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by Alexander Totzek and Hans-Werner Wohltmann

Barro-Gordon revisited: Reputational equilibria in a New Keynesian Model

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Abstract

The aim of this paper is to solve the inconsistency problem à la Barro and Gordon within a New Keynesian model and to derive time-consistent (stable) interest rate rules of Taylor-type. We find a multiplicity of stable rules. In contrast to the Kydland/Prescott-Barro/Gordon approach, implementing a monetary rule where the cost and benefit resulting from inconsistent policy coincide – which implies a net gain of inconsistent policy behavior equal to zero – is not optimal. Instead, the solution can be improved by moving into the time-consistent area where the net gain of inconsistent policy is negative. We moreover show that under a standard calibration, the standard Taylor rule is stable in the case of a cost-push shock as well as under simultaneous supply and demand shocks.

JEL classification: A20; E52; E58
Keywords: Optimal monetary policy; New Keynesian macroeconomics; Reputational equilibria; time-consistent simple rules

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1 Introduction

The IS/LM-AD/AS-model is still the standard framework for teaching undergraduate macroeconomics. However, it gets more and more criticized for several aspects. The most import points in our opinion are that the model is unable to deal with an explicit monetary policy rule with the interest rate as the central bank's policy instrument [see Romer (2000)] and that it is expressed in terms of the price level and not of inflation [see Walsh (2002)].[1] To get rid of these problems, Bofinger, Meyer, and Wollmershäuser (2006) introduce a static approximation of the standard New Keynesian model where the interest rate is the instrument of the monetary authority.[2] Within this framework, they are able to discuss different monetary policy regimes at an intermediate level.

Our paper moreover offers an approach which enables us to discuss both the commitment vs. discretion debate of the New Keynesian literature and the time-inconsistency problem à la Barro and Gordon (1983a,b) combined in a unified framework.[3] This can be worthwhile especially for teaching intermediate macroeconomics. The aim of this paper is thus to solve the inconsistency problem within the static New Keynesian model and to derive time-consistent (or: stable) interest rate rules of Taylor-type. In contrast to the famous Kydland/Prescott-Barro/Gordon approach, New Keynesian models also consider the demand side of the economy and the central bank cannot directly control for the inflation rate.[4] Instead, the mechanism is as follows. (i) The central bank commits itself to follow an interest rate rule of Taylor-type. (ii) Private agents form inflation expectations. (iii) The central bank sets the interest rate and the households adjust their consumption expenditures according to the Euler consumption equation. (iv) Inflation is then determined by expected future inflation and the output realization via the New Keynesian Phillips curve.

Our main findings are as follows. Under a completely standard calibration in-

[1] Other unsatisfactory features are the lack of a microfoundation and the exogeneity of expectations [see Romer (2000)].

[2] See Bofinger, Meyer, and Wollmershäuser (2009) for the corresponding open economy framework.

[3] Bofinger, Meyer, and Wollmershäuser (2006) already highlight that their approach can be extended for this exercise. However, they only point out that an output gap target above zero as in Barro and Gordon (1983a) leads to an inflation bias which was already shown by Clarida, Galí, and Gertler (1999) within the dynamic New Keynesian model.

[4] More precisely, Barro and Gordon (1983a,b) assume that the policymaker controls an instrument which has a direct connection to the inflation rate – for instance, the money growth rate. The economy is just represented by a Phillips curve and does not consider any demand effects.

cluding a time preference rate of the monetary authority equal to the long-run interest rate, the standard Taylor rule is stable in the presence of a cost-push shock. The central bank thus does not have an incentive to deviate from the announced rule and to switch over to the inconsistent policy regime. However, there exists a multiplicity of stable Taylor rules which are superior to the standard one. In contrast to the Kydland/Prescott-Barro/Gordon approach, implementing a monetary rule such that the cost and benefit resulting from inconsistent policy coincide – which implies a net gain of inconsistent policy behavior equal to zero – is not optimal. Instead, the solution can be enhanced by moving into the time-consistent area where the net gain of inconsistent monetary policy is negative. Moreover and in contrast to Barro and Gordon (1983a,b), there exists no stable monetary policy rule maximizing the welfare. The stable area furthermore becomes larger when assuming an additional term in the social loss function concerning interest rate stability. This implies that the reputation of the central bank naturally improves if the policy maker is also concerned about stabilizing the interest rate. Our results remain robust with respect to the analysis of simultaneous supply and demand shocks.

The remainder is organized as follows. Section 2 shortly describes the applied model. In Section 3, we turn to monetary policy issues including the optimal discretionary monetary policy, simple Taylor rules, and the incentive to deviate from the announced policy rule. We moreover derive the continuum of time-consistent Taylor rules, discuss the problem of finding an optimal stable rule, and check our results for robustness. The last section concludes.

2 The Model

Following Bofinger, Meyer, and Wollmershäuser (2006), we apply a static approximation of the microfounded canonical New Keynesian model. The model can be represented by a three-dimensional equation system including an IS curve, a Phillips curve, and a monetary policy rule. The IS curve is given by

$$x = a - br + \varepsilon_1 \tag{1}$$

where x denotes the output gap which is defined as the deviation of output from its efficient level. a is a constant. b represents the intertemporal elasticity of substitution. r is the real interest rate and ε_1 denotes a demand shock. As shown in the Appendix, the demand shock can also be interpreted as a shock to aggregate technology or innovation.[5]

The second building block of the model is the static approximation of the New Keynesian Phillips curve

$$\pi = \pi^e + \delta x + \varepsilon_2 \tag{2}$$

where π and π^e represent current and expected future inflation, respectively. δ is the slope of the Phillips curve. ε_2 represents a cost-push shock.

In contrast to the demand shock, the supply shock causes a trade-off for the monetary authority between stabilizing output and inflation. Therefore, we will restrict our analysis to cost-push shocks. However, we will re-consider the demand shock for a robustness check at the end of our analysis.

3 Monetary Policy

In the following we will discuss different types of policy regimes, namely the optimal discretionary monetary policy, D, the commitment regime à la Taylor, TR, and the regime under inconsistent policy, IP. Independently of the assumed type of monetary policy, the central bank seeks to minimize a social loss function.

