EUROPEAN CENTRAL BANK

EUROSYSTEM

WORKING PAPER SERIES

NO 1092 / SEPTEMBER 2009

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In 2009 all ECB publications feature a motif taken from the €200 banknote.

1 Comments and suggestions from Krisztina Molnar and seminar participants at the European Central Bank, Bank of Finland, Norges Bank and University of Birmingham are gratefully acknowledged. Martin Ellison acknowledges support from an ESRC Research Fellowship, "Improving Monetary Policy for the 21st Century" (RES-000-27-0126). The views expressed in this paper are those of the authors and do not necessarily reflect those of the ECB.

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ISSN 1725-2806 (online)

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Abstract

The assumption of asymmetric and incomplete information in a standard New Keynesian model creates strong incentives for monetary policy transparency. We assume that the central bank has better information about its objectives than the private sector, and that the private sector has better information about shocks than the central bank. Transparency has the potential to trigger a virtuous circle in which all agents find it easier to make inferences and the economy is better stabilised. Our analysis improves upon existing work by endogenising the volatility of both output and inflation. Improved transparency most likely manifests itself in falling output volatility.

Keywords: Imperfect credibility, Asymmetric information, Signal extraction

JEL Classification: E32, E37, E52

Non-Technical Summary

There is a large consensus among policymakers and academics about the benefits of credibile and transparent monetary regimes in fostering overall macroeconomic stability. A general argument is that such regimes, by anchoring inflation expectations around the objective of the central bank, make price and wage setting less responsive to temporary shocks and allow the central bank to ensure greater macroeconomic stability through only moderate policy actions. In contrast, poor transparency weakens the anchor on expectations and leads to a deterioration in the stochastic properties of both inflation and output. For example, Erceg and Levin (2003) show that a lack of transparency about the inflation target creates unwanted inflation persistence and increased costs of deflation in an otherwise standard DSGE model. This paper introduces an additional channel through which transparent monetary policy promotes a more stable macroeconomic environment. The idea is that anchoring private inflation expectations to a transparent target makes it is easier for the central bank to infer the state of the economy, in which case the central bank is better informed and can fine-tune stabilisation policy to increase stability in the macroeconomic environment. In particular, we consider a situation in which the central bank is unable to observe either private inflation expectations or the natural rate of interest that summarises the shocks hitting the economy. Private agents in turn have to infer the inflation target of the central bank. Information in our model is *asymmetric* because the central bank and private agents know different things. Information is *incomplete* because neither the central bank nor private agents know everything about the state of the economy. Such asymmetry and incompleteness of information means that the central bank and private agents solve different yet interrelated inference problems, in which case transparency has the potential to create a virtuous circle where all agents become better informed. Transparency makes it easier for private agents to infer the inflation target, thereby anchoring inflation expectations and making it easier for the central bank to infer the natural rate of interest. But if the central bank has better knowledge of the natural rate of interest then policy will be more predictable and private agents will find it easier to infer the inflation target. This exchange of information continues until all the gains from the complementarity of the inference processes have been exploited. Our paper builds on the idea in Aoki and Kimura

(2008) that a central bank finds it difficult to infer the state of the economy if private agents are uncertain about the inflation target. Their paper stresses how, in an endowment economy with flexible prices, asymmetric and incomplete information causes unwanted volatility and persistence in inflation. An important limitation of their analysis is that output is exogenous and so by construction unaffected by any inferences made by the central bank and private agents. We relax this assumption by considering a DSGE model where both inflation and output are determined endogenously, and find that asymmetric and incomplete information has first order implications for the stochastic properties of output. Moreover, the unwanted volatility in output is considerably larger than that in inflation. That inflation and output volatility exhibit time-variation has been convincingly argued by McConnell and Perez-Quiros (2000), Cogley and Sargent (2005) and many others over the last decade. The mechanism we identify in this paper contributes to this debate by showing how transparency of monetary regimes promotes macroeconomic stability of both output and inflation.

1 Introduction

Many policymakers and academics believe that transparent monetary policy regimes have a role to play in fostering macroeconomic stability. A general argument is that such regimes anchor inflation expectations around the objective of the central bank, making prices and wages less responsive to temporary shocks and allowing the central bank to ensure greater macroeconomic stability through only moderate policy actions. In contrast, poor transparency weakens the anchor on expectations and leads to a deterioration in the stochastic properties of both inflation and output. For example, Erceg and Levin (2003) show that a lack of transparency about the inflation target creates unwanted inflation persistence and increased costs of deflation in an otherwise standard DSGE model.

This paper introduces an additional channel through which transparent monetary policy promotes macroeconomic stability. The idea is that anchoring private inflation expectations to a transparent target makes it is easier for the central bank to infer the state of the economy, in which case the central bank is better informed and can fine-tune stabilisation policy to increase stability in the macroeconomic environment. In particular, we examine a situation in which the central bank is unable to observe either private inflation expectations or the natural rate of interest that summarises the shocks hitting the economy. The more well-anchored private inflation expectations are in such a world the easier it is for the central bank to infer and react to the natural rate of interest. Private agents in turn know the natural rate of interest but have to infer the inflation target of the central bank. Information in our model is *asymmetric* because the central bank and private agents know different things. Information is *incomplete* because neither the central bank nor private agents know everything about the state of the economy.

