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The Rise in Danish Unemployment: Reallocation or Mismatch?

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Abstract

Two key relations in theoretical work on labor market flows are analyzed; the hiring function and the uv-curve. The relations are identified in a cointegrated VAR-model framework. The system comprising unemployment, vacancies and hirings is driven by a stochastic trend in the vacancies, corresponding to the status of vacancies as the driving force in models of labor market flows. The drift in the relations is modeled by smooth transition functions and is identified as increased mismatch problems in the Danish labor market as opposed to increased structural change.

Keywords: Hiring function, UV-curve, cointegrated VAR-model, smooth transition functions, mismatch.

JEL classification: J41, J61

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1. Introduction

Most economic models of aggregate labor market flows are based on two key relations; the hiring function and the uv-curve. The hiring function describing hires as a function of unemployment and vacancies is an important building block in theoretical models while the existence and the properties of the function is an empirical question. The uv-curve and especially analyses of movements in the uv-plane has played an important role in distinguishing between alternative hypotheses about the causes of changes in unemployment.

Empirical analyses of hiring functions are of a recent date, the seminal contribution is Pissarides (1986) followed by Layard, Nickell, and Jackman (1991) and Burgess (1993) on British data, Burda and Wyplosz (1994) on data for several European countries, and Blanchard and Diamond (1989,1990) on US data. Empirical work on the uv-curve has a longer tradition. Pioneering work was carried out by Dow and Dicks-Mireaux (1958), recent contributions include Pissarides (1986) and Jackman, Layard, and Pissarides (1989) for Britain, Abraham (1987) and Blanchard and Diamond (1989,1990) for the US, and Franz (1987) for Germany.

In the present paper the relations between hirings, unemployment, and vacancies are analysed jointly in a cointegrated VAR-model. The main contributions are identifications of (1) the theoretical relations, the matching function and the uv-curve, (2) the driving force in the system, this role is taken by the supply of vacancies analogous the role of vacancies in theoretical analysis, and (3) the source of the outward shift in the uv-curve, which has been attributed to either increased mismatch or increased structural change.

Section 2 establish the theoretical relations of interest for this study, the hiring function and the uv-curve, and discuss their properties and interrelations. In particular, we look at the possibility of identifying the causes of the shifts in the uv-curve. At the outset of section 3, two long-run relations between the variables are identified in a cointegrated VAR-model. The two cointegration relations seem to have non-constant parameters, therefore we proceed to an analysis of a cointegrated model with smooth shifts in the cointegration relations in section 4. We test if the shifts in the two relations have the same time pattern, and as this is confirmed we conclude that the shifts are due to increased mismatch. Section 5 offers a brief discussion of the findings in section 4. Exogeneity of the vacancies, which is assumed throughout, is tested in section 6. This section also illustrate the dynamic properties of the model using impulse-response functions. Finally, section 7 gives concluding remarks.

2. A model of labor market flows

In this section we discuss a model for the flows into and out of unemployment. The aim of the model is to describe equilibrium relations between hirings, unemployment and vacancies, given the data available for the empirical analysis. Special emphasis in the derivations is placed on the sources of shifts in the uv-curve and the hiring function.

The stock of workers (the labor force) in each period is denoted by N_t. The workers are either employed or unemployed such that $N_t \equiv E_t + U_t$, where E_t is the stock of employed workers and U_t is the stock of unemployed workers.

The flows into and out of employment are given by the number of hirings (H_t) and separations (S_t), and we have the following relation between the flow rates

$$\Delta e_t \equiv h_t - s_t \left(1 - u_t\right), \tag{1}$$

where h_t is the rate of new hirings to the labor force (H_t/N_t), s_t is the rate of separations to the number of employed workers (S_t/E_t), and $1 - u_t \equiv e_t \equiv E_t/N_t$ is the employment rate.

The forces governing the flow rates on the right hand side of (1) are described by two behavioral relations. For hirings, we assume that the matching of vacant jobs and unemployed workers can be described by a hiring function. This function maps the stock of vacancies and the stock of unemployed into the flow of workers entering employment from unemployment. The hiring function is assumed to be approximated by a Cobb-Douglas function with constant returns to scale.[1] The data for vacancies are the number of vacant positions registered at the employment offices. These vacancies are posted by firms who wish to hire unemployed workers. The outcome of the match between unemployed workers and vacancies registered at the unemployment offices is an unemployment-employment transition. However, the hiring data includes all transitions into employment, hence employment-employment transitions are included. Denoting the fraction of hires from unemployment as F_t, unemployment-employment transitions become $F_t h_t$. Consequently, we get the following hiring function, describing the matching between vacancies and unemployed workers

$$\ln h_t = m_t - f_t + \phi \ln u_t + (1 - \phi) \ln v_t + \varepsilon_{1t}, \tag{2}$$

where $v_t = V_t/N_t$ is the vacancy rate, $f_t = \ln F_t$, ϕ is the weight of the unemployed in the matching process, and ε_{1t} is an error term. The term m_t measures

[1] Increasing returns to scale had an important place in the early development of search theory as it can generate multiple equilibria, see Diamond (1982). In the empirical analysis we will present evidence for the reasonabless of the constant returns to scale assumption.

the effectiveness of the matching process. Lower levels of m_t signifies increased mismatch between unemployed workers and vacant jobs. Another movement in the hiring function happens if the unemployed increase their share of hirings (f_t goes up) this shows up as downward shift in the hiring function.

As our hiring data contains employment-employment transitions, we turn to separations out of employment. These consist of two flows: i) employment to employment transitions (Q_t) and ii) employment to unemployment transitions (L_t). Hence the separation rate is $s_t \equiv q_t + l_t$ where $q_t = Q_t/E_t$ and $l_t = L_t/E_t$.