As shown by Galí (2008, Chapter 4) and Woodford (2003, Chapter 6), the second order approximation of the households' utility function delivers a quadratic loss function which represents flexible inflation targeting in the spirit of Svensson (1999). The static approximation of this function is given by

$$V = (\pi - \pi^T)^2 + \lambda x^2 \tag{3}$$

where π^T represents the target inflation rate and $\lambda \in [0, 1]$ is the central bank's

[5] In the dynamic version of the standard New Keynesian model, it can moreover be shown that in the case of an expansionary but persistent technology shock, the resulting demand shock is contractionary because the current output level reacts less expansionary than its natural counterpart. The output gap consequently declines. In the case of a permanent innovation, the resulting technology shock however has an expansionary impact on the output gap.

preference parameter on stabilizing the output gap as in Bofinger, Meyer, and Wollmershäuser (2006, 2009). In the case $\lambda = 0$, the central bank's preferences represent a strict inflation targeting regime, i.e. the monetary authority is just concerned about stabilizing inflation.

Following Barro and Gordon (1983a,b), we additionally assume that the monetary authority's target of the output gap is positive, i.e. $x^T > 0$.[6] An economic rationale is that e.g. monopolistic distortions or taxes keep potential output below its efficient level [see Clarida, Galí, and Gertler (1999)]. Then the social loss is given by

$$V = (\pi - \pi^T)^2 + \lambda(x - x^T)^2 \tag{4}$$

3.1 The discretionary monetary policy regime

In this section, we will derive the optimal discretionary monetary policy. In this regime, the expected inflation rate is taken as given for the central bank since the monetary authority applies a sequential optimization. Therefore, it is unable to make credible announcements concerning the design of monetary policy that could influence private expectations.

The central bank minimizes the social loss (4) subjected to the Phillips curve (2).[7] Inserting the Phillips curve (2) in the social loss function (4) and optimizing the resulting equation with respect to the output gap yields the following first order condition:

$$\begin{aligned} & 2\delta(\pi^e + \delta x + \varepsilon_2 - \pi^T) + 2\lambda(x - x^T) = 0 \\ \Leftrightarrow \quad & x = -\frac{\delta}{\delta^2 + \lambda}(\pi^e - \pi^T + \varepsilon_2) + \frac{\lambda}{\lambda + \delta^2} x^T \end{aligned} \tag{5}$$

Following Barro and Gordon (1983a,b), we assume that private expectations about inflation are formed before the shocks occur. This implies that when forming expectations about inflation, the shocks (ε_1 and ε_2) are not included in the information

[6] An alternative approach to include the problem of time-inconsistency into the model would be to assume an asymmetric loss function [see Cukiermann and Gerlach (2003), Nobay and Peel (2003), or Ruge-Murcia (2003)]. However, there is no micro-foundation for such a loss function at all. We moreover want to remain as close as possible to the Barro and Gordon approach. In addition, there is empirical evidence for both approaches [see e.g. Ireland (1999) and Gerlach (2003)].

[7] Note that the IS curve is not a binding restriction in this case.

set of private agents [see also Walsh (2003), Ch. 8].

Inserting (5) in the Phillips curve and taking rational expectations conditional on the set of private information, I, with $E[\varepsilon_2|I] = 0$, yields the expected inflation rate under discretionary monetary policy.

$$\pi^e|_D = \pi^T + \frac{\lambda}{\delta}x^T \tag{6}$$

Note that in the case of a flexible inflation targeting regime implying $\lambda > 0$, expected inflation is above the central bank's target level when the monetary authority aims for a positive output gap.

Combining (5) and (6) yields the solution path of the output gap.

$$x|_D = -\frac{\delta}{\delta^2 + \lambda}\varepsilon_2 \tag{7}$$

The solution of the output gap is independent of the corresponding target level, x^T, and moreover coincides with the discretionary solution in the case where the central bank does not target a positive output gap, i.e. $x|_D^{x^T>0} = x|_D^{x^T=0}$ [cf. Clarida, Galí, and Gertler (1999)].

However, this does not hold for the solution of inflation. By inserting (6) and (7) in the Phillips curve, we obtain

$$\pi|_D = \pi^T + \frac{\lambda}{\delta}x^T + \frac{\lambda}{\lambda + \delta^2}\varepsilon_2 \tag{8}$$

The central bank's target level of the output gap represents an additional term [or: inflation bias] in the solution of inflation which pushes inflation above its target level. Since under rational expectations the model structure including the loss function is known by private agents, the intention of the central bank to push the output gap above its natural level fails. Instead, the solution of output remains unchanged and that of inflation is 'biased'.

This result also holds in the absence of a supply shock, i.e. $\varepsilon_2 = 0$. Moreover, equation (8) implies that inflation only coincides with its target level when the central bank's preferences represent strict inflation targeting ($\lambda = 0$). This is a very intuitive result since in this case, the central bank is not concerned about the output gap at all.

When combining (7) and (8), discretionary monetary policy can be expressed as a targeting rule [see Svensson (1999)] given by

$$x|_D - x^T = -\frac{\lambda}{\delta}\left[\pi|_D - \pi^T\right] \tag{9}$$

implying a negative relationship between the stabilization of inflation and the output gap at the respective target level.

Finally, the social loss under discretionary monetary policy can be derived by inserting the solutions of the output gap (7) and inflation (8) in the welfare function (4).

$$V|_D = \frac{\lambda}{\lambda + \delta^2}\left[\frac{\delta^2 + \lambda}{\delta} x^T + \varepsilon_2\right]^2 \tag{10}$$

If a cost-push shock is existent, i.e. $\varepsilon_2 \neq 0$, the loss is strictly positive as long as $\lambda > 0$.

3.2 Simple rule

In this section, we will derive the social loss when the central bank credibly commits itself to follow a simple monetary policy rule of Taylor-type and thus influences private expectations.

The Taylor rule is commonly represented as

$$i = i^T + k_\pi(\pi - \pi^T) + k_x(x - x^T) \tag{11}$$

where k_x and k_π are the elasticities of the nominal interest rate, i, with respect to the deviation of the output gap and the inflation rate from their respective target level. In the following, we will refer to them as Taylor rule coefficients. The real interest rate which is the argument of the IS curve (1) is then obtained from the nominal interest rate via the well-known Fisher equation.

