Staff Papers Series

Staff Paper P88-39 November 1988

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The Buffer Role of Groundwater When Surface Water Supplies Are Uncertain: The Implications For Groundwater Development

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The helpful comments of Arie Issar, K. William Easter and Ted Graham-Tomasi are gratefully acknowledged.

Abstract

When used in conjunction with surface water for irrigation, groundwater serves two roles: to increase water supply; and to mitigate fluctuations in the supply of water. The latter is the buffer role. This paper identifies and evaluates the economic benefit associated with the buffer role of groundwater. Implications for the development of groundwater resources are investigated. An estimate is given of the buffer benefit to wheat growers of the fossil water aquifer underlying the Israeli Negev. It is found that, under the prevailing variability in the supply of surface water, this benefit may well exceed the groundwater benefit associated with the increase in water supply.

The Buffer Role of Groundwater When Surface Water Supplies Are Uncertain: The Implications For Groundwater Development

1. Introduction

When used in conjunction with surface water for irrigation, groundwater serves two roles: first, to increase water supply; second, to mitigate undesirable fluctuations in the water supply. We refer to the latter as the buffer role of groundwater. The purpose of this paper is to assign an economic value to the buffer role of groundwater and to investigate its effect on the development of groundwater resources.

The task of how to use groundwater in conjunction with surface water has been the subject of much research, most notably that of Burt (1964). These analyses generally are carried out within a dynamic framework, seeking a rule for allocating the groundwater over time when the demand for groundwater varies according to available supply of surface water. The buffer role of groundwater and its implications for the development of groundwater resources remain implicit in these analyses. The analysis here is presented within a single period model. The results may be interpreted as the solution of a dynamic system that have reached a steady state or as an approximation of a more general dynamic model (on the conditions that justify such an approximation see Gisser [1983]).

The term "conjunctive ground- and surface-water system" is applied to a number of systems; they differ according to the surface water source. One extreme is a system in which the only source of surface water is stream flows emanating from aquifers. A situation similar to this is considered in Young and Bredehoeft (1972). In the other extreme, surface water is independent of groundwater sources. Cummings and Winkelman (1970) analyze a system near this extreme. The system considered in this paper is at the second extreme, in which surface water derives solely from precipitation. This is the situation in the region that concerns us, the northern part of the Negev desert in Israel.

While many of the factors affecting the development of groundwater resources are likely to be uncertain (see e.g.,Szidarovsky et al. [1976], Taylor and North [1976]), the present paper focuses only on the effect of uncertainty in the supply of surface water. The analysis applies, perhaps with some modifications, to any deterministic stock of water, such as a reservoir or a dam, playing the role of groundwater.

The next section (an extension of Section 3 of Tsur and Issar [1987]) identifies the benefit associated with the buffer role of groundwater, denoted as the buffer value of groundwater. In section 3, the task of choosing the appropriate capacity for a groundwater project is analyzed. It is shown that, in general, the capacity of the groundwater project should increase with the variability in the supply of surface water. This result may explain observed cases of apparent over-investment in well capacity (Bredehoeft and Young [1983]). Section 4 applies the analysis to the fossil water aquifer underlying the Negev desert in Israel. It demonstrates that, under the prevailing rainfall variability, the buffer value of groundwater to growers of unirrigated wheat in the northern Negev may well exceed the benefit associated with the increased water supply. Implications for the development of the aquifer are discussed. A brief summary, Section 5, concludes the paper.

2. The Buffer Value of Groundwater

Let S indicate the stochastic supply of surface water distributed according to some cumulative distribution function H. It is assumed that H is uniquely defined by the vector of moments $\theta=(\mu,\sigma,\ldots)$, where $(\mu,\sigma,\ldots)$ represents the mean, standard deviation and higher moments of S. In the absence of groundwater, growers use the realized amount of surface water and enjoy the operating profit per hectare (ha) of pF(S), where $F(\cdot)$ represents per hectare yield response to water and p is the net unit value of output. When operating in a certain environment profit is $pF(\mu)$. The water response function $F(\cdot)$ depends on other variable inputs, but these can be suppressed by a conditional optimization over these inputs given prices and the level of water input.

Suppose now that groundwater becomes available at a fixed price of $\$z/m^3$. Let K(z) indicate the level of water input satisfying $pF'(K(z))=z$ (see Figure 1). In a stable environment, where surface water supply is fixed at μ, the demand for groundwater is $K(z)-\mu$ m^3/ha, provided $K(z)>\mu$. The net benefit of groundwater, obtained by subtracting from total profit the cost of groundwater and the profit of surface water, is equal to (\$/ha)

$$B_c(p,z,\mu) = \begin{cases} pF(K(z)) - z[K(z)-\mu] - pF(\mu) & \text{if } K(z) > \mu \\ 0 & \text{otherwise} \end{cases} . \quad (2.1)$$

In Figure 1, B_c equals the area (abc).

