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Estimation of supply response in CAPRI

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Abstract

The primary objective of this paper is to estimate behavioural parameters of the quadratic regional supply models in the modelling system CAPRI, using the time series data in the CAPRI database. A secondary objective is to replace the constant yields of the original model by functions that depend on input use. Due to lack of statistical robustness, the second objective is not achieved, thus yields remain constant. A Bayesian highest posterior density estimator is developed to address the primary objective. After discarding regions with insufficient data, parameters for up to 23 crop production activities with related inputs, outputs, prices and behavioural functions are estimated for 165 regions in EU-15. The results are systematically compared to the outcomes of other studies. For crop aggregates (e.g. cereals, oilseeds etc.) on the level of nations, the estimated own price elasticities of supply are found to be in a plausible range. On a regional level and for individual crops, the picture is much more diverse. Whether the regional results are plausible or not is difficult to judge, since no other study of similar regional and product coverage is known to the author.

Keywords: Bayesian estimation, errors-in-variables, PMP
JEL-classification: Q11, C32.

1. Introduction

The primary objective of this research is to develop a robust method for estimating the behavioural parameters of the supply module in the regionalised European agricultural sector model CAPRI, utilizing the time series of observations available in the CAPRI database and the optimality conditions of the model. As a secondary objective, the current model assumption of constant yields will be reviewed and, if feasible, revised.

The CAPRI model is a constrained quadratic programming model for NUTS2 regions in 34 European countries, where agriculture in each region is represented by an instance of a template programming model.

In this context we only consider the arable annual crop producing part of the representative regional farm, keeping other parts (husbandry, permanent grassland and permanent crops) fixed when necessary or leaving them out altogether when possible. We also ignore the fertilization constraints of the full model, working with Leontieff fertilizer input coefficients. With those restrictions, we need to

[1] This paper is an excerpt from the dissertation "Econometric specification of constrained optimization models" (Jansson, 2007).

estimate parameters for a maximum of 23 land use activities using ten inputs in 172 regions in EU-15 (thus excluding new member states).

Since most regions produce only a subset of the 23 crops, and some regions have too short time series of data, the actual extent of the exercise is somewhat smaller. Still, it is a large scale application that requires a method equally applicable to all regions and that is robust to data problems. The full list of crops and crop groups (see following sections) is provided in appendix 1, table 16. The ten inputs are listed in table 17.

Data for the model is provided by the CAPRI database. The part of the dataset that is relevant for this research has been compiled from the *Economic Accounts for Agriculture* (EAA, production values and volumes at national level) and *New Cronos Regio* (acreages and yields on regional level), both databases from Eurostat, completed with policy information from regulations and expert data where necessary. The dataset has been processed by econometric/heuristic software of the CAPRI system to be made complete (no holes in time series) and consistent (with respect to physical and economical interrelations) on member state as well as NUTS2 level.

The report is outlined as follows: In section two we describe the structure of the template regional representative farm model that is used for all regions. The existing model has *fixed* input and output coefficients. In order to check whether that is a good specification, two sections follow that investigate two different extensions of the model to endogenous yields. In section three we test for all regions of the model whether yield significantly depends on inputs. Section four analyses in greater depth for one single region, selected for its good data quality (long time series, many crops produced) whether changed acreages lead to changed yields. Since it is concluded that none of the extensions in section three and four is statistically reliable, we return in section five to the primary objective to estimation of the model with fixed yield coefficients. We propose a general estimation approach based on Bayesian technique that finds the posterior mode, similar to the methods described by DeGroot (1970). In section six, results are presented for selected regions, and compared to the results of other studies.

2. A regional supply model

The regional representative farm is assumed to act as if solving a linearly constrained quadratic programming problem (1) in every time period t. Throughout this paper we generally use lower case bold face letters to represent items that are column vectors for each t, upper case bold face letters to represent matrices and italic letters to represent scalars. The dimensions of vectors and matrices are denoted by upper case letters, where a lower case version of the same letter denotes the indices of the elements in that dimension, so that for instance the "J-vector of acreages **x**" means a vector of length J, with elements x_j, $j = 1 \dots J$. The prime character (′) denotes the ordinary transpose of a vector or a matrix.

All regional models have identical structure, and no cross-regional constraints or relationships are assumed, in order to keep down the complexity of the estimation. Thus, indices for regions can be omitted. The producer is assumed to solve the optimization problem in each period independently of other periods, thus all items that change across periods obtain an index t, so that for example $\mathbf{x}_t$ denotes the vector **x** in period t. This implies that **x** also can be considered a 3-dimensional array with dimensions with only one column, or dim(**x**) = (J,1,T). At some occasions it is convenient to denote the time series for some element j of **x**, and this is done somewhat sloppy as $\mathbf{x}_j$, where the reader is assumed to remember that **x** also has another dimension T, which is now in the rows[2].

The model can then be written for each period as

$$\begin{aligned} &\max_{\mathbf{x}_t} \mathbf{x}_t' \left[\mathbf{Y}_t \mathbf{p}_t + \mathbf{s}_t - \mathbf{A}_t \mathbf{w}_t \right] - \mathbf{x}_t' \left[q_t \mathbf{c} - \tfrac{1}{2} l_t \left[\mathbf{D} + \mathbf{GBG}' \right] \mathbf{x}_t \right] \\ &\text{s.t.} \quad \mathbf{R}_t \mathbf{x}_t = \mathbf{v}_t \end{aligned} \tag{1}$$

where for each t,

$\mathbf{x}_t$	vector of acreages for each of J land uses
$\mathbf{Y}_t$	$J \times J$ diagonal matrix with yields on the diagonal
$\mathbf{p}_t$	J vector of prices
$\mathbf{s}_t$	J vector of direct subsidies
$\mathbf{A}_t$	$J \times I$ matrix of input coefficients for I inputs
$\mathbf{w}_t$	I vector of input prices
q_t	price index
$\mathbf{c}$	J vector of parameters
l_t	a land availability index (described further below)
$\mathbf{D}$	$J \times J$ diagonal matrix of parameters
$\mathbf{G}$	$J \times M$ matrix that sums up land use by each of M = 6 crop groups, i.e. with $g_{jm} = 1$ if crop j belongs to group m, else $g_{jm} = 0$
$\mathbf{B}$	6×6 matrix of parameters
$\mathbf{R}_t$	$2 \times J$ matrix of constraint coefficients, where $r_{1j} = 1$ for $j = 1 \dots J$ and r_{2j} is the net set-aside contribution of crop j
$\mathbf{v}_t$	2 vector with v_1 total land available, $v_2 = 0$.

The model implies that the producer maximises the sum of gross margins (the first term) minus a quadratic function (the second term), subject to a land constraint and set-aside requirement. The quadratic function in the objective function is a behavioural term in the tradition of *positive mathematical programming*

[2] I.e. we perform a generalised transpose of the 3-D array **x** where the first and last dimensions are swapped, and signal this only by a switch of indices. In general, symbols are better thought of as 3-D arrays where the index denotes the 3rd dimension.

(PMP, see e.g. Horner et al. 1992 or Howitt 1995) that is intended to capture the aggregated influence of economic factors that are not explicitly included in the model, like land heterogeneity and additional resource constraints (Heckelei 2002). The function is in what follows sometimes referred to as the *PMP-function*, and the parameters **c**, **D** and **B** as the *PMP parameters*, or the behavioural parameters of the model. It is the objective of this work to estimate those parameters.

In order to reduce the number of parameters to estimate, we assume that the quadratic function has a special structure: Cross-crop effects are only permitted between *groups of crops*, so that for instance an increase in the area of potatoes plus sugar beet may influence the cost of producing cereals and increase the cost of producing both sugar beet and potatoes. In order to provide each individual crop with increasing marginal costs[3], we also admit a quadratic term that depends only on the individual crop. The structure described is implemented using a vector **c** of linear effects, a diagonal matrix **D** of quadratic own-crop effects, and a matrix **B** of cross-group effects. The $J \times M$ matrix **G** is used to sum the acreages within each group, substantially reducing the number of parameters compared to estimation of a full $J \times J$ matrix.

The prices **p** and **w** in the model are nominal, and since the quadratic function is assumed to capture, among other things, the opportunity cost of resources not explicitly modelled, it should be inflated. This is obtained by multiplication of **c** by the general price index q_t.

The total amount of land fluctuates slightly between years, in general with a downward trend due to migration of land into other sectors (fallow land is modelled explicitly as a land use activity). We do not know if it is productive or unproductive land that migrates, so to avoid that land migration strongly influences land rent (the dual value of the first constraint), we use land shares instead of absolute land use in the quadratic term of the PMP-function. This is equivalent to scaling the matrix [$\mathbf{D} + \mathbf{GBG}'$] by the square inverse of total land available in each period. In order to obtain values approximately interpretable as “marginal cost change in euro per hectare” it is also multiplied by ½ times square of total land available in year 2000, or $(v_1)_{2000}$. Thus, the $l_t = ((v_1)_{2000}/(v_1)_t)^2$.

The optimization model (1) can be equivalently described by the following first- and second order conditions for optimal **x**

$$\mathbf{Y}_t\mathbf{p}_t + \mathbf{s}_t - \mathbf{A}_t\mathbf{w}_t - q_t\mathbf{c} - l_t\left[\mathbf{D} + \mathbf{GBG}'\right]\mathbf{x}_t - \mathbf{R}_t'\boldsymbol{\lambda}_t = \mathbf{0} \tag{2}$$

$$\mathbf{R}_t\mathbf{x}_t = \mathbf{v}_t \tag{3}$$

[3] More precisely, to ensure a strictly definite Hessian matrix.

$$\mathbf{B} = \mathbf{U}'\mathbf{U} \tag{4}$$

$$d_{jj} \geq 0 \text{ for } j = 1\ldots J \text{ (and } d_{ij} = 0 \text{ for } i \neq j) \tag{5}$$

$\boldsymbol{\lambda}_t$ is the 2×1 vector of dual values for the constraints. Note that for positive semi-definiteness of the Hessian matrix it is sufficient that **B** is positive semi-definite, which is satisfied by the Cholesky factorisation with the upper triangular matrix **U**, and that all elements of **D** are non-negative[4].

The primary objective of the paper can now be more precisely formulated as estimating the PMP parameters by using the optimality conditions as estimating equations.

The secondary objective of evaluating the assumption of constant yields can be restated as an attempt to lift some of the non-linearity out of the PMP-function and explicit it in the form of a non-constant marginal gross value added, i.e. to estimate the relationship between yields and input use.. The first such extension is a variant of the model where yield depends endogenously on input use (land counting as an input). A second extension is the lesser modification that yields depend on allocated acreage.

3. Should yield depend on input use?

3.1. Motivation

The purpose of this section is to determine if prices of outputs and inputs are important determinants of yields of major agricultural crops in the EU. If a significant relationship between prices and yields can be identified, yields should be an endogenous function of input use in the CAPRI model, else input use and yields should be treated as exogenous to the model. The underlying idea is that perhaps some of the nonlinearity of the model, which is currently modelled only by the quadratic cost component, can be explained more explicitly (cf. Heckelei 2002). To decide which of those two alternative formulations to use, we estimate a yield function.

We start from the microeconomic model (1), and augment it with yields endogenously depending on **x** and **A** as in equation (6). We thus assume that yield Y_{jt} of crop j in period t can be approximated by a function that is quadratic in inputs $\mathbf{A} = (a_{ij})$, linearly dependent on planned number of hectares **x** and on trend T and with a random term $\boldsymbol{\varepsilon}$:

$$Y_{jt} = \gamma_{0j} + \gamma_{1j}T_t + \gamma_{2j}x_{jt} + \alpha_{1ij}a_{ijt} + \alpha_{2ij}a_{ijt}^2 + \varepsilon_{jt} \tag{6}$$

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[4] In fact, we will use a stronger restriction of $d_{jj} \geq \delta_{ij} > 0$ in estimations to avoid numerical problems when estimating elasticities.

In this estimation, it is assumed that the acreage allocation **x** is the optimal solution to the maximization problem at some expected prices and yield. We may then use the envelope theorem to obtain the optimality conditions for input use.

The first order condition for profit maximum of the extended model with respect to **A** at the expected output prices **p** and input prices **w** can be written

$$\frac{\partial Y_{jt}}{\partial a_{ijt}} p_{jt} = \left(\alpha_{1ij} + 2\alpha_{ij} a_{2ijt}\right) p_{jt} = w_{it}$$

Solving for the optimal input quantities gives $a_{ijt}^{*} = (w_{it}/p_{jt} - \alpha_{1ij})/(2\alpha_{2ij})$. Substituting that expression into the yield function (6) and defining

$$\beta_{0j} = \gamma_{0j} - \sum_i \frac{\alpha_{1ij}^2}{4\alpha_{2ij}}$$

$\beta_{1j} = \gamma_{1j}$

$\beta_{2j} = \gamma_{2j}$

$\beta_{3ij} = \frac{1}{4\alpha_{2ij}}$ and

$r_{ijt} = w_{it}/p_{jt}$

gives us an expression for yields that depends on the square price ratio r_{ijt}:

$$Y_{jt} = \beta_{0j} + \beta_{1j} T_t + \beta_{2j} x_{jt} + \sum_i \beta_{3ij} r_{ijt}^2 + \varepsilon_{it} \tag{7}$$

The second-order condition for a profit maximum is that $\alpha_{2ij} < 0$, so we expect β_3 to be negative. Without that condition holding true, we will not obtain useable estimates, and we would better choose exogenous yields. Thus, we want to test the hypothesis that $\beta_3 < 0$ versus $\beta_3 \geq 0$.

3.2. Data

The estimation is carried out on NUTSII level for the EU15 member states. All input prices have been aggregated to a single input price index by first computing the Laspeyres price index of the aggregates “plant protection” (PLAP) and “all other inputs” (REST), with the average total input quantities 2001-2003 as weights, and then merging them into a single input price index for each crop by computing the Laspeyres price index using the average 2001-2003 crop specific input coefficients as weights (input coefficients coming from the CAPRI database). Expected output prices were observed prices lagged one year (naïve price expectation), whereas input prices entered without lag.

It is crucial to be able to separate the effect of *trend* from that of the other explanatory variables., The squared price ratio is, however, likely to contain a trend component as well, which we will not be able to separate from the pure trend. To be on the safe side, i.e. not to find a significant influence of prices that is really only the influence of the trend in prices, we subtract linear trends from the explanatory variables **x** and **r**. This is done by fitting and subtracting a simple trend from each variable $\boldsymbol{\xi}$ using the equation

$$\boldsymbol{\xi}^* = \boldsymbol{\xi} - \mathbf{C}\left(\mathbf{C}'\mathbf{C}\right)^{-1}\mathbf{C}'\boldsymbol{\xi}$$

where **C** is the $n \times 2$ matrix with ones in the first column and the sequence from 1 to n in the second column, and $\boldsymbol{\xi}$ a time series for some exogenous variable to clear of trend.

3.3. Estimation method

The principal estimation method used is Least Squares. Three problems are likely to be present in the data set, so that some modifications of the ordinary least squares (OLS) seem appropriate. Firstly, there may be problems with endogeneity, because the lagged price ratio is likely to influence acreage. To avoid obtaining biased estimates, we try an alternative estimation where the trend free acreage is instrumentalized by lagged acreage, lagged output price, lagged price index of substitutes, direct subsidies and the other explanatory variables in (7) except for lagged squared price ratio. Denoting, for now, the $T \times K$ matrix of K explanatory variables T years for each crop j by $\mathbf{X}_j$ (not to confuse with acreage $\mathbf{x}_j$) the instrumental variables matrix by $\mathbf{Z}_j$, and the vector of coefficients by $\boldsymbol{\beta}_{IV}$, we estimate

$$\hat{\boldsymbol{\beta}}_{IVj} = \left(\hat{\mathbf{X}}'_j \mathbf{X}_j\right)^{-1} \mathbf{X}'_j \mathbf{y}_j, \ \ \hat{\mathbf{X}}_j = \mathbf{Z}_j \left(\mathbf{Z}'_j \mathbf{Z}_j\right)^{-1} \mathbf{Z}'_j \mathbf{X}_j \ \text{ for } j = 1 \ldots J$$

The correlation between acreage and instrumentalised acreage should be rather high for the instrumentation to make sense. The coefficients of correlation are shown for all relevant crops in table 1. Albeit there are some cases with low correlation, the general impression is that the instrumentation is good, with 55% of the correlations greater than 0.80.

Table 1: Correlation between acreage and instrumentalized acreage

	BL	DK	DE	EL	ES	FR	IR	IT	NL	AT	PT	SE	FI	UK
SWHE	0.72	0.84	0.86	0.87	0.87	0.70	0.54	0.61	0.39	0.83	0.79	0.78	0.90	0.54
DWHE			0.84	0.67	0.89	0.92		0.85		0.90	0.84			0.95
RYEM	0.89	0.55	0.87	0.61	0.83	0.96		0.42	0.56	0.89	0.79	0.77	0.61	0.83
BARL	0.92	0.74	0.87	0.94	0.78	0.88	0.96	0.89	0.70	0.85	0.52	0.90	0.79	0.91
OATS	0.79	0.65	0.96	0.46	0.97	0.99	0.63	0.91	0.87	0.77	0.85	0.74	0.88	0.66
MAIZ			0.78	0.97	0.78	0.53		0.86	0.83	0.85	0.74			
OCER	0.60		0.63	0.92	0.78	0.90		0.81			0.95			0.90
RAPE		0.86	0.87		0.93	0.79	0.84	0.94		0.96		0.87	0.26	0.41
SUNF			0.91	0.97	0.96	0.82		0.86		0.88	0.77			
SOYA					0.86	0.71		0.74		0.69				
PULS	0.78	0.40	0.96	0.98	0.80	0.95		0.93	0.89	0.93	0.79	0.53	0.84	0.73
POTA	0.88	0.88	0.99	0.69	0.94	0.74	0.53	0.87	0.68	0.89	0.43	0.64	0.73	0.62
SUGB	0.77	0.94	0.76	0.54	0.94	0.68	0.82	0.72	0.77	0.99	0.54	0.68	0.88	0.90
MAIF		0.98	0.80	0.64	0.91	0.53		0.97	0.80	0.84	0.85	0.58		
OFAR	0.72	0.69	0.95	0.99		0.72	0.82	0.62	0.71	0.93	0.63	0.85	0.81	

Secondly, a strong correlation between error terms of certain crops should be expected due to the similar influence of weather on similar crops. For example, one should expect a positive correlation between the yields of barley and rye, because their vegetative periods are similar and they have similar requirements regarding weather and soil. Thus, a seemingly unrelated regression (SUR) would be appropriate. Such an estimator would be more efficient than OLS would the covariance matrix be known. In the current case, the covariance matrix is not known, but has to be estimated, which may hamper efficiency considerably. This was tried out using iterated SUR with and without the instrumentation above. The estimation was carried out in three steps: (i) instrumentation of $\mathbf{X}$ by $\hat{\mathbf{X}}$ as above, (ii) iterated SUR of $\mathbf{Y}$ on $\hat{\mathbf{X}}$ to obtain stable weights matrix $\mathbf{W}$, which was computed from the inverse covariance matrix $\mathbf{\Sigma}$ of the error terms of the regression of $\mathbf{Y}$ on $\hat{\mathbf{X}}$, weighting each element of the covariance matrix by the harmonic mean of the degrees of freedom of the relevant equations, $DF_{ij} = \sqrt{(N_i - K_i)(N_j - K_j)}$, and (iii) computation of estimator $\boldsymbol{\beta}_{SUR} = \left(\hat{\mathbf{X}}'\mathbf{W}\mathbf{X}\right)^{-1}\hat{\mathbf{X}}'\mathbf{W}\mathbf{y}$. The index free matrices represent the stacked system as in Greene (2003, p. 342). $\mathbf{X}$ is the $(JT) \times (JK)$ partitioned matrix with matrix $\mathbf{X}_j$ on the j^{th} diagonal position and zeros elsewhere, and similar for $\hat{\mathbf{X}}$. $\mathbf{W} = \mathbf{\Sigma}^{-1} \otimes \mathbf{I}$, and $\mathbf{y}$ the vertically concatenated vectors $\mathbf{y}_j$.

