The Business Cycle Effects of Seasonal Shocks*

Yi Wen
Department of Economics
Cornell University
Ithaca, NY 14853
yw57@cornell.edu

November 1999

Abstract

Conventional wisdom emphasizes supply and demand shocks as the major sources of the business cycle. Yet the most visible, most synchronized, and most frequently encountered supply and demand shocks take place at the seasons. The central question to be addressed in this paper is to what extent impulses at the seasonal frequency are responsible for business cycles? Despite strong empirical evidence suggesting that seasonal fluctuations and business cycle fluctuations are closely related, questions like this are difficult to answer because of the lack of effective methods for identifying seasonal versus non-seasonal shocks. Traditional methods of measurement and identification (such as the use of seasonal dummies to isolate the seasonal and non-seasonal components) are inadequate and inappropriate because they fail to take into account the possible interactions between seasonal fluctuations and business-cycle fluctuations. In this paper, we develop a procedure that allows us to identify seasonal shocks versus non-seasonal shocks. We found that seasonal shocks account for the bulk of business-cycle fluctuations in US output (roughly 50%). The finding suggests that models relying heavily on technology shocks to explain the business cycle are miss-specified, that using seasonally adjusted data to evaluate business cycle models can lead to incorrect conclusions, and that theories of the business cycle, as well as government policies concerning the business cycle ought to address seasonal fluctuations seriously.

* I thank Jeff Miron, Karl Shell, and especially, Peter Pedroni, as well as seminar participants at Cornell, Penn State and SUNY Albany for comments. I also thank Joseph Beaulieu for providing the seasonally unadjusted data.

What shocks are responsible for economic °uctuations? Conventional wisdom emphasizes supply and demand shocks as the major sources of the business cycle. Yet the most visible, most synchronized, and most frequently encountered supply and demand shocks take place at the seasons. Most research on the business cycle, however, has worked only with seasonally adjusted data. Underlying this practice is the view that business °uctuations are generated by a fundamentally di®erent mechanism than seasonal °uctuations. Such views have been questioned by a series of empirical studies (e.g., Barsky and Miron, 1989, Beaulieu, MacKie-Mason, and Miron, 1992, Beaulieu and Miron, 1992, Miron, 1994, 1996, and 1998, to name just a few). These authors documented that the two types of °uctuations display striking similarities. In particular, with respect to most stylized facts about the business cycle, the seasonal cycle displays the same characteristics as the business cycle, in many cases even more dramatically than the business cycle. Further more, countries and industries with large seasonal cycles also have large business cycles.

An important distinction, however, must be made between two fundamental problems regarding the business cycle: the propagation mechanism and the impulse mechanism. The propagation mechanism pertains to the endogenous properties of the business cycle, while the impulse mechanism pertains to the exogenous properties of the business cycle. To trigger the business cycle, exogenous impulses are needed to activate the economy's propagation mechanism so that recurrent °uctuations at the macroeconomic level become visible. Two distinct sets of questions, therefore, are involved in understanding the relationship between business cycle °uctuations and seasonal °uctuations. First, do they share the same underling propagation mechanism? Second, do they share the same sources of impulse? The primary interest of this paper is to address the second question.

Business cycles have characteristic frequencies (e.g., the spectrum of US GDP growth has maximum power centered around the 4-6 year cycle frequency).[1] If the economy is a dynamic system with endogenous propagation mechanism that determines the characteristic frequencies of the business cycle, then any external forces ought to be able to trigger dynamic responses from the system at those characteristic frequencies determined by it's internal propagation mechanism.[2] The seasonal impulses are simply one particular kind of those forces which regu-

[1] See Mark Watson (1993).

[2] The post-war US investment to output ratio provides one of the most striking evidence of such kind of endogenous propagation mechanism that generates dampened oscillations (e.g., Y. Wen, 1998).

larly hit the economy with an exogenously determined frequency – the seasonal frequency. Thus, we should expect that under the influence of seasonal impulses, the economy may exhibit not only fluctuations at the exogenously determined frequency (the seasonal frequency), but also fluctuations at the endogenously determined frequency (the characteristic business cycle frequency).[3]

The intriguing and fundamental question to ask then is, to what extent impulses at the seasonal frequency are responsible for the observed business cycles? In other words, besides the conventional supply and demand shocks, what proportion of the aggregate fluctuations at the business cycle frequency is due to seasonal shocks? Questions like these are fundamental because if seasonal shocks do cause business cycles, then theories that rely so heavily on technology shocks to explain the business cycle are flawed. Further more, using seasonally adjusted data to test or evaluate business cycle models may lead to incorrect conclusions because seasonal adjustment does not handle the interaction between seasonal fluctuations and business cycle fluctuations properly.

Unfortunately, despite strong empirical evidence suggesting that seasonal fluctuations and business cycle fluctuations are closely related, questions like these are difficult to answer because of the lack of effective methods for identifying seasonal versus non-seasonal shocks. Consequently, it is not possible to assess the contributions of seasonal shocks to the business cycle. Traditional methods of measurement and identification (such as the use of seasonal dummies to isolate the seasonal and non-seasonal components in a time series) are inadequate and inappropriate because they fail to take into account the possible interactions between seasonal fluctuations and business-cycle fluctuations.

This paper proposes a procedure to identify seasonal and non-seasonal shocks. The method provides a more sensible way of measuring the seasonal components of a time series than traditional methods, hence making it possible to correctly assess the contributions of seasonal shocks to the business cycle. To illustrate, suppose there are two types of aggregate impulses, one is seasonal and the other

[3] An example often encountered in engineering is a mechanical system with internal vibration frequency (such as a spring). If the system is subject to an external vibrational force, then oscillations at two different frequencies will be observed. One is determined by the system's internal structure, and the other determined by the external periodic force.

