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Interest Rates, Promotional Prizes and Competition in the Banking Industry*

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Abstract

Some Colombian commercial banks have used the strategy of offering promotional prizes in order to attract new savings customers. In this paper we develop a two-stage game model that allows us to understand the effects of this promotional strategy on the deposit interest rates, the deposit market shares and the intermediation spreads. We find that under this strategy it is possible for the bank that offers the highest prize to segment the deposit market serving only customers that assign high subjective probabilities to winning prizes. More importantly we show that the bank that offers the highest promotional prize not only pays the lowest deposit interest rate but also has the largest deposit market share and the widest intermediation spread.

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JEL Classifications: D21, D43, G21 and L13.

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1 Introduction

Since the mid 1990's, some Colombian commercial banks have offered prizes to attract new customers who want to open savings accounts. Cars, houses, and cash have been some of the prizes offered to try to capture the attention of new depositors. The new promotion strategy works like a lottery or a raffle, in the sense that it is not certain that potential new customers will win the prize. Despite the uncertainty of the prize, depositors have welcomed the new banking promotional strategy. This fact suggests that Colombian commercial banks have found a new strategy to compete in the deposit market as an alternative to using interest rates.

It is clear that this innovative activity of promotional prizes must respond to some economic forces. At first glance the motivation of this activity can be explained by the banks' desire of acquiring a bigger deposit market share. However, the explanation is not as simple.

As suggested by Silber (1983), new financial practices are in general designed to lessen the constraints imposed on banks in order to achieve a particular goal. These constraints can be external and internal such as government regulations, high interest rates, high inflation rates and the type of market structure, among others. In this sense, Colombian banks' strategy of offering prizes can be understood as a way to loosen some of their internal and external constraints in order to increase their deposits or their access to funds sources.

The type of constraints may vary across banks. However there are some constraints that are common for all the banks. For instance, after implementing diverse financial innovations, banks repeatedly face the internal constraint of having the interest rate as the only simple instrument to stimulate deposits. But offering higher interest rates to attract deposits also implies rising the costs paid by the bank. In addition if market interest rates are already high or the government regulates them, then the use of the interest rates as a tool to capture more savings can be externally constrained.

Some empirical evidence from Colombia supports the importance of the aforementioned external constraints. In particular it has been argued that the interest rates have been high during the last decade. According to Greco (1999) in the period 1990 to 1997, the nominal annual interest rate on loans oscillated roughly between 34% and 47%, while the annual nominal interest rate on deposits oscillated between 23% and 37%. These levels of interest rates were considered high since in the same period there were several episodes in which the Central Bank tried to lower and regulate them.

It is important to observe that the Colombian banks prize strategy since the mid 1990's can be understood not only as a strategy to loosen some contraints in order to stimulate deposits. It can be also understood as an attempt to relax these constraints to increase their intermediation spread up to their historical high levels. In fact, Barajas et al. (1999) mention that the interest rate spread declined steadily from an initial level of about 25% in 1991 to 19% in 1996.

Based on all these stylized facts, in this paper we develop a model that explains how offering prizes to their new savings customers, banks can reduce the interest rate paid on deposits, increase their intermediation spread, and stimulate the demand for deposits.

We model Banks as firms that compete in the deposits market. They supply savings accounts that can be characterized by two features: first, their interest rate and second, their potential prize.

More formally we construct a model in which banks differentiate horizontally and vertically.[1] The horizontal differentiation is based on the different and exogenous physical location of the banks, which affects their interest rate on deposits due to transportation costs. The vertical differentiation comes from the banks' strategy of offering promotional prizes to their customers. In this sense the prize can be viewed as an improvement in the quality of the savings accounts.

We model competition between banks as a two-stage game duopoly. In the first stage banks simultaneously decide the value of the prize they offer in order to attract new savings customers. In the second stage, banks compete with the interest rate that they pay on deposits.

It is important to note that we do not explicitly model the decision of offering and not-offering a prize. This decision is implicitly taken by each bank when it decides to offer a positive value of the prize or a prize whose value is zero.[2] Moreover, the goal of our model is not to explain why some banks have not adopted the prize strategy. Instead we want to use the model to explain how the aforementioned strategy has affected interest rates spreads and deposits in the banks that adopted the strategy.

The prize strategy can be seen as an instrument that segments the depositors according to the subjective probability that these customers assign to win the prize. This probability is

[1] Two products are horizontally differentiated when there is no ranking among consumers based on their willingness-to-pay for the two products. On the other hand two products are vertically differentiated when there exits such a ranking. See Neven and Thisse (1982).

[2] To model the decision of offering and not-offering a prize we can include another stage at the beginning of the game.

a measure of the their taste for raffles and lotteries, and can be considered as the probability-type of each individual.[3] Depending on the unitary cost of transportation and the difference between the prizes' values, there are two types of segmentation or dominance. If the unitary cost of transportation for the savings customers is greater than the difference between the prizes' values, then the banks serve all the customers' probability-types, regardless of the subjective probability that the depositors assign to winning the prize. This case is described as *horizontal dominance*. On the other hand, if the unitary cost of transportation is less than the difference between the prizes' values, then the bank that offers the highest prize attracts only customers that assign a subjective high probability of winning the prize. In other words, this bank ends with a zero market share for low probability-type depositors. This situation corresponds to *vertical dominance*.

We find that the equilibrium of the two-stage game depends on the type of dominance. Under *horizontal dominance* we obtain a symmetric equilibrium. That is, both banks offer the same prize, the same deposit interest rates and have the same deposit market share. Under *vertical dominance* we derive an asymmetric equilibrium where only one bank offers a prize. This bank pays the lowest interest rate and has the largest deposit market share and the widest intermediation spread. Moreover this bank specializes completely in serving customers that assign a subjective high probability to winning the prize, while the bank whose prize is zero specializes in serving low probability-type customers.

We also define and solve a *benchmark* game. This is a simple game in which banks only compete with deposit interest rates. Comparing the equilibrium of this game with the equilibrium of the two-stage game lead to some interesting results. We find that under *horizontal dominance*, the equilibrium interest rates and the market shares of the two stage game are the same as the ones that are obtained under the *benchmark* game. On the other hand, under *vertical dominance*, the bank that offers a positive prize reduces its deposit interest rate and increases its deposit market share in comparison with the ones that it has under the *benchmark* game. Furthermore the bank that does not offer any prize pays a higher interest rate and has a lower deposit market share than those it has under the *benchmark* game.

Our model is based on the large literature of industrial organization that has studied price competition among firms under horizontal differentiation and/or under vertical differ-

[3]The word probability-type is introduced since the type of each individual is defined by the location of each individual and the probability that she assigns to win the prize (taste for raffles). It will be discussed in section 2.

entiation. Some examples of models of horizontal differentiation, also known as the "address location" approach, are Hotelling (1929), d'Aspremont et al. (1979), Salop (1979) and Economides (1989a). Some prototypes of models of vertical differentiation are Mussa and Rossen (1978) and Shaked and Sutton (1983). In addition, Economides (1989b), Economides (1993), Neven and Thissen (1990) and Dos Santos and Thisse (1996) are models that simultaneously study and integrate vertical and horizontal differentiation considering solely two characteristics of the products: variety and quality.

There are also articles that have been written to explain the industrial organization of banking including the concepts of vertical or horizontal differentiation. Some of them relied on previous results derived in the price competition papers aforementioned. In general the structure of these banking models is based upon the circular-city model of Salop (1979). Adapting this structure to banking competition has allowed researchers to study different problems. For instance, Freixas and Rochet (1997) use this structure to find the optimal number of banks while Chiappori et al. (1995) apply it to study the impact of deposit rate regulation on credit rates. In addition, Matutes and Padilla (1994) utilize this structure to find the appropriate level of interbank cooperation in automated teller machine (ATM) networks, whereas Bouckaert and Degryse (1995) apply it to analyze the consequences of the introduction of phone banking. Most of these models do not consider vertical differentiation among banks. If they consider it, like in Bouckaert and Degryse (1995), they assume that the depositors have the same taste for the quality-option that the banks offer. Therefore it is important to emphasize that the model that we develop in this paper makes explicit the interaction between vertical and horizontal product differentiation as Degryse (1996) does to analyze the conditions under which banks offer remote access.

The rest of the paper is organized as follows: in Section 2 we describe the set up of the model; in Section 3 we solve the two-stage game and state and analyze the main results of this game; finally, in Section 4 we present some concluding remarks.

2 The Model

We assume that there is a duopolistic industry of banks in the deposit market.[4] The two banks are denoted by Bank 1 and Bank 2. Each bank has a single branch that is located on a circle with unit circumference. For simplicity we suppose that the banks are already located at a distance of 1/2 from each other.

We model competition for deposits as a two-stage game. In the first stage banks simultaneously decide the value of the prize, $q_i \in [0, \infty)$, that they offer to attract new savings customers. In the second stage, banks compete with deposit interest rates, $r_i \in [0, \infty)$.

Following Economides (1993), we argue that this timing is justified by the fact that all strategic variables are not equally flexible. In the short-run, interest rates can be easier to change than the prize strategy. The reason is that there is a promotional and advertising campaign associated with the prize strategy that takes time to design and it may be difficult to change.

From the timing of the game and the fact that banks' locations are given exogenously it is evident that our purpose in this model is to focus on the introduction of prizes and their interaction with interest rates and deposit market shares. However it is important to notice that although banks do not choose their location, their deposit accounts are still characterized by two features: first, the exogenously given location of the bank that implicitly defines the physical accessibility to it and affects the deposit interest rate through transportation costs; second, the value of the prize.

As in many of the papers of banking, that were mentioned above, we take the existence of banks as given and concentrates only on their liability side.[5] The deposits, D_i, that the banks attract, are invested to obtain an identical and fixed return $R \geq r_i$ per unit of money. Therefore the intermediation spread can be defined as $(R - r_i)$.

