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ABSTRACT

Optimal Redistributive Taxation with Both Extensive and Intensive Responses*

This paper characterizes optimal income taxation when individuals respond along both the intensive and extensive margins. Individuals are heterogeneous across two dimensions: specifically, their skill and disutility of participation. Preferences over consumption and work effort can differ with respect to the level of skill, with only the Spence-Mirrlees condition imposed. Employing a tax perturbation approach, we derive an optimal tax formula that generalizes previous results by allowing for income effects and extensive margin responses. We provide a sufficient condition for optimal marginal tax rates to be nonnegative everywhere. We discuss the relevance of this condition with analytical examples and numerical simulations using U.S. data.

JEL Classification: H21, H23

Keywords: optimal tax formula, tax perturbation, random participation

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I Introduction

This paper provides an optimal nonlinear income tax formula that solves the redistribution problem when individuals respond along both the intensive (in-work effort) and extensive (participation) margins. For this purpose, we consider an economy where individuals are heterogeneously endowed with two unobserved characteristics: their skill level and disutility of participation. Because of the former heterogeneity, employed workers typically choose different earnings levels, while because of the latter heterogeneity, at any skill level, only some individuals choose to work. The government can only condition taxation on endogenous earnings and not on the exogenous characteristics whose heterogeneity in the population lies at the origin of the redistribution problem.[1] Therefore, positive marginal tax rates are necessary to transfer income from rich to poor individuals, while inherently distorting intensive labor supply decisions. Moreover, when individuals of a given skill level experience an increase in either the tax level they paid when employed or in the benefit for the nonemployed, some of them leave the labor force. Such an increase in the so-called *participation tax*[2] thereby generates distortions along the extensive margin of the labor supply.

Since Mirrlees (1971), the optimal tax problem has usually been solved by searching for the best incentive-compatible allocation using optimal control. However, while this method has proved successful, it lacks economic intuition. We instead derive the optimal tax formula by measuring the effects of a change in marginal tax rates on a small interval of income levels.[3] This"tax perturbation approach" emphasizes the economic mechanisms at work, it also faces a difficulty in that because of the nonlinearity of the tax schedule, when an individual responds to a tax perturbation with a change in her labor supply, the induced change in gross income affects in turn the marginal tax rate, thereby inducing a further labor supply response. To take this"circular process"into account, we define behavioral elasticities along the optimal nonlinear tax schedule. Because of this redefinition, we can then intuitively express optimal marginal rates as a function of the social welfare weights, the skill distribution and the behavioral elasticities. The formula obtained then generalizes previous results by allowing for income effects and extensive margin responses.

We also provide a sufficient condition under which optimal marginal tax rates are nonnegative. Clarifying the restrictions that ensure this result remains an issue in the optimal income

[1] Because the latter heterogeneity matters only for participation decisions, the government faces a multidimensional screening problem that reduces to the "random participation" model introduced by Rochet and Stole (2002).

[2] Which equals the tax level plus the benefit for the nonemployed, such that each additional worker increases the government's revenue by the level of the participation tax.

[3] We verify in Appendix B that the solution derived using the tax perturbation approach is consistent with the Mirrleesian approach in terms of incentive-compatible allocations.

tax literature with only intensive responses.[4] Intuitively, the optimality of nonnegative marginal tax rates holds whenever social welfare weights are decreasing along the skill distribution, so that the distortions induced by positive marginal tax rates are compensated for by the equity gains of transferring income from high-skilled to low-skilled workers. By adding an extensive margin response, we find a condition on the ratio one minus the social welfare weights over the extensive behavioral response. Strikingly, the optimal participation tax equals this ratio when individuals respond only along the extensive margin. When both margins are included, we show that optimal marginal tax rates are nonnegative whenever this ratio decreases along the skill distribution. While our sufficient condition is expressed in terms of endogenous variables, we discuss its relevance in practice and provide examples of specifications on primitives where this condition holds. For instance, when the government has a Maximin objective, we argue that the additional restrictions are fairly weak.

Using U.S. data, we calibrate the model to illustrate the quantitative implications of our optimal tax formula. These simulations suggest that a more responsive extensive margin reduces marginal tax rates by a significant amount without qualitatively changing its profile. In our sensitivity analysis, marginal tax rates are always positive. However, for the least skilled workers, participation taxes are typically negative under a Benthamite criterion, while they are always positive under Maximin. The literature on optimal taxation in the pure extensive model has typically found the latter. The optimality of a negative participation tax at the bottom of the earnings distribution is interpreted as a case for an Earned Income Tax Credit (EITC) rather than Negative Income Tax (NIT) (see Saez 2002). We provide numerical examples with a strictly positive lower bound for the earnings distribution,[5] with a negative participation tax at this minimum (as for the EITC) and nonnegative marginal tax rates above this minimum (as for the NIT).

Our paper contributes to the literature aiming to improve the usefulness of optimal income taxation useful for applied public finance. For many years following Mirrlees' (1971) seminal work, the various theoretical developments focused on useful technical refinements but provided little economic intuitions. The first important advance in this regard was made when Atkinson (1990), Piketty (1997) and Diamond (1998) reexpressed optimality conditions derived from the Mirrlees model in terms of behavioral elasticities in the absence of income effects. Saez (2001) provided a second important advance by deriving an optimal tax formula using the tax perturbation approach.[6] He took into account the above-mentioned "circular process" by expressing

[4] See, e.g., Mirrlees (1971), Sadka (1976), Seade (1982), Werning (2000) or Hellwig (2007) or the counterexamples given by Choné and Laroque (2009b).

[5] We assume a strictly positive minimum for the skill distribution.

[6] Christiansen (1981) first introduced the tax perturbation approach. However, he did not derive any implications for the optimal income tax, his focus being instead on the optimal provision of public goods and the structure of commodity taxation. Revecz (1989) also proposed a method to derive an optimal income tax formula in terms

the optimal tax formula in terms of the unappealing notion of a"virtual"[7] earnings distribution and verified the consistency of his solution with that in Mirrlees (1971). Further, Saez (2001) allowed for income effects. In this study, we avoid the use of virtual densities because of our redefinition of the behavioral elasticities.

The above-mentioned studies neglected labor supply responses along the extensive margin, while the empirical labor supply literature emphasizes that labor supply responses along the extensive margin are much more important (see, e.g., Heckman (1993)). Saez (2002) derived an optimal tax formula in an economy with both intensive and extensive margins. For this purpose, he developed a model where agents choose from a finite set of occupations, each associated with an exogenous level of earnings. However, Saez (2002) provided no analytical result for the mixed case where both the extensive and intensive margins matter. Moreover, Saez (2002) focused essentially on the EITC/NIT debate about whether the working poor should receive greater transfers than nonemployed individuals, while we discuss the conditions under which marginal tax rates should be nonnegative. Moreover, our formula allows for income effects.[8] Finally, our treatment of the intensive margin is more standard and allows us to consider a continuous earnings distribution. In our view, this appears more appropriate for the study of marginal tax rates than the discrete occupation setting of Saez (2002).[9]

The paper is organized as follows. Section II presents the model. Section III derives the optimal tax formula in terms of behavioral elasticities using the tax perturbation method. This section also compares the tax formula obtained with the existing the literature. Section IV provides a sufficient condition to obtain nonnegative optimal marginal tax rates and gives examples where this condition is satisfied. Section V presents the simulations for the U.S. In the appendix, we develop the formal model. In particular, we solve for the optimal allocations using the typical optimal control approach. We then verify that this solution is consistent with that derived in the main body of the text.

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of elasticities but did not consider the above-mentioned circular process. Hence, his solution was inconsistent with Mirrlees (1971) (see Revecz (2003) and Saez (2003)). Using a tax perturbation method, Piketty (1997) derived the optimal nonlinear income tax schedule under Maximin. However, he too neglected to take into account the circular process, though this had no consequence as he assumed away the income effects. Roberts (2000) derived an optimal tax formula under Benthamite preferences.

[7] Saez (2001, p.215) defines the virtual density at earnings level z as "the density of incomes that would take place at z if the tax schedule $T(.)$ were replaced by the linear tax schedule tangent to $T(.)$ at level z".

[8] The formal model in the appendix in Saez (2002) allows for the possibility of income effects. Moreover, the appendix in Saez (2000) (the NBER version of Saez (2002)) extends his optimal tax formula with both extensive and intensive responses to the case of a continuum of earnings but still without income effects.

[9] Boone and Bovenberg (2004) introduce search decisions in the Mirrlees model. This additional margin has a similar flavor to the participation decision. However, their specification of the search technology implies that any individual with a skill level above (below) an endogenous threshold searches (does not search) at the maximum intensity (does not search).

II The model

II.1 Individuals

Each individual derives utility from consumption C and disutility from labor supply or effort L. More effort implies higher earnings Y, the relationship between the two also depends on the individual's skill endowment w. The literature typically assumes that $Y = w \times L$. To avoid this unnecessary restriction on the technology, we express individuals' preferences in terms of the observables (C and Y) and the individuals' exogenous characteristics (particularly w). This also enables us to consider cases where the preferences over consumption C and effort L are skill dependent. The skill endowments are exogenous, heterogeneous and unobserved by government. Hence, consumption C is related to earnings Y through the tax function $C = Y - T(Y)$.

The empirical literature has emphasized that a significant part of the labor supply responses to tax reforms are concentrated along the extensive margin. We integrate this feature by considering a specific disutility of participation, which makes a difference in the level of utility only between *workers* (for whom $Y > 0$) and the *nonemployed* (for whom $Y = 0$). This disutility may arise from commuting, job-search effort, or the reduced amount of time available for home production. However, for some people, employment has value *per se*, as at least some enjoy working (see, e.g., Polachek and Siebert (1993, p. 101)). Some individuals would even feel stigmatized if they had no job. Let χ denote an individual's disutility of participation net of this intrinsic job value. We assume that people are endowed with different positive or negative (net) disutility of participation χ. As for the skill endowment, χ is exogenous and unobserved by the government. Because of this additional heterogeneity, individuals with the same skill level may take different participation decisions. This is consistent with the observation that in all OECD countries, skill-specific employment rates always lie inside $(0, 1)$.

For tractability, we require that the *intensive* labor supply decisions Y of individuals that have chosen to work depend only on their skill and not on their net disutility of participation. To obtain this simplification, we need to impose some separability in individuals' preferences. We specify the utility function of an individual of type (w, χ) as:

$$\mathcal{U}(C, Y, w) - \mathbb{I}_{Y>0} \cdot \chi \tag{1}$$

where $\mathbb{I}_{Y>0}$ is an indicator variable equal to one if the individual works and zero otherwise. The *gross* utility function $\mathcal{U}(.,.,.)$ is twice-continuously differentiable and concave with respect to (C, Y). Individuals derive utility from consumption C and disutility from labor supply, so $\mathcal{U}'_C > 0 > \mathcal{U}'_Y$. Finally, we impose the strict-single crossing (Spence-Mirrlees) condition. We assume that, starting from any positive level of consumption and earnings, more skilled workers need to be compensated with a smaller increase in their consumption to accept a unit rise in earnings. This implies that the marginal rate of substitution $-\mathcal{U}'_Y(C, Y, w) / \mathcal{U}'_C(C, Y, w)$

decreases in the skill level. Hence we have:

$$\mathcal{U}''_{Yw}(C,Y,w)\cdot\mathcal{U}'_C(C,Y,w)-\mathcal{U}''_{Cw}(C,Y,w)\cdot\mathcal{U}'_Y(C,Y,w)>0 \tag{2}$$

The distribution of skills is described by the density $f(.)$, which is continuous and positive over the support $[w_0, w_1]$, with $0 < w_0 < w_1 \leq +\infty$. It is worth noting that the lowest skill is positive. The size of the total population is normalized to 1 so $\int_{w_0}^{w_1} f(w)\,dw = 1$. The distribution of χ conditional on the skill level w is described by the conditional density $k(.,w)$ and the cumulated density function $K(.,w)$, with $k(\chi,w) \stackrel{\text{def}}{\equiv} \partial K(\chi,w)/\partial\chi$. The density is continuously differentiable. It is worth noting that w and χ can be distributed independently or not. The support of the distribution is $(-\infty, \chi^{\max}]$, with $\chi^{\max} \leq +\infty$. The assumption about the lower bound is made for tractability as it ensures a positive mass of employed individuals at each skill level.