Note that the following condition is necessary to ensure that the dynamic counterpart of our model has a unique and stable equilibrium [see Bullard and Mitra

(2002) or Walsh (2003), Chap. 5]

$$\delta(k_\pi - 1) + (1-\beta)k_x > 0 \tag{12}$$

where $\beta \in [0,1]$ is the discount factor of private households. The stability of the whole system thus crucially depends on the Taylor rule coefficients. In the following, we will assume that the Taylor principle, $k_\pi > 1$, and the condition $k_x > 0$ hold. Then the stability condition (12) is obviously satisfied.

i^T is the central bank's target level of the nominal interest rate which follows

$$i^T = r^T + \pi^T \tag{13}$$

The corresponding target level of the real interest rate, r^T, follows from the IS equation and is given by

$$r^T = \frac{1}{b}(a - x^T) \tag{14}$$

Note that the target level of the real interest rate coincides with its natural level, $r^n = \frac{a}{b}$, in the borderline case $x^T = 0$.[8]

Inserting the Taylor rule (11) with (13) and (14) in the IS curve (1), yields

$$\begin{aligned} x &= (1+bk_x)x^T - bk_x x + b(\pi^e - \pi^T) - bk_\pi(\pi - \pi^T) \\ \Leftrightarrow \quad x &= x^T + \frac{1}{1+bk_x}\left[b(\pi^e - \pi^T) - bk_\pi(\pi - \pi^T)\right] \end{aligned} \tag{15}$$

By plugging this expression into the Phillips curve (2) and taking rational expectations, we obtain the expected inflation rate under the monetary policy regime TR.

$$\pi^e|_{TR} = \pi^T + \frac{1+bk_x}{b(k_\pi - 1)}x^T \tag{16}$$

Under our assumptions concerning k_π and k_x, expected inflation exceeds the target level in the case of a positive target level of the output gap. However, this holds independently of the central bank's preferences on stabilizing output. By contrast

[8] Remark: A natural level denotes the value of a variable in a non-distorted framework.

and as shown in the last section, under discretionary monetary policy expected inflation exceeds its target level only in the case of flexible inflation targeting, $\lambda > 0$ [cf. equation (6)].

When combining (16) and the Phillips curve (2), we obtain the solution path of the output gap and the inflation rate

$$x|_{TR} = -\frac{bk_\pi}{\alpha}\varepsilon_2 \tag{17}$$

$$\pi|_{TR} = \pi^T + \frac{1 + bk_x}{b(k_\pi - 1)}x^T + \frac{1 + bk_x}{\alpha}\varepsilon_2 \tag{18}$$

where $\alpha \equiv 1 + b(k_x + \delta k_\pi)$. Equivalently to the case of the discretionary monetary policy, the solution of the output gap is independent of its target level. Hence, equation (17) also represents the solution of the borderline case $x^T = 0$ where the central bank targets a closed output gap. Again, this result does not hold for the solution of inflation since equation (18) is not only a function of the target level of inflation but also of the output gap target. The introduction of a positive target level of the output gap thus leads to higher inflation while the resulting output gap remains unchanged.

In order to obtain the social loss under the policy regime TR for arbitrary coefficients k_π and k_x, we finally insert the solutions of the output gap and inflation in the welfare function (4).

$$V|_{TR} = (1 + bk_x)^2 \left[\frac{1}{b(k_\pi - 1)}x^T + \frac{1}{\alpha}\varepsilon_2\right]^2 + \lambda \left[x^T + \frac{bk_\pi}{\alpha}\varepsilon_2\right]^2 \tag{19}$$

From (19) it directly follows that the social loss approaches infinity, if k_π tends to unity. In this limit case the social loss exceeds that under discretionary monetary policy (10). The rationale is the reaction of expected inflation which according to (16) also tends to infinity, if $k_\pi \to 1$.

The following numerical example however shows that when applying a standard calibration, the social loss under discretionary monetary policy clearly exceeds that under TR. The Taylor rule regime is then preferable. As standard in the New Keynesian literature, we assume the intertemporal elasticity of substitution, b, to be equal to one. Under commonly chosen deep parameters, the slope of the Phillips curve is 0.0858 implying an average price duration of four quarters and an annual nominal interest rate of about 4%. Following Svensson (1999), we set the flexible inflation targeting parameter λ to 0.5. We arbitrarily assume the target level of the output gap to be 0.1 implying that the potential output level is 10% higher than the current one. The shock impact is normalized to one.

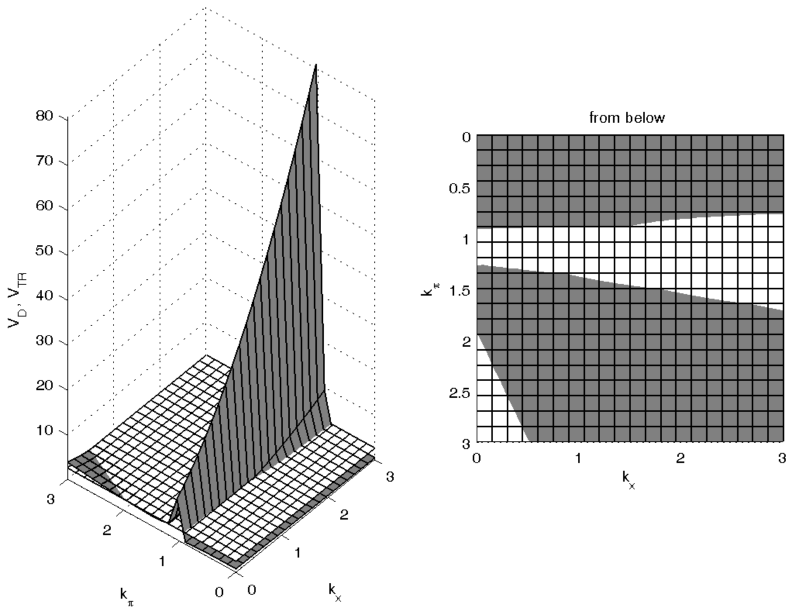

Figure 1: Comparing the social loss under the regime TR [grey area] and D [white area]

Under this standard calibration, Figure 1 depicts the social loss in the regime TR for different combinations of k_π and k_x [grey area] as well as the loss under regime D [white shaded area]. The social loss under TR can exceed that under D, when the Taylor rule coefficient k_π tends to unity. However, if k_π is sufficiently larger than one and k_x is sufficiently larger than zero, $V|_{TR}$ is preferable to $V|_D$.[9]

In the case of standard Taylor rule, i.e. $k_\pi = 1.5$ and $k_x = 0.5$, the social loss in the regime TR is significantly lower than in the discretionary case. The numerical evaluation of $V|_D$ and $V|_{TR}$ yields

$$V|_{TR} = 2.0120 < 2.4956 = V|_D \qquad (20)$$

The credible commitment to the standard monetary policy rule yields a significant welfare gain via the expectation channel which is not active in the discretionary

[9]Note that there also exists a small area with $V|_{TR} > V|_D$, if k_x is sufficiently small and k_π is sufficiently large.

case.[10] The social loss under D exceeds that under TP by about 24%.

3.3 Inconsistent Policy

In this section, we will show that the central bank has an incentive to deviate from the announced Taylor rule if the monetary authority is faced with a purely static one-period optimization approach.