Figure 1

The demand for groundwater in an uncertain environment, with stochastic surface water supplies S, depends on whether growers make their decisions before or after the actual realization of S is observed. Each situation gives rise to a different buffer values of groundwater. The buffer value of groundwater corresponding to situations where decisions on groundwater demand are made after the actual realization of S is observed will be labeled as *ex post*, while that corresponding to situations where decisions must be made in advance of the realization of S will be referred to as *ex ante*. Throughout we assume risk neutrality on the part of growers.[1]

Given observations on the actual realization of surface water, and provided this level does not exceed K(z),[2] the *ex post* demand for groundwater is K(z)-S m^3/ha. It provides the net benefit: pF(K(z))-z[K(z)-S]-pF(S). The mean of this net benefit, when the randomness of S is accounted for, is given by ($/ha):

$$B_u^p(p,z,\theta) = p[F(K(z))-E\{F(S)\}] - z[k(z)-\mu]. \tag{2.2}$$

where E is the expectation operator with respect to the distribution H and is therefore a function of θ.

In the *ex ante* scenario the demand for groundwater, g_a, is determined from $\underset{g\geq 0}{MAX}\{E\{pF(S+g)\}-zg\}$ and yields the expected net benefit

$$B_u^a(p,z,\theta) = pE\{F(S+g_a)-F(S)\} - zg_a. \tag{2.3}$$

The buffer value of groundwater is defined as the difference between the net benefit of groundwater in the uncertain and certain environments. It is the amount a grower facing an uncertain surface water supply, S, would be willing to pay for groundwater over and above the corresponding amount he or she would be willing to pay had the surface water supplies been certain at μ. In other words, it is a measure of how much a producer would be willing to pay to move from a situation in which surface water supplies fluctuate about a mean μ to a stable environment in which this supply is fixed at the level μ.

Given the definitions above and provided K(z)>μ, the *ex post* buffer value is given as

$$BV^p(p,z,\theta) = B_u^p(p,z,\theta)-B_c(p,z,\mu) = p[F(\mu) - E\{F(S)\}]. \tag{2.4}$$

the *ex ante* buffer value is (after some algebraic manipulations)

$$BV^a(p,z,\theta) = B_u^a(p,z,\theta) - B_c(p,z,\mu) =$$
$$= BV^p(p,z,\theta) + \left\{pE\{F(S+g_a)\} - pF(K(z)) - z[g_a-(K(z)-\mu)]\right\}. \tag{2.5}$$

By assuming that $F(\cdot)$ is strictly concave and using Jensen's inequality, it follows that BV^p is positive. The introduction of groundwater has shifted the uncertainty from production to costs. Since costs are linear in groundwater quantity, producers are indifferent to uncertainty in this component. Production, on the other hand, is concave in water input and stability in the water input is therefore desirable.

The following simple example illustrates this point. Suppose there are only two possible states of the nature, which are equally likely to occur. In the first $S=s_1$, corresponding to a drought year, and in the second $S=s_2$, where $s_1<s_2<K$ (the argument z is dropped from $K(\cdot)$ for notational convenience). At the beginning of the year, before the realization of S is known, the farmer enjoys the random profit $pF(s_1)$ or $pF(s_2)$ with equal probabilities. In Figure 1, $pF(s_1)$=area$\{hgs_1o\}$ and $pF(s_2)$=area$\{hds_2o\}$. In comparison with the situation where S is stable at the level μ, this may be viewed as an uncertain prospect in which the farmer loses the amount given by area$\{ga\mu s_1\}$ with probability 0.5 and gains the amount given by area$\{ads_2\mu\}$ with probability 0.5. Since the derived demand for water slopes downward (resulting from the strict concavity of F), this is an unfavorable bet entailing a negative expected gain. Even a risk neutral grower prefers the stabilized situation under such circumstances. The amount a grower would be willing to pay to

ensure stabilization is BV^p \$/ha, which is given by e-f in Figure 1.

By expanding F(S) about μ, letting $\gamma = -F''(\mu)/F'(\mu)$, BV^p can be approximated as:

$$BV^p \quad .5pF'(\mu)\gamma\sigma^2.$$

This illuminates the dependence of BV^p on: a) the value of marginal productivity of water at input level μ ($pF'(\mu)$); b) the degree of concavity of F at μ (γ); and c) the variance in surface water supply (σ^2).

The *ex ante* index BV^a differs from BV^p by the term inside the large curly brackets on the right hand side of (2.5). It readily is verified that this term is negative; thus $BV^p > BV^a$.[3] In contrast to the *ex post* scenario, in the *ex ante*, when growers commit themselves to a certain amount of groundwater before observing the actual realization of surface water, some of the uncertainty in water input to production is retained, leading to a lower buffer value.

Whether the *ex ante* or *ex post* concept is appropriate depends on how the market for groundwater operates. An argument can be made in favor of the *ex post* procedure. Empirical evidence suggests that, in many cases, the derived demand for irrigation water is convex toward the origin (see, e.g., Howitt et al. [1980]). This is an indication that the water response function F(·) has a positive third derivative. In such cases, g_a exceeds $K-\mu$, which is the average groundwater demand under the *ex post* procedure.[4] Thus, on average, more groundwater is consumed under the *ex ante* allocation mechanism than under the *ex post* one. Conservation of groundwater resources has been, and will continue to be, an issue of great concern. Furthermore, in the *ex post* scenario farmers are better informed, hence use groundwater more efficiently. Thus, for example, a farmer operating under the *ex ante* mechanism uses the pre-committed (pre-paid) quantity of groundwater g_a even when the supply of surface water eventually happens to be so large that the groundwater entails losses.