Each agent solves the following maximization problem:

$$\max_Y \quad \mathcal{U}(Y-T(Y),Y,w)-\mathbb{I}_{Y>0}\cdot\chi$$

where the choice of Y can be decomposed into a *participation* decision (i.e., $Y=0$ or $Y>0$) and an *intensive* choice (i.e., the value of Y when $Y>0$). For a worker of type (w,χ), selecting a positive earnings level Y to maximize $\mathcal{U}(C,Y,w)$ subject to $C=Y-T(Y)$ amounts to solving:

$$U_w \stackrel{\text{def}}{\equiv} \max_Y \quad \mathcal{U}(Y-T(Y),Y,w) \tag{3}$$

In particular, two workers with the same skill level but with a different disutility of participation χ face the same intensive choice program, thereby taking the same decisions along the intensive margin.[10] Let Y_w be the intensive choice of a worker of skill w, and let C_w be the corresponding consumption level, so $C_w = Y_w - T(Y_w)$. The gross utility of workers of skill w therefore equals $U_w = \mathcal{U}(C_w, Y_w, w)$. We ignore the non-negativity constraint on Y_w when solving the intensive choice program. We verify in our simulations that the minimum of the earnings distribution is always positive (given that we assume $w_0>0$). Therefore, the possibility of bunching from the nonnegativity constraint can be neglected.

[10] The key assumption for this result is that preferences over consumption and earnings for employed agents vary only with skill and do not depend on the net disutility of participation χ. Such a property is obtained under weakly separable preferences of the form:

$$W(C,Y,w,\chi)=\begin{cases} V(\mathcal{U}(C,Y,w),w,\chi) & if \quad Y>0 \\ \mathcal{U}^0(C,w,\chi) & \quad\quad\;\; Y=0 \end{cases}$$

where W is discontinuous at $Y=0$. $V(.,.,.)$ is an aggregator increasing in its first argument. Function $\mathcal{U}(.,.,.)$ verifies $\mathcal{U}'_C > 0 > \mathcal{U}'_Y$ and (2). $\mathcal{U}^0(.,.,.)$ describes the preference of the non-employed and increases in its first argument. Functions $\mathcal{U}(.,.,.)$, $\mathcal{U}^0(.,.,.)$ and $V(.,.,.)$ are twice-continuously differentiable over respectively $\mathbb{R}^+\times\mathbb{R}^+\times[w_0,w_1]$, $\mathbb{R}^+\times[w_0,w_1]\times\mathbb{R}^+$ and $\mathbb{R}\times[w_0,w_1]\times\mathbb{R}^+$. Finally, we assume that for given levels of C, Y, w and b, the function $\chi \mapsto V(\mathcal{U}(C,Y,w),w,\chi)-\mathcal{U}^0(b,w,\chi)$ is decreasing and tends to $+\infty$ whenever χ tends to the lowest bound of its support. All results derived in this paper can be obtained under this more general specification, the additional difficulty being only notational.

We now turn to the participation decisions. Let $b = -T(0)$ denote the consumption level for individuals out of the labor force. We refer to b as the welfare benefit. If an individual of type (w, χ) chooses to work, she obtaines utility $U_w - \chi$. If she chooses not to participate she obtains $\mathcal{U}(b, 0, w)$. An individual of type (w, χ) then chooses to work if $U_w - \chi \geq \mathcal{U}(b, 0, w) \Leftrightarrow \chi \leq U_w - \mathcal{U}(b, 0, w)$. Therefore, the density of workers of skill w is given by $h(w)$ defined as:

$$h(w) \stackrel{\text{def}}{\equiv} K(U_w - \mathcal{U}(b, 0, w), w) \cdot f(w) \tag{4}$$

with some abuse of notation as $h(w)$ does not make explicit the dependence of $h(.)$ with respect to b and U_w. The function $h(w)$ is twice-continuously differentiable, increasing in U_w and decreasing in b, with respective derivatives $h'_U(w)$ and $h'_b(w)$. The cumulative distribution is $H(w) = \int_{w_0}^{w} h(n) \cdot dn$. There are then $H(w_1)$ employed individuals and $1 - H(w_1)$ nonemployed individuals.

II.2 Behavioral elasticities

We define the behavioral elasticities from the intensive choice program (3) and the extensive margin decision (4). When the tax function is differentiable, the first-order condition associated with the intensive choice (3) implies:

$$1 - T'(Y_w) = -\frac{\mathcal{U}'_Y}{\mathcal{U}'_C} \tag{5}$$

where the derivatives of $\mathcal{U}(.)$ are evaluated at (C_w, Y_w, w). When the tax function is twice differentiable, the second-order condition is:[11]

$$\mathcal{U}''_{YY} - 2\left(\frac{\mathcal{U}'_Y}{\mathcal{U}'_C}\right)\mathcal{U}''_{CY} + \left(\frac{\mathcal{U}'_Y}{\mathcal{U}'_C}\right)^2 \mathcal{U}''_{CC} - T''(Y_w) \cdot \mathcal{U}'_C \leq 0 \tag{6}$$

Whenever the second-order condition (6) strictly holds, which we henceforth assume throughout the remainder of this section, the first-order condition (5) implicitly defines[12] earnings Y_w as a function of the skill level and the tax function. The elasticity α_w of earnings with respect to the skill level equals:[13]

$$\alpha_w \stackrel{\text{def}}{\equiv} \frac{w}{Y_w} \cdot \dot{Y}_w = -\frac{\frac{w}{Y_w} \cdot \left[\mathcal{U}''_{Yw} \cdot \mathcal{U}'_C - \mathcal{U}''_{Cw} \cdot \mathcal{U}'_Y\right]}{\left[\mathcal{U}''_{YY} - 2\left(\frac{\mathcal{U}'_Y}{\mathcal{U}'_c}\right)\mathcal{U}''_{CY} + \left(\frac{\mathcal{U}'_Y}{\mathcal{U}'_c}\right)^2 \mathcal{U}''_{CC} - T''(Y_w) \cdot \mathcal{U}'_C\right] \cdot \mathcal{U}'_C} \tag{7}$$

[11] By the concavity of $U(.,.,.)$ on (C, Y), the second-order condition is satisfied if the tax schedule is locally linear or convex (so that $T''(.) \geq 0$), or is not "too concave".

[12] In addition, one has to assume that among the possible multiple local maxima of $Y \mapsto U(Y - T(Y), Y, w)$, a single maximum corresponds to the global maximum. If program $Y \mapsto U(Y - T(Y), Y, w^*)$ admits two global maxima for a skill level w^*, workers of a skill level w slightly above (below) w^* would strictly prefer the higher (lower) maximum because of the strict single-crossing condition (see Equation (2)). Hence, function $w \mapsto Y_w$ exhibits a discontinuity at skill w^*. Moreover, once again through the strict single-crossing condition, function $w \mapsto Y_w$ is nondecreasing. Therefore, it is discontinuous on a set of skill levels that is at worst countable (and at best empty). Because the skill distribution is assumed continuous without any mass point, the latter set is of zero measure.

[13] See Appendix A.

Let $\hat{h}(Y)$ and $\hat{H}(Y)$ denote respectively the density and cumulated density function of the earnings distribution among employed individuals, with $\partial \hat{H}(Y)/\partial Y = \hat{h}(Y)$. For all skill levels, one has $\hat{H}(Y_w) \equiv H(w)$. From Equation (7), $h(w)$ and $\hat{h}(Y_w)$ are thus related by:

$$\frac{Y_w}{w} \cdot \alpha_w \cdot \hat{h}(Y_w) \equiv h(w) \tag{8}$$

If the left-hand side of (6) is nil, then the function $Y \mapsto \mathcal{U}(Y - T(Y), Y, w)$ becomes typically constant around w. Therefore, individuals of type w are indifferent between a range of earnings levels, so the function $n \mapsto Y_n$ becomes discontinuous at skill $n = w$. The same phenomenon also occurs when the tax function is downward discontinuous at Y_w ($T''(Y)$ tends to minus infinity, so (6) is violated). Conversely, bunching of types occurs when $\alpha_w = 0$ (i.e. $T''(Y)$ tends to plus infinity). This corresponds to a kink in the tax function. From here on, we assume that $T(.)$ is differentiable and hence exclude bunching. However, this assumption is relaxed in the appendix, where we solve the model in terms of incentive-compatible allocations and consider what happens when bunching takes place.

We now consider different elementary tax reforms and compute how they affect the intensive (3) and extensive (4) choices. The first elementary tax reform captures the *substitution* effect around the actual tax schedule. The marginal tax rate $T'(Y)$ is decreased by a small amount τ over the range of earnings $[Y_w - \delta, Y_w + \delta]$. In so doing, the level of tax at earnings level Y_w is kept constant, as is C_w. This reform is illustrated by the left-hand side panel in Figure 1.

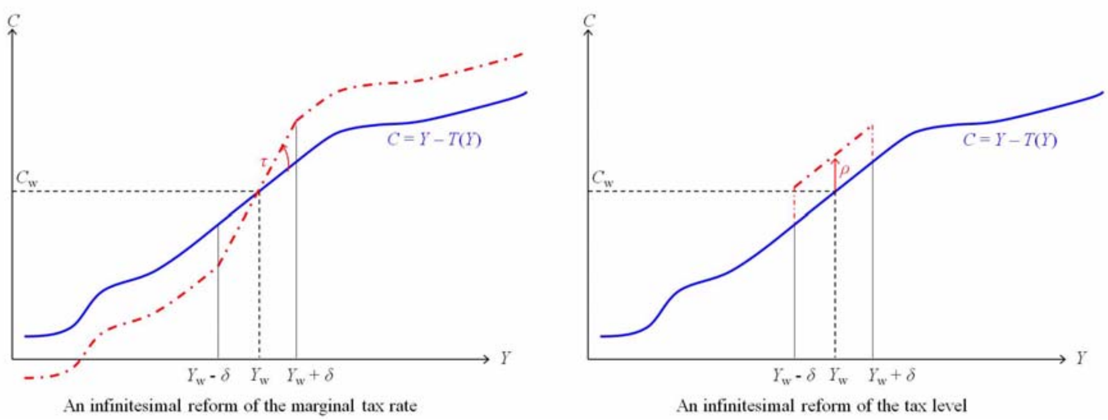

Figure 1: Tax reforms around Y_w defining behavioral responses ε_w and α_w

The behavioral response to this reform by a worker of skill w is captured by the *compensated* elasticity of earnings with respect to $1 - T'(Y)$:[14]

$$\varepsilon_w \stackrel{\text{def}}{\equiv} \frac{1 - T'(Y_w)}{Y_w} \cdot \frac{\partial Y}{\partial \tau} = \frac{\mathcal{U}'_Y}{Y_w \cdot \left[\mathcal{U}''_{YY} - 2\left(\frac{\mathcal{U}'_Y}{\mathcal{U}'_c}\right)\mathcal{U}''_{CY} + \left(\frac{\mathcal{U}'_Y}{\mathcal{U}'_c}\right)^2 \mathcal{U}''_{CC} - T''(Y_w) \cdot \mathcal{U}'_C\right]} > 0 \tag{9}$$

[14]The elasticity ε_w is *compensated* in the sense that the tax level is unchanged at earnings level Y_w.

When the marginal tax rate is decreased by τ, a unit rise ΔY_w in earnings generates a higher gain $\Delta C_w = (1 - T'(Y_w) + \tau)\,\Delta Y_w$ of consumption. Therefore, workers substitute earnings for leisure. Finally, this reform only has a second-order effect on U_w and thereby on the participation decisions.[15]

The next elementary tax reform captures the *income* effect around the actual tax schedule. The level of tax decreases by a small lump sum ρ over a range in earnings $[Y_w - \delta, Y_w + \delta]$. This reform is illustrated by the right-hand side panel in Figure 1. Along the intensive margin, the behavioral response for a worker of skill w to this reform is captured by the income effect:

$$\eta_w \stackrel{\text{def}}{\equiv} \frac{\partial Y}{\partial \rho} = \frac{\left(\frac{\mathcal{U}'_Y}{\mathcal{U}'_C}\right)\mathcal{U}''_{CC} - \mathcal{U}''_{CY}}{\mathcal{U}''_{YY} - 2\left(\frac{\mathcal{U}'_Y}{\mathcal{U}'_c}\right)\mathcal{U}''_{CY} + \left(\frac{\mathcal{U}'_Y}{\mathcal{U}'_c}\right)^2\mathcal{U}''_{CC} - T''(Y_w)\cdot\mathcal{U}'_C} \tag{10}$$

This term can be either positive or negative. However, when leisure is a normal good, the numerator is positive; hence, the income effect (10) is negative.

