QUANTITATIVE IMPLICATIONS OF THE CREDIT CONSTRAINT IN THE KIYOTAKI-MOORE (1997) SETUP

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October, 2003

Abstract

The Kiyotaki-Moore (1997) framework is a prominent macro model that features credit constraints as an important factor that propagates and magnifies the effects of shocks. However, the quantitative importance of these constraints in this setup remains an open question. This paper introduces the Kiyotaki-Moore (1997) setup into an otherwise standard dynamic general equilibrium model to explore the quantitative properties of credit constraints. I take a Hansen (1985)-type RBC model and introduce a banking sector that intermediates savings and investment. After calibrating the model to post-1959 U.S. data, I evaluate the propagation and magnification effects of a standard TFP shock to the aggregate economy. I find that the quantitative importance is very small. I then ask if the propagation and magnification effects are stronger if the shock originates in the banking sector. I therefore introduce TFP shocks into financial intermediation. I find that the constraints are also quantitatively unimportant. I conclude that the quantitative significance of the credit constraint in the Kiyotaki-Moore setup is small. The reason underlying this result has to do, theoretically, with asset market dynamics and, empirically, with the low participation of loans in economic activity in the U.S.

Key Words: credit constraint, credit multipliers, financial accelerator, amplification, propagation.


* aarias@minhacienda.gov.co. This paper is part of my PhD dissertation. I want to thank my advisor Lee Ohanian and also Gary Hansen for their constant help and support. Suggestions from Costas Azariadis, Andy Atkeson, Hal Cole and Dairo Estrada also helped me improve significantly the paper. Comments from assistants to the macro and international/development proseminars at UCLA have also been very helpful. All errors are my own.
1 INTRODUCTION

A huge volume of theoretical literature suggests that frictions/constraints in financial markets play a critical role in propagating and amplifying macroeconomic shocks.\(^1\) The basic story is that the firm’s ability to finance its production plan is an increasing function of the value of its assets. When the value of these assets increases (either because the price of assets increases or because the firm reinvests more profits), the firm is able to expand its production plan (either because credit constraints become less stringent or because some external finance premium falls). A higher level of production and investment increases asset demand (and asset prices) and/or earnings (and reinvestment of profits) thus increasing even further the value of the firm’s assets and its ability to expand its production plan, and so on. A credit multiplier or financial accelerator is articulated as the basic source of propagation and amplification of shocks.

The Kiyotaki-Moore (1997) -KM hereafter- framework is a prominent macro model among this type of literature. It features credit constraints as the essential factor that propagates and magnifies the effects of macro shocks. Indeed, their propagation/amplification mechanism consists of static and dynamic credit multipliers engineered around a credit constraint that is an increasing function of asset prices. Hence, action in the asset market fuels the amplification/propagation effects of their macro dynamics. However, the quantitative significance of the macroeconomic effects coming from the credit constraint in the KM setup remains an open question.

This paper introduces and simplifies the KM framework into an otherwise standard dynamic general equilibrium model to explore the quantitative properties of credit constraints. I take a Hansen (1985)-type RBC model and introduce a banking sector

that intermediates savings and investment. Credit operates as an intermediate input to final good production. The KM credit multipliers are activated by constraining the outstanding value of the firm’s bank debt with the value of the firm owner’s collateral. Asset prices, quantities and the loan rate will determine the volume of collateral.

After calibrating the model to post-1959 U.S. data, I evaluate the propagation and magnification effects of a standard TFP shock to the aggregate (non-banking) economy. I find that the quantitative importance of the propagation/amplification effects of the credit constraint is very small. I then ask if these effects are stronger when the shock originates in the banking sector. I therefore introduce TFP shocks into financial intermediation. I also find that the propagation/amplification effects of the credit constraint are quantitatively unimportant. I conclude that the quantitative importance of the credit constraint in the KM setup is small.