Aside from the above remarks, no further evaluations or comparisons will be made of the two groundwater allocation mechanisms. From now on the focus is exclusively on the *ex post* situation, which is the one prevailing in the case we wish to study--the Negev region in Israel. The *ex post* buffer value will simply be referred to as BV.

3. Capacity Choice

The presence of a positive buffer value implies that groundwater is worth more in an uncertain environment than in a certain one, even though the usage rate is, on average, the same in both cases. A cost benefit argument should thus support larger capacity investment in groundwater pumping facilities in regions where fluctuations in surface water supplies are larger. This section undertakes an analysis of this proposition. The significance of the error made in the capacity choice when the buffer role of groundwater is neglected (i.e., when surface water supplies are regarded as fixed at their mean level) depends on the size of the buffer value of groundwater. The case of the Israeli Negev region, discussed in the next section, suggests that this error may be substantial.

The capacity of a groundwater project is the maximum quantity of groundwater that can be supplied in a given year and is denoted by Q. Let C(Q) be the annual imputed cost of the investment in a groundwater project of capacity Q. The cost function C(·) is generated by the technology of groundwater extraction and is assumed to be non-decreasing in Q. The criterion for choosing the capacity Q depends mainly on the structure of the groundwater supply industry including the extraction technology, who owns the groundwater and the rules governing well placement and extract rates--an interesting subject on its own, but one that lies outside the scope of this paper. Here we assume that the capacity of the groundwater project is chosen so as to maximize expected net benefit.

Let A be the number of cultivated hectares and $q=Q/A$ denote the capacity per hectare. In an uncertain environment, the use of groundwater (per hectare) and the resulting benefit it generates depend on the state of the nature S: if $S>K$, no groundwater is used; in the event $K-q<S\leq K$, K-S m^3/ha of groundwater are applied and the aggregate benefit is $A[pF(K)-z(K-S)]-C(Q)$; in the event $S\leq K-q$, the entire capacity level q is utilized providing the benefit $A[pF(S+q)-zq]-C(Q)$.[5] The expected total net benefit of groundwater, with stochastic surface water supplies S, is thus given by

$$W_u(Q) = A\Big[E\{pF(K)-z(K-S)\,|\,K-q<S\leq K\}\text{Prob}(K-q<S\leq K) + E\{pF(S+q)-zq\,|\,S\leq K-q\}\text{Prob}(S\leq K-q)\Big] - C(Q) - A\cdot E\{pF(S)\,|\,S\leq K\}\text{Prob}(S\leq K). \quad (3.1)$$

With stable surface water supplies, the level of groundwater use is q, provided that $q\leq K-\mu$, and the associated aggregate benefit is

$$W_c(Q) = A[pF(\mu+q)-zq] - C(Q) - A\cdot pF(\mu) \quad (3.2)$$

Maximizing W_u and W_c with respect to Q, recalling that $q=Q/A$, yields the necessary conditions:

$$E\{pF'(S+q_u)-z\,|\,S\leq K-q_u\}\text{Prob}(S\leq K-q_u) = C'(Q_u) \quad (3.3)$$

and

$$pF'(\mu+q_c) - z = C'(Q_c), \quad (3.4)$$

where Q_u and Q_c are the capacity levels that maximize W_u and W_c, respectively, and $q_u = Q_u/A$, $q_c = Q_c/A$.[6] The total net benefits of groundwater are determined by evaluating W_u and W_c at the capacity choices Q_u and Q_c: $W_u^* = W_u(Q_u)$ and $W_c^* = W_c(Q_c)$. These benefit indexes serve as the economic criteria for the decision on whether or not to develop a given groundwater aquifer. They may differ substantially from each other, as the example of the next section indicates.

To gain insight into the difference between the capacity choices q_u and q_c, suppose first that $C'(Q)$ is negligible. Condition (3.3) requires $q_u = K$ and condition (3.5) yields $q_c = K - \mu$.[7] Thus the groundwater capacity in the unstable environment is sufficient to fulfill the groundwater demand in the worse possible case where $S=0$. This finding explains, to some extent, apparent over-investment in well capacity in locations with high variability in supply of surface water. Bredehoeft and Young (1983) attribute such observations to effects of risk aversion. Here it is shown that such outcomes may be justified without assuming risk aversion.

In cases where $C'(Q_u) > 0$, conditions (3.3) and (3.4) require $q_u < K$ and $q_c < K - \mu$. As expected, capacity decreases with the marginal capacity costs. However, it can be shown that, in most cases, the result that q_u exceeds q_c is preserved.[8] Thus, investments in developing groundwater resources, if desirable at all, are likely to be larger under the unstable environment.

