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NOVIEMBRE DE 2005****INTERNATIONAL TRADE, MIGRATION AND INVESTMENT  
WITH HORIZONTAL PRODUCT DIFFERENTIATION  
AND FREE ENTRY AND EXIT OF FIRMS****HERNÁN VALLEJO G.<sup>1</sup>****Abstract**

This paper builds on a circular road model of the world with horizontal product differentiation and free entry and exit of firms, to derive results that can be applied in industrial organization, international trade and political economy. The model shows that freer international trade increases welfare -with ideal variety preferences- through the exploitation of economies of scale and better allocative efficiency; that all participating countries gain from trade, and that smaller countries have more to win from free trade than larger countries. The model also explains that there may be adjustment costs when liberalizing trade and thus, political resistance to trade liberalization. International migration can also be analyzed with the model, showing the possibility of suboptimal migration flows and political barriers to the exit of national citizens. The model suggests that foreign direct investment will be welfare improving for the source country in the short run, and for the receiving country in the long run. Finally, the model provides a microfoundation for the use of demand curves with constant and negative slopes.

**Key Words:** Monopolistic competition, horizontal product differentiation, international trade, international migration and foreign direct investment.

**JEL Classification:** F12, F13.

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# COMERCIO, MIGRACIÓN E INVERSIÓN INTERNACIONAL CON DIFERENCIACIÓN HORIZONTAL DE PRODUCTO Y LIBRE ENTRADA Y SALIDA DE FIRMAS

## Resumen

Este documento se basa en un modelo de carretera circular del mundo con diferenciación horizontal de producto y libre entrada y salida de firmas, para derivar resultados que pueden ser aplicados en la organización industrial, el comercio internacional y la economía política. El modelo muestra que un comercio internacional más libre aumenta el bienestar –con preferencias de variedad ideal– por medio de la explotación de economías de escala y de una mejor asignación de recursos, que todos los países participantes ganan con el comercio, y que los países más pequeños tienen más que ganar del libre comercio que los países grandes. El modelo también explica que puede haber costos de ajuste al liberar el comercio y que por lo tanto, puede haber resistencia política a la liberación del comercio. La migración internacional también puede ser analizada con el modelo, identificando la posibilidad de que haya flujos subóptimos de migración y barreras de política a la salida de ciudadanos nacionales de un país. El modelo sugiere que la inversión extranjera directa mejorará el bienestar del país fuente en el corto plazo, y el bienestar del país receptor en el largo plazo. Finalmente, el modelo provee una microfundamentación para el uso de curvas de demanda con pendientes constantes y negativas.

**Palabras clave:** Competencia monopolística, diferenciación horizontal de producto, comercio internacional, migración internacional e inversión extranjera directa.

**Clasificación JEL:** F12, F13

## I. INTRODUCTION

This paper builds on the circular road model of horizontal product differentiation of Eaton *et al.* (1975) with free entry and exit of firms, to derive results that can be applied in industrial organization, international trade and political economy.

The model shows that freer international trade increases welfare -with ideal variety preferences- through the exploitation of economies of scale and improved allocative efficiency; that all participating countries gain from trade, and that smaller countries have more to win from free trade than larger countries. Furthermore, the model explains that there may be adjustment costs when liberalizing trade and thus, political resistance to trade liberalization. International migration can also be analyzed with the model, showing the possibility of suboptimal migration flows and political barriers to the exit of national citizens. The model suggests that foreign direct investment will be welfare improving for the source country in the short run, and for the receiving country in the long run.

## II. PREVIOUS LITERATURE

Many of the insights generated in this paper have been derived before in a monopolistic competition model within a general equilibrium setting, by Lancaster (1979) –with ideal variety preferences- and by Krugman (1979) –with love of variety preferences-. In fact, the model presented in this paper can be interpreted –at least in part- as a formal and partial equilibrium representation of the model presented in an intuitive manner by Lancaster (1979). A similar approach but with some differences in the parameters used and emphasising the effects of tariffs was followed by Schmitt (1990). Other authors that have made written in this area include Salop (1979), Schmitt (1995), and Boccard and Wauthy (2000).

