

ELASTICITIES FOR U.S. WHEAT FOOD USE BY CLASS

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Abstract: We conceptualize wheat for food use as an input into flour production and derive demand functions to quantify price responsiveness and economic substitutability across wheat classes. Cost, price, and substitution elasticities are estimated for hard red winter, hard red spring, soft red wheat, soft white winter, and durum wheat. In general, hard red winter and spring wheat varieties are much more responsive to their own price than are soft wheat varieties and durum wheat. Morishima elasticities indicate that hard red winter and hard red spring wheat are economic substitutes for milling purposes.

JEL Classification: C15, C30, Q11

Key Words: elasticities, wheat by class, economic substitution, Monte Carlo

Introduction

In the United States (U.S.), the use of wheat as an input into food production has been increasing over the past several decades. The U.S. Department of Agriculture estimates that per-capita flour consumption, including semolina, has increased from 111 pounds in 1974 to 148 pounds in 1998. Concurrently, per-capita wheat used for food increased from 2.55 bushels in 1974 to 3.89 bushels in 1998. While physical substitutability among wheat classes has been extensively studied (see, for example, Faridi and Faubion 1995), economic substitutability among classes of wheat for domestic food use has yet to be addressed in a theoretically consistent and empirically rigorous manner. The importance of economic substitutability of wheat between classes is evident by recent publications highlighting the impact of substitutability on the flour milling industry (Sosland), on university research programs developing new varieties of hard white wheat (Boland, Schumacher, and Johnson 2000), and on government wheat research and programs (Barnes and Shields 1998). For example, the Commodity Credit Corporation (CCC) just released market loan rates by class “to establish loan rates that are in line with market forces in order to avoid over-production of wheat in a county in response simply to the benefits that are available under the marketing loan program” (U.S. Department of Agriculture 2002).¹ Given the importance of economic substitutability of wheat between classes, the purpose of the current study is to obtain cost, price, and substitution elasticity estimates from an industry cost function of the flour milling industry in the U.S. using aggregate food use data.

There are five major classes of wheat grown in the U.S. for food consumption, including hard red winter (*HRW*), hard red spring (*HRS*), soft red winter (*SRW*), soft white (*SWW*), and durum (*DUR*) wheat.² Most *HRW* is grown in the central and southern Great Plains, *HRS* in the northern Great Plains, *SRW* east of the Mississippi River, *SWW* in the Pacific Northwest and *DUR* in North Dakota and

Montana. Historically, wheat for food in the U.S. has been predominately used as input into flour production.³ The hard wheat classes have higher protein content, which is desirable for baking. The higher protein content in hard red spring and hard red winter is suited for the production of bread and rolls. Durum is used in the production of semolina flour and a variety of pasta products. The soft wheat classes have lower protein content. Soft red winter is used in flat breads, cakes, crackers, and pastries. Soft white wheat is processed into crackers, cookies, pastries, muffins, and flour for cakes. *HRW* has the widest range of protein content and is often mixed with *HRS* and *SRW*. This illustrates the wide variety of food items consumed in the primary market that are produced with wheat flour from different classes of wheat produced in geographically distinct parts of the U.S.

Several studies have attempted to differentiate among end uses by estimating primary or consumer demand for wheat by class.⁴ Demand for each wheat class was specified as a function of its own-price, prices of competing classes, and income. Chai (1972) estimated domestic demand for wheat by class over the period from 1929 to 1963. Linear equation-by-equation OLS demand models were estimated for *HRW*, *HRS*, *SRW*, *SWW*, and *DUR* using wheat cash prices from major markets. Chai concluded that price elasticities were more elastic for hard classes than soft classes of wheat. Barnes and Shields (1998) estimated a double-log demand system for wheat by class. The wheat classes examined were *HRW*, *HRS*, *SRW*, *SWW*, and *DUR*. Annual data from 1981 to 1998 were used in a demand system analysis with regional prices at the farm level. Inelastic own-price elasticities were reported for each of the five wheat classes, but different from Chai, *SWW* was reported as being the most elastic and *DUR* being the least elastic. Barnes and Shields (1998) also estimated linear equation-by-equation OLS models that yielded results qualitatively consistent with Chai. Mohanty and Peterson (1999) estimated demand for wheat by class and origin for the U.S. and European Union (EU) using a

dynamic AIDS model. They examined several classes of wheat, separating *DUR* from spring wheat and other wheat. Reported price elasticities indicate that *DUR* was more price responsive than spring wheat, which was more price responsive than other wheat.

Other studies have examined aspects of wheat quality. For example, Bale and Ryan (1977) applied a “Lancaster” production characteristics approach to differentiate classes of wheat by their protein content. Estimates of relative wheat prices were obtained from simple measures of protein supply. Wilson and Gallagher (1990) investigated the effects of relative prices on shifts of imported wheat class market shares. They found quality differentials and prices both are competitive factors in international markets. Espinosa and Goodwin (1991) estimated a hedonic price model for Kansas wheat characteristics. They concluded that wheat prices are responsive to differences in the quality of wheat, as measured both at the farm gate and in milling and baking enduses. Parcell and Stiegert (1998) also estimated the marginal value of hard red spring and red winter wheat-grading characteristics and wheat protein in a spatially competitive framework. The marginal values of protein in Kansas *HRW* and North Dakota *HRS* were affected by the level of protein in other districts within and across regions. Dahl and Wilson (2000) examined changes in exports of hard wheat across grades and classes in the U.S. and Canada. These studies illustrate the important role that quality, and in particular protein content, plays in markets for wheat food use.

Relative to the previous demand studies, deriving price and substitution elasticity estimates from an industry cost function of the flour milling industry provides several advantages. First, U.S. consumers typically do not utilize raw grain products for direct consumption. Rather, raw wheat is processed into flour before consumption. Moreover, only 15% of the flour processed is directly sold to consumers, while the other 85% is used in baked goods (Harwood et al. 1989). Second, specification of

consumer income in the demand for raw wheat product is not generally consistent with economic theory. Rather, changes in income are signaled through retail prices of flour and baked goods to flour millers and processors. Third, use of farm level prices in a consumer demand model is also problematic. Consumers respond to retail level prices of flour and baked goods. In contrast, in the processing sector, flour millers respond to farm level prices. Consequently, price elasticities and measures of substitutability across wheat classes derived from an industry cost function of the flour industry are consistent with economic theory and, as will be discussed ahead, are consistent with the data available with which to estimate empirical relationships.⁵ In all, deriving demand from a cost function of the flour industry better differentiates market characteristics for wheat food use that can be used to enhance industry, university, and government decision making.