The variability in the supply of surface water causes some of the groundwater capacity to stand idle some of the time (the entire capacity is idle when $S>K$ and a fraction is idle when $K-q_u<S<K$). With stable surface water supplies the groundwater capacity is fully employed. The cost of idle capacity provides an offsetting disincentives to invest in groundwater

development when surface water supplies are unstable. Nevertheless, the presence of the buffer role of groundwater is sufficient to outweigh this effect and to make investment in developing groundwater resources more desirable when surface water supplies fluctuate. Indeed it can be shown that the difference $W_u^* - W_c^*$ and the buffer value of groundwater are related through

$$W_u^* - W_c^* = A \cdot BV(q_u, q_c) - [C(Q_u) - C(Q_c)], \tag{3.5}$$

where $BV(q_u, q_c)$ is the buffer value in the presence of the capacity limits q_u and q_c. The verification of (3.5) and the extension of the buffer value to situations involving capacity constraints is left for the Appendix.

The capacity choice rule described above implicitly assumes a steady state situation, i.e., that the average rate of water recharge into the aquifer is at least as big as the average annual withdrawal of groundwater. If this is not the case, a decline in the groundwater table takes place, which causes an increase in pumping costs, z, which in turn reduces the demand for groundwater over time. This process will continue until the system reaches a steady state.

It may happen that the rate of water replenishment is negligible; examples are the so called fossil water aquifers (Margat and Saad [1984]). In such cases, to pump water means to mine water and such activities must be determined within an intertemporal framework to account for water scarcity, in addition to the user cost of the water. The intertemporal allocation of groundwater resources has been addressed by numerous authors (Burt [1964b], Burt and Cummings [1970], Cummings and Winkelman [1970], Domenico et al [1968], among others); the task of analyzing the buffer role of groundwater within this framework is left for future research.

Closely related to the case of a nonrenewable aquifer is the issue of irreversibility of capital investments in pumping capacity. If depletion of the aquifer is eventually to occur, much of the pumping facility will then become idle and the future cost of idleness should effect present capacity decisions.

The problem of capital immalleability in long-term resource allocation is considered, among others, in Cummings and Burt (1969) (also in their 1970 paper), and Clark et al. (1979). A discussion of how well a static allocation scheme approximates an intertemporal one can be found in Gisser and Sanchez (1980), and Gisser (1983). The task of capacity choice within a dynamic model is treated, in a related context, in Hochman et al. (1984). These questions are also beyond the scope of this paper.

4. An illustrative example

The Negev desert comprises the southern, arid part of Israel and is part of the desert belt extending from the Sahara through Egypt to the Arabian Desert. The northern part of the Negev contains large amounts of arable land and is characterized by high variability of precipitation. Table 1 presents data on annual rainfall in the period 1949-50 to 1986-87 as measured in *Kibbutz* Beit-Qama, located in the center of the northern Negev.

Table 1

A large portion of the cultivated land is allocated to unirrigated wheat. In 1985, the total cultivated area in the Negev was 77,551.5 hectares (ha), out of which 52,008.5 ha were allocated to unirrigated wheat (Israel Ministry of Agriculture and the Jewish Agency [1985]).

Except for the heavens, additional water sources are local aquifers and the national water conveyor that carries water from the northern part of the country. Both sources are exploited to their limits. The use of local aquifers is limited by considerations of water recharge and water quality; shipping water from the north is simply an expensive operation.

A potential water source, utilized to some extend in the Arava valley of the southern Negev, is the large aquifer of fossil water underlying the Negev and the Sinai peninsula (see Issar [1985] for a hydrological account). Though feasibility studies on the development and use of this aquifer in the northern Negev can be found, most notably that of Tushia (1981), many cost components are still unknown. Thus, various levels of pumping cost, z, and marginal capacity cost, $C'(Q)=c$, will be considered.

To evaluate the buffer value of the fossil groundwater associated with unirrigated wheat in the northern Negev, requires first to estimate the wheat response to water. To that end, data of wheat yield and rainfall quantities were collected from unirrigated fields of the *kibbutzim* Beit-Qama and Mishmar-Hanegev. These data are reported in Table 2.

Table 2.

The water production function is specified as

$$F(x) = \begin{cases} \alpha-\beta/x & \text{if } x > \alpha/\beta \\ 0 & \text{otherwise} \end{cases} . \qquad (4.1)$$

For its non-vanishing part, this is a concave function which, as the water input x (m^3/ha) is increased, approaches the maximum yield α (kg/ha) at a rate which depends on β. Allowing the water production function $F(\cdot)$ to vanish on some positive interval of water input requires some modifications of the analysis of Sections 2 and 3. These modifications are presented in the

Appendix.