Each bank i maximizes profits, π_i

$$\pi_i = (R - r_i)D_i - C(q_i) \tag{1}$$

[4]For the purpose of this paper it is sufficient to consider only two banks. An extension to more banks is straightforward. However graphical analysis becomes cumbersome for models with more than two banks.

[5]We are excluding the possible competition that banks can have in the credit market. This exclusion simplifies the model and makes it more tractable. Note that in reality banks, like intermediaries of any kind, face a double competition, i.e. simultaneous competition on loans (credit market) and deposits (deposit market). To model this kind of competition some assumptions in terms of timing are required . But the timing of these games can affect the market structure. For instance, an oligopolistic competition in the deposit market can produce equilibria in which there is a monopolist in the credit market. See Stahl (1988).

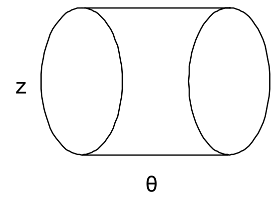

Figure 1:

where $C(q_i)$ represents the cost function for the bank of offering the prize q_i. We assume that the function $C(q_i)$ is increasing at an increasing rate ($C'(q_i) > 0$ and $C''(q_i) > 0$). The intuition of this assumption is that each bank is willing to offer higher prizes to attract more customers; but this implies that it also has to invest more resources in advertising and promotion to reach not only the individuals near the bank but also the individuals that are closer to its competitor.

2.1 The Demand for Deposits

We assume that each depositor is endowed with one unit of money that she invests at only one of the two banks. Depositor preferences vary along two dimensions. First, each depositor has a unique location z on the circumference with $z \in [0, 1]$ and measured with respect to the bank i. Second, each depositor has a taste for raffles or prizes. This taste is represented by the subjective probability that individuals assign to winning the prize. This probability is denoted by $\theta \in [0, 1]$ and also describes the probability-type of each depositor. In this sense individuals differ in their probability-type.

It is important to emphasize that it is not certain that when a depositor opens a new savings account in a bank she will win the prize offered by this bank. She only gets the option of participating in a raffle.

Under these assumptions we can characterize each depositor by the type (θ, z). As in Economides (1993) and Degryse (1996), we can represent the space of the depositors' characteristics by the cylinder $[0, 1] \times [0, 1]$ (see Figure 1). Moreover, we suppose that depositors are uniformly distributed over the surface of the cylinder with probability density equal to one. Therefore the total mass of these depositors corresponds to one.

We assume that depositors are risk-neutral. The expected utility for each individual of depositing one unit of money in the bank i can be expressed as

$$U(q_i; z, \theta) = E(u(q_i; z, \theta)) = \theta[\omega + r_i - tz + q_i] + (1 - \theta)[\omega + r_i - tz]$$

Thus,

$$U(q_i; z, \theta) = \omega + r_i - tz + \theta q_i \tag{2}$$

where ω is the reservation value that we suppose to be large enough such that the deposit market is covered; and t is the unitary cost of transportation. As Matutes and Vives (1996) point out, it is interesting to observe that the transportation costs, tz, can be understood in different ways. They do not only represent the depositors' cost of time spent traveling to the bank but also represent the bank's provision of diverse services such as ATM network sizes or consumer credit facilities, among others.

We proceed to derive the demand for deposits. As it was stated before we assume that there are only two banks $i = 1, 2$. Without loss of generality we suppose that the Bank 1 offers a higher prize than the Bank 2, $q_1 \geq q_2$. Using the utility function in (2), it is possible to derive the set of depositors that are indifferent between Banks 1 and 2. For any probability-type $\theta \in [0, 1]$, the marginal depositor is found by solving for the location z that makes her indifferent between the two banks. In other words, using the expected utility (2) the location z solves[6]

$$\omega + r_1 - tz + \theta q_1 = \omega + r_2 - t\left(\frac{1}{2} - z\right) + \theta q_2$$

and defining $x(\theta) = 2z(\theta)$ we obtain

$$x(\theta) = 2z(\theta) = \frac{1}{t}\left(r_1 - r_2 + \frac{t}{2} + \theta(q_1 - q_2)\right) \tag{3}$$

where $x(\theta)$ represents the market share of Bank 1 for the probability-types θ. This is a linear and increasing function in θ that partitions the total deposit market in two groups of depositors. It defines the market area of each bank, as illustrated in Figure 2. An increase in r_1 (decrease in r_2), shifts the function to the left increasing the market area of Bank 1 and reducing the market area of Bank 2.

[6]Note that for each individual the assigned probability of winning the prize $q_1$is the same as the assigned probability of winning the prize q_2.

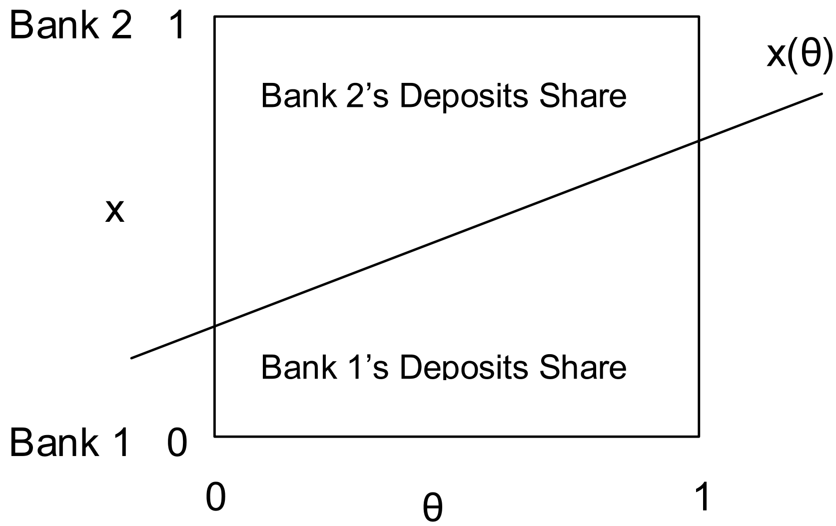

Figure 2:

To determine the demand for deposits of Bank 1, D_1,as a function of the interest rate r_1,we integrate the function $x(\theta)$ of the equation (3) over $[0,1]$ taking into account the appropriate range of r_1. There are 5 ranges and we continue describing them.

Given r_2, for "very low" values of r_1 ,and regardless of the magnitude of the slope $\frac{(q_1-q_2)}{t}$, the line $x(\theta)$ does not cross the unit square in Figure 2. This defines the segment of the demand that we call D_1^I. For "low" values of r_1, given r_2 and regardless of $\frac{(q_1-q_2)}{t}$, the line $x(\theta)$ crosses the bottom and the right side of the unit square in Figure 2. This determines the segment of the demand that corresponds to D_1^{II}. Given r_2 and for "intermediate values" r_1 the function $x(\theta)$ can cross either the vertical sides or the horizontal sides of the unit square. The first possibility implies that

$$\left|\frac{\partial x(\theta)}{\partial \theta}\right| = \frac{(q_1-q_2)}{t} < 1 \qquad \Longleftrightarrow \qquad (q_1-q_2) < t$$

This means that the difference between the value of the prizes offered by the banks (q_1-q_2) is less than the unitary cost of transportation, t. This situation is known as *horizontal dominance* $(q_1-q_2<t)$.[7] Under this situation, Bank 1 attracts a strictly positive

[7] See Degryse (1996).

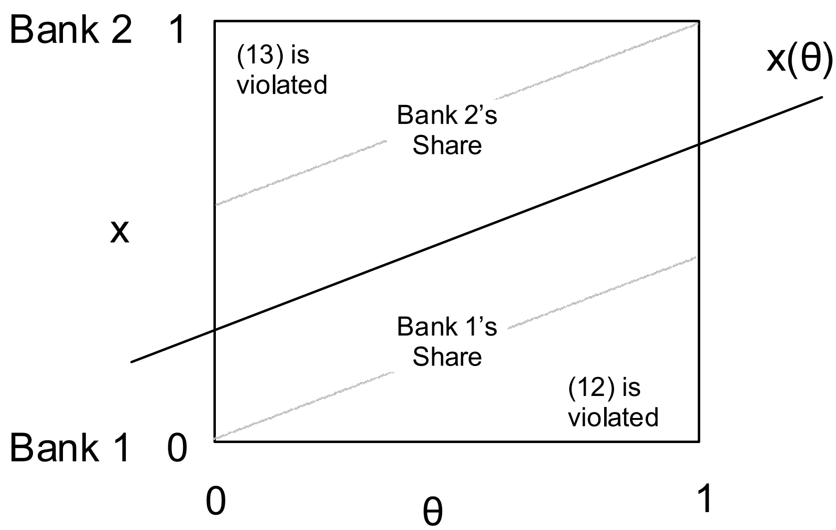

Figure 3:

market of the low probability-type depositors, but it does not serve the entire market for the high probability-type depositors (see Figure 3).

In the second possibility

$$\left|\frac{\partial x(\theta)}{\partial \theta}\right| = \frac{(q_1 - q_2)}{t} > 1 \qquad \Longleftrightarrow \qquad (q_1 - q_2) > t$$

In this situation, the difference between the value of the prizes offered by the banks $(q_1 - q_2)$ is greater than the linear rate cost of transportation, t. This situation is referred to as *vertical dominance* $(q_1 - q_2 > t)$. Under this situation, Bank 1 attracts the entire market of the high probability-type depositors and has a zero market share for the low probability-type depositors (see Figure 4).

It is important to notice that if

$$\left|\frac{\partial x(\theta)}{\partial \theta}\right| = \frac{(q_1 - q_2)}{t} = 1 \qquad \Longleftrightarrow \qquad (q_1 - q_2) = t$$

there is neither vertical nor horizontal dominance.

Under these two situations, *vertical dominance* and *horizontal dominance,* the segment of the demand function for deposits will be called D_1^{III}.