Deriving Demand for Wheat Food Use

Following Wholgenant (1989), Goodwin and Brester (1995), and others, we consider raw product as an input into food production. Hence, we specify an industry cost function for the flour milling industry and derive factor demand equations. In specification of the cost function, we do not differentiate between types of flour produced, but rather assume flour output is a homogeneous product. Although this is a simplification, the assumption is empirically practical because of limited quantity data for flour. Finally, millfeed output is not considered in the conceptual model specification. This is because millfeed is a by-product of flour milling that is used as feed input in the livestock industry and prices typically follow other feed stuffs such as corn prices (Harwood et al., 1989).

Conceptual Model

The cost function is defined by

$$(1) \quad C = C(\mathbf{w}, y)$$

where y is a (1×1) scalar representing flour output and $\mathbf{w}=(w_1, \dots, w_k)'$ is a $(k \times 1)$ vector of input prices. The standard properties of a cost function are that it is homogenous of degree one, nondecreasing, and concave in input prices, as well as nondecreasing and convex in y (Varian 1992, Chambers 1988). The underlying behavioral assumption is a bundle of input quantities are chosen so as to minimize cost of producing y . It is convenient for the purposes of this paper to assume the cost function is weakly separable in inputs, partitioning inputs into two subgroups of wheat and other inputs

$$(2) \quad C = C(\mathbf{w}, y) = C\left(c^1(\mathbf{w}^1, y), c^2(\mathbf{w}^2, y), y\right)$$

In (2), c^1 and c^2 are micro-functions that possess properties of a cost function, $\mathbf{w}^1=(w_1, \dots, w_n)'$ is a $(n \times 1)$ vector of input prices representing the different classes of wheat, and $\mathbf{w}^2=(w_{n+1}, \dots, w_k)'$ is a $(k-n \times 1)$ vector of prices for the remaining inputs (e.g., capital, labor, energy).⁶ From this framework, conditional factor demand equations for wheat by class may be obtained by applying Shephard's Lemma to the micro-function c^1

$$(3) \quad \frac{\partial c^1}{\partial \mathbf{w}^1} = \mathbf{x}^1(\mathbf{w}^1, y)$$

where $\mathbf{x}^1=(x_1, \dots, x_n)'$ is a $(n \times 1)$ vector of input quantities of wheat by class.

Price and substitution elasticities derived from the conditional demand equations in (3) reflect theoretically consistent behavioral responses across the different classes within the wheat group, but the weak separability assumption imposes specific restrictions across wheat and other inputs. Weak separability in inputs is akin to assuming a two-step cost minimization process where the first step is to minimize the cost of producing a single unit of an aggregate input composed of a subgroup of the inputs and the second step is to combine the aggregate inputs in a cost minimizing manner to produce the final

product (Chambers 1988). The implications of maintaining weak separability are equivalent to restricting the degree and direction of the substitution relationships. For example, if input prices w_i and w_j in the wheat group are separable from price w_k outside of the wheat group, then inputs x_i and x_k are Allen substitutes only if x_j and x_k are also Allen substitutes. Although weak separability imposes restrictions, we contend the separability assumption is flexible enough to estimate price and substitution elasticities across wheat classes and, yet, retains sufficient degrees of freedom for estimation in the empirical model discussed below.⁷

Empirical Model

To complete the model specification, the factor demand equations in (3) are derived from a normalized quadratic cost function. The normalized quadratic is a flexible functional form that allows estimation of price and substitution elasticities, as well as the explicit investigation of the interactions between input prices and output quantity (e.g., Shumway, Saez, and Gottret 1988, Featherstone and Moss 1994). The normalized quadratic function is given by

$$(4) \quad c^*(w, y) = b_0 + \sum_{i=1}^{n-1} b_i w_i^* + \frac{1}{2} \left(\sum_{i=1}^{n-1} \sum_{j=1}^{n-1} b_{ij} w_i^* w_j^* \right) + b_y y + \sum_{j=1}^{n-1} b_{iy} w_i^* y + \frac{1}{2} b_{yy} y^2$$

where normalized cost and input prices are defined by $c^* = c/w_n$ and $w_i^* = w_i / w_n$.⁸ Hence, the input demand equations are given by

$$(5) \quad x_i = b_i + \sum_{j=1}^{n-1} b_{ij} w_j^* + b_{iy} y \text{ for } i = 1, \dots, n-1$$

The complete system of equations consists of the cost function in (4) and $n-1$ demand equations in (5). Homogeneity follows from normalizing the input prices and cost. Concavity of input prices can be imposed by reparameterizing the matrix \mathbf{B} of input price coefficients into \mathbf{B}^* as

$$(6) \quad \mathbf{B}^* = \begin{bmatrix} b_{1,1}^* & \cdots & b_{1,n-1}^* \\ \vdots & \ddots & \vdots \\ b_{n-1,1}^* & \cdots & b_{n-1,n-1}^* \end{bmatrix} = \begin{bmatrix} a_{1,1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ a_{n-1,1} & \cdots & a_{n-1,n-1} \end{bmatrix} \begin{bmatrix} a_{1,1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ a_{n-1,1} & \cdots & a_{n-1,n-1} \end{bmatrix}' = -\mathbf{A}\mathbf{A}'$$

where \mathbf{B}^* is a negative semi-definite matrix (Lau 1978). Symmetry requires that $b_{ij}^* = b_{ji}^*$. Convexity of output is imposed by $b_{yy}^* = a_{yy}^* a_{yy}$.