The regression of yield (kg/ha) on the inverse of water input ($1/(m^3/ha)$), using the data of Table 2, provides the following estimates (t values are in parentheses):

$$\hat{\alpha} = \underset{(10.3)}{4{,}548.809}; \quad \hat{\beta} = \underset{(6.9\ 7)}{7{,}145{,}701}; \quad R^2 = .65; \quad DW = 1.85; \quad 28 \text{ observations.}$$

The estimates imply that a minimum water application of $\hat{\beta}/\hat{\alpha} = 1570.9$ m^3/ha (equivalent to 157.09 mm rain) is required to obtain a positive yield, and that the maximum attainable yield equals, on average, $\hat{\alpha} = 4548.809$ kg/ha.

We consider two levels of wheat price (p) and four levels of groundwater price (z): p= \$0.193/kg, the price received by Israeli wheat growers in 1985; p= \$0.12/kg, which better reflects the market price of wheat in 1985; z=0.05, 0.1, 0.15 and 0.2 $\$/m^3$. We assume first that the groundwater capacity does not constitute a constraint and calculate the (per hectare) buffer value for each p,z combination using formula (A.3) of the Appendix. The results are presented in Table 3, which also presents: estimates of the groundwater benefit attributed to the increase in the water supply, i.e. the groundwater benefit that would prevail if rainfall were stable at $\mu = 2931.18$ m^3/ha, denoted B_c [cf. equation (2.1)]. Table 3 also contains the average groundwater demand per hectare $K - \mu$. Details on the method of calculation of the buffer value are provided in the Appendix.

Table 3

Table 3 reveals that when wheat price is 0.193 \$/kg and groundwater costs $\$0.1/m^3$, the buffer value of groundwater in the northern Negev is BV = \$48.4/ha and the benefit due to increase in water supply is B_c = \$20.89/ha.

The benefit of groundwater due to its buffer role is more than twice the benefit associated with its role in increasing the water supply. The total groundwater benefits (per hectare) is the sum of the two: 48.4 + 20.89 = \$69.29/ha.

In calculating the capacity choices we consider the case where the cost function C(Q) is linear in the capacity Q. This constant marginal capacity choice is denoted by c. For various levels of p, z and c, Table 4 presents: q_u--the capacity choice under the prevailing rainfall variability; q_c--the capacity choice calculated under the assumption that rainfall quantities are stable at $\mu = 2931.18$ m^3/ha (equivalent to 293.118 mm rainfall); B_u and B_c--the net benefits from groundwater (abstracting from fixed costs) associated with q_u and q_c, defined in equations (A.2) and (2.1); and $BV = B_u - B_c$--the buffer value of groundwater with the capacity constraints q_u, q_c.

Table 4

The results of Table 4 reveal that when the prices of wheat and groundwater are \$0.12/kg and \$0.1/m^3, respectively, the development of groundwater would not be desirable if annual rainfall were taken to be stable at its mean. Under the prevailing rainfall variability, the desired groundwater capacity is $q_u = 2{,}130.28$ m^3/ha, 1,720.29 m^3/ha or 0 m^3/ha as c = 0, 0.001 or 0.01, respectively. Multiplying by A=52,008.5 ha (the area of unirrigated wheat recorded in the northern Negev at 1985) yields the respective aggregate capacities.

Consider, for the sake of illustration, the case where c = 0.001, p = \$0.193/kg and z = \$0.1/m^3. The total net benefit of groundwater to growers of unirrigated wheat in the northern Negev (abstracting from the fixed costs

$C(Q_u)$) is $B_u \cdot A$ = \$2.9645 million per year. Under the same scenario but with surface water supplies regarded as fixed at μ=2,931.18 m^3/ha, total net benefit of groundwater is $B_c \cdot A$ = \$1.086 million per year. If the annual (imputed) capacity cost levels $C(Q_c)$ and $C(Q_u)$ fall between these two levels, developing the groundwater aquifer cannot be justified on economic grounds when surface water supplies are assumed stable; whereas with the prevailing instability the development is warranted. With wheat price at the level p=\$.12/kg and everything else remaining the same, developing the fossil water aquifer may still be warranted as long as the annual fixed costs do not exceed \$764,000.

5. **Summary**

When the supply of surface water is uncertain, groundwater, in addition to its role in increasing the water supply, serves also as a buffer that mitigates undesirable fluctuations in water supply. In this paper, we evaluate the benefit associated with the buffer role of groundwater. Implications for the development of groundwater resources are then investigated. It was found that, in general, the investment in groundwater should increase with the variability of the supply of surface water. Thus, depending on the cost structure of the groundwater pumping technology, a pumping capacity sufficient to irrigate the entire region may indeed be desirable if the supply of surface water falls to zero. Application of the analysis to the fossil water aquifer underlying the Negev desert in Israel reveals that, with the prevailing variability in annual rainfall, the magnitude of the buffer value of groundwater may well exceed the groundwater benefit attributed to the average increase in water supply.

The present analysis can be extended in various directions. First, by accounting for intra-season variability in the supply of surface water. In general, yield response to water depends not only on the quantity of water available, but also on how this quantity is applied during the growing season (see, e.g., Yaron et al. [1973]). We expect that allowing the groundwater to improve the timing of water application within the growing season, would even magnify the buffer role of groundwater. Second, by considering the effects of various crop insurance schemes that are known to exist. We expect that allowing farmers to insure against low levels of surface water supplies would mitigate the buffer role of groundwater. Finally, the output and groundwater prices may be considered to be determined endogenously; the first depending on the level of output and the second on the level of groundwater consumed.

