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## **Trade, Competition, and Efficiency**

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**Abstract:**

We present a general equilibrium model of monopolistic competition featuring pro-competitive effects and a competitive limit, and investigate the impact of trade on welfare and efficiency. Contrary to the constant elasticity case, in which all gains from trade are due to product diversity, our model allows for a welfare decomposition between gains from product diversity and gains from pro-competition effects. We then show that the market outcome is not efficient because too many firms operate at an inefficiently small scale by charging prices above marginal costs. Using pro-competitive effects and the competitive limit, we finally illustrate that trade raises efficiency by narrowing the gap between the equilibrium utility and the optimal utility.

**Keywords:** Pro-competitive effects, competitive limit, excess entry, trade and efficiency, monopolistic competition

**JEL Classification:** D43, D51, F12

# 1 Introduction

Few trade theorists would disagree with the statement that product diversity, scale economies, and pro-competitive effects are central to any discussion about gains from trade and efficiency with differentiated goods under imperfect competition.<sup>1</sup> Yet, it is fair to say that these questions have not been fully and jointly explored within a simple and solvable general equilibrium model of monopolistic competition.<sup>2</sup> This is largely due to the fact that the workhorse approach to international trade under monopolistic competition, namely the constant elasticity of substitution (henceforth, CES) framework, displays two peculiar features. First, it does not allow for pro-competitive effects so that “there is no effect of trade on the scale of production, and the gains from trade come solely through increased product diversity” (Krugman, 1980, p.953). Second, the equilibrium in the CES model is usually constrained (second-best) optimal, i.e., the market provides the socially desirable number of varieties at an efficient scale (Dixit and Stiglitz, 1977). Consequently, trade is not efficiency enhancing in the CES model because it does not correct the only existing market failure, pricing above marginal cost.

In order to more fully explore gains from trade and efficiency under monopolistic competition, we must depart from the standard CES model. Doing so, however, has long been difficult since variable elasticity of substitution (henceforth, VES) models have “not proved tractable, and from Dixit and Norman (1980) and Krugman (1980) onwards, most writers have used the CES specification” (Neary, 2004, p.177). Building on the new general equilibrium model of monopolistic competition by Behrens and Murata (2007), we present a simple VES model of international trade featuring pro-competitive effects (i.e., profit-maximizing prices are decreasing in the mass of competing firms) and a competitive limit (i.e., profit-maximizing prices converge to marginal costs when the mass of competing firms becomes arbitrarily large). Within this framework, we investigate the impact of trade on welfare and efficiency.

Our results can be summarized as follows. First, unlike in the standard CES model, the market outcome is not efficient under autarky because too many firms operate at an inefficiently small scale by charging prices above marginal costs.<sup>3</sup> Second, due to pro-competitive effects, markups in autarky are no longer the same across countries of different sizes. Nevertheless,

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<sup>1</sup>Dixit (2004, p.128) summarizes the gains from trade under monopolistic competition as follows: (i) availability of greater variety; (ii) better exploitation of economies of scale; and (iii) greater degree of competition, driving prices closer to marginal costs. More recently, the World Trade Organization (2008, pp.48-50) provides a similar classification: (i) gains from increased variety; (ii) gains from increased competition; and (iii) gains from increased economies of scale.

<sup>2</sup>For instance, the World Trade Organization (2008, p.48) states that “(a)s far as the gains from intra-industry trade are concerned, most studies have focused on either one of the variety, scale or pro-competitive (price) effects of trade opening”.

<sup>3</sup>Note that in a more general CES model, where market power and taste for variety are disentangled, the equilibrium mass of firms can be larger or smaller than the optimal one (Benassy, 1996). However, that discrepancy is not due to pro-competitive effects because the model displays constant markups.

we show that free trade leads not only to the equalization of price-wage ratios but also to both product and factor price equalization, even when products are differentiated. Third, we concisely illustrate that trade leads to an increase in the mass of varieties consumed, and to a decrease in the mass of varieties produced in each country. In addition, we show that such exit of firms due to pro-competitive effects is accompanied by an increase in output per firm, which leads to a better exploitation of firm-level scale economies. Fourth, contrary to the CES case, in which all gains from trade are due to greater product diversity, our model allows for a welfare decomposition between gains from product diversity and gains from pro-competitive effects. Finally, using pro-competitive effects and the competitive limit, we illustrate that trade raises efficiency by narrowing the gap between the equilibrium utility and the optimal utility.

The remainder of the paper is organized as follows. Section 2 develops the model, and Section 3 focuses on the autarky case. Section 4 analyzes the trade equilibrium, decomposes the gains from trade, and shows that trade integration drives prices closer to marginal costs, thus enhancing efficiency. Section 5 concludes.

## 2 Model

Consider a world with two countries, labeled  $r$  and  $s$ . Variables associated with each country will be subscripted accordingly. There is a mass  $L_r$  of workers/consumers in country  $r$ , and each worker supplies inelastically one unit of labor. Thus,  $L_r$  also stands for the total amount of labor available in country  $r$ . We assume that labor is internationally immobile and that it is the only factor of production.

### 2.1 Preferences

There is a single monopolistically competitive industry producing a horizontally differentiated consumption good with a continuum of varieties. Let  $\Omega_r$  (resp.,  $\Omega_s$ ) be the set of varieties produced in country  $r$  (resp.,  $s$ ), of measure  $n_r$  (resp.,  $n_s$ ). Hence,  $N \equiv n_r + n_s$  stands for the endogenously determined mass of available varieties in the global economy. A representative consumer in country  $r$  solves the following consumption problem, with ‘constant absolute risk aversion’ (CARA) sub-utility functions (Behrens and Murata, 2007):

$$\begin{aligned} \max_{q_{rr}(i), q_{sr}(j)} \quad U_r &\equiv \int_{\Omega_r} [1 - e^{-\alpha q_{rr}(i)}] di + \int_{\Omega_s} [1 - e^{-\alpha q_{sr}(j)}] dj \\ \text{s.t.} \quad &\int_{\Omega_r} p_{rr}(i) q_{rr}(i) di + \int_{\Omega_s} p_{sr}(j) q_{sr}(j) dj = E_r, \end{aligned} \tag{1}$$

where  $\alpha > 0$  is a utility parameter;  $E_r$  stands for the expenditure;  $p_r(i)$  denotes the price of variety  $i$ , produced in country  $r$ ; and  $q_{sr}(j)$  stands for the per-capita consumption of variety  $j$ , produced in country  $s$  and sold in country  $r$ .