The results of this paper support the findings of Kocherlakota (2000). He showed that even though he could engineer arbitrarily high degrees of amplification using credit constraints (in a KM setup), the parameter values required to articulate such amplification degrees did not seem plausible. Since I use plausibly calibrated parameter values (those that replicate long-run empirical regularities in U.S. post-Korean war macro data), the quantitative significance of the action coming from the KM credit constraint could not be very high.

The reason that explains why credit constraints in the KM framework are not delivering any action has to do with the asset market. Any typical investment demand

\footnote{This captures the idea that firms usually need to pay for some intermediate inputs (or labor services) in advance of production and must rely on liquid funds provided by banks to do so. Without these liquid external funds, firms cannot operate their technologies and, in this sense, bank loans can be understood as a different input of production.}

\footnote{e.g.: discount factor very close to one, collateralizable inputs’ share (e.g. land share) close to one and non-collateralizable inputs’ share close to zero. When he introduces labor and sets parameter values that are closer to observed input shares, the amplification degree is very small.}
function is downward sloping in asset prices. However, the shape of investment supply as a function of asset prices is conditioned to whether the credit constraint binds or not. If the credit constraint binds, investment supply is upward sloping in asset prices. Otherwise, as in a standard RBC model, investment supply is a horizontal line at a fixed asset price. Consider a positive productivity shock that shifts investment demand outwardly. Even though asset prices increase more in the credit constrained environment, the quantity invested rises more in the unconstrained environment. Thus, the overall (i.e. price \times quantity) effect over investment value could be very similar under both environments. If so, the quantitative significance of the constraint will be very low. As I show in this paper, with a plausible calibration of the KM setup this is the result that is observed.

Recent work has confronted other financial friction stories with macro data. The results also question the quantitative significance of macro dynamics stemming from financial frictions. For instance, using the standard business cycle model as theoretical framework, Cole and Ohanian (2000) find that the traditional Bernanke (1983) financial disintermediation/bank failure story can only explain a very small portion of the Great Depression (at most, a 1% output reduction between 1929 and 1933).\footnote{This small number comes from two sources: i) the banking shock in terms of the model (share of deposits in failed/suspended banks) is small (19\% between 1930 and 1933), and ii) the elasticity of aggregate output with respect to a banking shock in terms of the model (banking’s share of value added) is very small also (1\%).} Moreover, they find that some of the model’s predictions following a banking shock are at variance with the data. Chakraborty and Lahiri (2001) modify the standard, one-sector neoclassical growth model to incorporate financial frictions using agency costs. They calibrate the model to cross-country, 1990-1997 data for 79 countries and report that financial frictions can typically explain less than 5\% of the income gap between the five richest and five poorest countries in the world.

The paper is organized as follows. Section 2 presents the theoretical model. Section
3 discusses the calibration strategy. Sections 4 presents the results of the paper. The last section concludes.

2 MODEL

In each period the economy is inhabited by a large number \((N)\) of identical, infinitely-lived, risk-averse households that discount the future at rate \(1/\beta - 1\). Population grows at rate \(\eta\) and the initial population level is normalized to 1 \([N_t = (1 + \eta)^t]\). Each household is endowed with one unit of time which can be allocated to leisure or to labor. Labor is indivisible like in Hansen (1985). The shift length is fixed at \(h < 1\) units of time and the household sends a fraction \(n\) of its members to work while the remaining fraction \((1 - n)\) does not work at all. Households have log utility in consumption \((c)\) and leisure:

\[
U = \log(c) + n \log(1 - h)
\]

Households supply their labor services to a competitive market at wage \(w\). Each household also owns capital \((k)\) and land \((l)\), which it can rent out in competitive markets at rental rates \(r\) and \(s\), respectively. The final good of this economy, which is the numeraire, can be consumed \((c)\) or accumulated as additional capital \((k)\) by each household. Land, on the other hand, is a different good and its total supply is equal to the population level. Hence, land supply is fixed at 1 at the per capita level. To purchase an additional unit of land a household must pay \(q\). The stock of capital depreciates at rate \(\delta\) and the stock of land does not depreciate.