Appendix

In the example of Section four, the production function $F(\cdot)$ vanishes for some positive interval of water inputs and the supply of surface water exceeds K with a positive probability. This Appendix provides the extension needed to account for such cases as well as a description of the procedures used to obtain the estimates reported in Tables 3 and 4.

In cases where $F(x)=0$ for some positive interval of water inputs, groundwater that costs $\$z/m^3$ will be demanded only when $L \le S \le K$, where L is defined from

$$pF(K) - z(K-L) = 0, \tag{A.1}$$

and it is recalled that K, defined in Section 2, is the level satisfying $pF'(K)=z$. (It is easy to verify that when $S<L$, any quantity of groundwater bought at a price $\$z/m^3$ entails losses.) When Prob($S<L$ or $S>K$) is strictly positive, the net benefit defined in equation (2.2) is modified as

$$B_u = E\{pF(K)-z(K-S) - pF(S)|L\leq S\leq K\}\cdot Prob(L\leq S\leq K).$$

Using the relation $E\{X\}=E\{X|A\}Prob(A)+E\{X|not\ A\}Prob(not\ A)$ for any random variable X and an event A, B_u can be rewritten as

$$B_u = pF(K) - z(K-\mu) - pE\{F(S)\} - E\{pF(K)-z(K-S)-pF(S)|S<L \text{ or } S>K\}Prob(S<L \text{ or } S>K). \quad (A.2)$$

By subtracting B_c, of equation (2.1), the buffer value becomes

$$BV = p[F(\mu)-E\{F(S)\}] - E\{pF(K)-z(K-S)-pF(S)|S<L \text{ or } S>K\}Prob(S<L \text{ or } S>K) \quad (A.3)$$

Note that whenever $Prob(S<L)=Prob(S>K)=0$, the buffer value defined in (A.3) coincides with that of (2.4).

We turn now to the capacity choice task. Under a capacity constraint q, the level of surface water supply below which groundwater will not be demanded is denoted by L(q) and is determined from

$$pF(L(q)+q) - zq = 0. \quad (A.4)$$

As in the definition of L above, it is easy to verify that, with a capacity limit q, when $S < L(q)$, any quantity of groundwater that costs $\$z/m^3$ entails losses. Thus L(q) replaces L. W_u of equation (3.1) becomes

$$W_u(Q) = E\{A[pF(K)-z(K-S)]|K-q<S\leq K\}Prob(K-q<S\leq K) + E\{A[pF(S+q)-zq]|L(q)<S\leq K-q\}Prob(L(q)<S\leq K-q) - C(Q) - - E\{pF(S)|S\leq K\}Prob(S\leq K), \quad (A.5)$$

and the necessary condition (3.3) is modified accordingly as

$$E\{pF'(S+q_u)-z|L(q_u)\leq S\leq K-q_u\}Prob(L(q_u)\leq S\leq K-q_u) = C'(Q_u) \quad (A.6)$$

The sufficient conditions for Q_u and Q_c to be local maxima of W_u and W_c are, respectively, $\int_{L(q_u)}^{K-q_u} pF''(S+q_u)h(S)dS<C''(Q_u)$ and $pF''(\mu+q_c)<C''(Q_c)$. The

strengthened condition

$$pF''(x) < C''(y) \text{ for all } 0 \le x \le K,\ 0 \le y \le A \cdot K \qquad (A.7)$$

guarantees global maxima.

With the capacity constraints q_u and q_c, further modification in the definition of the buffer value is required. B_u, of equation (A.2), with a capacity constraint q_u, becomes:

$$B_u(q_u) = E\{pF(K)-z(K-S)|K-q_u \le S \le K\}\text{Prob}(K-q_u \le S \le K) + \\ + E\{pF(S+q_u)-zq_u|L(q_u) \le S < K-q_u\}\text{Prob}(L(q_u) \le S < K-q_u) - \\ - E\{pF(S)|S \le K\}\text{Prob}(S \le K). \qquad (A.8)$$

Likewise, B_c defined in equation (2.1), is modified, in the presence of a capacity limit $q_c \le K-\mu$, to:

$$B_c(q_c) = \begin{cases} pF(\mu+q_c) - zq_c - pF(\mu) & \text{if } K > \mu \\ 0 & \text{otherwise} \end{cases}. \qquad (A.9$$

The buffer value with capacity constraints q_u and q_c is given by

$$BV(q_u,q_c) = B_u(q_u) - B_c(q_c). \qquad (A.10)$$

Using (A.8)-(A.10) and that $W_u^* = W_u(Q_u)$, $W_c^* = W(Q_c)$ [W_u and W_c are defined in equations (3.1)-(3.2)], equation (3.5) can be verified.