The main contributions of this paper are that the results generated here are derived within a partial equilibrium framework, highlighting the different effects of trade in goods between larger and smaller countries; giving a more careful look at the political economy of trade and migration; and providing insights into the potential benefits of foreign direct

investment. The paper also provides a microfoundation to the use of demand curves with constant and negative slopes.

## II THE MODEL

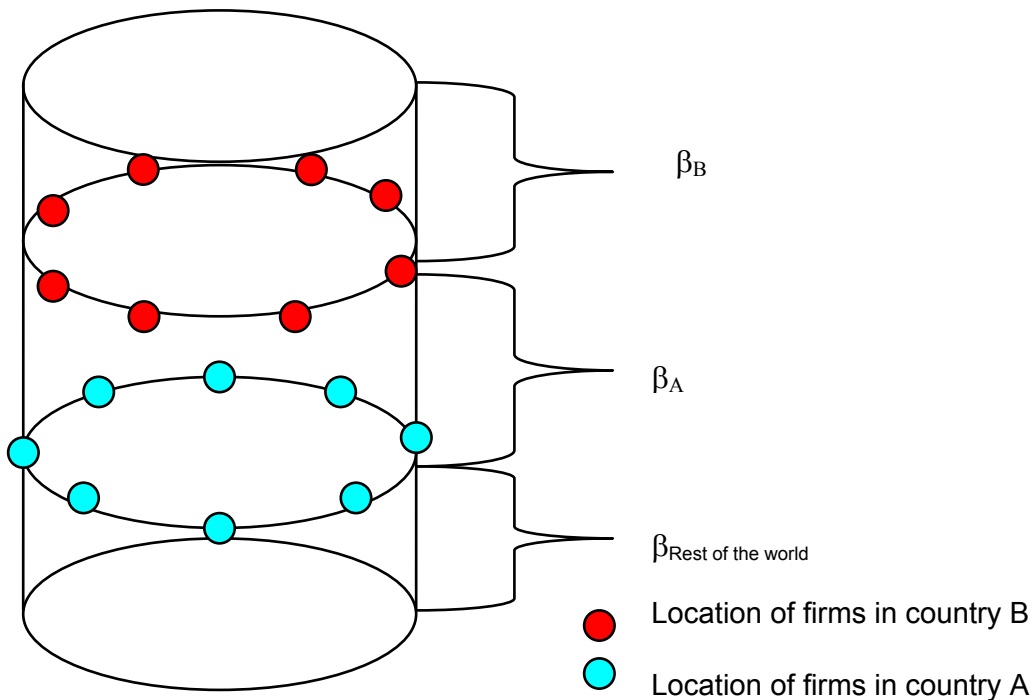
### A. Assumptions and notation

The basic assumptions of the model presented in this paper are:

- i. The world can be represented as a circular road of extension equal to  $\alpha$ , where all countries are located one on top of the other (see figure 1).

Figure 1

#### A CIRCULAR ROAD WORLD



- ii. There is one industry (this is a partial equilibrium model).
- iii. The good produced is homogeneous.

- iv. There are  $N$  producers of the homogeneous good, that represent  $N$  varieties of that good in terms of location.
- v. Firms play a two stage game: on the first stage, they determine locations, and on the second stage they determine their prices.
- vi. To solve the model by backward induction, in the second stage of the game firms are assumed to be located at a distance  $\frac{\alpha}{N}$  of each other.
- vii. The sales of the representative producer are noted by  $x$ .
- viii. Each firm has the same cost structure  $TC = f + ex$ , where  $f$  is the fixed cost and  $e$  is the constant marginal cost.
- ix. Consumers are homogeneous and uniformly distributed along the circular road (there are  $\beta$  consumers on every unit of distance of the road, with  $\beta > 0$ ).
- x. Consumers have identical ideal variety preferences and they all consume one unit of the good, as long as the utility they receive from that consumption is non-negative. Thus, the representative consumer will have the following utility function:

$$U = u^* - p - td \tag{1}$$

where

- $u^*$  = Utility derived from consuming one unit of the good.
- $P$  = Unit price of the good.
- $t$  = Unit transport cost for the consumers, with  $t > 0$ .
- $d$  = Distance to the nearest producer.
- xi. There are no international transport costs.