Price elasticities are given by the equation

$$(7) \quad \mathbf{e}_{ij} = \frac{b_{ij}^* w_j^*}{x_i} \quad \text{for } i, j=1, \dots, n-1$$

using the estimated b_{ij}^* and the predicted x_i . Morishima elasticities of substitution are

$$(8) \quad \mathbf{s}_{ij}^M = \mathbf{e}_{ji} - \mathbf{e}_{ii} \quad \text{for } i, j=1, \dots, n-1$$

that measure the effect of varying the input price ratio p_i / p_j in the i th direction on the input quantity ratio x_i / x_j (Blackorby and Russell 1989). The cost elasticity of output is defined by

$$(9) \quad E_{cy} = \left[\mathbf{a}_y + \sum_{i=1}^m b_{iy} w_i^* + \mathbf{a}_{yy} y \right] \left[y / c^* \right]$$

which measures the change in the micro-function c^* due to an incremental change in output given that all factors are held fixed. Output elasticities of demand are defined by

$$(10) \quad E_{iy} = b_{iy} [y / x_i]$$

and provide a measure of a shift in demand for wheat class x_i given an incremental change in output y .

Data

Annual prices and quantities for the empirical analysis for each of the five wheat classes are based on June to May marketing years, from 1974/1975 to 1999/2000. Descriptive statistics are provided in Table 1. Wheat quantity and price data were collected from U.S. Department of Agriculture's Economic

Research Service, *Wheat Year Book*, annually from 1974 to 2001. Total flour production increased from 251 million cwt in 1974 to 412 million cwt in 1999, averaging 332 million cwt over the period. Total wheat food use (the sum of *HRW*, *HRS*, *SRW*, *SWW*, and *DUR* food use) has increased from 545 million bushels in 1974 to 925 million bushels in 1999. Figure 1 presents food use by wheat class, showing food use has been trending upwards over time. From 1974 to 1999 the average (standard deviation) proportion of total food use was 0.42 (0.03), 0.25 (0.02), 0.19 (0.01), 0.07 (0.01), and 0.07 (0.01) for *HRW*, *HRS*, *SRW*, *SWW*, and *DUR*, respectively.

For completeness, and to compare with previous studies, we considered two different series to represent wheat prices in the empirical analysis. First, given the importance of protein content for hard wheat varieties in flour production, we estimated the empirical model with wheat cash prices from major markets. In particular, the *HRW* price is represented by Kansas City, No.1 (13% protein); *HRS* price by Minneapolis, dark No.1 spring (13% protein); *SRW* price by Chicago, No. 2; *SWW* price by Portland No.1; and *DUR* by Minneapolis, No.1 hard amber durum. Second, and primarily for comparison purposes, we re-estimated the empirical model using average price data by region from the U.S. It is anticipated that the empirical model with cash prices from major markets will yield more elastic responses (especially for *HRW* and *HRS* with 13% protein content). This is because *HRW* and *HRS* prices are sensitive to protein content across regions (Parcell and Stiegert 1998) and that these quality impacts from protein may likely be averaged out in the regional price data.⁹

Estimation Issues

The weakly separable, complete system consisted of the cost function in equation (4) and four factor demand equations in equation (5), including *HRW*, *HRS*, *SRW*, and *SWW*. *DUR* price was used to normalize cost and the remaining input prices. Several econometric issues were addressed prior to

selecting the final model, including statistical tests on exogeneity of prices, residuals, and symmetry and curvature restrictions. Furthermore, to draw inferences on the price and substitution elasticities bootstrapped confidence intervals were constructed. These issues are discussed in more detail below. The final version of the five-equation system was estimated imposing symmetry and curvature restrictions using an iterative seemingly unrelated regression estimator with a first-order autocorrelation correction. Price data used were from major market locations to account for quality impacts through protein content. Elasticities were recovered for the *DUR* equation using standard properties of general demand restrictions.

Exogeneity of Prices

Tests for exogeneity of own prices were conducted on each demand equation using the Hausman-Wu test statistic, which is asymptotically distributed chi-square with 1 degree of freedom (Hausman 1978). The null hypothesis is that prices are exogenous. The Hausman-Wu test statistics for the *HRW*, *HRS*, *SRW*, and *SWW* equations were 0.0025, 0.4136, 0.9164, and 0.2491, respectively. For a critical value of 3.84 at the 0.05 level of significance, exogeneity of own prices could not be rejected for each equation.

Diagnostic and Autocorrelation Tests of the Residuals

Diagnostic testing of the residuals for each equation of the system was completed with several nonparametric test statistics (see Table 2). The Kolmogorov-Smirnov-Lilliefors test statistic was used to test for normality of residuals equation-by-equation with the null hypothesis that the residuals are normally distributed (Mittelhammer 1996). Normality of residuals could not be rejected in all cases at the 0.05 level of significance.

The Wald-Wolfowitz (*WW*) runs test (Mittelhammer 1996), which under the null assumes independent and identically distributed (*iid*) residuals and has an asymptotic normal distribution, was also completed equation-by-equation. The *iid* hypothesis was rejected for the soft red and white wheat equations (with critical value of 1.96 at the .05 level). The system was then corrected for first-order autocorrelation using the Berndt and Savin (1975) approach, which has been adopted by Piggott et. al. (1996), Holt and Goodwin (1997), and others. The autocorrelation coefficient, r , was positive and significant at the 0.05 level (Table 3). After correcting for first-order autocorrelation, the *WW* test statistics the hypothesis of *iid* residuals was not rejected for each of the equations at the .05 level.

Symmetry and Curvature Restrictions

To test hypothesis consistent with symmetry and curvature restrictions, we used a log-likelihood ratio (*LR*) test between unrestricted and restricted models.¹⁰ The *LR* tests were completed for both the models with and without the autocorrelation correction. Under the null hypothesis that symmetry holds (with no curvature restrictions), the *LR* test is asymptotically chi-square distributed with $L=6$ degrees of freedom. The *LR* tests for symmetry yielded test statistic values of 8.79 with no autocorrelation and 3.18 with first-order autocorrelation (Table 4). Comparing these results with the 0.05 level critical value of 12.59, symmetry restrictions could not be rejected in either case.

To test curvature restrictions, we used a log-likelihood ratio (*LR*) test between an unrestricted model with no curvature restrictions and restricted models imposing concavity or curvature. Symmetry restrictions are maintained for both the unrestricted and restricted models. Under the null hypothesis that curvature restrictions hold, the *LR* test is asymptotically chi-square distributed with $L=10$ degrees of freedom for concavity in prices and $L=1$ degree of freedom for convexity in output. *LR* test statistics in

Table 4 indicate that curvature restrictions (concavity, convexity, and concavity-convexity) could not be rejected with and without the autocorrelation correction.

