \xi_i') \in T_i$.*
  (iii) *The set Y satisfies $Y - \mathbf{R}_+^l \subset Y$ (free disposal) and contains 0 in its interior.*

It may be noticed at this point that since we have posed no restriction on l and L and the nature of the characteristics, it may well be the case that some or all of the commodities correspond to characteristics in a 1-1 way. Thus, there is no loss of generality in having consumers' preferences defined only on characteristics bundles rather than on goods-and-characteristics bundles.

An allocation in the economy $\mathcal{E}$ is a pair (x, ξ), where $x = (x_i)_{i=1}^m$ is a commodity allocation and $\xi = (\xi_i)_{i=1}^m$ a characteristics allocation, that is for each consumer i, a characteristics bundle ξ_i. An allocation (x, ξ) is individually feasible if

$$x_i \in \mathbf{R}_+^l, \ (x_i, \xi_i) \in T_i \text{ for each } i,$$

and aggregate feasible if $\sum_{i=1}^m x_i \in Y$; it is feasible if it is both individually and aggregate feasible.

An allocation (x, ξ) in $\mathcal{E}$ is Pareto optimal if it is feasible and there is no other feasible allocation (x', ξ') such that $u_i(\xi_i') \geq u_i(\xi_i)$ for all i with at least one strict inequality.

An *equilibrium* in $\mathcal{E}$ is an array (x, ξ, p), where (x, ξ) is a feasible allocation and $p \in \mathbf{R}_+^l \setminus \{0\}$ is a price system, such that
  (i) for each consumer i, ξ_i maximizes u_i on

$$\{\xi_i' \mid \exists x_i' : (x_i', \xi_i') \in T_i, \ p \cdot x_i' \leq p \cdot x_i\},$$

(ii) $\sum_{i=1}^{m} x_i$ maximizes $p \cdot y$ on Y.

The equilibrium concept used is standard (corresponding to the notion of a compensated equilibrium in Debreu (1959)) except of course for the particular feature of our model, that preferences are defined on characteristics and not on commodity bundles.

The classical first theorem of welfare economics holds for our model. We state it below without proof, since it follows easily using the standard method of proof (cf. e.g. Green, MasColell and Whinston, 1995).

Theorem 1. *Let $\mathcal{E}$ be an economy satisfying Assumption 1, and let $((x, \xi), p)$ be an equilibrium in $\mathcal{E}$. Then (x, ξ) is Pareto optimal.*

As will be shown below, also the second theorem of welfare economics holds in $\mathcal{E}$ (provided that it satisfies Assumption 1). This means – as usual – that prices (of commodities) may be used for decentralizing decisions in $\mathcal{E}$, and that values at such decentralizing prices may be used to decide on the desirability of small displacements of the allocation from a Pareto optimal situation.

However, this may not be good enough for the purposes at hand. Assume for example that a new technique for producing characteristics (in the case of health characteristics, this may be a new treatment or a new type of medicine) must be evaluated; what is known then is the input-output relationships of the new techniques but not the changes in overall allocation of commodities brought about by the introduction of the technique. For an a priori evaluation of the welfare effects of the new technique the shadow prices of characteristics are called for. We therefore introduce another equilibrium concept where these shadow prices are made explicit.

An *extended equilibrium* is an array (x, ξ, p, q), where (x, ξ) is a feasible allocation p a price system on commodities, and $q = (q_i)_{i=1}^{m}$ a family of (individual) price vectors $q_i \in \mathbf{R}_+^L$ on characteristics, such that

(i') for each consumer i, ξ_i maximizes u_i on

$$\{\xi_i'' \mid \exists (x_i', \xi_i') \in T_i, \ q_i \cdot \xi_i'' \leq p \cdot x_i + (q_i \cdot \xi_i' - p \cdot x_i')\},$$

(ii') $\sum_{i=1}^{m} x_i$ maximizes $p \cdot y$ on Y.

Condition (i') may need some comment: In the equilibrium, the consumer is assumed to be endowed with an income for buying commodities in the market which exactly makes it possible to buy x_i. Performing a household production means that the bundle of value $p \cdot x_i$ is given up but that a characteristics bundle of shadow value $q_i \cdot \xi_i$ is obtained. The individual optimization condition tells us that the consumer cannot improve by any household production, in the sense that the shadow profit obtained will not give the consumer a higher shadow value for consumption of characteristics.