We show in Appendix A that the demand functions for country- $r$  consumers are given as follows:

$$q_{rr}(i) = \frac{E_r - \frac{1}{\alpha} \int_{\Omega_r} \ln \left[ \frac{p_r(i)}{p_r(j)} \right] p_r(j) dj - \frac{1}{\alpha} \int_{\Omega_s} \ln \left[ \frac{p_r(i)}{p_s(j)} \right] p_s(j) dj}{\int_{\Omega_r} p_r(j) dj + \int_{\Omega_s} p_s(j) dj}, \quad (2)$$

$$q_{sr}(j) = \frac{E_r - \frac{1}{\alpha} \int_{\Omega_r} \ln \left[ \frac{p_s(j)}{p_r(i)} \right] p_r(i) di - \frac{1}{\alpha} \int_{\Omega_s} \ln \left[ \frac{p_s(j)}{p_s(i)} \right] p_s(i) di}{\int_{\Omega_r} p_r(i) di + \int_{\Omega_s} p_s(i) di}. \quad (3)$$

Mirror expressions hold for country- $s$  consumers. Because of the continuum assumption firms are negligible, so that the own-price derivatives of the demand functions are given as follows:

$$\frac{\partial q_{rr}(i)}{\partial p_r(i)} = -\frac{1}{\alpha p_r(i)} \quad \frac{\partial q_{sr}(j)}{\partial p_s(j)} = -\frac{1}{\alpha p_s(j)}, \quad (4)$$

which then yields the variable demand elasticities  $\varepsilon_{rr}(i) = [\alpha q_{rr}(i)]^{-1}$  and  $\varepsilon_{sr}(j) = [\alpha q_{sr}(j)]^{-1}$ . Mirror expressions hold again for country  $s$ .

## 2.2 Technology

All firms have access to the same increasing returns to scale technology. To produce  $Q(i)$  units of any variety requires  $l(i) = cQ(i) + F$  units of labor, where  $F$  is the fixed and  $c$  is the marginal labor requirement. We assume that firms can costlessly differentiate their products and that there are no scope economies. Thus, there is a one-to-one correspondence between firms and varieties, so that the mass of varieties  $N$  also stands for the mass of firms operating in the global economy. There is free entry and exit in each country, which implies that  $n_r$  and  $n_s$  are endogenously determined by the zero profit conditions. Consequently, the expenditure  $E_r$  equals the wage  $w_r$  in country  $r$ .

International markets are assumed to be integrated, so that firm  $i \in \Omega_r$  sets a unique free-on-board price  $p_r(i)$  for consumers in both countries. Its profit is then as follows:

$$\Pi_r(i) = [p_r(i) - cw_r] Q_r(i) - Fw_r, \quad (5)$$

where  $Q_r(i) \equiv L_r q_{rr}(i) + L_s q_{rs}(i)$  stands for its total output.

## 2.3 Equilibrium

Country- $r$  (resp., country- $s$ ) firms maximize their profit (5) with respect to  $p_r(i)$  (resp.,  $p_s(j)$ ), taking the vectors  $(n_r, n_s)$  and  $(w_r, w_s)$  of firm distribution and factor prices as given.<sup>4</sup> This

<sup>4</sup>It is well known that price and quantity competition yield the same outcome in monopolistic competition models with a continuum of firms (Vives, 1999, p.168).

yields the following first-order conditions:

$$\frac{\partial \Pi_r(i)}{\partial p_r(i)} = Q_r(i) + [p_r(i) - cw_r] \left[ L_r \frac{\partial q_{rr}(i)}{\partial p_r(i)} + L_s \frac{\partial q_{rs}(i)}{\partial p_r(i)} \right] = 0, \quad (6)$$

$$\frac{\partial \Pi_s(j)}{\partial p_s(j)} = Q_s(j) + [p_s(j) - cw_s] \left[ L_s \frac{\partial q_{ss}(j)}{\partial p_s(j)} + L_r \frac{\partial q_{sr}(j)}{\partial p_s(j)} \right] = 0. \quad (7)$$

Conditions (6) and (7) highlight a fundamental property of monopolistic competition models: although each firm is negligible to the market, it must take into account the aggregate pricing decisions of the other firms since their prices enter its first-order condition.

We define a *price equilibrium* as a distribution of prices satisfying (6) and (7) for all  $i \in \Omega_r$  and  $j \in \Omega_s$ . We will discuss its existence, uniqueness, and some other properties in the following sections.<sup>5</sup>

An *equilibrium* is a price equilibrium and vectors  $(n_r, n_s)$  and  $(w_r, w_s)$  of firm distribution and factor prices such that national factor markets clear, trade is balanced, and firms earn zero profits. More formally, an equilibrium is a solution to the following three conditions:

$$\int_{\Omega_r} [cQ_r(i) + F] di = L_r, \quad (8)$$

$$\int_{\Omega_s} [cQ_s(j) + F] dj = L_s, \quad (9)$$

$$L_s \int_{\Omega_r} p_r(i) q_{rs}(i) di = L_r \int_{\Omega_s} p_s(j) q_{sr}(j) dj, \quad (10)$$

where all quantities are evaluated at a price equilibrium. It is readily verified that firms earn zero profits when conditions (8)–(10) hold. One may set either  $w_r$  or  $w_s$  as the numeraire. However, we need not choose a numeraire, since the model is fully determined in real terms.<sup>6</sup>

### 3 Autarky

Assuming that the two countries cannot trade initially with each other, we first characterize the equilibrium and the optimum in the closed economy, and show that there is excess entry in equilibrium. Without loss of generality, we consider country  $r$  in what follows.

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<sup>5</sup>As shown by Roberts and Sonnenschein (1977), the existence of (price) equilibria is usually problematic in monopolistic competition models, since firms' reaction functions may be badly behaved. Because our model relies on a continuum of firms, which are individually negligible, we do not face this problem. In a similar spirit, Neary (2003) uses a general equilibrium model of oligopolistic competition with a continuum of sectors, in which firms are 'large' in their own markets but 'negligible' in the whole economy. This also allows to restore equilibrium since firms cannot directly influence aggregates of the whole economy.

<sup>6</sup>The choice of the numeraire is immaterial in our monopolistic competition framework. This is an important departure from general equilibrium oligopoly models, where the choice of the numeraire is usually not neutral (Gabszewicz and Vial, 1972).