## B. Autarky Equilibrium

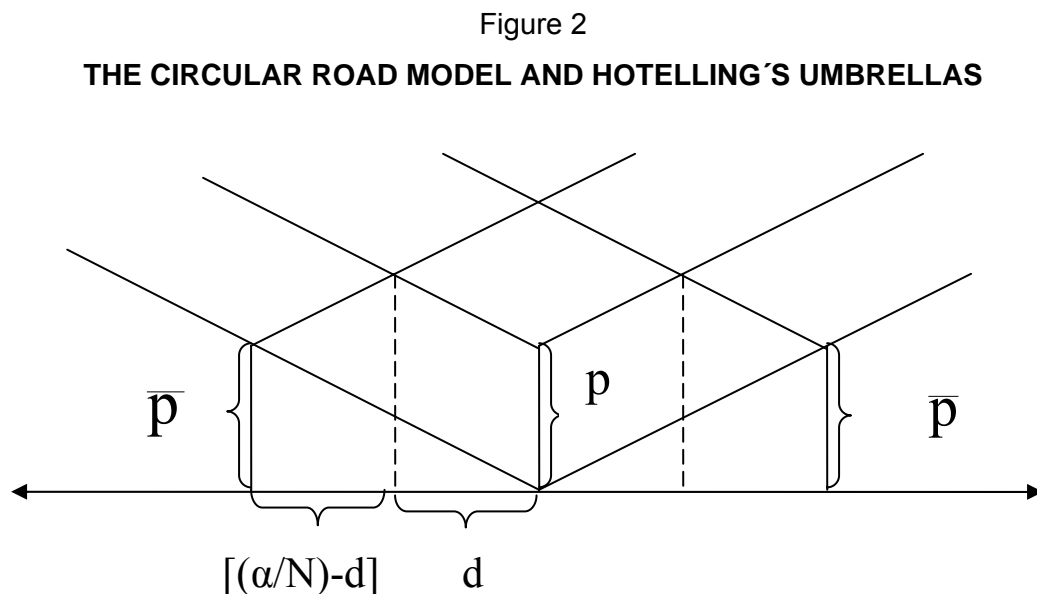
In this paper, a symmetrical equilibrium is searched. Thus, the second stage of the game identifies what is a symmetric Nash Equilibrium in prices. Then, by backward induction, the first stage of the game identifies a Nash Equilibrium in locations. The paper also

identifies what is the number of producers, what are their sales, and what is the level of welfare per capita at equilibrium.

To do so, the model works with the long run monopolistic competition conditions as follows: first, the sales of a typical firm in a circular road model are derived. Then, the optimal strategy price and the corresponding quantities are obtained from the profit maximization condition. These price and quantities -and the locations initially assumed-, are shown to be not only optimal strategies for the firm, but also a Nash Equilibrium for the market. The next step is to find the endogenous number of firms (varieties) from the zero profit conditions. Finally, the autarky equilibrium is completed by expressing the utility function of the average consumer in terms of the parameters of the model, and analysing the impacts that changes in the parameters of the model have on key variables.

1. Deriving the sales of a typical firm in a circular road model.

Given the assumptions, it is possible to construct Hotelling's umbrellas, as shown in figure 2.



The vertical bases of the umbrellas show the location of each producer, and the extension of the vertical bases represent the prices charged by the producers located at each base.

The slope of the arms of the umbrella is the uniform unit transport cost of the consumers. Given that the total cost incurred by a consumer located at a given point over the circular road -when buying a unit of the good- is the price paid plus the unit transport cost multiplied by the distance travelled, the heights of the arms of Hotelling's umbrellas show the total cost for a given consumer of buying the good from the producer located at the base corresponding to each umbrella.