Bootstrapped Confidence Intervals

To measure the significance of price and substitution elasticities, bootstrapped confidence intervals were constructed. Bootstrap procedures are convenient for intractable inference problems and are often equivalent or superior to first-order asymptotic results (Mittelhammer, Judge, and Miller 2000). Bootstrap estimates were obtained by (a) resampling the residuals of the model corrected for autocorrelation, (b) predicting cost and quantities of wheat with the autocorrelated model,¹¹ (c) reestimating the five-equation system with predicted values, and (d) then recalculating the elasticities. This process was repeated 1000 times to generate distributions of cost, price, and substitution elasticities. Then 90% confidence intervals for each elasticity were constructed based on the percentile method, which requires ordering the estimated elasticities and then selecting outcome 50 ($0.05 \cdot 1000$) for the lower critical value and outcome 950 ($0.95 \cdot 1000$) for the upper critical value. For hypothesis testing, if the bootstrapped confidence interval for the elasticity contains zero, then the elasticity value is not considered significantly different from zero at the 0.10 level.

Results and Discussion

Parameter estimates, asymptotic standard errors, and asymptotic z-values are presented in Table 3 for the model that was corrected for first-order autocorrelation with symmetry and curvature imposed. Based on asymptotic z-values, eleven of the twenty-one estimated coefficients and the autocorrelation coefficient are statistically significant at the 0.05 level. The output coefficients are positive and significant at the 0.05 level for each demand equation. R-square values, which explain variation in

quantity of wheat for food use, ranged from 0.84 for the *HRS* equation to 0.98 for the cost equation (Table 2).

Table 5 contains the price elasticities evaluated at mean values for each demand equation along with 90% bootstrapped confidence intervals. Signs of the own-price coefficient estimates were negative as required with the imposition of concavity. The results indicate that the own-prices are inelastic for *SRW*, *SWW*, and *DUR* and elastic for *HRW* and *HRS*. The most inelastic is *SRW* at -0.05. The most elastic is *HRS* at -1.72, followed by *HRW*, *DUR*, and *SWW*, respectively. Cross-price effects are inelastic, except between *HRW* and *HRS*. *SRW* exhibited the most inelastic response to cross-price effects. For the *DUR* equation, the cross-price effects were larger for *HRW* and *HRS* prices relative to the soft wheat classes. Except for the cross-effects between *HRW* and *HRS*, the 90% confidence intervals for the cross-price elasticities included zero at the 0.10 level of significance.

To better interpret substitution across wheat classes, Table 6 reports the Morishima substitution elasticities evaluated at mean values with bootstrapped 90% confidence intervals. These results indicate that all the wheat classes are Morishima substitutes at the mean and that there appears to be more potential for substitution between *HRW* and *HRS* than the other wheat classes. For the *HRW* and *HRS* equations, excluding *SWW* in the *HRW* equation, the 90% confidence intervals for the substitution elasticities do not contain zero. For the *SRW* and *SWW* equations, the 90% confidence intervals indicate the substitution elasticities are almost all not statistically different from zero. The exception being in the *SWW* equation, which has a significant substitution elasticity for *SRW*. Similar to the price elasticities, the substitution elasticities for the *SRW* equation are the most inelastic. Finally, consider the *DUR* equation. It has significant substitution elasticities for *HRW* and *SRW*, but not for *HRS* or *SWW*.

Interestingly, the Morishima substitution elasticities in Table 6 exhibit asymmetrical responses. For example, the substitution elasticities for the *SRW* equation are very inelastic and insignificant. In contrast, changes in price of *SRW* induce significant substitution effects across the other classes of wheat. More generally, the significance (insignificance) of the substitution elasticities for the *HRW* and *HRS* (*SRW* and *SWW*) equations are likely due to the nature of the price data. Under a cost minimizing assumption, one would not expect a miller to substitute (or mix off) the higher protein *HRW* and *HRS* wheat for lower protein *SRW* and *SWW* wheat.

Elasticities of cost and demand with respect to output are reported in Table 7 with 90% bootstrapped confidence intervals. The cost elasticity estimate from (9) is 1.1212 with a lower critical value of 1.0840 and upper critical value of 1.2125 for the bootstrapped 90% confidence interval. That is, a 1% increase in output yields a 1.12% increase in costs. This does not imply there are decreasing economies of scale for the milling industry, but simply that the micro-function for producing a single unit of aggregate input composed of the wheat subgroup decrease with increasing output.¹² Output demand elasticities in (10) are all elastic, with *HRS* the most elastic at 1.7848 and *SWW* the least elastic at 1.2960. For instance, 1% increase in output yields a 1.78% shift upwards in the demand for *HRS*.

Further Results

For comparison purposes, the own-price elasticities from the current study and the results from Chai (1972) and Barnes and Shields (1998) are presented in Table 8. Using prices from major market locations, Chai found that the soft wheat types were least responsive to own price changes and the hard wheat types the most responsive to own price changes. These estimates were obtained using equation-by-equation OLS for two study periods from 1929 to 1941 and 1946 to 1963. Using average prices for each wheat class by U.S. region, Barnes and Shields reported elasticities using an equation-by-equation

OLS estimator from 1977 to 1995 and using a double-log demand system estimated with seemingly unrelated regression from 1981 to 1997. The equation-by-equation OLS estimates were qualitatively similar to the results reported by Chai. In contrast, the elasticities from the double-log demand system indicated that *SWW* wheat was the most price responsive. The next most price responsive to own price from the double-log system was the *HRW* equation, followed by *SRW*, *HRS*, and *DUR*. Interestingly, except for *DUR*, the own-price elasticity estimates from the Barnes and Shields double-log demand system fall outside of the 90% confidence intervals reported in Table 5.