There are strong incentives for transparency in our model. The asymmetry and incompleteness of information means that the central bank and private agents solve different yet interrelated inference problems, in which case transparency has the potential to create a virtuous circle where all agents become better informed. Transparency makes it easier for private agents to infer the inflation target, thereby anchoring inflation expectations and making it easier for the central bank to infer the natural rate of interest. But if the central bank has better knowledge of the natural rate of interest then policy will be more predictable and private agents will find it easier to infer the inflation target. This exchange of information continues until all the gains from the complementarity of the inference processes have been exploited. A lack of transparency in monetary policy similarly triggers a vicious circle in which the difficulty faced by private agents when trying to infer the inflation target makes it harder for the central bank to infer the natural rate of interest and so on.

Our paper builds on the idea in Aoki and Kimura (2008) that a central bank finds it difficult to infer the state of the economy if private agents are uncertain about the inflation target. Their paper stresses how, in an endowment economy with flexible prices, asymmetric and incomplete information causes unwanted volatility and persistence in inflation. An important limitation of their analysis is that output is exogenous and so by construction unaffected by any inferences made by the central bank and private agents. We relax this assumption by considering a DSGE model where both inflation and output are determined endogenously, and find that asymmetric and incomplete information has first order implications for the stochastic properties of output. Moreover, the unwanted volatility in output is considerably larger than that in inflation. The DSGE model we use draws on earlier work by Erceg and Levin (2003) that shows how inflation becomes persistent if private agents have to learn the inflation target in a DGE model with staggered nominal contracts. Our contribution uses a similar framework, but imposes the additional complication that inflation expectations are unobservable so the central bank only infers the natural rate of interest and does not know it with certainty.

The structure of asymmetric and incomplete information we assume implies that the central bank does not have ready access to measures of the inflation expectations of private agents. We find our assumption realistic. Whilst central banks can and do use survey data and information contained in asset prices to quantify the inflation expectations of market participants, such indicators will at best be noisy and at worst may be uncorrelated with the inflation expectations of private agents that actually drive the economy. For example, Bekaert et al. (2006) and Kosicki and Tinsley (2005) disagree in empirical studies as to whether US inflation expectations were 14% or 8% in the late 1970s to early 1980s. Such large discrepancies give credence to our assumption that central banks only have limited information about the inflation expectations of private agents.

An alternative view of our contribution is that it improves our understanding of how forward-looking agents make optimal inferences when faced with only limited information about the state of the economy. The seminal papers in this literature by Pearlman (1986, 1992) and Pearlman et al. (1986) derive optimal monetary policy under incomplete (*partial* in this literature) yet symmetric information. More recent contributions by Aoki (2003) and Svensson and Woodford (2004) solve for optimal policy under asymmetric information, but assume that private agents have complete information so information is only incomplete on the part of the central bank. Whilst we do not address the question of optimal policy in this paper, our analysis allows for information sets that are both incomplete and asymmetric.

The paper is organised as follows: In Section 2 we outline our DSGE model of the economy and specify what information is held by the central bank and private agents. Section 3 describes the interrelated inference problems of the central bank and private agents, and solves for equilibrium dynamics using a variant of the method of undetermined coefficients. A quantitative assessment of the model is provided in Section 4. A final Section concludes.

2 The economic environment

A full characterisation of the economic environment requires us to specify what information different agents hold, how agents make inferences on the basis of that information, and how inferences affect aggregate outcomes in the economy. In this Section we start the process from the last of these, by first describing the DSGE model that maps inferences to outcomes. We then define information sets and add the assumption that agents have rational expectations subject to their information and knowledge of the economy.

2.1 The model economy

The link between inferences and outcomes in our economy is described by the standard New Keynesian model originally introduced by Calvo (1983) and extensively reviewed by Woodford (2003).[1] The model consists of an intertemporal IS equation (2.1) and an expectations-augmented aggregate supply equation (2.2), which themselves are log-linear approximations of the optimal behaviour of households and firms:

$$y_t = E_t^p y_{t+1} - \sigma(i_t^p - r_t^n - E_t^p \pi_{t+1}), \tag{2.1}$$

$$\pi_t = \kappa y_t + \beta E_t^p \pi_{t+1}. \tag{2.2}$$

The equations simultaneously determine the output gap y_t and inflation rate π_t as functions of expectations and the short-term nominal interest rate i_t^p as perceived by private agents.[2] The expectations in (2.1) and (2.2) are superscripted ^{p} to indicate that households and firms form expectations conditional on the information set of private agents. We assume that households and firms have the same private information set I_t^p. The term r_t^n is the Wicksellian natural rate of interest, namely the equilibrium real rate of interest that would prevail if the economy had flexible prices. It is assumed to follow an exogenous first-order autoregressive process:

$$r_t^n = \delta r_{t-1}^n + u_t, \tag{2.3}$$

where u_t is iid normally distributed with mean zero and variance σ_u^2. The short-term nominal interest rate i_t^p is assumed to be that set by the central bank i_t^c plus an iid normally-distributed perceptions error ε_{tq} with mean zero and variance $\sigma_{\varepsilon q}^2$, such that:

$$i_t^p = i_t^c + \varepsilon_{tq}. \tag{2.4}$$

To close the model we assume that the central bank uses a simple rule to set the short-term nominal interest rate:

$$i_t^c = \pi_t^* + E_t^c r_t^n + \phi(E_t^c \pi_t - \pi_t^*), \tag{2.5}$$

with π_t^* a time-varying inflation target, $E_t^c r_t^n$ the central bank's current estimate of the natural rate of interest and $E_t^c \pi_t$ the central bank's estimate of current inflation. The central bank's

[1] See also Goodfriend and King (1997) and Clarida, Galí and Gertler (1999).