We note that an extended equilibrium is an equilibrium (so that the notion 'extended' refers only to the number of variables which are made explicit in the equilibrium): If (x, ξ, p, q) is an extended equilibrium, then (ii) is trivially satisfied; to check (i), let $\xi_i' \succ_i \xi_i$, with $(x_i', \xi_i') \in T_i$ for some x_i'. From (i') we get that

$$q_i \cdot \xi_i' > p \cdot x_i + (q_i \cdot \xi' - p \cdot x_i')$$

6

or $p \cdot x_i' > p \cdot x_i$ which is (i).

Clearly, the allocation (x, ξ) belonging to an extended equilibrium is Pareto optimal (given Assumption 1); what is more important is that with our new concept, we get a version of the second theorem of welfare theory with explicit shadow prices on characteristics.

Theorem 2. *Let $\mathcal{E}$ be an economy satisfying Assumption 1, and let (x, ξ) be a Pareto optimal allocation in $\mathcal{E}$, where $\xi_i \in \mathbf{R}_{++}^L$ for all i. Then there is price system p on commodities and a system of individual characteristics prices $q = (q_i)_{i=1}^m$ such that*

(1) (x, ξ, p, q) is an extended price equilibrium;

(2) if (x', ξ') is an individually feasible allocation with

$$\sum_{i=1}^m q_i \cdot \xi_i' > \sum_{i=1}^m q_i \cdot \xi_i,$$

then (x', ξ') is a potential Pareto improvement in the sense that there is an individually feasible allocation (x'', ξ'') with $\sum_{i=1}^m p \cdot x_i'' = \sum_{i=1}^m p \cdot x_i'$ and such that $\xi_i'' \succ_i \xi_i$ for all i.

The proof of Theorem 2, which uses the standard method of embedding the given economy in an ordinary economy with a larger commodity space, is given in the appendix. The notion of a potential Pareto improvement is adapted to the present model: incomes for commodity purchases (evaluated at the equilibrium prices p) can be redistributed in such a way that everyone can buy a commodity bundle which is as good as (and for some consumers strictly better than) the bundles prescibed by (x, ξ).

The result of theorem 2 provides the basis for using characteristics or shadow prices in welfare considerations: Changes in overall welfare may be evaluated by computing the aggregate value of the characteristics consumption. However, this welfare measure uses individual shadow prices, and as such it is hardly of any practical use. In order to be operational, a welfare measure of individual quantity displacements should use shadow prices which are independent of individuals. Unless the shadow prices q_i turn out to be equal or proportional in at least fairly many Pareto optimal allocations, measuring welfare changes by changes in shadow values has no theoretical foundation. The question therefore is whether such equality obtains reasonably often, and this will be investigated in the following sections.

3. Equality of individual characteristics prices almost never obtains

In this section we show that the equality of characteristics prices for different individuals, which as we argued, is a precondition for their use in welfare comparisons, is an event which in general economies happen very rarely.

We start by presenting the ideas in a simple example.

Example 1. The economy $\mathcal{E}$ below has two consumers; there is only one commodity, $l = 1$. Consumers transform amounts of this commodity into bundles of two different

characteristics (so $L = 2$) using a a household technology T, assumed to be the same for both consumers and given by $(x, (\xi_1, \xi_2)) \in T$ if

$$(\xi_1^2 + \xi_2^2)^{1/2} \leq x.$$

Thus, the characteristics are made under conditions of joint production, and there are constant returns to scale.

Define the utility functions by

$$u_1(\xi_{11}, \xi_{12}) = \xi_{11}\xi_{12}^4$$
$$u_2(\xi_{21}, \xi_{22}) = \xi_{21}^4\xi_{22};$$

there is an initial endowment of 1 unit of the commodity, i.e. $Y = \{y \in \mathbf{R} \mid y \leq 1\}$.

We obtain all Pareto optimal allocations (x, ξ) by dividing the initial endowment between consumers and letting them produce characteristics from this endowment in such a way as to maximize utility. Thus, let $x_1 \in [0, 1]$; then ξ_1 and ξ_2 satisfy

$$\xi_1^2 + \xi_2^2 = x_1^2, \ \xi_1^2 + \xi_2^2 = (1 - x_1)^2,$$

and adding the individual maximization conditions, we get that

$$\xi_1 = \left(\frac{x_1}{\sqrt{5}}, \frac{2x_1}{\sqrt{5}} \right), \ \xi_2 = \left(\frac{2(1 - x_1)}{\sqrt{5}}, \frac{(1 - x_1)}{\sqrt{5}} \right).$$

We may now find the marginal rate of substitution between characteristics for consumer 1 as

$$\mathrm{MRS}_1 = \frac{\partial u_1}{\partial \xi_1} \Big/ \frac{\partial u_1}{\partial \xi_2} = \frac{1}{4}\frac{\xi_{12}}{\xi_{11}} = \frac{1}{2},$$

and similarly, the marginal rate of substitution for consumer 2 is found as $\mathrm{MRS}_2 = 2$. Thus, the marginal rates of substition are independent of the parameter x_1 and the same in all the Pareto optimal allocations (except for $x_1 = 0$ or $x_1 = 1$, where any individual price vector is a support for the preferred set of the individual getting nothing). In particular, the two marginal rates of substitutions are different, no matter which Pareto optimal allocation we consider.