The buffer values reported in Table 3 are calculated as follows. For each level of p and z, we calculate $K = \sqrt{p\beta/z}$ and $L = K - pF(K)/z$, where $F(\cdot)$ is defined in (4.1) with its parameters evaluated at $\hat{\alpha},\hat{\beta}$ and $\hat{\beta}/\hat{\alpha} = 1579.9$ m^3/ha as the level of water input below which production ceases. Corresponding to the rainfall series S_t, $t=1,2,\ldots,38$, presented in Table 1, the series $F(S_t)$ is constructed and the mean μ is estimated by $\hat{\mu} = \sum_{t=1}^{38} S_t/38 = 2931.18$ m^3/ha. The first, second and third terms on the right hand side of (A.3) are estimated, respectively, by $I = p[F(\hat{\mu}) - \sum_{t=1}^{38} F(S_t)/38]$, $II = [pF(K)-zK]N(K,L)/38 + z \sum_{N(K,L)} S_t/38$ and $III = p \sum_{S_t \geq K} F(S_t)/38$, where $N(K,L)$ = (# of cases with $S_t<L$ or $S_t>K$) and $\sum_{N(K,L)}$ represents summation over these cases. BV is estimated by $I - II + III$.

The estimation of the terms reported in Table 4 is now described. The definition of L(q) in (A.4) and the form of F(·) given in (4.1) imply that $L(q)=\hat{\beta}/(\hat{\alpha}-zq/p)-q$. Given values for p and z and the estimates $\hat{\alpha}$, $\hat{\beta}$, the series $R_t(q) = pF'(S_t+q)-z$ can be constructed for any value q, where $F'(x) = 0$ or $\hat{\beta}/x^2$ as x is less than or greater than $\hat{\beta}/\hat{\alpha}$=1570.9 m^3/ha, respectively [cf. equation (4.1)]. Following (A.6) and given c (=C′), q_u is found as the level of q satisfying

$$\sum_{L(q)\leq S_t \leq K-q} R_t(q)/38 = c. \tag{A.11}$$

Following (3.4) and given the form of F, q_c is estimated as

$$q_c = \sqrt{p\hat{\beta}/(z+c)} - \hat{\mu}, \tag{A.12}$$

provided the right hand side of (A.12) is positive; q_c=0 otherwise. The third item reported in Table 4 is $B_u(q_u)$. It is Estimated, following (A.8), by

$$\sum_{K-q_u<S_t\leq K} [pF(K)-z(K-S_t)]/38 + \sum_{L(q_u)<S_t\leq K-q_u} [pF(S_t+q_u)-zq_u]/38 - \sum_{S_t\leq K} pF(S_t)/38.$$

$B_c(q_c)$ is estimated directly, using (A.9), by $pF(\hat{\mu}+g_c) - zq_c - pF(\hat{\mu})$, provided $q_c>0$; it is equal to zero otherwise. Finally, the buffer value when capacity limits are q_u and q_c is evaluated as the difference between the estimates of $B_u(q_u)$ and $B_c(q_c)$.

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Footnotes

[1]Obviously, allowing for risk aversion would affect the demand for water (see e.g., Lyon [1983]) and, thereby, the buffer value of groundwater. However, as will soon become apparent, the presence of a positive buffer value is not at all a result of risk aversion (even though risk aversion may affect its magnitude). We therefore maintain risk neutrality.

[2]For clarity of presentation, it is assumed throughout this section that the level of surface water supply may not exceed K(z), i.e., that Prob(S>K(z))=0, and that the water response function $F(\cdot)$ is strictly concave for any positive level of water input. The general case, where S may exceed K(z) and F may be convex for low levels of water input, is left for the Appendix.

[3]Define $D(x,g_a)=p[F(x+g_a)-F(K)]-z[g_a-(K-\mu)]$ and note, from (2.5), that $BV^a-BV^p=E\{D(S,g_a)\}$. Hence the strict concavity of F, using Jensen's inequality again, implies that $BV^a-BV^p < D(\mu,g_a)$. Now note that for all levels of g_a, $D(\mu,g_a)\leq 0$ with equality holding only for $g_a=K-\mu$.

[4]Recall that g_a solves $\underset{g\geq 0}{MAX}\{pE\{F(S+g)\}-zg\}$ and satisfies $pE\{F'(S+g_a)\}=z$, whereas K satisfies $pF'(K)=z$. Expanding $F'(S+g_a)$ about $\mu+g_a$, using the strict concavity of F and assuming $F'''>0$, verifies $g_a>K-\mu$.

[5]If obtaining positive yield requires a minimal positive amount of water, it is possible that groundwater will not be demanded when S falls short of some critical level. This situation is treated in the Appendix.

[6]To verify (3.3), rewrite (3.1), disregarding its rightmost term, as $W_u = A\left\{\int_{K-q}^{K}[pF(K)-z(K-S)]h(S)dS + \int_{0}^{K-q}[pF(S+q)-zq]h(S)dS\right\} - C(Q)$, where $h(\cdot)$ is the density function of S. Now differentiate W_u with respect to Q, recalling $q=Q/A$, to obtain $\partial W_u/\partial q = \int_{0}^{K-q}[pF'(S+q)-z]h(S)dS - C'(Q) =$ $E\{pF'(S+q)-z|S\le K-q\}Prob(S\le K-q) - C'Q)$.