The "ρ-reform"illustrated by the right-hand side panel in Figure 1 also induces some individuals of skill w to enter the labor market. We capture this *extensive* response for individuals of skill w with:

$$\kappa_w \stackrel{\text{def}}{\equiv} \frac{1}{h(w)}\cdot\frac{\partial h(w)}{\partial \rho} = \frac{h'_U(w)}{h(w)}\cdot\mathcal{U}'_C = \frac{k(U_w - \mathcal{U}(b,0,w))}{K(U_w - \mathcal{U}(b,0,w))}\cdot\mathcal{U}'_C > 0 \tag{11}$$

which stands for the percentage of variation in the number of workers with skill level w. Finally, we measure the elasticity of participation when, combined with a uniform decrease in the tax level by ρ, the welfare benefit b rises by ρ (i.e., when $T(Y) + b$ is kept constant). This reform captures income effects along the extensive margin. The (endogenous) semi-elasticity of the number of employed individuals of skill w with respect to such a reform equals:

$$\nu_w \stackrel{\text{def}}{\equiv} \kappa_w + \frac{h'_b(w)}{h(w)} = \frac{k(U_w - \mathcal{U}(b,0,w))}{K(U_w - \mathcal{U}(b,0,w))}\cdot\left[\mathcal{U}'_C(C_w, Y_w, w) - \mathcal{U}'_C(b,0,w)\right] \tag{12}$$

When the utility function $\mathcal{U}(.,.,.)$ is additively separable and concave in consumption and if $C_w > b$, $\mathcal{U}'_C(C_w, Y_w, w)$ is lower than $\mathcal{U}'_C(b,0,w)$. Income effects along the extensive margin are then negative, which corresponds to the “normal” case.

The behavioral responses given in (7), (9), (10), (11) and (12) are endogenous in that they depend on the skill level w, the earnings level Y and the tax function $T(.)$. In particular, the various responses along the intensive margin given in (7), (9) and (10) are standard (see, e.g., Saez (2001)), except for the presence of $T''(.)$ in the denominators. An exogenous increase in either w, τ, or ρ induces a direct change in earnings $\Delta_1 Y_w$. However, this change in turn modifies

[15] Decreasing $T'(.)$ by τ implies a rise ΔY_w of earnings, which itself increases C_w by $\Delta C_w = (1 - T'(Y_w) + \tau)\,\Delta Y_w$. Therefore the impact on U_w is given by $\Delta U_w = \Delta\mathcal{U}(C_w, Y_w, w) = [(1 - T'(Y_w) + \tau)\mathcal{U}'_C + \mathcal{U}'_Y]\,\Delta Y_w = \mathcal{U}'_C\cdot(\varepsilon_w Y_w/(1 - T'(Y_w)))\,\tau^2$ where the second equality follows (5) and (9) through $\Delta Y_w = (\varepsilon_w Y_w/(1 - T'(Y_w)))\,\tau$.

the marginal tax rate by $\Delta_1 T' = T''(Y_w) \times \Delta_1 Y_w$, thereby inducing a further change in earnings $\Delta_2 Y_w$. Therefore, a "circular process"takes place: the earnings level determines the marginal tax rate through the tax function, and the marginal tax rate affects the earnings level through the substitution effect. The term $T''(Y_w) \cdot \mathcal{U}'_C$ captures the indirect effects from to this circular process (in the words of Saez (2001), see also Saez (2003, p. 483) and Appendix A). Unlike Saez (2001), we do not define the behavioral responses along a hypothetical linear tax function, but along the actual (or later optimal) tax schedule, which we allow to be nonlinear. Therefore, our behavioral response parameters (7), (9) and (10) take into account the circular process and exhibit the term $T''(.)$ in their denominators.[16]

II.3 The government

The government's budget constraint takes the form:

$$b = \int_{w_0}^{w_1} (T(Y_w) + b) \cdot h(w) \cdot dw - E \tag{13}$$

where E is an exogenous amount of public expenditures. For each additional worker of skill w, the government collects taxes $T(Y_w)$ and saves welfare benefit b.

Turning now to the government's objective, we adopt a welfarist criterion that sums over all types of individuals a transformation $G(v, w, \chi)$ of individuals' utility v, with $G(.,.,.)$ twice-continuously differentiable and $G'_v > 0$. Given the labor supply decisions, the government's objective is:

$$\begin{aligned} \Omega \;=\; & \int_{w_0}^{w_1} \left\{ \int_{-\infty}^{U_w - \mathcal{U}(b,0,w)} G(U_w - \chi, w, \chi) \cdot k(\chi, w)\, d\chi \right. \\ & \left. + \int_{U_w - \mathcal{U}(b,0,w)}^{\chi^{\max}} G(\mathcal{U}(b,0,\chi), w, \chi) \cdot k(\chi, w)\, d\chi \right\} f(w)\, dw \end{aligned} \tag{14}$$

The social transformation function $G(.,.,.)$ depends not only on the utility levels v of individuals, but also on their exogenous type (w, χ). Our social welfare function generalizes the Bergson-Samuelson social objective, which does not depend on the individuals' type. With the latter criterion, the preferences for redistribution would be induced by the concavity of $G(.)$; that is, by $G''_{vv} < 0$. Our specification also encompasses the case where function $G(.,.,.)$ equals a type-specific exogenous weight times the individuals' level of utility. The government's desire to compensate for heterogeneous skill endowments would require $G''_{vw} < 0$.

Let λ denote the marginal social cost of public funds E. For a given tax function $T(.)$,we denote g_w (respectively g_0) the (average and endogenous) marginal social weight associated with

[16] See also Blumquist and Simula (2010).

employed individuals of skill w (to the nonemployed), expressed in terms of public funds by:

$$g_w \overset{\text{def}}{\equiv} \mathbb{E}_\chi \left[\frac{G'_v\left(U_w - \chi, w, \chi\right) \cdot \mathcal{U}'_C\left(C_w, Y_w, w\right)}{\lambda} | w, \chi \leq U_w - \mathcal{U}\left(b, 0, w\right) \right] \tag{15}$$

$$g_0 \overset{\text{def}}{\equiv} \mathbb{E}_{w,\chi} \left[\frac{G'_v\left(\mathcal{U}\left(b, 0, w\right), w, \chi\right) \cdot \mathcal{U}'_C\left(b, 0, w\right)}{\lambda} | \chi > U_w - \mathcal{U}\left(b, 0, w\right) \right] \tag{16}$$

The government values an additional dollar to the $h(w)$ employed individuals of skill w (to the $1 - H(w_1)$ nonemployed) as g_w times $h(w)$ dollars (g_0 times $1 - H(w_1)$ dollars). The government thus wishes to transfer income from individuals whose social weight is below 1 to those whose social weight is above 1. As will be made clear below, g_0 and the shape of the marginal social weights $w \mapsto g_w$ entirely summarize how the government's preferences influence the optimal tax policy. The only properties we have are that g_0 and g_w are positive. In particular, the shape of $w \mapsto g_w$ can be non-monotonic, decreasing or increasing and we can have g_0 above or below g_{w_0}. However, a government that has a redistributive motive would typically adopt a decreasing shape $w \mapsto g_w$ of social welfare weights, as discussed in Section IV.

III Optimal marginal tax rates

III.1 Derivation of the optimal marginal tax formula

The government's problem comprises finding a nonlinear income tax schedule $T(.)$ and a welfare benefit b to maximize the social objective (14), subject to the budget constraint (13) and the labor supply decisions along both margins. In this section, we directly derive the optimal tax formula through a small perturbation of the optimal tax function. Following Mirrlees (1971), Appendix B solves the government's problem in terms of incentive-compatible allocations, using optimal control techniques and verifies that both methods lead to the same optimal tax formulae.

Proposition 1 *The optimal tax policy must verify*

$$\frac{T'(Y_w)}{1 - T'(Y_w)} = \mathcal{A}(w) \cdot \mathcal{B}(w) \cdot \mathcal{C}(w) \tag{17}$$

$$0 = \mathcal{C}(w_0) \tag{18}$$

$$1 - g_0 \left(1 - \int_{w_0}^{w_1} h(n) \cdot dn\right) - \int_{w_0}^{w_1} g_n \cdot h(n) \cdot dn = \int_{w_0}^{w_1} \left\{\eta_n \cdot T'(Y_n) + \nu_n \cdot \left(T(Y_n) + b\right)\right\} \cdot h(n) \cdot dn \tag{19}$$

where

$$\mathcal{A}(w) \overset{def}{\equiv} \frac{\alpha_w}{\varepsilon_w} \qquad \mathcal{B}(w) \overset{def}{\equiv} \frac{H(w_1) - H(w)}{w \cdot h(w)}$$

$$\mathcal{C}(w) \overset{def}{\equiv} \frac{\int_w^{w_1} \left\{1 - g_n - \eta_n \cdot T'(Y_n) - \kappa_n \left(T(Y_n) + b\right)\right\} \cdot h(n) \cdot dn}{H(w_1) - H(w)}$$

Equation (17) summarizes the trade-off underlying the choice of the marginal tax rate at earnings level Y_w. We consider the effects of an infinitesimal perturbation of the tax function as depicted in the left hand-side panel of Figure 2. Marginal tax rates are uniformly decreased by an amount τ over a range of earnings $[Y_w - \delta, Y_w]$. Therefore, the tax levels uniformly decrease by an amount $\rho = \tau \times \delta$ for all skill levels n above w. This tax reform has four effects: a *substitution* effect for taxpayers whose earnings before the reform are in $[Y_w - \delta, Y_w]$, and some *mechanical*, *income* and *participation* response effects for taxpayers with skill n above w.

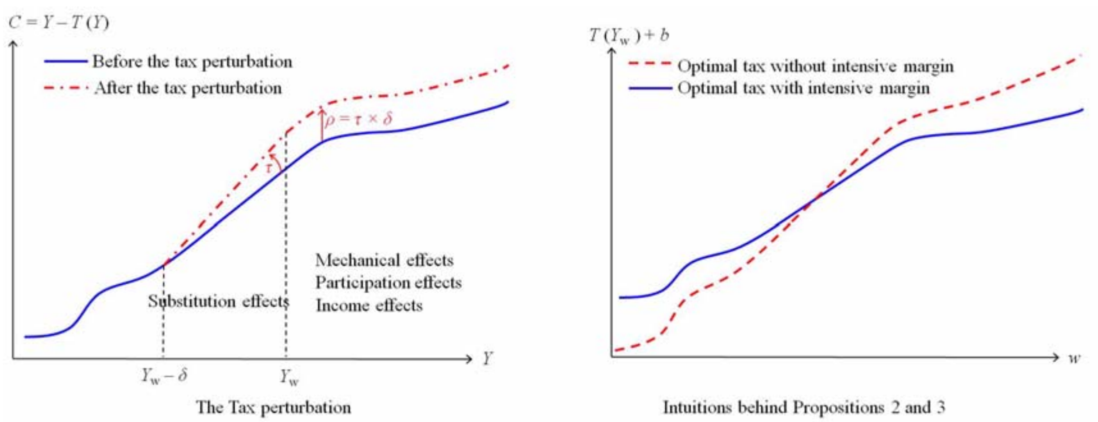

Figure 2: The optimal tax schedule

Substitution effect The substitution effect takes place on the range of gross earnings $[Y_w - \delta, Y_w]$. The mass of workers affected by the substitution effect is $\hat{h}(Y_w) \cdot \delta$. For these workers, according to Equation (9), the decrease by τ of the marginal tax rate induces a rise ΔY_w of their earnings, with

$$\Delta Y_w = \frac{\varepsilon_w \cdot Y_w}{1 - T'(Y_w)} \cdot \tau$$

The tax reform has only a second-order effect on U_w and thereby on the participation decision and its contribution to the government objective. However, the rise in earnings increases the government's tax receipt by $T'(Y_w) \cdot \Delta Y_w$. Hence, given that $\tau \times \delta = \rho$, the total substitution effect equals:

$$\mathcal{S}_w = \frac{T'(Y_w)}{1 - T'(Y_w)} \cdot \varepsilon_w \cdot Y_w \cdot \hat{h}(Y_w) \cdot \rho \tag{20}$$

Workers of skill n above w face a reduction ρ in their tax level with no change in the marginal tax rate. This has three consequences.

Mechanical effects First, in the absence of any behavioral response from these workers, the government gets ρ units of tax receipts less from each of the $h(n)$ workers of skill n. However, the tax reduction induces a higher consumption level C_n, which is valued g_n by the government. Hence the total mechanical effect at skill w is:

$$\mathcal{M}_w = -\int_w^{w_1} (1 - g_n) \cdot h(n) \cdot dn \cdot \rho \tag{21}$$

Income effects Second, the tax reduction induces each of the workers of skill n to change her intensive choice by $\Delta Y_n = \eta_n \cdot \rho$ (see Equation (10)). This income response has only a first-order effect on the government's budget: each of the $h(n)$ workers of skill n pays $T'(Y_n) \cdot \Delta Y_n$ additional tax. Hence, the total income effect at skill n equals:

$$\mathcal{I}_w = \int_w^{w_1} \eta_n \cdot T'(Y_n) \cdot h(n) \cdot dn \cdot \rho \tag{22}$$

Participation effects Finally, the reduction in tax levels induces $\kappa_n \cdot h(n) \cdot \rho$ individuals of skill n to enter employment (see Equation (11)). The change in participation decisions then has only a first-order effect on the government's budget. Each additional worker of skill n pays $T(n)$ taxes, and the government saves the welfare benefit b. Hence, the total participation effect at skill w equals:

$$\mathcal{P}_w = \int_w^{w_1} \kappa_n \cdot (T(Y_n) + b) \cdot h(n) \cdot dn \cdot \rho \tag{23}$$

The sum of $\mathcal{S}_w$, $\mathcal{M}_w$, $\mathcal{I}_w$ and $\mathcal{P}_w$ should be zero if the original tax function is optimal. Rearranging terms then gives:

$$\frac{T'(Y_w)}{1 - T'(Y_w)} = \frac{1}{\varepsilon_w} \times \frac{\int_w^{w_1} \{1 - g_n - \eta_n \cdot T'(Y_n) - \kappa_n (T(Y_n) + b)\} \cdot h(n) \cdot dn}{Y_w \cdot \hat{h}(Y_w)} \tag{24}$$

which gives (17) because of (8).