[2] Preston (2004) argues that equations such as (2.1) and (2.2) are not properly microfounded when agents do not have complete information. We acknowledge this point but follow the traditional approach of Honkapohja, Mitra and Evans (2003) and impose our information structure on aggregate relationships derived under complete information.

estimate of current inflation enters because measurement errors $v_{t\pi}^c \sim N(0, \sigma_{v\pi}^2)$ and $v_{ty}^c \sim N(0, \sigma_{vy}^2)$ prevent it from knowing either current inflation or the output gap with certainty. This idea follows Orphanides (2002) and is motivated by practical constraints that real-time policy often has to act before actual outcomes are known. The expectations in (2.5) are superscripted ^{c} to indicate that policy is set by the central bank subject to its own information set I_t^c. We assume $\phi > 1$ so the short-term nominal interest rate responds strongly to expected inflation and the Taylor principle is satisfied. With determinacy of equilibrium thus ensured, it can be shown as in Woodford (2001) that our simple policy rule is consistent with the optimal equilibrium. The time-varying inflation target π_t^* is persistent as in Erceg and Levin (2003):

$$\pi_t^* = \rho \pi_{t-1}^* + \varepsilon_{tp}, \tag{2.6}$$

where ε_{tp} is *iid* normally distributed with mean zero and variance $\sigma_{\varepsilon p}^2$. The term π_t^* is expressed in terms of the percentage deviation from the constant steady-state inflation target around which the model is log-linearised, and is assumed to be sufficiently persistent that the economy takes a long time to return to steady state after an ε_{tp} shock. The structure and parameters in equations (2.1) - (2.6) are assumed to be common knowledge, as are the distributions of all shocks.

2.2 Information structure

The information set of private agents includes inflation, the output gap, their perception of the short-term nominal interest rate, the natural rate of interest, the central bank's current estimate of inflation and the central bank's current estimate of the natural rate of interest, i.e. $I_t^p \equiv \{\pi_t, y_t, i_t^p, r_t^n, E_t^c \pi_t, E_t^c r_t^n\}$. The presence of inflation, the output gap and the private perception of the short-term nominal interest rate follows immediately from the role of private agents as households and firms. That private agents know the natural rate of interest can be motivated by the island model of Aoki and Kimura (2008), where information is aggregated in equilibrium such that private agents are better informed about the natural rate of interest than the central bank.[3] Private agents are assumed to use publicly available documents to obtain

[3]Our assumption that private agents have complete information about the natural rate of interest can be interpreted as a special case of Aoki and Kimura (2008). Moreover, relaxing the assumption of complete

the central bank's current estimates of inflation and the natural rate of interest. It is not unusual for central banks to publish such information. Indeed, the Bank of England publishes its assessment of current and future inflation and output growth in the quarterly *Inflation Report*, alongside measures of capacity utilisation that can be interpreted as estimates of the output gap. The *Monthly Bulletin* of the European Central Bank similarly makes public a wide range of macroeconomic data including projections of inflation within the current quarter. Finally, the Congressional Budget Office compiles data on current and future real potential gross domestic product that is subsequently published by the Federal Reserve Board. Data on potential real gross domestic product can be thought of as a proxy for the central bank's estimate of the natural rate of output in the economy.

The information set of the central bank is comprised of the inflation target, the short-term nominal interest rate it sets and noisy indicators of inflation and the output gap contaminated by measurement errors, i.e. $I_t^c \equiv \{\widetilde{\pi}_t^*, i_t^c, \pi_t + v_{t\pi}, y_t + v_{ty}\}$. Whilst it is obvious that the inflation target and the short-term nominal rate of interest should be included, the absence of the natural rate of interest from the information set of the central bank warrants further comment. In theory, if a central bank could perfectly observe the expectations of private agents then it would be much easier for it to calculate the natural rate of interest. In practice, expectations are never perfectly observed and the central bank has to rely on surveys or attempt to extract expectations from asset market data. Neither method is wholly satisfactory and it is likely to be difficult to find robust estimates of private sector expectations. Even with the benefit of hindsight and *ex post* data, empirical studies such as Bekaert et al. (2006) and Kosicki and Tinsley (2005) still disagree wildly about the level of inflation expectations some 25 years ago. We argue that tracking current expectations is even more problematic so it is reasonable to assume that the central bank has no way of observing the expectations of private agents, in which case it cannot use them to infer the natural rate of interest. The central bank can of course use the other variables in its information set to make inferences about the expectations of private agents and the natural rate of interest.