In the case of economic models, however, perfectly anticipated exogenous movements may have very different impact compared to unanticipated shocks. Seasonal impulses contain both anticipated and unanticipated factors (e.g., the Christmas is an anticipated seasonal event, but the magnitude of Christmas spending differs in each year in a random way). Seasonal shocks pertain to the random component of seasonal impulses.

non-seasonal,[4] and that innovations in the two types of impulses are orthogonal to each other. Identification can then be achieved by imposing the restriction that the non-seasonal innovations have minimal contributions to seasonal fluctuations in the time series. This identifying assumption is based on the understanding that seasonal cycles are exogenous, and are caused primarily by impulses that take place regularly at the seasons (such as the Christmas holiday or the winter). Therefore, non-seasonal shocks (e.g., the conventional business cycle shocks) should have little responsibility for the seasonal cycle. The orthogonality assumption, on the other hand, is based on the understanding that most business cycle shocks such as technological innovations, oil price crises, or unexpected monetary policy changes are non-seasonal and are independent of the season.[5]

With these identifying assumptions, we can then uniquely decompose a time series (seasonally unadjusted) into two components, one pertains to the seasonal disturbances, the other pertains to the non-seasonal disturbances. By comparing the partial spectra of the two components around the business-cycle frequency, we can then answer the question of how important seasonal shocks are to the business cycle.

Our econometric method is a generalized version of the method proposed by Blanchard and Quah (B-Q, 1989). We generalize the B-Q method in the frequency domain so that any identifying restrictions on dynamic impulse response functions can be viewed as restrictions on spectral density functions at a partic-

[4] Here, seasonal impulses are defined as stochastic processes with seasonal cycles. For example, the following random variable S_t is one type of seasonal impulses:

$$S_t = \rho_1 S_{t-1} + \rho_2 S_{t-2} + \rho_3 S_{t-3} + \varepsilon_t$$

where ε_t is an $i.i.d$ innovation. The assumption that seasonalities can be modeled (or approximated) as indeterministic processes is a subtle one and may be controversial. But it simplifies our analysis a lot, because deterministic processes do not have well-defined spectral density functions. Extending our method to allow for both deterministic and indeterministic seasonal cycles is left to future researches.

[5] This does not exclude the possibility that monetary policy is seasonal. For example, since 1914, the Fed has adopted the policy of accommodating seasonal fluctuations in money demand in order to smooth the nominal interest rates. Such kind of seasonality in money supply, however, is an endogenous response to seasonal shocks, not the shocks themselves.

On the other hand, even if there are innovations in the monetary policy that are seasonal, they should then be identified as seasonal shocks. Since the primary question of this paper is to understand how important those seasonal shocks are to the business cycle, our identifying assumptions serve as a benchmark for further investigations (relaxing the orthogonality assumption is something worth pursuing in the future). Answers to the above question, hopefully, will shed light on the legitimacy of using seasonally adjusted data to study the business cycle, as if seasonal fluctuations are irrelevant to the business cycle.

ular set of frequencies. For example, the long-run restriction imposed by B-Q to identify the transitory demand shock is a restriction imposed on the spectrum of output growth at frequency zero. Our generalized method, which renders the B-Q method as a special case, identifies seasonal shocks and the business cycle effects of seasonal shocks by imposing restrictions at the seasonal frequency.

Applying the procedure to the US economy, we found that seasonal shocks have substantial contributions to the business cycle components of many US aggregates. For example, about 50% of the variance of GDP growth at the business cycle frequency is due to seasonal shocks. And similar results hold also for aggregate consumption and aggregate investment. Such empirical findings explain Cochrane's failure to find large, identifiable, exogenous shocks in seasonally adjusted data to account for the bulk of business cycle fluctuations (J. Cochrane, 1994). They also suggest that theories of the business cycle, as well as government policies concerning the business cycle ought to address the seasonal shocks seriously.[6]

In what follows, we describe our identification method in Section 2. Section 3 shows how the method can be applied to studying the business cycle effects of seasonal shocks. Section 4 provides an economic model that rationalizes the identifying assumptions adopted in Section 3. Section 5 reports the estimated business cycle effects of the seasonal shocks in the US economy. Finally, Section 6 concludes the paper.

1. A Generalized Identification Method

Let x_t be a stationary time series with moving average representation:

$$x_t = a_1(L)\varepsilon_{1t} + a_2(L)\varepsilon_{2t}; \quad var(\varepsilon) = I; \tag{1}$$

where $a_1(L) \equiv \sum_{j=0}^{\infty} a_{1j} L^j$, $a_2(L) \equiv \sum_{j=0}^{\infty} a_{2j} L^j$, and ε_{1t} and ε_{2t} are orthogonal $i.i.d$ structural shocks whose economic properties remain to be specified. We are interested in recovering this presentation from the data.

Since x_t is stationary, it has a Wold-moving average representation:

$$x_t = b_1(L)v_{1t} + b_2(L)v_{2t}; \quad var(v) = \Sigma; \tag{2}$$

which can be uniquely estimated from the data using a bivariate VAR (we'll deal with the choice of the second variable later). Since the covariance matrix Σ is not

[6] Chatterjee and Ravikumar (1992), and Braun and Evans (1995) evaluate equilibrium business cycle models with regard to seasonal cycles.

diagonal in general, in order to recover presentation (1) from the data, we need to find a mapping:

$$v = A_0 \varepsilon;$$

(where A_0 is a 2×2 real matrix with full rank) so that (2) can be written as:

$$x_t = \begin{bmatrix} b_1(L) & b_2(L) \end{bmatrix} A_0 \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}:$$

Given A_0, the structural representation can be completely recovered from the data under the identity,

$$\begin{bmatrix} a_1(L) & a_2(L) \end{bmatrix} = \begin{bmatrix} b_1(L) & b_2(L) \end{bmatrix} A_0;$$

and the identity

$$\varepsilon = A_0^{-1} v:$$

Economic data will impose the following identifying restrictions on A_0:

$$A_0 A_0' = \Sigma: \tag{3}$$

As was pointed out by Blanchard and Quah (1989), however, information in Σ is not sufficient for uniquely identifying the four elements of A_0 (since Σ is symmetric). We need one extra assumption regarding the properties of A_0 in order to exactly identify it.