Equation (18) describes the effects of providing a uniform transfer ρ to all employed individuals. This tax perturbation does not affect marginal tax rates, so it only induces the mechanical, income and participation effects. The sum of (21), (22) and (23) evaluated for $w = w_0$ should be nil at the optimum, which leads to (18). Equations (17) and (18) imply that the optimal marginal tax rate is nil at the minimum earnings level.[17]

To grasp the intuition behind Equation (19), consider a unit increase in welfare benefit b and a unit lump-sum decrease in the tax function for all skill levels. This reform changes neither the marginal nor the participation tax rates. Hence, it has only mechanical and income effects along the intensive and extensive margins. This reform induces a (mechanical) loss of the tax revenues valued 1 by the government and a gain in the social objective. The latter amounts to

[17] Intuitively, increasing the marginal tax rate at a skill level w' improves equity when the extra tax revenue can be redistributed towards a positive mass of people with skill levels less than or equal to w'. Given that the mass of agents with a skill level less than or equal to w_0 is nil, a positive marginal tax rate at w_0 does not improve equity. It does, however, distort the labor supply. The optimal marginal tax rate at the lowest skill level then equals zero. This result no longer holds if there is bunching at the bottom of skill distribution (Seade (1977)).

$g_0 \cdot \left(1 - \int_{w_0}^{w_1} h(n) \cdot dn\right)$ for the nonemployed and $\int_{w_0}^{w_1} g_n \cdot h(n) \cdot dn$ for employed. Therefore, the mechanical effect corresponds to the left-hand side of (19). The right-hand side captures the income effects along both margins.[18] First, through the income response along the intensive margin, earnings change by $\Delta Y_n = \eta_n$. This affects tax revenues by the weighted integral of $\Delta Y_n \cdot T'(Y_n) = \eta_n \cdot T'(Y_n)$. Second, participation decisions change through the income effect by $\Delta h(n) = \nu_n \cdot h(n)$. Given that for each additional worker of skill n, tax revenues increase by $T(Y_n) + b$, the total impact is the weighted integral of $\nu_n \cdot (T(Y_n) + b)$. In the normal case, $\eta_n < 0$ and $\nu_n < 0$. Therefore, as $T(Y_n) + b$ is typically positive for most workers, we expect that larger income effects along both margins increase the average of the social welfare weights (g_0 and g_n's) above 1.

III.2 Comparison with the optimal tax literature

Equation (17) decomposes the determinants of the optimal marginal tax rates into three components. $\mathcal{A}(w)$ is the *efficiency term*. $\mathcal{B}(w)$ captures the role of the *skill distribution* among employed individuals. Finally, $\mathcal{C}(w)$ stands for the *social preferences for income redistribution*, taking into account the induced responses through income effects and along the participation margin.

There are two apparent differences between our formulation of the *efficiency term* $\mathcal{A}(w)$ and that in the literature. The first is the presence of $T''(Y_w)$ in the definitions (7) and (9) of α_w and ε_w. This is because of our definitions of behavioral responses along a potentially nonlinear income tax schedule and the induced endogeneity of marginal tax rates. However, in the ratio α_w/ε_w, these additional terms cancel out. Consequently, the term $\mathcal{A}(w)$ is the same whether we define behavioral elasticities α_w and ε_w along the optimal tax schedule (as in the present study) or along a "virtual" linear tax schedule (as common in the literature; see, e.g., Piketty 1997, Diamond 1998 and Saez 2001). The second difference is induced by our assumption on preferences (1). The literature typically restricts this to the case where preferences over consumption and in-work effort do not vary with the skill level, and are described by $\mathfrak{U}(C, Y/w)$. Then, it happens that the numerator of $\mathcal{A}(w)$ coincides with one plus the *uncompensated* elasticity of the labor supply. This is counterintuitive, as it suggests that *ceteris paribus* marginal tax rates increase with the latter elasticity. Our more general assumption on preferences enables us to stress that what matters is the elasticity α_w of earnings with respect to the skill level. Marginal tax rates are then inversely related to the compensated elasticity in the vein of theRamsey's "inverse elasticity"rule.

The term $\mathcal{B}(w)$ captures the role of the *skill distribution*. Consider an increase in the

[18] Diamond (1975), Sandmo (1998) and Jacobs (2009) emphasize that the social value of public funds should only take into account the behavioral responses from the income effects. Equation (19) shows that only income effects along the intensive η_w and extensive ν_w margins matter.

marginal tax rate around the earnings level Y_w (the left hand-side of Figure 2). The induced distortions along the intensive margin is proportionnal to the product of the skill level w times the number of workers at that skill level, $w \cdot h(w)$ (Atkinson 1990). However, the gain in tax revenues is proportional to the number $H(w_1) - H(w)$ of employed individuals of skill n above w. Two differences with the literature are worth noting. First, because of the extensive margin responses, what matters is the distribution of skills among employed individuals, instead of the one over the entire population. Given that $h(w)/f(w)$ equals the employment rate of workers of skill w and $(H(w_1) - H(w))/(1 - F(w))$ equals the aggregate employment rate above skill w, one can further decompose $\mathcal{B}(w)$ into its exogenous and endogenous components:

$$\mathcal{B}(w) = \frac{1 - F(w)}{w \cdot f(w)} \cdot \frac{\frac{H(w_1) - H(w)}{1 - F(w)}}{\frac{h(w)}{f(w)}}$$

The first term on the right-hand side equals the exogenous skill distribution term of Diamond (1998).[19] Second, the distribution term in (Saez (2001), Equation (19)) concerns the (virtual) distribution of *earnings* and not the *skill* distribution. This is how Saez (2001) removes the counterintuitive presence of the uncompensated labor supply elasticity in the numerator of his efficiency term. Using (7), one then obtains that $\alpha_w \mathcal{B}(w) = \left(\hat{H}(Y_{w_1}) - \hat{H}(Y_w)\right) / \left(Y_w \cdot \hat{h}(Y_w)\right)$, so our optimal tax formula can also be expressed in terms of the earnings distribution, as in (24). Both formulations have their advantage. The earnings distribution has the advantage that earnings are directly observable. However, earnings are endogenous, and hence the observed and optimal earnings distributions may differ. To compute optimal tax rates, one then has to specify the utility function. Once this is done, the individual's first-order condition (5) enables us to recover the individual's skill level w from her observed earnings Y (and from knowledge of the tax function). Accordingly, the advantage of the formulation in terms of the earnings distribution disappears. Nevertheless, we present both formulations and leave it to interested readers to choose which they prefer.

The term $\mathcal{C}(w)$ captures the influence of *social preferences for income redistribution*, taking into account the induced responses through income effects and along the participation margin. $\mathcal{C}(w)$ equals the average of mechanical, income and participation effects for all workers of skill n above w. Diamond (1998) considers the case where participation is exogenous and there is no income effect.[20] Introducing income effects or participation responses in the analysis then amounts to modifying the social weight to:

$$\breve{g}_n \stackrel{\text{def}}{\equiv} g_n + \kappa_n \cdot (T(Y_n) + b) + \eta_n \cdot T'(Y_n)$$

[19] Diamond's (1998) $\mathcal{C}(w)$ corresponds to our $\mathcal{B}(w)$ and vice versa.

[20] Under redistributive preferences, the marginal social weights g_w are decreasing in skill levels w. Then, $\mathcal{C}(w)$ is increasing but remains below 1. When preferences are also Maximin (see Atkinson 1975, Piketty 1997, Salanié 2005, Boadway and Jacquet 2008 among others), then the marginal social weights for workers g_w are nil, so $\mathcal{C}(w)$ is constant and equals 1.

Saez (2002, p. 1055) has explained why the government is more willing to transfer income to groups of employed individuals for which the participation response κ_n or the participation tax $T(Y_n) + b$ is larger. The behavioral parameter κ_n is positive, so a decrease in the level of tax paid by workers of skill n induces more of them to work. Whenever the participation tax $T(Y_n) + b$ is positive, tax revenues increase, which is beneficial to the social planner. We can make a similar interpretation can be made for the income effect. Typically, leisure is a normal good (hence $\eta_n < 0$). Then, through the income effect, a decrease in the level of tax paid by workers of skill n induces them to work less through the income effect. Whenever workers face a positive marginal tax rate, this response decreases the tax they pay, which is detrimental to the government. Therefore, the government is more willing to transfer income to groups of employed individuals with either lower income effects (i.e. higher η_n) or lower marginal tax rates (Saez 2001).

IV Properties of the second-best optimum

IV.1 Sufficient condition for nonnegative marginal tax rates

We first consider the special case where the labor supply decisions take place only along the extensive margin, as assumed in Diamond (1980) and Choné and Laroque (2005, 2009a), so $\varepsilon_w = \eta_w = 0$. The optimal tax formula then verifies:[21]

$$T(Y_w) = \frac{1 - g_w}{\kappa_w} - b \tag{25}$$

The optimal level of tax then trades off the mechanical effect (captured by the social weight g_w) and the participation response effect (captured by the participation response κ_w) of a rise in the level of tax. Marginal tax rates are then everywhere nonnegative if along the optimal allocation, the function $Y \mapsto (1 - g_w) / \kappa_w$ is increasing. The following Proposition shows that this result remains valid in the presence of responses along the intensive margin.

Proposition 2 *If along the optimal allocation, $w \mapsto \frac{1-g_w}{\kappa_w}$ is increasing, marginal tax rates are always nonnegative. Furthermore, they are almost everywhere positive, except at the two extremities Y_{w_0} and Y_{w_1}.*

This Proposition is proved in Appendix C. The intuition is illustrated in the right hand-side panel of Figure 2. This figure depicts the level of the participation tax $T(Y_w) + b$ paid by a worker of skill w, as a function of her skill level. When labor supply responses are only along the extensive margin, the optimal tax schedule is represented by the dashed curve. This corresponds

[21] In the absence of a response along the intensive margin, substitution effects $\mathcal{S}_w$ in (20) and income effects $\mathcal{I}_w$ in (22) are nil at each skill level. Therefore, the sums of the mechanical $\mathcal{M}_w$ and the participation $\mathcal{P}_w$ effects must be nil at each skill level, which gives (25).

to the optimal trade-off between the mechanical and participation effects. If $w \mapsto (1 - g_w)/\kappa_w$ is increasing in w, this function is increasing in the skill level. However, when workers can also decide along their intensive margin, the increasing tax function and its positive marginal tax rates induce distortions of the intensive choices. Hence, the optimal tax function depicted by the solid curve is flatter than the optimal curve without an intensive margin to limit the distortions along the intensive margin. It also has to be as close as possible to the optimal curve without intensive margin to limit the departures from the optimal trade-off between the participation and mechanical effects.

Proposition 3 *If along the optimal allocation, $w \mapsto \frac{1-g_w}{\kappa_w}$ is increasing in w, and if $g_w \leq 1$ for all skill levels, then the in-work benefits (if any) are smaller than the welfare benefit b.*

This Proposition is proved in Appendix D. The assumption that $g_w \leq 1$ for all skills is restrictive as it implies that in the case without intensive responses, the optimal tax is characterized by providing the least-skilled workers with a lower benefit than the nonemployed (hence, a Negative Income Tax is optimal). This result remains valid in the presence of intensive responses as the optimal tax function under unobserved skills is flatter than that under observed skills. Proposition 3 emphasizes this result.

In the absence of behavioral responses along the intensive margin, in-work benefits for the working poor (of skill w_0) are larger than the welfare benefits if and only if $g_{w_0} > 1$. By continuity, as long as the compensated elasticity (along the intensive margin) ε_{w_0} is sufficiently small, in-work benefits should remain higher than welfare benefits; hence, an EITC is optimal,as already stressed by Saez (2002).

IV.2 Examples

The sufficient condition in Propositions 2 and 3 depends on the patterns of social weights g_w and extensive behavioral responses κ_w, both of which are endogenous. This subsection provides examples where the primitives of the model guarantee the sufficient conditions in Propositions 2 and 3.