Our information structure differs in several respects from that typically found in the ex-

information does not as such undermine the message of our paper.

isting literature. In contrast to the early literature of Pearlman (1986, 1992) and Pearlman et al. (1992), we do not assume that information is symmetric between private agents and the central bank. Of the studies that do allow for asymmetric information, we depart from Aoki (2003) and Svensson and Woodford (2004) by allowing private agents to have incomplete rather than complete information. The closest information structure to ours is found in Aoki and Kimura (2008), although their assumption that the central bank observes inflation and the output gap makes inference trivial if output is endogenous. Our assumption that the central bank only has access to noisy indicators of inflation and the output gap ensures that inference still plays a role even though output is determined endogenously in our model.

2.3 The inference problem of private agents

The assumption that central bank inferences are available to private agents is useful because it restricts the degree of higher order beliefs that matter in equilibrium. In Section 3 we find that a truncated state vector with 12 elements is sufficient to describe the equilibrium dynamics of the economy:

$$X_t \equiv \begin{pmatrix} \pi_t & y_t & r_t^n & \pi_t^* & \varepsilon_{tq} & E_t^p\pi_t^* & E_t^p\varepsilon_{tq} & E_t^c\pi_t & E_t^c y_t & E_t^c r_t^n & E_t^c E_t^p \pi_t^* & E_t^c E_t^p \varepsilon_{tq} \end{pmatrix}'$$

The problem of private agents is to infer the unknown elements of X_t. In our model private agents do not know π_t^* and ε_{tq}, but do know $E_t^c r_t^n$ and $E_t^c \pi_t$ through knowing central bank expectations. Combining the perceived short-term nominal interest rate (2.4) with the policy rule (2.5):

$$i_t^p = \pi_t^* + E_t^c r_t^n + \phi(E_t^c \pi_t - \pi_t^*) + \varepsilon_{tq},$$

and noting that the sum on the right hand side is observable, the inference problem of private agents is a question of whether movements in their perceived short-term nominal interest rate i_t^p are most likely due to perception errors ε_{tq} or persistent shocks to the inflation target ε_{tp}. This is a standard Kalman filter problem. In terms of the definition of the state vector X_t, we can write the measurement and transition equations as follows:

$$Z_t^p = L^p X_{t-1} + M^p V_t, \quad (2.7)$$

$$X_t = A X_{t-1} + B V_t, \quad (2.8)$$

where $Z_t^p = i_t^p$ is the observed variable and L^p and M^p picks out the required elements of X_{t-1} and V_t. At this stage we simply conjecture that private agents know the transition equation is of the form (2.8) with $V_t = (\ \varepsilon_{tq} \quad \varepsilon_{tp} \quad u_t \quad v_{t\pi} \quad v_{ty}\ )'$. Confirmation of this conjecture is postponed until Section 3, where we use a method of undetermined coefficients argument to show that the equilibrium transition equation is indeed of this form.[4] Applying the Kalman filter to (2.7) and (2.8) implies that private agents form inferences according to:

$$E_t^p X_t = E_{t-1}^p A X_{t-1} + k^p (Z_t^p - E_{t-1}^p L^p X_{t-1}), \tag{2.9}$$

where k^c and P^c can be calculated recursively from the Kalman filtering equations:

$$\begin{aligned} k^p &= (AP^p L^{p\prime} + B\Sigma_V M^{p\prime})(L^p P^p L^{p\prime} + M^p \Sigma_V M^{p\prime})^{-1}, \\ P^p &= AP^p A' + B\Sigma_V B' - k^p (AP^p L^{p\prime} + B\Sigma_V M^{p\prime})'. \end{aligned}$$

with Σ_V the variance-covariance matrix of V_t.

2.4 The inference problem of the central bank

The information set of the central bank includes all elements of the state vector X_t except $\pi_t, y_t, r_t^n, \varepsilon_{tq}, E_t^p \pi_t^*$ and $E_t^p \varepsilon_{tq}$. It also has noisy indicators of inflation and output and is aware that private agents use equation (2.9) to make inferences about the inflation target. The inference problem it faces is hence whether observed inflation and output fluctuations are due to changes in the natural rate of interest, changes in inferences made by private agents, or measurement errors. This is again a standard problem that can be solved by applying the Kalman filter to an appropriately defined state space form. From the perspective of the central bank, the measurement and transition equations are:

$$Z_t^c = L^c X_{t-1} + M^c V_t, \tag{2.10}$$

$$X_t = A X_{t-1} + B V_t, \tag{2.11}$$

[4] Strictly speaking, we do not need to conjecture a transition equation to solve the inference problem of private agents. The only transition dynamics of interest to private agents are the exogenous evolution of the inflation target, so we could work with a simpler model. We prefer the more general specification to economise on notation later in the paper.

where $Z_t^c = (\; \pi_t + v_{t\pi} \quad y_t + v_{ty} \quad \pi_t^* \;)'$ is a vector of observed variables. The matrices L^c and M^c select the appropriate elements to map X_{t-1} and V_t into the observed variables. Since the central bank and private agents are both assumed to know the structure and parameters of the model, we use the same conjectured form for the transition equation as we did in the inference problem of private agents.[5] The central bank applies the Kalman filter to the state space form (2.10) and (2.11) and makes inferences according to:

$$E_t^c X_t = E_{t-1}^c X_t + k^c (Z_t^c - E_{t-1}^c L^c X_{t-1}), \tag{2.12}$$

where k^c and P^c satisfy the Kalman filtering equations:

$$\begin{aligned} k^c &= (AP^cL^{c\prime} + B\Sigma_V M^{c\prime})(L^cP^cL^{c\prime} + M^c\Sigma_V M^{c\prime})^{-1}, \\ P^c &= AP^cA' + B\Sigma_V B' - k^c(AP^cL^{c\prime} + B\Sigma_V M^{c\prime})'. \end{aligned}$$

A key feature of the model is that the central bank internalises the inference process of private agents when making its own inferences. We cannot see this explicitly in our exposition, but in equilibrium one row of the transition equation (2.11) is precisely the inference process of private agents (2.9). In this way the matrix A in the transition equation is a function of κ^p and L^p and the inference process of private agents is internalised.