The results obtained in the example are, as may be seen immediately, not connected with the choice of dimensions or the particular numerical values of the parameters chosen. Indeed, the following result generalizes the insights of the example to all economies where all consumers have the same constant-returns-to-scale technology, and where there are at least two consumers differing in their (homothetical) preferences for characteristics bundles:

Theorem 3. *Let $\mathcal{E}$ be an economy satisfying Assumption 1 and such that*
(i) $T_1 = \ldots = T_m = T$ (all household technologies are identical), and T is a convex cone in $\mathbf{R}_+^l \times \mathbf{R}_+^L$ (constant returns to scale),

(ii) for each i, the preference relation $\succsim_i$ is homothetical in the sense that if $\xi_i \succsim_i \xi_i'$ then $\lambda\xi_i \succsim_i \lambda\xi_i'$ for all $\lambda > 0$.

Assume that

(1) For each $p \in \mathbf{R}_+^l$, the set

$$T(p) = \{\xi \in \mathbf{R}_+^L \mid \exists x \in \mathbf{R}_+^l, p \cdot x = 1, (x, \xi) \in T\}$$

is strictly convex in $\mathbf{R}_+^L$,

(2) there are two consumers, say 1 and 2, such that for all $\xi \in \mathbf{R}_{++}^L$, the sets

$$\{q \in \mathbf{R}_+^L \mid \xi' \succsim_1 \xi \Rightarrow q \cdot \xi' \geq q \cdot \xi\} \text{ and } \{q \in \mathbf{R}_+^L \mid \xi' \succsim_2 \xi \Rightarrow q \cdot \xi' \geq q \cdot \xi\}$$

of supports of the preferred sets at ξ for consumers 1 and 2 are disjoint.

Then there are no extended equilibria (x, ξ, p, q) of $\mathcal{E}$ with $p \cdot x_i > 0$ for all i, such that $q_1 = \cdots = q_m$.

Proof: Assume to the contrary that there is an extended equilibrium in $\mathcal{E}$ with $q_1 = \cdots = q_m = q$. Consider the two consumers 1 and 2 satisfying the conditions (2) of the theorem. Then ξ_i is maximal for $\succsim_i$ on the set of characteristics bundles which can be produced in T from some input x_i' satisfying $p \cdot x_i' \leq p \cdot x_i$, $i = 1, 2$. Clearly, we must have that $\xi_1 \neq \lambda\xi_2$ for all $\lambda > 0$ by the condition (2).

Let $\lambda_i = 1/p \cdot x_i$, and consider the bundles $\lambda_i\xi_i$ for $i = 1, 2$. By homotheticity of $\succsim_i$ and constant returns to scale of T, we have that $\lambda_i\xi_i$ is maximal for $\succsim_i$ on $T(p)$, $i = 1, 2$. We conclude that q supports $T(p)$ at $\lambda_i\xi_i$ for $i = 1, 2$, and consequently, by the strict convexity of $T(p)$ there is some $\xi'' \in T(p)$ with

$$q \cdot \xi'' > q \cdot (\lambda_1\xi_1) = q \cdot (\lambda_2\xi_2),$$

contradicting that $\lambda_i\xi_i$ is maximal for $\succsim_i$ on $T(p)$, $i = 1, 2$. We conclude that $q_1 \neq q_2$, and this proves the theorem. $\qquad\square$

The assumptions of the theorem may perhaps look restrictive; however, they all express ordinary properties of well-behaved economies (constant returns to scale, homotheticity of preferences) and they are usually satisfied in textbook examples. Thus, the main message to be obtained from the theorem is that equality of characteristics prices does not obtain , unless the consumers are all identical, in which case the problem was trivial from the outset.