For "high" values of r_1, given r_2 and independently of the slope $\frac{(q_1-q_2)}{t}$, the line $x(\theta)$ crosses the top and the left side of the unit square. This determines the segment of the

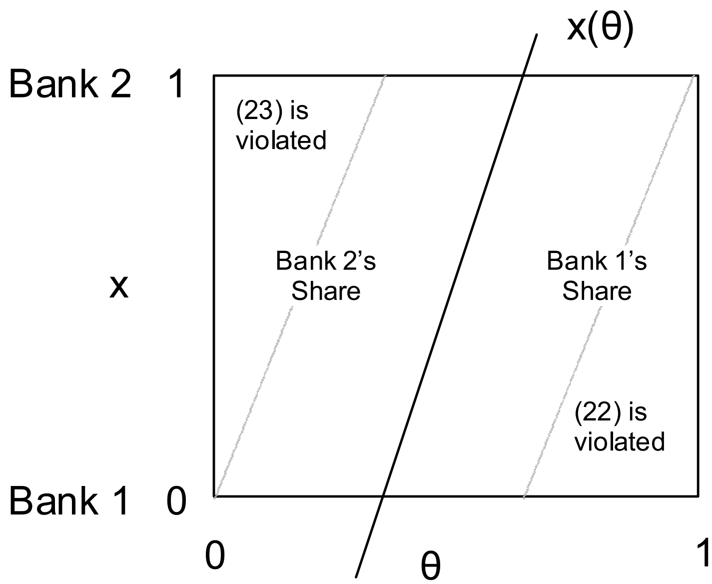

Figure 4:

demand that we call D_1^{IV}. Finally, given r_2, for "very high" values of r_1, and independently of the slope $\frac{(q_1-q_2)}{t}$, the line $x(\theta)$ does not cross the unit square. This defines the segment of the demand that corresponds to D_1^V.

The details for the derivation of the demand for deposits for bank 1, D_1, can be found in the Appendix 1. In this part of the paper we only present the derived functional forms for this demand and give specific meaning to the aforementioned terms "very low", "low", "intermediate", "high" and "very high", through closed intervals for the deposit interest rate r_1.

The results (derived functional form) can be summarized as follows. First, for a "very low" interest rate, $r_1 \in [0, r_2 - \frac{t}{2} - (q_1 - q_2)] = [0, r_1^{\text{min}}]$, and regardless of the type of dominance, it can be established that

$$D_1^I = 0 \tag{4}$$

Second, if *horizontal dominance* $(q_1 - q_2 < t)$ prevails, then a "low" interest rate satisfies $r_1 \in [r_2 - \frac{t}{2} - (q_1 - q_2), r_2 - \frac{t}{2}] = [r_1^{\text{min}}, r_{1,h}^1]$. On the other hand, if *vertical dominance* $(q_1 - q_2 > t)$ prevails, then a "low" interest rate satisfies $r_1 \in [r_2 - \frac{t}{2} - (q_1 - q_2), r_2 + \frac{t}{2} - (q_1 - q_2)] = [r_1^{\text{min}}, r_{1,v}^1]$. However, the same demand function applies for both types of dominance.

That is

$$D_1^{II} = \frac{1}{2t(q_1 - q_2)} \left(r_2 - r_1 - \frac{t}{2} - (q_1 - q_2) \right)^2 \tag{5}$$

Third, under *horizontal dominance* $(q_1 - q_2 < t)$, when the "intermediate" interest rate satisfies $r_1 \in [r_2 - \frac{t}{2}, r_2 + \frac{t}{2} - (q_1 - q_2)] = [r_{1,h}^1, r_{1,h}^2]$, the demand function for deposits is

$$D_{1,h}^{III} = \frac{1}{t} \left(r_1 - r_2 + \frac{t}{2} \right) + \frac{(q_1 - q_2)}{2t} \tag{6}$$

Under *vertical dominance* $(q_1 - q_2 > t)$, when the "intermediate" interest rate satisfies $r_1 \in [r_2 + \frac{t}{2} - (q_1 - q_2), r_2 - \frac{t}{2}] = [r_{1,v}^1, r_{1,v}^2]$, the demand function for deposits corresponds to

$$D_{1,v}^{III} = \frac{1}{(q_1 - q_2)} [r_1 - r_2 + (q_1 - q_2)] \tag{7}$$

Fourth, we find that if *horizontal dominance* $(q_1 - q_2 < t)$ prevails then a "high" interest rate satisfies $r_1 \in [r_2 + \frac{t}{2} - (q_1 - q_2), r_2 + \frac{t}{2}] = [r_{1,h}^2, r_1^{\max}]$. On the other hand, if *vertical dominance* $(q_1 - q_2 > t)$ prevails then a "high" interest rate satisfies $r_1 \in [r_2 - \frac{t}{2}, r_2 + \frac{t}{2}] = [r_{1,v}^2, r_1^{\max}]$. However the same functional form applies for both types of dominance. That is

$$D_1^{IV} = 1 - \frac{\left(r_2 - r_1 + \frac{t}{2}\right)^2}{2t(q_1 - q_2)} \tag{8}$$

Finally, for the last range, under both types of dominance and for a "very high" interest rate $r_1 \in [r_2 + \frac{t}{2}, \infty) = [r_1^{\max}, \infty)$, we establish that

$$D_1^V = 1 \tag{9}$$

It is important to observe that under neither *vertical dominance* nor *horizontal dominance* $(q_1 - q_2 = t)$, the demand function for deposits can be described solely by the segments D_1^I, D_1^{II}, D_1^{IV} and D_1^V. Moreover regardless of the type of dominance, the segments D_1^{II}, D_1^{III} and D_1^{IV} are strictly convex, linear and strictly concave, respectively. However differentiability is not assured.[8] Figure 5 illustrates a typical demand function for the deposits of Bank 1.

The demand function for deposits of Bank 2 with respect to r_2 can be derived similarly as we derived the demand function for Bank 1. The shape is the same as the one in Figure 5 . Moreover note that the relation $D_2 = 1 - D_1$ holds, since the whole market for deposits is assumed to be covered.

[8]See Appendix 1

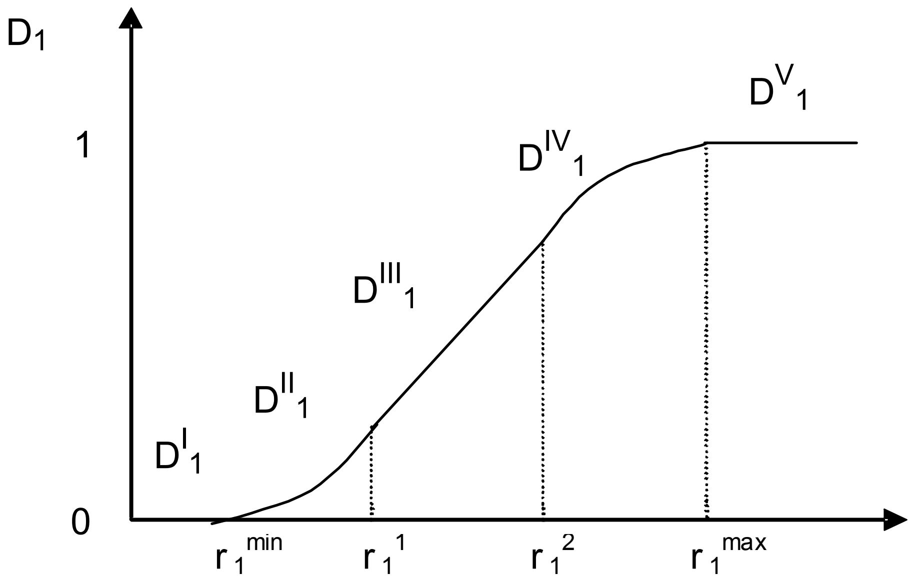

Figure 5:

3 Solving The Game

The game of this model can be solved applying the principle of backward induction. This means that we first, solve the deposit interest rate subgame (second stage) and then we solve the prize game (first stage).

3.1 The Interest Rate Subgame

This subgame is a typical Bertrand competition game applied to the Banking industry. Given t, q_1 and q_2, there are six types of equilibria in this subgame or second stage: three under *vertical dominance* and three under *horizontal dominance.*[9]

In other words for each *type of dominance* there are three cases that determine the type of equilibrium achieved in this stage:

- Case 1*:* The equilibrium appears on the linear segments of the demand for deposits of each bank, that is D_1^{III} and D_2^{III}.

[9]This part follows Neven and Thisse (1990).

- Case 2*:* The equilibrium occurs on the strictly concave segment of D_1 and on the strictly convex segment of D_2. This means that the equilibrium occurs on D_1^{II} and D_2^{IV}.

- Case 3*:* The equilibrium rises on the strictly convex segment of D_1 and on the strictly concave segment of D_2. This means that the equilibrium is present on D_1^{IV} and D_2^{II}.

Before calculating these equilibria it is important to notice their existence can be proven by the following argument. As is shown in the Appendix 2, the profits functions, for the banks, π_i^{II}, π_i^{III} and π_i^{IV} where $i = 1, 2$, are strictly convex, strictly concave and strictly concave in r_i respectively. This implies that these functions are also strictly quasiconcave along their respective interval of interest rates. Moreover they are continuous on the interest rates of the banks but not necessarily differentiable. In addition, the intervals of the interest rates over which these profit functions are defined, are non-empty, convex and compact subsets of the Euclidean Space R. Therefore all the conditions are satisfied to assure the existence of a Nash Equilibrium in pure strategies in each subgame.[10] Furthermore, since the profit functions are strictly quasiconcave in r_i a unique equilibrium is assured for the respective subgames.

The equilibria of the subgames are presented in this part of the paper. In each case, vertical and horizontal dominance are analyzed sequentially.