To provide a closer comparison to the results reported by Barnes and Shields study, we re-estimated equations in (4) and (5) using the average price by region at the farm level. These results are reported in Table 9. Most notable differences occurred for the magnitudes of the *HRW* and *HRS* price elasticities, which decreased with *HRW* having a larger magnitude than *HRS* and both being inelastic. One plausible explanation is that the average price data from U.S. regions does not fluctuate as much with changes in protein content. In all, results of the current study remain qualitatively consistent with results of Chai's and Barnes and Shields' single-equation OLS results in that *HRW* and *HRS* are most price elastic. On the contrary, results of the current study differ with those the Barnes and Shields' double-log demand system.¹³

To test the sensitivity of the results from the normalized quadratic model, we re-estimated the same cost and input demand equations with the translog cost model.¹⁴ Homogeneity and symmetry conditions were imposed following standard procedures. Curvature restrictions are imposed in the translog system at every data point using Geweke's (1986) Bayesian framework. More specifically, we follow closely the systems estimation approach outlined by Griffith, O'Donnell, and Cruz (2000) using the Metropolis-Hastings algorithm to impose curvature.¹⁵ The autocorrelation structure imposed on the

normalized quadratic model is not included in this analysis, but both concavity of input prices and convexity of output are imposed.¹⁶ Price elasticities from the translog model are presented in Table 9 for both major market and regional prices. The own-price elasticities from the translog based on major markets remained elastic for *HRW* and *HRS*, but increased in magnitude for *SRW*, *SWW*, and *DUR*, relative to the elasticities from the normalized quadratic in Table 5. For regional prices, the translog own-price elasticities are more elastic than those from the normalized quadratic.

Implications

Findings from the current research are of importance to industry agents, university researchers, and policymakers. In contrast to previous studies on demand for wheat food by class, which reported that post World War II demand for wheat is inelastic, we find that high protein *HRS* and *HRW* wheat used for food and flour production are own-price elastic.¹⁷ This has important implications that can help understand better the wheat market and be used to answer questions related to government programs. For example, millers often blend hard red spring and hard winter wheat to produce flour for breads.¹⁸ Interestingly, this observation is consistent with the substitution elasticities between *HRW* and *HRS* wheat presented in Table 6. In hindsight, the substitution response between *HRW* and *HRS* is apparent in the quantity series shown in Figure 1. As a result, we contend that this more thorough understanding of the economic substitutability between wheat classes can help prepare flour millers to better anticipate and respond to future changes in the quantity demanded for wheat food use.

Alternatively, consider government price support and export programs. Chai (1972) argued that using an elasticity estimate from all wheat has limited and possibly misleading implications when applied to analysis of individual classes for wheat in domestic food use. Farnsworth (1961) went a step further and emphasized some consequences of ignoring wheat by class relationships. She identified

surplus problems that arose from government price support and export programs that kept price spreads between different types and qualities of wheat narrower and less variable. Interestingly, the recent action by the Commodity Credit Corporation to released market loan rates by class reflects the government's recognition of problems that arise by treating wheat as a homogeneous product (U.S. Department of Agriculture 2002).

Of particular importance to university and government policymakers, is the introduction and distribution into the agricultural sector of newly developed varieties of hard white wheat (*HWW*). *HWW* and *HRW* are reportedly close substitutes in baking quality with the primary difference that *HRW* carries the polyphenol oxidase that may cause discoloration in the processed product. Boland and Howe indicate that short-run economic incentives for *HWW* will likely be driven by domestic flour millers in the form of price premiums from millers to growers. For sake of discussion, suppose *HRW* and *HWW* are perfect economic substitutes. Consider a 1.00% increase in price of *HRW*; this yields a decrease of 1.10% in quantity demanded of *HRW* and an increase of 1.72% of *HRS*. In 1999/2000 this would have resulted in a decrease of 4.00 million bushels of *HRW* demanded and an increase of 3.74 million bushels of *HRS* demanded. Under the perfect substitute scenario, any price increase for *HWW* would likely induce a shift out of *HWW* into *HRS* wheat by millers. The potential substitution to *HRS* wheat further complicates the introduction of *HWW* and could potentially nullify any short-run economic incentive for *HWW* coming from the milling industry.

These findings also have implications based on provisions of the 2002 Farm Bill. Under Section 1616, Subtitle F, Title I – Commodity Programs, the CCC will make available \$US20 million dollars a year for incentive payments to producers to encourage production of *HWW* for food use. To be eligible the *HWW* must meet a specific quality criteria and the producer must demonstrate that buyers and

endusers are available. If *HWW* and *HRW* are perfect economic substitutes for a specific region then an overall affect of this farm policy program may be to increase the number of acres planted to *HWW* but not necessarily directly impact allocations of cost minimizing expenditures by millers for wheat food use.

Conclusions

Policymakers in the U.S. have been recently altered and introduced farm programs that recognize differences in demand and supply responses for wheat classes. The Commodity Credit Corporation (CCC) just released market loan rates by wheat class and the 2002 Farm Bill will make available \$US20 million dollars a year for incentive payments to producers to encourage production of new hard white wheat varieties for food use. To better understand market responses for wheat food use, we conceptualized and specified an industry cost function with the different wheat classes as an input into flour production. The cost function and factor demand system, derived from the normalized quadratic function, were estimated, elasticities derived, and results compared to findings of previous studies.

Empirical results of this study are important to policymakers. Results indicate that own-prices are more elastic for *HRW* and *HRS* than for *SRW*, *SWW*, and *DUR*. This is consistent with results from linear equation-equation models reported in Chai, Barnes and Shields, and Terry, but not consistent with results from the Barnes and Shields double-log demand system. In contrast to previous studies, which reported inelastic own-price elasticities across all wheat classes, we find that *HRW* and *HRS* with higher protein levels (13%) have elastic own-price effects. In addition, there appears to be more substitution between *HRW* and *HRS* than among the soft wheat varieties and *DUR*. The larger substitution effect between *HRW* and *HRS* has important policy implications relevant to industry, government wheat programs, and the introduction of new hard white wheat varieties in the U.S.

Finally, there are several important limitations of this study. First, the empirical results are based on a weakly separable cost function and conditional factor demand equations. Ideally the derived demand system should accommodate all factor input prices. Second, we dealt with primarily wheat quantity issues and not quality (e.g., protein) issues that are in need of further research. We suggest future research such as the demand for wheat protein be investigated at a more disaggregate level. Even with such limitations, the findings of this study provide an important step towards understanding the economic substitution between wheat classes.

