CEDE

DOCUMENTO CEDE 2005-61
ISSN 1657-7191 (Edición Electrónica)
NOVIEMBRE DE 2005

A GENERALIZED INDEX OF MARKET POWER

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Abstract

This paper analyses two approaches to measuring market power –the commonly used Lerner index and a range of exploitation measures-. It is argued that the Lerner index is designed to quantify market power from the supply side, and the exploitation measures are designed to quantify market power from the demand side, and that those two approaches do not always behave in a symmetric way, since they do not always have the same bounds. To sort out these potentially undesirable properties, this paper proposes a new general index to measure market power, which is symmetrical in the sense that it is bounded between cero and one, regardless of whether the market power comes from the supply or the demand side. The index proposed allows for the presence of more than one firm and for the existence of conjectural variations.

Keywords: Market Power, Mark Up, Mark Down, Lerner Index, Exploitation Measures, Industrial Organization, Conjectural Variations.

JEL Classification: D49, L10, L11.

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1 I am grateful to Luis Eduardo Quintero for his research assistance, and to Jorge Tovar and Jean Pietro Bonaldi for their suggestions. Support from Universidad de los Andes and its Department of Economics is also gratefully acknowledged. All remaining errors are mine. E-mail: hvallejo@uniandes.edu.co

UN ÍNDICE GENERALIZADO DE PODER DE MERCADO

Resumen

Este documento analiza dos enfoques para medir poder de mercado –el frecuentemente utilizado índice de Lerner y un conjunto de medidas de explotación-. Se argumenta que el índice de Lerner está diseñado para cuantificar el poder de mercado por el lado de la oferta, y que las medidas de explotación están diseñadas para cuantificar el poder de mercado por el lado de la demanda, y que esos dos enfoques no siempre tienen los mismos límites. Para corregir estas propiedades potencialmente no deseables, este documento propone un nuevo índice general para medir poder de mercado, que es simétrico -estando restringido a valores entre cero y uno-, independientemente de si el poder de mercado proviene del lado de la oferta o de la demanda. El índice propuesto permite la presencia de más de una firma y la existencia de variaciones conjeturales.

Palabras clave: Poder de mercado, mark up, mark down, índice de Lerner, medidas de explotación, organización industrial, variaciones conjeturales.

Clasificación JEL: D49, L10, L11.

I. INTRODUCTION

This paper analyses two approaches to measuring market power –the commonly used Lerner index and a range of exploitation measures-. It is argued that the Lerner index is designed to quantify market power from the supply side, and the exploitation measures are designed to quantify market power from the demand side, and that those two approaches do not always behave in a symmetric way, since they do not always have the same bounds.

To sort out these potentially undesirable properties, this paper proposes a new general index to measure market power, which is symmetrical in the sense that it is bounded between cero and one, regardless of whether the market power comes from the supply or the demand side. The index proposed allows for the presence of more than one firm and for the existence of conjectural variations.

The paper is organized as follows: the next section presents a revision of some of the most relevant literature for the purpose at hand. Then, the theoretical framework derives the Lerner index and three alternative –and related- exploitation measures, highlighting their main properties. The following section proposes an index that overcomes some of the limitations of the above measures, describing the properties of such new index. The paper ends with the main conclusions.

II PREVIOUS LITERATURE

The most widely used measure of market power is the Lerner mark up index -or Lerner index for short, proposed by Abba Lerner (1934), and defined as[2]:

$$L = \frac{P_G - MgC}{P_G}$$

2 This index has been criticized among others, because estimating it is complex, since it is difficult to obtain measures of marginal costs, and since prices may be affected by cyclical economic behaviour. Thus, in general, it is useful to think of the Lerner index –and of market power quantifications in general- as measuring average mark up.

where

L	=	Lerner index
P_G	=	Per unit price of the good analyzed
MgC	=	Marginal cost

Cabral (2000) explains how measures of market power may be required for industries and not just firms, and shows how when this is taken into account, the Lerner index can be represented by

$$L_H = \frac{H}{|\eta|}$$

where

L_H	=	Lerner index with more than one firm
H	=	Herfindahl market concentration index
η	=	Price elasticity of demand