- Case 1

Under *horizontal dominance* ($q_1 - q_2 < t$) and if $r_1 \in [r_2 - \frac{t}{2}, r_2 + \frac{t}{2} - (q_1 - q_2)] = [r_{1,h}^1, r_{1,h}^2]$ and $r_2 \in [r_1 - \frac{t}{2} + (q_1 - q_2), r_1 + \frac{t}{2}] = [r_{2,h}^1, r_{2,h}^2]$, then the profits maximization problems for the banks can be formulated as

$$\underset{r_1}{Max}\ \pi_{1,h}^{III} = \underset{r_1}{Max}(R - r_1)D_{1,h}^{III} - C(q_1)$$

$$\underset{r_2}{Max}\ \pi_{2,h}^{III} = \underset{r_2}{Max}(R - r_2)(1 - D_{1,h}^{III}) - C(q_2)$$

The equilibrium for this Bertrand competition is characterized by

$$r_{1,h}^* = R - \frac{t}{2} - \frac{(q_1 - q_2)}{6} \qquad (10)$$

[10] See Proposition 8.D.3 in Mas-Colell et al (1995).

$$r^*_{2,h} = R - \frac{t}{2} + \frac{(q_1 - q_2)}{6} \tag{11}$$

In addition this equilibrium is unique and satisfies that $r^*_{1,h} \in [r^*_{2,h} - \frac{t}{2}, r^*_{2,h} + \frac{t}{2} - (q_1 - q_2)]$ and $r^*_{2,h} \in [r^*_{1,h} - \frac{t}{2} + (q_1 - q_2), r^*_{1,h} + \frac{t}{2}]$ if and only if

$$\frac{3t}{2} \geq (q_1 - q_2) \tag{12}$$

and

$$\frac{3t}{4} \geq (q_1 - q_2) \tag{13}$$

In other words (10) and (11) represent the interest rate equilibrium for the parameter region defined by (12) and (13). See Figure 3. Based on this equilibrium we can calculate the deposits demands, profits and intermediation spreads for both banks. They are

$$D^{III*}_{1,h} = \frac{1}{2} + \frac{(q_1 - q_2)}{6t} \tag{14}$$

$$D^{III*}_{2,h} = \frac{1}{2} - \frac{(q_1 - q_2)}{6t} \tag{15}$$

$$\pi^{III*}_{1,h} = t\left(\frac{1}{2} + \frac{(q_1 - q_2)}{6t}\right)^2 - C(q_1) \tag{16}$$

$$\pi^{III*}_{2,h} = t\left(\frac{1}{2} - \frac{(q_1 - q_2)}{6t}\right)^2 - C(q_2) \tag{17}$$

$$R - r^*_{1,h} = \frac{t}{2} + \frac{(q_1 - q_2)}{6} \tag{18}$$

$$R - r^*_{2,h} = \frac{t}{2} - \frac{(q_1 - q_2)}{6} \tag{19}$$

On the other hand, under *vertical dominance* $(q_1 - q_2 > t)$ and if $r_1 \in [r_2 + \frac{t}{2} - (q_1 - q_2), r_2 - \frac{t}{2}] = [r^1_{1,v}, r^2_{1,v}]$ and $r_2 \in [r_1 + \frac{t}{2}, r_1 - \frac{t}{2} + (q_1 - q_2)] = [r^1_{2,v}, r^2_{2,v}]$, then the profits maximization problems for the banks can be stated as

$$\underset{r_1}{Max}\ \pi^{III}_{1,v} = \underset{r1}{Max}(R - r_1)D^{III}_{1,v} - C(q_1)$$

$$\underset{r_2}{Max}\ \pi_{2,v}^{III} = \underset{r2}{Max}(R - r_2)(1 - D_{1,v}^{III}) - C(q_2)$$

The equilibrium for this Bertrand competition corresponds to

$$r_{1,v}^{*} = R - \frac{2(q_1 - q_2)}{3} \tag{20}$$

$$r_{2,v}^{*} = R - \frac{(q_1 - q_2)}{3} \tag{21}$$

This equilibrium is also unique and satisfies that $r_{1,v}^{*} \in [r_{2,v}^{*} + \frac{t}{2} - (q_1 - q_2), r_{2,v}^{*} - \frac{t}{2}]$ and $r_{2,v}^{*} \in [r_{1,v}^{*} + \frac{t}{2}, r_{1,v}^{*} - \frac{t}{2} + (q_1 - q_2)]$ if and only if

$$(q_1 - q_2) \geq \frac{3t}{4} \tag{22}$$

and

$$(q_1 - q_2) \geq \frac{3t}{2} \tag{23}$$

In other words (20) and (21) represent the interest rate equilibrium for the parameter region defined by (22) and (23). See Figure 4. We can use these interest rates to derive the deposits demands, profits and intermediation spreads for both banks. They are

$$D_{1,v}^{III*} = \frac{2}{3} \tag{24}$$

$$D_{2,v}^{III*} = \frac{1}{3} \tag{25}$$

$$\pi_{1,v}^{III*} = \frac{4(q_1 - q_2)}{9} - C(q_1) \tag{26}$$

$$\pi_{2,v}^{III*} = \frac{(q_1 - q_2)}{9} - C(q_2) \tag{27}$$

$$R - r_{1,v}^{*} = \frac{2(q_1 - q_2)}{3} \tag{28}$$

$$R - r_{2,v}^{*} = \frac{(q_1 - q_2)}{3} \tag{29}$$

- Case 2

For this case the equilibrium deposit interest rates have the same functional form under both types of dominance. The only particular feature for each type of dominance is the range or intervals of the interest rates for which this equilibrium is valid. Moreover, the intermediation spreads, the profits functions and the demand functions evaluated at the equilibrium interest rates have the same functional form under both types of dominance.

The profits maximization problems for the banks can be stated as

$$\underset{r_1}{Max}\ \pi_1^{II} = \underset{r_1}{Max}(R - r_1)D_1^{II} - C(q_1)$$

$$\underset{r_2}{Max}\ \pi_2^{IV} = \underset{r_2}{Max}(R - r_2)(1 - D_1^{II}) - C(q_2)$$

Define for simplicity $\Delta q = (q_1 - q_2)$. The equilibrium for this Bertrand competition is defined by[11]

$$r_1^{**} = R - \frac{t + 2\Delta q + \sqrt{(t + 2\Delta q)^2 + 32\Delta q t}}{16} \tag{30}$$

$$r_2^{**} = R - \frac{3\sqrt{(t + 2\Delta q)^2 + 32\Delta q t}}{16} + \frac{5(t + 2\Delta q)}{6} \tag{31}$$

Using these equilibrium interest rates we can determine the deposits demands, profits and intermediation spreads for both banks. This is accomplished in the Appendix 3. We do not present these results in this part since as we will discuss below, Case 2 lacks of importance. Under *horizontal dominance* $(q_1 - q_2 < t)$ the equilibrium described by equations (30) and (31) are valid if $r_1 \in [r_2 - \frac{t}{2} - (q_1 - q_2), r_2 - \frac{t}{2}] = [r_1^{\min}, r_{1,h}^1]$ and $r_2 \in [r_1 + \frac{t}{2}, r_1 + \frac{t}{2} + (q_1 - q_2)] = [r_{2,h}^2, r_2^{\max}]$. Under *vertical dominance* $(q_1 - q_2 > t)$ the equilibrium characterized by equations (30) and (31) hold if $r_1 \in [r_2 - \frac{t}{2} - (q_1 - q_2), r_2 + \frac{t}{2} - (q_1 - q_2)] = [r_1^{\min}, r_{1,v}^1]$ and $r_2 \in [r_1 - \frac{t}{2} + (q_1 - q_2), r_1 + \frac{t}{2} + (q_1 - q_2)] = [r_{2,v}^2, r_2^{\max}]$. Therefore using (30) and (31) we can conclude that under *horizontal dominance* the equilibrium interest rate is unique and satisfies that $r_1^{**} \in [r_2^{**} - \frac{t}{2} - (q_1 - q_2), r_2^{**} - \frac{t}{2}]$ and $r_2^{**} \in [r_1^{**} + \frac{t}{2}, r_1^{**} + \frac{t}{2} + (q_1 - q_2)]$ if and only if

$$(q_1 - q_2) \geq \frac{3t}{2} \tag{32}$$

[11] See the First Order Conditions in the Appendix 2.

Note that in this case condition (12) is violated.

On the other hand, under *vertical dominance* the equilibrium given by (30) and (31) is unique and satisfies that $r_1^{**} \in [r_2^{**} - \frac{t}{2} - (q_1 - q_2), r_2^{**} + \frac{t}{2} - (q_1 - q_2)]$ and $r_2^{**} \in [r_1^{**} - \frac{t}{2} + (q_1 - q_2), r_1^{**} + \frac{t}{2} + (q_1 - q_2)]$ if and only if

$$\frac{3t}{4} \geq (q_1 - q_2) \tag{33}$$

Note that in this case condition (32) is violated.

Finally it is important to notice that when (32) is satisfied with equality ($q_1 - q_2 = \frac{3t}{2}$) then $r_1^{**} = r_{1,h}^{*}$ and $r_2^{**} = r_{2,h}^{*}$. On the other hand if (33) is satisfied with equality ($q_1 - q_2 = \frac{3t}{4}$) then $r_1^{**} = r_{1,v}^{*}$ and $r_2^{**} = r_{2,v}^{*}$. This means that the equilibrium interest rates vary continuously as q_1, q_2, and t change.

- Case 3

For this case, the equilibrium deposit interest rates also have the same functional forms under both types of dominance. As in Case 2, the only particular characteristic for each type of dominance is the range or intervals of the interest rates for which this equilibrium holds. Moreover the intermediation spreads, the profits functions and the demand functions evaluated at the equilibrium have the same functional form under both types of dominance.