It is rather obvious that the similar examples of economies where all equilibria have different individual characteristics prices can be obtained with identical preferences but different technologies. Even if both household technologies and preferences are identical among consumers we may still get such results, provided that we drop the assumption of constant returns to scale in the household technology:

Example 2. Suppose that the common household technology T is such that $(x, \xi) \in T$ if

$$(t\xi_1^2 + \xi_2^2)^{1/2} \leq t, \ t \leq x,$$

and that both consumers have the utility function u with

$$u(\xi_1, \xi_2) = \xi_1 \xi_2.$$

As before, we find all Pareto optimal allocations by choosing $x_1 \in [0, 1]$ and maximizing u under each of the constraints

$$x_1 \xi_{11}^2 + \xi_{12}^2 = x_1^2, \ (1 - x_1)\xi_{21}^2 + \xi_{22}^2 = (1 - x_1)^2.$$

This gives us the characteristics bundles

$$\xi_1 = \left(\sqrt{\frac{x_1}{2}}, \frac{x_1}{\sqrt{2}} \right), \ \xi_2 = \left(\sqrt{\frac{(1 - x_1)}{2}}, \frac{1 - x_1}{\sqrt{2}} \right).$$

Once more we have inequality of marginal rates of substitution, which with the particular utility function are $\mathrm{MRS}_1 = \sqrt{x_1}$ and $\mathrm{MRS}_2 = \sqrt{1 - x_1}$, respectively, differing except in the case $x_1 = (1 - x_1)$ where the identical consumers are treated equally.

Summing up, we have shown that in well-behaved economies, consumers end up with different shadow prices on non-marketed goods in all except in a few, exceptional cases. It is fairly clear that the results generalize to less well-behaved economies, as long as the common household technology features some kind of strict convexity, and some consumers differ in their characteristics preferences. Thus, it is to be expected that many or most equilibria will be such that characteristics prices of at least two consumers differ, thus making the use of a common characteristics price in cost-benefit analysis illusory.

4. Conditions for common evaluation of characteristics

In the previous sections, our main emphasis has been on nonequality of marginal rates of substitution for characteristics. In doing so, a particular condition on the household technology emerged, as stated in (1) of Theorem 3. We show in the present section that this condition is crucial: if it fails, then the ensuing economy will exhibit equality of characteristics prices in its extended equilibria.

Throughout this section, we assume constant returns to scale in the household production, which is assumed identical for all consumers. As an intuitive starting point, we notice that if household production is separable in the sense that production of each characteristics is carried out separately, with a technology that does not depend on the level of production of the other characteristics, then each characteristic will have a unique shadow price depending only on the commodity prices, so that characteristics prices of individual consumers are indeed the same.

In this section, we show that separability in the above sense of the common household technology is both a necessary and a sufficient condition for equality of individual characteristics prices. We start with an intermediate result which uses the sets

$$T(p) = \{\xi \in \mathbf{R}_+^L \mid \exists x \in \mathbf{R}_+^l, p \cdot x = 1, (x, \xi) \in T\},$$

for $p \in \mathbf{R}_{++}^l$ introduced in the previous section.

Theorem 4. *Let $T \subset \mathbf{R}_+^l \times \mathbf{R}_+^L$ be a household technology satisfying constant returns to scale, and let $\mathbf{E}_T$ be the class of economies satisfying Assumption 1 such that all consumers have the household technology T. Then the following conditions are equivalent:*

(i) for all economies $\mathcal{E} \in \mathbf{E}_T$, all extended equilibria (x, ξ, p, q) of $\mathcal{E}$ with $\xi_i \neq 0$ for all i satisfy $q_1 = \cdots = q_m$,

(ii) for all $p \in \mathbf{R}_{++}^l$, the set $T(p)$ is the intersection with $\mathbf{R}_+^L$ of a half-space in $\mathbf{R}^L$.

Proof: (ii) $\Rightarrow$ (i): Let $\mathcal{E} \in \mathbf{E}_T$, and let (x, ξ, p, q) be an extended equilibrium. Then by the condition (i') of extended equilibria, for each i the characteristics bundle ξ_i is maximal for $\succsim_i$ on the set of all

$$\{\xi_i'' \mid q_i \cdot \xi_i'' \leq p \cdot x_i + \max_{(x_i', \xi_i') \in T_i} [q_i \cdot \xi_i' - p \cdot x_i']\} = \{\xi_i'' \mid q_i \cdot \xi_i'' \leq q_i \cdot \xi_i\};$$

clearly, this set contains

$$T_i(p) = \{\xi_i' \mid \exists x_i' : p \cdot x_i' \leq p \cdot x_i, (x_i', \xi_i') \in T_i\},$$

and ξ_i belongs to the boundary of $T_i(p)$. Now, by constant returns to scale of T, we have that $T_i(p) = \lambda_i T(p)$, where $\lambda_i = p \cdot x_i$, and using (ii) we get that

$$T(p) = \lambda^{-1}\{\xi_i'' \mid q_i \cdot \xi_i'' \leq q_i \cdot \xi_i\}.$$

Since the left hand side is independent of i, so is the right hand side, and we conclude that $q_1 = \cdots = q_m$.