Table 1. Descriptive statistics for nominal price and quantity data from 1974 to 1999.

<u>Variable</u>	<u>Mean</u>	<u>St. Dev.</u>	<u>Min</u>	<u>Max</u>
QFL (1000 cwt)	332090.00	51405.00	251100.00	411970.00
PHRW (\$US/bu)				
Price of Hard Red Winter	3.93	0.66	2.81	5.69
PHRS (\$US /bu)				
Price of Hard Red Spring	3.94	0.67	2.83	5.64
PSRW (\$US /bu)				
Price of Soft Red Wheat	3.43	0.63	2.19	4.83
PSWW (\$US /bu)				
Price of Soft White Wheat	3.86	0.61	2.90	5.27
PDUR (\$US /bu)				
Price of Duru m	4.74	1.11	3.30	7.03
QHRW (million bu)				
Quantity of Hard Red Winter	305.35	45.95	251.00	387.00
QHRS (million bu)				
Quantity of Hard Red Spring	178.46	39.52	128.00	260.00
QSRW (million bu)				
Quantity of Soft Red Wheat	133.65	17.10	94.00	155.00
QSWW (million bu)				
Quantity of Soft White Wheat	54.23	14.50	31.00	85.00
QDUR (million bu)				
Quantity of Durum	53.15	17.63	32.00	80.00

Table 2. Summary and test statistics for normalized quadratic models.

	Equation				
	Cost	HRW	HRS	SRW	SWW
Model I ^a					
<i>WW</i> ^b	0.0000	-0.9624	0.0000	-4.2640*	-3.4514*
<i>KSL</i> ^c	0.1327	0.1281	0.1260	0.1418	0.1479
Model II ^d					
<i>WW</i> ^b	1.0031	1.2523	1.6697	-0.1117	-0.4174
<i>KSL</i> ^c	0.1060	0.1281	0.1130	0.1397	0.1742
R-square	0.9832	0.8996	0.8379	0.9580	0.8679

^aModel I - not corrected for autocorrelation with curvature and symmetry imposed. Log-Likelihood, LL= -418.592.

^bWald-Wolfowitz runs test. * reject *iid* residuals at the 0.05 level with critical value 1.96.

^c Kolmogorov-Smirnov-Lilliefors Normality Test of Errors. Critical value of 0.1772 at the 0.05 level with 25 observations.

^dModel II - corrected for first-order autocorrelation with curvature and symmetry imposed. Log-Likelihood, LL=-398.123.

^e Kolmogorov-Smirnov-Lilliefors Normality Test of Errors. Critical value of 0.1806 at the 0.05 level with 24 observations.

Table 3. Parameter estimates from the normalized quadratic system. Study period from 1974 to 1999.^a

Coefficient	Coefficient Estimate	Asymptotic Z-value	Asymptotic P-value
b_0	-70.02309	-4.6796	0.0000
b_1	48.66286	0.7463	0.4555
b_2	-85.92666	-1.3311	0.1831
b_3	59.80453	3.8818	0.0001
b_4	-21.47446	-1.0152	0.3100
b_5	34.65769	8.7591	0.0000
a_{11}	19.99827	6.4055	0.0000
a_{12}	-18.71125	-5.9336	0.0000
a_{13}	0.43579	0.3618	0.7175
a_{14}	0.45775	0.3036	0.7615
a_{22}	-4.08071	-2.0341	0.0419
a_{23}	-1.96406	-0.8424	0.3996
a_{24}	2.31600	0.9087	0.3635
a_{25}	2.15029	0.5367	0.5915
a_{33}	-2.39413	-1.0984	0.2720
a_{34}	0.00002	0.0000	1.0000
b_{15}	86.59887	5.0535	0.0000
b_{25}	72.50575	4.2553	0.0000
b_{35}	24.38608	6.1190	0.0000
b_{45}	21.43907	3.8599	0.0001
a_{yy}	-0.00003	0.0000	1.0000
r	0.65508	9.2181	0.0000

^a Quantity of flour was scaled by 100,000 in estimation.

Table 4. Likelihood ratio test results for symmetry and curvature restrictions.

Restrictions	LR Test Statistics		Degrees of Freedom	Chi-square 0.05 Critical Value
	Model I ^a	Model II ^b		
Symmetry	8.79	3.18	6.00	12.59
Concavity with Symmetry	0.95	1.57	10.00	18.31
Convexity with Symmetry	2.08	3.44	1.00	3.84
Concavity and Convexity with Symmetry	2.91	5.13	11.00	19.68

^aModel I - not corrected for autocorrelation.

^bModel II - corrected for first-order autocorrelation.

Table 5. Price elasticity estimates from the normalized quadratic system with bootstrapped 90% percentile confidence intervals.

Price	Equation				
	Price Elasticities				
	HRW	HRS	SRW	SWW	DUR
HRW	-1.1043*	1.7517*	-0.0545	-0.1396	0.5469
HRS	1.0351*	-1.7200*	0.0009	0.2753	-0.3251
SRW	-0.0211	0.0006	-0.0476*	0.1272	0.0774
SWW	-0.0251	0.0839	0.0591	-0.1715*	-0.0779
DUR	0.1155	-0.1162	0.0421	-0.0913	-0.2213*

90% Confidence Interval					
Upper Critical Value					
	HRW	HRS	SRW	SWW	DUR
HRW	-0.5313	3.2634	0.2212	1.0489	1.0493
HRS	1.9203	-0.7818	0.2919	1.1188	0.2408
SRW	0.0857	0.1920	-0.0017	0.3273	0.1413
SWW	0.1881	0.3436	0.1501	-0.0230	0.0873
DUR	0.2195	0.0849	0.0758	0.1010	-0.0357

90% Confidence Interval					
Lower Critical Value					
	HRW	HRS	SRW	SWW	DUR
HRW	-2.0822	0.8074	-0.3185	-0.8388	-0.0873
HRS	0.4797	-3.1828	-0.2498	-0.7361	-0.8009
SRW	-0.1249	-0.1636	-0.1126	-0.0262	-0.0472
SWW	-0.1509	-0.2189	-0.0120	-0.7261	-0.2218
DUR	-0.0182	-0.2851	-0.0254	-0.2562	-0.3623

* 90% confidence interval does not contain zero.