### 3.1 Equilibrium

Inserting (2)–(4) into (6), and letting  $q_{rs}(i) = \partial q_{rs}(i)/\partial p_r(i) = 0$  and  $q_{sr}(j) = \partial q_{sr}(j)/\partial p_s(j) = 0$ , one can show that the unique price equilibrium is symmetric and given as follows (see Behrens and Murata, 2007, for the proof):

$$p_r^a = \left(1 + \frac{\alpha}{cn_r^a}\right) cw_r^a, \quad (11)$$

where an  $a$ -superscript henceforth denotes autarky values. At the symmetric price equilibrium, the profit of each firm is given by  $\Pi_r^a = L_r q_{rr}^a (p_r^a - cw_r^a) - Fw_r^a$ . Using the consumer's budget constraint  $w_r^a = n_r^a p_r^a q_{rr}^a$ , the above expression can be rewritten as

$$\Pi_r^a = p_r^a q_{rr}^a [L_r (1 - cn_r^a q_{rr}^a) - Fn_r^a].$$

Zero profits then imply that the quantities must be such that

$$q_{rr}^a = \frac{1}{c} \left( \frac{1}{n_r^a} - \frac{F}{L_r} \right), \quad (12)$$

which are positive because  $n_r^a F < L_r$  must hold from the resource constraint when  $n_r^a$  firms operate. Utility is then given by

$$U(n_r^a) = n_r^a \left[ 1 - e^{-\frac{\alpha}{c} \left( \frac{1}{n_r^a} - \frac{F}{L_r} \right)} \right]. \quad (13)$$

Note that (12) and (13) hold whenever prices are symmetric and firms earn zero profit.

Inserting  $q_{rr}^a = w_r^a / (n_r^a p_r^a)$  into the labor market clearing condition (8), we get:

$$n_r^a = \frac{L_r}{F} \left( 1 - \frac{cw_r^a}{p_r^a} \right). \quad (14)$$

The equilibrium mass of firms can then be found by using (11) and (14), which yields:<sup>7</sup>

$$n_r^a = \frac{\sqrt{4\alpha c F L_r + (\alpha F)^2} - \alpha F}{2cF} > 0. \quad (15)$$

Finally, inserting (15) into (13), the equilibrium utility in autarky is given by

$$U(L_r) = \frac{\sqrt{4\alpha c F L_r + (\alpha F)^2} - \alpha F}{2cF} \left[ 1 - e^{-\frac{2\alpha F}{\sqrt{4\alpha c F L_r + (\alpha F)^2} + \alpha F}} \right] > 0, \quad (16)$$

which is a strictly increasing and strictly concave function of the population size  $L_r$  for all admissible parameter values, i.e.,  $\alpha > 0$ ,  $c > 0$ ,  $F > 0$ , and  $L_r > 0$ .

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<sup>7</sup>Note that the other root is negative and must, therefore, be ruled out.

## 3.2 Optimum

We now determine the socially optimal mass of varieties. The planner maximizes the utility, as given by (1), subject to the technology and resource constraints (8). The first-order conditions of this problem with respect to  $q_{rr}(i)$  show that the quantities must be symmetric. This, together with (8), implies that:

$$q_{rr} = \frac{Q_r}{L_r} = \frac{1}{c} \left( \frac{1}{n_r} - \frac{F}{L_r} \right). \quad (17)$$

Hence, the planner maximizes

$$U(n_r^o) = n_r^o \left[ 1 - e^{-\frac{\alpha}{c} \left( \frac{1}{n_r^o} - \frac{F}{L_r} \right)} \right], \quad (18)$$

with respect to  $n_r^o$ , where an  $o$ -superscript henceforth denotes the optimal values.<sup>8</sup> Standard calculations show that

$$\frac{\partial U}{\partial n_r^o} = 1 - \left( 1 + \frac{\alpha}{cn_r^o} \right) e^{-\frac{\alpha}{c} \left( \frac{1}{n_r^o} - \frac{F}{L_r} \right)} \quad (19)$$

and that  $U$  is a strictly concave function of  $n_r^o$ . Equating (19) to zero, utility maximization requires the first-order condition

$$\frac{cn_r^o}{\alpha + cn_r^o} = e^{-\frac{\alpha}{c} \left( \frac{1}{n_r^o} - \frac{F}{L_r} \right)} \implies - \left( 1 + \frac{\alpha}{cn_r^o} \right) e^{-\left( 1 + \frac{\alpha}{cn_r^o} \right)} = -e^{-1 - \frac{\alpha F}{cL_r}}. \quad (20)$$

By definition of the Lambert W function, the latter can be rewritten as<sup>9</sup>

$$- \left( 1 + \frac{\alpha}{cn_r^o} \right) = W \left( -e^{-1 - \frac{\alpha F}{cL_r}} \right).$$

Solving this equation for  $n_r^o$  yields a unique optimal mass of firms

$$n_r^o = - \frac{\alpha}{c \left[ 1 + W_{-1} \left( -e^{-1 - \frac{\alpha F}{cL_r}} \right) \right]} > 0, \quad (21)$$

where  $W_{-1}$  is the real branch of the Lambert W function satisfying  $W(-e^{-1-\alpha F/(cL_r)}) \leq -1$  (Corless *et al.*, 1996, pp.330-331).<sup>10</sup> Note that  $W_{-1}$  is increasing in  $L_r$  and that  $-\infty < W_{-1} < -1$  for  $0 < L_r < \infty$ . Furthermore, using (20) we can show the following excess entry result.

**Proposition 1** *There are too many firms operating at an inefficiently small scale in equilibrium, i.e.,  $n_r^a > n_r^o$ .*

**Proof.** See Appendix B. ■

<sup>8</sup>As shown in Behrens and Murata (2006, Appendix B), alternative policies: (i) marginal cost pricing and lump-sum transfers; and (ii) profit-maximizing prices and non-negative profits, boil down to the same problem.

<sup>9</sup>Formally, the Lambert W function is defined as the inverse of the function  $x \mapsto xe^x$  (Corless *et al.*, 1996).

<sup>10</sup>As  $-e^{-1} < -e^{-1-\alpha F/(cL_r)} < 0$ , there is another possible real value of  $W(-e^{-1-\alpha F/(cL_r)})$  satisfying  $-1 < W(-e^{-1-\alpha F/(cL_r)}) < 0$ . However, it leads to  $n_r^o < 0$  and must, therefore, be ruled out.



Note that excess entry arises because of pro-competitive effects ( $\partial p_r^a / \partial n_r^a < 0$  by (11)). The negative externality each firm imposes on the other firms gives rise to the ‘business-stealing effect’ (Mankiw and Whinston, 1986, p.49), i.e., the equilibrium output per firm declines as the number of firms grows ( $\partial Q_r^a / \partial n_r^a = \partial(L_r q_{rr}^a) / \partial n_r^a < 0$  by (12)).

Interestingly, this result contrasts starkly with the constant elasticity case, where the equilibrium mass of varieties is (second-best) optimal.<sup>11</sup> Stated differently, the basic CES model does not account for the tendency that too many firms produce at inefficiently small scale in autarky (the so-called ‘Eastman-Stykolt hypothesis’; Eastman and Stykolt, 1967), an argument often used to criticize import-substituting industrialization policies (Krugman and Obstfeld, 2003, pp.261-263) or tariff barriers (Horstmann and Markusen, 1986) on efficiency grounds.