(i) $\Rightarrow$ (ii): Let $C = \{c^1, \ldots, c^i, \ldots\} \subset \mathbf{R}_{++}^L$ be a countable dense subset of $\mathbf{R}_+^L$, and let $\hat{p} \in \mathbf{R}_{++}^l$ be arbitrary. For each natural number m, we define an economy $\mathcal{E}_m \in \mathbf{E}_T$ with m consumers $i = 0, 1, \ldots, m-1$ as follows: Consumer 0 has preferences such that $\xi' \succsim_0 \xi$ if and only if $\sum_{h=1}^L \xi_h' \geq \sum_{h=1}^L \xi_h$. For $i = 1, \ldots, m-1$, let the preferences of consumer i be described by the utility function

$$u_i(\xi) = \min_{h=1,\ldots,L} \frac{\xi_h}{c_h^i}.$$

The aggregate availability set $Y \subset R^l$ is defined by

$$Y = \{y \in \mathbf{R}^l \mid p \cdot y \leq 0, y_1 \leq 1\},$$

(so that commodity 1 is available in the magnitude 1 and can be used as input, whereas the other commodities can only be obtained as output).

Let (x, ξ, p, q) be an extended equilibrium in $\mathcal{E}_m$. By our assumption, $q_0 = \cdots = q_{m-1} = \bar{q}$. We claim that $\sum_{i=0}^{m-1} \xi_i > 0$. Suppose not, then there is $h \in \{1, \ldots, L\}$ such that $\xi_{ih} = 0$ for all i and consequently $u_i(\xi_i) = 0$ for $i \geq 1$. If $\xi_i \neq 0$, then also $x_i \neq 0$ (here we use relevance, cf. Assumption 1(ii)), and then by monotonicity of T (also in Assumption 1(ii)) there is $\xi_i' \succ_0 \xi_0$ with $(x_0 + x_i, \xi_0') \in T$, contradicting the Pareto

11

optimality of (x, ξ) (if consumer i gets utility 0 anyway then her commodities could just as well be given to consumer 0, who would become better off). This proves our claim.

By Assumption 1(ii) (relevance), we now conclude that $\sum_{i=1}^{m} x_i > 0$, and it follows from equilibrium condition (ii') that $p = \hat{p}$. Arguing as above, we have that ξ_i maximizes $\bar{q} \cdot \xi$ on

$$\{\xi \mid \exists x : \hat{p} \cdot x \leq \hat{p} \cdot x_i, \ (x, \xi) \in T_i\},$$

and by constant returns to scale, we have that $\lambda_i^{-1} \xi_i$ maximizes $\bar{q} \cdot \xi$ on $T(\hat{p})$, where $\lambda_i = \hat{p} \cdot x_i$, each i. We conclude that $\text{conv}(\{\lambda_i^{-1} \xi_i \mid i = 1, \ldots, m-1\})$ belongs to the intersection of bd $T(\hat{p})$ with a hyperplane $\{\xi \mid \bar{q} \cdot \xi = M\}$, where $M = \lambda_i^{-1} \bar{q} \cdot \xi_i$ is independent of i.

Next, we notice that $\bar{q}_k > 0$ for all k since otherwise consumer 0 would not satisfy the individual optimality constraint (i') at ξ_0. But this means that for $i \geq 1$,

$$\lambda_i^{-1} \xi_i = \frac{M}{\bar{q} \cdot c_i} c_i,$$

so that for m large enough, the set $\text{conv}(\{\lambda_i^{-1} \xi_i \mid i = 1, \ldots, m-1\})$ gets as close as desired to $\{\xi \in \mathbf{R}_+^L \mid \bar{q}\cdot = M\}$. But then

$$T(\hat{p}) = \{\xi \in \mathbf{R}_+^L \mid \bar{q} \leq M\},$$

and since $\hat{p}$ was arbitrary, we have shown (ii). $\qquad\qquad\square$

A household technology $T \subset \mathbf{R}_+^l \times \mathbf{R}_+^L$ is *separable* if $T = \sum_{k=1}^{L} T^k$ with