If the central bank credibly announces to follow a specifically calibrated Taylor rule, expected inflation is tied at a given level according to (16). However, the central bank can then achieve a welfare gain by re-optimizing in a discretionary manner. In this case, the monetary authority will not implement the announced policy rule. We will refer to this policy regime as inconsistent monetary policy, IP.

The maximization problem under IP is given by

$$\begin{aligned} \max_{x,\,\pi} \quad & L = (\pi - \pi^T)^2 + \lambda(x - x^T)^2 \\ \text{s.t.} \quad & \pi = \pi^e + \delta x + \varepsilon_2 \\ & \pi^e = \pi^e|_{TR} \end{aligned} \tag{21}$$

As in the discretionary case, the first order condition with respect to the output gap is given by

$$x|_{IP} = -\frac{\delta}{\lambda + \delta^2}\left[\pi^e|_{TR} - \pi^T + \varepsilon_2\right] + \frac{\lambda}{\lambda + \delta^2} x^T \tag{22}$$

Equation (22) just deviates from (5) via the formation of the expected inflation rate.

By inserting (16) in (22), we obtain the solution of the output gap under the inconsistent policy regime, IP.

$$x|_{IP} = \frac{1}{\lambda + \delta^2}\left[\lambda - \frac{\delta(1 + bk_x)}{b(k_\pi - 1)}\right] x^T - \frac{\delta}{\lambda + \delta^2}\varepsilon_2 \tag{23}$$

[10] Remark: In the dynamic New Keynesian framework, Clarida, Galí, and Gertler (1999) and Woodford (1999) show that a commitment strategy can be advantageous even in the absence of the inflation bias. See Dennis (2010) for an insightful discussion of this topic.

Again, the solution of inflation is then obtained via the Phillips curve (2).

$$\pi|_{IP} = \pi^T + \frac{\lambda}{\lambda + \delta^2}\left[\frac{\alpha - \delta b}{b(k_\pi - 1)}x^T + \varepsilon_2\right] \tag{24}$$

In contrast to the purely discretionary monetary policy and the regime under commitment to a Taylor rule, the solution of the output gap (23) now depends on its target level. This implies that the central bank's intention to push output above its natural level can now be achieved. However and in line with our previous finding under discretionary monetary policy, inflation only coincides with its target level when the central bank follows strict inflation targeting.

The combination of (23) and (24) yields the same targeting rule as in the discretionary case [cf. (9)]:

$$x|_{IP} - x^T = -\frac{\lambda}{\delta}\left[\pi|_{IP} - \pi^T\right] \tag{25}$$

The social loss under inconsistent monetary policy can finally be obtained by inserting (23) and (24) in (4).

$$V|_{IP} = \frac{\lambda}{\lambda + \delta^2}\left[\frac{\alpha - \delta b}{b(k_\pi - 1)}x^T + \varepsilon_2\right]^2 \tag{26}$$

By definition, $V|_{TR}$ must exceed $V|_{IP}$, i.e. the deviation from the announced Taylor rule yields a welfare enhancement. The numerical evaluation for the two policy regimes under the standard parameterizations delivers

$$V|_{IP} = 1.6875 < 2.0120 = V|_{TR} \tag{27}$$

The welfare gain resulting from the inconsistent policy regime when announcing a standard Taylor rule is thus about 19%.

3.4 Time-Consistent Simple Rules

In this section, we will derive a continuum of time-consistent (or: stable) simple rules. This is done by assuming a long-run planning horizon of the monetary authority as in Barro and Gordon (1983a,b).

As shown in last section, the central bank has an incentive to re-optimize, if it credibly announces to follow a commitment strategy. If its announcements are not credible, private expectations are given for the central bank and the monetary authority must follow a discretionary monetary policy. By assuming that the central bank looses its reputation, if it deviates once from its announcement, i.e. if the central bank switches over to the regime IP, one can find both a continuum of time-consistent and inconsistent simple rules. More precisely, we assume that the central bank looses its reputation for exactly one period when deviating once, i.e. a punishment interval of one period.[11] The announcements of the central bank will then no longer be credible such that private agents will form their expectations as in the discretionary case.

In this framework à la Barro and Gordon (1983a,b), the central bank is faced with a simple cost-benefit calculation where the benefit, B, is the welfare gain resulting from the inconsistent policy in comparison to the implementation of the announced Taylor rule, $V|_{TR} - V|_{IP}$. The cost, C, on the other hand is the discounted next period welfare loss resulting from the sacrifice in the central bank's reputation, $V|_D - V|_{TR}$. The net gain, N, of the inconsistent policy is then given by[12]

$$N = B - C = (V|_{TR} - V|_{IP}) - \frac{1}{1+z}(V|_D - V|_{TR}) \tag{28}$$

Equation (28) implies that there exists a subjective time preference rate, z, such that the net gain is zero. In this case, the monetary authority is indifferent between switching over to IP and executing the announced rule. This critical time preference rate, z^*, is given by

$$z^* = \frac{V|_D - V|_{TR}}{V|_{TR} - V|_{IP}} - 1 \qquad \Leftrightarrow \qquad N = 0 \tag{29}$$

[11] Alternatively, one can analyze the case where the central bank looses its reputation for all times when deviating once. However, the qualitative results remain unchanged.

[12] By assuming that the central bank looses its reputation for all times when deviating once from the announcement, the total gain resulting from IP would be given by

$$N' = B - C' = (V|_{TR} - V|_{IP}) - \sum_{i=1}^{\infty}\left(\frac{1}{1+z'}\right)^i (V|_D - V|_{TR})$$
$$\Leftrightarrow \quad N' = (V|_{TR} - V|_{IP}) - \frac{1}{z'}(V|_D - V|_{TR})$$

A central bank with a rate of time preference larger than z^*, i.e. a monetary authority whose planning horizon is rather short, consequently expects $N > 0$. Correspondingly, a more long-run oriented central bank, $z < z^*$, expects $N < 0$, i.e. it would not switch over to the regime IP.

In our numerical example, the critical time preference rate z^* is 0.4903. The corresponding discount factor, $1/(1 + z^*)$, is then about 0.67 implying a central bank whose planning horizon is rather short. In the monetary economics literature, a basic assumption is that the subjective preference rate coincides with the long-run real interest rate.[13] The latter is typically chosen to be equal to 4% which is clearly below the critical subjective rate of time preference. Hence, such a central bank is farsighted and does not yield a net gain from inconsistent monetary policy. Assuming $z = 0.04$, the standard Taylor rule is stable.