Thirdly, there could be an aggregation bias. It may well be that for example a price increase has a greater production response in a sub region with generally low yields. The weight of the low yield region in the aggregate would increase, leading to reduced aggregate yield although the yield in each sub-region increased as response to the higher price. To investigate this effect to the extent possible by the available data, the regressions were re-run on sub national level (NUTS2 where possible, UK NUTS1). Prices are only available on national level. They were mapped down to the respective sub regions. Acreages and yields, on the other hand, are also available for NUTS2 regions.

Alltogether, eight different estimations were run to account for each of the three problems. The estimation setups are shown in table 2. The estimations were evaluated based on the number of significant coefficients using t-tests on the 5% level. The t-tests were computed for the test β_{3j} = 0 using standard deviations of the vector of estimators computed as the square root of the diagonal elements of $\mathrm{Cov}(\boldsymbol{\beta}) = \left(\hat{\mathbf{X}}'\mathbf{W}\hat{\mathbf{X}}\right)^{-1}$. Note that in the case of no instrumentation, $\hat{\mathbf{X}} = \mathbf{X}$, and $\mathbf{W}^{-1}$ becomes the standard variance estimator with degrees of freedom correction.

Table 2: Different estimation methods tried

Estimation nr.	Regional resolution	Acreage instrumentation	SUR
1	national	no	no
2	sub regions	no	no
3	national	yes	no
4	sub regions	yes	no
5	national	no	yes
6	sub regions	no	yes
7	national	yes	yes
8	sub regions	yes	yes

3.4. Results

The results indicate that there is a relationship between yields and input prices and also between acreages and yields in some regions for some crops., The relationships ,however, can not be statistically detected for all crops in all regions. For the *major share* of all crops no significant influence at all of neither input prices nor acreages on yields is found. The results also show that the sophistication of the estimation method by the instrumentation of acreages, use of sub regions and SUR covariance structure is worthwhile, because the number of significant coefficients increase by their introduction, and the signs of the price influence tend to be more conform with theory (which suggests a negative influence of the output-input price ratio). Table 3 shows the number of estimated equations, the number of coefficients with positive and negative signs and the number of coefficients significantly different from zero with each sign.

Since a rather large number of t-tests were carried out at the 5% level, one would expect 5% of the tests to show a significant $\beta_3 \neq 0$ even if the true $\beta_3 = 0$. For example, in the estimations with sub regions, 1858 t-tests were carried out. We would then expect 2.5% of 1858 = 93 tests to show b significantly different from zero in each direction even if the true $b = 0$. Even with this in mind, it seems that the number of significant coefficients is too large to be a pure random outcome (e.g. 184 negative significant to 96 positive significant out of 1858 tests for regionalised iterated SUR estimation with instrumentation). Therefore, we conclude that there is indeed a general influence of prices on yields, but that the influence is so hard to detect statistically that it does not seem worthwhile to try to estimate an economic model with endogenous yields.

Table 3: Summary of results for different estimation setups.

Est. nr.	Eq.	b_3<0	b_3>0	b_3<0*	b_3>0*	b_2<0	b_2>0	b_2<0*	b_2>0*
1	163	90	73	12	8	95	68	20	6
2	1858	1097	745	137	63	1051	807	204	114
3	163	95	68	9	7	96	67	14	4
4	1858	1112	730	125	61	1024	834	117	103
5	163	98	65	24	9	84	79	29	11
6	1858	1032	810	189	117	995	863	273	192
7	163	98	65	28	6	90	73	31	21
8	1858	1062	780	184	96	986	871	176	178

Est. nr. refers to estimation number in table2, b_3 *is the coefficient of price ratio,* b_2 *the coefficient of acreage, and a star refers to significance of 5 % level double sided t-test.*

Why is there no statistically reliable influence of prices on yields? It is well known that yield of most crops is a concave function of inputs. Given profit maximizing behaviour of producers, a relationship similar to that estimated here should result. There are, however, at least five major obstacles involved.

(1) The quadratic yield model implied here may be wrong. In reality, yield also depends on a lot of other factors that are all collected in the error term. Crop rotation is certainly a significant determinant of yield that is not controlled for in these estimations. This could be introduced by a careful selection of substitute acreages. A share of this influence should already be represented by the inclusion of own acreage, and introducing further explanatory variables would reduce the degrees of freedom and aggravate the problems with endogeneity (acreages depending on prices)

(2) The producers may not be rational in the way assumed here. Output price expectations may not be naïve, and the decision on input use may have to be taken with some time lag so that an input price expectation is required as well. It may also be the case that the yield function is largely unknown to the producer, so that rational behaviour as in the conceptualized model is impossible. Producers are perhaps more likely to choose input amounts from a table or heuristic with very few, if any, alternative levels of inputs. As an alternative price expectation, the formula $0.67P_{t-1} + 0.33P_{t-2}$ was tried, but without improvement in fit.

(3) The yield function may have a shape that implies almost the same input use and yield for a wide range of price ratios, so that there are almost only two different profit maximizing solutions: either “zero” or “full” input use. That would be the case if the graph of yield to inputs has an almost linear initial part and then bends sharply downwards at some point. Then the influence of the price ratio

would be “almost” discontinuous, with almost no change in yield for moderate price ratio changes and a big leap at some point. Then, for most price ratios, the optimal yield choice is almost the same.

(4) It may well be the case that the sub regional level used in the estimations 2, 4, 6 and 8 is still too aggregated so that an aggregation bias remains.

(5) The data sampling model underlying the estimations is inappropriate. Actually, observed acreages and prices are only indicators of the true (latent) *planned acreage* and *expected price*. Because the errors on acreages and prices now (erroneously) are attributed to measurement errors in yields, the estimated variance is too large, and thus the tests likely weaker. The coefficients are also likely to be biased in unknown directions (Fuller 1987). In addition, the observed yield is the *average* yield, whereas if yield really is endogenous the decision is based on the *expected marginal* yield. Actually, a model including measurement errors and marginal yield expectation together with the full optimality conditions (2-5) was the starting point of the estimation, but proved too complex to handle efficiently. Thus, the estimations of yield functions were performed in this separate step to determine whether endogenous yield should be part of the final model. We return to the measurement error model and yield expectations below, though without endogenous input coefficients.

4. Should yields depend on land allocation?

4.1. Problematic marginal cost curves

If the zero-profit condition (2) is solved for $\mathbf{x}$ we find that acreages are linearly depending on prices according to the relation

$$\mathbf{x}_t = l_t^{-1}\left[\mathbf{D} + \mathbf{GBG}'\right]^{-1}\left[\mathbf{Y}_t\mathbf{p}_t + \mathbf{s}_t - \mathbf{A}_t\mathbf{w}_t - q_t\mathbf{c} - \mathbf{R}_t'\boldsymbol{\lambda}_t\right] \tag{8}$$

Because the matrix $l_t^{-1}\left[\mathbf{D} + \mathbf{GBG}'\right]^{-1}$ is required to be positive semi-definite by the second order conditions, we expect the graph of $\mathbf{x}_t$ to gross margin $\mathbf{m}_t = \mathbf{Y}_t\mathbf{p}_t + \mathbf{s}_t - \mathbf{A}_t\mathbf{w}_t$, to be an upward sloping curve, so that increasing gross margin leads to increased acreage. Figure 1 shows the development of rye acreage and gross margin (nominal prices) between 1985 and 2003 for one of the most important cereals producing regions in France, the Nuts 2 region with code FR24 (Centre). Obviously, it would be difficult to fit acreage to gross margin with a positive slope if no other information is included, because the gross margin has increased whereas production decreased. In fact, the coefficients in an OLS regression of acreage on constant and gives the slope coefficient -0.0122 with a p-value of 0.0152. The points and the fitted line are shown in figure 2.

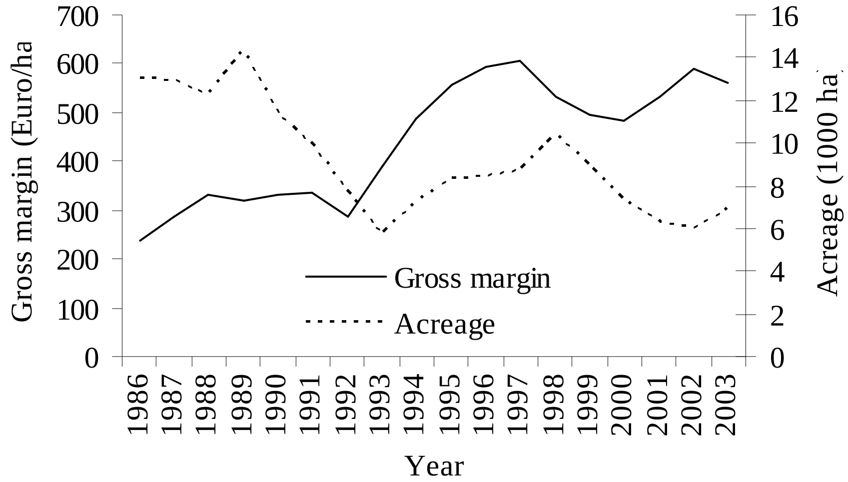

Figure 1. Gross margin and acreage of rye in FR24.

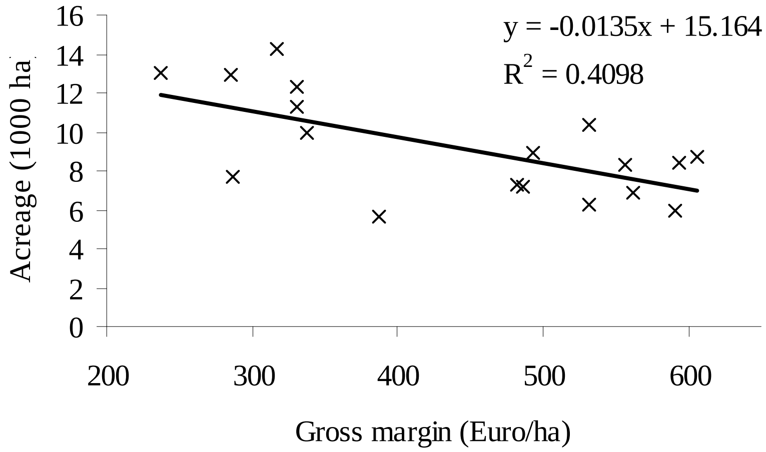

Figure 2. Acreage linearly fitted to gross margin for rye in FR24.

Thus, something more is influencing the producer decision to decrease rye production despite apparently increasing gross margin. Several auxiliary hypotheses come to mind. For instance, we tacitly assumed that the dual vector $\boldsymbol{\lambda}$ was constant, whereas it in fact $\boldsymbol{\lambda}$ depends on the gross margins of all other crops. Perhaps gross margins in, say the most important crop soft wheat, has increased enough to increase land price enough to force back rye. Figure 3 shows acreage and gross margin in soft wheat in the same region and time period. As can be seen in the figure, the gross margin in soft wheat has decreased slightly during the time period, which is not favourable for that hypothesis (though it is not enough to reject it; soft wheat may have been the wrong crop).

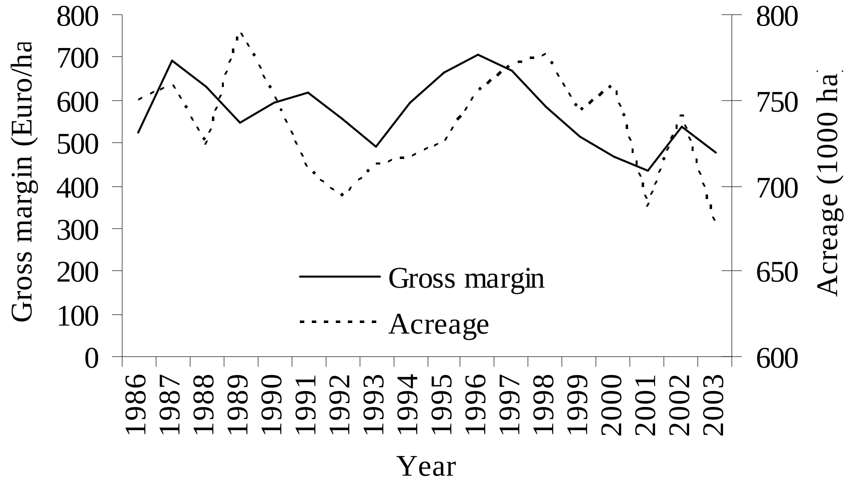

Figure 3. Gross margin and acreage of soft wheat in FR24.

A second assumption in the simple regression of acreage on gross margin was that the coefficient is constant over time. Comparison of the regression model with the equation (8) derived from the first order conditions reveals that the coefficient contains the parameters **c**, **D** and **B** which change over time with price index and total area. Thus the cost component **c** actually increases in nominal terms over time, which also helps alleviate the problem of reverse reaction of rye. A proper analysis should thus include at least the full first order conditions.

Estimation of (2-5) for all crops simultaneously, with a measurement error approach[5] allowing for errors on **x**, **Y**, **p**, **A** and **w**, and endogenous dual values with prior information for identification, did however result in a boundary solution for **D** and/or **B**. The boundary solution is such that rye obtains as small a coefficient as possible, still yielding a positive definite matrix. That implies an elasticity of supply of rye of close to infinity in the resulting simulation model, which is simply not plausible. That model is further discussed in the next section.

The rest of this section discusses a third auxiliary hypothesis that is sufficient to estimate rye parameters with the expected sign. The hypothesis is based on the fact that our yield data are really *average* yields, whereas the producer is assumed to base his production decision on *expected marginal* yield. Then gross margins **m** were computed in the wrong way above, using average yields. In fact, a closer

[5] The estimation also uses linear trends for expected yield and expected input requirements to remove stochastic weather influences, and uses prior information of 0.5 times gross margin of soft wheat for land price dual value and similar for set-aside for identification of the model.

look on the components **p**, **Y**, **s**, **A** and **w** of gross margins reveals that output prices have dropped steadily, and that the main reason for the increasing gross margin is that rye yields have risen sharply from about three to about five tons per hectare (figure 4). If the marginal yield is actually depending on **x**, then the development of marginal yields may be a qualitatively different from that of averages.

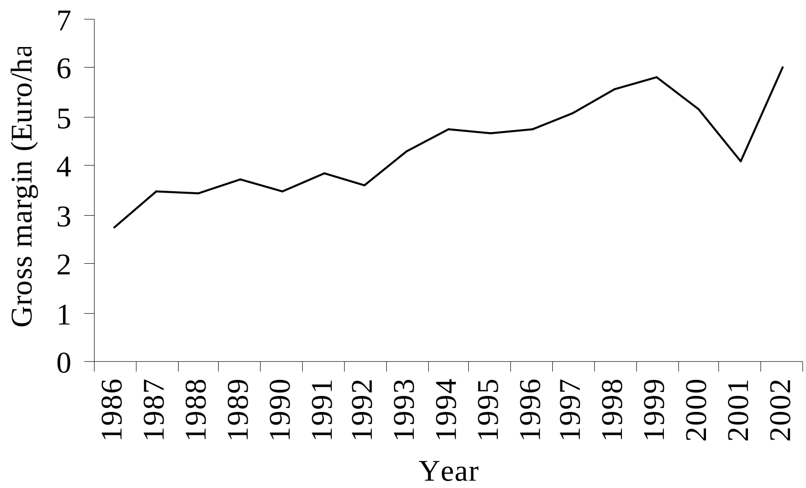

Figure 4. Average yields of rye in FR24

4.2. Motivation for endogenous yield

It could be the case that rye, which is grown on a considerably smaller area than soft wheat, is treated as an inferior alternative of many producers, and is thus grown on soil less suitable for cereals production. If prices increase, rye becomes an increasingly competitive alternative to soft wheat on the better soils. In that case, the marginal yield of rye with respect to acreage would be an *upward* sloping function.

One could also motivate a *downward* sloping yield function (of acreage) by assuming that first the soil that is best suited for rye is used, or that there is some rotational effect favouring smaller land use for rye. To investigate which is the case, we attempt to estimate the relation between yield and acreage.

Assume that marginal yield is approximated by the linear model

$$f_{jt}(x_{jt}) = \beta_{0j} + T_t\beta_{1j} + 2x_{jt}\beta_{2j}$$

with T a linear trend, and that observations of average yields arise according to

$$y_{jt} = \frac{1}{x_{jt}} \int_0^{x_{jt}} f_{jt}(z)dz + \varepsilon_{jt} \ .$$

Integration gives the model to estimate,

$$y_{jt} = \beta_{0j} + T_t\beta_{1j} + x_{jt}\beta_{2j} + \varepsilon_{jt}. \tag{9}$$

Note that the coefficient β_2 in the expression for the marginal yield enters with twice its estimated value. Thus, if β_{2j} is positive and x_{jt} decreases over time, then the marginal yield decreases over time compared to average yield. If the β_{2j} is big enough, this may be enough to turn the apparent positive gross margin development for example in rye in the case study region FR24 into a negative one.

4.3. Pitfalls when estimating the expected marginal yield

A straightforward least squares estimation of (9) gives a β_2 for rye of 0.04577, which is supporting the hypothesis that gross margin actually has been increasing less rapidly than indicated by the average yields. The t-test for $\beta_2 = 0$ gives a poor p-value of 0.544. The estimation, however, has at least two pitfalls that potentially make the estimation less efficient and reduces the power of the t-test of $\beta_2 = 0$.

(i) The yields of all crops tend to be correlated.

(ii) We ignore that acreage is measured with errors.

The first pitfall makes the LS estimation inefficient, because a more efficient estimator would recognise that if, say, all cereals have a low yield in 2003 (which was the actual case), then error terms in that year should have less weight in the estimation. That is, a *seemingly unrelated regression* (SUR) could be a more appropriate model (as in the previous section).

The second pitfall must be further explained. Above it was briefly mentioned that the ultimate goal is to perform an estimation with errors on the acreages **x**. So we should not now ignore that our observations of acreages may not be the true planned acreages, but acknowledge that a measurement error may be involved. If we assume the simple model that observed acreages **X** relate to true planned acreages **x** with a simple additive error model,

$$\mathbf{X}_j = \mathbf{x}_j + \mathbf{u}_j$$

then the estimates of β_2 are likely to be biased and the variances of the estimates are likely to be biased too (see Fuller 1987 for a thorough treatment of the linear measurement error model). In a simple linear model with a single explanatory variable, the coefficient is biased towards null by a factor $\kappa = \sigma_{xx}(\sigma_{xx} + \sigma_{uu})^{-1}$, and the estimated variance of the coefficient is biased by κ^{-2}. (but t-test $\beta = 0$ is not weakened). Unfortunately the situation becomes more complicated when there are two explanatory variables (TREND and ACREAGE), one of which is measured without error (TREND). To correct for these biases, a measurement error model seems to be the appropriate method.

4.4. A seemingly unrelated regression

The SUR estimator requires knowledge of the covariance matrix of yields. If that is not available, it can be estimated in a feasible generalised least squares estimation (FGLS). In this analysis we use an iterated SUR. In the first step, we estimate the model with independent error terms (identity matrix as weighing matrix). The residuals are used to estimate the yield covariance matrix $\boldsymbol{\Sigma}_e$. The inverse covariance matrix $\boldsymbol{\Sigma}_e^{-1}$ is used in the second step to estimate the FGLS model by minimising the generalised sum of squares

$$\min \sum_{jkt} \left(Y_{jt} - \beta_{0j} - \beta_{1j} T_t - \beta_{2j} X_{jt}\right)\left(\Sigma_e^{-1}\right)_{jk} \left(Y_{kt} - \beta_{0k} - \beta_{1k} T_t - \beta_{2k} X_{kt}\right)$$

where j,k are alternative indices for J crops and t the index for time.

In order for the coefficient vector to converge, certain limitations are required to bring down the number of elements in $\boldsymbol{\Sigma}_e$. This was done by subdividing the crops into five groups that were conjectured to react similarly or perform a similar function in the rotation. This is equivalent to a separate SUR estimation for each group. The groups are the ones shown in table 16 in the appendix, except of course for the group NOCR (crops with no physical yield) which was not included. In FR24 there was sufficient data for 15 cropping activities.