3 Equilibrium dynamics

The equilibrium dynamics of the model are determined by the structural equations, the exogenous processes, and the processes by which private agents and the central bank infer the state of the economy. In the previous section we conjectured that equilibrium dynamics could be described by a transition equation of the form:

$$X_t = AX_{t-1} + BV_t. \tag{3.1}$$

[5] In other words, private agents and the central bank share common knowledge about the equilibrium laws of motion of the economy and the distribution of shocks. What they do not have common knowledge about is the precise values of some endogenous variables at a particular point in time. Confirmation of the validity of the conjectured transition equation (2.11) is again postponed until Section 3.

We now verify that equilibrium dynamics do have this form and that we can apply the method of undetermined coefficients to identify the matrices A and B.

3.1 Structural equations

The structural equations (2.1) and (2.2) can be expressed in terms of the state vector X_t by using the perceived short-term nominal interest rate (2.4) and the policy rule (2.5) to substitute out for the short-term nominal interest rate. The equilibrium dynamics are then:

$$I_{1\_2}E_t^p X_{t+1} = \begin{pmatrix} \beta & 0 \\ \sigma & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1 & -\kappa & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\sigma & \sigma(1-\phi) & \sigma & 0 & 0 & \sigma\phi & 0 & \sigma & 0 & 0 \end{pmatrix} X_t, \tag{3.2}$$

where I_{1_2} is an indicator matrix that selects the first two elements of $E_t^p X_{t+1}$. From the definition of the state vector X_t we know that $E_t^p X_t \subset X_t$ and we can write $E_t^p X_t = \Upsilon^p X_t$ for an appropriately-defined matrix Υ^p. From the transition equation (3.1) we also know that $E_t^p X_{t+1} = AE_t^p X_t$, so the left hand side of (3.2) is equivalent to $A_{1\_2}\Upsilon^p X_t$ with A_{1_2} defined as the first two rows of A. When written in this way, equation (3.2) has the same form as that conjectured in the transition equation (3.1) and it is valid to apply the method of undetermined coefficients. Comparing coefficients, we have:

$$A_{1\_2}\Upsilon^p = \begin{pmatrix} \beta & 0 \\ \sigma & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1 & -\kappa & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\sigma & \sigma(1-\phi) & \sigma & 0 & 0 & \sigma\phi & 0 & \sigma & 0 & 0 \end{pmatrix},$$

which defines the first two rows of the A matrix. The absence of any coefficients on V_t in (3.2) implies that all the elements in the first two rows of B are zero.

3.2 Exogenous processes

The exogenous processes (2.3) and (2.6) for r_t^n and π_t^* and the definition of ε_{tq} can be written in terms of the state vector X_t and the shock vector V_t as follows:

$$I_{3\_5}X_t = \begin{pmatrix} 0 & 0 & \delta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} X_{t-1} + \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} V_t,$$

where the indicator matrix I_{3_5} selects the third, fourth and fifth elements of X_t. Equilibrium dynamics are again consistent with the conjectured form of the transition equation (3.1) and comparing coefficients gives:

$$A_{3\_5} = \begin{pmatrix} 0 & 0 & \delta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$B_{3\_5} = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix},$$

so the exogenous processes define the third, fourth and fifth rows of A and B respectively.

3.3 Inference processes

The inference processes of private agents (2.9) and the central bank (2.12) state that equilibrium dynamics are given by:

$$E_t^p X_t = E_{t-1}^p A X_{t-1} + k^p (Z_t^p - E_{t-1}^p L^p X_{t-1}),$$
$$E_t^c X_t = E_{t-1}^c A X_{t-1} + k^c (Z_t^c - E_{t-1}^c L^c X_{t-1}),$$

where the observables are defined by $Z_t^p = L^p X_{t-1} + M^p V_t$ and $Z_t^c = L^c X_{t-1} + M^c V_t$. From the definition of the state vector X_t we have that $E_t^c X_t \subset X_t$ so we can write $E_t^c X_t = \Upsilon^c X_t$ and $E_{t-1}^c X_{t-1} = \Upsilon^c X_{t-1}$ for an appropriately-defined matrix Υ^c. Substituting these definitions, $E_t^p X_t = \Upsilon^p X_t$ and $E_{t-1}^p X_{t-1} = \Upsilon^p X_{t-1}$ into the inference processes implies:

$$X_t = (\Upsilon^p)^{-1} \left((A - k^p L^p)\Upsilon^p + k^p L^p \right) X_{t-1} + (\Upsilon^p)^{-1} k^p M^p V_t, \quad (3.3)$$
$$X_t = (\Upsilon^c)^{-1} \left((A - k^c L^c)\Upsilon^c + k^c L^c \right) X_{t-1} + (\Upsilon^c)^{-1} k^c M^c V_t, \quad (3.4)$$

which verifies the conjectured transition equation and validates the use of the method of undetermined coefficients. The sixth and seventh rows of (3.3) describe the inferences made by private agents and define A_{6_7} and zeros for B_{6_7} when coefficients are compared. The eight to twelfth rows of (3.4) describe the inferences made by the central bank and allow us to identify A_{8_12} and $B_{8\_12.}$ The other rows in equations (3.3) and (3.4) contain no new information as they simply replicate restrictions that have already imposed on A and B by the structural relationships and exogenous processes.

Our analysis demonstrates that the transition equation postulated in (2.8), (2.11) and (3.1) is consistent with equilibrium dynamics and that the matrices A and B are just identified by the method of undetermined coefficients. Equilibrium dynamics are characterised by a fixed point mapping in which the matrices A and B are a function of the Kalman gain parameters k^p and k^c, which themselves are a function of the matrices A and B and so on. The quantitative analysis of the next section is based on an iterative algorithm that solves for this fixed point.

4 Quantitative analysis

4.1 Calibration

The baseline calibration of our model is presented in Table 1, where the first three parameters follow Clarida, Galí and Gertler (2000). The remaining parameters are calibrated so that the inferences of both private agents and the central bank play a role in equilibrium. For example, our calibration implies that private agents assign a weight of 0.36 to prediction errors, a value somehwat higher than that estimated using US data by Erceg and Levin (2003). The key to making inference non-trivial in equilibrium is suitable calibration of the model's exogenous processes. We experimented with many alternatives, but only found inference mattering when the process for the inflation target was calibrated as more persistent than the process for the natural rate of interest. In our baseline calibration we therefore make the inflation target highly persistent and the natural rate of interest close to *iid*.

Parameters	k	σ^{-1}	β	ϕ_π	ρ	$\sigma_{\varepsilon p}$	$\sigma_{\varepsilon q}$	δ	σ_u	σ_π	σ_y
Value	0.3	1	0.99	1.5	0.95	0.05	0.1	0.35	0.2	0.1	0.1

Table 1: Calibration

The standard deviations of innovations of the exogenous processes also matter for inference in equilibrium. Innovations to the persistent component of the inflation target are calibrated to a low value so that both persistent and transitory components are important for the dynamics of the overall inflation target. The results are less sensitive to the precise calibration of the standard deviations of other innovations, with our baseline calibration taking a broadly neutral stand as to the relative contribution of each innovation to equilibrium dynamics.

4.2 Results

The asymmetric and incomplete information in our model causes misperceptions in expectations that impact upon aggregate dynamics. Misperceptions in the expectations of private agents affect output and inflation through the IS and aggregate supply equations, whereas misperceptions in central bank expectations have an effect through the short-term nominal interest rate. A further complication is that misperceptions arise and persist endogenously in our model as a result of the interrelated inference processes of private agents and the central bank. To explain these mechanisms in the calibrated model we examine the impact of each exogenous shock in turn. We then quantify the amount of volatility they cause in the aggregate economy.

4.2.1 Shocks to private perceptions

A negative shock ε_{tq} to private perceptions causes the nominal short-term interest rate perceived by private agents to be lower than expected. Private agents react to this news by attributing the lower nominal interest rate to either a negative misperceptions shock ε_{tq} or a positive shock to the inflation target ε_{tp}. Optimal filtering requires private agents to put at least some weight on both these possibilities and partially revise up their estimate of the central bank inflation target π_t^*. In this way the perceptions shock causes private agents to make an error in their inference. The error leads to a revision of private expectations of inflation and output in the next period, and consequently equilibrium inflation and output are higher than what the central bank expected in the next period.

From the perspective of the central bank, the news that inflation and output are higher than expected can be rationalised in three different ways. Firstly, it could be due to pure measurement errors $v_{t\pi}$ and v_{ty} in inflation and output, in which case there is no effect on private beliefs and there is no need for the central bank to revise its beliefs. Secondly, it could be due to a negative perceptions shock ε_{tq}, which the central bank knows causes private agents to partially revise up their estimate of the inflation target. In this case the central bank needs to change its beliefs $E_t^c E_t^p \pi_t^*$. Thirdly, it could be due to a positive natural rate of interest shock u_t, which requires the central bank to update its beliefs $E_t^c r_t^n$. That there are three different ways of rationalising news causes problems for central bank inference. The central bank inevitably makes errors, even when filtering optimally as prescribed by the Kalman filter. In the case of a perceptions shock examined here, the central bank mistakenly attributes at least part of the unexpected change in inflation and output to a natural rate of interest shock.