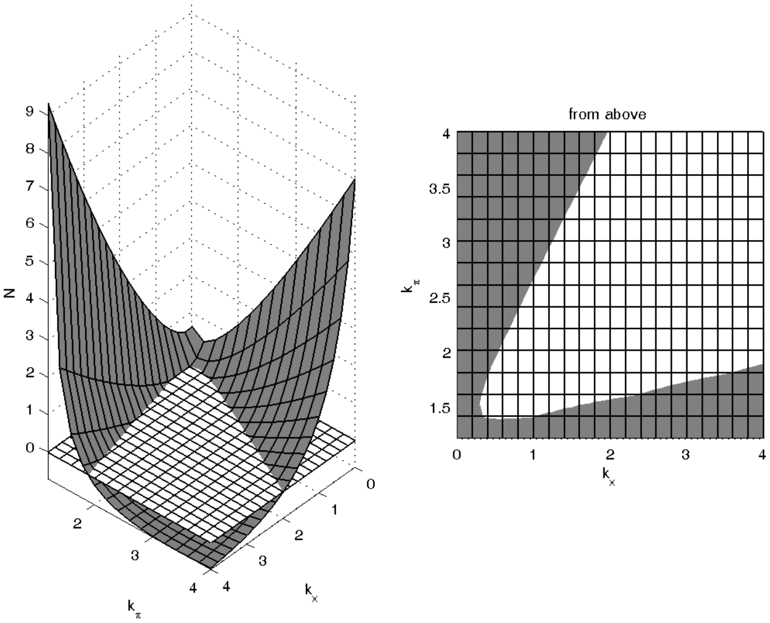

Figure 2: Stable and unstable simple rules

Figure 2 illustrates the net gain resulting from the deviation from the announced simple rule for different combinations of Taylor rule coefficients. The white shaded area indicates the zero plane. The intersection of this area and the net gain function consequently delivers the specific k_π/k_x-combinations which result in $N = 0$. This

[13] As already mentioned, the social loss function (3) can be derived by a second order approximation of the household's utility function. Hence, the discount factors of the central bank and the households must coincide. Further note that when assuming a zero-inflation steady state, the real interest rate is equal to its nominal counterpart in the long-run.

implies that the k_π/k_x-combinations within the white area on the right-hand side of Figure 2 deliver $N < 0$ since the cost exceeds the benefit of inconsistent policy. These rules are consequently time-consistent.[14] The limit case $N = 0$ just holds for the boundary of this area. It can moreover be observed from Figure 2 that a Taylor rule with $k_x = 0$ is never stable given our assumptions.

As in Barro and Gordon (1983a), there exists a continuum of reputational equilibria where the central bank has no incentive to deviate from the announced monetary policy. The monetary authority will consequently not switch over to inconsistent policy. Announcing such a rule is necessarily credible. The next step is to find an optimal stable interest rate rule.

Since the policy maker follows a Taylor rule – satisfying the Taylor principle and $k_x > 0$ – the monetary authority has to control for two parameters, k_π and k_x. In contrast to the Kydland/Prescott-Barro/Gordon approach where the central bank can directly control the inflation rate, the optimal stable policy must now be determined in a three-dimensional space. Moreover, the optimal choice in Barro and Gordon (1983a) is a point in the intersection of the function B and C since it minimizes the social loss. In our approach, the social loss resulting from the k_π/k_x-combinations implying $N = 0$ can however be improved when moving into the stable area.

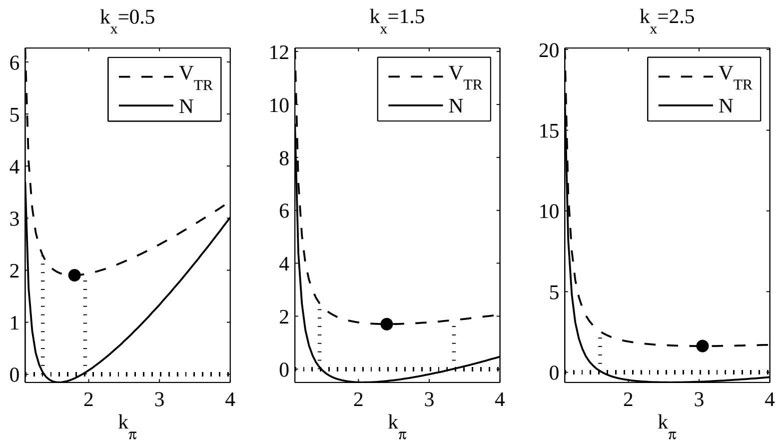

Figure 3: The net gain N and the social loss under TR for different Taylor rule coefficients

For illustrating this issue, we will fix one Taylor coefficient for the moment

[14] Remark: When disregarding the Taylor principle, i.e. by allowing $k_\pi < 0$, we moreover obtain a second continuum of stable simple rules. This is a plausible result since the social loss under TR (19) is a function in k_π of fourth-order implying that for a given k_x there can exist up to four different real-valued solutions of k_π.

in order to obtain a two-dimensional decision problem. Figure 3 illustrates the corresponding partial social loss function under TR (dashed lines) and the net gain from the deviation from the announced rule (solid lines) when holding the Taylor rule coefficient on the output gap constant.[15] The figure indicates that under constant k_x there always exists a coefficient on inflation such that the resulting social loss is optimal. These (restricted) optima are denoted with a black dot. Moreover, Figure 3 shows that the k_π/k_x-combinations which result in $N = 0$ can be enhanced as they do not yield an optimal loss for given k_x.

For instance, the combination $k_x = 0.5$ and $k_\pi = 1.35$ yields $N = 0$. However, this combination of Taylor rule coefficients is not optimal as the social loss declines when increasing the Taylor rule coefficient on inflation up to 1.8. This combination is within the stable area which is a general result as all (restricted) optima are within the stable area and not in the intersection of B and C. By contrast, the optimal policy choice in Barro and Gordon (1983a) is in the intersection of cost and benefit resulting from inconsistent monetary policy. Furthermore, Figure 3 indicates that the larger the Taylor rule coefficient, k_x, the larger is the second one, k_π, in the optimum. This result also holds vice versa when k_π is constant.