The equation that describes the marginal consumer, i.e., the consumer that is indifferent between buying to the producer located nearest to the right hand side, and the producer located nearest to the left hand side is:

$$p + td = \bar{p} + t \left[ \frac{\alpha}{N} - d \right]$$

Solving for d:

$$2td = \bar{p} - p + \frac{t\alpha}{N}$$

$$d = \frac{\bar{p} - p}{2t} + \frac{\alpha}{2N}$$

Given that the model is symmetric, and in particular, given that the per unit transport costs are uniform, the sales of the producer that is nearest to the right of the marginal consumer in figure 1 are:

$$x = 2d\beta \tag{2}$$

$$2d\beta = \left[ \frac{\bar{p} - p}{t} \right] \beta + \frac{\alpha\beta}{N}$$

$$x = \left[ \frac{\bar{p} - p}{t} \right] \beta + \frac{\alpha\beta}{N} \tag{3}$$

## 2. Profit maximization condition

The profit maximization condition for the typical firm is given by:

$$\begin{aligned}\Pi &= px - f - ex \\ \Pi &= p \left[ \left[ \frac{\bar{p} - p}{t} \right] \beta + \frac{\alpha\beta}{N} \right] - f - e \left[ \left[ \frac{\bar{p} - p}{t} \right] \beta + \frac{\alpha\beta}{N} \right] \\ \Pi &= \left[ \frac{\bar{p}p - p^2}{t} \right] \beta + \frac{p\alpha}{N} \beta - f - \left[ \frac{\bar{p}e - pe}{t} \right] \beta - \frac{\alpha\beta e}{N} \\ \frac{\partial \Pi}{\partial p} &= \left[ \frac{\bar{p} - 2p}{t} \right] \beta + \frac{\alpha\beta}{N} + \frac{\beta e}{t} = 0\end{aligned}$$

In a symmetric equilibrium, if all other firms charge the same price, the optimal strategy price will be:

$$\begin{aligned}-\frac{\beta p}{t} + \frac{\alpha\beta}{N} + \frac{\beta e}{t} &= 0 \\ p &= \frac{\alpha t}{N} + e\end{aligned}\tag{4}$$

Repeating this process for all other firms, and assuming a symmetric equilibrium in which all other firms charge the same price  $\bar{p}$ , a Nash Equilibrium in prices will occur when all firms charge a price equal to the one shown in (4).

In such equilibrium and from equation (3), the optimal quantities sold by each firm will be<sup>2</sup>

$$x = \frac{\alpha\beta}{N}\tag{5}$$

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<sup>2</sup> In the symmetric equilibrium and also from (3), the demand function faced by a typical firm will be:

$$x = \left[ \frac{2\alpha}{N} \beta + \frac{\beta e}{t} \right] - \frac{\beta p}{t}$$



Note that in the circular road model, if all firms charge identical prices, locating at a distance  $\frac{\alpha}{N}$  will be a Nash Equilibrium in location. Thus, a Nash Equilibrium has been found for location in the first stage, and for prices in the second stage.

### 3. The zero profit condition

Free entry and exit of firms ensures that in the long run, firms will have zero profits. The zero profit condition of profit maximizing firms is met when the average costs equal the per unit price, as follows:

$$\frac{f}{x} + e = \frac{\alpha t}{N} + e$$

$$N^2 = \frac{\alpha^2 \beta t}{f}$$

Thus, the total number of firms (varieties) expressed in terms of the parameters of the model, will be:

$$N = \alpha \sqrt{\frac{\beta t}{f}} \tag{6}$$

Replacing (6) in (4), the equilibrium price can also be expressed in terms of the parameters of the model.

$$p = \sqrt{\frac{ft}{\beta}} + e \tag{7}$$

Since the price includes a mark-up over marginal cost, resources are not allocated efficiently. The lower the fixed and transport costs, and the higher the market density, the better the allocative efficiency and the lower the price. Falls in the marginal cost will also lower the equilibrium price.