Cabral also explains that when firm behaviour is incorporated, the Lerner index can be expressed as:

$$L = \theta \frac{H}{|\eta|}$$

where θ is a conjectural variations coefficient, such that if:

$\theta = 0$, players play Bertrand and price equals marginal costs

$\theta = 1$, players play Cournot

$\theta = 1/H$, players play Collusion or Cartel and replicate Monopoly power

Arthur Pigou (1924) proposed an exploitation measure defined as:

$$PEM = \frac{MRP}{P_F}$$

where

PEM	=	Pigou's exploitation measure
MRP	=	Marginal revenue product
P_F	=	Factor or input unit price

Scully (1974) estimated a rate of monopsonistic exploitation as

$$RME = \frac{MRP - P_F}{MRP}$$

where

RME = Rate of monopsonisitc exploitation

and all the other variables are defined as before.

Similarly, Boal (1995) estimated an exploitation measure as

$$E = \frac{MRP - P_F}{P_F} = \frac{1}{\varepsilon} \tag{1}$$

where

ε = Price elasticity of supply.

Boal and Ransom (1997) show that that index can be generalized when there is more than one firm, as:

$$E_H = \frac{H}{\varepsilon}$$

where

E_H	=	Boal exploitation measure with more than one firm
H	=	Herfindahl market concentration index

Bresnahan (1989) reviews a vast literature on measuring market power, and provides a summary of Lerner index estimations, shown in table 1. Note that all of the estimations provided in that table are between cero and one.

Table 1

Summary of existing empirical work on the Lerner index

Author	Industry	Lerner Index
Lopez (1984)	Food Processing	0.504
Roberts (1984)	Coffee roasting	0.055/0.025 (a)
Appelbaum (1982)	Rubber	0.049 (c)
Appelbaum (1982)	Textile	0.072 (c)
Appelbaum (1982)	Electrical machinery	0.198 (c)
Appelbaum (1982)	Tobacco	0.648 (c)
Porter (1983)	Railroads	0.40 (b)
Slade (1987)	Retail gasoline	0.1
Bresnahan (1981)	Automobiles (1970s)	0.10/0.34 (d)
Suslow (1986)	Aluminum (interwar)	0.59
Spiller - Favaro (1984)	Banks "before" (e)	0.88/0.21 (f)
Spiller - Favaro (1984)	Banks "after" (e)	0.40/0.16 (f)

a Largest and second largest firm, respectively
b When cartel was succeeding: 0 in reversionary periods.
c. At sample midpoint.
d. Varies by type of car; larger in standard, luxury segment.
e. Uruguayan banks before and after entry deregulation
f. Large firms / small firms

Source: Bresnahan (1989) pp 1051.

Boal and Ransom (1997) collect information on a range of exploitation measures, and express them in terms of E, the Boal exploitation measure. Such information is summarized in table 2. Note that all of the estimations provided in that table are above cero and eventually, larger than one.

Table 2

Summary of existing empirical work on Pigou's exploitation measure

Author	Industry	Pigou's Exploitation Measure (a)
Scully (1974)	Professional Baseball	4-7
Medoff (1976)	Professional Baseball	1-2 (b)
Boyd (1994)	Coal Mining (c)	0.24 (d)
Sullivan (1989)	Nurses in Hospitals	0.75-0.26 short run; close to zero long run; 0.04-0.13 total
Hansen (1992)	Nurses in Hospitals (California only)	Less than 0.05
Boal (1995)	Coal Mining	0.15-0.53 short run; essentially zero in long run; 0.03-0.09 total.
Machin, Manning, and Woodland (1993)	Employment by residential homes (e	0.04. 0.15 with correction for hires and quits.
Ransom (1993)	Senior College teachers	5 to 18 (f)
Van den Berg and Ridder (1993)	General Labour Market	0.13-0.15 (g)
Brown and Meedof (1989)	General Labour Market	0.01-0.03

a. Boal and Ransom adjust original extimates to the exploitation measure E.
b. Data from the Reserve Clause era. The Reserve Clause is a legal clause that teams adopted, which prohibited competition for the hiring players between teams.
c. From isolated Coal Mining towns
d. Measure not significantly different from zero statistically.
e. Homes for the elderly in England
f. Black and Lowenstein (1991) say Ransom's estimates should be upper bound, because movers may be paid more than MRP.
g. Allowing for heterogeneity among workers lower the estimations in comparison to other papers'.