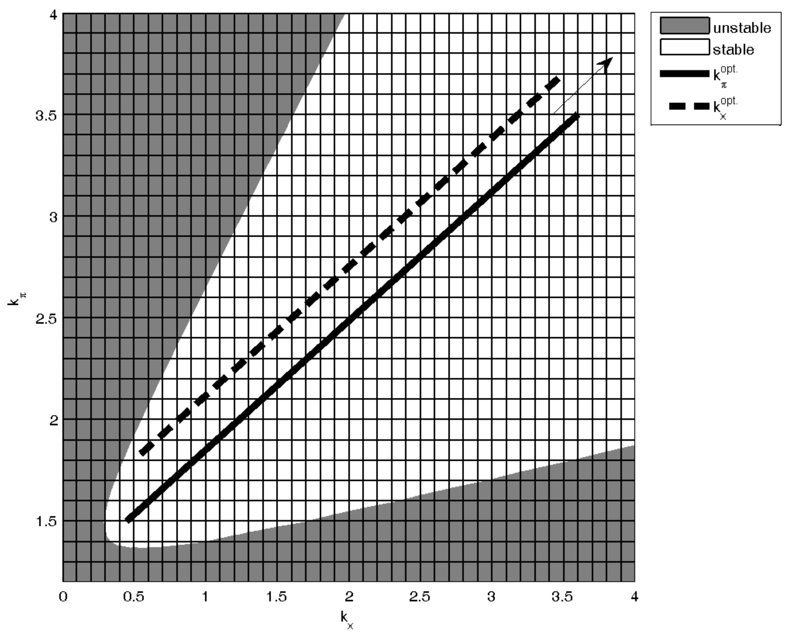

Figure 4: Optimal Taylor rule coefficients

[15] When holding k_π constant, we obtain a totally equivalent outcome.

Figure 4 depicts these optimal coefficients, $k_\pi^{opt.}$ and $k_x^{opt.}$, when respectively taking k_x and k_π as given. It can directly be observed that the corresponding lines of partial optima do not have an intersection. Consequently, there does not exists a globally optimal choice of Taylor rule parameters when both coefficients are variable. Since the loss decreases in both coefficients [cf. Figure 3], the minimal loss is obtained in the limit case $k_\pi \to \infty$ and $k_x \to \infty$. In Figure 4, this fact is indicated with a black arrow.

All in all, the unsatisfactory feature that there exists a multiplicity of stable monetary policy rules remains [see Clarida, Galí, and Gertler (1999)]. However, Taylor rule coefficients resulting in $N = 0$ can be improved when moving into the stable area which contrasts with Barro and Gordon (1983a). Moreover, there exists no globally optimal stable Taylor rule.

3.5 Extensions

For checking the robustness of the previous results, we will now turn to a monetary authority which is also concerned about stabilizing the interest rate as in Svensson (2000). The extended social loss function then looks as follows

$$V' = (\pi - \pi^T)^2 + \lambda(x - x^T)^2 + \gamma(i - i^T)^2 \tag{30}$$

with $\lambda > \gamma > 0$. Although (30) cannot be derived from the household's utility function in the canonical New Keynesian framework, Kobayashi (2008) and Teranishi (2008) show that in a framework where the financial sector has a non-trivial role, the social loss function should include a positive weight on a financial variable. We set γ to 0.05.

The proceeding for obtaining the social loss for the different policy designs is equivalent to that in the previous sections.[16]

[16] See the Appendix for the different social losses in the different policy regimes with $\gamma > 0$ and the respective derivations.

3.5.1 The Impact of Stabilizing the Interest Rate

The numerical evaluation of the different policy regimes in the case of the cost-push shock yields

$$V|'_D = 6.5586 > V|'_{TR} = 2.1044 > V|'_{IP} = 1.6987 \tag{31}$$

resulting in a critical rate of time preference equal to 9.9790. the standard Taylor rule is only unstable when assuming very myopic considerations of the central bank. Under our standard calibration including $z = 0.04$, the standard Taylor rule however remains stable when extending the social loss function.

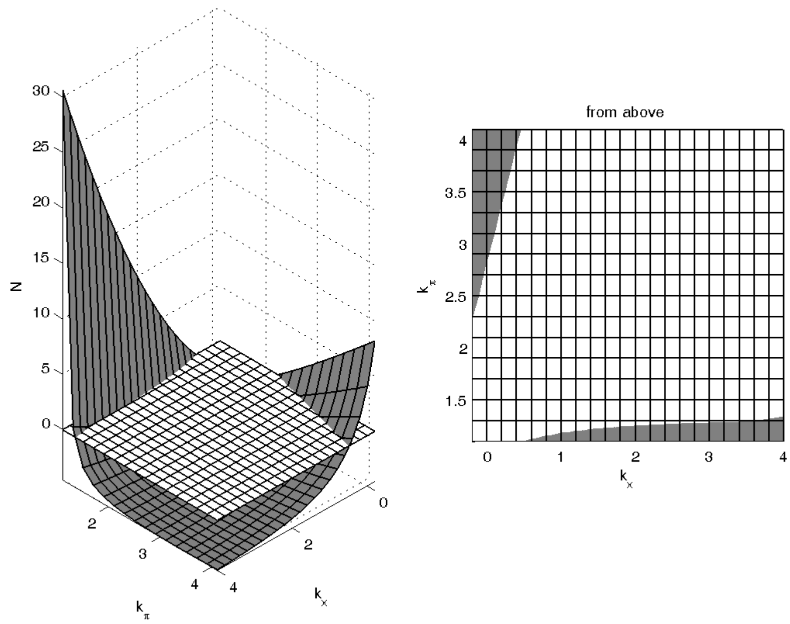

Figure 5: Stable and unstable simple rules with $\gamma > 0$

Figure 5 shows the net gain resulting from the deviation of the announced Taylor rule for different k_π/k_x-combinations. The qualitative results also remain unchanged in comparison to the case with $\gamma = 0$. However, it is worth mentioning that the stable area, i.e. the set of k_π/k_x-combinations that do not cause any incentive for the monetary authority to switch over to inconsistent policy, becomes larger when assuming $\gamma > 0$. This implies that the reputation of the central bank naturally improves if the policy maker is also concerned about stabilizing the interest rate. The rationale is that the social loss under discretionary monetary policy relatively increases more than those under inconsistent policy and under the Taylor rule. As a result, the cost of inconsistent policy, C, increases more than the benefit, B.

In contrast to the case $\gamma = 0$, a Taylor rule with $k_x = 0$ may be time-consistent if the Taylor rule coefficient on inflation is rather small.

3.5.2 Simultaneous Supply and Demand Shocks

As already mentioned and as shown in the Appendix, the demand shock can be interpreted as a shock to aggregate technology which is typically assumed to be expansionary ($\psi > 0$). In our notation this implies $\varepsilon_1 < 0$ since a technology shock causes the natural level, y^*, to increase more than the current one, y, leading to a decline in the output gap, $x \equiv y - y^*$.

It is a well-known that a pure demand shock can be totally compensated by discretionary monetary policy in the case $\gamma = 0$. This can be directly observed from the IS curve (1).[17] Hence, we will analyze simultaneous supply and demand shocks in the following. For the sake of simplicity, we will also normalize the impact of the demand shock to one, i.e. $\varepsilon_1 = -1$.