Our first example specifies the primitives of the model in such a way that g_w and κ_w become exogenous. For this purpose, individuals' preferences are quasilinear: $\mathcal{U}(C, Y, w) = C - \mathcal{V}(Y, w)$ with $\mathcal{V}'_Y, \mathcal{V}''_{YY} > 0 > \mathcal{V}''_{Yw}$. The marginal utility of consumption $\mathcal{U}'_C(C, Y, w)$ is then always equal to one. Moreover, we specify the distribution of the disutility of participation χ conditional on any skill level w as $K(\chi, w) = \exp(a_w + \kappa \cdot \chi)$, where a_w is a skill-specific parameter adjusted to keep some individuals out-of-the labor force at the optimum. Then, according to Equation (11), κ_w is always equal to parameter κ and thereby constant along the skill distribution. Finally, the social objective is linear in utility levels with skill-specific weights γ_w. Given that the specification of the individuals' utility rules out income effects, we have $g_w = \gamma_w / \int_{w_0}^{w_1} \gamma_w dw$ (see (15), (16) and (19)). Therefore, under redistributive social preferences, $w \mapsto \gamma_w$ is decreasing, so $(1 - g_w) / \kappa_w$ is decreasing. The marginal tax rates are then nonnegative according to Proposition 2. Note that in this example, g_{w_0} is necessarily strictly greater than one, so the optimal participation tax may be negative at the bottom. A negative participation tax at the bottom is nevertheless consistent with nonnegative marginal tax rates over the entire income distribution as we assume a positive lower bound for the skill distribution. Hence, the lowest earnings level is positive and the tax function can jump between $Y = 0$ and Y_{w_0}.

This first example is very specific. In general, we consider that it is plausible that $w \mapsto 1 - g_w$ is nonincreasing and $w \mapsto \kappa_w$ is strictly decreasing. First, a redistributive government typically places a higher social welfare weight on the consumption of the least-skilled workers. Second, there is some empirical evidence that the elasticity of participation, which equals $(Y_w - T(Y_w) - b)\,\kappa_w$, is typically a nonincreasing function (see, e.g., Juhn *et alii* (1991), Immervoll *et alii* (2007) or Meghir and Phillips (2008)). Given that consumption $Y_w - T(Y_w)$ is an increasing function, one could expect κ_w to decrease along the skill distribution.

We now provide more general specifications of the primitives where these two properties hold. Assume that the utility function is additively separable, i.e.:

$$\mathcal{U}(C, Y, w) = u(C) - \mathcal{V}(Y, w) \tag{26}$$

with $u'_C, \mathcal{V}'_Y, \mathcal{V}''_{YY} > 0 > u''_{CC}, \mathcal{V}''_{Yw}$. The additive separability restriction is only made for technical convenience. However, showing within the pure intensive model that marginal tax rates are positive without imposing the additive separability assumption (26) was a real issue (see, e.g., Sadka 1976, Seade 1982, Werning 2000). We add another restriction on preferences. For an employed individual, the more skilled the worker, the lower the effort to obtain a given earnings level. However, for the nonemployed, no effort is supplied. Hence, a larger skill does not improve utility. We therefore assume:

$$\mathcal{V}'_w(Y, w) \ \substack{< \\ =} \ 0 \quad \text{if} \quad Y \ \substack{> \\ =} \ 0 \tag{27}$$

As a result, the skill-specific threshold $U_w - \mathcal{U}(b, 0, w)$ of χ is constrained to be an increasing function of the skill level (See Equation (31) in Appendix B). The following properties are shown in Appendix E.

Property 1 *If $K(\chi, w)$ is strictly log-concave with respect to χ, $w \mapsto k(\chi, w) / K(\chi, w)$ is nonincreasing in w and (26)-(27) hold, then the function $w \mapsto \kappa_w$ is strictly decreasing.*

The log-concavity of $K(., w)$ is a property verified by most distributions commonly used. It is equivalent to assuming that $k(\chi, w) / K(\chi, w)$ is decreasing in χ. That $k(\chi, w) / K(\chi, w)$ is nonincreasing in w encompasses the specific case where w and χ are independently distributed.

Property 2 *Under either Maximin or Benthamite social preferences and (26)-(27), the function* $w \mapsto g_w$ *is nonincreasing*

Maximin (i.e., maximizing $u(b)$) and Benthamite (i.e., $G(U_w - \chi, w, \chi) \equiv U_w - \chi$) social preferences are polar specifications. Combining Properties 1 and 2, the relation $w \mapsto (1 - g_w)/\kappa_w$ is increasing provided that g_w remains below 1. Therefore, Propositions 2 and 3 hold under the joint following assumptions: a maximin social objective, the utility functions verifyies (26) and (27), $K(\chi, w)$ is strictly log-concave with respect to χ and $k(\chi, w)/K(\chi, w)$ is nonincreasing in w. Moreover, if the government is instead Benthamite and if $g_{w_0} \leq 1$, then Propositions 2 and 3 are again ensured.

V Numerical simulations for the U.S.

This section implements our optimal tax formula to U.S data to analyze whether and to what extent, optimal schedules resemble real-world schedules, and if not, how they can be reformed. This exercise also allows to check whether our sufficient condition for nonnegative marginal tax rates is empirically reasonable.

V.1 Calibration

To calibrate the model, we need to specify social and individual preferences and the distribution of characteristics (w, χ). We consider Benthamite and Maximin social preferences. We select a specification of individual preferences that enables us to control behavioral responses along the intensive margin. Following Diamond (1998), we assume away income effects along the intensive margin (hence $\eta_w \equiv 0$) and assume the compensated elasticities to be constantly equal to ε along a linear tax schedule. Moreover, individuals' preferences are concave such that a Benthamite government prefers to transfer income from high- to low- income earners. Hence, we specify:

$$\mathcal{U}(C, Y, w) = \frac{\left(C - \left(\frac{Y}{w}\right)^{1+\frac{1}{\varepsilon}} + 1\right)^{1-\sigma}}{1-\sigma}$$

The parameter ε corresponds to the compensated elasticity along a linear tax schedule (see Equation (9)), while parameter σ drives the redistributive preferences of a Benthamite government. Saez *et alii* (2009) survey the recent literature estimating the elasticity of earnings to one minus the marginal tax rate. They conclude that "The most reliable longer-run estimates range from 0.12 to 0.4" in the U.S. We take a central value of $\varepsilon = 0.25$ for our benchmark. For the concavity of preferences, we take $\sigma = 0.8$ in the benchmark case. We conduct a sensitivity analysis with respect to both of these parameters.

To calibrate the skill distribution, we employ the earnings distribution from the Current Population Survey (CPS) for May 2007. We use the first-order condition (5) of the intensive program to infer the skill level from each observation of earnings. We consider only single individuals to avoid the complexity of interrelated labor supply decisions within families. Using the OECD tax database, the real tax schedule of singles without dependent children is well approximated by a linear tax function at rate 27.9% with an intercept of $-\$4,024.9$ on an annual basis.[22] We use a quadratic kernel with a bandwidth of $\$3,822$ to smooth $h(w)$. High-income earners are underrepresented in the CPS. Diamond (1998) and Saez (2001) argue that the skill distribution actually exhibits a fat upper tail in the U.S., which has dramatic consequence for the shape of optimal marginal tax rates. We therefore expand (in a continuously differentiable way) our kernel estimation by taking a Pareto distribution, with an index[23] $a = 2$ for skill levels between $w = \$20,374$ and $w_1 = \$40,748$. This represents only the top 3.1% of our approximation of the skill distribution. The lower bound of the skill distribution is $w_0 = \$202$.

One final need is to calibrate the conditional distribution of χ. For numerical convenience, we choose a logistic and skill-specific specification of the form:

$$K(\chi, w) = \frac{\exp(-a_w + \beta_w\ \chi)}{1 + \exp(-a_w + \beta_w\ \chi)}$$

Parameters a_w and β_w are calibrated to obtain empirically plausible skill-specific employment rates, denoted by L_w, and elasticities of employment rates with respect to the difference in disposable incomes $C_w - b$, denoted π_w.[24] We take:

$$L_w = 0.7 + 0.1\left(\frac{w - w_0}{w_1 - w_0}\right)^{1/3} \qquad \pi_w = \pi_0 - \pi_1\left(\frac{w - w_0}{w_1 - w_0}\right) \text{ with } \pi_0 = 0.5 \text{ and } \pi_1 = 0.1$$

These specifications are consistent with the empirical evidence that the employment rate L_w is higher for the highly skilled. The average employment rate in the current economy equals 75.3%. The elasticity π_w is equal to 0.45 on average. Unreported simulations indicate that the properties of the optimal tax schedule are robust with respect to changes in the parameters of the $w \to L_w$ relationship. A sensitivity analysis is used to illustrate how the calibration of π_w modifies the optimal tax profile.

We take $b = \$2,381$, as the net replacement ratio for a long-term unemployed worker whose previous earnings equal 67% of the average wage is 9% in 2007 according to the OECD tax-benefit calculator.[25] Given this calibration of the current economy, we find that the budget constraint (13) is verified only when we set the exogenous revenue requirement to $E = \$6,110$ per capita.

[22] We multiply by 52 the weekly earnings given by the CPS survey.
[23] An (untruncated) Pareto distribution with Pareto index $a > 1$ is such that $\Pr(w > \widehat{w}) = C/\widehat{w}^a$ with $a, C \in \mathbb{R}_0^+$.
[24] $\pi_w = \kappa_w\ (Y_w - T(Y_w) - b)$ in the current economy.
[25] See http://www.oecd.org/document/18/0,3343,en_2649_34637_39717906_1_1_1_1,00.html

V.2 Benchmark simulations

Figure 3 plots the optimal marginal tax rates (Panel (a)) and participation tax levels (Panel (b)) as functions of earnings, under the Benthamite (solid line) and Maximin (dotted line) criteria. We focus on annual earnings below $\$100,000$.[26] Consistent with Proposition 2, the marginal tax rates are always positive under both criteria. Moreover, there is no distortion at the lower end of the earnings distribution whose value is $Y_{w_0} = \$508$. Under the Maximin, the latter result contrasts with the optimal positive marginal tax rate in a model with intensive margin only (Boadway and Jacquet 2008). In this latter case, the social objective values only the utility of employed individuals at $Y = Y_{w_0}$. When both extensive and intensive margins are modeled, the Maximin objective values only the utility of the nonemployed. Panel (a) illustrates that the absence of distortion at the bottom is a very local property: when $Y = \$2,150$, the marginal tax rate climbs to 60.5% (58.8%) under Benthamite (Maximin) preferences. Beyond this point, marginal tax rates follow the usual U-shaped profile (Salanié 2003) under both objective functions. Under the Maximin, the marginal tax rates are higher than under the Benthamite criterion, except at the bottom end (for Y lower than $Y = \$5,900$). Remarkably, the optimal marginal tax rates are significantly higher than the current 27.9%, except for the very low end of the earnings distribution. This is valid under both objectives. However, our optimal marginal tax rates are much lower than those found by Saez (2001).

Figure 3(b) illustrates that the participation tax levels at the bottom of the earnings distribution are typically negative under a Benthamite criterion. The optimality of a negative participation tax on the poorest workers is usually interpreted as a case for an EITC (Saez 2002). We find $b = \$2,665$ and $-T(Y_{w_0}) = \$9,345$. Contrastingly, Figure 3(b) also emphasizes that the participation tax levels at the bottom of the earnings distribution are positive under Maximin. An NIT then prevails. This is a standard result of the pure extensive margin model (Choné Laroque 2005), which is still valid here when considering extensive and intensive margins together.[27] Intuitively, it is hardly desirable to transfer income to the least-skilled workers, since their well-being does not matter under Maximin. At the Maximin optimum, we find $b = \$4,190$ and $-T(Y_{w_0}) = \$3,860$.

Figure 4(a) describes how the negative participation tax on least-skilled workers enables employment rates to be to boosted well above their values in the current economy under Benthamite social preferences. Moreover, Panel (b) illustrates how these negative participation tax rates (in the Bentham economy) increase the *gross* utility levels U_w of low-skilled workers significantly beyond their values in the current economy.

[26] Income earners above $\$100,000$ represent 4.65%, 3.73% and 5.66% of the population in the current economy, at the Benthamite optimum and at the Maximin optimum, respectively.

[27] Saez (2002) points this result for his mixed model.