Source: Boal and Ransom (1997).

III. THEORETICAL FRAMEWORK

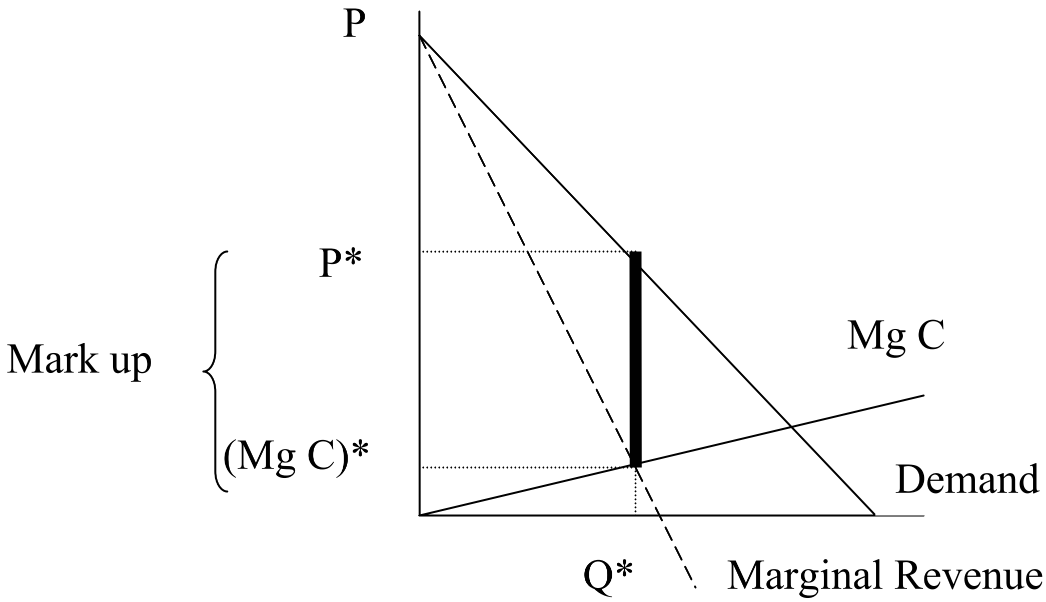
This section derives measures of market power from profit maximization. In order to make general statements, assume that there are n firms that play Cournot to start with, and keep in mind that this assumption will be relaxed later on, through the introduction of a conjectural variations coefficient.

A. Market power from the supply side

When market power is generated from the supply side (with prices greater than marginal costs, as in monopoly, monopolistic competition and -often- oligopoly), firms apply a mark up. This case is shown in figure 1.

Figure 1

MARKET POWER FROM THE SUPPLY SIDE:
MONOPOLY, MONOPOLISTIC COMPETITION
AND -OFTEN- OLIGOPOLY



Consider the following profit equation

$$\prod_i = P_G(q_i + Q_{-i})q_i - TC_i$$

where

Π_i	=	profits of firm i
P_G	=	Per unit price of the good produced by firm i
q_i	=	quantity produced by firm i
Q_{-i}	=	quantity produced by all firms in the market, except firm i
Q	=	quantity produced by all firms in the market
TCi	=	Total costs of firm i

From profit maximization with n firms, it is possible to conclude that the Lerner index with n firms (L_H) is:

$$\frac{\partial \prod_i}{\partial q_i} = P(q_i + Q_{-i}) + P'(q_i + Q_{-i})q_i - \frac{\partial TC}{\partial Q} = 0$$

$$P(Q) + QP'(Q)\frac{q_i}{Q} = MgC$$

$$P\left[1 + \frac{s_i}{\eta}\right] = MgC$$

$$1 - \frac{s_i}{|\eta|} = \frac{MgC}{P}$$

$$\frac{P - MgC}{P} = \frac{s_i}{|\eta|}$$

$$L_H = \frac{s_i}{|\eta|}$$

where

si	=	Market share of firm i
$\lvert\eta\rvert$	=	Absolute value of the price elasticity of demand faced by firm i

and all other variables defined as before.