Then, the numerical evaluation of the different policy regimes yields

$$V|'_D = 6.4250 > V|'_{TR} = 2.7489 > V|'_{IP} = 1.6992 \qquad (32)$$

implying a critical subjective discount factor equal to 2.5020. Hence, the standard Taylor rule still remains stable when additionally considering an technological innovation.

Figure 6 shows the net gain resulting from a deviation from the announced Taylor rule for a continuum of k_π/k_x-combinations. The continuum of stable Taylor rules now becomes smaller when considering the contractionary demand shock [cf. Figure 5].[18] The rationale is that the social loss under the commitment strategy increases relative to those under inconsistent and discretionary policy. Consequently, the cost resulting from the loss in the central bank's reputation declines leading to an increasing incentive to switch over to inconsistent policy.

[17] In this specific case, the discretionary monetary policy would obviously be the first best solution.
[18] As expected, the stable area becomes larger when assuming the demand shock to be expansionary, too, i.e. $\varepsilon_1 = \varepsilon_2 = 1$.

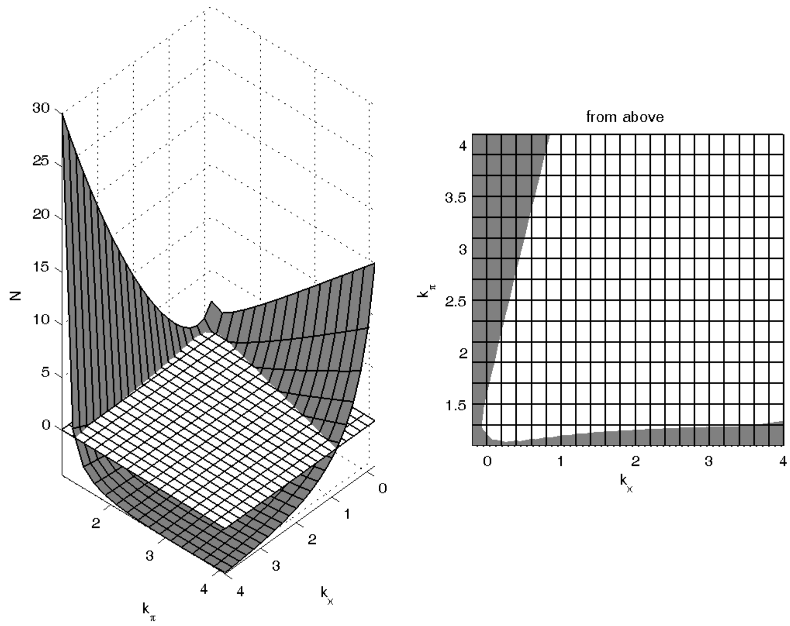

Figure 6: Stable and unstable simple rules – simultaneous supply and demand shocks ($\varepsilon_1 < 0$ and $\varepsilon_2 > 0$)

4 Conclusion

We implement the Kydland/Prescott-Barro/Gordon approach in a static approximation of the canonical New Keynesian model. Within this framework, we are able to discuss both the commitment vs. discretion debate of the New Keynesian literature and the time-inconsistency problem of Barro and Gordon (1983a,b) in a unified framework. This can be worthwhile especially for teaching macroeconomics at an intermediate level.

We first show that commitment strategies can be advantageous to discretionary monetary policy. Second, we show that these policy rules cause the monetary authority to deviate from their announcements since the re-optimization yields a welfare gain. By assuming a long-run planning horizon of the central bank and that the monetary authority looses its reputation when switching over to inconsistent policy, we find a continuum of stable interest rate rules of Taylor-type. In contrast to the Kydland/Prescott-Barro/Gordon approach, implementing a monetary rule such that the cost and benefit resulting from inconsistent policy coincide, is not optimal. Instead, the solution can be enhanced by moving into the stable area where the net gain of inconsistent monetary policy behavior is negative. By introducing an additional term in the social loss function concerning interest rate stabilization, the continuum of stable Taylor rules becomes larger. This implies that the reputation of the monetary authority naturally improves when it is also concerned about stabilizing the interest rate. Third, we find that under a standard calibration including a time preference rate equal to the long-run interest rate, the standard Taylor rule is time-consistent for the cost-push shock as well as for simultaneous supply and demand shocks. Fourth, there does not exist a stable Taylor rule in explicit form which minimizes the social loss.

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Appendix

A Derivation of the demand shock

Firms need only one input factor, labor which is denoted by N. The production function is then simply given by

$$Y = \Psi N \tag{A.1}$$

where Y is output and Ψ represents a shock to aggregate productivity.[19]

Under flexible prices the marginal costs, MC, are constant. They are obtained by cost minimization

$$MC = \frac{W}{\Psi} = const. \qquad \Leftrightarrow \qquad w = \psi \tag{A.2}$$

The Euler consumption equation and the labor supply equation follow from the utility maximization of the representative household.

$$y = a - br \tag{A.3}$$

$$w = \eta n + \frac{1}{b} y \tag{A.4}$$

where η represents the Frisch elasticity of labor supply.

Subtracting the natural level from the Euler equation yields[20]

$$x \equiv y - y^* = a - br - y^* \tag{A.5}$$

Log-linearizing the production function (A.1) expressed in natural levels and insert-

[19] In the following, capital letters denote variables in non-log-linearized form, while small latter denote log-linearized variables.

[20] In the following, an asterisk denotes a natural variables, i.e. without any nominal or real rigidity.

ing the resulting equation in (A.5) considering (A.2) and (A.4), yields

$$x = a - br - \frac{b + b\eta}{1 + b\eta}\psi \tag{A.6}$$

Since $\frac{b+b\eta}{1+b\eta} > 0$, a shock to aggregate technology can thus be interpreted as a contractionary demand shock. The rationale is that the natural level increases more to an expansionary shock to productivity than the distorted actual output level such that the difference – the output gap – decreases.

B Social Losses with $\gamma > 0$ and $\varepsilon_1 \neq 0$ and $\varepsilon_2 \neq 0$

The modified loss function now contains an additional term concerning nominal interest rate stabilization.

$$V' = (\pi - \pi^T)^2 + \lambda(x - x^T)^2 + \gamma(i - i^T)^2 \tag{B.1}$$

B.1 Simple rule

The proceeding for deriving the social loss is equivalent to that in the main text.