The SUR estimator $\hat{\beta}_2$ for β_2 in rye is considerably smaller than the OLS estimator, 0.01878 instead of 0.04577, and the t-statistic indicates an even less significant coefficient, P(abs($\hat{\beta}_2$)≥0.01878|β_2=0) = 0.598. The block wise covariance matrix and the estimated coefficients are shown in the following tables (4-9). One can see in the table that the assumption of covariation of yields across crops within the groups is reasonable, because all items except for the covariation PULS.POTA in table 8 are positive. Nevertheless, the estimated $\hat{\beta}_2$ are significantly different from zero only in 4 out of 15 cases (determined by Student's t-test), rye not being one of them. So even if the coefficient on rye tends to have the right sign, the effect could just as well be coincidence in most cases.

Table 4. Coefficients for TREND and ACREAGE in SUR estimation

	$\hat{\beta}_1$ .value	$\hat{\beta}_1$ .p	$\hat{\beta}_2$ .value	$\hat{\beta}_2$ .p	Significance of $\hat{\beta}_2$
SWHE	0.0688	0.0090	-0.0011	0.5870	
DWHE	0.0564	0.0250	-0.0022	0.1210	
RYEM	0.1492	0.0000	0.0188	0.5980	
BARL	0.0689	0.0050	0.0001	0.9770	
OATS	0.0534	0.0140	0.0177	0.0010	***
MAIZ	0.1785	0.0001	-0.0009	0.7880	
OCER	-0.0318	0.2620	0.0511	0.0190	*
RAPE	-0.0013	0.9680	0.0012	0.7070	
SUNF	0.0150	0.4100	-0.0020	0.3220	
PULS	0.0214	0.2760	0.0066	0.1460	
POTA	0.7869	0.0030	-0.6038	0.4310	
SUGB	0.9813	0.0000	-2.1111	0.0008	***
MAIF	-0.0790	0.7810	-0.2972	0.0440	
OFAR	-0.4271	0.1190	-0.1502	0.0000	***
NONF	0.0731	0.0020	0.0072	0.3830	

Table 5. Covariance matrix of SUR residuals for Cereals

	SWHE	DWHE	RYEM	BARL	OATS
SWHE	0.305	0.244	0.202	0.210	0.151
DWHE	0.244	0.274	0.183	0.184	0.133
RYEM	0.202	0.183	0.228	0.147	0.146
BARL	0.210	0.184	0.147	0.233	0.149
OATS	0.151	0.133	0.146	0.149	0.147

Table 6. Covariance matrix of SUR residuals for Cereals2

	MAIZ	OCER
MAIZ	0.413	0.256
OCER	0.256	0.263

Table 7. Covariance matrix of SUR residuals for Oilseeds

	RAPE	SUNF	NONF
RAPE	0.129	0.018	0.038
SUNF	0.018	0.067	0.021
NONF	0.038	0.021	0.061

Table 8. Covariance matrix of SUR residuals for Other Arable Crops

	PULS	POTA	SUGB
PULS	0.205	-0.034	0.835
POTA	-0.034	9.426	2.167
SUGB	0.835	2.167	11.051

Table 9. Covariance matrix of SUR residuals for Fodder

	MAIF	OFAR
MAIF	22.212	12.228
OFAR	12.228	15.024

4.5. A measurement error model

To include the assumption that $\mathbf{X}_j = \mathbf{x}_j + \mathbf{u}_j$ into the estimation, a total least squares estimation is performed by minimising the following extremum estimation criterion, scaled by the inverse of the number of observations $n = JT$ (for J crops and T periods):

minimize

$$\begin{aligned} &\tfrac{1}{n}\sum_{jkt}\left(Y_{jt}-\beta_{0j}-\beta_{1j}T_t-\beta_{2j}x_{jt}\right)\left(\Sigma_e^{-1}\right)_{jk}\left(Y_{kt}-\beta_{0k}-\beta_{1k}T_t-\beta_{2k}x_{kt}\right) \\ &+\tfrac{1}{n}\sum_{jkt}\left(X_{jt}-x_{jt}\right)\left(\Sigma_u^{-1}\right)_{jk}\left(X_{kt}-x_{kt}\right) \end{aligned} \tag{10}$$

Here $\mathbf{\Sigma}_e$ denotes the covariance matrix between the residuals obtained from the SUR estimation mentioned previously, whereas $\mathbf{\Sigma}_u$ is a prior covariance matrix of acreages. $\mathbf{\Sigma}_u$ only contains diagonal entries that are constructed following the principle that the standard deviation always is 6⅔ percent of the sample mean (over time for each crop), implying that virtually all outcomes are within ± 20% of the observations. That is, for σ_{jj} diagonal element of $\mathbf{\Sigma}_u$,

$$\sigma_{jj} = \left(\overline{X}_{j\bullet}\frac{0.20}{3}\right)^2 .$$

The model 10 with errors in the explanatory variables is referred to as a measurement error model (Fuller 1987), or sometimes *Errors-In-Variables-model* (EIV). The coefficients of the EIV estimation are solved for using a non-linear programming (NLP) solver software, and the results shown in the following table (10). The signs and sizes of the coefficients are generally similar to those of the SUR estimators.

Table 10. Coefficients in EIV estimation

	B0	B1	B2
SWHE	37.21941	0.00644	-0.04172
DWHE	5.58560	0.05500	-0.00250
RYEM	2.82318	0.14463	0.00699
BARL	4.91312	0.06313	0.00195
OATS	2.60144	0.05768	0.01952
MAIZ	6.17032	0.18124	-0.00051
OCER	3.05974	-0.04094	0.06173
RAPE	2.90948	-0.00335	0.00137
SUNF	2.74305	0.01259	-0.00229
PULS	3.68404	0.02101	0.00965
POTA	32.54321	0.85206	-0.86423
SUGB	166.77830	0.67172	-3.90225
MAIF	55.23074	-0.29928	-0.44824
OFAR	69.80859	-0.68218	-0.18152
NONF	0.67922	0.07145	0.00797

It would be desirable to obtain an estimator of the standard deviations of the EIV coefficient estimators. Fuller (1987) finds that he is unable to establish the exact distribution of the estimators even in the simple case with one explanatory variable. He instead derives an approximate (normal) distribution for the coefficient vector in large samples.

Here we follow another approach using asymptotic properties of extremum estimators as described in Mittelhammer, Judge, Miller (2000, ch. 7).

4.6. Asymptotic properties of the estimators in the EIV model

We start off by putting the model (10) in matrix form. Rewrite it separating the exogenous variable "acreages" that is measured with errors from the matrix of exogenous **Z** that is known with certainty; constant and trend. Denote the coefficients of **x** by $\boldsymbol{\gamma}$ and the coefficients for **Z** by $\boldsymbol{\beta}$. We denote the true planned acreages by lower case **x** and the observed values from the statistics by the random variable upper case **X**. Then the model can be written in matrix form as

$$\begin{aligned} &\min_{\beta,\gamma,x} n^{-1}\left(\mathbf{Y}-\mathbf{z}_b\boldsymbol{\beta}-\mathbf{x}_b\boldsymbol{\gamma}\right)'\boldsymbol{\Omega}_e^{-1}\left(\mathbf{Y}-\mathbf{z}_b\boldsymbol{\beta}-\mathbf{x}_b\boldsymbol{\gamma}\right)+\left(\mathbf{X}-\mathbf{x}\right)'\boldsymbol{\Omega}_u^{-1}\left(\mathbf{X}-\mathbf{x}\right) \\ &\Leftrightarrow \min_{\beta,\gamma,x} m(\boldsymbol{\beta},\boldsymbol{\gamma},\mathbf{x}\,|\,\mathbf{Y},\mathbf{z},\mathbf{X}) \end{aligned} \qquad (11)$$

where, for $\mathbf{I}_T$ of size T, and $\otimes$ the Kronecker product,

$$\boldsymbol{\Omega}_e^{-1}=\boldsymbol{\Sigma}_e^{-1}\otimes\mathbf{I}_T\,,\boldsymbol{\Omega}_u^{-1}=\boldsymbol{\Sigma}_u^{-1}\otimes\mathbf{I}_T$$

The vectors/matrices $\mathbf{Y}$, $\mathbf{x}$, $\mathbf{z}$, $\mathbf{X}$, and $\boldsymbol{\beta}$ are the vertically concatenated vectors/matrices $\mathbf{Y}_j$, $\mathbf{x}_j$, $\mathbf{z}_{j,}$, $\mathbf{X}_j$, and $\boldsymbol{\beta}_j$. $\boldsymbol{\gamma}$ is the vector of γ_j for crops $j = 1\ldots J$, the subscript b denotes the block-wise diagonalisation where the j^{th} diagonal block of the $JT \times JK$ (for K columns in $\mathbf{z}$) matrix $\mathbf{z}_b$ is $\mathbf{z}_j$ (and similar for $\mathbf{x}_b$), so that the function m can be written alternatively as

$$m = \frac{1}{n}\left(\begin{bmatrix}\mathbf{Y}_1\\ \vdots \\ \mathbf{Y}_J\end{bmatrix} - \begin{bmatrix}\mathbf{z}_1 & 0 & 0\\ 0 & \ddots & 0\\ 0 & 0 & \mathbf{z}_J\end{bmatrix}\begin{bmatrix}\boldsymbol{\beta}_1\\ \vdots \\ \boldsymbol{\beta}_J\end{bmatrix} - \begin{bmatrix}\mathbf{x}_1 & 0 & 0\\ 0 & \ddots & 0\\ 0 & 0 & \mathbf{x}_J\end{bmatrix}\begin{bmatrix}\gamma_1\\ \vdots \\ \gamma_J\end{bmatrix}\right)' \begin{bmatrix}\mathbf{I}_T\sigma_e^{11} & \cdots & \mathbf{I}_T\sigma_e^{1J}\\ \vdots & & \vdots \\ \mathbf{I}_T\sigma_e^{J1} & \cdots & \mathbf{I}_T\sigma_e^{JJ}\end{bmatrix}\left(\cdots\right)$$

$$+\frac{1}{n}\left(\begin{bmatrix}\mathbf{X}_1\\ \vdots \\ \mathbf{X}_J\end{bmatrix} - \begin{bmatrix}\mathbf{x}_1\\ \vdots \\ \mathbf{x}_J\end{bmatrix}\right)' \begin{bmatrix}\mathbf{I}_T\sigma_u^{11} & \cdots & \mathbf{I}_T\sigma_u^{1J}\\ \vdots & & \vdots \\ \mathbf{I}_T\sigma_u^{J1} & \cdots & \mathbf{I}_T\sigma_u^{JJ}\end{bmatrix}\left(\begin{bmatrix}\mathbf{X}_1\\ \vdots \\ \mathbf{X}_J\end{bmatrix} - \begin{bmatrix}\mathbf{x}_1\\ \vdots \\ \mathbf{x}_J\end{bmatrix}\right)$$

The rightmost vector (…) in the first term is the same as the first bracketed expression, omitted to save space. The extremum estimator defined in (11) is equivalent to an element-wise weighted total least squares estimator, shown to be consistent in Kukush and Huffel (2004).

We will now attempt to obtain a Lagrange Multiplier (LM) test of the hypothesis that $\boldsymbol{\gamma} = \mathbf{0}$, following the procedure described in Mittelhammer, Judge and Miller (2000) (section 7.6). One can show that the conditions in theorem 7.3.3 are satisfied, so that $\hat{\boldsymbol{\theta}}$ is asymptotically normally distributed, with $n^{½}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}_{true}) \xrightarrow{d} \mathrm{N}(\mathbf{0},\mathbf{H}^{-1}\boldsymbol{\Sigma}\mathbf{H}^{-1})$, with $\mathbf{H}^{-1}$ the Hessian of m and $\boldsymbol{\Sigma}$ the covariance matrix of $n^{½}$ times the Jacobian of m, both w.r.t. $\boldsymbol{\theta}$ evaluated at $\boldsymbol{\theta}_{true}$.

We may then use the operational Lagrange Multiplier (LM) test with the test statistic

$$LM = n\boldsymbol{\Gamma}_r'\left[\mathbf{c}\hat{\mathbf{H}}^{-1}\mathbf{c}'\right]\left[\mathbf{c}\hat{\mathbf{H}}^{-1}\hat{\boldsymbol{\Sigma}}\hat{\mathbf{H}}^{-1}\mathbf{c}'\right]^{-1}\left[\mathbf{c}\hat{\mathbf{H}}^{-1}\mathbf{c}'\right]\boldsymbol{\Gamma}_r \sim \chi^2(J,0)$$

for $\boldsymbol{\Gamma}_r$ the Lagrange Multipliers associated with the model (11) restricted by the J restrictions $\gamma_j = 0$, or in matrix form as a linear restriction of the entire parameter vector, $\mathbf{c}\boldsymbol{\theta} = \mathbf{0}$, where $\mathbf{c}$ is a $J \times (3J + JT)$ matrix of zeros and ones constructed by horizontal concatenation of $(J \times 2)$ zeros, $\mathbf{I}_J$ and $(J \times JT)$ zeros. Differentiation of m gives the Jacobian $\mathbf{J}(m)$ as column vector as

$$\mathbf{J}(m)=\begin{bmatrix}\frac{\partial m}{\partial \boldsymbol{\beta}}\\ \frac{\partial m}{\partial \boldsymbol{\gamma}}\\ \frac{\partial m}{\partial \mathbf{z}}\end{bmatrix}=n^{-1}2\begin{bmatrix}\mathbf{z}_b'\boldsymbol{\Omega}_e^{-1}\left(\mathbf{Y}-\mathbf{z}_b\boldsymbol{\beta}-\mathbf{x}_b\boldsymbol{\gamma}\right)\\ \mathbf{z}_b'\boldsymbol{\Omega}_e^{-1}\left(\mathbf{Y}-\mathbf{z}_b\boldsymbol{\beta}-x_b\boldsymbol{\gamma}\right)\\ \boldsymbol{\gamma}_d\boldsymbol{\Omega}_e^{-1}\left(\mathbf{Y}-\mathbf{z}_b\boldsymbol{\beta}-\mathbf{x}_b\boldsymbol{\gamma}\right)+\boldsymbol{\Omega}_u^{-1}\left(\mathbf{x}-\mathbf{X}\right)\end{bmatrix}$$

$$=n^{-1}2\begin{bmatrix}\mathbf{z}_b'\boldsymbol{\Omega}_e^{-1}\mathbf{e}\\ \mathbf{x}_b'\boldsymbol{\Omega}_e^{-1}\mathbf{e}\\ \boldsymbol{\gamma}_d\boldsymbol{\Omega}_e^{-1}e-\boldsymbol{\Omega}_u^{-1}\mathbf{u}\end{bmatrix}=n^{-1}2\begin{bmatrix}\mathbf{z}_b'\boldsymbol{\Omega}_e^{-1} & 0\\ \mathbf{x}_b'\boldsymbol{\Omega}_e^{-1} & 0\\ \boldsymbol{\gamma}_d\boldsymbol{\Omega}_e^{-1} & -\boldsymbol{\Omega}_u^{-1}\end{bmatrix}\begin{bmatrix}\mathbf{e}\\ \mathbf{u}\end{bmatrix}$$

where $\boldsymbol{\gamma}_{\mathrm{d}}$ denotes the $JT \times JT$ diagonal vector with $\boldsymbol{\gamma} \otimes \boldsymbol{\iota}_T$ (for $\boldsymbol{\iota}$ vector of “1”) on the diagonal. Since E(**e**) = E(**u**) = **0**, we have that E(**J**(m)) = **0**. The Jacobian is a linear combination of the (assumed) normally distributed random variables in $[\mathbf{e}'\ \mathbf{u}']'$, the covariance matrix

$$\text{est.cov}\left(\begin{bmatrix}\mathbf{e}\\ \mathbf{u}\end{bmatrix}\right)=\hat{\sigma}^2\boldsymbol{\Omega}=n^{-1}\left(\mathbf{e}'\boldsymbol{\Omega}_e^{-1}\mathbf{e}+\mathbf{u}'\boldsymbol{\Omega}_u^{-1}\mathbf{u}\right)\begin{bmatrix}\boldsymbol{\Omega}_e & \mathbf{0}\\ \mathbf{0} & \boldsymbol{\Omega}_u\end{bmatrix}=m(\boldsymbol{\beta},\boldsymbol{\gamma},\mathbf{x}\mid\mathbf{Y},\mathbf{z},\mathbf{X})\begin{bmatrix}\boldsymbol{\Omega}_e & \mathbf{0}\\ \mathbf{0} & \boldsymbol{\Omega}_u\end{bmatrix}$$

so the covariance $\boldsymbol{\Sigma}$ of $n^{1/2}\mathbf{J}(m(\boldsymbol{\theta}))$ evaluated at the estimated $\boldsymbol{\theta}$ computed is given by

$$\boldsymbol{\Sigma}=m(\boldsymbol{\beta},\boldsymbol{\gamma},\mathbf{x}\mid\mathbf{Y},\mathbf{z},\mathbf{X})n^{-1/2}2\begin{bmatrix}\mathbf{z}_b'\boldsymbol{\Omega}_e^{-1} & 0\\ \mathbf{x}_b'\boldsymbol{\Omega}_e^{-1} & 0\\ \boldsymbol{\gamma}_d\boldsymbol{\Omega}_e^{-1} & -\boldsymbol{\Omega}_u^{-1}\end{bmatrix}\begin{bmatrix}\boldsymbol{\Omega}_e & \mathbf{0}\\ \mathbf{0} & \boldsymbol{\Omega}_u\end{bmatrix}\begin{bmatrix}\mathbf{z}_b'\boldsymbol{\Omega}_e^{-1} & 0\\ \mathbf{x}_b'\boldsymbol{\Omega}_e^{-1} & 0\\ \boldsymbol{\gamma}_d\boldsymbol{\Omega}_e^{-1} & -\boldsymbol{\Omega}_u^{-1}\end{bmatrix}'n^{-1/2}2$$

$$=m(\boldsymbol{\beta},\boldsymbol{\gamma},\mathbf{x}\mid\mathbf{Y},\mathbf{z},\mathbf{X})n^{-1}4\begin{bmatrix}\mathbf{z}_b'\boldsymbol{\Omega}_e^{-1}\mathbf{z}_b & \mathbf{z}_b'\boldsymbol{\Omega}_e^{-1}\mathbf{x}_b & \mathbf{z}_b'\boldsymbol{\Omega}_e^{-1}\boldsymbol{\gamma}_d\\ \mathbf{x}_b'\boldsymbol{\Omega}_e^{-1}\mathbf{z}_b & \mathbf{x}_b'\boldsymbol{\Omega}_e^{-1}\mathbf{x}_b & \mathbf{x}_b'\boldsymbol{\Omega}_e^{-1}\boldsymbol{\gamma}_d\\ \boldsymbol{\gamma}_d\boldsymbol{\Omega}_e^{-1}\mathbf{z}_b & \boldsymbol{\gamma}_d\boldsymbol{\Omega}_e^{-1}\mathbf{x}_b & \boldsymbol{\gamma}_d\boldsymbol{\Omega}_e^{-1}\boldsymbol{\gamma}_d+\boldsymbol{\Omega}_u^{-1}\end{bmatrix}$$

The Hessian matrix is obtained by differentiation of the Jacobian, to obtain

$$\mathbf{H}(m)=\begin{bmatrix}\frac{\partial m^2}{\partial\boldsymbol{\beta}\partial\boldsymbol{\beta}'} & \frac{\partial m^2}{\partial\boldsymbol{\beta}\partial\boldsymbol{\gamma}'} & \frac{\partial m^2}{\partial\boldsymbol{\beta}\partial\mathbf{z}'}\\ \frac{\partial m^2}{\partial\boldsymbol{\gamma}\partial\boldsymbol{\beta}'} & \frac{\partial m^2}{\partial\boldsymbol{\gamma}\partial\boldsymbol{\gamma}'} & \frac{\partial m^2}{\partial\boldsymbol{\gamma}\partial\mathbf{z}'}\\ \frac{\partial m^2}{\partial\mathbf{z}\partial\boldsymbol{\beta}'} & \frac{\partial m^2}{\partial\mathbf{z}\partial\boldsymbol{\gamma}'} & \frac{\partial m^2}{\partial\mathbf{z}\partial\mathbf{z}'}\end{bmatrix}=-2n^{-1}\begin{bmatrix}\mathbf{z}_b'\boldsymbol{\Omega}_e^{-1}\mathbf{z}_b & \mathbf{z}_b'\boldsymbol{\Omega}_e^{-1}\mathbf{x}_b & \mathbf{z}_b'\boldsymbol{\Omega}_e^{-1}\boldsymbol{\gamma}_d\\ \mathbf{x}_b'\boldsymbol{\Omega}_e^{-1}\mathbf{z}_b & \mathbf{x}_b'\boldsymbol{\Omega}_e^{-1}\mathbf{x}_b & 2\mathbf{x}_b'\boldsymbol{\Omega}_e^{-1}\boldsymbol{\gamma}_d\\ \boldsymbol{\gamma}_d\boldsymbol{\Omega}_e^{-1}\mathbf{z}_b & 2\boldsymbol{\gamma}_d\boldsymbol{\Omega}_e^{-1}\mathbf{x}_b & \boldsymbol{\gamma}_d\boldsymbol{\Omega}_e^{-1}\boldsymbol{\gamma}_d-\boldsymbol{\Omega}_u^{-1}\end{bmatrix}$$

The model (11) is then solved twice, once unconstrained and once constrained by $\gamma_j = 0$ for $j = 1\ldots15$. The LM test statistic is computed using the Lagrange Multipliers obtained in the constrained model and the estimated **H** and **Σ**. The resulting test statistic is 26.7, which is asymptotically distributed as chi-square(15) if the constraints are true. For a test on 5% level we compare LM with the 5% tabular value of the chi-square distribution, which is 25.0, and conclude that the null hypothesis is rejected at the 5% level (the exact p-value is 0.031).