The employment-unemployment transition rate l_t is assumed to follow the relation

$$l_t = e^{r_t}(v_t/u_t)^{-\theta}e^{\varepsilon_{2t}}, \quad \theta > 0. \tag{3}$$

The first factor on the right hand side, e^{r_t}, measures structural changes in the economy causing changes in the rate of layoffs at a given level of economic activity, *i.e.*, r_t captures a changing level of shifts in employment between different sectors in the economy as in Lilien (1982). Alternatively, increased levels of reallocation of employment between firms within sectors could cause higher levels of layoffs, given the level of economic activity. The second factor, v_t/u_t, is labor market tightness. This term measures the relation between the level of economic activity and the rate of layoffs. The formulation entails that increased tightness on the labor market implies a smaller amount of layoffs, whereby layoffs move countercyclical. The last factor, ε_{2t}, is an error term.

As the share of hirings originating from employment is $(1 - F_t)$, the relation between the employment-employment transition rate q_t and the hiring rate is given by

$$q_t = (1 - F_t)h_t\,(1 - u_t)^{-1}. \tag{4}$$

Inserting (3) and (4) into the flow relation (1) yields

$$\Delta e_t = F_t h_t - (1 - u_t)(v_t/u_t)^{-\theta}e^{r_t}e^{\varepsilon_{2t}}. \tag{5}$$

The first term on the right hand side is unemployment-employment transitions while the second term is employment-unemployment transitions. In contrast to (1), the terms on the right hand side of (5) does not include job to job transitions.

Steady state is characterized by constant employment and unemployment rates, $\Delta e_t = \Delta u_t = 0$. Application of this condition in (5) and inserting the expression for $F_t h_t$ from the hiring function (2) yields

$$(\phi - \theta)\ln u_t - \ln(1 - u_t) = r_t - m_t - (1 - \phi + \theta)\ln v_t + \varepsilon_{1t} - \varepsilon_{2t}, \tag{6}$$

Here the uv-curve is implicitly defined as a relation between vacancies and unemployment. A Taylor expansion of $\ln(1 - u_t)$ implies that the factor to $\ln u_t$ on the left hand side of (6) changes from $\phi - \theta$ to $d = \phi - \theta + \bar{u}/(1-\bar{u})$, where $\bar{u}$ is the point of expansion. The entity d is likely to be positive, which will be assumed in the following. With this reformulation, we get a log-linear formulation of the uv-curve

$$\ln u_t = (r_t - m_t + \mu)\, d^{-1} + \kappa \ln v_t + \varepsilon_{3t}, \quad \kappa = -\,(1 - \phi + \theta)\, d^{-1} \tag{7}$$

where the term μ is a function of $\bar{u}$, and the error term is $\varepsilon_{3t} = (\varepsilon_{1t} - \varepsilon_{2t})/d$. Given $d > 0$, the slope of the uv-curve is negative if $\phi - \theta < 1$.

From the first term on the right hand side in (7) it is clear that increased reallocation (larger r_t) will shift the uv-curve outwards. The same is the case with increased mismatch in the economy (smaller m_t).

One of the main reasons for the interest in analyzing the uv-curve and movements in the uv-plane is the potential of identifying the causes of changes in unemployment. In the present model comprising of a hiring function in (2) and a uv-curve in (7), identification of changes in the different factors affecting the transitions processes can be obtained from the fact that they have a differential impact on the levels of the two curves.

Consider first increased mis-match on the labor market given as a reduction of m_t. Increased mis-match can be caused by increased heterogeneity on the labor market, increased reluctance by the unemployed workers to accept vacant jobs or increased choiceness from employers towards unemployed workers. From (2) this will result in a decreased level of hirings given unemployment and vacancies; a downward shift in the hiring function. From (7) increased mis-match will result in an increased level of unemployment given vacancies; an outward shift in the uv-curve. So a downward shift in the hiring function and a parallel outward shift in the uv-curve is an indication of increased mismatch on the labor market.

Increased structural change in the economy given the level of economic activity can be described as a change in r_t. An increase in r_t could be due to increases in the change of employment between different sectors in the economy, or an increased level of job reallocation within sectors. As r_t does not enter (2), the level of hirings h_t is unchanged, while it follows from (7) that unemployment u_t increases given the level of vacancies; an outward shift in the uv-curve. So the presence of a constant hiring function and an outward shift in the uv-curve in an indication of increased structural change on the labor market.

Consider finally an increased share of hirings from unemployment given by an increase f_t. This will occur if search from unemployment gains a larger weight relative to search from employment. In this case hirings decrease in (2) while unemployment in (7) is unaffected as f_t does not enter the uv-relation. So a downwards shift in the hiring function and a constant uv-curve is an indication of a relative larger weight of search from unemployment relative to search from employment.[2]

Stated in another way, factors affecting the flow from unemployment into employment will lead to changes in both the hiring function and the uv-curve. Factors affecting the flow out of employment to unemployment only give rise to changes in the uv-curve leaving the hiring function unaltered. Factors affecting the flow from employment to employment, given the level of unemployment, vacancies and the flow from unemployment to employment, will affect the hiring function but not the uv-curve. These properties of the model will be pursued in an analysis of the changes over time in the two relations (2) and (7).

The hiring function and the uv-curve determines the equilibrium values of hirings and unemployment for given values of vacancies. To close the model a relation for the supply of vacancies is needed. In theoretical work the model is closed by modeling the 'production sector' of the economy as opposed to the 'trade sector' describing the flow of workers. The outcome is a relation determining the supply of vacancies as a function of employment, unemployment, and wage formation, see Pissarides (1990). Here, we limit the analysis to the trade sector which is why we consider the vacancies to be the driving force in the model.

Vacancies can be considered as a jump variable in the adjustment process towards equilibrium; firms can instantaneously change the number of posted vacancies in an attempt to obtain the desired future path of employment and production. Unemployment moves more slowly because the hiring process takes time. Therefore, upturns are characterized by an increase in the number of vacancies, subsequently leading to increased hirings and decreasing unemployment. Conversely, downturns begin with a decrease in the number of vacancies followed by a reduction in hirings and increasing unemployment. That is, the cyclical movements in the uv-plane follow a counterclockwise path around the uv-curve.