The profits maximization problems for the banks can be stated as

$$\underset{r_1}{Max}\ \pi_1^{IV} = \underset{r_1}{Max}(R - r_1)D_1^{IV} - C(q_1)$$

$$\underset{r_2}{Max}\ \pi_2^{II} = \underset{r_2}{Max}(R - r_2)(1 - D_1^{IV}) - C(q_2)$$

Define for simplicity $\Delta q = (q_1 - q_2)$. The equilibrium for this Bertrand competition is described by[12]

$$r_1^{***} = R + \frac{5t - 3\sqrt{t^2 + 32\Delta q t}}{16} \tag{34}$$

$$r_2^{***} = R - \frac{t + \sqrt{t^2 + 32\Delta q t}}{16} \tag{35}$$

[12]See the First Order Conditions in the Appendix 2.

Using these equilibrium interest rates we can determine the deposits demands, profits and intermediation spreads for both banks. See Appendix 3. We do not present these results in this part since as we will argue below, Case 2 lacks of importance. Under *horizontal dominance* $(q_1 - q_2 < t)$ the results presented by equations (34) and (35) are valid if $r_1 \in [r_2 + \frac{t}{2} - (q_1 - q_2), r_2 + \frac{t}{2}] = [r_{1,h}^2, r_1^{\max}]$ and $r_2 \in [r_1 - \frac{t}{2}, r_1 - \frac{t}{2} + (q_1 - q_2)] = [r_1^{\min}, r_{2,h}^1]$. Under *vertical dominance* $(q_1 - q_2 > t)$ the results presented by equations (34) and (35) hold if $r_1 \in [r_2 - \frac{t}{2}, r_2 + \frac{t}{2}] = [r_{1,v}^2, r_1^{\max}]$ and $r_2 \in [r_1 - \frac{t}{2}, r_1 + \frac{t}{2}] = [r_1^{\min}, r_{2,h}^1]$. Therefore using (34) and (35) we can derive that under *horizontal dominance* this equilibrium is unique and satisfies that $r_1^{***} \in [r_2^{***} + \frac{t}{2} - (q_1 - q_2), r_2^{***} + \frac{t}{2}]$ and $r_2^{***} \in [r_1^{***} - \frac{t}{2}, r_1^{***} - \frac{t}{2} + (q_1 - q_2)]$ if and only if

$$(q_1 - q_2) \geq \frac{3t}{4} \tag{36}$$

Note that in this case condition (13) is violated. On the other hand, under *vertical dominance* the equilibrium given by (34) and (35) is unique and satisfies that $r_1^{***} \in [r_2^{***} - \frac{t}{2}, r_2^{***} + \frac{t}{2}]$ and $r_2^{***} \in [r_1^{***} - \frac{t}{2}, r_1^{***} + \frac{t}{2}]$ if and only if

$$\frac{3t}{2} \geq (q_1 - q_2) \tag{37}$$

Note that in this case condition (23) is violated.

Finally it is important to notice that when (36) is satisfied with equality $(q_1 - q_2 = \frac{3t}{4})$ then $r_1^{***} = r_{1,h}^*$and $r_2^{***} = r_{2,h}^*$. On the other hand if (37) is satisfied with equality $(q_1 - q_2 = \frac{3t}{2})$ then $r_1^{***} = r_{1,v}^*$ and $r_2^{***} = r_{2,v}^*$. As was mentioned before this means that the equilibrium interest rates vary continuously as q_1, q_2, and t change.

3.1.1 Comparative and Static Analysis

It is interesting to analyze and compare some of the partial results from the second stage of the game. We only analyze the results from Case 1, that is, the results of the interest rate equilibrium that correspond to the linear segments of the demand for deposits of each bank, D_1^{III} and D_2^{III}. We restrict the analysis to this case, because as will be shown later, for the first stage of the game there are no values of the prizes and the rate of the transportation cost that support the interest rate equilibrium of cases 2 and 3. In this sense these two cases lack of importance.

The following proposition summarizes the main result of the comparative and static analysis. We warn the reader about the interpretation of the proposition since this is a partial analysis in the sense that we have not yet completed the task of solving the two-stage game. In this sense the following proposition and results should always be understood taking into account that the prizes q_1 and q_2 are given in the second stage of the game. However so far we cannot say anything about the optimality of the given prizes.

Proposition 1 *In the second stage of the game, regardless of the type of dominance, vertical or horizontal, the bank that offered the highest prize in the first stage pays the lowest deposit interest rate, acquires the largest deposit market share and enjoys the widest intermediation spread.*

Proof. *The proof is straightforward. Use (10), (11), (14) and (15) to derive the following. If* $q_1 > q_2$ *then* $r^*_{1,h} < r^*_{2,h}$ *and* $r^*_{1,v} < r^*_{2,v}$. *If* $q_1 > q_2$ *then* $D^{III*}_{1,h} > D^{III*}_{2,h}$ *and* $D^{III*}_{1,v} > D^{III*}_{2,v}$. *In addition, since the intermediation margin has the simple form:* $R - r_i$ *then the bank that pays the lowest interest rate enjoys also the widest intermediation spread* ($R - r^*_{1,h} > R - r^*_{2,h}$ *and* $R - r^*_{1,v} > R - r^*_{2,v}$). ■

Even if Proposition 1 is derived from a partial analysis in the sense that we have not solved the complete two-stage game, it has an important implication. This proposition suggests that under both types of dominance a bank has an incentive to offer higher prizes than its competitor since this strategy assures it a wider intermediation spread and a larger deposit market share.

There are other interesting results that come from this partial analysis. In terms of the equilibrium market share there is a difference between the two kinds of dominance. In qualitative terms, under *vertical dominance* there is a probability-type segmentation of the market. The bank that offers the highest prize specializes in serving high probability-type depositors while the bank that offers the lowest prize serves low probability-type individuals (see Figure 4). Under *horizontal.dominance*, both banks serve all the different probability-types individuals. In quantitative terms, note that the market share of the bank that offers the highest prize is greater under *vertical dominance* than the one under *horizontal dominance.* This is due to the fact that under the latter $q_1 - q_2 < t$, and therefore

$$D^{III*}_{1,v} = \frac{2}{3} > D^{III*}_{1,h} = \frac{1}{2} + \frac{q_1 - q_2}{6t}$$

This suggests that the bank that offers better prizes than its competitor also has an incentive to segment the market according to the probability-type of the individuals.

Other results are summarized as follows. Under *horizontal dominance we* find that

$$\frac{\partial r^*_{1,h}}{\partial q_1} < 0 \qquad \frac{\partial r^*_{1,h}}{\partial q_2} > 0 \qquad \frac{\partial r^*_{2,h}}{\partial q_2} < 0 \qquad \frac{\partial r^*_{2,h}}{\partial q_1} > 0$$

$$\frac{\partial D_{1,h}^{III*}}{\partial q_1} > 0 \qquad \frac{\partial D_{1,h}^{III*}}{\partial q_2} < 0 \qquad \frac{\partial D_{2,h}^{III*}}{\partial q_2} > 0 \qquad \frac{\partial D_{2,h}^{III*}}{\partial q_1} < 0$$

Under *vertical dominance* we find that

$$\frac{\partial r_{1,v}^{*}}{\partial q_1} < 0 \qquad \frac{\partial r_{1,v}^{*}}{\partial q_2} > 0 \qquad \frac{\partial r_{2,v}^{*}}{\partial q_2} < 0 \qquad \frac{\partial r_{2,v}^{*}}{\partial q_1} > 0$$

$$\frac{\partial D_{1,v}^{III*}}{\partial q_1} = 0 \qquad \frac{\partial D_{1,v}^{III*}}{\partial q_2} = 0 \qquad \frac{\partial D_{2,v}^{III*}}{\partial q_2} = 0 \qquad \frac{\partial D_{2,v}^{III*}}{\partial q_1} = 0$$

It is important to notice that under both types of dominance the equilibrium interest rate paid by a bank decreases as the value of the offered prize by this bank increases. On the other hand, the equilibrium interest rate for a bank increases as the value of the prize offered by its competitor increases. Furthermore, under *horizontal dominance,* the demand for deposits of a bank increases as the value of the prize offered by this bank increases. However, this demand for deposits decreases as the value of the prize offered by its competitor increases. Under *vertical dominance* is it striking that there is no effect of the prizes on the demand for deposits of each bank.

3.2 The First Stage - Selecting the Prize

Once the equilibrium of the second stage is found, the first stage of the problem can be solved to derive the equilibrium prize. In Section 2, we proposed an increasing and convex function $C(q_i)$ in q_i to describe the costs of the promotional prize strategy. For simplicity we assume the following functional form for the costs incurred by bank $i = 1, 2$

$$C(q_i) = q_i^2$$

It is important to notice that we are implicitly assuming that both banks have the same technology to design a promotion campaign for the prize that they are offering. The only difference in their costs is the prize q_i that they are offering.

The introduction of the aforementioned cost function brings the attention upon some previous works in the banking literature. In particular we do not understand why models of banking like Matutes and Padilla (1994), Bouckaert and Degryse(1995) and Degryse (1996), explicitly omit the costs of the banking technological innovations. In other words it is not clear why these papers implicitly claim that the assumption of a costless technology is not relevant for their results. We believe the introduction of these costs can affect the results that they obtain.

If we apply the assumption of a costless technology of these papers to the present model, it is necessary to constrain the value of the prizes to an interval $[0, \bar{q}]$, otherwise the banks can end choosing unrealistically an infinite prize in the first stage. This is the same assumption used in models where the vertical differentiation is related with quality, like in Neven and Thisse (1990), Economides (1989b) and Economides (1993).

Once more, the equilibria for this first stage can be characterized according to the type of dominance. Therefore we analyze Cases *1, 2* and *3* sequentially.