If for simplicity purposes, all firms have identical market shares,

$$L_H = \frac{s}{|\eta|}$$

$$s = \frac{1}{n}$$

$$H = \sum_{i=1}^{n} s_i^2$$

$$H = \frac{n}{n^2} = \frac{1}{n} = s \tag{2}$$

$$L_H = \frac{H}{|\eta|} \tag{3}$$

The Lerner index can also be adapted to the conjectural variations model described in Cabral (2000), by multiplying the equation (3) by the conjectural variations coefficient θ

$$L_H^{\theta} = \theta \frac{H}{|\eta|}$$

where, as in Cabral (2000), if

$\theta = 0$, players play Bertrand and price equals marginal costs.

$\theta = 1$, players play Cournot

$\theta = 1/H$, players play Collusion or Cartel and replicate Monopoly power.

Note that

$$\frac{1}{n} \le H \le 1$$

Note also that profit maximizing firms always operate in the elastic region of the demand curve[3].

$$1 \le |\eta| < \infty$$

Thus,

$$\lim_{\theta \to 0}\left(L_H^{\theta}\right) = 0 \qquad \lim_{\theta \to 1}\left(L_H^{\theta}\right)\Big|_{|\eta|=1;H=1} = 1$$

$$\lim_{|\eta| \to \infty}\left(L_H^{\theta}\right) = 0 \qquad \lim_{|\eta| \to 1}\left(L_H^{\theta}\right)\Big|_{\theta=1;H=1} = 1$$

$$\lim_{H \to \frac{1}{n}}\left(L_H^{\theta}\right)\Big|_{|\eta|=1;\theta=1} = \frac{1}{n} \qquad \lim_{H \to 1}\left(L_H^{\theta}\right)\Big|_{|\eta|=1;\theta=1} = 1$$

and

$$0 \le L_H^{\theta} \le 1$$

Appendix 1 presents a simulation of L_H^{θ} for different values of the parameters. In general the n firms Lerner index with conjectural variations, has the following properties:

i. It is designed to measure market power from the supply side

ii. It is bounded between cero and one

iii. The index is cero when there is no market power (perfect competition)

---

3 Profit maximization implies that MRP = MgC. Since $MRP = P_G\left[1 - \frac{1}{|\eta|}\right]$, and since MgC ≥ 0, profit maximizing firms only operate on the elastic region of the demand curve.

iv. The maximum market power generates a value of one

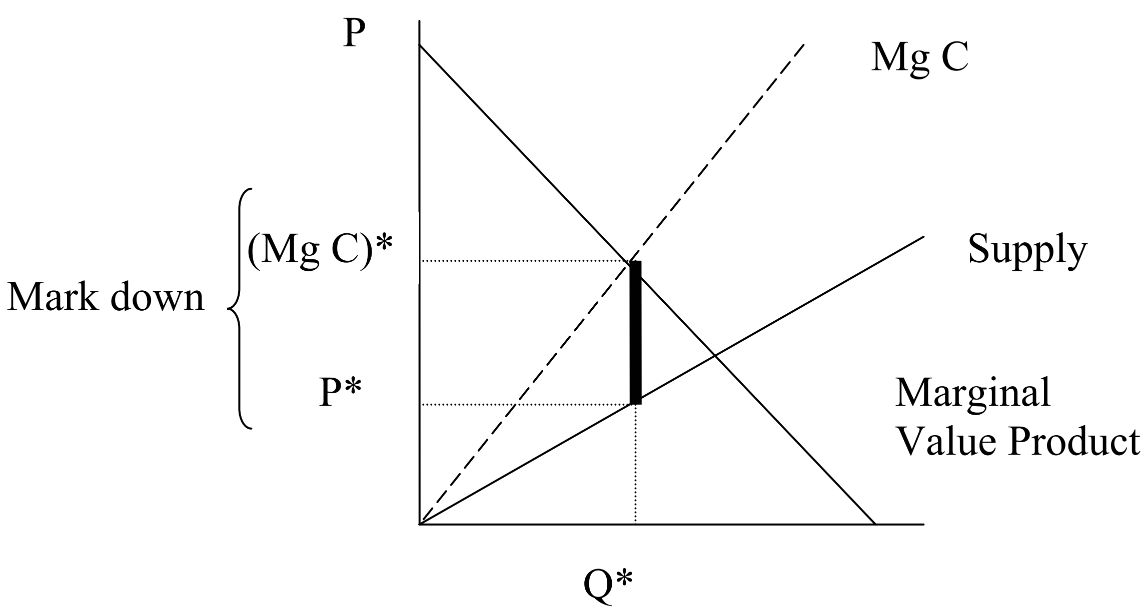
v. The index is positively monotonic to market power