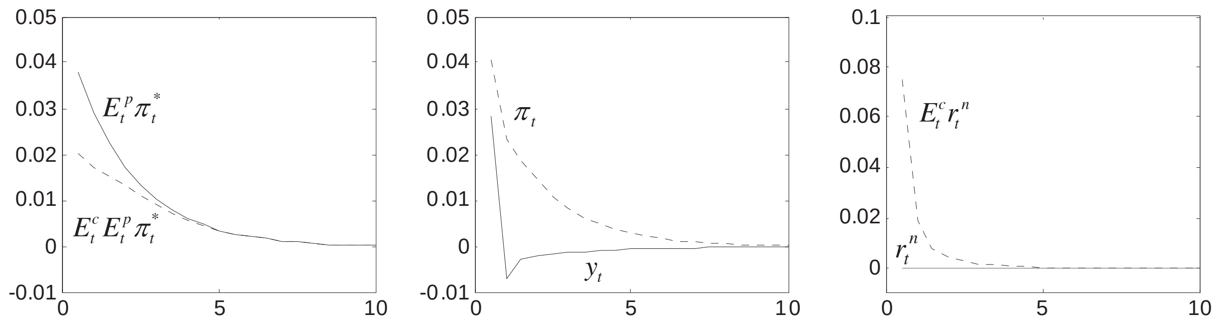

Figure 1: Response to a perceptions shock ε_{tq}

Figure 1 shows how the error made by private agents translates into an error made by the central bank in the baseline calibration of our model. The first panel shows how private agents mistakenly revise up their belief about the inflation target after a one standard deviation negative perceptions shock. This leads in the second panel to a rise in inflation and a rise in output, which in the third panel the central bank incorrectly attributes to a positive shock to the natural rate of interest.

The error made by the central bank is a direct consequence of the error made by private agents. It is only because private agents mistakenly revise up their expectation of the inflation target that the central bank mistakenly infers a change in the natural rate of interest. This is the central mechanism in our paper; asymmetric and incomplete information creates a vicious circle in which the difficulty private agents have in inferring the inflation target makes it harder for the central bank to infer the natural rate of interest.

4.2.2 Shocks to the natural rate of interest

The natural rate of interest is observed directly by private agents but can only be inferred by the central bank on the basis of its observations of output and inflation. The initial response to a positive u_t shock is therefore an increase in inflation and output as consumers and firms react to the increase in aggregate demand implied by a higher natural rate of interest. As was the case for the shock to private perceptions, there are three different ways in which the central bank can rationalise rising inflation and output. Optimal inference is once more destined to make errors, in this case the central bank mistakenly attributes at least part of the rise in inflation and output to measurement errors and a negative perceptions shock when in reality it is all due to a positive natural rate of interest shock. The errors made by the central bank are unavoidable when information is asymmetric and incomplete as in our model. What is needed is a transparent monetary policy regime in which the beliefs of private agents are anchored to the inflation target. Such a regime would improve policy as the central bank would no longer attribute changes in inflation and output to the wrong type of shock.

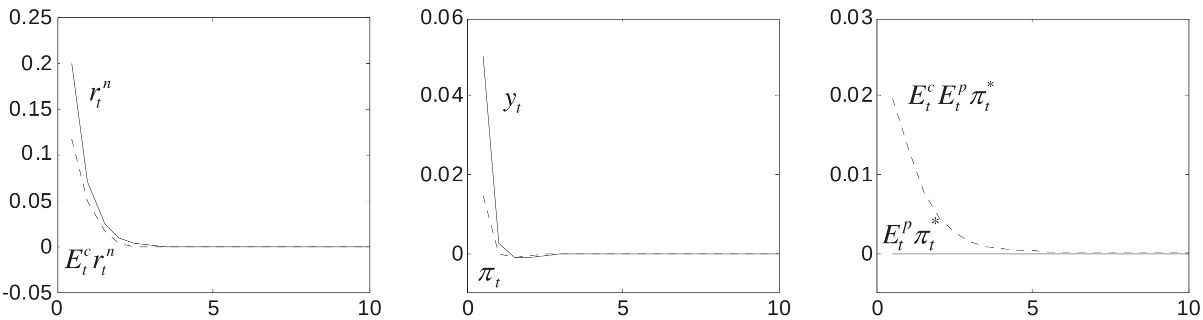

Figure 2: Response to a natural rate of interest shock u_t

Figure 2 traces out the effects of a positive natural rate of interest shock in the baseline calibration of the model. The first panel shows how the shock is initially not fully picked up by the central bank. The behaviour of consumers and firms - who do observe the shock

- leads to an increase in inflation and output in the second panel, which the central bank partially attributes to a perceptions shock in the third panel. Returning to the second panel, it is apparent that the shock to the natural rate of interest has an effect on inflation. This is entirely driven by the interrelated inference problems of private agents and the central bank. If the central bank could observe the natural rate of interest directly then monetary policy could stabilise the economy and there would be no impact on inflation and output.

4.2.3 Shocks to the inflation target

A positive shock to the inflation target ε_{tp} has the same initial effect as a negative perceptions shock ε_{tq} in that the short-term nominal interest rate observed by private agents is lower than expected. Private agents react to this news as before by partially revising up their estimates of the central bank's inflation target, which *ceteris paribus* would lead to higher inflation and output. However, in contrast to the previous two cases the central bank now knows that the increase in inflation and output is caused by a change in its inflation target. This knowledge enables the central bank to adjust policy in real time to ensure that inflation and output are immediately stabilised. As a consequence, the (unreported) impulse response functions for inflation and output are completely flat.