The assumption is often made with regard to the dynamic effects of the structural shocks. Blanchard and Quah (1989) assumes that shock 1 (ε_1) has no long-run effect on x:

$$\begin{bmatrix} b_1(1) & b_2(1) \end{bmatrix} A_0 \begin{bmatrix} \varepsilon_{1t} \\ 0 \end{bmatrix} = 0: \tag{4}$$

This implies:

$$[A_0]_{11} b_1(1) + [A_0]_{21} b_2(1) = 0; \tag{5}$$

where $[A_0]_{ij}$ indicates the ijth element in A_0. Obviously, conditions (5) and (3) uniquely determine A_0.

From the point of view of spectral analysis, what identifying restriction (4) amounts to is to assert that shock 1 (ε_1) has minimum contribution to the variance of x at frequency zero. To see what this implies in general, consider the spectral representation of specification (1) (the power spectrum of x)[7]:

$$\begin{aligned} f(e^{-i\omega}) &= \left[a_1(e^{-i\omega}) \quad a_2(e^{-i\omega}) \right] \left[a_1(e^{i\omega}) \quad a_2(e^{i\omega}) \right]' \\ &= \left| a_1(e^{i\omega}) \right|^2 + \left| a_2(e^{-i\omega}) \right|^2 ; \end{aligned} \tag{6}$$

where $|a(e^{-i\omega})|^2 \equiv a(e^{-i\omega})a(e^{i\omega})$: This shows that the power spectrum of x can be decomposed into two components at each frequency ω, the first one measures the independent contribution of ε_1 to the power spectrum of x, and the second one measures the independent contribution of ε_2 to the power spectrum of x.

Similarly, the spectral representation of the data (equation 2) is given by:

$$\begin{aligned} f(e^{-i\omega}) &= \left[b_1(e^{-i\omega}) \quad b_2(e^{-i\omega}) \right] \Sigma \left[b_1(e^{i\omega}) \quad b_2(e^{i\omega}) \right]' \\ &= \left[b_1(e^{-i\omega}) \quad b_2(e^{-i\omega}) \right] A_0A_0' \left[b_1(e^{i\omega}) \quad b_2(e^{i\omega}) \right]' ; \end{aligned} \tag{7}$$

in which the partial spectrum with respect to shock 1 (ε_1) is given by:

$$f_{11}(e^{-i\omega}) = \left| [A_0]_{11} b_1(e^{-i\omega}) + [A_0]_{21} b_2(e^{-i\omega}) \right|^2 :$$

A general method of identifying A_0 in the frequency domain is to impose restrictions on the dynamic properties of ε_1 at a set of particular frequencies, $\{\omega_1; \omega_2; ...; \omega_n\} \in [0; \pi]$; such that the partial spectrum of x with respect to ε_1 over the domain $\Omega = \{\omega_1; \omega_2; ...; \omega_n\}$ is minimized. Namely, we choose $[A_0]_{11}$ to solve:[8]

$$\min \int_{\Omega} \left| [A_0]_{11} b_1(e^{-i\omega}) + [A_0]_{21} b_2(e^{-i\omega}) \right|^2 d\omega;$$

[7] The power spectrum (spectral density function) decomposes the total variance of a stationary time series into "variance density distribution" across frequencies. The power at each frequency measures the contribution of cycles at that frequency to the total variance.

[8] The spectrum is always non-negative at each frequency ω, hence the quadratic function,

$$f(\alpha_{11}; \alpha_{21}) = \left| \alpha_{11} b_1(e^{-i\omega}) + \alpha_{21} b_2(e^{-i\omega}) \right|^2$$

is always convex with respect to α_{ij}. Therefore, the extra identifying restriction can be obtained by choosing α_{11} (or α_{21}) to minimize $f(\cdot)$. Notice that the minimized spectral density may not necessarily be zero. It is zero, for example, at the zero frequency. Because when $\omega = 0$, we have $e^{-i\omega} = 1$, hence $\alpha_{11} b_1(e^{-i\omega}) + \alpha_{21} b_2(e^{-i\omega})$ and its conjugate are both real. Consequently, the minimum can be found simply by setting $\sqrt{f} = \alpha_{11} b_1(1) + \alpha_{21} b_2(1) = 0$. When $\omega \neq 0$, the expression, $\alpha_{11} b_1(e^{-i\omega}) + \alpha_{21} b_2(e^{-i\omega})$, is generally a complex quantity. And the only way to ensure a zero value for $f(\cdot)$ at arbitrary frequency ω is to have $\alpha_{11} = \alpha_{21} = 0$, which results in overidentification.

which gives

$$[A_0]_{11} = -[A_0]_{21}\left(\frac{\int_\Omega \left\{b_1(e^{-i\omega})b_2(e^{i\omega}) + b_2(e^{-i\omega})b_1(e^{i\omega})\right\} d\omega}{2\int_\Omega |b_1(e^{-i\omega})|^2 d\omega}\right). \tag{8}$$

This relationship (equation 8), together with the condition $AA' = \Sigma$, uniquely determines the matrix A_0. Hence, we are able to recover representation (1) along with the identification of two structural shocks (ε_1 and ε_2) from the data, the first of which has minimal effect on the power spectrum of x over the frequency set Ω.[9] In the special case of $\Omega = 0$ (namely, $\omega = 0$ is the only frequency at which the identifying restriction is imposed), equation (8) simplifies to

$$[A_0]_{11} = -[A_0]_{21}\frac{b_2(1)}{b_1(1)},$$

which is identical to (5). Therefore, the B-Q (1989) identifying scheme is just a special case of our general method.

2. Application

In this section, we show how the general method can be applied to identifying the business cycle effects of seasonal shocks for the US economy. As in B-Q (1989), we must choose a second variable as an instrument to help accomplishing the identification task in a bivariate VAR system. Since our identifying restriction requires that the non-seasonal shocks have minimum effect on the variance of a time series at the seasonal frequency, a sensible instrument is a seasonally unadjusted variable that has little seasonal component in it. Empirical studies found that the US interest rates have very little seasonal component (e.g., see Miron, 1996), we choose the 3-month T bill rate as our instrument variable.[10]

Let Y denote the logarithm of a seasonally unadjusted aggregate US time series (e.g., the real GDP), and r denote the instrument variable (e.g., the 3-month T bill rate). And let ε_s and ε_n be the seasonal and non-seasonal disturbances (s = seasonal, n = non-seasonal). In addition, let X be the vector $(\Delta Y, \Delta r)'$ and ε be the vector $(\varepsilon_n, \varepsilon_s)'$. The stationarity assumption of X implies the moving

[9] When the set Ω contains discrete points of frequencies, we can replace the integral $\int_\Omega$ by summation $\sum_\Omega$. Note that $[A_0]_{11}$ is always real valued as $b_2(e^{-i\omega})b_1(e^{i\omega})$ is the complex conjugate of $b_1(e^{-i\omega})b_2(e^{i\omega})$.