Finally, combining (18) and (20) yields  $U^o(n_r^o) = \alpha n_r^o / (\alpha + c n_r^o)$ . Inserting (21) into this expression, the optimal utility is given by

$$U^o(L_r) = -\frac{\alpha}{cW_{-1}\left(-e^{-1-\frac{\alpha F}{cL_r}}\right)} > 0, \quad (22)$$

which is a strictly increasing function of the population size  $L_r$  for all admissible parameter values, i.e.,  $\alpha > 0$ ,  $c > 0$ ,  $F > 0$ , and  $L_r > 0$ .

## 4 Free trade

We now analyze the impacts of trade on welfare and efficiency in a world with pro-competitive effects and a competitive limit. Section 4.1 analyzes the equilibrium. Section 4.2 then shows the existence of gains from trade and decomposes them into gains from product diversity and gains from pro-competitive effects. Section 4.3 finally illustrates that trade narrows the gap between the equilibrium utility and the optimal utility by driving prices closer to marginal costs.

### 4.1 Equilibrium

We have shown that the profit-maximizing price under autarky is given by (11), where  $n_r^a$  is given by (15). Therefore, unlike in the standard CES model, markups in autarky are no longer the same across countries of different sizes. Nevertheless, we now show that free trade leads not only to the equalization of price-wage ratios, but also to product and factor price equalization. Assume that both countries can trade freely. The profits and the first-order conditions are still given by (5)–(7), respectively. Using these expressions, we establish the following proposition.

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<sup>11</sup>This can be seen from Dixit and Stiglitz (1977, p.301), when letting  $s = 1$  and  $\theta = 0$  in their equations (20) and (21), since there is no homogeneous good in our setting. Note that the two-factor two-sector CES trade model by Lawrence and Spiller (1983, p.68) even displays insufficient entry. This runs against the general tendency of excess entry obtained under “a range of very plausible situations” (Vives, 1999, p.176).

**Proposition 2** *Free trade leads to product and factor price equalization for all admissible parameter values.*

**Proof.** See Appendix C. ■

Note that there is a priori no reason for product price equalization to hold in our setting, even under free trade.<sup>12</sup> This is because firms sell differentiated varieties, factor markets are segmented, and firms are imperfectly competitive.

From Proposition 2, we know that  $p_r = p_s = p$  and  $w_r = w_s = w$ , which allows us to rewrite (2)–(4) as follows:

$$q_{rr} = q_{sr} = q_{ss} = q_{rs} = \frac{w}{Np} \quad (23)$$

and

$$\frac{\partial q_{rr}}{\partial p_r} = \frac{\partial q_{sr}}{\partial p_s} = \frac{\partial q_{ss}}{\partial p_s} = \frac{\partial q_{rs}}{\partial p_r} = -\frac{1}{\alpha p}. \quad (24)$$

Inserting (23) and (24) into the first-order condition (6), we obtain the price equilibrium:

$$p = \left(1 + \frac{\alpha}{cN}\right) cw, \quad (25)$$

which is an extension of the autarky case (11). Since prices and wages are equalized, all firms sell the same total quantity  $Q = (L_r + L_s)q$ . Labor market clearing then implies that  $n_r/n_s = L_r/L_s$ , which yields

$$n_r = \frac{L_r}{F} \left(1 - \frac{cw}{p}\right). \quad (26)$$

Plugging (25) into (26) and the analogous expression for country  $s$ , we obtain two equations with two unknowns  $n_r$  and  $n_s$ . Solving for the equilibrium masses of firms, we get

$$\begin{aligned} n_r &= \frac{L_r}{L_r + L_s} \frac{\sqrt{4\alpha cF(L_r + L_s) + (\alpha F)^2} - \alpha F}{2cF} \\ n_s &= \frac{L_s}{L_r + L_s} \frac{\sqrt{4\alpha cF(L_r + L_s) + (\alpha F)^2} - \alpha F}{2cF}. \end{aligned}$$

Thus, the equilibrium mass of firms in the global economy is given by

$$N = n_r + n_s = \frac{\sqrt{4\alpha cF(L_r + L_s) + (\alpha F)^2} - \alpha F}{2cF}, \quad (27)$$

which is an extension of the autarky expression (15). It is readily verified that  $N > \max\{n_r^a, n_s^a\}$ , thus implying that  $p/w < \min\{p_r^a/w_r^a, p_s^a/w_s^a\}$  from (11) and (25). Finally, from expressions (14) and (26), we obtain  $n_r < n_r^a$  and  $n_s < n_s^a$ . Hence, the relationship between trade and product diversity can be summarized as follows:

**Proposition 3** *When compared with autarky, the mass of varieties produced in each country decreases under free trade, whereas the mass of varieties consumed in each country increases.*

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<sup>12</sup>Most studies assume, rather than prove, that product price equalization holds under free trade (e.g., Helpman, 1981).

Proposition 3 illustrates exit of firms due to the pro-competitive effects of international trade. Once trade occurs, the price-cost margins in both countries decrease, thus driving some firms out of each national market (Krugman, 1979; Feenstra, 2004, Ch.5).<sup>13</sup> Factor market clearing then makes sure that firm-level and total production expands, as labor is reallocated from the fixed requirements of closing firms to the marginal requirements of surviving firms. This is an important departure from the CES model, in which such an effect does not arise. Note also that Proposition 2 holds regardless of country size. In autarky, a smaller country tends to have a smaller mass of firms, which implies a higher price-wage ratio. Therefore, the price-wage ratio in a small country decreases more than that in a large country under free trade, i.e., we observe convergence in price-wage ratios across countries.

Note, finally, that although there is a growing literature on firm heterogeneity in international trade (e.g., Melitz, 2003), the price-cost margin for each firm is usually assumed to be constant in these models because of the CES specification.<sup>14</sup> By contrast, our model captures the ‘old idea’ that international trade under imperfect competition leads to lower markups and hence to exit of firms even without firm heterogeneity (Dixit and Norman, 1980).

## 4.2 Welfare decomposition and gains from trade

We now discuss gains from trade by decomposing welfare as in Krugman (1981). Since varieties are symmetric under both free trade and autarky, the utility difference is given by:

$$U_r - U_r^a = N \left( 1 - e^{-\frac{\alpha w}{Np}} \right) - n_r^a \left( 1 - e^{-\frac{\alpha w_r^a}{n_r^a p_r^a}} \right).$$

Adding and subtracting  $n_r^a e^{-\alpha w/(n_r^a p)}$ , and rearranging the resulting terms, we obtain the following welfare decomposition:

$$U_r - U_r^a = \underbrace{N \left( 1 - e^{-\frac{\alpha w}{Np}} \right) - n_r^a \left( 1 - e^{-\frac{\alpha w}{n_r^a p}} \right)}_{\text{Product diversity}} + \underbrace{n_r^a \left( e^{-\frac{\alpha w_r^a}{n_r^a p_r^a}} - e^{-\frac{\alpha w}{n_r^a p}} \right)}_{\text{Pro-competitive effects}}, \quad (28)$$

which isolates the two channels, namely product diversity and pro-competitive effects, through which gains from trade materialize in our model.