So, yields could depend on acreages. However, a look at the estimated coefficients in table 10 shows that the estimations are not sufficiently robust to use on a large scale: Sugar beet (SUGB) obtains the (significant at 0.1% level in a test using the asymptotic normal distribution for $\hat{\boldsymbol{\theta}}$) coefficient of minus 3.90 tons per thousand hectares. This implies that the marginal yield at the observed acreage (about 25’000 hectares) is negative, which is unacceptable. At the same time, the coefficient on rye is very close to zero and not significant, so the original problem is not solved. Thus, we decide to discard the model with yield depending on acreage *despite* the failure to reject the hypothesis that $\boldsymbol{\gamma} = \mathbf{0}$.

5. A Bayesian estimator based on highest posterior density

5.1. Principles of estimator

After having discussed two different versions of yield endogeneity in sections three and four, we now return to the primary objective and model (1). The basic assumption underlying the data sampling model is that there exists a set of true parameters $\boldsymbol{\psi} = (\mathbf{p},\mathbf{Y},\mathbf{s},\mathbf{A},\mathbf{w},\mathbf{q},\mathbf{l},\mathbf{c},\mathbf{D},\mathbf{B},\mathbf{R},\mathbf{v})$ of the model, satisfying the second order conditions (4-5), a vector of true planned acreages $\mathbf{x}^*$ and a vector of dual values $\boldsymbol{\lambda}^*$ such that $(\mathbf{x}^*,\boldsymbol{\lambda}^*)$ is the unique optimal solution to the model parametrized by $\boldsymbol{\Psi}$. We may thus write $\mathbf{x}^* = \mathbf{x}^*(\boldsymbol{\psi})$ and $\boldsymbol{\lambda}^* = \boldsymbol{\lambda}^*(\boldsymbol{\psi})$. Furthermore, the values $\mathbf{z} = (\mathbf{x}^{obs},\mathbf{p}^{obs},\mathbf{Y}^{obs},\mathbf{s}^{obs},\mathbf{A}^{obs},\mathbf{w}^{obs},\mathbf{q}^{obs},\mathbf{l}^{obs},\mathbf{R}^{obs},\mathbf{v}^{obs})$ in the CAPRI database are considered the outcome of a random variable vector **Z** that is conditional on $\boldsymbol{\Psi}$, i.e. there exists a probability density function $f(\mathbf{z}|\boldsymbol{\Psi})$.

We have prior beliefs regarding the parameter $\boldsymbol{\Psi}$ that are not contained in the CAPRI database. We expect the dual values of the constraints and the price elasticities implied by $\boldsymbol{\psi}$ to be of “reasonable size”. If we are express those beliefs as a prior density function $\xi(\boldsymbol{\Psi})$, we may use Bayes’ rule to derive the posterior density function of $\boldsymbol{\Psi}$ conditional on the outcome $\mathbf{z}$:

$$\xi(\boldsymbol{\Psi}|\mathbf{z}) \propto f(\mathbf{z}|\boldsymbol{\Psi})\xi(\boldsymbol{\Psi})$$

In the following sections, we first develop an error model that relates $\mathbf{z}$ to $\boldsymbol{\Psi}$ in order to derive the function f. We discuss the chosen error model and compare it to alternatives. Then we formulate our prior beliefs regarding elasticites and dual values in terms of the unconditional density function ξ. Finally, we devise an estimation method that chooses as an estimate the parameter vector $\boldsymbol{\Psi}$ that maximises the conditional density $\xi(\boldsymbol{\Psi}|\mathbf{z})$. DeGroot (1970) calls this estimator the generalised maximum likelihood estimator. Other authors have called it the posterior mode estimator, the maximum a-posteriori estimator or the highest posterior density estimator.

5.2. Data sampling process

The distribution of $\mathbf{Z}$ is based on the following assumptions, which are detailed further below:

(i) All elements in $\mathbf{Z}$ are independent.

(ii) Subsidies, price index, set-aside rate and total land constraint are known with certainty, i.e. are degenerate random variables.

(iii) Errors are additive.

(iv) Producers have naïve price expectations.

(v) Expected yields and input requirements follow linear trends.

(i) The covariance matrix Σ only contains diagonal elements. This is discussed further in the following section on prior distributions.

(ii) We assume that set-aside rate, subsidies, price index and total land constraint are known with certainty. Since the outcomes of those items in the random vector $\mathbf{Z}$ will be the corresponding items of $\boldsymbol{\Psi}$ itself, they are from now on removed from $\mathbf{Z}$. An outcome of $\mathbf{Z}$ is thus written $\mathbf{z} = (\mathbf{x}^{obs},\mathbf{p}^{obs},\mathbf{w}^{obs},\mathbf{Y}^{obs},\mathbf{A}^{obs})$.

(iii) We write an outcome of $\mathbf{Z}$ as the sum of its conditional expectation $\boldsymbol{\mu}(\boldsymbol{\psi}) = (\boldsymbol{\mu}_x,\boldsymbol{\mu}_p,\boldsymbol{\mu}_w,\boldsymbol{\mu}_Y,\boldsymbol{\mu}_A)$, (with appropriate dimensions), and the random error vector $\boldsymbol{\varepsilon}$, so that, $\mathbf{Z} = \boldsymbol{\mu}(\boldsymbol{\psi}) + \boldsymbol{\varepsilon}$. For acreages, we have

$$\boldsymbol{\mu}_x = \mathbf{x}^*(\boldsymbol{\psi})$$

(iv) Naïve price expectations imply that the expectation of the price measurement in period t-1 equals the producer price in that period, or conversely,

$$\mathbf{p}_t = (\mu_p)_{t-1}$$

$$\mathbf{w}_t = (\mu_w)_{t-1}$$

where the expression on the right hand side denotes the expected value of the output and input prices for all crops in period t-1.

(v) The producers expect the yield in each period to equal $(\mu_y)_t$, which increases over time by an exogenous linear trend. The same assumption is made for input coefficients, We thus have that

$$\mathbf{Y}_t = (\mu_y)_t = \beta_0 + \beta_1 T_t \tag{12}$$

$$\mathbf{A}_t = (\mu_A)_t = \alpha_0 + \alpha_1 T_t$$

with T being a linear trend and $\beta = (\beta_0,\beta_1)$ and $\alpha = (\alpha_0,\alpha_1)$ new parameters to estimate. Unfortunately, there are no observations available for actual input application. Instead, we use estimated input coefficients (available in the CAPRI database), that are based on total input use in the agricultural sector in combination with farm level data, economic reasoning and engineering knowledge. Those expert coefficients are denoted by $\mathbf{A}^{obs}$. The actual amount of inputs applied in any given year may differ from the expected value due to unexpected climatic conditions, just as the yield may deviate from expected yield, though the hypothesis is that the agricultural production plan is made up with the expected values in mind[6].

5.3. Discussion of alternative error models

The error model developed above is fairly sophisticated in the sense that it attempts to take into account that all measurements are likely to be subject to errors[7]. The sophistication comes at a cost, because it requires information about the covariance matrix of **Z**. Ideally, this information would be supplied by replicate measurements or external datasets (Carroll, Ruppert and Stefanski 1995). In the case at hand, no such replicates are available, and instead, the relative variability of the different errors is based on assumptions.

Although the error model is sophisticated on the side of the researcher (measurement errors), it is very simple on the side of the economic agent. We assume

[6] This implies a general error model, but the resulting formulation is indistinguishable from the measurement error model.

[7] Griliches and Ringstad (1970 p. 370) conclude, in relation to measurement errors in nonlinear models, that "*In short, errors in variables are bad enough in linear models. They are likely to be disastrous to any attempts to estimate additional nonlinearity or curvature parameters*."

that the agent has perfect information about the true parameters, and that he is able to determine the optimal production decision exactly. That is, no part of the errors enter the model equations, thereby influencing production. A more general error model, as discussed by McElroy (1987) and Pope and Just (2002) would also take into account the possibility that the producer may not correctly appreciate the true parameters and/or is not able to determine exactly the optimal supply decision. Let us look at the implications of neglecting those errors.

The exogenous (in this model) parameters that are subject to considerable uncertainty are prices ($\mathbf{p}$,$\mathbf{w}$) and I/O coefficients ($\mathbf{A}$,$\mathbf{Y}$). Saying that the producer does not correctly appreciate those is silly, since they are *defined* as the producer's expectation. It may however be the case that the *expectation model* is not the correct one (the possibility that the producer does not base his expectation on the same observations as the researcher is already included in the error term). In those cases, the producer bases his land allocation decision not on the true parameters ($\mathbf{p}$,$\mathbf{w}$,$\mathbf{A}$,$\mathbf{Y}$) (which can then no longer be called "true") but on stochastic ($\mathbf{p} + \boldsymbol{\delta}_p$,$\mathbf{w} + \boldsymbol{\delta}_w$,$\mathbf{A} + \boldsymbol{\delta}_A$,$\mathbf{Y} + \boldsymbol{\delta}_Y$) for some deviations $\boldsymbol{\delta}$. This is a kind of specification error of the model. If we at this point assume that the producer solves the optimization problem correctly, we can substitute the disturbed parameters into the first order conditions and rearrange to obtain

$$\mathbf{Y}_t\mathbf{p}_t + \mathbf{s}_t - \mathbf{A}_t\mathbf{w}_t - q_t\mathbf{c} - l_t\left[\mathbf{D} + \mathbf{GBG}'\right]\mathbf{x}_t - \mathbf{R}_t'\boldsymbol{\lambda}_t = \boldsymbol{\Delta}_t$$

with $\boldsymbol{\Delta} = \mathbf{A}\boldsymbol{\delta}_w + \mathbf{w}\boldsymbol{\delta}_A + \boldsymbol{\delta}_A\boldsymbol{\delta}_w - \mathbf{Y}\boldsymbol{\delta}_p - \mathbf{p}\boldsymbol{\delta}_Y - \boldsymbol{\delta}_p\boldsymbol{\delta}_Y$ (time indices omitted). This makes the relationship between the true parameters stochastic. It is not clear what effect the omission of $\boldsymbol{\Delta}$ has on the estimation of the parameters of interest, ($\mathbf{c}$,$\mathbf{B}$,$\mathbf{D}$).

The producer may also commit an optimization error, i.e. instead of choosing the optimal acreage vector $\mathbf{x}^*$, which would solve the optimization problem, he allocates $\mathbf{x}^* + \boldsymbol{\delta}_x$, which does not solve it, but satisfies the constraints. That kind of error would be impossible to distinguish from a pure measurement error on the side of the researcher, except that we would require $\mathbf{R}\boldsymbol{\delta}_x = \mathbf{0}$ (because $\mathbf{Rx}^* = \mathbf{v} = \mathbf{R}(\mathbf{x}^* + \boldsymbol{\delta}_x)$).

Since the general error model requires an increased amount of prior information and is anyway difficult to distinguish from the measurement error model, we choose to limit ourselves to measurement errors. We now proceed with explicit assumptions regarding the data sampling processes.

5.4. Augmented parameter vector and its prior distribution

In the ex-post perspective, the outcome $\mathbf{e}$ of the error vector $\boldsymbol{\varepsilon}$ has actually already been determined, but the outcome is not directly observable. We thus choose to consider $\mathbf{e}$ yet another unknown parameter. If the density function f for the random vector $\mathbf{Z}$ is conditional also on $\mathbf{e}$ and the yield and input parameters $\boldsymbol{\beta}$, and $\boldsymbol{\alpha}$

defined above, then there are no random components left, and f becomes the degenerate density function

$$f(\mathbf{z} \mid \boldsymbol{\psi}, \boldsymbol{\beta}, \boldsymbol{\alpha}, \mathbf{e}) = \begin{cases} 1: & \mathbf{z} = \boldsymbol{\mu}(\boldsymbol{\psi}, \boldsymbol{\beta}, \boldsymbol{\alpha}) + \mathbf{e},\ g(\boldsymbol{\psi}, \mathbf{x}^*, \boldsymbol{\lambda}^*) = 0 \\ 0: & \text{else} \end{cases}$$

One can immediately see that there must be a large number of vectors ($\boldsymbol{\psi}$,$\boldsymbol{\beta}$,$\boldsymbol{\alpha}$,$\mathbf{e}$) that give the density value “1” for almost any outcome $\mathbf{z}$ of $\mathbf{Z}$. Without further information, there is no way of discriminating between any two such vectors by saying that one is any more likely than the other to be the true parameter vector. This is why we require a prior distribution $\xi(\boldsymbol{\Psi},\mathbf{e},\boldsymbol{\alpha},\boldsymbol{\beta})$. In this section, we define the prior distribution based on the following assumptions, detailed below:

(i) $\xi(\boldsymbol{\Psi},\mathbf{e},\boldsymbol{\alpha},\boldsymbol{\beta}) = \xi(\mathbf{e})\xi(\boldsymbol{\lambda}^*(\boldsymbol{\Psi},\boldsymbol{\alpha},\boldsymbol{\beta}))\xi(\boldsymbol{\eta}(\boldsymbol{\Psi},\boldsymbol{\alpha},\boldsymbol{\beta}))$, with $\boldsymbol{\eta}(\boldsymbol{\Psi},\boldsymbol{\alpha},\boldsymbol{\beta})$ denoting the vector of implied own price supply elasticities. That is, we assign prior distributions to error terms, dual values and implied point price elasticities of supply, and assume that those are functionally independent.

(ii) The errors $\mathbf{e}$ are independent and normally distributed with standard deviations equal to a fix share of the observed value of the respective parameter.

(iii) The dual values are independent, with means proportional to average observed gross margins over all crops in each region each year, and standard deviations proportional to a fix share of that.

(iv) We believe that the parameter vector is such that the implied point price elasticity of supply matrix $\boldsymbol{\eta}(\boldsymbol{\Psi},\boldsymbol{\alpha},\boldsymbol{\beta})$ is normally distributed with mean depending on the crop mix (rotational shares) and standard deviation independent for each item. For non-diagonal elements of $\boldsymbol{\eta}$, the prior distribution is non-informative (i.e. we have no specific beliefs regarding cross price elasticities).

Regarding (ii): Specifically we assume that $\mathbf{e} \sim N(0,\boldsymbol{\Sigma}_e)$ with $\boldsymbol{\Sigma}_e$ a diagonal matrix with $\sigma_{ei}^2 = \left(0.20/3z_i\right)^2$ on the i^{th} position. This means that we assume that errors are independent normally distributed with mean zero covariance matrix such that three standard deviations cover 20% of the observed value of the related parameter.

Regarding (iii): In order for the posterior density to have a unique maximum, we require informative priors also for the dual values $\boldsymbol{\lambda}$ in order to be able to identify $\boldsymbol{\Psi}$ (since for example $\mathbf{c}$ and $\boldsymbol{\lambda}$ enter the first order conditions additively). We make the assumptions

$$\lambda_{1t} \sim N\left(0.25\overline{m}_t, \left(\frac{0.20}{3}0.25\overline{m}_t\right)^2\right)$$

$$\lambda_{2t} \sim N\left(0.25\left(m_{"OSET"t}(1-\rho_t)-\overline{m}_t\rho_t\right), \left(\frac{0.20}{3}0.25\left(m_{"OSET"t}(1-\rho_t)-\overline{m}_t\rho_t\right)\right)^2\right),$$

where $m_{"OSET"t}$ is the observed gross margin in compulsory set aside, $\overline{m}_t$ the average gross margin over all crops and ρ_t the general set-aside rate in period t. The prior mode (mean of normal distribution) of $\boldsymbol{\lambda}$ is thus based on the assumption that the expected land rent is approximately 25% of the average observed gross margin $\overline{m}_t$ in the respective year taken over all crops except sugar beet (whereas sugar quota rents are missing in the model). For the case study region FR24 this fits reasonably well with data on land rental prices obtained from Eurostat for France, shown in table 11. The priors for dual values of the set-aside constraint were derived in a similar manner, but also including. The variances of $\boldsymbol{\lambda}$ were assumed to be such that 20% of the prior means equal three standard deviations.

Table 11. Land rents in France (Euro per ha)

	Eurostat*	λ_1 prior	λ_2 prior
1986	102	86	
1987	104	132	
1988	106	94	
1989	109	113	
1990	111	134	
1991	113	119	
1992	115	101	-116
1993	117	91	-23
1994	119	124	-5
1995	121	175	-8
1996	122	163	-27
1997	125	148	11
1998	129	155	-98
1999	132	193	-6
2000	132	139	39
2001	131	145	74
2002	131	181	24

**Source: Eurostat (2003)*

Regarding (iv): There are cases when the observations imply a supply elasticity that is far outside any plausible range, e.g. > 1000. One case when this would happen is when the observations imply a downward sloping supply function, as in the case of rye in FR24 discussed in a previous section. Given the second order conditions for optimality, the best fit is obtained by a horizontal supply curve, implying an infinite elasticity. Such a simulation behaviour of the model is unacceptable, and we firmly believe that the aggregate supply response of regions in reality is much smoother. Put differently, we believe that the parameter vector comes from a distribution that makes such extreme values utterly improbable, but is rather indifferent for elasticities within some plausible range. For this purpose, we choose a very wide normal distribution, with mean and standard deviation derived below.

Most studies (see comparison to other studies below) find supply elasticities in the range of, say, 0.1 to 5. More specifically, we see that the elasticity is typically around unity for major crops, but that it is higher for crops that occupy a small share of the total area. One motivation for such a relation is that if a small crop expands with a certain percentage, that should have less effect on the value of fixed resources, like pushing other crops out of the rotation on the constrained land, compared to if a major crop expands by the same percentage.

Letting r_j denote the share of land allocated to crop j, we believe that the own price supply elasticities have means $0.5 r_j^{-1/3}$ and standard deviations such that three standard deviations cover 1000% of the mean (the standard deviation relative to mean is thus fifty times that of the acreages, prices or yields). There are no priors at all for cross price elasticities. In the result section below, the priors are compared to elasticities from literature for the Netherlands, Denmark and France, and found to be in a plausible range.