B. Market power from the demand side

This section derives the Pigou index designed to measure market power from the demand side (with prices lower than marginal costs, as in monopsony, monopsonistic competition, and -often- oligopsony). This case, where firms apply a mark down, is shown in figure 3.

Figure 3

MARKET POWER FROM THE DEMAND SIDE: MONOPSONY, MONOPSONISTIC COMPETITION AND –OFTEN- OLIGOPSONY



Consider the following profit equation

$$\Pi_i = P_G Q(f_i) - P_F(f_i + F_{-i}) f_i$$

where

fi = amount of factor or input used by firm i.

F_{-i} = total amount of factor or input used in the market, except for firm i.

F = total amount of factor or input used in the market

and the rest of the variables are defined as before.

Profit maximization with n firms, implies that

$$\frac{\partial \prod_i}{\partial f_i} = MRP - P_F\left(F\right) - \frac{\partial P_F\left(F\right)}{\partial f_i} f_i = 0$$

$$MRP = P_F\left(F\right) + \frac{F \partial P_F\left(F\right)}{\partial f_i} \frac{f_i}{F} = MgC$$

$$MgC = P_F\left(F\right)\left[1 + \frac{P_F\left(F\right)}{F} \frac{\partial P_F\left(F\right)}{\partial f_i} s_i\right]$$

$$MgC = P_F\left(F\right)\left[1 + \frac{s_i}{\varepsilon_i}\right] \qquad (4)$$

where

∂_i = price elasticity of supply faced by firm i

and all other variables are defined as before.

Note that under profit maximization for a monopsonist, the marginal revenue product is equal to the marginal cost, and for simplicity purposes, assume that the firms are symmetric, and recall that by (2), *s* = *H*. Thus, from (4) it is possible to derive the following measures of exploitation:

1. Pigou's exploitation measure for n firms

$$PEM_H = \frac{MgC}{P_F} = \left[\frac{\varepsilon + H}{\varepsilon}\right]$$

Recall that

$$\frac{1}{n} \leq H \leq 1$$

However, as opposed to the elasticity of demand, the elasticity of supply has the following characteristic[4]:

$$0 \leq \varepsilon < \infty$$

Thus,

$$\lim_{\varepsilon \to \infty} \left(PEM_H\right)\Big|_{H=1} = 1 \qquad \lim_{\varepsilon \to 0} \left(PEM_H\right)\Big|_{H=1} = \infty$$

$$\lim_{H \to \frac{1}{n}} \left(PEM_H\right)\Big|_{\varepsilon=1} = \frac{n+1}{n} \qquad \lim_{H \to 1} \left(PEM_H\right)\Big|_{\varepsilon=1} = 2$$

and

$$1 \leq PEM_H < \infty$$

---

4 Profit maximization implies that MRP = MgC. Since MRP > 0 and $MgC = P_G\left[1 + \frac{s}{\varepsilon}\right]$, profit maximizing firms can operate on the elastic and the inelastic regions of the demand curve.

Appendix 2 presents simulations of PEM_H for different values of the parameters. In general the n firms Pigou's exploitation measure with conjectural variations has the following properties:

i. It is designed to measure market power from the demand side

ii. The measure is positively monotonic to market power

iii. The measure is bounded by one when there is no market (perfect competition)

iv. The measure is unbounded as the market power rises

2. Scully's rate of monopsonistic exploitation for n firms

$$1 - RME_H = \frac{P_F}{MgC} = \left[\frac{\varepsilon}{\varepsilon + H}\right]$$

$$RME_H = \frac{MgC - P_F}{MgC} = \frac{\varepsilon + H - \varepsilon}{\varepsilon + H}$$

$$RME_H = \frac{MgC - P_F}{MgC} = \frac{H}{\varepsilon + H} = \frac{1}{\frac{\varepsilon}{H} + 1}$$