[10] The results are generally robust to the choice of different interest rates series.

average representation:[11]

$$X_t = \left(A_0 + A_1 L + A_2 L^2 + \cdots\right)\varepsilon_t = A(L)\varepsilon_t,$$

where L is the lag operator, $\mathrm{var}(\varepsilon) = I$ ($\sigma_n = \sigma_s = 1$), and $A(L) = \begin{bmatrix} a_{11}(L) & a_{12}(L) \\ a_{21}(L) & a_{22}(L) \end{bmatrix}$ with $a_{ij}(L) = \sum_{k=0}^{\infty} a_{ij}(k)L^k$ for $i, j = 1, 2$. The corresponding spectral density function is given by

$$F_x(e^{-i\omega}) = \left|A(e^{-i\omega})\right|^2,$$

where the upper left-hand entry (the spectrum of output growth) is given by

$$F_{11}(e^{-i\omega}) = \sigma_n^2\left|a_{11}(e^{-i\omega})\right|^2 + \sigma_s^2\left|a_{12}(e^{-i\omega})\right|^2, \quad (9)$$

and the partial spectrum of ΔY with respect to the non-seasonal shock ε_n is given by the first term in $F_{11}(e^{-i\omega})$. The identifying assumption that ε_n has minimum contributions to seasonal cycles in ΔY implies that $\left|a_{11}(e^{-i\omega})\right|^2$ attains its minimum at the seasonal frequency $\omega = \frac{\pi}{2}$.[12]

To recover representation (9) from the data, let the Wold-moving average representation of X be given by

$$X_t = \left(I + B_1 L + B_2 L^2 + \cdots\right)\varepsilon_t = B(L)v_t, \quad (10)$$

where $B(L) = \begin{bmatrix} b_{11}(L) & b_{12}(L) \\ b_{21}(L) & b_{22}(L) \end{bmatrix}$, and $\mathrm{var}(v) = \Sigma$. This representation can be obtained by first estimating and then inverting the vector autoregressive representation of X in the usual way. Taking the Fourier transform of (10), the corresponding spectral representation is given by:

$$F_x(e^{-i\omega}) = B(e^{-i\omega})A_0A_0'B(e^{i\omega}),$$

where $A_0A_0' = \Sigma$. The spectrum of output growth ΔY is given by the upper left-hand entry:

$$\left|[A_0]_{11}b_{11}(e^{-i\omega}) + [A_0]_{21}b_{12}(e^{-i\omega})\right|^2 + \left|[A_0]_{12}b_{11}(e^{-i\omega}) + [A_0]_{22}b_{12}(e^{-i\omega})\right|^2; \quad (11)$$

[11] We assume that seasonalities can be modeled (or be approximated) as indeterministic processes so that the seasonal patter can be captured by a VAR. This assumption may not be 100% realistic, but it can nevertheless serve as a benchmark for further investigations.

[12] If there exist seasonal cycles at harmonic frequencies as well, then the identifying assumption requires that $\left|a_{11}(e^{-i\omega})\right|^2$ attains minima at both the fundamental frequency and harmonic frequencies.

in which the first term is the partial spectrum of ΔY with respect to the non-seasonal innovation ε_n; and the second term is the partial spectrum of ΔY with respect to the seasonal innovation ε_s. Choosing $[A_0]_{11}$ to minimize the first term in (11) at the seasonal frequency $\omega = \frac{\pi}{2}$ (for quarterly data) gives

$$[A_0]_{11} = -[A_0]_{21}\left(\frac{b_{11}(e^{-i\frac{\pi}{2}})b_{12}(e^{i\frac{\pi}{2}}) + b_{12}(e^{-i\frac{\pi}{2}})b_{11}(e^{i\frac{\pi}{2}})}{2\left|b_{11}(e^{-i\frac{\pi}{2}})\right|^2}\right). \qquad (12)$$

The system of equations that can be solved for the four elements in A_0 is then given by the identifying restriction (12) and the relation $A_0A_0' = \Sigma$.[13]

With the knowledge of A_0, we can then examine the business-cycle effects of seasonal shocks using representation (11), which decomposes the total spectrum of ΔY into two parts: the part due to non-seasonal shocks and the part due to seasonal shocks. If seasonal shocks are important for triggering business-cycle fluctuations in the US economy, then the partial spectrum of ΔY with respect to seasonal shocks should constitute a significant fraction of the total spectrum of ΔY at the business cycle frequency.

3. Interpretation

Our interpretation of disturbances with minimal effects at the seasonal frequency as non-seasonal shocks (i.e., as conventional business cycle shocks), and of disturbances with maximum effects at the seasonal frequency as seasonal shocks that may also have effects at the business cycle frequency is motivated by a traditional multiplier-accelerator model of the business cycle (P. Samuelson, 1939, and J. Hicks, 1950).[14] The model has three equations:

$$Y_t = C_t + I_t + G_t; \qquad (A)$$

[13] When seasonal cycles also exist at harmonic frequencies, the identifying restriction (12) becomes:

$$[A_0]_{11} = -[A_0]_{21}\left(\frac{\sum_j\left[b_{11}(e^{-i\frac{2\pi j}{4}})b_{12}(e^{i\frac{2\pi j}{4}}) + b_{12}(e^{-i\frac{2\pi j}{4}})b_{11}(e^{i\frac{2\pi j}{4}})\right]}{2\sum_j\left|b_{11}(e^{-i\frac{2\pi j}{4}})\right|^2}\right);$$

where $j = 1, 2$; indicating that nonseasonal shocks have minimal effect at both the fundamental frequency, $\frac{\pi}{2}$; and the harmonic frequency, π.