Table 6. Morishima substitution elasticities from the normalized quadratic system with bootstrapped 90% percentile confidence intervals.

Price	Equation				
	Substitution Elasticities				
	HRW	HRS	SRW	SWW	DUR
HRW		2.7551*	0.0265	0.1464	0.3368*
HRS	2.8560*		0.0482	0.2555	0.1051
SRW	1.0498*	1.7209*		0.2306*	0.2635*
SWW	0.9647	1.9952*	0.1748		0.1300
DUR	1.6512*	1.3949*	0.1250	0.0937	
90% Confidence Interval Upper Critical Value					
	HRW	HRS	SRW	SWW	DUR
HRW		5.0768	0.1459	0.8195	0.5183
HRS	5.3239		0.2575	0.8972	0.3480
SRW	2.0769	3.2709		0.8494	0.3936
SWW	2.9502	3.6791	0.4235		0.3423
DUR	2.8723	3.0489	0.2269	0.6312	
90% Confidence Interval Lower Critical Value					
	HRW	HRS	SRW	SWW	DUR
HRW		1.2749	-0.0801	-0.0265	0.0598
HRS	1.3626		-0.1240	-0.0430	-0.1296
SRW	0.4328	0.8165		0.0168	0.0337
SWW	-0.0174	0.7119	-0.0138		-0.0774
DUR	0.7525	0.4248	-0.0235	-0.0067	

* 90% confidence interval does not contain zero.

Table 7. Output cost and demand elasticities with bootstrapped 90% percentile confidence intervals.

Equation	Elasticity	Lower Critical Value	Upper Critical Value
Cost	1.1212*	1.0840	1.2125
HRW	1.5963*	1.2653	1.7491
HRS	1.7848*	1.5622	2.2192
SRW	1.4644*	1.4483	1.8202
SWW	1.2960*	1.0841	1.6095

* 90% confidence interval does not contain zero.

Table 8. Estimated own-price elasticities from previous studies.

	Equation				
	HRW	HRS	SRW	SWW	DUR
Chai:					
OLS single-equation 1929-1941	-1.808 ^a	-0.759	-0.447	-0.428 ^a	-0.087 ^a
OLS single-equation 1946-1963	-0.617 ^a	-0.725	-0.091 ^a	-0.022	-0.106 ^a
Barnes and Shields:					
OLS single-equation 1977-1995	-0.746 ^a	-0.468	-0.024 ^a	-0.137 ^a	-0.146 ^a
ITSUR double-log system 1981-1997	-0.420	-0.205	-0.239	-0.769	-0.161 ^a

^a own-price elasticities contained in 90% bootstrapped confidence interval reported in Table 5.

Table 9. Further price elasticity estimates from 1974 to 1999.

Price	Equation				
	Price Elasticities				
	Normalized Quadratic				
	U.S. Regional Prices				
	HRW	HRS	SRW	SWW	DUR
HRW	-0.5107	0.4839	0.0531	0.2511	0.5517
HRS	0.2984	-0.2837	-0.0226	-0.1522	-0.3303
SRW	0.0232	-0.0160	-0.0542 ^b	0.0225	0.0235
SWW	0.0487	-0.0479	0.0100	-0.0339 ^b	-0.0668
DUR	0.1404	-0.1363	0.0137	-0.0875	-0.1782 ^b

	Translog ^a				
	U.S. Major Market Prices				
	HRW	HRS	SRW	SWW	DUR
HRW	-1.1400 ^b	1.7558	-0.0363	0.4846	0.2238
HRS	1.0238	-1.8736 ^b	0.1411	-0.1594	0.2038
SRW	-0.0141	0.0942	-0.3629	0.4113	0.1398
SWW	0.0847	-0.0478	0.1847	-0.6801 ^b	-0.0482
DUR	0.0457	0.0714	0.0734	-0.0564	-0.5191

	Translog ^a				
	U.S. Regional Prices				
	HRW	HRS	SRW	SWW	DUR
HRW	-0.7199 ^b	0.6454	0.3552	0.8295	0.0341
HRS	0.3976	-0.5597	-0.1473	-0.2588	0.2467
SRW	0.1561	-0.1051	-0.3400	0.1089	0.1505
SWW	0.1578	-0.0799	0.0471	-0.7170 ^b	0.0287
DUR	0.0085	0.0993	0.0849	0.0374	-0.4600

^a homogeneity, symmetry, and curvature conditions imposed with no autocorrelation structure.

^b own-price elasticities contained in 90% bootstrapped confidence interval reported in Table 5.