We now examine the role and the sign of each component in expression (28) in more details, both from a theoretical and an empirical point of view.

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<sup>13</sup>In Lawrence and Spiller (1983, Proposition 7), international trade leads to a redistribution of existing firms between the two countries while the total mass of firms remains unchanged. This result is driven by changes in relative factor prices under free trade and, as pointed out by the authors, need not hold under variable markups.

<sup>14</sup>One notable exception is Melitz and Ottaviano (2008) who recently proposed a model that explains trade-induced exit by combining pro-competitive effects and firm heterogeneity in a quasi-linear framework. Yet, the quasi-linear specification rules out income effects and gives their model a partial equilibrium flavor.

**Product diversity:** As shown in Proposition 3, trade expands product diversity, as in the CES case, despite the exit of some domestic producers. Ceteris paribus this raises utility via ‘love-of-variety’, which can be seen as follows. Given the wage-price ratio under free trade,  $w/p$ , we have

$$U_r = N \left( 1 - e^{-\frac{\alpha w}{Np}} \right), \quad \frac{\partial U_r}{\partial N} = 1 - e^{-\frac{\alpha w}{Np}} \left( 1 + \frac{\alpha w}{Np} \right) > 0 \quad \forall N.$$

To obtain the last inequality, let  $z \equiv \alpha w/(Np)$  and  $h(z) \equiv 1 - e^{-z}(1+z)$ . Clearly,  $h(0) = 0$  and  $h'(z) > 0$  for all  $z > 0$ , which shows that for any given wage-price ratio  $w/p$ , utility increases with the range of varieties consumed.

Despite its central theoretical role in new trade theory, little is known until now about the empirical importance of gains from product diversity (Feenstra, 1995). Using extremely disaggregated data, Broda and Weinstein (2006) document that the number of product varieties in US imports rose by 212% between 1972 and 2001, and according to their estimates this maps into US welfare gains of about 2.6% of GDP. This finding suggests that product diversity is an important real-world channel through which gains from trade materialize.

**Pro-competitive effects:** The second term in (28) captures the beneficial effects of increased price competition in the product market, driving prices closer to marginal costs. This can be seen by comparing expressions (11) and (25). Note that  $w_r^a/p_r^a = w/p$  would hold in the CES case, i.e., there would be no gains from trade due to pro-competitive effects.

It is well known from various industrial organization studies that prices in many imperfectly competitive industries are increasing functions of producer concentration (see Schmalensee, 1989, pp.987-988, for a survey). In our symmetric equilibrium, the Herfindahl-index of concentration, defined as the sum of squared market shares, reduces to  $H = N(1/N)^2 = 1/N$ . Since the mass of firms is increasing in market size in our model, markups are lower in larger markets (see Campbell and Hopenhayn, 2005, for empirical evidence). Similarly, by increasing the number of competitors in each market, import competition decreases concentration, which maps into lower consumer prices. Several case studies confirm this ‘imports-as-market-discipline hypothesis’ (e.g., Levinsohn, 1993; Harrison, 1994; Tybout, 2003). More recently, Badinger (2007) finds solid evidence that the Single Market Programme of the EU has reduced markups by 26% in aggregate manufacturing of 10 member states.

Let us summarize our results as follows:

**Proposition 4** *Free trade raises welfare both by expanding consumers’ choice sets (‘product diversity’) and by driving prices closer to marginal costs (‘pro-competitive effects’).*

**Proof.** To prove our claim, it is sufficient to examine the sign of the two components in (28). As shown above, they are both positive, which ensures gains from trade. ■

### 4.3 Competition and efficiency

In Section 3.2 we have considered the case in which each national government maximizes its own welfare under its domestic resource constraint (‘decentralized’ first-best). As shown by Behrens and Murata (2006, Appendix D), doing so is equivalent to letting a global planner maximize world welfare under a global resource constraint (‘centralized’ first-best). Put differently, if  $n_r^o$  and  $n_s^o$  is a decentralized optimum, then  $N^o = n_r^o + n_s^o$  is a centralized optimum. Conversely, if  $N^o$  is a centralized optimum, then  $n_r^o = (L_r/L)N^o$  and  $n_s^o = (L_s/L)N^o$  is a decentralized optimum, where  $L \equiv L_r + L_s$ . The optimal utilities under both regimes are the same.

Since in our model free trade amounts to increasing the population size, the result on excess entry established in Section 3.2 continues to hold, even under free trade. Stated differently, there is a unique optimal mass of firms  $N^o < N$  satisfying the first-order condition (20) under free trade. Hence, there are too many firms operating at an inefficiently small scale and the market outcome is not efficient. Figure 1 depicts a typical example of  $N/N^o$  as a function of  $L$ .<sup>15</sup> It shows that excess entry gets larger as population increases. Yet, for any given value of  $L$ , the smaller the value of  $\alpha$  the narrower the gap between the equilibrium and the optimum. Furthermore,  $\lim_{L \rightarrow 0} N/N^o = 1$  and  $\lim_{L \rightarrow \infty} N/N^o = \sqrt{2}$  hold regardless of parameter values.

[Insert Figure 1 around here]

On the contrary, combining (25) and (27) we obtain pro-competitive effects as follows:

$$\frac{p}{cw} = 1 + \frac{2\alpha F}{\sqrt{4\alpha cFL + (\alpha F)^2} - \alpha F} \quad \text{and} \quad \frac{\partial}{\partial L} \left( \frac{p}{cw} \right) < 0. \quad (29)$$

It is then readily verified that our model exhibits a competitive limit

$$\lim_{L \rightarrow 0} \frac{p}{cw} = \infty \quad \text{and} \quad \lim_{L \rightarrow \infty} \frac{p}{cw} = 1,$$

so that prices converge to marginal costs as population gets arbitrarily large. Despite the fact that excess entry tends to get worse as population increases, such losses are dominated by efficiency gains from the competitive limit, thus yielding the following efficiency result.<sup>16</sup>

**Proposition 5** *When the population gets arbitrarily large in the integrated economy, the equilibrium utility converges to the optimal utility, i.e.,*

$$\lim_{L \rightarrow \infty} U(L) = \lim_{L \rightarrow \infty} U^o(L) = \frac{\alpha}{c}.$$

<sup>15</sup>The parameter values are as follows:  $\alpha = 0.1, 1, 5$  from above;  $c = 0.5$ ; and  $F = 1$ . Other admissible parameter values yield qualitatively similar figures, thus suggesting that the underlying property is robust.