The system comprising hirings, unemployment, and vacancies is a minimal model for the flows on the labor market. Such a system enables us to investigate the interrelationships between the variables $\ln h_t$, $\ln u_t$, and $\ln v_t$ and it opens the possibility to identify a hiring function and a uv-curve. The differences in the timing of the changes in the vacancy and the unemployment rates requires an estimation strategy in which the difference between the short run and the long run is modeled explicitly. This requirement is fulfilled if the two relations can be identified as being cointegration relations between the three variables. Therefore, the cointegrated VAR-model framework is particularly suited for this kind of analysis.

[2]I should be noted that the competition for jobs between the employed and the unemployed is not modeled explicitly in the present analysis as this requires separate data for unemployment-employment and employment-employment transitions, see Burgess (1993) for a treatment of this topic.

3. A minimalist model of hirings, unemployment, and vacancies

The line of thought behind the hiring function in equation (2) is that members of the stock of unemployed and members of the stock of vacancies meet and perform a match into employment. Correspondingly the empirical analysis is based on stock data for vacancies and unemployment. Vacancies are the number of unfilled orders at the labor exchange at the last day in each quarter. Unemployment is a similar quarterly stock measure, whereas hires is a constructed series based on the official employment series and on measures of exit and entry of workers among members of the Danish Confederation of Employers. Hirings include all hires irrespective of previous labor market status. The general presumption is that hirings from outside the labor force on the Danish labor market are limited compared to e.g. the US.

Quarterly observations of the three series for the period 1974q3 - 1988q4 are shown in Figure 1, in which the vacancy rate is multiplied by 100 to match the range of hirings and unemployment.[3]

[Figure 1 about here]

The first question of interest is whether the series should be modeled as stationary or difference stationary. To test this hypothesis a partial vector autoregressive model, in which the vacancy rate is assumed to be weakly exogenous, is estimated. The model is formulated in the error correction form as

$$\Delta y_t = \Gamma_0 \Delta \ln v_t + \Gamma_1 \Delta x_{t-1} + \alpha\beta' x_{t-1} + \alpha\delta + \Phi S_t + \eta_t. \tag{8}$$

Here $y_t' = (\ln h_t, \ln u_t)$, $x_t' = (\ln h_t, \ln u_t, \ln v_t)$, and S_t is a vector of centered seasonal dummies. β is a matrix of cointegration vectors, of dimension $3 \times r$ where r is the number of cointegration vectors, and α is a $2 \times r$ matrix of feed back coefficients. δ is an r-dimensional vector of intercepts in the cointegration space, and Γ_0 measures the contemporaneous effects between $\Delta \ln v_t$ and $\Delta \ln h_t, \Delta \ln u_t$. ($\Delta$ is the difference operator). The error term η_t is assumed to be Gaussian with mean zero and a constant positive definite variance Ω.

[3]The labour exchange offices in Denmark do not register all vacancies. A change in the registration system from 1989 and onwards resulted in a break in the vacancy serie. For the present purpose, we consequently stop the analysis by 1988.

[Table 1 about here]

The number of cointegration vectors is determined by comparing the rank test statistics in Table 1 with the quantiles of the asymptotic distributions given in Harboe et al. (1998). If the hiring function and the uv-curve are to be equilibrium relations between non-stationary variables this implies stationarity of both relations. Thus, if the series are non-stationary we expect to find two cointegration vectors. As seen from Table 1 there is quite strong empirical support for a choice of two cointegration vectors. This implies the precence of one common stochastic trend in the system.

Judging from Figure 1 the hiring rate might be stationary with a strong seasonal component. The hypothesis about stationarity of the hiring rate can be investigated within this model by testing whether the hiring rate is one of the two cointegration vectors spanning the cointegration space. The likelihood ratio test statistic for this hypothesis is shown in Table 1 along with test statistics for the hypotheses of stationarity of the unemployment rate and the vacancy rate. The tests for stationarity are all in the extreme upper tail of the asymptotic distribution, implying that the three series should be modeled as I(1) series in this system.

The weak exogeneity of the vacancy rate with respect to the parameters in the two cointegration relations is crucial to the following analysis because it implies that the cointegration space does not describe a relation for the supply of vacancies. Thus, the exogeneity assumption is actually an identifying restriction used to identify the cointegration space as consisting of the hiring function and the uv-curve while the common stochastic trend in the system is a function of the cumulated shocks to the vacancy rate.

Identification of the two vectors spanning the cointegration space in the model (8) requires at least one restriction on each vector. The hiring function is identified by imposing the homogeneity assumption, and the identification of the uv-curve is done by excluding the hiring rate from the cointegration vector. The estimated coefficients of the long-run hiring function and the uv-relation are shown in Table 2.

As the assumption of constant returns is used for the identification of the hiring function, the question of whether the hiring function actually has constant returns to scale has to be addressed indirectly. Increasing or decreasing returns to scale would imply that the two relations, (2) and (7), are non-stationary if the three series are individually non-stationary. Therefore, acceptance of two cointegration vectors implies acceptance of a homogeneous hiring function because the labor force is not required to make the two relations stationary.[4] Another indirect check is whether the estimated coefficients are inside the parameter space $[0, 1]$ which is required for a well behaved hiring function. Table 2 shows that the point estimates are well inside the acceptable parameter space, and the estimated standard errors shows that the coefficients are significantly different from $\{0, 1\}$ and $\{1, 0\}$ whereby the estimated hiring function is theoretically acceptable.

[TABLE 2 ABOUT HERE]

The deviations from the long-run hiring function and the long-run uv-curve are shown in Figure 2. Panel A shows the deviations from the hiring function and Panel B shows the deviations from the uv-curve. The series 'Actual deviations' are plots of $\hat{\beta}' x_t$ whereas the series 'Smoothed deviations' are four quarter moving averages of $\hat{\beta}' x_t$. The latter series reveals possible drifts. The estimated uv-curve is above the moving average values in the period 1974q3 to 1984q2, except for the period 1978q2-1979q3, and the relation is below the moving average values in the period 1984q3 to 1988q4. We interpret the drifts as an indication of changing intercepts in the two relations, as outlined in Section 2.