[7]To verify that $q_u=K$ in this case note that $pF'(S+q_u)>z$ for all levels of S satisfying $S<K-q_u$. Thus, $E\{pF'(S+q_u)-z|S\le K-q_u\} > 0$ for all $q_u > 0$ (cf. Figure 1) and the left hand side of (3.3) vanishes only when $Prob(S\le K-q_u)=0$. Assuming that S can take any positive value with a positive probability, $Prob(S\le K-q_u)=0$ requires $q_u=K$.

[8]Suppose $C'(Q)$ is constant, say $C'(Q)=c$, and that the derived demand for water is convex toward the origin so that $F'''>0$. Then $pF'(\mu+q)-z < E\{pF'(S+q)-z\} =$ $E\{pF'(S+q)-z|S<K-q\}Prob(S<K-q)+E\{pF'(S+q)-z|S\ge K-q\}Prob(S\ge K-q)$. But the last conditional expectation term is non-positive because $pF'(S+q)\le z$ for all $S\ge K-q$ (cf. Figure 1). Hence $pF'(\mu+q)-z < E\{pF'(S+q)-z|S<K-q\}Prob(S<K-q)$ for all levels of q. Now if q_u satisfies (3.3), it is clear that setting $q_c=q_u$ causes a violation of (3.4); q_c must be smaller than q_u for (3.4) to hold. This result can be extended to more general capacity cost functions.

Table 1.

The series S_t: annual amounts of rain (mm) as measured in *Kibbutz* Beit-Qama located in the center of the northern Negev.

Year	Rain	Year	Rain	Year	Rain	Year	Rain
1949-50	407.5	1959-60	116.8	1969-70	222.3	1979-80	543.6
1950-51	189.5	1960-61	304.7	1970-71	326.2	1980-81	278.6
1951-52	334.3	1961-62	139.4	1971-72	415.2	1981-82	213.8
1952-53	246.6	1962-63	79.8	1972-73	281.1	1982-83	477.1
1953-54	303.4	1963-64	461.2	1973-74	422.7	1983-84	173.1
1954-55	222.2	1964-65	448.6	1974-75	273.6	1984-85	239.5
1955-56	374.3	1965-66	177.3	1975-76	194.8	1985-86	205.4
1956-57	504.9	1966-67	364.1	1976-77	272.1	1986-87	360.8
1957-58	233.3	1967-68	362.2	1977-78	253.6		
1958-89	239.2	1968-69	233.6	1978-79	242.4		

mean (S_t) = 293.118
standard deviation (S_t) = 110.957

Table 2.

Average yield (kg/ha) and amount of rainfall (translated into m^3/ha) of unirrigated wheat fields of Beit-Qama and Mishmar-Hanegev

Yield (kh/ha)	Water (m^3/ha)	Yield (kh/ha)	Water (m^3/ha)	Yield (kg/ha)	Water (m^3/ha)	Yield (kh/ha)	Water (m^3/ha)
900	2200	360	1780	610	2780	1940	4070
1660	2300	1960	3440	1530	2450	470	1760
1350	2575	520	1500	1280	2500	2530	3930
1720	2670	800	2050	3090	3180	350	1550
800	2190	500	1760	3300	3170	1380	2721
3000	5436	2155	2786	737	2138	4952	4771
1100	1731	1090	2395	1230	2054	3575	3608

Table 3.

Buffer value (BV), groundwater benefit due to increase in water supply (Bc) and average groundwater consumption E(k-s) with no capacity constraints

	Z:	0.05	0.1	0.15	0.2
	BV (\$/ha)	52.56	48.40	33.26	14.7
P=.193	B_c(\$/ha)	91.87	20.89	0.52	0.0
	E{k-s}(m_3/ha)	2325.56	970.05	511.34	242.45
	BV (\$/ha)	31.85	17.77	4.29	0.74
P=.12	B_c(\$/ha)	24.97	0.0	0.0	0.0
	E{k-s}(m_3/ha)	1308.57	451.19	144.11	25.05

Table 4.

Capacity choices (q_u and q_c,) net benefits from groundwater (B_u and B_c) and the buffer value for growers of unirrigated wheat in the northern Negev region.

	C:	0.0	0.001	0.01
	q_u(m^3/ha)	2915.65	2436.29	1556.37
P=.193	q_u(m^3/ha)	782.47	764.04	609.64
	$B_u(q_u)$(\$/ha)	69.29	57.00	49.42
	B_c(q)(\$/ha)	20.89	20.88	20.04
	BV($q_u q_c$)(\$/ha)	48.40	36.12	29.38
	q_u(m^3/ha)	2130.28	1720.29	715.44
P=.12	q_u(m^3/ha)	0	0	0
	B_c(\$/ha)	0	0	0
	BV(\$/ha)*	17.77	14.69	11.81

*When B_c=0, B_u and BV coincide.

Figure 1