<sup>16</sup>This result is reminiscent of Mankiw and Whinston (1986, Proposition 3) who establish conditions for the equilibrium utility in a partial equilibrium model to converge to the optimal utility when excess entry gets large.

**Proof.** Applying the l'Hospital's rule to (16), it is readily verified that  $\lim_{L \rightarrow \infty} U(L) = \alpha/c$ . Furthermore, by definition of the Lambert W function,  $\lim_{L \rightarrow \infty} W_{-1}(-e^{-1-\alpha F/(cL)}) = -1$  holds. Hence, taking the limit of expression (22) yields  $\lim_{L \rightarrow \infty} U^o(L) = \alpha/c$ .<sup>17</sup> ■

The limit result established in Proposition 5 may be extended to a finite economy by investigating whether

$$\max \left\{ \frac{U(L_r)}{U^o(L_r)}, \frac{U(L_s)}{U^o(L_s)} \right\} < \frac{U(L)}{U^o(L)} < 1, \quad (30)$$

where the last inequality comes from the definition of the optimum. To this end, we check whether the ratio of equilibrium to optimal utility  $U/U^o$  monotonically increases in  $L$ . This is not a trivial question since both the equilibrium and the optimal utility increase in  $L$ , as can be seen from (16) and (22).

[Insert Figure 2 around here]

Figure 2 depicts a typical example of  $U/U^o$  as a function of  $L$ .<sup>18</sup> As  $U/U^o < 1$  the market outcome remains inefficient for finite population sizes. However, for any given value of  $L$ , the smaller the value of  $\alpha$ , the narrower the gap between the equilibrium and the optimum. The reason is that a smaller value of  $\alpha$  implies lower  $p/(cw)$  for all  $L$  by (29). Furthermore, since  $U/U^o$  increases monotonically with  $L$ , trade between larger countries yields higher efficiency than trade between smaller countries.

## 5 Conclusions

We have developed a VES model of international trade displaying pro-competitive effects and a competitive limit and investigated the impact of trade on welfare and efficiency. First, we have shown that, unlike in the standard CES model, there is excess entry due to pro-competitive effects. Second, even in the presence of pro-competitive effects and product differentiation, free trade leads to product and factor price equalization between countries of different sizes. Third, we have illustrated the well-known property that trade leads to an increase in the mass of varieties consumed, and to a decrease in the mass of varieties produced, in each country. In addition, we have shown that exit due to pro-competitive effects is accompanied by an increase in output per firm because of resource reallocation from the fixed to the marginal labor requirement, leading to a better exploitation of firm-level scale economies. Fourth, contrary to the CES case, our model allows for a welfare decomposition between gains from product diversity and gains from pro-competitive effects. Finally, we have illustrated that trade is

<sup>17</sup>See Behrens and Murata (2006, Appendix E) for an alternative proof without using the Lambert W function.

<sup>18</sup>The parameter values are as follows:  $\alpha = 0.1, 1, 5$  from above;  $c = 0.5$ ; and  $F = 1$ . Other admissible parameter values yield qualitatively similar figures, thus suggesting that the underlying property is robust. Furthermore,  $\lim_{L \rightarrow 0} U/U^o = 1 - e^{-1} \approx 0.63$  holds regardless of parameter values.

efficiency enhancing through increased competition. In particular, as the population size gets arbitrarily large in the integrated economy, the equilibrium utility converges to the optimal utility because of a competitive limit.

The framework presented in this paper is flexible enough to allow for many extensions, including heterogeneous firms and trade costs (Behrens *et al.*, 2009) and heterogeneous consumers (Behrens and Murata, 2009). The next step is to extend our model to a multi-sector setting in order to fully explore the impact of trade on efficiency. In our model, trade tends to reduce within-sector efficiency losses, which disappear at the competitive limit. However, as is well known, markup heterogeneity across sectors is a source of between-sector distortions (Bilbiie *et al.*, 2008; Epifani and Gancia, 2009), which may become more important with freer trade. Whereas Epifani and Gancia (2009) show that trade may reduce efficiency, the existence of a competitive limit in our model suggests that trade ultimately leads to efficiency gains even with multiple sectors since prices in each sector converge to marginal costs as the mass of firms gets sufficiently large. Establishing this formally is left for future research.

Finally, as we have obtained the closed form solutions for the equilibrium utility and the optimal utility, our model can also be extended to a multi-region setting in a spatial economy. Doing so sheds new light on whether or not larger cities are more efficient. In such a setting, a larger population exacerbates not only excess entry but also congestion in cities while achieving prices closer to marginal costs. Although we have shown that efficiency gains from the competitive limit dominate efficiency losses from excess entry, it is not obvious whether or not our result carries over to a spatial economy with urban congestion. Exploring this formally is also left for future research.

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## Appendix A: Derivation of the demand functions

A representative consumer in country  $r$  solves problem (1). Letting  $\lambda$  stand for the Lagrange multiplier, the first-order conditions for an interior solution are given by:

$$\alpha e^{-\alpha q_{rr}(i)} = \lambda p_r(i), \quad \forall i \in \Omega_r \quad (31)$$

$$\alpha e^{-\alpha q_{sr}(j)} = \lambda p_s(j), \quad \forall j \in \Omega_s \quad (32)$$

and the budget constraint

$$\int_{\Omega_r} p_r(k) q_{rr}(k) dk + \int_{\Omega_s} p_s(k) q_{sr}(k) dk = E_r. \quad (33)$$

Taking the ratio of (31) with respect to  $i$  and  $j$ , we obtain

$$e^{-\alpha[q_{rr}(i)-q_{rr}(j)]} = \frac{p_r(i)}{p_r(j)} \implies q_{rr}(i) = q_{rr}(j) + \frac{1}{\alpha} \ln \left[ \frac{p_r(j)}{p_r(i)} \right] \quad \forall i, j \in \Omega_r.$$

Multiplying the last expression by  $p_r(j)$  and integrating with respect to  $j \in \Omega_r$  we obtain:

$$q_{rr}(i) \int_{\Omega_r} p_r(j) dj = \int_{\Omega_r} p_r(j) q_{rr}(j) dj + \frac{1}{\alpha} \int_{\Omega_r} \ln \left[ \frac{p_r(j)}{p_r(i)} \right] p_r(j) dj. \quad (34)$$

Analogously, taking the ratio of (31) and (32) with respect to  $i$  and  $j$ , we get:

$$e^{-\alpha[q_{rr}(i)-q_{sr}(j)]} = \frac{p_r(i)}{p_s(j)} \implies q_{rr}(i) = q_{sr}(j) + \frac{1}{\alpha} \ln \left[ \frac{p_s(j)}{p_r(i)} \right] \quad \forall i \in \Omega_r, \forall j \in \Omega_s.$$

Multiplying the last expression by  $p_s(j)$  and integrating with respect to  $j \in \Omega_s$  we obtain:

$$q_{rr}(i) \int_{\Omega_s} p_s(j) dj = \int_{\Omega_s} p_s(j) q_{sr}(j) dj + \frac{1}{\alpha} \int_{\Omega_s} \ln \left[ \frac{p_s(j)}{p_r(i)} \right] p_s(j) dj. \quad (35)$$

Summing (34) and (35), and using the budget constraint (33), we finally obtain the demands (2). The derivations of the demands (3) are analogous.