[FIGURE 2 ABOUT HERE]

There are different ways of resolving the problem of non-constant parameters in the long-run relations. One approach is to include more information into the system to account for the changes in the relations. This approach is the most informative with respect to the processes on the labor market, but it has shown to be difficult to find the relevant information in aggregate time series. Another approach is to model a deterministic change over time in some of the parameters. We follow the latter approach. Therefore the model is extented to allow for smooth deterministic changes in the intercepts.

[4] A more general matching function, which does not neccessary exhibit constant returns to scale, would take the form $\ln h_t = c_t + \phi_u \ln u_t + \phi_v \ln v_t - (1 - \phi_u - \phi_v) \ln N_t$, where c_t is the constant term, ϕ_u is the weight in the matching function corresponding to unemployement, ϕ_v is the weight for vacancies, and N_t is the labour force. If the matching function is homogeneous of degree one, it is possible to find a cointegration relationship between $\ln h_t$, $\ln u_t$, and $\ln v_t$. If $\phi_u + \phi_v \neq 1$, it is neccessary to include $\ln N_t$ in the equation.

4. A model with drifts in the long-run relations

Several authors include time trends in the hiring function and the uv-curve to capture increasing mismatch problems. Blanchard and Diamond (1989,1990) and Burda and Wyplosz (1994) augment the simple hiring function by a linear time trend and Pissarides (1986) includes a third order polynomial in time in addition to other measures of mismatch in the hiring function. Section 2 showed that a trend in the hiring function will give a trend in the uv-curve. Among the previous empirical analyses of the uv-curve, we note that Abraham (1987) include a discrete jump and a linear time trend in the estimated uv-curve and Blanchard and Diamond (1989,1990) include a linear trend in the system comprising unemployment, vacancies, and the labor force. Hence, modelling changes in the labor market relations using time trends seems to be the rule rather than the exception.

Our approach is to estimate a continuous deterministic change in the scale parameter in the hiring function and in the location of the uv-relation, keeping the elasticities constant over time. However, in contrast to the aforementioned analyses of the hiring function and the uv-curve we choose an estimation strategy which does not necessarily result in continued shifts in the relations outside the sample period. This is feasible by adopting the idea of a smooth transition model proposed by Lin and Teräsvirta (1994). Accordingly, a new statistical model for the changes in the hiring rate and the unemployment rate is given by

$$\Delta y_t = \Gamma_0 \Delta \ln v_t + \Gamma_1 \Delta x_{t-1} + \alpha\beta' x_{t-1} + \alpha\delta + \alpha\psi' \begin{bmatrix} F_h(t) \\ F_u(t) \end{bmatrix} + \Phi S_t + \eta_t, \qquad (9)$$

where ψ is a 2×2 matrix with zeros off the diagonal and unknown parameters on the diagonal. The $F_i(t)$ functions are given by

$$F_i(t) = \left(1 + \exp\{-\gamma_{i3}(\tilde{t}^3 + \gamma_{i2}\tilde{t}^2 + \gamma_{i1}\tilde{t} + \gamma_{i0})\}\right)^{-1}, \quad \tilde{t} = 0.1, 0.2, \ldots \qquad (10)$$

The function $F(t)$ has been introduced, in this context, by Lin and Teräsvirta (1994) to model parameter non-constancy in single equation regression models.[5] The specification of the transition functions has two advantages. First of all, the parameterization is sufficiently flexible to capture both the apparent cyclical movement in the period 1978-79 and the drift over the full sample. Second, the function is bounded in the interval $[0, 1]$. This will make the final model better suited for forecasting purposes.

[5] The degree of the time polynomial in (10) was determined following the proposal in Lin and Teräsvirta (1994). $F_i(t)$ was replaced by a first order Taylor approximation of $F_i(t) - \frac{1}{2}$ and the significance of each power of t was tested. In our model a third order polynomial was significant.

The parameters of model (9) are estimated jointly with two identifying restrictions imposed on each cointegration relation. The restrictions are imposed to identify the hiring function, the uv-relation, and a transition function for each of the relations.

Within the model it is possible to investigate whether the change in the location of the uv-curve can be attributed to altered economic conditions that govern the flow of workers either into employment or out of employment. As noted, changes in the matching process are captured by m_t, which enters in both $F_h(t)$ and $F_u(t)$. Structural changes are captured by r_t which only enters in $F_u(t)$. Therefore, if the two time functions $F_h(t)$ and $F_u(t)$ are equal, shifts in the uv-curve can be attributed to changes in the matching process.[6] A likelihood ratio test of the hypothesis $F_h(t) = F_u(t)$, which is asymptotically distributed as $\chi^2(4)$, results in a test statistic of 7.79, whereby the hypothesis of equal time patterns in the two relations is accepted at conventional levels of significance. This means that the model can be simplified to include only one common deterministic function. The estimated parameters of this model are reported in Table 3.

[Table 3 about here]

5. Interpretation of the changes in long-run relations

Although the model is formulated to restrict the transition function to the cointegration space it is not possible to identify the temporary and the permanent movements of the two long-run relations in the model.[7] The interpretation of the permanent changes in the long-run relations is therefore not based on statistical measures.

The estimated transition function start at zero in 1974. There is a local maximum in 1978 and a local minimum in 1983. The local maximum is well below 1, which is the value of the function from 1986 and onward.

Our interpretation of the results is that the sample can be divided into four subsamples with two periods of transition and two periods with stable uv-relations. The first sub-sample is the period 1974q3 - 76q1; a period of transition. The second sub-sample 1976q2 - 84q2 is regarded as a period with a stable uv-relationship,

[6] We recognize the possibility that the changes in m_t and r_t can have been identical in the sample period, and that in this case it is not possible to identify the cause of the drift in the uv-curve. We conjecture that the probability of this event is small enough to be neglected in the following.