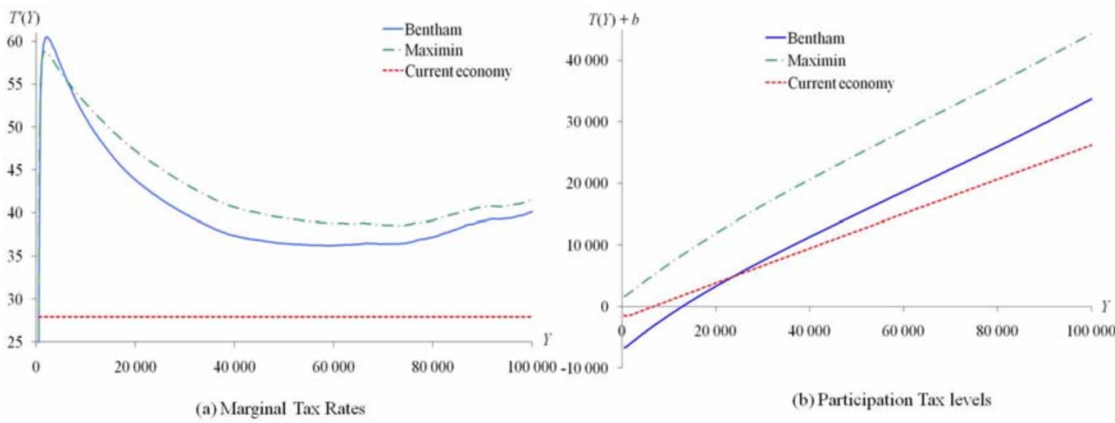

Figure 3: The simulation under the benchmark calibration

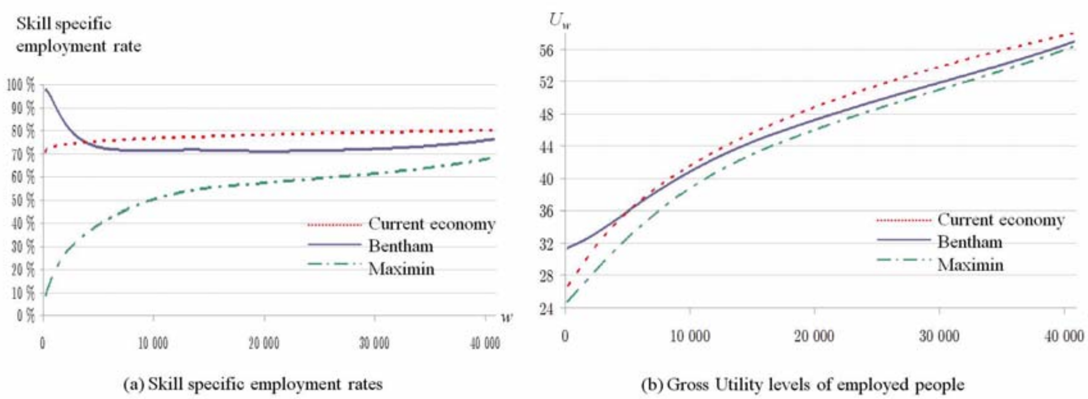

Figure 4: Optimal allocations

V.3 Sensitivity analysis

All our various sensitivity analyses point out that the U-shape profile is valid, and none display negative marginal tax rates. The only configuration where our sufficient condition for nonnegative marginal tax rate is violated requires an extremely low σ. Even then, the marginal tax rates remain positive. This section therefore focuses on the quantitative implications of the parameters on the optimal tax rates.

As illustrated in Figure 5(a), the levels of the marginal tax rates are quite sensitive to the parameter σ of individual preferences. This is because any rise in σ increases the marginal tax profile by a substantial amount as the planner becomes more averse to inequality. The participation tax levels increase (decrease) with σ below (above) Y around $\$20,000$. Higher redistributive tastes then increase transfers towards low-paid workers, and the other workers pay more taxes (see Panel (b)).

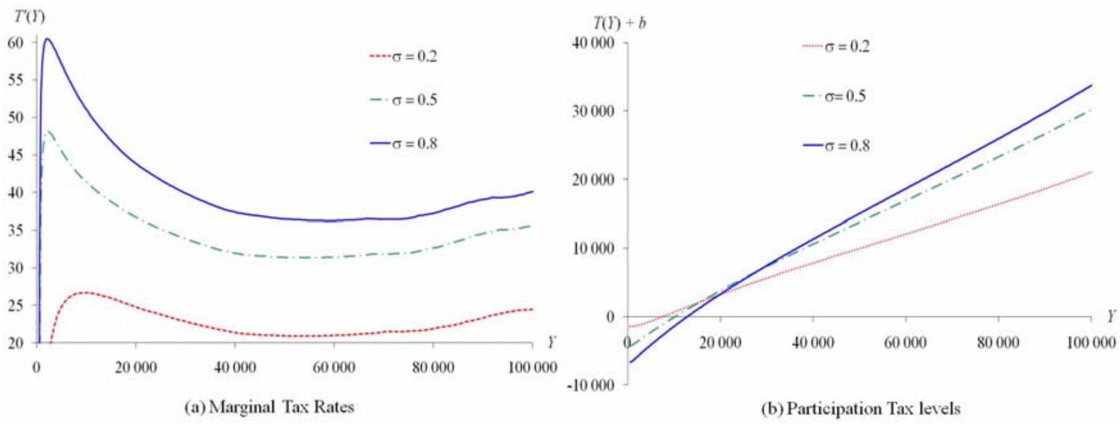

Figure 5: Sensitivity analysis with respect to σ for the Benthamite optimum

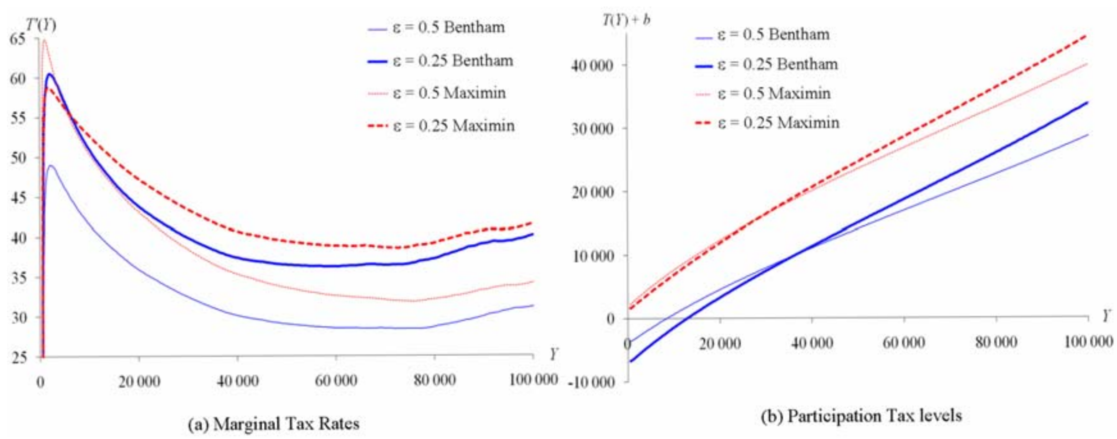

Figure 6: Sensitivity analysis with respect to ε

Figure 6(a) illustrates that the marginal tax rates decrease with the elasticity of earnings ε, as theoretically expected from the implied decrease of $\mathcal{A}(w)$ in Equation (17). Figure 6(a) illustrates this result with ε equal to 0.25 and 0.5, under Maximin and Benthamite preferences.[28] Figure 6(b) emphasizes that participation taxes decrease (increase) with ε for earnings above (below) roughly around \$30,000, under both criteria.

The next exercise studies the impact of reducing the participation response κ_w. Figure 7 plots the tax schedule when the parameter π_0 shrinks from 0.5 to 0.15. This reduction in the elasticities of the employment rates $w \mapsto \pi_w$ (hence the reduction of κ_w) significantly increases the marginal tax rates (see Panel (a)), as expected from the implied decrease of $\mathcal{C}(w)$ in Equation (17). Moreover, as also expected from theory, the participation tax levels increase (Panel (b)). This exercise highlights the quantitative implications of introducing the extensive margin.

[28] Under Maximin, the marginal tax rates decrease with ε except for earnings below \$5,249.

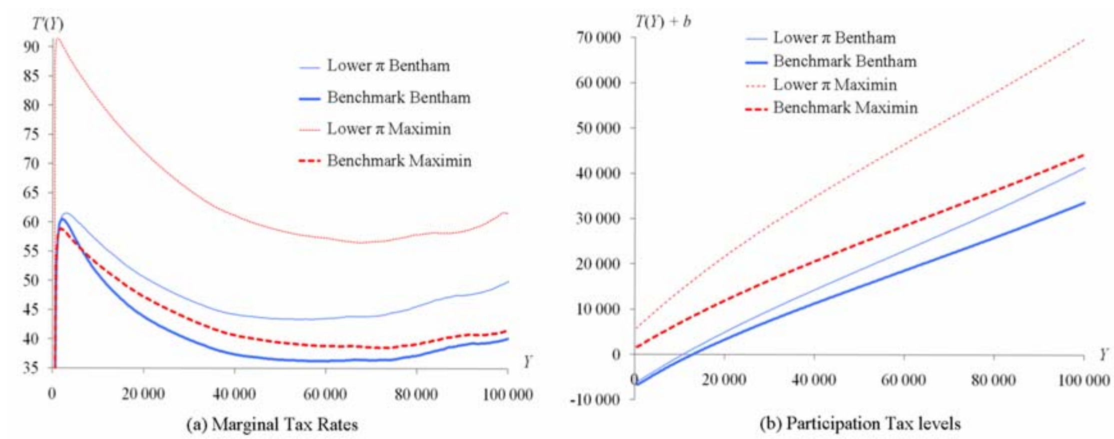

Figure 7: A lower $w \mapsto \pi_w$ in the calibration of the current economy

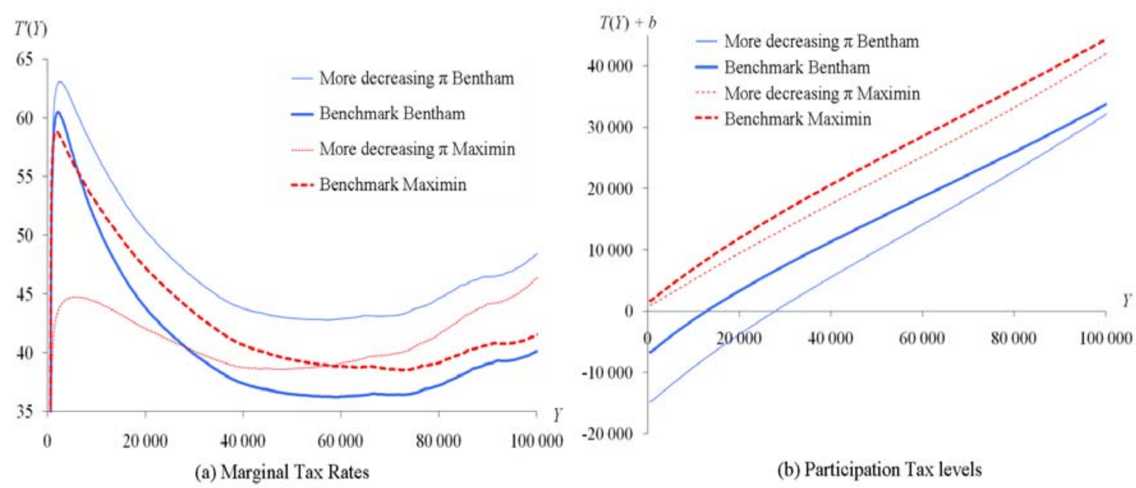

Figure 8: A more decreasing $w \mapsto \pi_w$ in the current economy

A further sensitivity analysis considers a more decreasing $w \mapsto \pi_w$ in the current economy, hence a more decreasing $w \mapsto \kappa_w$. Figure 8 plots the tax rates when $(\pi_0, \pi_1) \equiv (0.75, 0.6)$ (solid curves) instead of $(\pi_0, \pi_1) \equiv (0.5, 0.1)$ (dashed curves). As expected from the $\mathcal{C}(w)$ term in Equation (17), the marginal tax rates then increase. In addition, the participation tax curves become more increasing, under both criteria (Panel (b)), as expected from theory.

Importantly, our calibration abstracts from the income effects. For consistency with the theoretical framework, we also focus on single households and so abstract from the interactions between the labor supply decisions of couples (see Kleven *et alii* 2009 for a theory of the optimal taxation of couples). However, those dimensions are unnecessary to show how crucial it is to consider both labor supply margins when making tax policy recommendations.

VI Conclusion

This paper explored the optimal income tax schedule when labor supply responds simultaneously along both the extensive and intensive margins. Here, individuals are heterogeneous across two dimensions: their skills and their disutility of participation. We derived a mild sufficient condition for nonnegative marginal tax rates over the entire skill distribution. This condition is derived using a new method to sign distortions (along the intensive margin) in screening models with random participation. Our exercise illustrated that at the optimum, negative participation tax rates can coexist with positive marginal tax rates everywhere.