## Appendix B: Proof of Proposition 1

We need to compare the market outcome  $n_r^a$  and the optimum  $n_r^o$  defined as the solution of the equation:

$$f(n_r) = g(n_r), \quad \text{where} \quad f(n_r) \equiv \frac{cn_r}{\alpha + cn_r} \quad \text{and} \quad g(n_r) \equiv e^{-\frac{\alpha}{c} \left( \frac{1}{n_r} - \frac{F}{L_r} \right)}. \quad (36)$$

Note first that  $f$  is strictly increasing in  $n_r$ , taking values from 0 to 1, and that  $g$  is also strictly increasing, taking values from 0 to  $e^{\alpha F/(cL_r)} > 1$ . Some standard calculations show that there is a unique intersection since: (i) both functions are continuous; (ii)  $f$  is concave, whereas  $g$  is convex for  $n_r$  sufficiently small; (iii) the slope of  $f$  is strictly greater than that of  $g$  for  $n_r$

sufficiently small;<sup>19</sup> and (iv)  $g$  admits a single value for which its second-order derivative is equal to zero.

We next show that  $n_r^a > n_r^o$ . To prove our claim, we use a convexity argument. The equilibrium mass of varieties is given by (15), whereas the optimal mass of varieties is the unique solution to (36). First, evaluate  $f$  at  $n_r^a$ , which yields

$$f(n_r^a) = \frac{cn_r^a}{\alpha + cn_r^a} = \frac{-\alpha F + \sqrt{\alpha F(4cL_r + \alpha F)}}{\alpha F + \sqrt{\alpha F(4cL_r + \alpha F)}} = \frac{-2\alpha F + X_r}{X_r}, \quad (37)$$

where  $X_r \equiv \alpha F + \sqrt{\alpha F(4cL_r + \alpha F)}$ . Second, evaluate  $g$  at  $n_r^a$  to get

$$g(n_r^a) = e^{-\frac{\alpha}{c}\left(\frac{1}{n_r^a} - \frac{F}{L_r}\right)} = e^{-\frac{2\alpha F}{X_r}}. \quad (38)$$

Let  $Y_r \equiv (2\alpha F)/X_r < 1$  and  $g(Y_r) = e^{-Y_r}$ . Note that (37) can then be expressed as  $f(Y_r) = 1 - Y_r$ , which is tangent to (38) at  $Y_r = 0$ :

$$1 - Y_r = g(0) + g'(0)(Y_r - 0).$$

Since (38) is strictly convex, it lies strictly above its tangent. Put differently,  $f(Y_r) = 1 - Y_r < e^{-Y_r} = g(Y_r)$  holds for all  $Y_r > 0$  (see Figure A1). Hence, the right-hand side of (36) exceeds the left-hand side of (36) at the equilibrium mass of firms  $n_r^a$ . By uniqueness of the optimal mass of firms, and since the right-hand side of (36) exceeds the left-hand side if and only if  $n_r > n_r^o$ , we may conclude that  $n_r^a > n_r^o$ , which proves our claim (see Figure A2).

[Insert Figures A1 and A2 around here]

## Appendix C: Proof of Proposition 2

Conditions (6) and (7) must hold for both country- $r$  and country- $s$  firms at every price equilibrium which, using (2)–(4), yields

$$\frac{\partial \Pi_r(i)}{\partial p_r(i)} - \frac{\partial \Pi_s(j)}{\partial p_s(j)} = 0 \quad \iff \quad c \left[ \frac{w_r}{p_r(i)} - \frac{w_s}{p_s(j)} \right] = \ln \left( \frac{p_r(i)}{p_s(j)} \right). \quad (39)$$

---

<sup>19</sup>To check this, note that  $\lim_{n_r \rightarrow 0} f'(n_r) = c/\alpha > \lim_{n_r \rightarrow 0} g'(n_r) = 0$ . The last equality is obtained as follows. Noting that

$$\ln g'(n_r) = -\frac{2}{n_r} \left[ \frac{\ln(n_r)}{1/n_r} + \frac{\alpha}{2c} \right] + \ln \left( \frac{\alpha}{c} \right) + \frac{\alpha F}{cL},$$

and that  $\lim_{n_r \rightarrow 0} \ln n_r / (1/n_r) = 0$  by l'Hospital's rule, we have

$$\lim_{n_r \rightarrow 0} \ln g'(n_r) = -\lim_{n_r \rightarrow 0} \frac{2}{n_r} \times \lim_{n_r \rightarrow 0} \left[ \frac{\ln(n_r)}{1/n_r} + \frac{\alpha}{2c} \right] + \ln \left( \frac{\alpha}{c} \right) + \frac{\alpha F}{cL} = -\infty,$$

which, by continuity of the logarithmic function, implies  $\lim_{n_r \rightarrow 0} g'(n_r) = 0$ .

It is also readily verified that

$$Q_r(i) \begin{matrix} \geq \\ \leq \end{matrix} Q_s(j) \iff -\frac{L_r + L_s}{\alpha} \ln \left[ \frac{p_r(i)}{p_s(j)} \right] \begin{matrix} \geq \\ < \end{matrix} 0. \quad (40)$$

Furthermore, an equilibrium is such that firms earn zero profit, i.e.,

$$\begin{aligned} \Pi_r(i) &= w_r \left\{ \left[ \frac{p_r(i)}{w_r} - c \right] Q_r(i) - F \right\} = 0 \\ \Pi_s(j) &= w_s \left\{ \left[ \frac{p_s(j)}{w_s} - c \right] Q_s(j) - F \right\} = 0. \end{aligned}$$

Assume that there exists  $i \in \Omega_r$  and  $j \in \Omega_s$  such that  $p_r(i) > p_s(j)$ . Then condition (39) implies that

$$\frac{w_r}{p_r(i)} > \frac{w_s}{p_s(j)} \implies \frac{p_r(i)}{w_r} < \frac{p_s(j)}{w_s},$$

whereas condition (40) implies that  $Q_r(i) < Q_s(j)$ . Hence,  $\Pi_r(i) < \Pi_s(j)$ , which is incompatible with an equilibrium. We may hence conclude that  $p_r(i) = p_s(j)$  must hold for all  $i \in \Omega_r$  and  $j \in \Omega_s$ , which shows that product prices are equalized. Condition (39) then shows that  $w_r = w_s$ , i.e., factor prices are equalized whenever product prices are equalized, which proves our claim.