[7] We have attempted to identify the permanent changes using other deterministic functions, but none of the models with two different types of deterministic functions converged.

but with a cyclical movement which is partly captured by the swings in the transition function. 1984q3 - 85q4 is the second period of transition bringing the uv-relation to the final level which is reached in 1986q1.[8]

To give an idea of the effect of the transition function the uv-relations are plotted in Figure 3. Along with the actual observations of the unemployment and vacancy rates – measured as moving averages over a full year to remove seasonal fluctuations – Figure 3 shows three uv-curves. Two of these correspond to the uv-relation given in Table 3 where the values of the transition function are for 1984q2 and 1986q1 (interpreted as stable periods). The third curve is based on the model in Table 2 with constant intercepts. In addition, Table 4 reports predicted equilibrium values of the hiring rate and the unemployment rate at a given level of the vacancy rate (0.05%). The equilibrium rates are calculated for the two stable periods 1976q2-84q2 and 1986q1-88q4 corresponding to the location of the two uv-curves in Figure 3. The equilibrium rates for the model with constant intercepts, the third uv-curve in Figure 3, are given in the last column for reference.

[Figure 3 about here]
[Table 4 about here]

Figure 3 and Table 4 give a clear picture of increased mismatch in the labor market over the sample. The equilibrium rate of the unemployment has increased by almost 60% from 1975 to 1986. About half of the increase in the equilibrium level happened in the first period of transition and the other half in the second period of transition. The equilibrium level of the hiring rate has dropped by 15%. For the hiring rate around 80% of the total change happened in the period 75q1-76q2. The results indicate the occurrence of major changes and increasing mismatch problems in the aftermath of the first oil-crises. The second period of transition (1984-85) occurs at the same time as a jump in the vacancy rate from about 0.02% to about 0.1%. Because of the cyclical loops around the uv-curve, it is difficult to infer if the changes in the uv-curve are caused by increased mismatch or just very slow adjustments towards the equilibrium. However, the position of the unfinished branch after the last turning point in Figure 3 lends support to an interpretation of the movements as increased mismatch.

The specific dating of the changes is by no means certain and it is mainly

[8] It is common to find irregular patterns in the co-movement of vacancies and unemployment over time in different countries, see e.g. the sample of uv-curves in Layard, Nickell, and Jackman (1991). To the extent that the development in the Danish uv-curve in the late 1970'es is such an irregular pattern, this is reflected in the estimated transition function.

based on the belief that the cycle in 1978-79 is a non-structural phenomenon in which the full cycle is accidentally above the uv-curve. However, the findings of increased mismatch are significant, regardless of the partitioning of the transition function into temporary and permanent components.

6. The dynamics of the system

To complete the specification of the model we turn to the estimation of the short-run dynamics. As we do not have a theoretical model for the dynamics in the system, the identification of the equations is based on exclusions of insignificant parameters. The main objective is to test for weak and strong exogeneity of the vacancy rate.

The reduced form equation for $\Delta \ln v_t$ is given in Table 5. In the first row $\Delta \ln h_{t-1}$, $\Delta \ln u_{t-1}$ and the lagged deviations from the two long-run relations are included in the equation. None of the four variables are significant, neither individually nor jointly. The test of the significance of the two long-run relations can be considered as a Lagrange Multiplier test of the weak exogeneity assumption used in the estimation of the long-run relations. The test shows that the assumption is not rejected. Hence the results obtained for the hiring function and the uv-curve conditional on the vacancies seems to be valid. Furthermore, as neither $\Delta \ln h_{t-1}$, nor $\Delta \ln u_{t-1}$ are significant in the model, hirings and uneployment do not Granger-cause vacancies in this system.

[Table 5 about here]

Table 6 shows the estimated values of $\Gamma_0, \Gamma_1, \alpha$, and Φ in the model (9) with $\hat{\beta}, \hat{\psi}$, and $\hat{F}(t)$ as specified in Table 3.[9] In the hiring equation the lagged deviation from the estimated uv-curve was insignificant and, therefore, the parameter has been restricted to zero. The coefficient to the lagged change in the unemployment rate is restricted such that the one period lagged effect of $\ln u_t$ on h_t is zero (the unrestricted point estimate gave an insignificant negative effect). In the unemployment equation the contemporary change in $\ln v_t$, the lagged value of $\Delta \ln h_t$ and the lagged deviations from the estimated hiring relation were all insignificant, leading to a very simple model for the unemployment rate.

[9] The lagged values of Δv_t were insignificant in both equations which is why the variable has been left out of the model completely.

[Table 6 about here]
[Table 7 about here]

Finally, Table 7 reports the covariance and the correlation matrix for the residuals of the three equations. The covariances are small and a Wald test of the hypothesis of zero covariance between the three residual series is clearly accepted.

The conclusion from this analysis of the dynamics of the system is that the only shocks with a permanent effect on the equilibrium values of h_t, u_t, and v_t are the shocks to $\Delta \ln v_t$. Shocks to the hiring equation and the unemployment equation have only temporary effects. Furthermore, the vacancy rate is strongly exogenous to the parameters in the equation for the unemployment rate, and both the vacancy rate and the unemployment rate are strongly exogenous with respect to the parameters in the equation for the hiring rate. This means that the vacancy rate can be used to forecast the unemployment rate and subsequently the hiring rate.

As the parameters of error-correction models are often difficult to interpret the dynamics of the levels in the model are illustrated using impulse-response functions. We do not give the impulse-response functions of shocks to η_{ht} or η_{ut} as these do not have a clear economic interpretation apart from being unanticipated changes in the hiring rate and unemployment rate. Furthermore, the effects of shocks to η_{ht} and η_{ut} will be transitory because the shocks have no impact on $\Delta \ln v$.