Since we analyze calibrated instead of optimal simple rules, private expectations are thus not altered by the modified loss function and still follow (16)

$$\pi^e|_{TR} = \pi^T + \frac{1 + bk_x}{b(k_\pi - 1)}x^T \tag{B.2}$$

When combining (16) and the Phillips curve (2), we obtain the solution path of the output gap and the inflation rate.

$$x|'_{TR} = \frac{1}{\alpha}\varepsilon_1 - \frac{bk_\pi}{\alpha}\varepsilon_2 \tag{B.3}$$

$$\pi|'_{TR} = \pi^T + \frac{1 + bk_x}{b(k_\pi - 1)}x^T + \frac{\delta}{\alpha}\varepsilon_1 + \frac{1 + bk_x}{\alpha}\varepsilon_2 \tag{B.4}$$

The additional demand shock causes an upward-pressure on both inflation and the output gap.

Finally, we need the solution for the nominal interest rate. Therefore, we insert

(B.3) and (B.4) in the Taylor rule (11).

$$i|'_{TR} = i^T + \frac{k_\pi + bk_x}{b(k_\pi - 1)}x^T + \frac{\delta k_\pi + k_x}{1 + b(k_x + k_\pi \delta)}\varepsilon_1 + \frac{k_\pi}{1 + b(k_x + k_\pi \delta)}\varepsilon_2 \tag{B.5}$$

In order to obtain the social loss under the policy regime TR for arbitrary coefficients k_π and k_x, we insert the solutions of the output gap, inflation, and the interest rate in the welfare function (4).

$$\begin{aligned} V|'_{TR} = & \left[\frac{1 + bk_x}{b(k_\pi - 1)}x^T + \frac{\delta}{\alpha}\varepsilon_1 + \frac{1 + bk_x}{\alpha}\varepsilon_2\right]^2 + \lambda\left[\frac{1}{\alpha}\varepsilon_1 - \frac{bk_\pi}{\alpha}\varepsilon_2 - x^T\right]^2 \\ & + \gamma\left[\frac{k_\pi + bk_x}{b(k_\pi - 1)}x^T + \frac{\delta k_\pi + k_x}{1 + b(k_x + k_\pi \delta)}\varepsilon_1 + \frac{k_\pi}{1 + b(k_x + k_\pi \delta)}\varepsilon_2\right]^2 \end{aligned} \tag{B.6}$$

Naturally, the latter expression simplifies to (19), if $\gamma = 0$ and $\varepsilon_1 = 0$.

B.2 Optimal discretionary monetary policy

In contrast to the case where the demand shock is absent, the monetary authority must now consider the IS curve in the optimization approach.

Inserting the Phillips curve and the Euler consumption equation in the social loss function and optimizing the resulting expression with respect to the output gap delivers the following first-order condition

$$\begin{aligned} & \lambda(x - x^T) + \delta(\pi - \pi^T) - \frac{\gamma}{b}(i - i^T) = 0 \\ \Leftrightarrow \qquad & i - i^T = \frac{\lambda b}{\gamma}(x - x^T) + \frac{b\delta}{\gamma}(\pi - \pi^T) \end{aligned} \tag{B.7}$$

Inserting (1) and re-arranging yields

$$x = x^T + \frac{b\gamma}{\lambda b^2}(\pi^e - \pi^T) - \frac{\delta b^2}{\lambda b^2 + \gamma}(\pi - \pi^T) + \frac{\gamma\delta}{\lambda b^2 + \gamma}\varepsilon_1 + \varepsilon_2 \tag{B.8}$$

By inserting this expression in the Phillips curve and taking rational private expectations, we obtain

$$\pi^e|'_D = \pi^T + \frac{\lambda b^2 + \gamma}{b(\delta b - \gamma)}x^T \tag{B.9}$$

The solution of the output gap is then given by

$$\pi|_D' = \pi^T + \frac{\lambda b^2 + \gamma}{b(b\delta - \gamma)} x^T + \frac{\gamma\delta}{b^2(\lambda + \delta^2) + \gamma}\varepsilon_1 + \frac{\lambda b^2 + \gamma}{b^2(\lambda + \delta^2) + \gamma}\varepsilon_2 \tag{B.10}$$

Inserting (B.9) and (B.10) in the Phillips curve, yields the solution of the output gap

$$x|_D' = \frac{\gamma}{b^2(\lambda + \delta^2) + \gamma}\varepsilon_1 - \frac{\delta b^2}{b^2(\lambda + \delta^2) + \gamma}\varepsilon_2 \tag{B.11}$$

The solution of the nominal interest rate is finally obtained by inserting (B.10) and (B.11) in (B.7)

$$i = i^T + \frac{\delta + b\lambda}{b\delta - \gamma} x^T + \frac{b(\lambda + \delta^2)}{b^2(\lambda + \delta^2) + \gamma}\varepsilon_1 + \frac{\delta b}{b^2(\lambda + \delta^2) + \gamma}\varepsilon_2 \tag{B.12}$$

B.3 Inconsistent monetary policy

The proceeding is totally analogous to the main text. Therefore, we will only present the resulting solutions of the output gap, the inflation rate, and the nominal interest rate. They are given by

$$\begin{aligned}
\pi|_{IP}' - \pi^T &= \frac{(\lambda b^2 + \gamma)(1 + b(k_x + \delta k_\pi)) + \delta b^2(\gamma k_x - \lambda b)}{(b^2(\lambda + \delta^2) + \gamma) b(k_\pi - 1)} x^T \\
&\quad + \frac{\gamma\delta}{b^2(\lambda + \delta^2) + \gamma}\varepsilon_1 + \frac{\lambda b^2 + \gamma}{b^2(\lambda + \delta^2) + \gamma}\varepsilon_2 && \text{(B.13)} \\
x|_{IP}' - x^T &= \frac{\gamma(1 + bk_x) - \delta b(1 + b(k_x + \delta k_\pi) - \delta b)}{(k_\pi - 1)(b^2(\lambda + \delta^2) + \gamma)} x^T \\
&\quad + \frac{\gamma}{b^2(\lambda + \delta^2) + \gamma}\varepsilon_1 - \frac{\delta b^2}{b^2(\lambda + \delta^2) + \gamma}\varepsilon_2 && \text{(B.14)} \\
i|_{IP} - i^T &= \frac{\lambda b(1 + bk_x) + \delta(1 + b(k_x + \delta(k_\pi + bk_x)))}{(k_\pi - 1)(b^2(\lambda + \delta^2) + \gamma)} x^T \\
&\quad + \frac{b(\lambda + \delta^2)}{b^2(\lambda + \delta^2) + \gamma}\varepsilon_1 + \frac{\delta b}{b^2(\lambda + \delta^2) + \gamma}\varepsilon_2 && \text{(B.15)}
\end{aligned}$$