[14] For simplicity, we have adopted an ad hoc model. But the structural equations in the model can be interpreted as reduced-form equilibrium decision rules derived from a rational expectations general equilibrium model with fully specified preferences and technologies.

$$C_t = \alpha_0 + \alpha Y_{t-1} + S_t; \qquad \text{(B)}$$

$$I_t = \beta(C_t - C_{t-1}). \qquad \text{(C)}$$

The variable $Y; C; I; G$ denote output, consumption, investment, and government spending respectively. The time period is assumed to be one quarter. Equation (A) is the goods market equilibrium condition with G_t as the aggregate demand shock (the non-seasonal business cycle shock). Equation (B) is a simple consumption function where $\alpha_0 > 0$ is autonomous consumption, $\alpha \in (0; 1)$ is the marginal propensity to consume, and S_t is a seasonal forcing variable that impacts consumption demand (e.g., the Christmas effect). Equation (C) describes investment behavior as responding primarily to changes in aggregate consumption demand with the accelerator coefficient $\beta > 0$.

To close the model, we need to specify how G_t and S_t evolve. As an illustration, we assume that:

$$G_t = \varepsilon_{gt};$$

$$S_t = -\rho(S_{t-1} + S_{t-2} + S_{t-3}) + \varepsilon_{st}; \quad 0 < \rho \le 1; \qquad (13)$$

where ε_g and ε_s are the serially uncorrelated, orthogonal innovations to the non-seasonal and seasonal impulses respectively.[15] Solving for output in the above system gives:

$$Y_t = \alpha(1 + \beta)Y_{t-1} - \alpha\beta Y_{t-2} + G_t + S_t;$$

or

$$(1 - \lambda_1 L)(1 - \lambda_2 L)Y_t = G_t + S_t; \qquad (14)$$

where L is the lag operator, and λ_1 and λ_2 are the characteristic roots of equation (14) satisfying

$$\lambda_1 + \lambda_2 = \alpha(1 + \beta);$$
$$\lambda_1\lambda_2 = \alpha\beta.$$

It is well known that for reasonable values of α and β; the above system exhibits dampened endogenous business cycles (i.e., the characteristic roots λ_1 and λ_2

[15] There are many different ways to model seasonalities. An alternative model for the seasonal variable is $S_t = \rho S_{t-4} + \varepsilon_{st}; 0 < \rho \le 1$. For our analysis, which model to choose does not matter, but the indeterministic model in equation (13) seems to capture the seasonalities in the US data quite well. Our identifying procedure as well as the empirical results obtained in this paper, however, do not require knowledge about the true model of the seasonalities, except the assumptions that S_t possesses stochastic cycles at the seasonal frequency and has well-defined spectrum.

form a complex conjugate pair, $a \pm bi$). For example, when $\alpha = \beta = 0.9$, we have $\lambda = 0.855 \pm 0.281i$, implying dampened oscillations at frequency

$$\omega = \cos^{-1}\left(\frac{0.855}{\sqrt{0.855^2 + 0.281^2}}\right) = 0.05 \text{ (cycle per quarter)},$$

or periodicity (average cycle length) of 20 quarters per cycle.

Let the structural parameters α and β be such that the characteristic roots of the model are $a \pm bi$. To see how business cycle fluctuations in output Y depend on the non-seasonal and seasonal shocks, we rewrite equation (9) as

$$\begin{aligned} Y_t &= \frac{1}{(1-\lambda_1 L)(1-\lambda_2 L)}\varepsilon_{gt} + \\ &\quad \frac{1}{(1-\lambda_1 L)(1-\lambda_2 L)(1+\rho L+\rho L^2+\rho L^3)}\varepsilon_{st} \\ &\equiv B(L)\varepsilon_{gt} + B(L)S(L)\varepsilon_{st}, \end{aligned} \tag{15}$$

where $B(L) \equiv [(1-\lambda_1 L)(1-\lambda_2 L)]^{-1}$ represents the endogenous propagation mechanism, $S(L) \equiv \left[1+\rho L+\rho L^2+\rho L^3\right]^{-1}$ represents the exogenous propagation mechanism that transmits the impact of seasonal innovations (ε_s) in a manner that mimics seasonal cycles.

The corresponding power spectrum of Y is given by

$$\begin{aligned} F_y(e^{-i\omega}) &= \frac{\sigma_g^2}{|(1-\lambda_1 e^{-i\omega})(1-\lambda_2 e^{-i\omega})|^2} + \\ &\quad \frac{\sigma_s^2}{|(1-\lambda_1 e^{-i\omega})(1-\lambda_2 e^{-i\omega})|^2\,|1+\rho e^{-i\omega}+\rho e^{-2i\omega}+\rho e^{-3i\omega}|^2} \\ &\equiv B(e^{-i\omega})\sigma_g^2 + B(e^{-i\omega})S(e^{-i\omega})\sigma_s^2, \end{aligned} \tag{16}$$

where

$$B(e^{-i\omega}) \equiv \frac{1}{|(1-\lambda_1 e^{-i\omega})(1-\lambda_2 e^{-i\omega})|^2},$$

and

$$S(e^{-i\omega}) \equiv \frac{1}{|1+\rho e^{-i\omega}+\rho e^{-2i\omega}+\rho e^{-3i\omega}|^2}.$$

Equation (15) is the moving-average, time-domain representation of the dynamics of output, with the first component showing the dynamic effects of the non-seasonal shock on Y, and the second component showing the dynamic effects of the seasonal shock on Y. Equation (16) is the frequency-domain analogue of the decomposition.