Figure 1:  $N/N^o$  as a function of  $L$ .

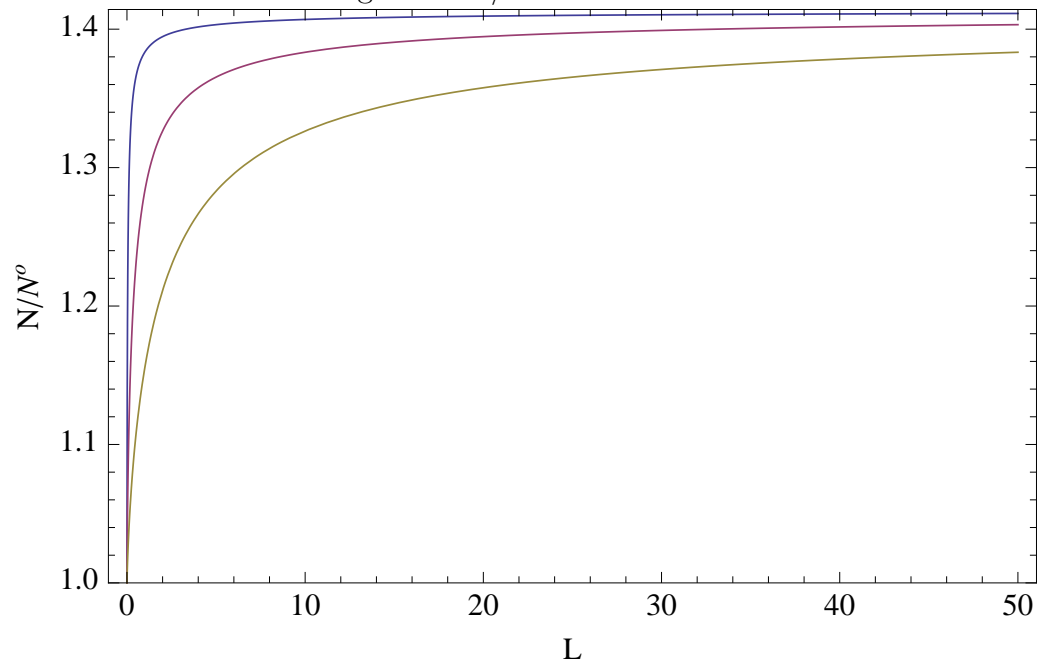


Figure 2:  $U/U^o$  as a function of  $L$ .

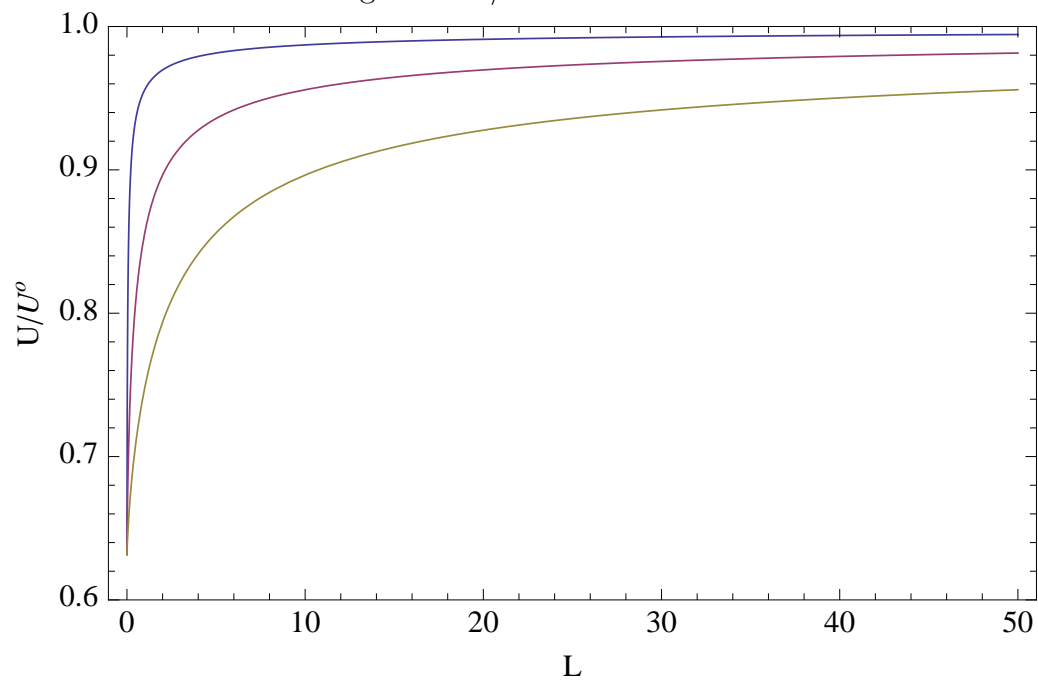


Figure A1:  $f$  and  $g$  as a function of  $Y_r$ .

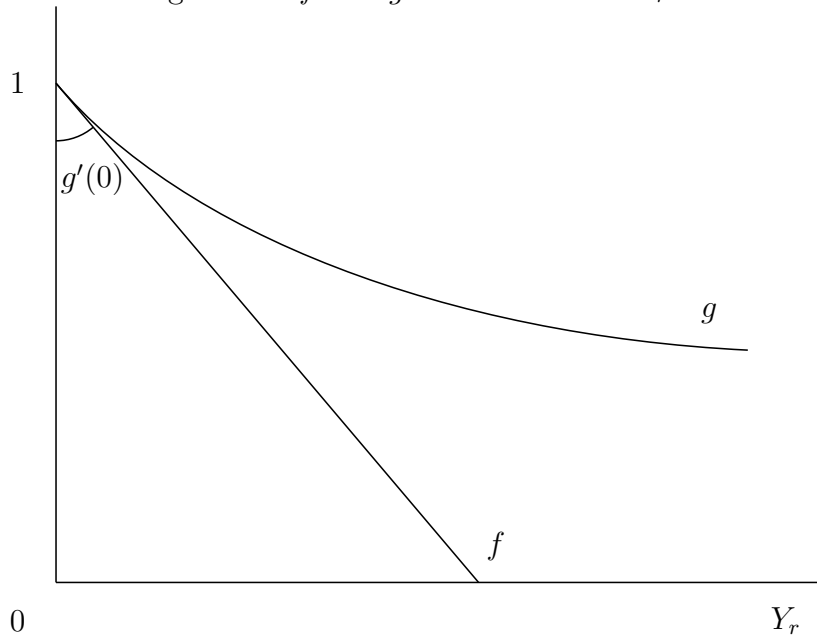


Figure A2: Excess entry.