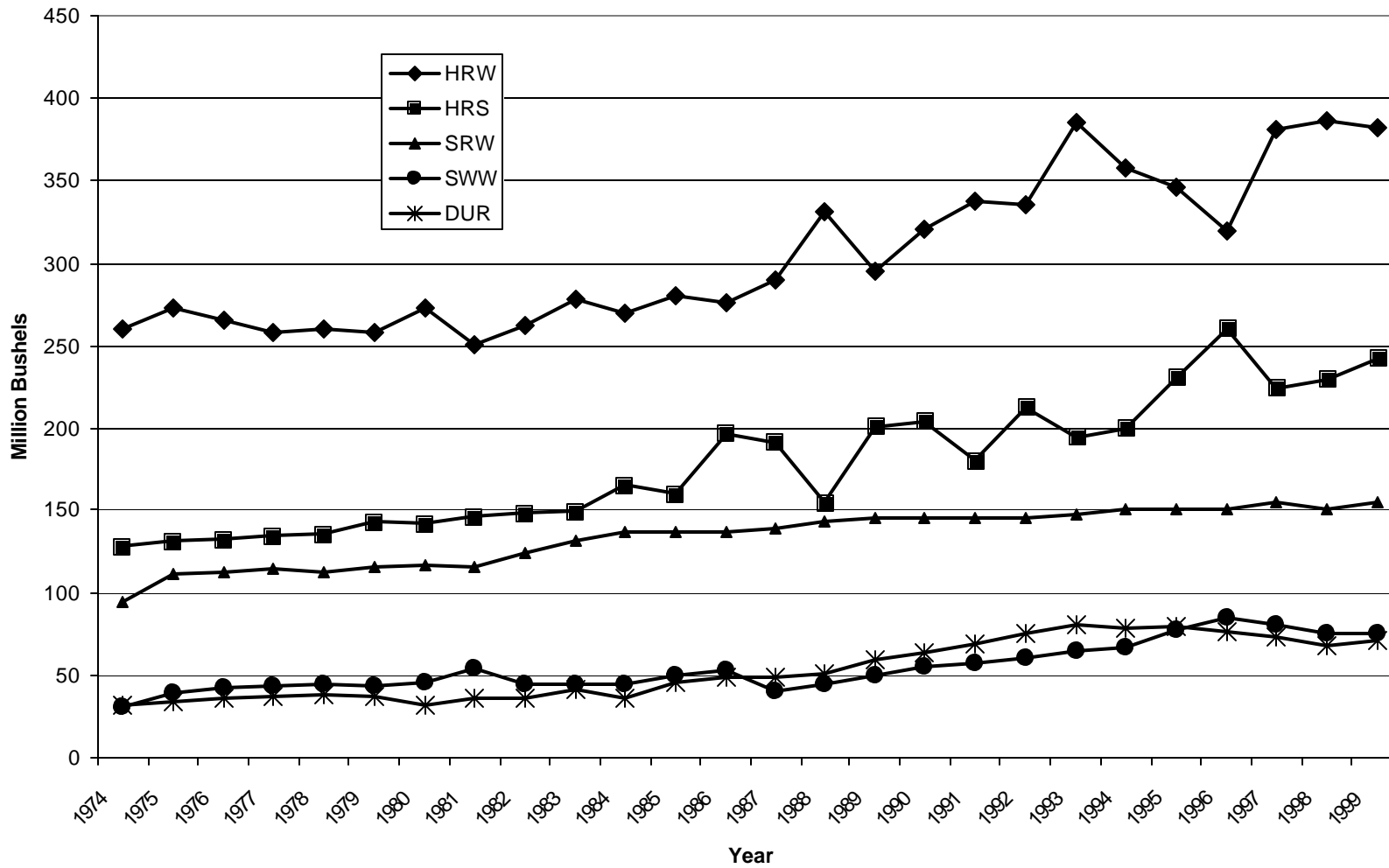


Figure 1. Domestic food use in the US by wheat class from 1974 to 1999.

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Footnotes

¹ The concern over perverse impacts of government intervention on wheat by class is not a recent matter. Farnsworth (1961) and Chai (1972) both discussed surpluses of hard red winter and white wheat, attributing them to government price support and export programs that kept price spreads between different types and qualities of wheat narrower and less variable.

² Hard white wheat is not explicitly examined in this analysis because of the lack of consistent time series of data.

³ Over the period from 1973 to 1998, the average ratio of bushels of wheat used for food to bushels of wheat ground for flour was 97% (U.S. Department of Agriculture).

⁴ Blakeslee (1980), and others, assumed perfect substitutability across wheat classes in studies that pursued alternative objectives.

⁵ Wholgenant (1989) provides theoretical and empirical insight into issues between primary and derived demand relationships for agricultural commodities.

⁶ From 1983 to 1998 at the Kansas City Milling center, the cost of wheat as input into flour production made up 91% of its wholesale price (Table 24, USDA-ERS).

⁷ Limited degrees of freedom are not the only obstacle to overcome if additional inputs were used in the empirical analysis. For instance, the marketing year for the annual quantity data are from June to May, which is not necessarily consistent with available industry data for other inputs.

⁸ Henceforth, the superscript notation is discarded as we deal only with the wheat inputs.

⁹ More specifically, a shortage of protein content for the *HRW* crop in the central and southern Great Plains has the potential of shifting up the demand for higher protein *HRS* in the northern Great Plains.

¹⁰ All likelihood ratio test statistics are calculated using the adjusted likelihood ratio test statistic for systems estimation $LR[MT-.5(Nu+Nr)-.5M(M+1)]/(MT)$ where LR-unadjusted log-likelihood value, M-# equations, T-# observations, Nu-#parameters in unrestricted model, Nr-#parameters in restricted model (Moschini, Moro, and Green 1994).

¹¹ Computation of predictions with the autocorrelated model follows standard methods discussed in Reinsel (1993).

¹² By examining the marginal effects of (2), it becomes evident why (9) can not be used to interpret the economies of scale for an industry. Taking a partial derivative of the cost function with weakly separable inputs in (2) with respect to output yields $\frac{\partial C}{\partial y} = C_1 \frac{\partial c^1}{\partial y} + C_2 \frac{\partial c^2}{\partial y} + C_3$, where C_i is the partial derivative with respect to the *i*th argument in the cost function. The cost elasticity in (9) depends on the marginal effect $\frac{\partial c^1}{\partial y}$ of the wheat micro-function c^1 , which is part of only

one term in the cost elasticity of the industry functions in (1).

¹³ Terry (2000) estimated several factor demand systems derived from an indirect profit function and a cost function for wheat by class, but combined *SRW* and *SWW* into a soft wheat class. Terry's results were consistent with findings in the current paper and the equation-by-equation OLS models in that soft wheat was less responsive to its own price than were the hard wheat varieties.

¹⁴ Initially, homogeneity and symmetry conditions were imposed without curvature restrictions. Here, the most notable change was the appearance of a positive own-price elasticity for *SRW* wheat. Terry (2000, Appendix A), using regional price data, also reported a positive own-price

elasticity for SRW wheat when using the translog model and when trying to replicate the Barnes and Shields double-log demand system.

¹⁵ Further discussion of applying the Metropolis-Hastings algorithm is provided in Chib and Greenberg (1996) and Robert and Casella (1999).

¹⁶ Additional details of this estimation process are available from the author upon request.

¹⁷ Blakeslee (1980) reported inelastic demand for all wheat from 1954 to 1974, with a price elasticity of -0.012. He also notes that wheat for food use had little variation over the study period.

¹⁸ Harwood, Leath, and Heid (1989) report that in periods when red winter wheat has low protein, millers of bread flour generally purchase large amounts of hard red spring wheat and blend the two classes.