$$T^k \subset \mathbf{R}_+^l \times \{\xi \in \mathbf{R}_+^L \mid \xi_j = 0, \ j \neq k\}$$

for $k = 1, \ldots, L$. Thus, a characteristics bundle $\xi = (\xi_1, \ldots, \xi_L)$ is obtained by producing each characteristic ξ_k separately in the technology T^k. For $p \in \mathbf{R}_{++}^l$ we have that

$$T(p) = \text{conv}(0, \xi^1(p), \ldots, \xi^L(p)),$$

where the characteristics vector $\xi^k(p)$ for $k = 1, \ldots, L$ are defined as the solution to the problem

$$\max\{\xi_k \mid (x, \xi) \in T^k, \ p \cdot x = 1\},$$

so $T(p)$ has the structure of an intersection of $\mathbf{R}_+^L$ with a half-space. Our final result shows that the converse holds as well, so that this property characterizes separable technologies.

Theorem 5. *Let $T \subset \mathbf{R}_+^l \times \mathbf{R}_+^L$ be a constant-returns-to-scale satisfying Assumption 1(ii), and suppose that for all $p \in \mathbf{R}_{++}^l$, the set*

$$T(p) = \{\xi \in \mathbf{R}_+^L \mid \exists x \in \mathbf{R}_+^l, \ p \cdot x = 1, \ (x, \xi) \in T\}$$

is the intersection of $\mathbf{R}_+^L$ with a half-space. Then T is separable.

Proof: For $k = 1, \ldots, L$, define T^k by

$$T^k = T \cap \{(x, \xi) \in \mathbf{R}_+^l \times \mathbf{R}_+^L \mid \xi_j = 0, j \neq k\}.$$

Then T^k is a convex cone contained in T, each k, and $\sum_{k=1}^{L} T^k \subset T$.

Assume that $(x, \xi) \in T$ but $(x, \xi) \notin \sum_{k=1}^{L} T^k$. Then for every array $(x^1, \ldots, x^L) \in (\mathbf{R}^l)^L$ such that $(x^k, e^k) \in T^k$ for each k (where e^k is the kth unit vector in $\mathbf{R}^L$), we have that $x \neq \sum_{k=1}^{L} \xi_k x^k$. The set

$$C = \left\{ \sum_{k=1}^{L} \xi_k x^k \mid (x^k, e^k) \in T^k, \ k = 1, \ldots, L \right\}$$

is closed and convex (by convexity of each of the sets T^k), and

$$\{x' \in \mathbf{R}^l \mid x' \leq x\} \cap C = \emptyset$$

(by the free disposal property of household technologies), so by separation of convex sets, there is $p \in \mathbf{R}_{++}^l$ with

$$p \cdot x = 1,$$
$$p \cdot x' > 1 \text{ for } x' \in C.$$

However, by the assumptions of the Theorem, we know that there are $(\hat{x}^k, \hat{\xi}^k) \in T^k$ with $p \cdot \hat{x}^k = 1$, $k = 1, \ldots, L$, such that

$$\xi = \sum_{k=1}^{L} \lambda_k \hat{\xi}^k$$

for some $\lambda_1, \ldots, \lambda_L \geq 0$ with $\sum_{k=1}^{L} \lambda_k = 1$. Clearly, for each k we have that $\lambda_k \hat{\xi}_k^k = \xi_k$, meaning that $(\lambda_k \hat{x}^k, \xi_k e^k) \in T_i$, and

$$\sum_{k=1}^{L} \lambda_k \hat{x}^k \in C.$$

However, since $p \cdot \hat{x}^k = 1$ for each k, we get that

$$p \cdot \left(\sum_{k=1}^{L} \lambda_k \hat{x}^k \right) = \sum_{k=1}^{L} \lambda_k (p \cdot \hat{x}^k) = 1,$$

contradicting that $p \cdot x' > 1$ for each $x' \in C$. $\qquad\qquad\square$

5. Discussion

The results above, in particular those of the previous section, tell us that for equality of individual assessments of characteristics to obtain not only in particular cases but in all extended equilibria, the household technology will have to be separable, so that each of the characteristics are produced from suitable commodity bundles without any mutual interaction. This does not seem reasonable, in particular not in the application of the model to health, where different aspects of health usually come together. Thus, the basic (implicit) assumptions for constructing an index of health, namely that all individuals value the different aspects of health in the same way, must be considered as rather too restrictive.