[Figure 4 about here]

Figure 4 shows the response in the levels of the three series of a one standard error shock to η_{vt}.[10] This shock has permanent effects on the equilibrium values. The dynamic path towards the new equilibrium is monotonous for the unemployment rate and the vacancy rate whereas the path for the hiring rate has a peak after one year from which it converge to the new equilibrium from above. The new equilibrium values are reached after three years.

The lower right corner of Figure 4 shows the dynamic paths of the unemployment rate and the vacancy rate in the uv-plane. In this plot the counterclockwise movement towards the new equilibrium is clear.[11]

[10]The standard error of the impulse is calculated from the covariance matrix in Table 7 but the covariances are set to zero.

[11]The initial response in the uv-plane is vertical simply because we were not able to find signif-

7. Conclusion

In this paper the joint behavior of hirings, unemployment, and vacancies in Denmark is analysed using a cointegrated vector autoregressive model as the statistical framework. This enables an empirical identification of two key relationships in models of aggregate labor market flows; the hiring function and the uv-curve. The identification of these theoretical concepts is done by imposing restrictions on the long-run parameters in the cointegrated VAR-model. The source of the non-stationarity in the model is identified as a stochastic trend in the vacancies. This corresponds to modeling the vacancies as the driving force in theoretical models.

A close investigation of the two long-run relations reveals structural changes in the Danish labor market. The percieved movements in the equilibrium relations are modeled using smooth transition functions to capture the changes. By imposing restrictions on the transition functions we identify the source of the changes in the labor market as being increased mismatch problems, as opposed to increased structural change. This means that the increase in the equilibrium level of unemployment at a given level of vacancies is caused by changes in the process governing the inflow to employment.

The identification of vacancies as the driving force in the model is tested and vacancies are shown to be strongly exogenous for the parameters in the equations explaining hirings and unemployment. Furthermore both vacancies and unemployment are strongly exogenous for the parameters in the equation for hirings. In fact, it is shown that unemployment is well described by it's own past and the slowly drifting uv-relation.

Finally, it must be stressed that although the vacancy rate is strongly exogenous to the parameters in the unemployment equation, this does not imply a horizontal vacancy supply curve in the uv-plane. If there exist a real wage curve (or some variety of a Phillips-correlation) relating wages and unemployment, and if wages have an impact on the supply of vacancies, then unemployment has an impact on the supply of vacancies. This effect is not captured in our model because the vacancy supply curve is a cointegration relation between (at least) vacancies, unemployment, and wages. An extension of the analysis to include a modelling of the formation of vacancies is a topic for future research.

icant contemporaneous correlations between unemployment and vacancies. The point estimate is negative, though, as can be seen from the Table 7.

References

ABRAHAM, K. (1987): "Help-wanted Advertising, Job Vacancies, and Unemployment," *Brookings Papers on Economic Activity*, 1, 207–248.

BLANCHARD, O. J., AND P. A. DIAMOND (1989): "The Beveridge Curve," *Brookings Papers on Economic Activity*, 1, 1–60.

——— (1990): "The Aggregate Matching Function," in *Growth/Productivity/Unemployment*, ed. by P. A. Diamond. MIT Press.

BURDA, M., AND C. WYPLOSZ (1994): "Gross worker and job flows in Europe," *European Economic Review*, 38, 1287–1315.

BURGESS, S. M. (1993): "A Model of Competition Between Unemployed and Employed Job Searchers: An Application to the Unemployment Outflow Rate in Britain," *Economic Journal*, 103, 1190–1204.

DIAMOND, P. A. (1982): "Aggregate Demand Management in Search Equilibrium," *Journal of Political Economy*, 90, 881–894.

DOW, J. C. R., AND L. DICKS-MIREAUX (1958): "The excess Demand for Labour: A study of Conditions in Great Britain," *Oxford Economic Papers*, 10, 1–33.

FRANZ, W. (1987): "The End of Expansion in Employment in Germany," in *Unemployment: Theory, Policy and Structure*, ed. by P. J. Pedersen, and R. Lund, pp. 81–103. deGruyter, Berlin.

HARBOE, I., S. JOHANSEN, B. NIELSEN, AND A. RAHBEK (1998): "Asymptotic Inference on Cointegration Rank in Partial Systems," *Journal of Business and Economics Statistics*, 16(4), 388–399.

JACKMAN, R., R. LAYARD, AND C. PISSARIDES (1989): "On Vacancies," *Oxford Bulletin of Economics and Statistics*, 51, 377–394.

LAYARD, R., S. NICKELL, AND R. JACKMAN (1991): *Unemployment. Macroeconomic Performance and the Labour market*. Oxford University Press.

LILIEN, D. M. (1982): "Sectoral Shifts and Cyclical Unemployment," *Journal of Political Economy*, 90, 777–793.

LIN, C.-F. J., AND T. TERÄSVIRTA (1994): "Testing the constancy of regression parameters against continous change," *Journal of Econometrics*, 62, 211–228.

PISSARIDES, C. A. (1986): "Unemployment and Vacancies in Britain," *Economic Policy*, 3, 499–559.

——— (1990): *Equilibrium Unemployment Theory*. Basil Blackwell.

Table 1: Tests for cointegration rank and stationarity.

Rank	Test statistics	5% Critical values	
0	56.4	25.5	
1	14.0	12.3	
Tests of stationarity, $\chi^2(1)$			
Series	$\ln h_t$	$\ln u_t$	$\ln v_t$
Test statistics	6.27	11.82	13.26

The critical values are from Harboe et al. (1995), Table 3.

Table 2: The long-run relations.

		$\ln u_t$	$\ln v_t$	const.
The hiring function	$\ln h_t =$	0.740	0.260	0.829
		(0.030)	(0.030)	(0.162)
The uv-curve	$\ln u_t =$		-0.187	-3.697
			(0.030)	(0.229)

Estimated standard errors in parentheses.