The flat impulse responses misleadingly suggest that inflation target shocks have no role to play in inflation and output dynamics. In practice they do once we consider how inflation target shocks interact with other shocks in the economy. For example, if past realisations of shocks have left the central bank over-optimistic about its reputation, i.e., $E_t^c E_t^p \pi_t^* < E_t^p \pi_t^*$, then the central bank underestimates the effect of inflation target shocks on the beliefs of private agents. The unwanted volatility that results does not show up in simple orthogonalised impulse response analysis. To gain further insight we therefore need either generalised impulse response functions or more sophisticated summary statistics. In what follows we take the latter route and present equilibrium dynamics under different shock assumptions.

4.2.4 Volatility

The volatility of all variables in the model can be calculated directly from the transition equation (3.1). Table 1 shows the resulting unconditional standard deviations as a function of the standard deviation of shocks to private perceptions. We equate smaller values of the latter to a more transparent monetary policy regime in which the central bank is better able to communicate its inflation target. The analogy is appropriate since in both cases the private sector finds it easier to infer the inflation target and the central bank finds it easier to infer the natural rate of interest. Under this interpretation, the transparency of policy improves as we move down Table 1 and comparison of successive rows reveals the incentives for central bank transparency in the model.

ε_{tq}	$\pi_t^* - E_t^p \pi_t^*$	$r_t^n - E_t^c r_t^n$	y_t	$\pi_t - \pi_t^*$	π_t
0.300	0.087	0.169	0.180	0.098	0.071
0.250	0.083	0.162	0.152	0.091	0.067
0.200	0.080	0.152	0.127	0.085	0.066
0.150	0.074	0.140	0.109	0.078	0.068
0.100	0.065	0.124	0.097	0.069	0.073
0.000	0.000	0.081	0.089	0.030	0.090

Table 1: Unconditional standard deviations

As expected from the impulse response analysis above, Table 1 shows that an increase in transparency makes it easier for private agents to infer the inflation target of the central bank. This is apparent in the second column, where the falling standard deviation of $\pi_t^* - E_t^p \pi_t^*$ shows that the beliefs of private agents become more closely anchored to the inflation target as transparency improves. The central tenet of our paper is that the ability of the central bank to infer the natural rate of interest is intrinsically linked to this anchoring of beliefs. Such interrelatedness shows up in the third column, where the standard deviation of $r_t^n - E_t^c r_t^n$ falls in tandem with beliefs becoming more closely anchored to the inflation target. We therefore have our result that the central bank finds it easier to track the natural rate of interest when the beliefs of private agents are more closely anchored to the inflation target. A transparent monetary policy regime can then promote a virtuous circle in which both private agents and the central bank find it easier to perform their respective inferences.[6]

In terms of aggregate outcomes, columns four and five of Table 1 demonstrate that transparency reduces the volatility of output and brings inflation closer to its target π_t^*. We do not have an explicit welfare metric in our model, but the incentives for transparency are strong in the sense that it unambiguously reduces any convex combination of the standard deviations of output and $\pi_t - \pi_t^*$. The final column of Table 1 shows that transparency has a non-monotonic impact on the volatility of inflation. The non-monotonicity arises because there are two channels through which transparency affects inflation volatility in the model. Firstly, transparency decreases volatility because it is easier for the central bank to fine-tune stabilisation policy when it is better informed about the natural rate of interest in the economy. This is the central mechanism that we have already stressed. Secondly, transparency increases volatility as it becomes possible for the central bank to ensure that inflation closely follows the time-varying inflation target. At low levels of transparency the first channel dominates and the volatility in inflation represents undesirable deviations of inflation from its target. At high levels of transparency the second channel dominates and the resulting volatility reflects desirable correlation between inflation and its target. It is therefore likely that transparency is good for welfare even though its effects on inflation volatility are ambiguous. The finding that transparency unambiguously reduces output volatility vindicates our original desire to relax the assumption in Aoki and Kimura (2008) that output follows an exogenous process. Furthermore, the ambiguity with which transparency affects inflation volatility in our results suggests that the strongest evidence of improved transparency would be an moderation in output rather than inflation volatility.

[6]Note that the central bank makes errors in its inference even under full transparency due to the presence of measurement errors.

5 Conclusions

That inflation and output volatility exhibit time-variation has been convincingly argued by McConnell and Perez-Quiros (2000), Cogley and Sargent (2005) and many others over the last decade. The mechanism we identify in this paper contributes to this debate by showing how small changes in the transparency of monetary policy can have important implications for aggregate volatility. Our reasoning is that improvements to transparency create a virtuous circle in which private agents find it easier to infer the objectives of the central bank and the central bank is better placed to identify the shocks hitting the economy. If the central bank can improve its knowledge in this way then it is in a better position to fine-tune policy and stabilise the economy. At the heart of our contribution is an assumption that information is asymmetric and incomplete; we assume that the central bank has better information about its own objectives than the private agents and that private agents have better information about the natural rate of interest than the central bank. Neither of these assumptions seems unreasonable. With such an information structure such as this in place the inference problems of the central bank and private agents become interrelated and the effects of changes in transparency are magnified. We therefore identify a new channel through which transparency promotes macroeconomic stability of both output and inflation in a DSGE model.

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