Replacing (6) in (5), the equilibrium quantities can be expressed as:

$$x = \sqrt{\frac{\beta f}{t}} \quad (8)$$

This means that higher population density and fixed costs, and lower unit transport costs, will lead to higher equilibrium quantities. Note also that in the symmetric equilibrium and also from (3), the demand function faced by a typical firm will be:

$$x = \left[ \frac{2\sqrt{f\beta}}{\sqrt{t}} + \frac{\beta e}{t} \right] - \frac{\beta}{t} p$$

which is a demand curve with a constant and negative slope. In this sense, the model developed in this paper provides a microfoundation to the use of this simple type of demand curves.

From (2) and from (8)

$$d = \frac{1}{2} \sqrt{\frac{f}{\beta t}} \quad (9)$$

As such, the utility function of the marginal consumer in terms of the parameters of the model, can be obtained by substituting (6), (7) and (9) in (1)

$$U = u^* - \sqrt{\frac{ft}{\beta}} - e - \frac{t}{2} \sqrt{\frac{f}{\beta t}}$$

$$U = u^* - \frac{3}{2} \sqrt{\frac{ft}{\beta}} - e \quad (10)$$

Now, in the symmetric Nash Equilibrium in prices, the utility of the consumer located exactly in the same place where a producer is located is  $u^* - p$ . This means that there will

be transactions in this economy only as long as  $u^* \geq \sqrt{\frac{ft}{\beta}} + e$ . On the other hand, the

utility of the marginal consumer is given by (10). This implies that all consumers will buy

the good only if  $u^* \geq \frac{3}{2}\sqrt{\frac{ft}{\beta}} - e$

Besides, the utility of the average consumer will be given by:

$$U = \frac{u^* - p + u^* - p - td}{2}$$

$$U = u^* - \frac{5}{4}\sqrt{\frac{ft}{\beta}} - e \tag{11}$$

Thus, a nation's welfare per capita in autarky will increase (decrease) the lower (greater) the fixed, marginal and transport costs, and the greater (lower) the population density.

Note that the number of varieties is endogenous, and since individuals have ideal variety preferences, it does not have an ultimate impact on welfare. Note also that the size of the circular road ( $\alpha$ ) has a direct effect on the number of firms, but it does not have an impact on the level of production per firm and the level of equilibrium prices and so, it does not affect the level of welfare.

### C. International Trade.

International trade can be introduced in this model as an increase in population density perceived by the producers of the good<sup>3</sup>. To do so, assume that there are two countries as follows.

Country	Population density
A	$\beta_A$

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<sup>3</sup> The size of the circular road ( $\alpha$ ) is kept constant and equal in all countries since as mentioned above, it affects the number of producers but it does not affect output per firm, prices and welfare.