- Case 1

Under *horizontal dominance* ($q_1 - q_2 < t$) the profit maximization problems for the banks can be stated as

$$\underset{q_1}{Max}\ \pi_{1,h}^{III*} = \underset{q_1}{Max}\ t\left(\frac{1}{2} + \frac{(q_1 - q_2)}{6t}\right)^2 - q_1^2$$

$$\underset{q_2}{Max}\ \pi_{2,h}^{III*} = \underset{q_2}{Max}\ t\left(\frac{1}{2} - \frac{(q_1 - q_2)}{6t}\right)^2 - q_2^2$$

and the equilibrium is

$$q_{1,h}^* = q_{2,h}^* = \frac{1}{12} \tag{38}$$

Using equations (10), (11) and (14)-(19) the equilibrium interest rates, the equilibrium demands for deposits, the equilibrium profits and the equilibrium intermediation spreads are

$$r_{1,h}^* = r_{2,h}^* = R - \frac{t}{2} \tag{39}$$

$$D_{1,h}^* = D_{2,h}^* = \frac{1}{2} \tag{40}$$

$$\pi_{1,h}^* = \pi_{2,h}^* = \frac{t}{4} - \frac{1}{144} \tag{41}$$

$$R - r_{1,h}^* = R - r_{2,h}^* = \frac{t}{2} \tag{42}$$

Under *vertical dominance* $(q_1 - q_2 > t)$ the profit maximization problems for the banks can be described as

$$\underset{q_1}{Max}\ \pi_{1,v}^{III*} = \underset{q_1}{Max}\ \frac{4}{9}(q_1 - q_2) - q_1^2$$

$$\underset{q_2}{Max}\ \pi_{2,v}^{III*} = \underset{q_2}{Max}\ \frac{1}{9}(q_1 - q_2) - q_2^2$$

and the equilibrium is described by

$$q_{1,v}^* = \frac{2}{9} \qquad q_{2,v}^* = 0 \tag{43}$$

Using equations (20), (21) and (24)-(29) the equilibrium interest rates, the equilibrium demands for deposits, the equilibrium profits and the equilibrium intermediation spreads can be expressed as

$$r_{1,v}^* = R - \frac{4}{27} \qquad r_{2,v}^* = R - \frac{2}{27} \tag{44}$$

$$D_{1,v}^* = \frac{2}{3} \qquad D_{2,v}^* = \frac{1}{3} \tag{45}$$

$$\pi_{1,v}^* = \frac{4}{81} \qquad \pi_{2,v}^* = \frac{2}{81} \tag{46}$$

$$R - r_{1,v}^* = \frac{4}{27} \qquad R - r_{2,v}^* = \frac{2}{27} \tag{47}$$

These results for both types of dominance are summarized in the following two propositions.

Proposition 2 *For the two-stage game, if there is horizontal dominance $(q_1 - q_2 < t)$ and if the cost functions are described by $C(q_i) = q_i^2$ where $i = 1, 2$ then a symmetric equilibrium is obtained in which both banks select the same prizes and the same deposit interest rates. These in turn imply that both banks acquire the same deposit market shares and receive the same profits.*

Proposition 3 *For the two-stage game, if there is vertical dominance $(q_1 - q_2 > t)$ and if the cost functions are described by $C(q_i) = q_i^2$ where $i = 1, 2$ then Bank 1 offers a positive prize while Bank 2 does not offer any prize. Moreover Bank 1 not only pays the lowest deposit interest rate but also has the largest deposit market share and the widest intermediation spread.*

This last proposition is more robust than Proposition 1, since it is a proposition that is derived based on the whole two-stage game. Therefore in contrast to Proposition 1, these Proposition 2 and 3 suggest that only under *vertical dominance* a bank has an incentive to offer higher prizes than its competitor since this strategy assures it a wider intermediation spread and a larger deposit market share. Moreover notice that under *vertical dominance,* Bank 1 segments the deposit market serving only high probability-type customers while Bank 2 specializes in serving low probability-type customers (see Figure 4).

We proceed analyzing Cases 2 and 3. In both cases we argue that there is not a pair of prizes (q_1^*, q_2^*) that supports the equilibrium interest rates derived for the second stage

- Case 2

The following propositions are useful to discard an equilibrium for this case.

Proposition 4 *For the parameter region defined by* $(q_1 - q_2) \geq \frac{3t}{2}$ *and under horizontal dominance,* $(q_1 - q_2) < t$*, there is not a pair of prizes* (q_1^*, q_2^*) *that supports the equilibrium of the interest rate defined by (30) and (31).*

Proof. The proof is straightforward. Take the inequalities $2(q_1 - q_2) \geq 3t$ and $2(q_1 - q_2) < 2t$ and note that there are no positive values of $(q_1 - q_2)$ that satisfy both inequalities at the same time. Therefore it is not possible to support the equilibrium of the interest rate defined by (30) and (31). ■

Proposition 5 *For the parameter region defined by* $\frac{3t}{4} \geq (q_1 - q_2)$ *and under vertical dominance,* $(q_1 - q_2) > t$*, there is no a pair of prizes* (q_1^*, q_2^*) *that supports the equilibrium of the interest rate defined by (30) and (31).*

Proof. *The proof is straightforward. Take the inequalities* $3t \geq 4(q_1 - q_2)$ *and* $3(q_1 - q_2) > 3t$ *and note that there are no positive values of* $(q_1 - q_2)$ *that satisfy both inequalities at the same time. Hence it is not possible to support the equilibrium of the interest rate defined by (30) and (31).* ■

- Case 3

This case is the most difficult because it is not possible to derive an explicit expression for the equilibrium prizes. The reason is that under both types of dominance, the First Order Conditions of the profit maximization problems are cumbersome and non-linear equations on the prizes. Therefore they cannot be solved analytically.[13] However it is possible to solve numerically for the equilibrium prizes. Simulations for different values of the unitary cost of transportation, t, suggest that the equilibrium is asymmetric. We obtain that $q_1^* > 0$ and $q_2^* = 0$, regardless of the type of dominance. However for all the simulations we find that (q_1^*,q_2^*) does not satisfy (36) under *horizontal dominance,* and (37) under *vertical dominance*. Thus we argue that under this case it is not possible to find equilibrium prizes in the first stage that support the equilibrium interest rates described by (34) and (35).

The previous analysis is useful to emphasize that the only equilibria that are relevant for the two-stage game are those derived in the Case 1. Hence we will continue focusing on this case.

3.3 A Comparison with a Benchmark Game

In order to understand the impact of the introduction of "prizes" in the banking industry upon interest rates, demand deposits and intermediation spreads, it is useful to pursue the following exercise. We will compare the equilibria of two different duopolistic banking industries. The first industry is characterized by two banks that compete using solely their deposits interest rates. Banks do not offer prizes or raffles. This is defined as the *benchmark* game The second industry is characterized by the two banks that are involved in the two-stage game that we solved above.

We proceed analyzing the *benchmark* game. In this game, two banks are involved in a Bertrand competition using as strategies their deposits interest rates. This game is the same game as in Salop (1979) that is applied for the case of banking in Freixas and Rochet (1997).

As before we can find the marginal depositor by solving for the location z that makes her indifferent between the two banks. In other words using the utilities derived from the services of each bank, the location z solves

$$\omega + r_1 - tz = \omega + r_2 - t\left(\frac{1}{2} - z\right)$$

and defining $x = 2z$ we obtain

$$x = 2z = \frac{1}{t}\left(r_1 - r_2 + \frac{t}{2}\right) \tag{48}$$

where x represents the market share of Bank 1. Note that in comparison to equation (3), x in (48) is an independent function of the probability-type of the individuals (θ).

[13] These conditions are available from the author upon request.

The banks' problem can be stated as follows. For bank $i \neq j$

$$\underset{r_i}{Max}\ \pi_i = \underset{ri}{Max}(R - r_i)D_i = \underset{ri}{Max}(R - r_i)\frac{1}{t}\left(r_i - r_j + \frac{t}{2}\right)$$

Solving this game we find the following Nash Equilibrium

$$r_1^B = r_2^B = R - \frac{t}{2} \tag{49}$$

and using it we can derive the demands for deposits, the profits and the intermediation spreads for both banks. They are

$$D_1^B = D_2^B = \frac{1}{2} \tag{50}$$

$$\pi_1^B = \pi_2^B = \frac{t}{4} \tag{51}$$

$$R - r_1^B = R - r_2^B = \frac{t}{2} \tag{52}$$

This is a symmetric equilibrium in which the market of deposits is divided in equal shares for each bank. The division of the deposits market is represented in Figure 6.

Some interesting results arise comparing the equilibria and outcomes of the *benchmark* game and those of the two-stage game (Case 1). The main results of this comparison are summarized in the following propositions.

Proposition 6 *If there is horizontal dominance in the described two-stage game then the deposit interest rates, the deposit market shares and the intermediation spreads for both banks are the same as those of the benchmark game.*

Proof. *Compare (39), (40) and (42) with (49),(50) and (52).* ■

Proposition 7 *If there is vertical dominance in the described two-stage game then the bank that offers a prize, pays a lower deposit interest rate, has a larger market share and enjoys a wider intermediation spread than those that it has under the benchmark game.*

Proof. *Compare (44), (45) and (47) with (49),(50) and (52).* ■

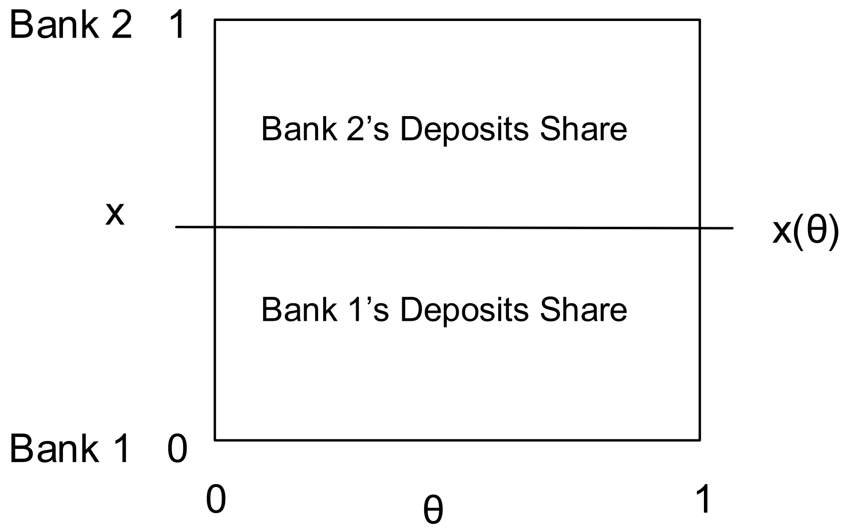

Figure 6:

The last proposition supports the argument by Silver (1983). In particular, notice that under vertical dominance the strategy of offering prizes loosens some of the internal and external constraints for a bank. The new strategy not only endows a bank with a new tool to compete in the deposit market. It also allows the bank to achieve a larger deposit market share and a wider intermediation spread if it offers the biggest prize.