Using U.S. data, we implemented our optimal tax formula. This exercise emphasized that the U-shaped optimal tax schedule found in the literature with an intensive margin only remains still valid when both labor supply margins are considered. However, introducing the extensive margin quite substantially reduces the marginal tax rates. Interestingly, the marginal tax rates are always positive in our simulations.

The work undertaken in this study identifies several possible extensions. First, the method to sign distortion along the intensive margin can be applied to other contexts of monopoly screening with *random participation à la* Rochet and Stole (2002). Second, it would also be interesting to extend the numerical simulations in this analysis to countries outside the U.S. Finally, in this paper, we ignored the interactions in labor supply decisions within couples (Kreiner *et alii* (2009)).

Appendices

A Behavioral elasticities

We define:

$$\begin{aligned}&\mathcal{Y}(Y,w,\tau,\rho) \stackrel{\text{def}}{\equiv} \left(1-T'(Y)+\tau\right)\cdot \mathcal{U}'_C\left(Y-T(Y)+\tau(Y-Y_w)+\rho,Y,w\right)\\ &+\mathcal{U}'_Y\left(Y-T(Y)+\tau(Y-Y_w)+\rho,Y,w\right)\end{aligned}$$

The first-order condition (5) is equivalent to $\mathcal{Y}(Y_w,w,0,0)=0$. When $T(.)$ is twice-differentiable, one has (using (5)):

$$\mathcal{Y}'_Y(Y_w,w,0,0) = \mathcal{U}''_{YY} - 2\left(\frac{\mathcal{U}'_Y}{\mathcal{U}'_c}\right)\mathcal{U}''_{CY} + \left(\frac{\mathcal{U}'_Y}{\mathcal{U}'_c}\right)^2\mathcal{U}''_{CC} - T''(Y_w)\cdot\mathcal{U}'_C \quad (28a)$$

$$\mathcal{Y}'_w(Y_w,w,0,0) = \left(1-T'\right)\cdot\mathcal{U}''_{Cw} + \mathcal{U}''_{Yw} = \frac{\mathcal{U}''_{Yw}\cdot\mathcal{U}'_C - \mathcal{U}''_{Cw}\cdot\mathcal{U}'_Y}{\mathcal{U}'_C} \quad (28b)$$

$$\mathcal{Y}'_\tau(Y_w,w,0,0) = \mathcal{U}'_C \quad (28c)$$

$$\mathcal{Y}'_\rho(Y_w,w,0,0) = \left(1-T'\right)\cdot\mathcal{U}''_{CC} + \mathcal{U}''_{CY} = \frac{\mathcal{U}''_{CY}\cdot\mathcal{U}'_C - \mathcal{U}''_{CC}\cdot\mathcal{U}'_Y}{\mathcal{U}'_C} \quad (28d)$$

The second-order condition is $\mathcal{Y}'_Y(Y_w,w,0,0)\leq 0$, which gives (6). When this condition holds with strict inequality, and when the global maximum in Y of $\mathcal{U}(Y-T(Y),Y,w)$ is unique, we can apply the implicit function theorem to $\mathcal{Y}(Y_w, w, 0, 0)$. Provided that the sizes of the changes in w, τ and ρ are small enough for the maximum of $Y \mapsto \mathcal{U}(Y - T(Y), Y, w)$ to change only marginally, one has for $x = w, \tau, \rho$, that $\partial Y/\partial x = -\mathcal{Y}'_x/\mathcal{Y}'_Y$ evaluated at $(Y_w, w, 0, 0)$. This leads directly to (7), (9) and (10).

We now make the link between our definitions of behavioral elasticities and the elasticities along a linear tax schedule used in Saez (2001). We denote the latter with a tilde. Rewriting (9) and (10) with $T''(.) = 0$ yields:

$$\tilde{\varepsilon}_w = \frac{\mathcal{U}'_Y}{Y_w\left[\mathcal{U}''_{YY} - 2\left(\frac{\mathcal{U}'_Y}{\mathcal{U}'_C}\right)\mathcal{U}''_{CY} + \left(\frac{\mathcal{U}'_Y}{\mathcal{U}'_C}\right)^2 \mathcal{U}''_{CC}\right]} \qquad \tilde{\eta}_w = \frac{\left(\frac{\mathcal{U}'_Y}{\mathcal{U}'_C}\right)\mathcal{U}''_{CC} - \mathcal{U}''_{CY}}{\mathcal{U}''_{YY} - 2\left(\frac{\mathcal{U}'_Y}{\mathcal{U}'_c}\right)\mathcal{U}''_{CY} + \left(\frac{\mathcal{U}'_Y}{\mathcal{U}'_c}\right)^2 \mathcal{U}''_{CC}} \tag{29}$$

Consider now a uniform decrease τ of marginal tax rates (respectively a rise ρ of the level of tax). Such a reform has a *first* impact on earnings $\Delta_1 Y_w$ that equals:

$$\Delta_1 Y_w = \tilde{\varepsilon}_w \times \frac{Y_w}{1 - T'(Y_w)} \times \tau \qquad \text{or} \qquad \Delta_1 Y_w = \tilde{\eta}_w \times \rho$$

which in turn implies a change in marginal tax rates of $-T''(Y_w) \times \Delta_1 Y_w$. Hence, the reform has a *second* impact on earnings that equals:

$$\Delta_2 Y_w = -\tilde{\varepsilon}_w \times \frac{Y_w}{1 - T'(Y_w)} \times T''(Y_w) \times \Delta_1 Y_w$$

This "circular process" takes place infinitely, with the n^{th} impact on earnings being linked to the $(n-1)^{\text{th}}$ impact through:

$$\Delta_n Y_w = -\tilde{\varepsilon}_w \times \frac{Y_w}{1 - T'(Y_w)} \times T''(Y_w) \times \Delta_{n-1} Y_w$$

Using the identity $1 - x + x^2 - x^3 ... = 1/(1 + x)$, the total impact equals $\sum_{i=0}^{+\infty} \Delta_i Y_w = \Delta_1 Y_w / \left(1 + \tilde{\varepsilon}_w \times \frac{Y_w}{1 - T'(Y_w)} \times T''(Y_w)\right)$. Hence ε_w, η_w, $\tilde{\varepsilon}_w$ and $\tilde{\eta}_w$ are linked through

$$\frac{\varepsilon_w}{\tilde{\varepsilon}_w} = \frac{\eta_w}{\tilde{\eta}_w} = \frac{\sum_{i=0}^{+\infty} \Delta_i Y_w}{\Delta_1 Y_w} = \frac{1}{1 + \tilde{\varepsilon}_w \times \frac{Y_w}{1 - T'(Y_w)} \times T''(Y_w)}$$

Using (5) and (29), one retrieves (9) and (10).

B The government's optimum

This appendix solves the government's problem in terms of allocation, as in Mirrlees (1971), and considers the possibility of bunching. Using the obtained government's optimality conditions, we show the equivalence between this formulation and the optimal tax formula of Proposition 1.

According to the *taxation principle* (Hammond 1979, Rochet 1985 and Guesnerie 1995), the set of allocations induced by the tax function $T(.)$ corresponds to the set of incentive-compatible allocations $\{Y_w, C_w, U_w\}_{w \in [w_0, w_1]}$ that verify:

$$\forall (w, x) \in [w_0, w_1]^2 \qquad U_w \equiv \mathcal{U}(C_w, Y_w, w) \geq \mathcal{U}(C_x, Y_x, w) \tag{30}$$

The incentive-compatible restrictions (30) impose the condition that when taking their intensive decisions, workers of skill w prefer the bundle (C_w, Y_w) designed for them rather then the bundle (C_x, Y_x) designed for workers of any other skill level x. We assume that $w \mapsto Y_w$ is continuous on $[w_0, w_1]$ and differentiable everywhere, except for a finite number of skill levels. Finally, $w \mapsto U_w$ is differentiable. Hence, $w \mapsto C_w$ is also continuous everywhere and differentiable almost everywhere. These assumptions are made for reasons of tractability and have been standard since Guesnerie and Laffont (1984).[29]

From Equation (2), the strict single-crossing condition holds. Hence, constraints (30) are equivalent to imposing the differential equation:

$$\dot{U}_w \stackrel{\text{a.e.}}{=} \mathcal{U}'_w \left(C_w, Y_w, w\right) \tag{31}$$

(31) and the monotonicity requirement that the earnings level Y_w be a nondecreasing function of the skill level w. We obtain:

Lemma 1 *The necessary conditions for the government's problem are:*[30]

- *if there is no bunching at skill* w :

$$\left(1 + \frac{\mathcal{U}'_Y}{\mathcal{U}'_C}\right) \cdot h\left(w\right) = Z_w \cdot \frac{\mathcal{U}''_{Yw}\mathcal{U}'_C - \mathcal{U}''_{Cw}\mathcal{U}'_Y}{\mathcal{U}'_C} \tag{32}$$

- *if there is bunching over* $[\underline{w}, \overline{w}]$:

$$\int_{\underline{w}}^{\overline{w}} \left(1 + \frac{\mathcal{U}'_Y}{\mathcal{U}'_C}\right) \cdot h\left(w\right) \cdot dw = \int_{\underline{w}}^{\overline{w}} Z_w \cdot \frac{\mathcal{U}''_{Yw}\mathcal{U}'_C - \mathcal{U}''_{Cw}\mathcal{U}'_Y}{\mathcal{U}'_C} \cdot dw \tag{33}$$

For all skill levels:

$$-\dot{Z}_w = \frac{(1 - g_w) \cdot h\left(w\right) + Z_w \cdot \mathcal{U}''_{Cw}}{\mathcal{U}'_C} - \left(T\left(Y_w\right) + b\right) \cdot h'_U\left(w\right) \tag{34}$$

with $Z_{w_1} = Z_{w_0} = 0$ *and:*

$$\left(1 - \int_{w_0}^{w_1} h\left(w\right) \cdot dw\right)\left(1 - g_0\right) = \int_{w_0}^{w_1} \left(Y_w - C_w + b\right) \cdot h'_b\left(w\right) \cdot dw \tag{35}$$

Proof. Given that $\mathcal{U}\left(.,.,.\right)$ is increasing in C, we define C_w as function $\Gamma\left(U_w, Y_w, w\right)$ such that:

$$u = \mathcal{U}\left(C, Y, w\right) \qquad \Leftrightarrow \qquad C = \Gamma\left(u, Y, w\right)$$

We obtain

$$\Gamma'_u = \frac{1}{\mathcal{U}'_C} \qquad \Gamma'_Y = -\frac{\mathcal{U}'_Y}{\mathcal{U}'_C} \qquad \Gamma'_w = -\frac{\mathcal{U}'_w}{\mathcal{U}'_C} \tag{36}$$

where the functions are evaluated at $\left(w, C = \Gamma\left(u, Y, w\right), u = \mathcal{U}\left(C, Y, w\right), Y\right)$, Next, we rewrite (31) as $\dot{U}_w = \Psi\left(U_w, Y_w, w\right)$, where

$$\Psi\left(u, Y, w\right) \stackrel{\text{def}}{\equiv} \mathcal{U}'_w\left(\Gamma\left(u, Y, w\right), Y, w\right)$$

[29] Hellwig (2008) explain how the same first-order conditions can be obtained under weaker assumptions on $w \mapsto Y_w$ and $w \mapsto U_w$.

[30] where the various derivatives of $\mathcal{U}$ are evaluated at (C_w, Y_w, w).

One has from (36):

$$\Psi'_Y = \frac{\mathcal{U}''_{Yw}\mathcal{U}'_C - \mathcal{U}''_{Cw}\mathcal{U}'_Y}{\mathcal{U}'_C} \qquad \Psi'_U = \frac{\mathcal{U}''_{Cw}}{\mathcal{U}'_C} \tag{37}$$

where the functions are evaluated at (w, C_w, U_w, Y_w). We consider Y_w as the control variable and U_w as the state variable. Then λ equals the Lagrange multiplier associated with the budget constraint (13). Let q_w be the costate variable associated to (31) and let $Z_w = -q_w/\lambda$. The Hamiltonian writes:

$$\begin{aligned}
&\mathcal{H}(Y, U, q, w, \lambda) \stackrel{\text{def}}{\equiv} \int_0^{U_w - \mathcal{U}(b,0,w)} G\left(V\left(U_w, w, \chi\right), w, \chi\right) \cdot k\left(\chi, w\right) \cdot d\chi \cdot f\left(w\right) \cdot dw \\
&+ \int_{U_w - \mathcal{U}(b,0,w)}^{+\infty} G\left(\mathcal{U}^0\left(b, w, \chi\right), w, \chi\right) \cdot k\left(\chi, w\right) \cdot d\chi \cdot f\left(w\right) \cdot dw - \lambda \cdot b \\
&+ \lambda\left[Y_w - \Gamma\left(U_w, Y_w, w\right) + b\right] \cdot h\left(w\right) + q_w \cdot \Psi\left(U_w, Y_w, w\right)
\end{aligned}$$

The first-order conditions of the government's program are:

- If there is no bunching at skill w, one must have:

  $$0 = \frac{\partial \mathcal{H}}{\partial Y}\left(Y_w, U_w, q_w, w, \lambda\right) = \lambda\left[1 - \Gamma'_Y\right] \cdot h + q_w \cdot \Psi'_Y$$

  Using $Z_w = -q_w/\lambda$, (36) and (37) leads to (32).