There is, of course, a way of justifying the use of QALYs or similar one-dimensional health status measures namely by postulating the individuals *are* indeed identical w.r.t. to preferences over characteristics bundles. This seems however a somewhat strange assumption which from the point of view of an economist calls for an explanation. We do not readily expect that people agree in their preferences over ordinary consumption goods (even if they do to some degree), and indeed economic theory would look quite differently if this were to be the basic assumption underlying demand theory. Similarly, there seems to be little a priori reason that all should agree in their ranking of different aspects of health. Moreover, the experience of practical QALY measurement does not readily support such a hypothesis.

When simple methods of aggregating fail, there is of course a need for other methods which do not use ad hoc aggregation. In the context of economic appraisals of medicine, where the tradition calls for a cost-effectiveness ratio, an alternative approach almost suggests itself: the basic setup, where several alternative treatments have to be compared, each of which are defined by a cost and a vector of outcome parameters, is formally identical to the problem of measuring efficiency in organizations with well-defined costs but outputs which are not marketed and therefore have no natural prices associated. In this context, the individual projects cannot be given a total ranking as was the case when cost-effectiveness ratios can be calculated meaningfully, but each project can be given a cost-effectiveness ranking *relative* to the other projects (which may include the status quo). Thus, an economic analysis may still give the decision maker valuable information, which moreover is not based on subjective aggregation performed by the analyst.

Summing up, existing methods of constructing health status measures used to calculate cost-effectiveness ratios is are based on implicit assumptions which are more restrictive than what is otherwise assumed when carrying out this type of economic appraisals. It is possible to avoid these assumptions if it is not insisted that the results of the appraisals shall take the form of a ratio expressing outcome per dollar invested, but only a measure of the position of any given project relative to the totality of other projects available.

References

Baldwin,S., C.Godfrey, and C.Propper (eds.) (1990), Quality of Life, Perspectives and

Policies, Routledge, London and New York.

Bleichrodt,H., P.Wakker and M.Johannesson (1997), Characterizing QALYs by risk neutrality, Journal of Risk and Uncertainty 215, 107 – 114.

Bleichrodt,H. and J.Quiggin (1997), Characterizing QALYs under a general rank dependent utility model, Journal of Risk and Uncertainty 15, 151 – 165.

Broome,J. (1993), QALYs, Journal of Public Economy 50, 149-167.

Culyer,A.J. (1990), Commodities, characteristics ofcommoditites, characteristics of people, utilities, and the quality of life, in Baldwin, Godfrey, and Popper (1990).

Debreu,G. (1959), Theory of value, Wiley, New York.

Green,J.R., A.Mas-Colell and A.M.D.Whinston (1995), Microeconomic theory, Oxford University Press, New York.

Gold,M.R., J.E.Siegel, L.B.Russell, M.C.Weinstein (eds.) (1996), Cost-effectiveness in health and medicine, Oxford University Press, New York.

Grossman,M. (1972), On the Concept of Health Capital and the Demand for Health, Journal of Political Economy 70, 223 – 255.

Lancaster,K. (1971), Consumer Demand, A New Approach, New York – London.

Loomes,G. and L.McKenzie (1989), The Use of QALYs in Health Care Decision Making, Social Science and Medicine 28, 299 - 308.

Mooney,G. (1986), Economics, medicine and health care, Harvester Wheatsheaf, London.

Mooney,G. (1994), Key Issues in Health Economics, Harvester Wheatsheaf, London.

Sintonen,H. (1981), An approach to measuring and valuing health states, Social Science and Medicine, 15C.

Torrance,G.W. (1986), Measurement of health state utilities for economic appraisal, Journal of Health Economics 12, 39 – 53.

Torrance,G.W., and D.Feeny (1989), Utlitites and Quality-Adjusted Life Years, International Journal of Technology Assessment in Health Care 5, 559 – 575.

Ware,J.E. (1995), The status of health assessment 1994, Annual Rev. Public Health 16, 327 – 354.

Williams,A. (1985), The value of QALYs, Health and Social Service Journal (July 18).

Appendix: Proof of Theorem 2

Below we give the proof of Theorem 2 stated in section 2:

Proof of Theorem 2: (1) For each consumer i, define the set

$$P_i^0 = \{\xi_i' \in \mathbf{R}_+^L \mid \xi_i' \succsim_i \xi_i\}$$

of as-good-or-better consumption programs; P_i is closed and convex by our assumptions on $\succsim_i$; we embed P_i in $\mathbf{R}^{l+mL}$ putting 0 in all places except in the ith of the m segments of L coordinates. Denote the resulting set by $\tilde{P}_i$.