We will see that the explicit expression for supply elasticities is a nonlinear function of the parameters. That makes its inclusion into the estimation difficult. Jansson (2005) solves a similar model for supply elasticities and includes the expression explicitly in the estimation. His model, however, did not have area constraints, and imposed land constrain only implicitly over curvature constraints on the Hessian matrix, which simplified the expressions for supply elasticities considerable. Heckelei and Wolff (2003) makes a similar estimation but with invented data for a didactic size problem, with a simultaneous incorporation of elasticity priors. Here, we have two constraints in most years and only one constraint in some years (before set-aside regime), which complicates things further. The elasticities of supply in our model can be obtained by solving the first order conditions for $\mathbf{x}_t$ (repeated here for convenience),

$$\mathbf{x}_t^*(\mathbf{p}_t, \boldsymbol{\lambda}_t) = l_t^{-1}\left[\mathbf{D} + \mathbf{GBG}'\right]^{-1}\left[\mathbf{Y}_t\mathbf{p}_t + \mathbf{s}_t - \mathbf{A}_t\mathbf{w}_t - q_t\mathbf{c} - \mathbf{R}_t'\boldsymbol{\lambda}_t\right]. \tag{13}$$

Let $\mathbf{E}_t = l_t\left[\mathbf{D} + \mathbf{GBG}'\right]$ and insert that expression into the constraints to obtain a solution for $\boldsymbol{\lambda}$,

$$\boldsymbol{\lambda}_t^*(\mathbf{p}_t) = \left[\mathbf{R}_t\mathbf{E}_t^{-1}\mathbf{R}_t'\right]^{-1}\left[\mathbf{R}_t\mathbf{E}_t^{-1}\left(\mathbf{Y}_t\mathbf{p}_t + \mathbf{s}_t - \mathbf{A}_t\mathbf{w}_t - q_t\mathbf{c}\right) - \mathbf{v}_t\right]. \tag{14}$$

Computing $\mathbf{x}^*_t(\mathbf{p}_t,\boldsymbol{\lambda}^*_t(\mathbf{p}_t))$ by inserting (14) into (13), taking derivatives and multiplying the result by yield gives us the following expression for marginal production[8]:

$$\frac{\partial(\mathbf{Y}_t\mathbf{x}_t)}{\partial\mathbf{p}_t} = \mathbf{Y}_t\left(\mathbf{E}_t^{-1}\mathbf{Y}_t - \mathbf{E}_t^{-1}\mathbf{R}_t'\left[\mathbf{R}_t\mathbf{E}_t^{-1}\mathbf{R}_t'\right]^{-1}\mathbf{R}_t\mathbf{E}_t^{-1}\mathbf{Y}_t\right) \tag{15}$$

Using the definition of elasticity, we finally obtain the expression

$$\boldsymbol{\eta}_t = \mathbf{X}_t^{-1}\left(\mathbf{E}_t^{-1}\mathbf{Y}_t - \mathbf{E}_t^{-1}\mathbf{R}_t'\left[\mathbf{R}_t\mathbf{E}_t^{-1}\mathbf{R}_t'\right]^{-1}\mathbf{R}_t\mathbf{E}_t^{-1}\mathbf{Y}_t\right)\mathbf{P}_t \tag{16}$$

where upper case $\mathbf{X}_t$ means the square diagonal matrix with $\mathbf{x}_t$ on the diagonal, and similar for upper case $\mathbf{P}_t$.

This expression is strongly non-linear in **D** and **B** (via **E**) and thus difficult to include as constraint in the estimation. In some models, the expression has been simplified by neglecting the second term in the bracket and only computing diagonal elements in **E**. That simplification was previously used in different model to compute only diagonal elements of the quadratic PMP-parameter, e.g. in the CAPRI model (not published), and by Helming (2005) in the DRAM model.

Nevertheless, with appropriate initialisation of the solution algorithm (CONOPT for GAMS) together with reasonable bounds for the variables, equation 16 turns out to be possible to solve simultaneously in the estimation, thus enabling us to include our prior beliefs regarding elasticities of supply in a transparent way.

5.5. Definition of the estimator

Putting all the pieces together, we can now formulate the estimation problem using Bayes' theorem as above and write

$$\hat{\boldsymbol{\Psi}} = \arg\max \xi(\boldsymbol{\psi},\boldsymbol{\beta},\boldsymbol{\alpha},\mathbf{e}|\mathbf{z}) \propto f(\mathbf{z}|\boldsymbol{\psi},\boldsymbol{\beta},\boldsymbol{\alpha},\mathbf{e})\xi(\boldsymbol{\psi},\boldsymbol{\beta},\boldsymbol{\alpha},\mathbf{e})$$

To repeat, the point estimate of $(\boldsymbol{\psi},\boldsymbol{\beta},\boldsymbol{\alpha},\mathbf{e})$ that we are looking for is the value that maximises the posterior density $\xi(\boldsymbol{\psi},\boldsymbol{\beta},\boldsymbol{\alpha},\mathbf{e}|\mathbf{z})$, i.e. the posterior mode. Note that with the degenerate density function this is equivalent to solving

$$\begin{aligned} &\max \quad \xi(\boldsymbol{\psi},\boldsymbol{\beta},\boldsymbol{\alpha},\mathbf{e}) \\ &\text{subject to} \quad \mathbf{z} = \boldsymbol{\mu}(\boldsymbol{\psi},\boldsymbol{\beta},\boldsymbol{\alpha}) + \mathbf{e} \\ &\qquad \mathbf{g}(\boldsymbol{\psi},\mathbf{x}^*,\boldsymbol{\lambda}^*) = \mathbf{0} \end{aligned}$$

[8] In this case, the marginal production could be solved for directly. In the general case with continuous derivatives, the implicit function theorem may be used instead.

Since the value that maximises some function h also maximises log (h), we may take the logarithm of the objective function (which is a multivariate normal density function with covariance matrix $\boldsymbol{\Sigma}$). Doing that and replacing the constraints with the equations derived above, we arrive at the following extremum estimation problem:

minimise

$$\operatorname{vec}\left(\mathbf{e}_x, \mathbf{e}_Y, \mathbf{e}_p, \mathbf{e}_w, \mathbf{e}_A, \left(\boldsymbol{\lambda} - \boldsymbol{\lambda}^{prior}\right), \left(\operatorname{diag}(\mathbf{v}) - \hat{\mathbf{v}}\right)\right)'$$
$$\times \boldsymbol{\Sigma}_{total}^{-1} \operatorname{vec}\left(\mathbf{e}_x, \mathbf{e}_Y, \mathbf{e}_p, \mathbf{e}_w, \mathbf{e}_A, \left(\boldsymbol{\lambda} - \boldsymbol{\lambda}^{prior}\right), \left(\operatorname{diag}(\mathbf{v}) - \hat{\mathbf{v}}\right)\right)$$

subject to

$$\mathbf{Y}_t\mathbf{p}_t + \mathbf{s}_t - \mathbf{A}_t\mathbf{w}_t - q_t\mathbf{c} - l_t\left[\mathbf{D} + \mathbf{GBG}'\right]\mathbf{x}_t - \mathbf{R}_t'\boldsymbol{\lambda}_t - MAC_t\delta = \mathbf{0}$$

$$\mathbf{R}_t\mathbf{x}_t = \mathbf{v}_t$$

$$\mathbf{B} = \mathbf{U}'\mathbf{U}$$

$d_{jj} \geq 0$ for $j = 1\ldots J$ (and $d_{ij} = 0$ for $i \neq j$)

$$\mathbf{x}^{obs} = \mathbf{x} + \mathbf{e}_x$$

$$\mathbf{Y}^{obs} = \mathbf{Y} + \mathbf{e}_Y$$

$$\mathbf{Y}_t = \boldsymbol{\beta}_0 + \boldsymbol{\beta}_1 T_t$$

$$\mathbf{p}_t = \mathbf{p}_{t-1}^{obs} + (\mathbf{p}_t^{adm} - \mathbf{p}_{t-1}^{adm}) + \mathbf{e}_{pt}$$

$$\mathbf{A}^{exp} = \mathbf{A} + \mathbf{e}_A$$

$$\mathbf{A} = \boldsymbol{\alpha}_0 + \boldsymbol{\alpha}_1 T_t$$

$$\nu_t = \mathbf{X}_t^{-1}\left(\mathbf{E}_t^{-1}\mathbf{Y}_t - \mathbf{E}_t^{-1}\mathbf{R}_t'\left[\mathbf{R}_t\mathbf{E}_t^{-1}\mathbf{R}_t'\right]^{-1}\mathbf{R}_t\mathbf{E}_t^{-1}\mathbf{Y}_t\right)\mathbf{P}_t$$

The dummy variable MAC_t with associated parameter δ in the first order condition was added to control for additional effects of the MacSharry reform. It is equal to 1 for year 1992 and earlier for regions that were member of the EU then, and zero from 1993 and on. This is motivated by an optical inspection of the time series. For example, looking again at the gross margin and acreages of rye if figure 1 suggests that there are two clouds of observations, which correspond to pre- and post MacSharry reform (1993). Thus the reform is likely to have influenced behaviour in some way not captured in the present model (the situation is similar for some other products).

5.6. Data preparations

The time series in the CAPRI database is different long for different crops even within regions. It also contains holes and obvious errors, especially for crops of residual character like “other cereals”, or when the area cropped is very small compared to other crops in the region. Thus, the estimations require data to be processed prior to estimation in order to make sure that no obvious data errors corrupt the estimations, we must select a strategy for choosing which regions, crops and years to include in estimation, and we must decide what to do with zeros in the data.

Selection of crops: A potentially different set of crops were estimated in each region. To start with, all acreages smaller than 1000 ha were set to zero. Then, the crops to be estimated were those satisfying all of the following three conditions: (1) There is acreage data in year 2000, (2) there is acreage data in at least five years, and (3) the sum of acreage over all years is at least 10 000 ha.

Selection of years: A year t was included in the estimation if the total acreage over all crops just selected was at least 10 000 ha in year t-1. The lag is necessary for the lagged prices to work. The longest possible time series was 1986 to 2003.

Selection of regions: A region was included in the estimation if the following three conditions were satisfied: (1) Year 2000 was included in the set of years to estimate for that region, (2) the set of crops to estimate contain at least three elements, and (3) the number of observations over all crops and years is at least 50. The number of regions to estimate determined in this way turned out to be 165.

Treatment of outliers: Outliers for prices, yields and input coefficients were detected and replaced with time series mean using the following procedure:

Do for i = 1,2

1. Compute mean $\bar{z}$ using all but the greatest and the smallest value.
2. If not ($ai \le z_t \le b/i$), then replace z_t with $\bar{z}$

where a and b are constants. The replacement was done twice, and with narrower bounds in the second repetition in order to alleviate the problem that the presence of two outliers biases the mean. Trial and error revealed that (a,b) = (0.1,6.0) worked fine for prices, (0.2,4.0) for yields and (0.25,4.0) for input coefficients.

Unbalanced panels: In the cases where some time series were shorter than the others, it was assumed that this was really due to missing data, perhaps truncated by the “1000 ha rule”, not that the data truly was zero (except in the case of the “political activities” compulsory set-aside and non-food production on set-aside). Then, the estimator was allowed to choose any value satisfying the equation system as the estimate, but the item did not enter the posterior density function. Since consecutive years are interlinked via the other parameters (yield, input requirement, PMP terms), this does not generally cause any problems. In most regions where some time series was shorter than the other, it was early years that were missing, which are of lesser importance for the intended use of the estimates.

6. Results

The estimation produced a large number of results: 1917 elements of the key parameters **c** and **D** respectively, and 5457 elements of the cross group effects matrix **B**. Furthermore, 329 092 price elasticities were computed, including the cross price elasticities. To this comes a very large number of fitted values and all other parameters in **Ψ**. It is impossible to give even an overview of all those results, and in this section we only present estimation results for the French case study region FR24 and for France as a whole. The results are evaluated following two criteria:

1. How well is the prior information recovered? To address this, a kind of R^2 measure is computed as the share of the explained variance observations or prior mode. In an appendix, we also provide a visual presentation of prior and posterior mode for selected items (plots).

2. How is the resulting model behaving in simulation? We discuss our estimated point price supply elasicities and compare them to estimates from literature.

6.1. Measures of fit

Table (12) shows the share of explained variation, R^2, for acreages, prices and yields for all land use activities in FR24. We see that in most cases, the fit of acreage is high, above 0.90. Exceptions are soft wheat, potatoes, sugar beet and voluntary set-aside. Only the last of those crops has an R^2 below 0.50 (0.393). The fit of prices is equally high in general. The fit of yields is lower because here a more restrictive error model is employed: the expected yields have to lie on a straight line (12). In four cases, the fit of yield is even negative. One can see on the plots in the appendix that the yields of those crops are highly variable.

Table 12. R^2 for acreages (X), prices (P) and yields(Y)

Crop	Item	N	R2	Crop	Item	N	R2
SWHE	P	18	0.928	PULS	X	18	0.907
DWHE	P	18	0.820	POTA	X	18	0.649
RYEM	P	18	0.927	SUGB	X	18	0.805
BARL	P	18	0.791	MAIF	X	18	0.987
OATS	P	18	0.915	OFAR	X	18	0.938
MAIZ	P	18	0.794	NONF	X	11	0.999
OCER	P	18	0.935	OSET	X	12	1.000
RAPE	P	18	0.923	VSET	X	14	0.393
SUNF	P	18	0.932	SWHE	Y	18	0.291
PULS	P	18	0.838	DWHE	Y	18	0.235
POTA	P	18	0.964	RYEM	Y	18	0.673
SUGB	P	18	0.455	BARL	Y	18	0.179
MAIF	P	18	0.716	OATS	Y	18	0.012
OFAR	P	18	0.685	MAIZ	Y	18	0.657
NONF	P	18	0.948	OCER	Y	18	-0.030
SWHE	X	18	0.591	RAPE	Y	18	-0.164
DWHE	X	18	0.995	SUNF	Y	18	0.234
RYEM	X	18	0.998	PULS	Y	18	-0.027
BARL	X	18	0.977	POTA	Y	18	0.490
OATS	X	18	0.997	SUGB	Y	18	0.717
MAIZ	X	18	0.909	MAIF	Y	18	-0.086
OCER	X	18	0.988	OFAR	Y	18	0.428
RAPE	X	18	0.979	NONF	Y	11	0.891
SUNF	X	18	0.934				

Source: Own estimations.

6.2. Elasticities

The point price elasticities of supply are computed simultaneous in the estimations by equation (16). In this section we present elasticities for individual crops and for crop groups for one selected subregion, FR24, and for the aggregate France, all in the year 2002. The aggregation from the 22 French regions estimated and whole of France was done by weighing the regional elasticities with the region's share of national crop area, or

$$\sum_r \eta_{rj} x_{rj} \Big/ \sum_r x_{rj}$$

Aggregation to crop groups was done similarly, by weighing with each crop's share in the crop group to which it belongs. The crop groups are the same that were used in the estimation, reported in table (16). Table (18) and (19) shows the elasticities of individual crops for FR24 and France respectively. Table (20) and (21) show the elasticities of the crop groups. Some of the elasticities, especially for individual crops of minor land share on regional level, are high. This is true for e.g. rye and durum wheat, which both have elasticities above 7 and small rotational shares. In contrast, soft wheat has the moderate elasticity of 0.79 for a land share of 36%; however, there are notable exceptions. Potatoes has a rotational share of only 0.36%, but only an elasticity of 0.38.

As one might expect, the crop groups generally show less elasticity to price changes than the individual crops. This is partly due to the land restriction, but also to the crop group structure of the model, that allows catching substitution effects between related crops. The most notable case for FR24 is perhaps oil seeds. In table (18) we see that rapeseed and sunflower are good substitutes, but table (20) reveals inelastic supply response as a group.

Aggregation from regions to the member state offers no great surprises. Most of the elasticities are of similar size at national as on regional level in the case studied. The greatest difference is for durum wheat, where the elasticity in FR24 is much higher than that in the member state aggregate. One reason for not finding greater differences between the region and the aggregate is probably that the rotational shares in the region are similar to those on national level.

Although there are several studies that present elasticities on national level, no other study that the author is aware of publishes elasticities for individual crops on regional level with this crop coverage. Below we compare our point elasticity estimates as well as our priors with four other studies. Two of the other studies are for France, one study is for the Netherlands and one is for Denmark. In all comparisons, we use our point price elasticities for the year 2002.

For France, we can compare our results to those in Heckelei and Britz (2000) (HB00) and Guyomard et al. (1996) (GBC96). This has been done in table (13), where also the land share and prior mode are printed. GBC96 estimates a model with seven outputs and three inputs based on a restricted profit function, using annual data for France. HB00 estimate a similar model as ours, but they use a cross-section data set of French regions for the year 1994 instead of time series for individual regions as we do.

We see that GBC96 finds considerable smaller elasticities for barley (0.35) and other coarse grains (0.76) than this study (2.24 and 2.53), HB00 (2.65 for barley) or the priors (1.11 and 1.55). For soft wheat the results are much more in line, with the priors (0.77) quite close to GBC96 (0.72) and the estimates (1.01) in between GBC96 and HB00 (1.32). For maize the estimates (1.68) are close to GBC96 (1.63) but much higher than HB00 (0.65), whereas the priors lie in between (1.07). Rapeseed and sunflower occupy small rotational shares, less than 5%, and as a consequence the priors are higher, about 1.5. The elasticity estimates for those crops are also much higher, 1.28 and 2.96, than GBC96, which finds values of 0.42 and 0.22, and more in line with HB00, which finds elasticities greater than unity. All of the three studies find high elasticities for soya, for which the rotational share is less than 0.5%.

Table 13. Comparison with other studies of own price supply elasticities in France

Crop	Land share[b]	Prior[c]	Own estimate	GBC96[d]	HB00[e]
Other coarse grains[a]	0.034	1.547	2.531	0.758	-.---
Soft wheat	0.273	0.771	1.009	0.715	1.322
Maize	0.102	1.070	1.680	1.630	0.653
Barley	0.092	1.109	2.243	0.351	2.647
Rapeseed	0.045	1.405	1.284	0.418	1.457
Sunflower	0.027	1.664	2.959	0.223	1.126
Soya	0.004	3.276	2.020	3.701	1.861

a: Aggregated from rye, oats and other cereals.
b: Computed from the data in CAPRI for 2002
c: Using the formula for priors reported above
d: Guyomard et al. (1996)
e: Heckelei and Britz (2000)

For the Netherlands, Oude Lansink and Peerlings (1996) (OLP96) estimate twelve farm type models producing three outputs (CO = Cereals and oilseeds, Rootcrops = Potatoes and sugar beet, and Other = all other crops). They estimate the model using panel data on individual farms, and also have a land constraint and a fixed area of rootcrops. In their table A3 they present supply elasticites, of which the own price effects are compared to our estimates for the Netherlands for similar product aggregates in table (14). To make the comparison, our individual crop elasticities have been aggregated with estimated planned rotational shares for 2002. The "other crops" aggregate in OLP96 could not be formed, since we have three crops, (voluntary and compulsory set-aside and fallow land) for which there is no output price.

Our estimates for CO (0.94) are quite close to OLP96 (0.90), but considerably higher for root crops (OLP96 find 0.34, our estimate 0.91). We must then keep in mind that in OLP96, the area used in root crops was fixed, so that the price elasticity can come only from a change in intensity. It then seems reasonable that their estimates for that aggregate turn out lower.

Table 14. Comparison with other own price supply elasticity estimates for the Netherlands

Crop group	Land share	Prior	Own estimate	OLP96[a]
CO	0.266	0.778	0.937	0.90
Root crops	0.342	0.715	0.909	0.24

Oude Lansink and Peerlings (1996)

Jensen (1996) estimates an econometric model of Danish agriculture, and also presents aggregated supply elasticities for three selected crop groups. In table (15) we have reprinted those elasticities and also our implied estimates for the corresponding aggregates. We see that for the first two groups, our elasticities are higher than those in ibid., though our prior for cereals is similar to the estimate in ibid. For the last group, root crops, the elasticities are very similar and more than twice as high as our prior.

Table 15. Comparison with other own price supply elasticity estimates for Denmark

Crop	Land share	Prior	Own estimate	Jensen (1996)
Cereals	0.575	0.601	1.073	0.60
Pulses + rapeseed	0.037	1.498	1.999	0.66
Root crops	0.035	1.522	3.772	3.80

6.3. Complete results and estimation program

The GAMS program and the data that were used for performing the estimations, and all the results are available in a compressed archive from the publications page of the website of the Institute for Food and Resource Economics, Bonn University (http://www.ilr.uni-bonn.de/publ/dispap_e.htm). The top level GAMS program file is ESTNLP.GMS, and all results are found in the GAMS data exchange file RESULTS.GDX. New results are written at the end of the program to EVAL.GDX. The input data used is stored in TIMESERIES02.GDX. That file was extracted from the CAPRI database “Warsaw version” using the GAMS program GETDATA.GMS, which is also found in the archive. The directory IMG contains plots of fit similar to those found in the appendix to this text, but for all regions.