Both representations clearly indicate that fluctuations in output at the characteristic business cycle frequency $\omega = \cos^{-1}\left(\frac{a}{\sqrt{a^2+b^2}}\right)$ are determined by the model's endogenous propagation mechanism (i.e., by the business cycle polynomial $B(\cdot)$), and that fluctuations in output at the seasonal frequency are determined by the exogenous propagation mechanism (i.e., by the seasonal cycle polynomial $S(\cdot)$). Hence, the power spectrum of Y has two maxima (spectral peaks). One of them centers at the business cycle frequency $\omega = \cos^{-1}\left(\frac{a}{\sqrt{a^2+b^2}}\right)$, at which the function, $B(e^{-i\omega}) = 1/\left|(1-\lambda_1 e^{-i\omega})(1-\lambda_2 e^{-i\omega})\right|^2$, attains its maximum; and the other one centers at the seasonal frequency, $\omega = \frac{\pi}{2}$, at which the function, $S(e^{-i\omega}) = 1/\left|1+\rho e^{-i\omega}+\rho e^{-2i\omega}+\rho e^{-3i\omega}\right|^2$, attains its maximum.[16]

Equations (15) and (16) also indicate clearly that the dynamic impact of the non-seasonal innovation (ε_g) is not propagated by the seasonal factor $S(\cdot)$, while the dynamic impact of the seasonal innovation (ε_s) is propagated both by the seasonal factor $S(\cdot)$ and by the business cycle factor $B(\cdot)$. In other words, the non-seasonal disturbance (ε_g) can generate fluctuations in Y only through the endogenous propagation mechanism, $B(\cdot)$. But the seasonal disturbance (ε_s) can generate fluctuations in Y through two different propagation mechanisms, $B(\cdot)$ and $S(\cdot)$, the first of which generates stochastic business cycles, and the second generates stochastic seasonal cycles.

Consequently, compared with ε_s, the non-seasonal innovation (ε_g) has little contribution to the spectrum of output at the seasonal frequency. In fact, the ratio of the partial spectrum of Y with respect to ε_g and that with respect to ε_s is given by:

$$\frac{1}{S(e^{-i\omega})}\frac{\sigma_g^2}{\sigma_s^2} = \left|1+\rho e^{-i\omega}+\rho e^{-2i\omega}+\rho e^{-3i\omega}\right|^2 \frac{\sigma_g^2}{\sigma_s^2},$$

which attains a minimum at the seasonal frequency $\omega = \frac{\pi}{2}$.[17]

[16] Since the power spectrum decomposes the total variance of a stationary time series into contributions across frequencies, if a stochastic time series contains characteristic cycles at frequency ω, then its spectrum will exhibit a peak (concentration of power) at frequency ω, indicating large contributions from the characteristic cycle to the total variance of that time series.

[17] This is so because $S(e^{-i\omega})$ attains its maximum at the seasonal frequency $\frac{\pi}{2}$. Note that we do not require the minimum to be necessarily zero in our identifying scheme. In empirical applications, the actual value of the minimum is influenced by the data. Hence ε_g may still have positive contribution to the variance of Y at the seasonal cycle frequency, although that contribution is minimized under our identifying assumptions.

On the other hand, the seasonal innovation ε_{st} not only has maximum contributions to the spectrum of Y at the seasonal frequency, but also has a potentially very large contribution to the spectrum of Y at the business cycle frequency. This is so because the partial spectrum of Y with respect to ε_s,

$$B(e^{-i\omega})S\left(e^{-i\omega}\right)\sigma_s^2;$$

has one maximum at the seasonal frequency $\omega = \frac{\pi}{2}$ (at which the seasonal factor $S(e^{-i\omega})$ attains its maximum), and another maximum at the characteristic business cycle frequency $\omega = \cos^{-1}\left(\frac{a}{\sqrt{a^2+b^2}}\right)$ (at which the spectral function $B(e^{-i\omega})$ attains its maximum).

Hence, the economic model as represented by equation (16) clearly satisfies the identifying restrictions of the previous section. The dynamic implications of (16) can be better appreciated in a graphic illustration. Figure 1 plots the spectrum and partial spectrum of output Y defined in (16) using the parameterization: $\alpha = \beta = \rho = 0.9$; and $\sigma_g = \frac{1}{4}\sigma_s = 1$. It shows that the spectrum of output (dashed lines) has two peaks, one centering at the seasonal frequency ($\omega = 0.25$ quarters per cycle), another centering at the business cycle frequency ($\omega = 0.05$ quarters per cycle). This means that cyclical fluctuations at these frequencies are the two major contributors to the variance of output. The crucial thing to notice, however, is that the partial spectrum of output with respect to seasonal shocks (solid line) is most responsible for the shape of the total spectrum (i.e., the two spectral peaks), indicating that seasonal shocks generate not only seasonal cycles, but also business cycles. In the figure, seasonal shocks explain not only virtually all of the variance in output around the seasonal frequency, but also a substantial fraction of the variance of output around the business cycle frequency, leaving non-seasonal shocks to explain only a very limited portion of the business cycle in output.

4. Estimation

This section reports the estimated business cycle effects of seasonal shocks for some post war US aggregates using our identifying procedure outlined in section 2 (we use the seasonally unadjusted 3 month treasury bill rate as the instrument variable). Since the US housing construction sector is most volatile both at the business cycle frequency and at the seasonal cycle frequency, and many related variables are quite stationary (see Figure 2), we first report our estimation results for housing starts for the US economy (1947:1 - 1996:2).[18]

Figure 3 shows the estimated dynamic responses of housing starts to innovations in the non-seasonal and seasonal impulses (the time period is a quarter). The left window shows the response of housing starts to a non-seasonal innovation. It exhibits the typical hump-shaped pattern, re°ecting the endogenous business cycle propagation mechanism. The right window shows the response of housing starts to a seasonal innovation. It exhibits large seasonalities, re°ecting the anticipated calender e®ect of the seasonal-cycle propagation mechanism of the data, which has been modeled as indeterministic processes.[19]

At the impact period, the magnitude of the response to a seasonal innovation is about 80 times larger than the response to a non-seasonal innovation, indicating that housing starts are extremely sensitive to seasonal disturbances and relatively not very sensitive to non-seasonal disturbances. Both type of responses show hump-shaped transition patterns in returning to the steady state. The maximum e®ect of non-seasonal shocks, however, is reached only 5 quarters after the impact. For seasonal shocks, the low-frequency e®ect decays gradually (interrupted by seasonal cycles) with the trough being reached only after 10 quarters, indicating that seasonal shocks also have a strong business-cycle e®ect.