For each i, let $\tilde{T}_i$ be the set T_i embedded in $\mathbf{R}^{l+mL}$ by putting 0 in all coordinates except the first l, where the sign of is reversed, and the ith segment of L coordinates. Again, T_i is closed and convex. Finally, the closed and convex set $Y \subset \mathbf{R}^l$ is embedded in $\mathbf{R}^{l+mL}$ as $\tilde{Y}$ by adding 0s in all but the first l coordinates.

Now consider the set

$$Z = \sum_{i=1}^m \tilde{P}_i - \sum_{i=1}^n \tilde{T}_i - \tilde{Y};$$

We have that Z is closed and convex, and that it contains the zero vector, which has the representation

$$0 = \sum_{i=1}^n \tilde{\xi}_i - \sum_{i=1}^n \widetilde{(x_i, \xi_i)} - \tilde{y}$$

for some $y \in Y$ (here $\tilde{a}$ denotes the image of a by the ˜-mapping). We claim that $Z \cap \mathrm{Neg} = \emptyset$, where Neg is the cone of negative vectors in $\mathbf{R}^{l+mL}$. Suppose to the contrary that $u \in Z \cap \mathrm{Neg}$, so that

$$u = \sum_{i=1}^n \tilde{\xi}_i' - \sum_{i=1}^n \widetilde{(x_i', \xi_i')} - \tilde{y}'$$

for some $y' \in Y$; but this means that the allocation (x', ξ') is at least as good as (x_i, ξ_i) and that there is a feasible allocation (x_i'', ξ_i) with $x_i'' > x_i'$ for all i. By the monotonicity properties of T_i and $\succsim_i$, $i = 1, \ldots, m$, there is then a feasible allocation (x_i'', ξ_i'') with $\xi_i'' \succsim_i \xi_i$, contradicting Pareto optimality.

Now, by separation of the convex sets Z and Neg, there exists a non-negative vector $(p, q) = (p, (q_i)_{i=1}^m) \in \mathbf{R}^{l+mL}$ such that the scalar product $(p, q) \cdot z$ for $z \in Z$ is minimized at $z = 0$. But this means that

(a) y maximizes $p \cdot y$ on Y, i.e. $\sum_{i=1}^m x_i \in Y_p$, which is condition (ii') of an extended price equilibrium, that

(b) ξ_i minimizes $q_i \cdot \xi_i'$ on P_i, which, by the interiority condition, implies that $q \cdot \xi_i'' > q \cdot \xi_i$ for all ξ_i'' with $\xi_i'' \succ_i \xi_i$, and that

(c) (x_i, ξ_i) maximizes $q_i \cdot \xi_i' - p \cdot x_i'$ on T_i.

To check equilibrium condition (i'), let $i \in \{1, \ldots, m\}$ and assume that ξ_i'' satisfies the inequality

$$q_i \cdot \xi_i'' \le p \cdot x_i + \max_{(x_i', \xi_i') \in T_i} [q_i \cdot \xi_i' - p' \cdot x_i'].$$

Then from (c) we get that $q_i \cdot \xi_i'' \le p \cdot x_i + [q_i \cdot \xi_i - p \cdot x_i] = q_i \cdot \xi_i$, and (b) implies that $\xi_i \succsim_i \xi_i''$, which is (i').

To show part (2) of the theorem, let (x', ξ') be an allocation with the properties stated. Since $q_i \cdot \xi_i' - p \cdot x_i' \le q_i \cdot \xi_i - p \cdot x_i$ by the equilibrium conditions, we have that

$$p \cdot x_i' \ge q_i \cdot (\xi_i' - \xi_i) + p \cdot x_i;$$

16

summing over i and using that $\sum_{i=1}^{m} \xi_i' > \sum_{i=1}^{m} \xi_i$, we get that

$$\sum_{i=1}^{m} p \cdot x_i' > \sum_{i=1}^{m} p \cdot x_i.$$

Choose positive numbers $w_1, \ldots, w_m$ with $\sum_{i=1}^{m} w_i = \sum_{i=1}^{m} p \cdot x_i'$, and let

$$x_i'' = x_i + \frac{w_i}{\sum_{h=1}^{l} p_h}(1, \ldots, 1)$$

for $i = 1 \ldots, m$. Then $\sum_{i=1}^{m} x_i'' = \sum_{i=1}^{m} x_i'$, and by monotonicity of household production and of preferences, there are ξ_i'' with $(x_i'', \xi_i'') \in T_i$ such that $\xi_i'' \succ_i \xi_i$, $i = 1, \ldots, m$, which proves that (x', ξ') is a potential Pareto improvement. $\square$