6.4. Conclusive remarks

No confidence regions for the estimates are established. Exact analytical confidence regions are very difficult to deduce. Approximations would in theory be possible. Reilly and Patino-Leal (1981) compute approximate probability contours of the posterior in a non-linear errors-in-variables model by iterated linearisations. In our case, analytical deduction of approximate confidence regions is more difficult than in ibid. due to the curvature constraints. Numerical computation by Monte Carlo simulations is not feasible because of the amount of computation time required with the present setup (several hours for a single simulation of all regions).

We conclude that the estimated elasticities compare well with estimates in the four cases from literature studied. Nevertheless, only a handful elasticities from three member states could be compared. The vast amount of estimates are for individual crops in NUTS2 regions, and for them, we have nothing to compare to. Some of those elasticities appear high, e.g. rye and durum wheat in FR24 (table 18). Such parameter settings will result in a model that reacts strongly on shocks in simulation compared to the current CAPRI model that in the past had inelastic supply. However, the high elasticities are most often found for crops with small rotational shares, where an elastic response is sensible.

With repeated future applied analyses with the full CAPRI modelling system and the new parameters, experiences will be gained regarding the performance of the estimates.

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Appendix 1. Activities and inputs in estimation

Table 16. Crop groups and activities modelled

Group	Description	Crop	Description
CERE	Cereals	SWHE	Soft wheat
		DWHE	Durum wheat
		RYEM	Rye
		BARL	Barley
		OATS	Oats
CER2	Cereals2	MAIZ	Maize
		OCER	Other cereals
OILS	Oil seeds	RAPE	Rapeseed
		SUNF	Sunflower
		SOYA	Soya
		OOIL	Other oilseeds
		NONF	Ind. rapeseed
OARA	Other arable crops	POTA	Potatoes
		PULS	Pulses
		SUGB	Sugar beet
		TEXT	Fibre crops
FARA	Fodder on arable land	MAIF	Fodder maize
		OFAR	Silage grass
		ROOF	Fodder root crops
NOCR	Non-yield crops	OSET	Obligatory set-aside
		VSET	Voluntary set-aside
		FALL	Fallow land

Table 17: Inputs in estimation

Seed	Repairs buildings	Fuel
Plant protection	Electricity	Lubricants
Fertilize	Gas for drying	Other inputs
Repairs machinery		

Appendix 2: Supply elasticity estimates in France

Table 18. Supply elasticities for FR24 in year 2002 for individual crops.

	Share	SWHE	DWHE	RYEM	BARL	OATS	MAIZ	OCER	RAPE	SUNF	PULS	POTA	SUGB	MAIF	OFAR	NONF
SWHE	36.17%	**0.786**	-0.127	-0.018	-0.280	-0.027	-0.061	-0.007	0.003	0.016	0.032	0.000	0.002	-0.016	-0.055	0.039
DWHE	1.80%	-2.226	**7.913**	-0.134	-2.037	-0.199	-0.441	-0.049	0.022	0.115	0.234	0.001	0.016	-0.118	-0.397	0.283
RYEM	0.29%	-3.366	-1.392	**7.733**	-3.079	-0.301	-0.667	-0.073	0.034	0.174	0.354	0.001	0.024	-0.179	-0.600	0.427
BARL	12.04%	-0.860	-0.356	-0.052	**2.261**	-0.077	-0.171	-0.019	0.009	0.045	0.090	0.000	0.006	-0.046	-0.153	0.109
OATS	1.08%	-2.320	-0.959	-0.140	-2.122	**2.903**	-0.460	-0.051	0.023	0.120	0.244	0.001	0.017	-0.123	-0.413	0.294
MAIZ	7.27%	-0.237	-0.098	-0.014	-0.217	-0.021	**3.168**	-0.261	-0.109	-0.559	-0.965	-0.001	-0.020	0.155	0.742	0.064
OCER	1.93%	-0.233	-0.096	-0.014	-0.213	-0.021	-2.334	**2.074**	-0.107	-0.549	-0.949	-0.001	-0.019	0.152	0.729	0.063
RAPE	9.18%	0.012	0.005	0.001	0.011	0.001	-0.109	-0.012	**1.659**	-1.265	0.043	0.000	0.003	-0.014	-0.033	-0.066
SUNF	4.94%	0.134	0.056	0.008	0.123	0.012	-1.214	-0.134	-2.751	**4.059**	0.480	0.001	0.036	-0.151	-0.366	-0.738
PULS	2.89%	0.465	0.192	0.028	0.425	0.042	-3.568	-0.392	0.159	0.817	**2.225**	-0.040	-1.264	-0.302	-1.434	-0.117
POTA	0.38%	0.001	0.000	0.000	0.001	0.000	-0.002	0.000	0.000	0.001	-0.031	**0.384**	-0.001	0.000	-0.001	-0.001
SUGB	1.22%	0.017	0.007	0.001	0.016	0.002	-0.040	-0.004	0.006	0.033	-0.683	-0.001	**3.083**	-0.003	-0.021	-0.015
MAIF	1.36%	-0.711	-0.294	-0.043	-0.650	-0.064	1.728	0.190	-0.151	-0.775	-0.913	0.000	-0.015	**6.590**	-6.639	0.135
OFAR	6.90%	-0.366	-0.151	-0.022	-0.335	-0.033	1.274	0.140	-0.056	-0.289	-0.666	-0.001	-0.018	-1.020	**2.108**	-0.056
NONF	1.43%	1.658	0.685	0.100	1.516	0.148	0.695	0.076	-0.723	-3.708	-0.344	-0.003	-0.083	0.132	-0.355	**3.944**
OSET	6.29%	-0.289	-0.120	-0.017	-0.265	-0.026	-0.246	-0.027	0.181	0.930	0.139	-0.001	-0.035	0.077	-0.135	-0.917
VSET	0.82%	-3.982	-1.646	-0.240	-3.643	-0.356	-3.266	-0.359	-0.563	-2.886	1.564	0.002	0.073	-0.125	-0.127	0.619
FALL	4.01%	-1.044	-0.432	-0.063	-0.955	-0.093	-0.857	-0.094	-0.129	-0.661	0.412	0.001	0.016	-0.025	-0.044	0.081

Table 19. Supply elasticities for France in year 2002 for individual crops.

	Share	SWHE	DWHE	RYEM	BARL	OATS	MAIZ	OCER	RAPE	SUNF	PULS	POTA	SUGB	MAIF	OFAR	NONF
SWHE	26.84%	**1.009**	-0.056	-0.010	-0.397	-0.048	-0.091	-0.006	-0.003	-0.011	-0.001	-0.010	-0.006	-0.029	-0.020	-0.090
DWHE	1.84%	-0.766	**2.102**	-0.054	-0.480	-0.072	-0.132	-0.013	0.004	0.004	0.001	0.029	-0.002	-0.014	-0.022	-0.230
RYEM	0.16%	-3.276	-1.086	**8.577**	-2.818	-0.970	-0.939	-0.397	-0.018	-0.055	-0.013	0.074	0.092	0.003	-0.115	-1.240
BARL	9.01%	-1.322	-0.112	-0.028	**2.243**	-0.113	-0.199	-0.032	-0.027	-0.035	-0.001	-0.007	-0.004	-0.023	-0.028	-0.144
OATS	1.14%	-2.202	-0.241	-0.133	-1.666	**2.884**	-0.391	-0.126	-0.015	-0.036	-0.005	0.002	0.021	-0.031	-0.059	-0.440
MAIZ	10.04%	-0.195	-0.022	-0.006	-0.129	-0.018	**1.680**	-0.285	-0.020	0.043	0.011	-0.160	-0.024	-0.143	0.020	-0.314
OCER	2.03%	-0.122	-0.018	-0.022	-0.181	-0.050	-2.384	**2.205**	0.016	-0.003	0.009	-0.047	-0.002	-0.043	0.009	-0.566
RAPE	4.44%	-0.011	0.003	0.000	-0.049	-0.001	-0.054	0.006	**1.284**	-0.539	-0.011	0.087	-0.008	0.061	-0.098	-0.404
SUNF	2.67%	-0.076	0.007	-0.001	-0.097	-0.006	0.084	-0.001	-1.016	**2.959**	-0.042	0.358	-0.008	0.087	-0.181	-1.727
PULS	0.35%	-0.058	0.008	-0.003	-0.034	-0.009	0.482	0.047	-0.196	-0.373	**2.020**	0.066	-0.056	0.002	-0.050	-1.443
POTA	2.40%	-0.130	0.044	0.003	-0.014	0.001	-0.993	-0.005	0.180	0.458	0.010	**2.113**	-0.234	-0.712	-0.200	-1.812
SUGB	0.89%	-0.020	0.000	0.002	-0.004	0.004	-0.065	0.000	-0.006	-0.006	-0.003	-0.099	**1.210**	-0.023	0.000	0.059
MAIF	2.40%	-0.114	-0.003	0.000	-0.026	-0.002	-0.205	-0.008	0.030	0.023	0.000	-0.172	-0.018	**2.434**	-0.043	-0.144
OFAR	7.74%	-0.089	-0.009	-0.002	-0.043	-0.006	0.051	0.002	-0.071	-0.071	-0.002	-0.076	-0.001	-0.063	**1.304**	-1.114
NONF	18.08%	-0.195	-0.039	-0.009	-0.102	-0.022	-0.203	-0.059	-0.121	-0.287	-0.036	-0.237	0.033	-0.064	-0.417	**2.059**
OSET	2.02%	1.329	0.146	0.025	0.785	0.092	0.897	0.286	-0.586	-1.196	-0.083	0.381	-0.250	0.209	-0.127	-1.973
VSET	5.17%	-0.442	-0.043	-0.006	-0.256	-0.027	-0.241	-0.092	0.279	0.603	0.048	-0.062	0.076	-0.163	0.206	0.124
FALL	1.15%	-1.355	-0.141	-0.026	-0.824	-0.071	0.012	0.030	-0.208	-0.430	-0.010	0.292	-0.018	-0.001	-0.664	-1.499

Note: Numbers in parentheses from Heckelei and Britz (2000 table 2), in square brackets from Guyomard et al. (1996 table 2).

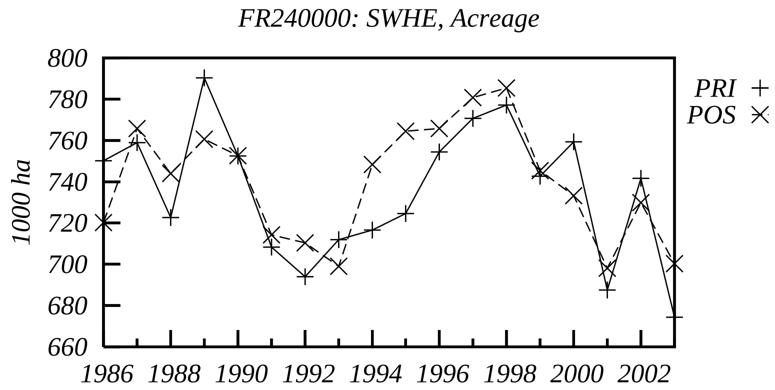
Table 20. Supply elasticities for FR24 for crop groups in 2002.

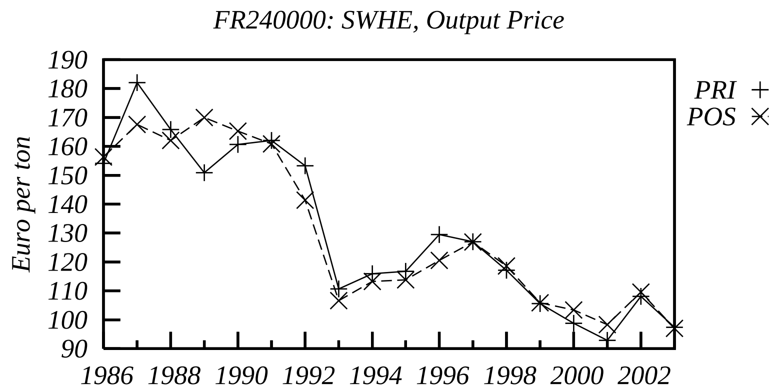
	CERE	CER2	OILS	OARA	FARA
CERE	**0.509**	-0.124	0.107	0.064	-0.131
CER2	-0.666	**2.554**	-0.685	-1.118	1.017
OILS	0.489	-0.418	**0.321**	0.148	-0.207
OARA	0.727	-2.473	0.540	**1.228**	-1.085
FARA	-0.999	1.428	-0.444	-0.691	**0.861**
NOCR	-2.066	-0.765	-0.354	0.334	-0.076

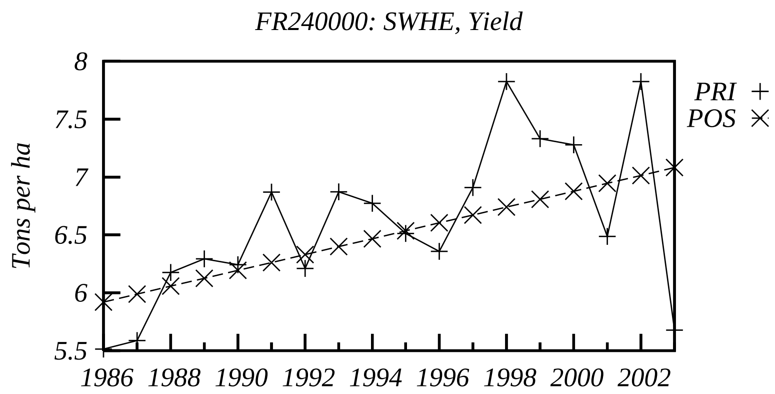
Table 21. Supply elasticities for France for crop groups in 2002.

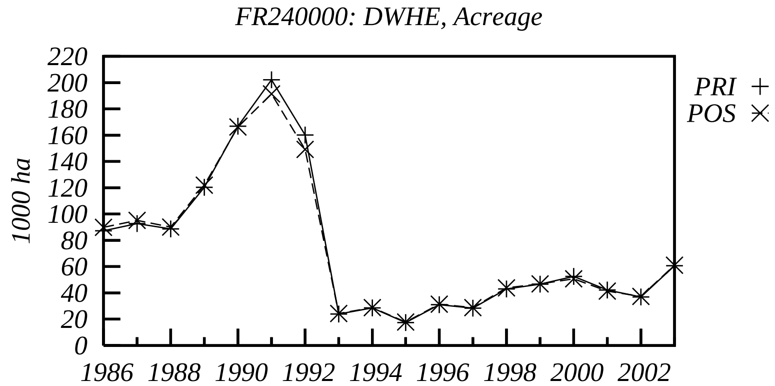
	CERE	CER2	OILS	OARA	FARA
CERE	**0.508**	-0.152	0.046	-0.038	-0.151
CER2	-0.395	**1.220**	0.076	-0.343	-0.343
OILS	0.352	0.209	**0.807**	0.240	-1.042
OARA	-0.138	-0.543	0.402	**1.623**	-0.895
FARA	-0.299	-0.167	-0.428	-0.231	**1.201**
NOCR	-1.273	-0.360	-0.353	-0.127	-0.656

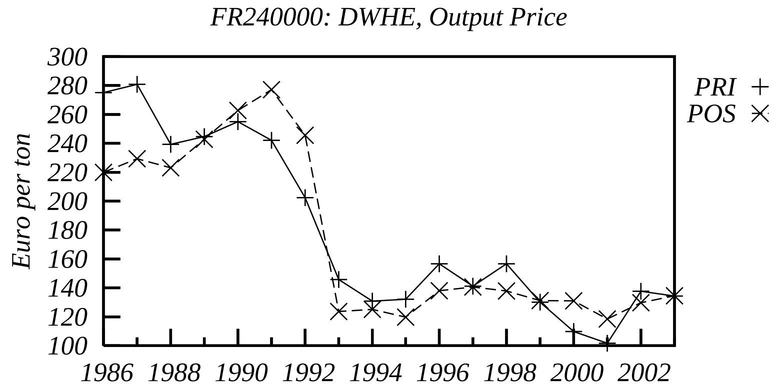
Appendix 3:Plots of prior versus posterior mode for FR24

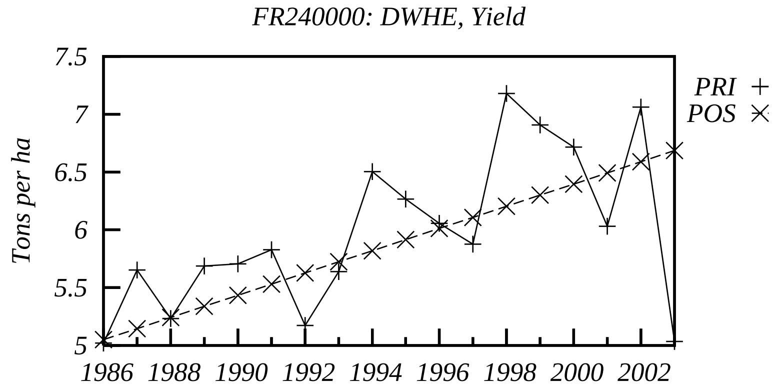


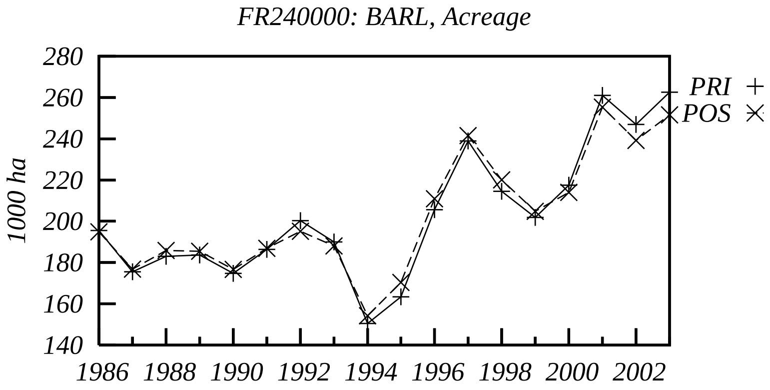





FR240000: DWHE, Acreage
1000 ha
220
200
180
160
140
120
100
80
60
40
20
0
1986 1988 1990 1992 1994 1996 1998 2000 2002
PRI
POS


FR240000: DWHE, Output Price
Euro per ton
300
280
260
240
220
200
180
160
140
120
100
1986 1988 1990 1992 1994 1996 1998 2000 2002
PRI
POS