Figure 4 shows the time-series decomposition for housing starts. The top window shows the time series representation of housing starts in the absence of seasonal disturbances. The bottom window shows the time series representation of housing starts due to seasonal disturbances. It is clear from the top window that housing starts would have been much less volatile if the seasonal shocks were absent. But the business cycle would still be present in housing starts even without the non-seasonal business cycle shocks (see the bottom window).

To better appreciate the business cycle e®ect of seasonal shocks, Figure 5 presents the spectral decomposition of the total variance of housing starts across frequencies. The total spectrum (dashed lines) has two peaks: one at the seasonal frequency (0:25 cycles per quarter or 4 quarters per cycle) and another at the busi-

[18] We use stationary data ¯rst to conduct our analyses, as it is well known that pre-¯ltering using either the ¯rst di®erence ¯lter or the HP ¯lter may cause distortional results. All variables are logged and 8 lags are used in the estimation.

[19] An indeterministic model of seasonality implies that the seasonal cycle can be observed only when the economy is subject to random shocks at the seasons. A deterministic model of seasonality, on the other hand, implies that the seasonal cycle can be observed in the steady state even without seasonal shocks. We have adopted the indeterministic approach in our identifying procedure. Consequently, the de¯nition for \seasonal shocks" is given by shocks that can trigger the seasonal propagation mechanism, and that for \nonseasonal shocks" is given by shocks that do not trigger the seasonal propagation mechanism.

ness cycle frequency (0.035 cycles per quarter or 28 quarters per cycle). Looking at the partial spectrum with respect to seasonal shocks (solid line), it is interesting to see how much the seasonal shocks can contribute to the business cycle movement in housing stars: an exceptionally large fraction of power around the business cycle frequency is due to seasonal shocks. Across all frequencies, seasonal shocks explain 81% of the total variance in housing starts. Around the business cycle frequencies (6-40 quarters per cycle), at which the conventional business cycle shocks are expected to dominate, seasonal shocks still explain about 69% of the total variance of housing starts.

Let's now turn to the case of real GDP, consumption and investment.[20] Figure 6 shows the real GDP growth for the US economy (1947:1 - 1987:4). Figures 7 and 8 show its decomposition in time and in frequency respectively.[21] In particular, the top window in figure 7 shows the time-series representation of GDP growth in the absence of seasonal innovations. The variance is only about 6% of the original series shown in Figure 6, indicating that GDP growth would have been much smoother if the seasonal disturbances had been absent from the US economy. The bottom window in figure 7 shows the time series of GDP due to seasonal disturbances. The variance is about 94% of the total variance of GDP. Clearly, seasonal movements are the most important source of fluctuations in GDP.

Figure 8 shows the effects of seasonal shocks across frequencies. There we see similar pictures to that of housing stars. Namely, the largest contributor to the total spectrum of GDP growth is the seasonal disturbance (solid line).[22] Conventional business-cycle disturbances explain only 6% of the total spectrum of GDP growth. Around the business cycle frequency (6-40 quarters per cycle), seasonal disturbances still explain about 48% of the variance in GDP growth, leaving the non-seasonal shocks to explain only about half of the business cycle in GDP. In addition, spectral decomposition at zero frequency indicates that seasonal disturbances have the dominant long-run effect on output.

The results are very similar for the case of consumption and investment. They are summarized in Table 1. For the first-differenced data, seasonal shocks explain about 94% of total variances in growth in consumption and investment across

[20] The price index used for deriving real quantities is seasonally unadjusted CPI. All variables (including the instrument) are logged before the first-difference filter or the HP filter was applied in deriving the growth rates. Six lags are included in the VARs.

[21] Figure 8 shows that the growth rate of GDP also has substantial power at the two-period cycle frequency. It turns out that minimizing the impact of nonseasonal shocks at the seasonal frequency is sufficient to induce a partial spectrum that also has minimal effect at the two-period cycle frequency.

[22] The spectral peak at $\omega = 0.5$ indicates harmonic cycles.

all frequencies, and explain about 50% of variances around the business cycle frequencies (6-40 quarters per cycle). For the HP filtered data, seasonal shocks explain about 70% or more of total variances in consumption and investment across all frequencies, and about 45% of variances around the business cycle frequencies.[23]

Table 1: Contributions of Seasonal Shocks to Variance (%)

variable	all frequencies	business cycle frequencies
first difference		
y	0.94	0.48
c	0.94	0.53
i	0.95	0.53
HP filter		
y	0.70	0.45
c	0.78	0.42
i	0.73	0.46

Despite the limited number of data series examined, we found consistent empirical results suggesting that seasonal shocks have surprisingly large contributions to the business cycle in the US economy. These results are generally robust to the instrument variable used. For example, if the GDP price index is used as the instrument variable instead, we find even larger contributions to the business cycle by seasonal shocks.

These empirical findings perhaps seem both astonishing and puzzling at first glance. But, pondering them in light of the economic model provided in section 3 makes it obvious that seasonal shocks ought to have large effects on the business cycle. This is so because no matter what measure we use, seasonal disturbances are by far the most frequent, most regular, most synchronized, and on average the severest shocks among all. What kind of endogenous propagation mechanism can possibly escape from the impact of such strikes?

[23] Because the HP filter puts more weight on business cycle frequencies and less weight on the high seasonal frequencies, the relative importance of seasonal fluctuations in the total variance is reduced.

5. Conclusions

What shocks are responsible for the business cycle? After careful examination using seasonally adjusted data, Cochrane (1994) concludes that none of the popular candidates for observable shocks accounts for the bulk of business-cycle °uctuations in output. The possible role for seasonal shocks, however, is excluded a priori. We developed an econometric procedure that allows us to identify seasonal shocks versus non-seasonal shocks, and the business cycle e®ects of seasonal shocks. Applying the procedure to the US data, we found that seasonal shocks account for the bulk of business-cycle °uctuations in output (roughly 50%), and that in general the larger the seasonal shocks are, the larger the business cycle is.[24] These ¯ndings suggest that models that rely heavily on technology shocks to explain the business cycle are missspeci¯ed, that using seasonally adjusted data to test or evaluate business cycle models may lead to incorrect conclusions, and that theories of the business cycle, as well as government policies concerning the business cycle ought to address seasonal °uctuations seriously.