Table 3: The long-run relations with smooth changes over time.

		$\ln u_t$	$\ln v_t$	const.	$\hat{F}(t)$
The hiring function	$\ln h_t =$	0.657	0.343	1.708	-0.599
		(0.017)	(0.017)	(0.075)	(0.119)
The uv-curve	$\ln u_t =$		-0.265	-4.647	0.486
			(0.016)	(0.053)	(0.139)
The transition function					
		$\hat{\gamma}_3$	$\hat{\gamma}_2$	$\hat{\gamma}_1$	$\hat{\gamma}_0$
$\hat{F}(t)$		0.643	-7.684	17.032	-9.087

Estimated standard errors in parentheses.

Table 4: Estimated equilibrium rates of hirings and unemployment

	1976q2 - 1984q2	1986q1 - 1988q4	constant intercept
v %	0.05	0.05	0.05
u %	9.23	11.69	10.27
h %	6.25	6.03	5.89

Table 5: The equation for $\Delta \ln v_t$

$\Delta \ln v_{t-1}$	$\Delta \ln v_{t-2}$	$\Delta \ln h_{t-1}$	$\Delta \ln u_{t-1}$	h_{ecm}	uv_{ecm}
-0.019	0.317	0.197	0.276	0.131	0.514
(0.162)	(0.150)	(0.385)	(0.360)	(0.310)	(0.313)
0.160	0.360	—	—	—	—
(0.125)	(0.123)				

Three seasonal dummise are included.

Estimated standard errors in parentheses.

LR test of the restrictions: $F(4, 49) = 1.28$.

Table 6: The short-run parameters in the hiring and the unemployment relations

	$\Delta \ln v_t$	$\Delta \ln h_{t-1}$	$\Delta \ln u_{t-1}$	h_{ecm}	uv_{ecm}
$\Delta \ln h_t$	0.148	-0.205	-0.313	-0.477	—
s.e.	(0.036)	(0.090)	(—)	(0.069)	
$\Delta \ln u_t$	—	—	0.285	—	-0.583
s.e.			(0.059)		(0.044)

h_{ecm} and uv_{ecm} are the lagged deviations from the estimated relations in Table 3. Three seasonal dummies are included in both models.

Test of the over-identifying restrictions: $\chi^2(5) = 1.33$.

Table 7: Covariance/Correlation matrix of the residuals

	$\hat{\eta}_{ht}$	$\hat{\eta}_{ut}$	$\hat{\eta}_{vt}$
$\hat{\eta}_{ht}$	0.0041	-0.0008	-0.0666
$\hat{\eta}_{ut}$	0.0000	0.0028	-0.0207
$\hat{\eta}_{vt}$	-0.0010	0.0002	0.0533

Wald test of zero-correlation between the three residual series: $\chi^2(3) = .27$.