FR240000: DWHE, Yield
Tons per ha
7.5
7
6.5
6
5.5
5
1986 1988 1990 1992 1994 1996 1998 2000 2002
PRI
POS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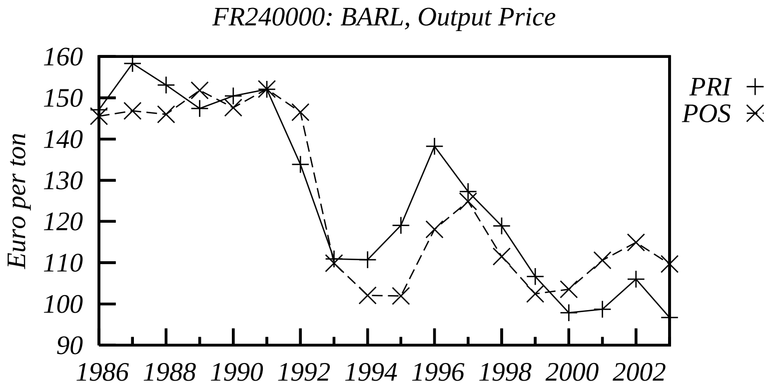
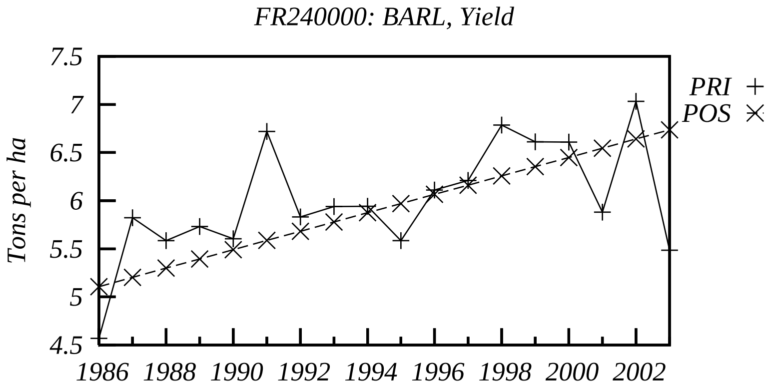
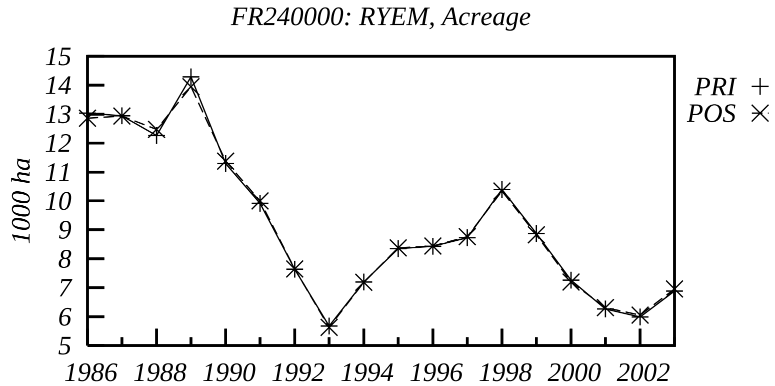
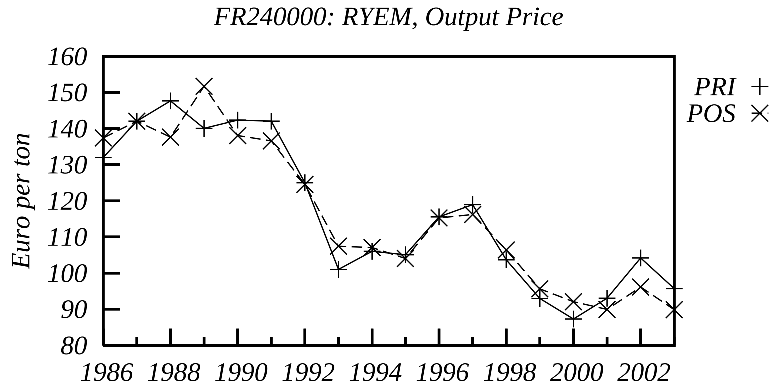
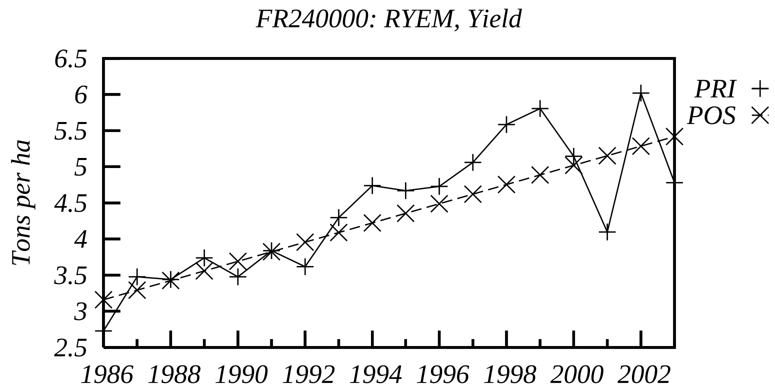
1000 ha
280
260
240
220
200
180
160
140
1986 1988 1990 1992 1994 1996 1998 2000 2002
PRI
POS

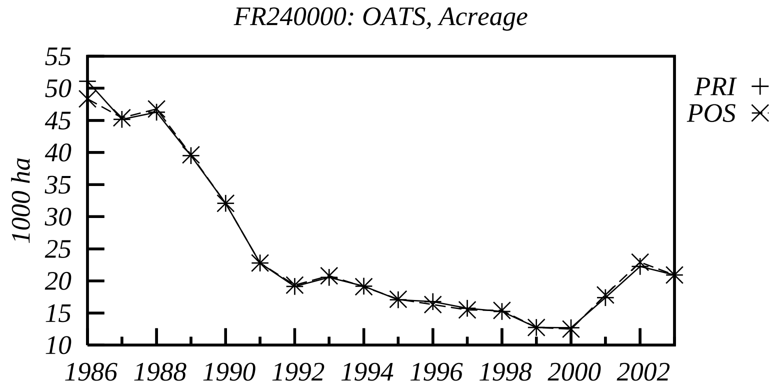

FR240000: BARL, Output Price
Euro per ton
160
150
140
130
120
110
100
90
1986 1988 1990 1992 1994 1996 1998 2000 2002
PRI
POS

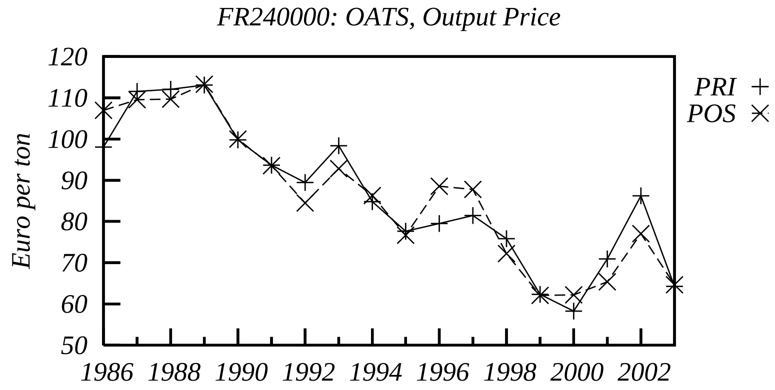

FR240000: BARL, Yield
Tons per ha
7.5
7
6.5
6
5.5
5
4.5
1986 1988 1990 1992 1994 1996 1998 2000 2002
PRI
POS

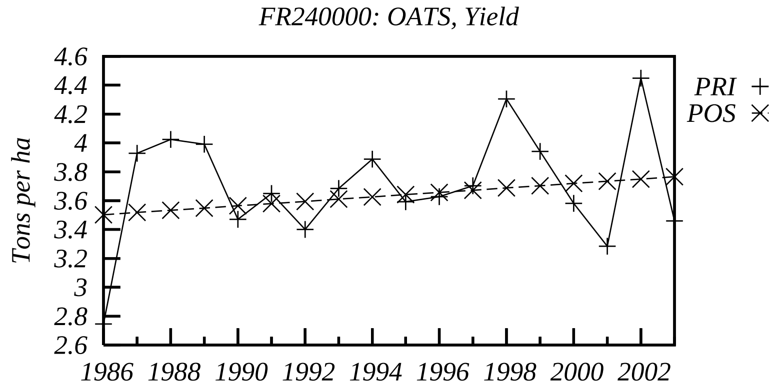

FR240000: RYEM, Acreage
1000 ha
PRI
POS

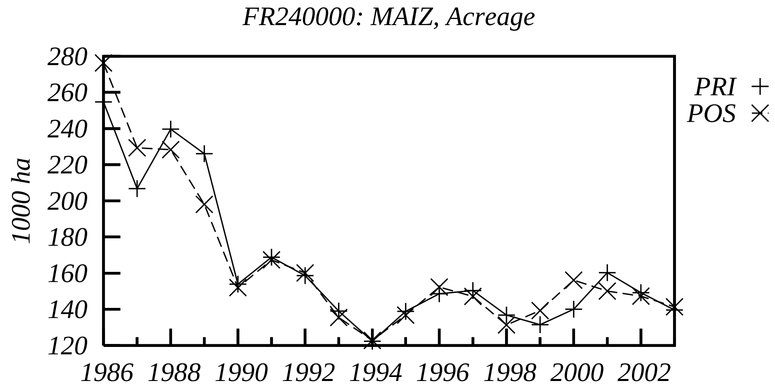

FR240000: RYEM, Output Price
Euro per ton
PRI
POS

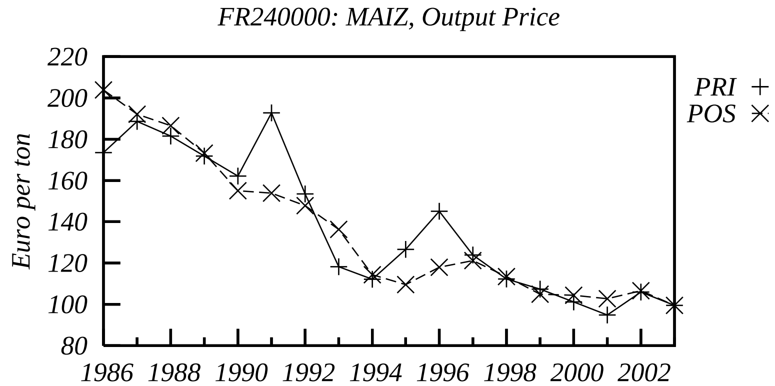

FR240000: RYEM, Yield
Tons per ha
PRI
POS

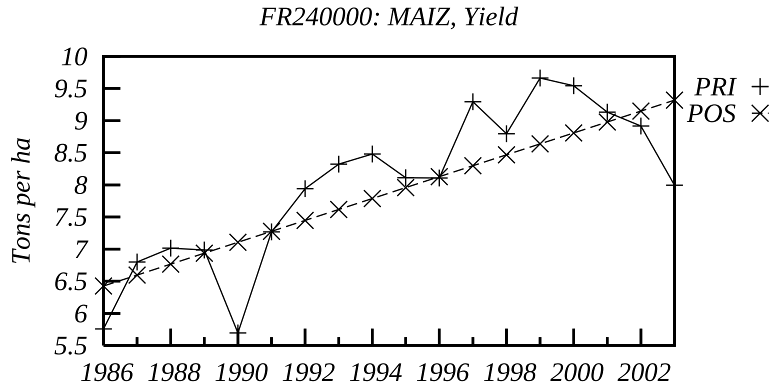

FR240000: OATS, Acreage
1000 ha
55
50
45
40
35
30
25
20
15
10
1986 1988 1990 1992 1994 1996 1998 2000 2002
PRI
POS

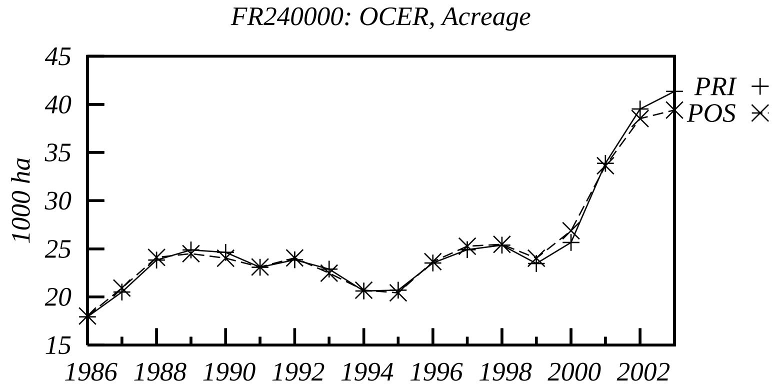

FR240000: OATS, Output Price
Euro per ton
120
110
100
90
80
70
60
50
1986 1988 1990 1992 1994 1996 1998 2000 2002
PRI
POS

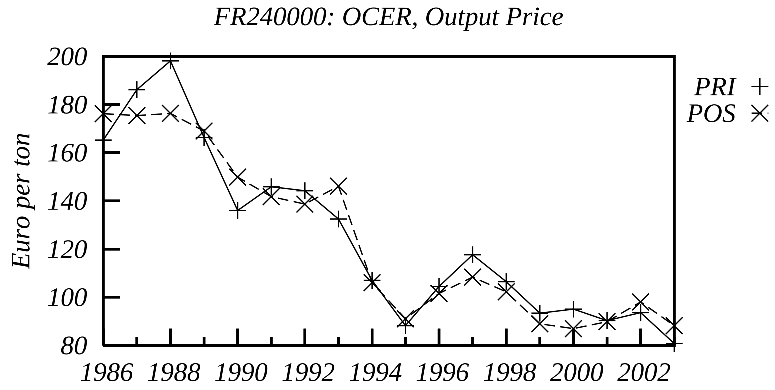

FR240000: OATS, Yield
Tons per ha
4.6
4.4
4.2
4
3.8
3.6
3.4
3.2
3
2.8
2.6
1986 1988 1990 1992 1994 1996 1998 2000 2002
PRI
POS

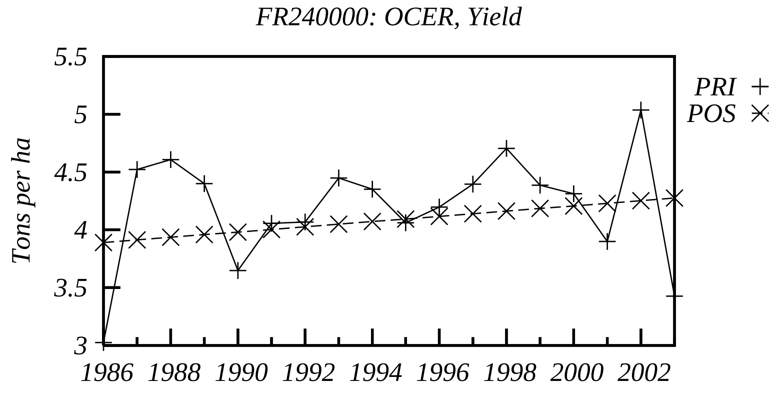

FR240000: MAIZ, Acreage
1000 ha
280
260
240
220
200
180
160
140
120
1986 1988 1990 1992 1994 1996 1998 2000 2002
PRI
POS


FR240000: MAIZ, Output Price
Euro per ton
220
200
180
160
140
120
100
80
1986 1988 1990 1992 1994 1996 1998 2000 2002
PRI
POS

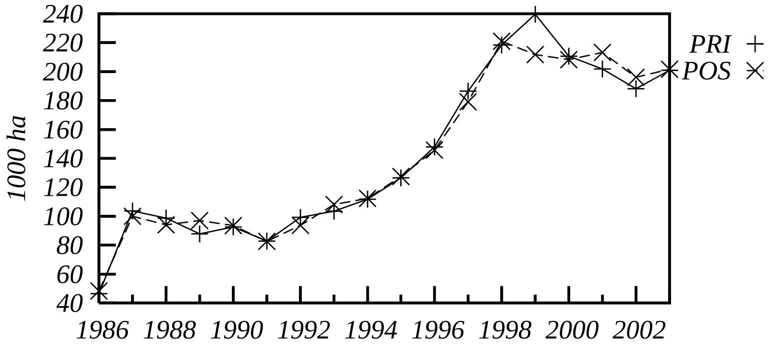

FR240000: MAIZ, Yield
Tons per ha
10
9.5
9
8.5
8
7.5
7
6.5
6
5.5
1986 1988 1990 1992 1994 1996 1998 2000 2002
PRI
POS


FR240000: OCER, Acreage
1000 ha
45
40
35
30
25
20
15
1986 1988 1990 1992 1994 1996 1998 2000 2002
PRI +
POS ×

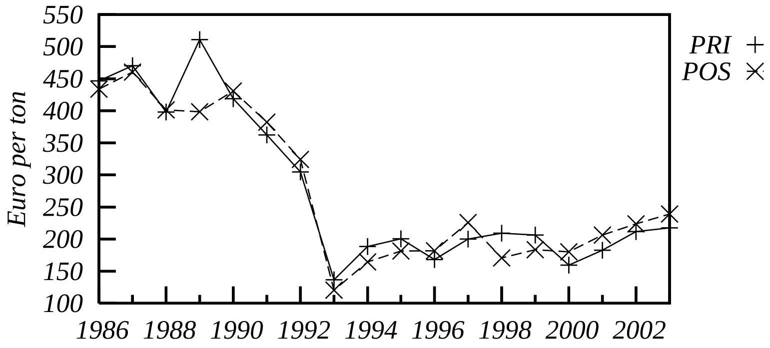

FR240000: OCER, Output Price
Euro per ton
200
180
160
140
120
100
80
1986 1988 1990 1992 1994 1996 1998 2000 2002
PRI +
POS ×


FR240000: OCER, Yield
Tons per ha
5.5
5
4.5
4
3.5
3
1986 1988 1990 1992 1994 1996 1998 2000 2002
PRI +
POS ×