It is relevant to point out that under *vertical dominance* the profits of the bank that offers a prize in the two-stage game are lower than the profits of the same bank under the *benchmark* game. This result can be explained by the introduction of costs associated with the prizes. It is clear that these costs play a key role in the model. Once more this result brings the attention upon some results of models of banking like Matutes and Padilla (1994), Bouckaert and Degryse(1995) and Degryse (1996) in which the costs of the banking technological innovations are omitted.

4 Conclusions

We construct a model that allows us to understand the motivation of some Colombian banks of offering some promotional prizes and raffles to new potential savings customers. The new strategy endows banks with a new tool to compete in the deposits markets without relying exclusively on the use of deposit interest rates.

We study the impact of this promotional strategy on the deposit interest rates, the deposit market shares and the intermediation spreads. We find that this impact can be analyzed by the equilibria of a two-stage game duopoly. In the first stage banks compete with prizes while in the second stage they compete with interest rates.

The types of equilibria that we find depend on the type of dominance that prevails. There are two types of dominance: *horizontal* and *vertical.* Under *horizontal dominance* the difference between the value of the prizes offered by the banks is less than the unitary cost of transportation of the customers. The equilibria under this type of dominance imply that both banks serve all the customers regardless of the subjective probability that these customers assign to winning prizes. Under *vertical dominance* the difference between the value of the prizes offered by the banks is greater than the unitary cost of transportation of the customers. In equilibrium this type of dominance allows one bank to segment the deposit market in terms of the customers probability-type. In general, the bank that offers the highest prize also serves only customers that assign high subjective probabilities to winning prizes. On the other hand the bank that offers the lowest prize, or no prize, specializes in serving customers that assign very low probability to win prizes.

From the equilibria of the two-stage game, we derive the following interesting results. Under *horizontal dominance* a symmetric equilibrium is obtained in which both banks not only offer the same prizes' value but also pay the same deposit interest rates, have the same deposit market shares and receive the same profits. Under *vertical dominance* only one bank offers a positive prize allowing it to segment the deposit market in terms of the customers probability-type. This bank not only pays the lowest deposit interest rate but also has the largest market share and enjoys the widest intermediation spread.

Finally we introduce a *benchmark* game to be able to understand what has changed in the Colombian banking structure with the introduction of promotional prize strategies. This *benchmark* game is defined as the situation in which banks do not offer any prize and compete only in deposit interest rates. Comparing the equilibrium of this game with equilibrium of the two-stage game we deduce the following results. Under the *horizontal dominance* equilibrium the deposit interest rates, the deposit market shares and the intermediation spreads are the same as the ones of the *benchmark* game. Under *vertical dominance*, the bank that offers the highest prize, pays a lower deposit interest rate, has a larger deposit market share and enjoys a wider intermediation spread than those that it has under the *benchmark* game.

5 Appendices

5.1 Appendix 1

In this appendix we derive the demand for deposits for Bank 1. To accomplish this task it is necessary to integrate the function $x(\theta)$ of equation (3) over $[0,1]$ taking into account the appropriate range of the interest rate r_1. There are 5 cases for each type of dominance.

Suppose that r_2 is given. For "very low" values of r_1, and independently of the type of dominance, the line $x(\theta)$ does not cross the unit square. This defines the segment of the demand that we call D_1^I. Equivalently when $r_1 \in [0, r_2 - \frac{t}{2} - (q_1 - q_2)] = [0, r_1^{\min}]$ we can establish that

$$D_1^I = 0 \tag{53}$$

For "low" values of r_1, given r_2 and independently of the slope $\frac{(q_1-q_2)}{t}$, the line $x(\theta)$ crosses the bottom and the right side of the unit square. This determines the segment of the demand that we call D_1^{II}. In this case the same demand functional form D_1^{II} holds for both types of dominance. However, the range of the interest rate in which this functional form is valid varies according to the type of dominance. If *horizontal dominance* $(q_1 - q_2 < t)$ prevails then $r_1 \in [r_2 - \frac{t}{2} - (q_1 - q_2), r_2 - \frac{t}{2}] = [r_1^{\min}, r_{1,h}^1]$. On the other hand if *vertical dominance* $(q_1 - q_2 > t)$ prevails then $r_1 \in [r_2 - \frac{t}{2} - (q_1 - q_2), r_2 + \frac{t}{2} - (q_1 - q_2)] = [r_1^{\min}, r_{1,v}^1]$. In both cases

$$D_1^{II} = \int_{\bar{\theta}}^1 x(\theta) d\theta = \int_{\bar{\theta}}^1 \left(r_1 - r_2 + \frac{t}{2} + \theta(q_1 - q_2) \right) d\theta$$

where

$$\bar{\theta} = \frac{1}{(q_1 - q_2)} \left(r_2 - r_1 - \frac{t}{2} \right)$$

Then

$$D_1^{II} = \frac{1}{2t(q_1 - q_2)} \left(r_2 - r_1 - \frac{t}{2} - (q_1 - q_2) \right)^2 \tag{54}$$

Given r_2 and for "intermediate values" r_1 the function $x(\theta)$ can cross either the vertical sides or the horizontal sides of the unit square. The first case implies *horizontal dominance* and the second case implies *vertical dominance.* Under *horizontal dominance* $(q_1 - q_2 < t)$ we have that $r_1 \in [r_2 - \frac{t}{2}, r_2 + \frac{t}{2} - (q_1 - q_2)] = [r_{1,h}^1, r_{1,h}^2]$ and the demand function for deposits can be calculated as

$$D_{1,h}^{III} = \int_0^1 x(\theta)d\theta = \int_0^1 \left(r_1 - r_2 + \frac{t}{2} + \theta(q_1 - q_2)\right) d\theta$$

Then

$$D_{1,h}^{III} = \frac{1}{t}\left(r_1 - r_2 + \frac{t}{2}\right) + \frac{(q_1 - q_2)}{2t} \tag{55}$$

On the other hand, if *vertical dominance* $(q_1 - q_2 > t)$ prevails then $r_1 \in [r_2 + \frac{t}{2} - (q_1 - q_2), r_2 - \frac{t}{2}] = [r_{1,v}^1, r_{1,v}^2]$ and the demand function can be derived as

$$D_{1,v}^{III} = \int_{\bar{\theta}}^{\hat{\theta}} x(\theta)d\theta = \int_{\bar{\theta}}^{\hat{\theta}} \left(r_1 - r_2 + \frac{t}{2} + \theta(q_1 - q_2)\right) d\theta$$

where

$$\begin{aligned} \bar{\theta} &= \frac{1}{(q_1 - q_2)}\left(r_2 - r_1 - \frac{t}{2}\right) \\ \hat{\theta} &= \frac{1}{(q_1 - q_2)}\left(r_2 - r_1 + \frac{t}{2}\right) \end{aligned}$$

Then

$$D_{1,v}^{III} = \frac{1}{(q_1 - q_2)}\left[r_1 - r_2 + (q_1 - q_2)\right] \tag{56}$$

For "high" values of r_1, given r_2 and independently of the slope $\frac{(q_1 - q_2)}{t}$, the line $x(\theta)$ crosses the top and the left side of the unit square. This defines the segment of the demand that we call D_1^{IV}. In this case the same demand functional form D_1^{IV} applies to both types of dominance. However, the range of the interest rate in which this demand holds varies with the type of dominance. If *horizontal dominance* $(q_1 - q_2 < t)$ prevails then $r_1 \in [r_2 + \frac{t}{2} - (q_1 - q_2), r_2 + \frac{t}{2}] = [r_{1,h}^2, r_1^{\max}]$. If *vertical dominance* $(q_1 - q_2 > t)$ prevails then $r_1 \in [r_2 - \frac{t}{2}, r_2 + \frac{t}{2}] = [r_{1,v}^2, r_1^{\max}]$. In both cases

$$D_1^{IV} = \int_0^{\hat{\theta}} x(\theta)d\theta + \int_{\hat{\theta}}^1 d\theta = \int_0^{\hat{\theta}} \left(r_1 - r_2 + \frac{t}{2} + \theta(q_1 - q_2)\right) d\theta + \int_{\hat{\theta}}^1 d\theta$$

where

$$\hat{\theta} = \frac{1}{(q_1 - q_2)} \left(r_2 - r_1 + \frac{t}{2} \right)$$

Then

$$D_1^{IV} = 1 - \frac{\left(r_2 - r_1 + \frac{t}{2}\right)^2}{2t(q_1 - q_2)} \tag{57}$$