While we ¯nd this simple exercise to have been worthwhile, we also believe that further work is needed, especially to validate and to re¯ne our de¯nition of seasonal shocks.[25] We have in mind three speci¯c extensions. The ¯rst is to model seasonalities as a mixture of both deterministic and indeterministic processes, so as to allow a further decomposition of the business-cycle e®ects of seasonal impulses into two parts: that due to the deterministic aspects of seasonalities, and that due to the stochastic aspects of seasonalities. We believe that both aspects of seasonalities can generate business cycles, but the mechanisms may be

[24] According to the economic model presented in Section 3, the relative contributions of seasonal shocks to the business cycle depends on the variance ratio of seasonal disturbances and nonseasonal disturbances, $\frac{¾^2_s}{¾^2_g}$. So for larger $¾^2_s$, not only is the business cycle larger, but also the fraction of the business cycle due to seasonal shocks.

A good empirical example is the comparison of housing starts and GDP. In terms of growth rates, the volatility of housing starts is 43 times that of GDP. Although for both series, seasonal movements account for about 95% of the total variance, the contribution of seasonal shocks to business cycle °uctuations is 70 percent for housing starts, and 48 percent for GDP. This suggests that housing starts would have been substantially less volatile not just at the seasonal frequency, but even more so at the business cycle frequency if the seasonal disturbances driving housing starts were not as severe.

These results are consistent with the empirical ¯ndings as well as theoretical conjectures of existing literature on seasonal cycles (e.g., Barsky and Miron, 1989, Beaulieu, MacKie-Mason and Miron, 1992, Miron 1996).

[25] In the paper, seasonal shocks are de¯ned as shocks that trigger °uctuations at the seasonal frequency, and non-seasonal shocks are de¯ned as shocks that do not trigger the seasonal cycles.

quite di®erent. The second extension is to ¯nd a way to identify the demand-side seasonal shocks (e.g., the \Christmas" e®ect) and the supply-side seasonal shocks (e.g., the weather e®ect). Research along this line can help address welfare questions regarding the issue of smoothing seasonal cycles. If seasonal cycles are largely demand driven, then clearly the welfare gains from smoothing them are very di®erent from that when they are largely supply driven. The third extension is to relax the orthogonality assumption with regard to seasonal and non-seasonal shocks. Although seasonal innovations (such as unusual weather conditions in each winter) and conventional business cycle innovations (such as a technological break through) may well be orthogonal, they may also be correlated. Extension along this line would also allow examination of seasonal monetary policies from an alternative perspective, as in Barsky, Mankiw, Miron, and Weil (1988).

References

[1] R. Barsky, G. Mankiw, J. Miron, and D. Weil, 1988, The worldwide change in the behavior of interest rates and prices in 1914, European Economic Review 32, 1123-1154.

[2] R. Barsky and J. Miron, 1989, The seasonal cycle and the business cycle, Journal of Political Economy 97, no. 3, 503-535.

[3] J. Beaulieu, J. MacKie-Mason, and J. Miron, 1992, Why do countries and industries with large seasonal cycles also have large business cycles? Quarterly Journal of Economics 107, no. 2, 621-656.

[4] J. Beaulieu and J. Miron, 1992, A cross country comparison of seasonal cycles and business cycles, Economic Journal 102, 772-788.

[5] O. Blanchard and D. Quah, 1989, The dynamic e®ects of aggregate demand and supply disturbances, American Economic Review 79, 655-673.

[6] R. Braun and C. Evans, 1995, Seasonality and equilibrium business cycle theories, Journal of Economic Dynamics and control 19, 503-531.

[7] S. Chatterjee and B. Ravikumar, 1992, A neoclassical model of seasonal °uctuations, Journal of Monetary Economics 29, 59-86.

[8] J. Cochrane, 1994, Shocks, Carnegie-Rochester Conference Series on Public Policy 41, 295-364.

[9] J. Hicks, 1950, A contribution to the theory of the trade cycle, Oxford University Press.

[10] J. Miron, 1996, The economics of seasonal cycles, The MIT Press, Cambridge.

[11] J. Miron and S Zeldes, Seasonality, cost shocks, and the production smoothing model of inventories, Econometrica 56, 877-908.

[12] P.A. Samuelson, 1939, Interaction between the multiplier analysis and the principle of acceleration, Review of Economic Statistics, May.

[13] M. Watson, 1993, Measures of ¯t for calibrated models, Journal of Political Economy 101, no. 6, 1011-1041.

[14] Y. Wen, 1998, Investment cycles, Journal of Economic Dynamics and Control 22, 1139-1165.

Figure 1: Spectral Decomposition of Output in the Theoretical Business Cycle Model. Dashed line represents the total spectrum under both shocks. Solid line represents the partial spectrum under the seasonal shock only.

Figure 2: US Housing Starts (1947:1 { 1996:2). Source: CITIBASE.

Figure 3: Impulse Responses of Housing Starts to Di®erent Shocks.

Figure 4: Time Series Decomposition of Housing Starts. Top: Fluctuations absent seasonal shocks. Bottom: Fluctuations due to seasonal shocks.

Figure 5: Spectral Decomposition of Housing Starts. Dashed lines represent the total spectrum. Solid lines represent partial spectrum with respect to seasonal shocks. Dotted lines represent partial spectrum with respect to nonseasonal shocks.

Figure 6: Real GDP Growth (1947:1 - 1987:4)

Figure 7: Time Series Decomposition of GDP growth. Top: Fluctuations absent seasonal shocks. Bottom: Fluctuations due to seasonal shocks.

Figure 8: Spectral Decomposition of GDP growth (bottom window: room-in). Dashed lines represent the total spectrum. Solid lines represent partial spectrum with respect to seasonal shocks. Dotted lines represent partial spectrum with respect to nonseasonal shocks.