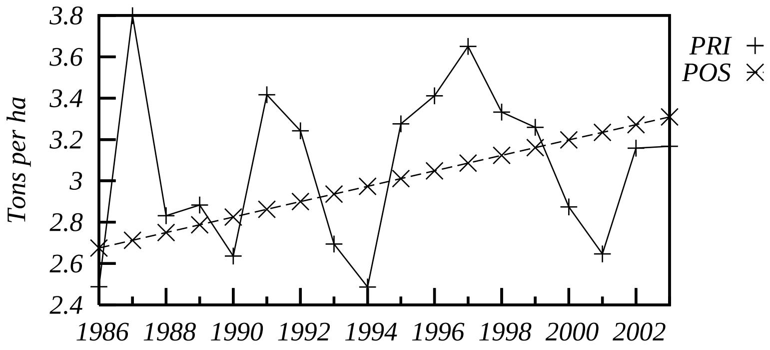
FR240000: RAPE, Acreage



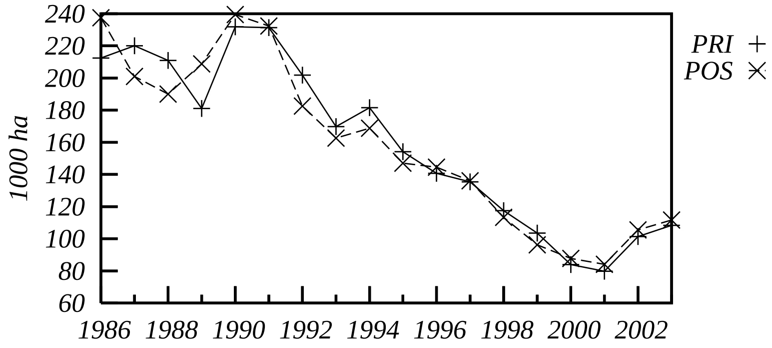
FR240000: RAPE, Output Price



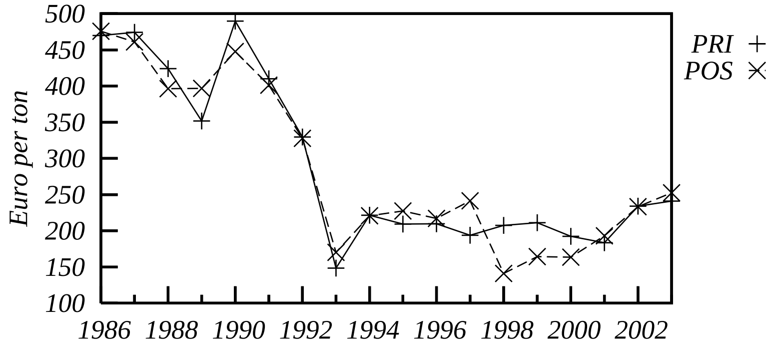
FR240000: RAPE, Yield



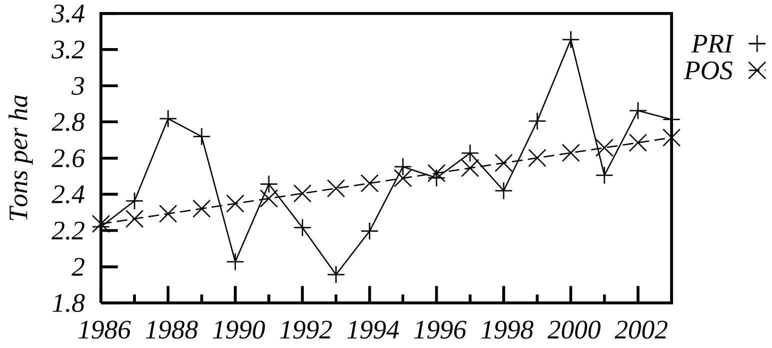
FR240000: SUNF, Acreage



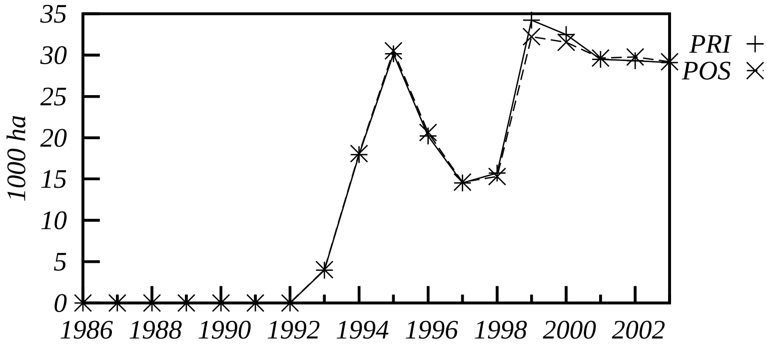
FR240000: SUNF, Output Price



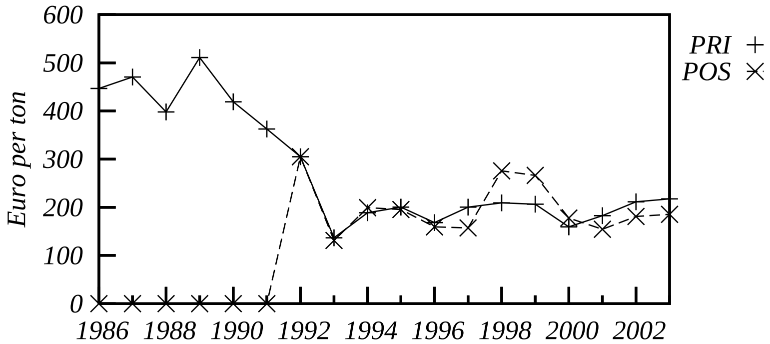
FR240000: SUNF, Yield



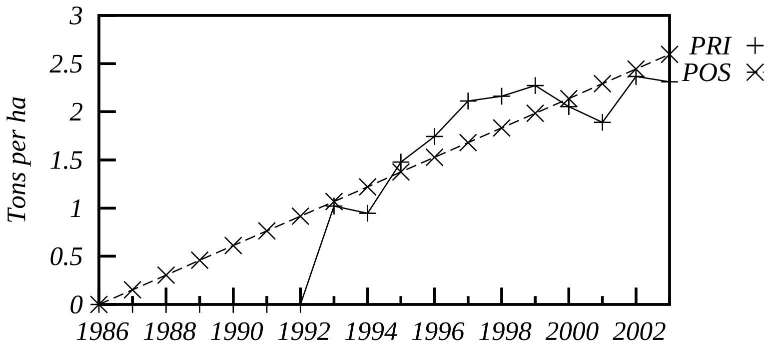
FR240000: NONF, Acreage



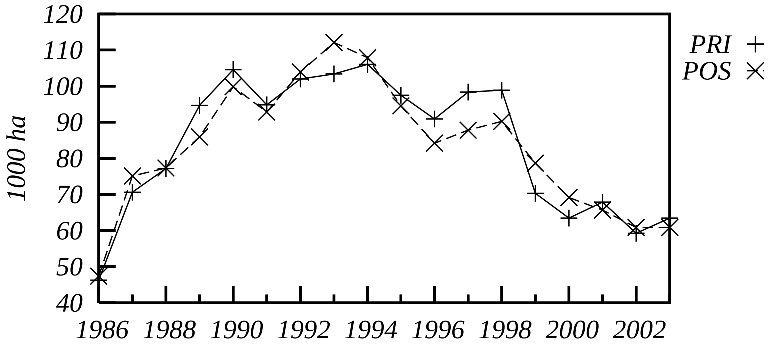
FR240000: NONF, Output Price



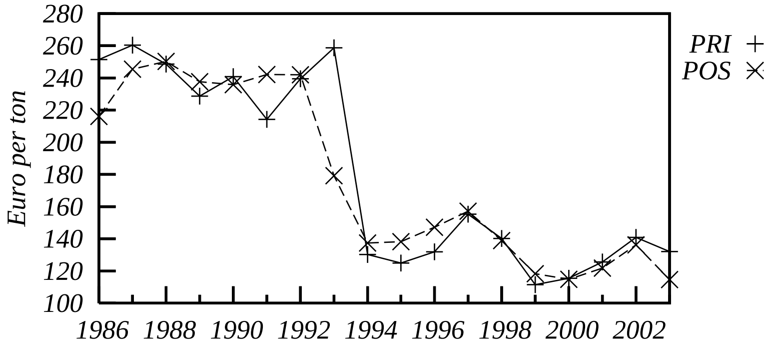
FR240000: NONF, Yield



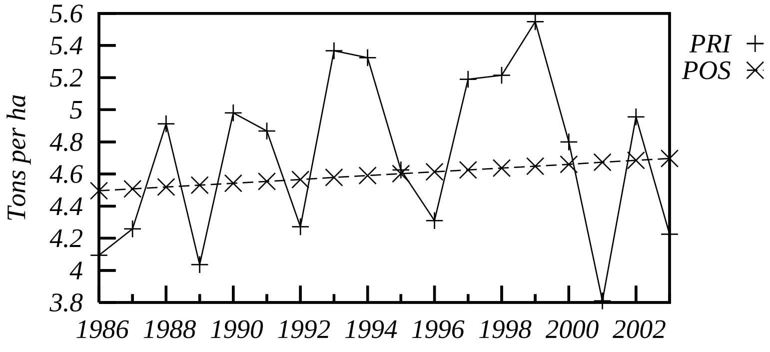
FR240000: PULS, Acreage

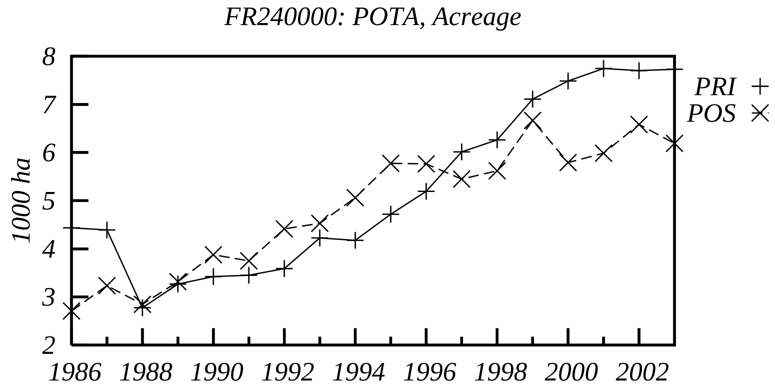


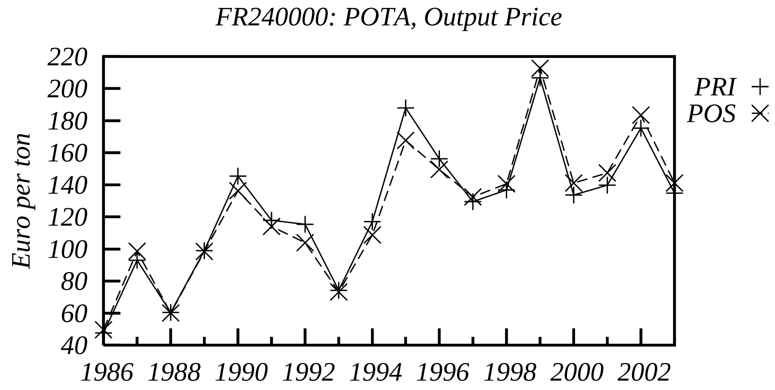
FR240000: PULS, Output Price

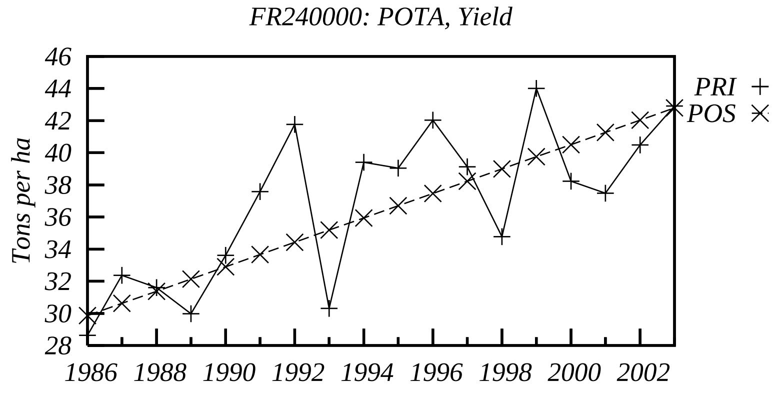


FR240000: PULS, Yield

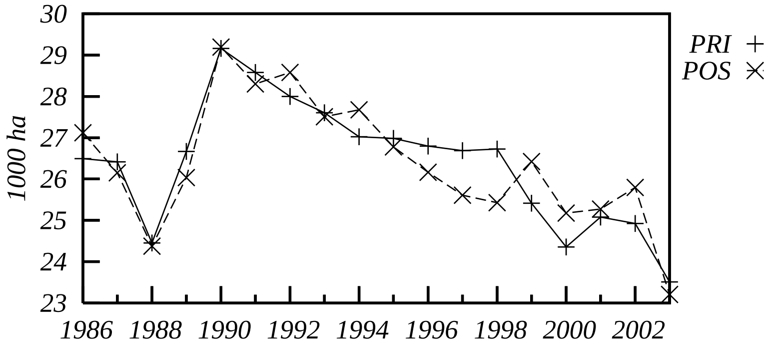



FR240000: POTA, Acreage
1000 ha
8
7
6
5
4
3
2
1986 1988 1990 1992 1994 1996 1998 2000 2002
PRI
POS

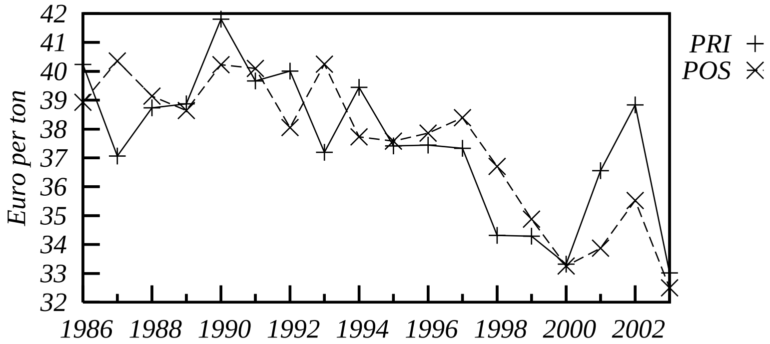

FR240000: POTA, Output Price
Euro per ton
220
200
180
160
140
120
100
80
60
40
1986 1988 1990 1992 1994 1996 1998 2000 2002
PRI
POS


FR240000: POTA, Yield
Tons per ha
46
44
42
40
38
36
34
32
30
28
1986 1988 1990 1992 1994 1996 1998 2000 2002
PRI
POS

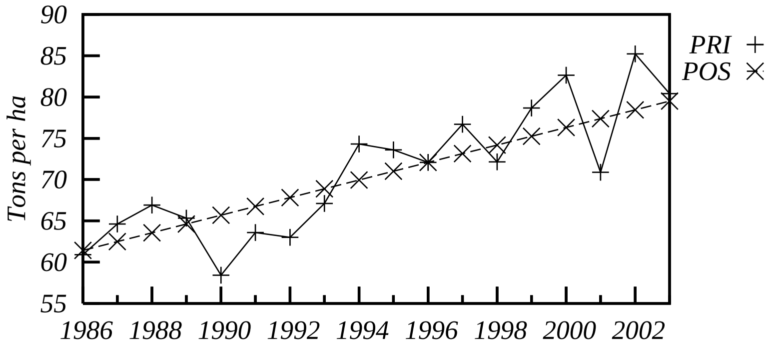
FR240000: SUGB, Acreage

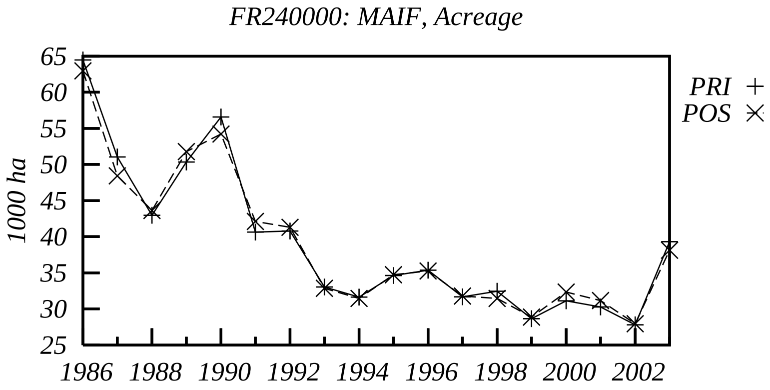


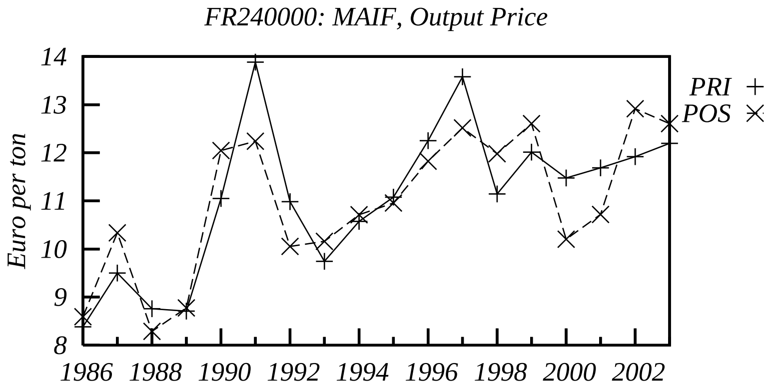
FR240000: SUGB, Output Price

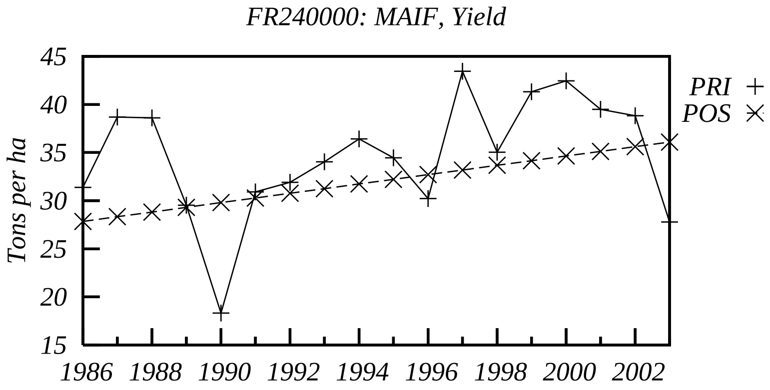


FR240000: SUGB, Yield

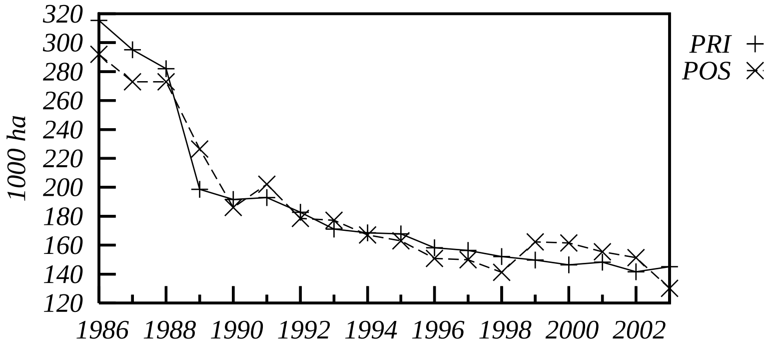



FR240000: MAIF, Acreage
1000 ha
65
60
55
50
45
40
35
30
25
1986 1988 1990 1992 1994 1996 1998 2000 2002
PRI
POS

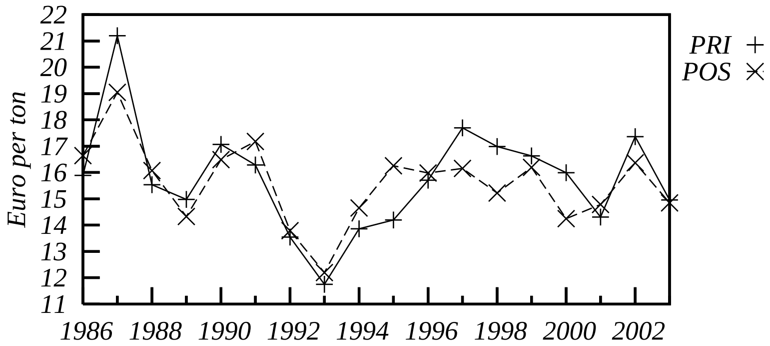

FR240000: MAIF, Output Price
Euro per ton
14
13
12
11
10
9
8
1986 1988 1990 1992 1994 1996 1998 2000 2002
PRI
POS


FR240000: MAIF, Yield
Tons per ha
45
40
35
30
25
20
15
1986 1988 1990 1992 1994 1996 1998 2000 2002
PRI
POS

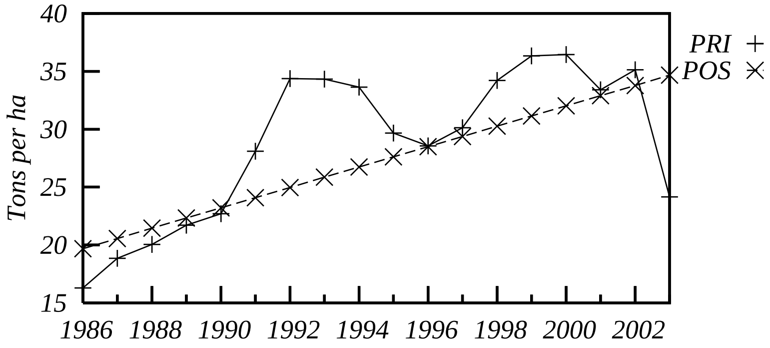
FR240000: OFAR, Acreage



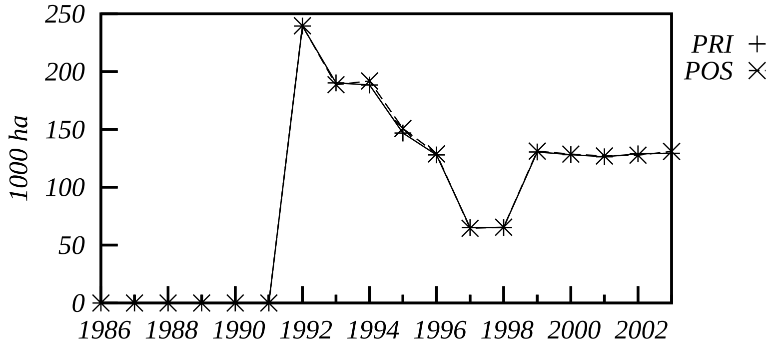
FR240000: OFAR, Output Price



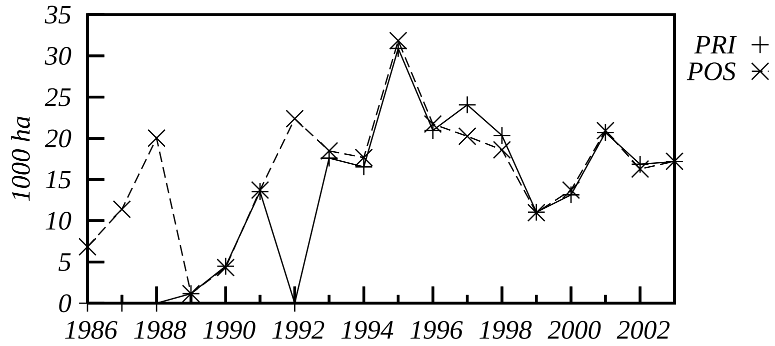
FR240000: OFAR, Yield



FR240000: OSET, Acreage



FR240000: VSET, Acreage



FR240000: FALL, Acreage