- If there is bunching over $[\underline{w}, \overline{w}]$, one must have $\int_{\underline{w}}^{\overline{w}} \partial\mathcal{H}/\partial Y\left(Y_w, U_w, q_w, w, \lambda\right) \cdot dw = 0$. Using again $Z_w = -q_w/\lambda$ (36) and (37) gives (33).

- The transversality conditions are $q_{w_0} = q_{w_1} = 0$ and, for any skill level where $w \mapsto Y_w$ is continuous, one obtains $-\dot{q}_w = \partial\mathcal{H}/\partial U\left(Y_w, U_w, q_w, w, \lambda\right)$. Using $Z_w = -q_w/\lambda$ and (15) gives (34).

- Finally, the first-order condition with respect to b gives (35).

■

We now show how to retrieve the formula in Proposition 1. Let:

$$X_w = Z_w \cdot \exp\left[\int_{w_0}^{w} \Psi'_U\left(U_x, Y_x, x\right) \cdot dx\right] \qquad \text{and} \qquad J_w = Z_w \cdot \mathcal{U}'_C\left(C_w, Y_w, w\right)$$

Z_w and J_w have the same sign as X_w. As $w \mapsto Z_w$, $w \mapsto X_w$ is moreover differentiable with a derivative:

$$\dot{X}_w = \left[\dot{Z}_w + Z_w \cdot \Psi'_U\left(U_w, Y_w, w\right)\right] \cdot \exp\left[\int_{w_0}^{w} \Psi'_U\left(U_x, Y_x, x\right) \cdot dx\right]$$

Therefore, from (11), (34) and (37):

$$-\dot{X}_w = \left\{1 - g_w - \kappa_w \cdot \left(T\left(Y_w\right) + b\right)\right\} \cdot \frac{h\left(w\right)}{\mathcal{U}'_C\left(C_w, Y_w, w\right)} \cdot \exp\left[\int_{w_0}^{w} \Psi'_U\left(U_x, Y_x, x\right) \cdot dx\right] \tag{38}$$

At skill levels for where there is no bunching, Equation (32) can be rewritten using (5), (28b) and (28c) as:

$$T'\left(Y_w\right) \cdot h\left(w\right) = Z_w \cdot \mathcal{Y}'_w = J_w \cdot \frac{\mathcal{Y}'_w}{\mathcal{Y}'_\tau}$$

Using (7), (9) (28b) and (28c) we obtain:

$$\frac{T'(Y_w)}{1-T'(Y_w)} \cdot h(w) = J_w \cdot \frac{\alpha_w}{\varepsilon_w \cdot w} \tag{39}$$

From (34) and (11) we obtain:

$$\begin{aligned} \dot{J}_w &= -\{1-g_w-\kappa_w\cdot(T(Y_w)+b)\}\cdot h(w) - Z_w\cdot\mathcal{U}''_{Cw}(C_w,Y_w,w) \\ &\quad + Z_w\left\{\mathcal{U}''_{CC}(C_w,Y_w,w)\,\dot{C}_w + \mathcal{U}''_{CY}(C_w,Y_w,w)\cdot\dot{Y}_w + \mathcal{U}''_{Cw}(C_w,Y_w,w)\right\} \end{aligned}$$

Assume now that the tax function is everywhere differentiable and there is no bunching. Differentiating $C_w = Y_w - T(Y_w)$ and using (5) gives:

$$\begin{aligned} \dot{J}_w &= -\{1-g_w-\kappa_w\cdot(T(Y_w)+b)\}\cdot h(w) \\ &\quad + Z_w\left\{\mathcal{U}''_{CY}(C_w,Y_w,w) - \mathcal{U}''_{CC}(C_w,Y_w,w)\frac{\mathcal{U}'_Y(C_w,Y_w,w)}{\mathcal{U}'_C(C_w,Y_w,w)}\right\}\cdot\dot{Y}_w \end{aligned}$$

Using (28c), (28d) and again (5):

$$\dot{J}_w = -\{1-g_w-\kappa_w\cdot(T(Y_w)+b)\}\cdot\hat{h}(w) + J_w\cdot\frac{\mathcal{Y}'_\rho}{\mathcal{Y}'_\tau}\cdot\dot{Y}_w$$

With (7), (9), (10), (28c) and (28d) :

$$\dot{J}_w = -\{1-g_w-\kappa_w\cdot(T(Y_w)+b)\}\cdot h(w) + J_w\cdot\frac{\eta_w\cdot\alpha_w}{\varepsilon_w\cdot w}\left(1-T'(Y_w)\right)$$

Finally, using (39):

$$\dot{J}_w = -\{1-g_w-\kappa_w\cdot(T(Y_w)+b)\}\cdot h(w) + \eta_w\cdot T'(Y_w)\cdot h(w)$$

Given $Z_{w_1}=0$, $J_{w_1}=0$, so $J_w = \int_w^{w_1}\left(-\dot{J}_n\right)dn$. Using the last equation and (39) gives (17). Equation (18) is obtained from the transversality condition $J_{w_1}=0$. Equation (19) is obtained by adding (35) to (18).

C Proof of Proposition 2

We return back to the case where $w \mapsto (C_w, Y_w)$ is continuous everywhere and differentiable everywhere except on a finite number of skill levels (so that bunching can occur on a finite number of skill intervals). Note that continuity of $w \mapsto Y_w$ implies that $w \mapsto U_w$ is continuously differentiable. We first show the following.

Lemma 2 *X_w (thereby Z_w) is everywhere nonnegative and almost everywhere positive within (w_0, w_1) whenever $w \mapsto \frac{1-g_w}{\kappa_w}$ is increasing.*

Proof. Assume by contradiction that $Z_{w'} \leq 0$ for some $w' \in (w_0, w_1)$. Then $X_{w'} \leq 0$. Through the continuity of $w \mapsto X_w$, and the transversality condition there exists a maximal interval $[w_2, w_3]$ where $X_w \leq 0$ for all $w \in [w_2, w_3]$ and $X_{w_2} = X_{w_3} = 0$. Moreover, as $w \mapsto C_w$ is also continuous everywhere and differentiable almost everywhere, X_w is everywhere differentiable with a derivative given by (38).

- Given $X_{w_2} = 0$ and $X_w \leq 0$ in the right neighborhood of w_2, one must have $\dot{X}_{w_2} \leq 0$. Hence, from (38):

$$\frac{1 - g_{w_2}}{\kappa_{w_2}} \geq T\left(Y_{w_2}\right) + b \tag{40}$$

- Given $X_{w_3} = 0$ and $X_w \leq 0$ in the left neighborhood of w_3, one must have $\dot{X}_{w_3} \geq 0$. By a symmetric reasoning, this leads to:

$$T\left(Y_{w_3}\right) + b \geq \frac{1 - g_{w_3}}{\kappa_{w_3}} \tag{41}$$

- One has:

$$T(w) + b = Y_w - \Gamma\left(U_w, Y_w, w\right)$$

Function $w \mapsto Y_w - \Gamma\left(U_w, Y_w, w\right)$ is continuous and, except at a finite number of points, differentiable with derivative:

$$\frac{d\left(Y_w - \Gamma\left(U_w, Y_w, w\right)\right)}{dw} = \dot{Y}_w\left(1 - \Gamma'_Y\left(U_w, Y_w, w\right)\right) - \Gamma'_U\left(U_w, Y_w, w\right) \cdot \dot{U}_w - \Gamma'_w\left(U_w, Y_w, w\right)$$
$$= \dot{Y}_w\left(1 + \frac{\mathcal{U}'_Y\left(U_w, Y_w, w\right)}{\mathcal{U}'_C\left(U_w, Y_w, w\right)}\right) - \frac{\mathcal{U}'_w\left(C_w, Y_w, w\right)}{\mathcal{U}'_C\left(C_w, Y_w, w\right)} + \frac{\mathcal{U}'_w\left(C_w, Y_w, w\right)}{\mathcal{U}'_C\left(C_w, Y_w, w\right)} = \dot{Y}_w\left(1 + \frac{\mathcal{U}'_Y\left(U_w, Y_w, w\right)}{\mathcal{U}'_C\left(U_w, Y_w, w\right)}\right)$$

where the second equality follows (31) and (36). If there is bunching at w, then $\dot{Y}_w = 0$. If there is no bunching at w, then Equation (32) applies. Condition (2) and $Z_w \leq 0$ then induce that $w \mapsto Y_w - \Gamma\left(U_w, Y_w, w\right)$ admits a nonpositive derivative. Hence, $w \mapsto Y_w - \Gamma\left(U_w, Y_w, w\right)$ is weakly decreasing over $[w_2, w_3]$, so:

$$T\left(Y_{w_2}\right) + b \geq T\left(Y_{w_3}\right) + b \tag{42}$$

Inequalities (40), (41) and (42) imply:

$$\frac{1 - g_{w_2}}{\kappa_{w_2}} \geq \frac{1 - g_{w_3}}{\kappa_{w_3}}$$

This is consistent with the assumption that $w \mapsto \left(1 - g_w\right)/\kappa_w$ is increasing if and only if $w_2 = w_3$. Therefore $w' = w_2 = w_3$ and $X_{w'} \geq 0$ for all skill levels and $X_w = 0$ only pointwise. ■

Given that X_w (hence Z_w) is nonnegative everywhere and can be nil only pointwise, then for skill levels where there is no bunching, according to (5) and (32), the marginal tax rate is nonnegative and can be nil only pointwise. Bunches of skills correspond to a mass point of the earnings distribution and to an upward discontinuity in the marginal tax rates. However, the discontinuity is between two marginal tax rates that correspond to skill levels without bunching for which we have shown that the marginal tax rates are nonnegative.

D Proof of Proposition 3

Given that $X_{w_0} = 0$ and for all w, $X_w \geq 0$ (from 2), then $\dot{X}_{w_0} \geq 0$. According to (38), this induces:

$$\frac{1 - g_{w_0}}{\kappa_{w_0}} \leq T\left(Y_0\right) + b$$

As $g_{w_0} \leq 1$, the left-hand side is positive, inducing that in-work benefit (i.e. $-T\left(Y_0\right)$ when $T\left(Y_0\right) < 0$) is lower than welfare benefit b.

E Proofs of Properties 1 and 2

Under (27), U_w is increasing in skill w. Then, a Maximin government values only the welfare of the nonemployed and $g_w = 0$ for all skill levels, which ensures property 2 for a Maximin government.

Under (26), $\mathcal{U}'_C$ depends only on the consumption level. From (2), incentive-compatible conditions (30) imply that $w \mapsto C_w$ is nondecreasing. Therefore, as $u''_{CC} < 0$, $w \mapsto \mathcal{U}'_C(C_w, Y_w, w)$ is nondecreasing, and increasing without bunching.

Under (26) and a Benthamite government, g_w simply equals $\mathcal{U}'_C(C_w, Y_w, w)/\lambda$ according to (15), which ensures property 2 for a Benthamite government.

Under Assumption (27), one has that the threshold value $U_w - \mathcal{U}(b, 0, w)$ of χ below which individuals of type (w, χ) choose to work, is decreasing in skill level w. So, when $K(\chi, w)$ is strictly log-concave with respect to χ and $w \mapsto k(\chi, w)/K(\chi, w)$ is nonincreasing in w, then $w \mapsto k(U_w - \mathcal{U}(b, 0, w), w)/K(U_w - \mathcal{U}(b, 0, w), w)$ is decreasing. Together with $w \mapsto \mathcal{U}'_C(C_w, Y_w, w)$ being nondecreasing, using (11) ensures that $w \mapsto \kappa_w$ is decreasing, even in the presence of bunching. Consequently, Property 1 is ensured.

E.1 Example 1

A Maximin government values only the welfare of the nonemployed, so $g_w = 0$ for all skill levels and $(1 - g_w)/\kappa_w = 1/\kappa_w$. As Property 1 holds, $(1 - g_w)/\kappa_w$ is therefore increasing in w and Proposition 2 applies. Moreover, as $g_w = 0$, Proposition 3 also applies.

E.2 Example 2

Combining Properties 1, 2 and $g_w \leq 1$ ensures that $(1 - g_w)/\kappa_w$ is increasing in w. As a result, Proposition 2 applies, and thereby Proposition 3 as it has been assumed that $g_w \leq 1$.

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