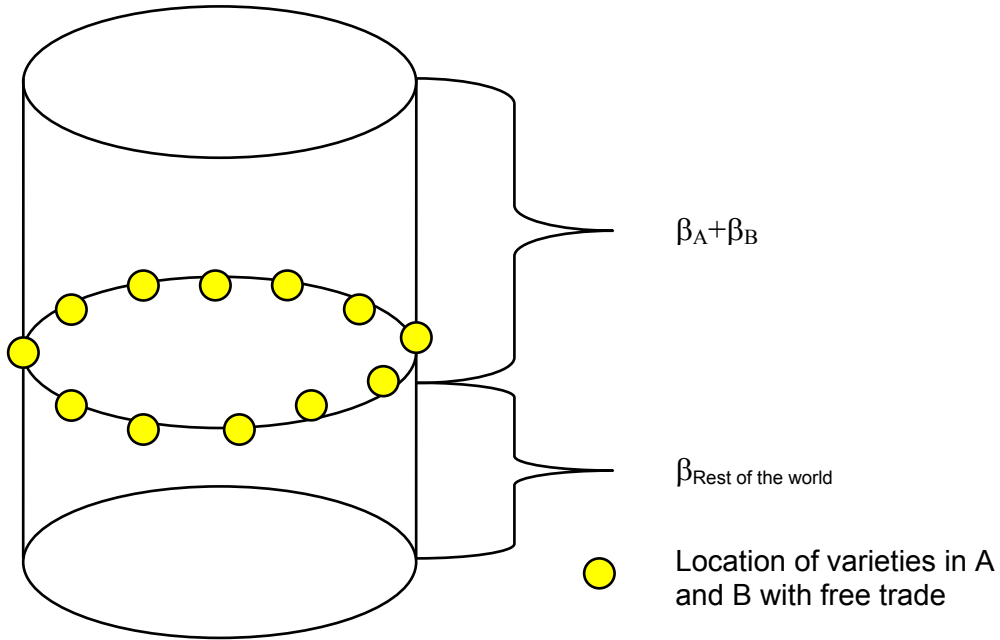
B

$\beta_B$

The circular road world with free trade between A and B will look as in figure 3.

Figure 3

**A CIRCULAR ROAD WORLD WITH FREE TRADE BETWEEN A AND B**



To start with, note that if both countries are identical, their autarky equilibrium conditions will also be identical. However, if one country is smaller than the other, the larger country will have in autarky more varieties, lower prices, higher output of each variety produced and greater welfare per capita, than the smaller country.

Note also that with free trade and no international transport costs between A and B, the change in population density perceived in A and B will lead to a new equilibrium that can be expressed as:

$$N_{FT} = \alpha \sqrt{\frac{(\beta_A + \beta_B)t}{f}} \quad (12)$$

$$p_{FT} = \sqrt{\frac{ft}{(\beta_A + \beta_B)}} + e \quad (13)$$

$$x_{FT} = \sqrt{\frac{(\beta_A + \beta_B)f}{t}} \quad (14)$$

$$U_{FT} = u^* - \frac{3}{4} \sqrt{\frac{ft}{(\beta_A + \beta_B)}} + \frac{e}{2} \quad (15)$$

This means that in both A and B, and regardless of whether A and B are large and/or small, the number of varieties available will increase, the per unit prices will fall, output of each variety will increase and welfare per capita will be higher. These gains from trade will be generated by the better exploitation of economies of scale (output per firm will be higher) and the improved allocative efficiency (the mark-up of prices over marginal costs will be lower) thanks to the free entry and exit of firms (lower unit costs of production and zero profits at equilibrium).

Furthermore, the free trade varieties, prices, output per variety and welfare per citizen will all be identical in both countries, regardless of their original size. As such, the gains from free trade will be larger for smaller countries, than for larger countries.

It has been shown that  $N_{FT} > N_A$  and that  $N_{FT} > N_B$  and under these conditions, the pattern of trade will be intra-industry –local varieties are exported and foreign varieties of the same good are imported–.

#### D. Political Economy of Trade Policy

The number of varieties in autarky for country A and for country B are:

$$N_A = \alpha \sqrt{\frac{\beta_A t}{f}}$$

$$N_B = \alpha \sqrt{\frac{\beta_B t}{f}}$$

The sum of the varieties available in autarky in country A, plus the number of varieties available in autarky in country B is:

$$N_A + N_B = \alpha \left[ \frac{\sqrt{\beta_A t} + \sqrt{\beta_B t}}{\sqrt{f}} \right]$$

The number of varieties with free trade in both A and B are:

$$N_{FT} = \alpha \frac{\sqrt{(\beta_A + \beta_B)t}}{\sqrt{f}}$$

$N_A + N_B$  will be higher than  $N_{FT}$  if:

$$(\sqrt{\beta_A} + \sqrt{\beta_B}) > \sqrt{(\beta_A + \beta_B)}$$

That is, if

$$2\sqrt{\beta_A \beta_B} > 0$$

which is true as long as  $\beta_A > 0$  and  $\beta_B > 0$ , i.e. as long as countries A and B exist.