This determines the segment of the demand that we call D_1^{IV}. Finally, given r_2, for "very high" values of r_1 ,and regardless of the type of dominance, the line $x(\theta)$ does not cross the unit square. This defines the segment of the demand that we call D_1^V. Equivalently when $r_1 \in [r_2 + \frac{t}{2}, \infty) = [r_1^{\max}, \infty)$, we can establish that

$$D_1^V = 1 \tag{58}$$

Note that D_1^{II} is strictly convex, $D_{1,h}^{III}$ and $D_{1,v}^{III}$ are linear, and D_1^{IV} is strictly concave in r_1. All of these segments are increasing in r_1. To prove it, assume that $q_1 > q_2$ and define $\Delta q = q_1 - q_2$, then we have that

$$\frac{\partial D_1^{II}}{\partial r_1} = \frac{1}{t\Delta q} \left[r_1 - r_1^{\min} \right] > 0 \qquad \frac{\partial^2 D_1^{II}}{\partial r_1^2} = \frac{1}{t\Delta q} > 0$$

$$\frac{\partial D_{1,h}^{III}}{\partial r_1} = \frac{1}{t} > 0 \qquad and \quad independent \quad of \quad r_1$$

$$\frac{\partial D_{1,v}^{III}}{\partial r_1} = \frac{1}{\Delta q} > 0 \qquad and \quad independent \quad of \quad r_1$$

$$\frac{\partial D_1^{IV}}{\partial r_1} = \frac{1}{t\Delta q} [r_1^{\max} - r_1] > 0 \qquad \frac{\partial^2 D_1^{IV}}{\partial r_1^2} = -\frac{1}{t(q_1 - q_2)} < 0$$

In addition notice that under *horizontal dominance* and *vertical dominance* the demand functions are continuous. That is

$$D_1^I = 0 = D_1^{II}(r_1 = r_1^{\min})$$

$$D_1^{II}(r_1 = r_{1,h}^1) = \frac{\Delta q}{2t} = D_{1,h}^{III}(r_1 = r_{1,h}^1)$$
$$D_1^{II}(r_1 = r_{1,v}^1) = \frac{t}{2\Delta q} = D_{1,v}^{III}(r_1 = r_{1,v}^1)$$

$$D_{1,h}^{III}(r_1 = r_{1,h}^2) = 1 - \frac{\Delta q}{2t} = D_1^{IV}(r_1 = r_{1,h}^2)$$
$$D_{1,v}^{III}(r_1 = r_{1,v}^2) = 1 - \frac{t}{2\Delta q} = D_1^{IV}(r_1 = r_{1,v}^2)$$

$$D_1^{IV}(r_1 = r_1^{\max}) = 1 = D_1^V$$

5.2 Appendix 2

It is straightforward to prove the continuity of the profit functions since the demand functions are continuous. Moreover the convexity of the profit functions can be stated as follows.

Under *horizontal dominance,* defining $\Delta q = q_1 - q_2$ and using the results from Appendix 1 we can deduce that

$$\frac{\partial^2 \pi_1^{II}}{\partial r_1^2} = -2\frac{\partial D_1^{II}}{\partial r_1} + [R - r_1]\frac{\partial^2 D_1^{II}}{\partial r_1^2} = \frac{1}{t\Delta q}\left[r_1 - r_1^{\min}\right] + \frac{[R - r_1]}{t\Delta q} > 0$$

$$\frac{\partial^2 \pi_{1,h}^{III}}{\partial r_1^2} = -2\frac{\partial D_{1,h}^{III}}{\partial r_1} < 0$$

$$\frac{\partial^2 \pi_1^{IV}}{\partial r_1^2} = -2\frac{\partial D_1^{IV}}{\partial r_1} + [R - r_1]\frac{\partial^2 D_1^{IV}}{\partial r_1^2} < 0$$

Under *vertical dominance,* defining $\Delta q = q_1 - q_2$ and using results from Appendix 1 we can deduce that

$$\frac{\partial^2 \pi_1^{II}}{\partial r_1^2} = -2\frac{\partial D_1^{II}}{\partial r_1} + [R - r_1]\frac{\partial^2 D_1^{II}}{\partial r_1^2} = \frac{1}{t\Delta q}\left[r_1 - r_1^{\min}\right] + \frac{[R - r_1]}{t\Delta q} > 0$$

$$\frac{\partial^2 \pi_{1,v}^{III}}{\partial r_1^2} = -2\frac{\partial D_{1,v}^{III}}{\partial r_1} < 0$$

$$\frac{\partial^2 \pi_1^{IV}}{\partial r_1^2} = -2\frac{\partial D_1^{IV}}{\partial r_1} + [R - r_1]\frac{\partial^2 D_1^{IV}}{\partial r_1^2} < 0$$

Therefore π_1^{II} is strictly convex and $\pi_{1,h}^{III}, \pi_{1,v}^{III}$ and π_1^{IV} are strictly concave. The analysis for the profits functions of Bank 2 leads to similar results.

5.3 Appendix 3

- Case 2

As was stated above under *horizontal dominance* the results presented by equations (30)and (31) are valid if $r_1 \in [r_1^{\min}, r_{1,h}^1]$ and $r_2 \in [r_{2,h}^2, r_2^{\max}]$. Under *vertical dominance* the results presented by equations (30)and (31) hold if $r_1 \in [r_1^{\min}, r_{1,v}^1]$ and $r_2 \in [r_{2,v}^2, r_2^{\max}]$. The profits maximization problem for the banks can be formulated as

$$\underset{r_1}{Max}\ \pi_1^{II} = \underset{r_1}{Max}(R - r_1)D_1^{II} - C(q_1)$$

$$\underset{r_2}{Max}\ \pi_2^{IV} = \underset{r_2}{Max}(R - r_2)(1 - D_1^{II}) - C(q_2)$$

Define for simplicity $\Delta q = (q_1 - q_2)$. The FOC's for this problem can be expressed as

$$\left(r_2 - r_1 - \frac{t}{2} - \Delta q\right)^2 + 2(R - r_1)\left(r_2 - r_1 - \frac{t}{2} - \Delta q\right) = 0$$

$$2t\Delta q - \left(r_2 - r_1 - \frac{t}{2} - \Delta q\right)^2 + 2(R - r_2)\left(r_2 - r_1 - \frac{t}{2} - \Delta q\right) = 0$$

and the solution is

$$r_1^{**} = R - \frac{t + 2\Delta q + \sqrt{(t + 2\Delta q)^2 + 32\Delta q t}}{16} \tag{59}$$

$$r_2^{**} = R - \frac{3\sqrt{(t + 2\Delta q)^2 + 32\Delta q t}}{16} + \frac{5(t + 2\Delta q)}{6} \tag{60}$$

Using (59) and (60) the demand functions for deposits and the profits functions can be written as

$$D_1^{II**} = \frac{t^2\mu^2}{2t\Delta q}$$

$$D_2^{IV**} = 1 - \frac{t^2\mu^2}{2t\Delta q}$$

$$\pi_1^{II**} = \frac{t^3\mu^3}{4t\Delta q} - C(q_1)$$

$$\pi_2^{IV**} = \left(\frac{3t\mu}{2} - \frac{t}{2} - \Delta q\right)\left(1 - \frac{t^2\mu^2}{2t\Delta q}\right) - C(q_2)$$

where

$$t\mu = \frac{t + 2\Delta q + \sqrt{(t+2\Delta q)^2 + 32\Delta q t}}{8}$$

while the intermediation spreads are

$$R - r_1^{**} = \frac{t + 2\Delta q + \sqrt{(t+2\Delta q)^2 + 32\Delta q t}}{16}$$

$$R - r_2^{**} = \frac{3\sqrt{(t+2\Delta q)^2 + 32\Delta q t}}{16} - \frac{5(t+2\Delta q)}{6}$$

- Case 3

Under *horizontal dominance* the results presented by equations (34) and (35) are valid if $r_1 \in [r_{1,h}^2, r_1^{\max}]$ and $r_2 \in [r_1^{\min}, r_{2,h}^1]$. Under *vertical dominance* the results presented by equations (34) and (35) hold if $r_1 \in [r_{1,v}^2, r_1^{\max}]$ and $r_2 \in [r_1^{\min}, r_{2,h}^1]$. The profit maximization problem for the banks can be stated as

$$\underset{r_1}{Max}\ \pi_1^{IV} = \underset{r_1}{Max}(R - r_1)D_1^{IV} - C(q_1)$$

$$\underset{r_2}{Max}\ \pi_2^{II} = \underset{r_2}{Max}(R - r_2)(1 - D_1^{IV}) - C(q_2)$$

Define $\Delta q = (q_1 - q_2)$. The FOC's for this problem can be expressed as

$$\left(r_2 - r_1 + \frac{t}{2}\right)^2 - 2t\Delta q + 2(R - r_1)\left(r_2 - r_1 + \frac{t}{2}\right) = 0$$

$$\left(r_2 - r_1 + \frac{t}{2}\right)^2 - 2(R - r_2)\left(r_2 - r_1 + \frac{t}{2}\right) = 0$$

and the solution is

$$r_1^{***} = R + \frac{5t - 3\sqrt{t^2 + 32\Delta q t}}{16} \qquad (61)$$

$$r_2^{***} = R - \frac{t + \sqrt{t^2 + 32\Delta qt}}{16} \tag{62}$$

Using (61) and (62) the demand functions for deposits and the profits functions can be written as

$$D_1^{IV***} = 1 - \frac{t^2\gamma^2}{2t\Delta q}$$

$$D_2^{II***} = \frac{t^2\gamma^2}{2t\Delta q}$$

$$\pi_1^{IV***} = \left(\frac{3t\gamma}{2} - \frac{t}{2}\right)\left(1 - \frac{t^2\gamma^2}{2t\Delta q}\right) - C(q_1)$$

$$\pi_2^{II***} = \frac{t^3\gamma^3}{4t\Delta q} - C(q_2)$$

where

$$t\gamma = \frac{t + \sqrt{t^2 + 32\Delta qt}}{8}$$

while the intermediation spreads are

$$R - r_1^{***} = -\frac{5t - 3\sqrt{t^2 + 32\Delta qt}}{16}$$

$$R - r_2^{***} = \frac{t + \sqrt{t^2 + 32\Delta qt}}{16}$$

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