This RME_H for n firms can be adapted to the conjectural variations model described in Cabral (2000), by multiplying it by the conjectural variations coefficient θ

$$RME_H = \frac{\theta}{\frac{\varepsilon}{H} + 1}$$

where θ is defined as before

Given that

$$0 \leq \theta \leq 1$$

$$\frac{1}{n} \leq H \leq 1$$

$$0 \leq \varepsilon < \infty$$

it follows that

$$\underset{\theta \to 0}{Lim}\left(RME_H^\theta\right)\Big|_{\varepsilon=1;H=1} = 0 \qquad \underset{\theta \to 1}{Lim}\left(RME_H^\theta\right)\Big|_{\varepsilon=1;H=1} = 1$$

$$\underset{\varepsilon \to \infty}{Lim}\left(RME_H^\theta\right)\Big|_{\theta=1;H=1} = 0 \qquad \underset{\varepsilon \to 0}{Lim}\left(RME_H^\theta\right)\Big|_{\theta=1;H=1} = 1$$

$$\underset{H \to \frac{1}{n}}{Lim}\left(RME_H^\theta\right)\Big|_{\varepsilon=1;\theta=1} = \frac{1}{n} \qquad \underset{H \to 1}{Lim}\left(RME_H^\theta\right)\Big|_{\varepsilon=1;\theta=1} = 1$$

and

$$0 \leq RME_H^\theta \leq 1$$

Appendix 3 presents a simulation of RME_H^θ for different values of the parameters. In general the n firms *RME* with conjectural variations has the following properties:

i. It is designed to measure market power from the demand side

ii. It is bounded between cero and one

iii. The index is positively monotonic to market power

iv. The index is cero when there is no market power (perfect competition)

v. The maximum market power generates a value of one

3. Boal's exploitation measure for n firms

$$1 + E_H = \frac{MgC}{P_F} = \left[\frac{\varepsilon + s}{\varepsilon}\right]$$

$$E_H = \frac{MgC - P_F}{P_F} = \frac{\varepsilon + s - \varepsilon}{\varepsilon}$$

$$E_H = \frac{MgC - P_F}{P_F} = \frac{H}{\varepsilon} \qquad (5)$$

This exploitation measure for n firms can be adapted to the conjectural variations model described in Cabral (2000), by multiplying the E_H index described in (5) by the conjectural variations coefficient θ

$$E_H^{\theta} = \theta \frac{H}{\varepsilon}$$

where θ is defined as before.

Since

$$0 \le \theta \le 1$$

$$\frac{1}{n} \le H \le 1$$

$$0 \le \varepsilon < \infty$$

it follows that

$$\underset{\theta \to 0}{Lim}\left(E_H^\theta\right)\Big|_{\varepsilon \neq 0} = 0 \qquad \underset{\theta \to 1}{Lim}\left(E_H^\theta\right)\Big|_{\varepsilon = 1; H = 1} = 1$$

$$\underset{\varepsilon \to \infty}{Lim}\left(E_H^\theta\right) = 0 \qquad \underset{\varepsilon \to 0}{Lim}\left(E_H^\theta\right)\Big|_{\theta \neq 0} = \infty$$

$$\underset{H \to \frac{1}{n}}{Lim}\left(E_H^\theta\right)\Big|_{\varepsilon = 1; \theta = 1} = \frac{1}{n} \qquad \underset{H \to 1}{Lim}\left(E_H^\theta\right)\Big|_{\varepsilon = 1; \theta = 1} = 1$$

and

$$0 \leq E_H^\theta < \infty$$

Appendix 4 presents simulations of E_H^θ for different values of the parameters. In general the n firms Boal's exploitation measure with conjectural variations has the following properties:

i. It is designed to measure market power from the demand side

ii. The index is positively monotonic to market power

iii. The index is bounded by cero when there is no market power (perfect competition)

iv. The index is unbounded as the market power rises

Thus, there is an asymmetry between the Lerner index and the exploitation measures analyzed used in the literature, since the Lerner index is designed to measure market power from the supply side, while the exploitation measures are designed to quantify market power from the demand side, and besides, the Lerner index is bounded for both increases and decreases in market power, while the exploitation measures are not always bounded for increases in market power.