Thus

$$N_A + N_B > N_{FT}$$

The number of varieties available with free trade is less than the sum of varieties available in A and in B in autarky. This means that since the remaining firms produce more, and the total density of the countries and the consumption per capita do not change, some firms have to exit the market.

Although the number of firms is endogenous in this model, and has no direct impact on welfare, this result is important for political economy reasons. In particular, there will be resistance to trade liberalization, because some firms will disappear -and some factors will have to move from some firms to other firms- introducing adjustment costs. However, such resistance may be milder with intra-industry trade than with inter-industry trade, since factors are moving within the same sector, as is the case in this paper. Besides, in the long run resources will remain fully employed with free trade, just as they were in autarky.

#### E. International Migration.

International migration can be introduced in this model as a shift in population density between countries. With no trade in goods and services and *ceteris paribus*, if countries A and B free up population movements, the following effects will occur:

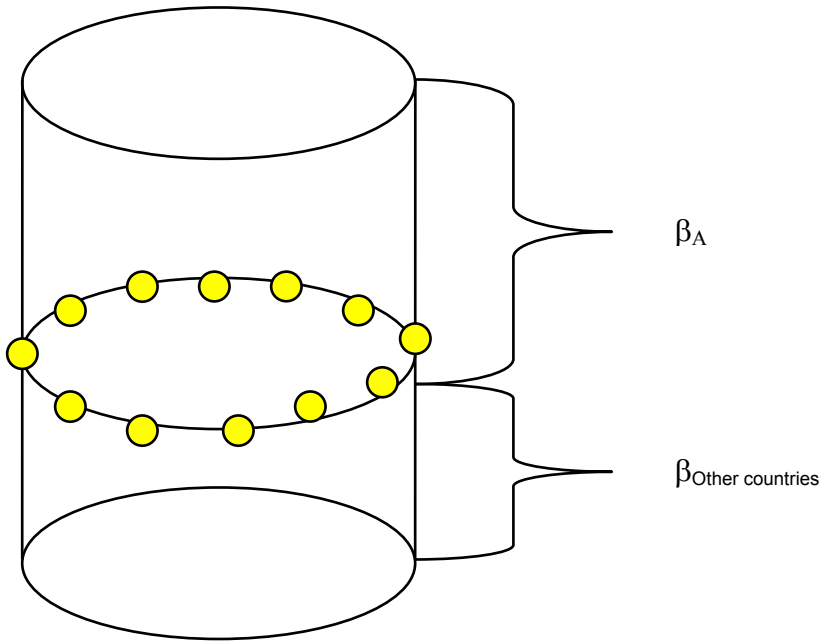
- i. As pointed out earlier, if one country has a higher population than the other, its citizens will have a higher welfare. For example, if  $\beta_A > \beta_B$ ,  $U_A > U_B$  because

$$\frac{5}{4} \sqrt{\frac{ft}{\beta_A}} < \frac{5}{4} \sqrt{\frac{ft}{\beta_B}}$$

Thus, if population movements are allowed *ceteris paribus*, all the people will move from the small country (B) to the large country (A). This situation is shown in figure 4.

Figure 4

**A CIRCULAR ROAD WORLD WITH FREE MOVEMENTS OF POPULATION BETWEEN A AND B, WHERE  $\beta_A > \beta_B$**



- ii. If A and B are identical, there will be no movements, since per capita welfare will be identical. However, this equilibrium will be unstable, since all that is required is that one consumer moves from one country to the other (to make welfare in the receiving country higher than in the country of origin), for all his fellow countrymen to do the same, just as in the previous case.
- iii. If one country has a technological advantage (lower  $f$ ,  $e$  and or  $t$ ), and both countries are identical in all other respects, the technologically advanced country will have higher autarky welfare. For example, if  $f_A > f_B$ ,  $U_A < U_B$  because

$$\frac{5}{4} \sqrt{\frac{f_A t}{\beta}} > \frac{5}{4} \sqrt{\frac{f_B t}{\beta}}$$

and all consumers will go from the less technologically advanced country (A) to the more technologically advanced country (B).