Figure 1: Hirings, Unemployment and Vacancies in Denmark 1974q3 - 1988q4

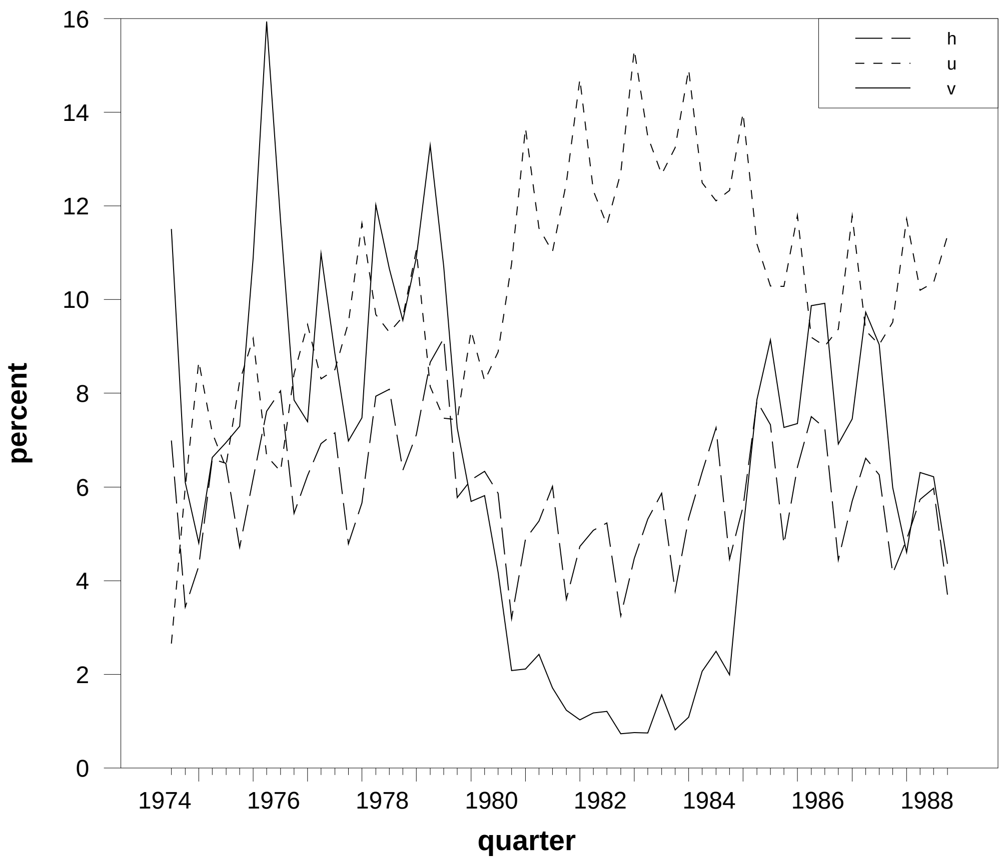

Figure 2: The deviations from the identified cointegration relations

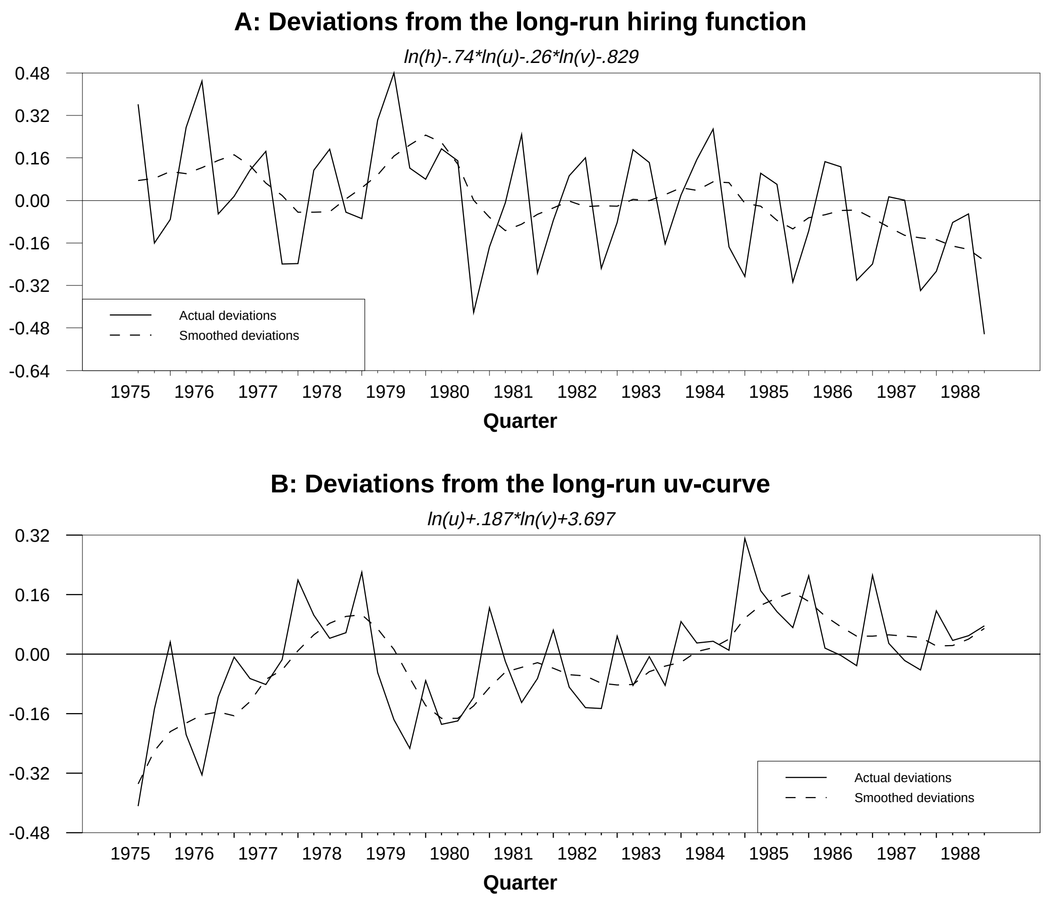

Figure 3: The uv-relations

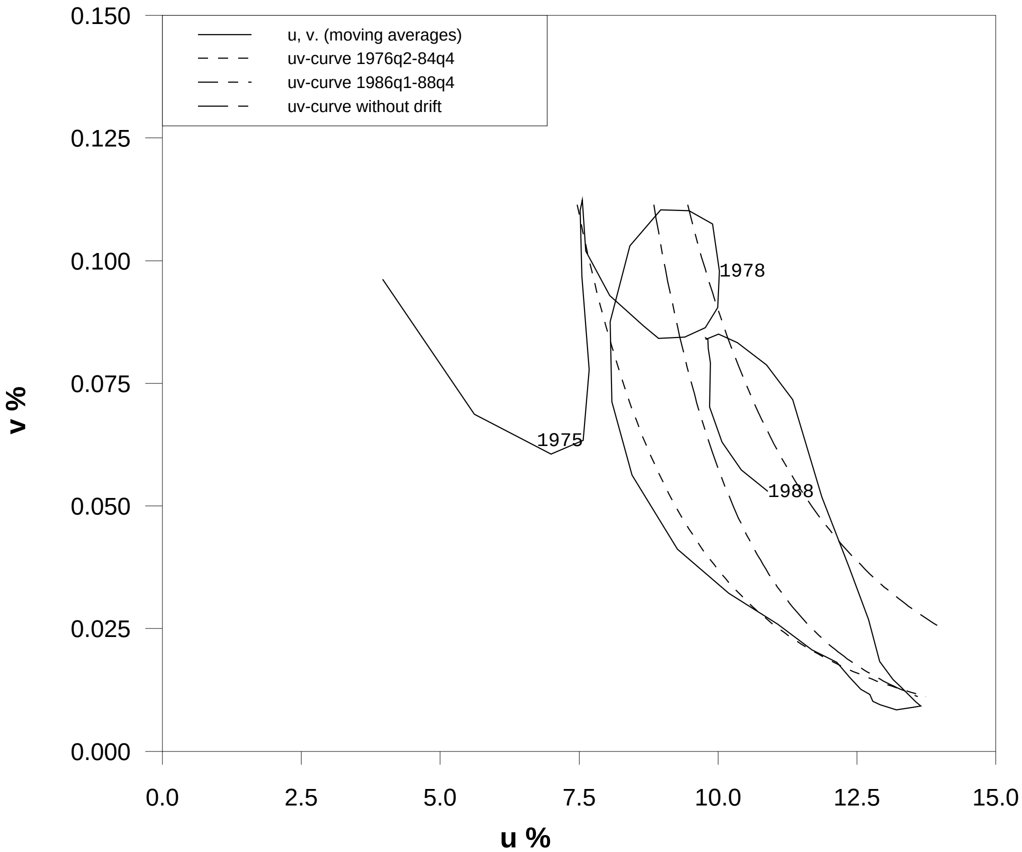

Figure 4: The responses of a one standard error shock to $\hat{\eta}_{vt}$