IV. THE GENERALIZED INDEX OF MARKET POWER

To sort out the potentially undesirable asymmetries of the Lerner index and the exploitation measures, this paper proposes a Generalized Index of Market Power *GIMP*, which is symmetrical and bounded between cero and one, regardless of whether the market power comes from the supply or the demand side. The index proposed is:

$$GIMP = \frac{|P - Mg\,C|}{Max[P, Mg\,C]}$$

where

P = P_G if market power comes from the supply side ($P > MgC$)

P = P_F if market power comes from the demand side ($P < MgC$).

When market power comes from the supply side, price is greater than the marginal cost and the index becomes $GIMP_{SS,}$ which is the same as the Lerner index.

$$GIMP_{SS} = \frac{P_G - Mg\,C}{P_G}$$

Thus, the microeconomic derivation and the properties of the *GIMP* when market power comes from the supply side, are the same as those of the Lerner index. Besides, the *GIMP* with n firms and conjectural variations can be expressed as $GIMP^{\theta}_{SS(H)}$

$$GIMP^{\theta}_{SS(H)} = \theta \frac{H}{|\eta|}$$

and

$$0 < GIMP^{\theta}_{SS(H)} < 1$$

This implies that the simulations reported in Appendix 1 apply to both the Lerner index and the $GIMP^{\theta}_{SS(H)}$.

When market power increases from the demand side, price is lower than marginal cost, and the index becomes $GIMP_{DD}$, which is the same as Scully's measure of exploitation.

$$GIMP_{DD} = \frac{Mg\ C - P_F}{Mg\ C}$$

The microeconomic derivation and the properties of the *GIMP* when market power comes from the demand side, are the same as those of Scully's rate of monopsonistic exploitation *RME*. Thus, the $GIMP_{DD}$ with n firms and conjectural variations can be expressed as $GIMP^{\theta}_{DD(H)}$

$$GIMP^{\theta}_{DD(H)} = \frac{\theta}{\frac{\varepsilon}{H} + 1}$$

with

$$0 \leq GIMP^{\theta}_{DD(H)} \leq 1$$

All of the above implies that the simulations reported in Appendix 3 apply to both Scully's *RME* and the $GIMP^{\theta}_{DD(H)}$.

In general the n firms GIMP index with conjectural variations has the following properties:

i. It is designed to measure market power from supply or demand

ii. It is bounded between cero and one

iii. The index is positively monotonic to market power

iv. The index is cero when there is no market power (perfect competition)

v. The maximum market power generates a value of one

A summary of the features of the market power measures analyzed in this paper is presented in table 3.

Table 3

Key features of the Market Power Measures Analyzed

Index	Type of Monotonicity Increased Market power from supply	Lower bound as market power falls	Upper bound as market power increase	Source of Market Power measured
Lerner	Positive	0	1	Supply
Pigou	Positive	1	Unbounded	Demand
Scully	Positive	0	1	Demand
Boal	Positive	0	Unbounded	Demand
GIMP	Positive	0	1	Supply and demand

V. CONCLUSIONS

This paper has analysed two approaches to measuring market power –the commonly used Lerner index and a range of exploitation measures-. It has argued that the Lerner index is designed to quantify market power from the supply side, while the exploitation measures are designed to quantify market power from the demand side, and that those two approaches do not always behave in a symmetric way, since they do not have the same bounds.

To sort out these potentially undesirable properties, a new general index to measure market power has been proposed, which is symmetrical in the sense that it is bounded between cero and one, regardless of whether the market power comes from the supply or the demand side. Besides, the index proposed has been extended to be used when there are several firms in the market, and to allow for conjectural variations.

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APPENDIX 1: SIMULATIONS OF THE LERNER INDEX (GIMP SS)
APPENDIX 2: SIMULATIONS OF THE PIGOU INDEX
APPENDIX 3: SIMULATIONS OF THE SCULLY INDEX (GIMP DD)
APPENDIX 4: SIMULATIONS OF THE BOAL INDEX