- iv. If one country has a technological advantage (lower  $f$ ,  $e$  and or  $t$ ), but at the same time that country is smaller than the other country, it is possible that the size advantage may more than compensate the technological advantage in terms of welfare. For example, if  $\beta_A > \beta_B$  and if  $f_A > f_B$ , and  $\frac{f_A}{\beta_A} < \frac{f_B}{\beta_B}$ ,  $U_A > U_B$  because

$$\frac{5}{4} \sqrt{\frac{f_A t}{\beta_A}} < \frac{5}{4} \sqrt{\frac{f_B t}{\beta_B}}$$

In this case, all consumers will go from the more technologically advanced country (B) to the less technologically advanced country (A), and the final equilibrium will be pareto dominated, i.e., there will be suboptimal migration flows. All the consumers in the world would have been better off if they had managed to live in the technologically advanced country.

#### F. Political Economy of International Migration

In this model, if international migration occurs at all, it will lead to emptying one country. Although this is an unlikely outcome, in this case such movements could lead political leaders –and even some economists- of the country whose consumers are leaving to put barriers to the exit (rather than barriers to entry) in order to avoid the massive departures of its citizens. To understand why, assume that A has a higher autarky welfare than B, and that all consumers in B want to migrate to A. If consumers in B go to A, all consumers in B will have higher welfare than before (the gross national income will be higher in both countries A and B). But the citizens located in B (none), will have zero welfare (the gross domestic product in B will be zero and lower than in autarky). Think of a dramatic but not complete migratory movement. Then, the citizens that for whatever reason remain in country B will be worse off than in autarky (again, B will have a lower gross domestic product).

This could explain -at least in part- although it may not justify, the exit restrictions that have prevailed in certain countries, or those that prevailed in former communist countries, given the welfare incentives for consumers and policy makers.

## G Foreign Direct Investment

In this model, with identical cost structures, without international trade and *ceteris paribus*, if firms have identical technologies, there will be no incentives for foreign direct investment (FDI) flows, regardless of the size of the countries. However, if firms in country B have a technological advantage over firms in country A, there would be an incentive for FDI flows to move from B to A since at the prices in A firms in B would be able to make a positive profit in the short run. However, if there is free entry and exit of foreign investors, eventually the firms in B investing in A would have zero economic profits in the long run. On the other hand, the long run gains in welfare per capita in A from receiving the FDI flows would be:

$$U_A^{\text{FDI}} - U_A \\ \frac{5}{4} \sqrt{\frac{f_A t}{\beta}} - \frac{5}{4} \sqrt{\frac{f_B t}{\beta}} > 0$$

## IV. CONCLUSIONS

This paper has built on the circular road model of horizontal product differentiation. The model derived shows that in autarky, larger countries have higher welfare than smaller countries. The model also shows that freer international trade increases welfare -with ideal variety preferences- through the exploitation of economies of scale and through better allocative efficiency, that benefit consumers thanks to the free entry and exit of firms.

In autarky, larger countries will have higher welfare than smaller countries, and although all countries that take part in international trade, gain from trade, with free trade all of the world's countries have the same welfare per capita, regardless of their population size. This implies that smaller countries have more to win from free trade than larger countries. As is usual in the literature of trade with imperfect competition, the pattern of trade in this model is intra-industry.

Furthermore, the model explains that there may be adjustment costs when liberalizing trade because some producers will exit the market and some factors will have to move from some firms to other firms. These adjustment costs are likely to generate political resistance to trade liberalization, although such resistance may be milder with intra-industry trade than with inter-industry trade, since factors are moving within the same sector. Besides, in the long run, all resources will be fully employed with free trade, as they were in the autarky equilibrium.

Allowing for international migration within the model has highlighted the possibility of both suboptimal migration flows, and the existence of political barriers to the exit of national citizens. Besides, foreign direct investment would provide short run gains for the source country and long run gains for the receiving country. As a by product, this paper has provided microfoundations to the use of demand curves with constant and negative slopes.

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