Besides households, two other actors play a role in this economy: a firm and a bank. The firm produces final output using labor, capital, land and the bank’s output as inputs to a constant returns to scale (crs) technology. Note then that the bank’s output is simply an intermediate input to the final good producing firm. To produce
Table 1:

<table>
<thead>
<tr>
<th>Market</th>
<th>Demand</th>
<th>Supply</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Final good</td>
<td>households</td>
<td>firm</td>
<td>1</td>
</tr>
<tr>
<td>2. Land</td>
<td>households</td>
<td>households</td>
<td>q</td>
</tr>
<tr>
<td>3. Labor services</td>
<td>firm and bank</td>
<td>households</td>
<td>w</td>
</tr>
<tr>
<td>4. Capital services</td>
<td>firm</td>
<td>households</td>
<td>r</td>
</tr>
<tr>
<td>5. Land services</td>
<td>firm</td>
<td>households</td>
<td>s</td>
</tr>
<tr>
<td>6. Credit</td>
<td>firm</td>
<td>bank</td>
<td>(1 + ρ)</td>
</tr>
</tbody>
</table>

this intermediate input the bank combines deposits and labor in another crs production function. Let the relative price of the bank’s output be denoted as $(1 + \rho)$. Given that the bank’s output has the interpretation of an intra-temporal loan, $(1 + \rho)$ has the interpretation of an intra-temporal, gross, loan rate. Deposits come from abroad at exogenous, intra-temporal, gross rate $(1 + R)$. Note that the bank plays no role in transferring purchasing power across periods. Households can do this internally by accumulating capital or by purchasing land.

At the end of the day the three actors of the economy revolve around six markets\(^5\):

Finally, it is assumed that the final good producing firm and the bank are subject to stochastic, AR(1), productivity shocks $z$ and $x$, respectively, and that they also exhibit deterministic, labor augmenting, technological progress at rates $u$ and $v$, respectively. This distinguishes continuous, permanent, technological improvement (e.g. discovery of a new technology) from random, temporary, productivity shocks (e.g. regulatory changes). This also allows the model to exhibit a constant loan-deposit interest rate

\(^5\)Actually there is a seventh market which is the market for deposits (banks demand and foreign investors supply). However its relative price $(1 + R)$ is exogenously determined by demand and supply conditions in foreign credit markets.
spread while the bank enjoys continuous productivity improvement.

The economy exhibits a balanced growth path along which the per-capita capital stock, per-capita consumption, per-capita final good output, per-capita deposits, the wage and the rental rate of land grow at a constant rate while per-capita landholdings, the employment rate and the rental rate of capital remain constant. The balanced growth path of the economy will be studied here.

2.1 Household

Let \( \bar{\beta} = \beta(1 + \eta) \). The following sequential problem for one household can be mapped into a social planning problem for the aggregate economy if the utility of each household is weighted equally by the planner:

\[
\begin{align*}
\max_{\{c_t, k_{t+1}, l_{t+1}, n_t\}} & \quad E_0 \sum_{t=0}^{\infty} \bar{\beta}^t [\log(c_t) + n_t \log(1 - h)] \\
\text{s.t.} & \quad c_t + (1 + \eta)k_{t+1} + q_t l_{t+1} = w_t n_t h + [r_t + (1 - \delta)]k_t + (s_t + q_t)l_t \\
& \quad q_t, w_t, r_t, s_t, \text{ given} \\
& \quad k_0, l_0 = 1 \text{ given}
\end{align*}
\]

where \( c, k, l, \) and \( n \) represent the household’s stock of capital, consumption level, stock of land and employment rate, respectively.

2.2 Final Good Producing Firm

In this economy the final good producing firm uses a Cobb-Douglas technology in four inputs of production: labor, capital, land and credit. Let \( k^d, n_1, b^d \) and \( l^d \) represent the per-capita volume of capital services, employees, credit and land services demanded by the firm. Thus, the firm’s per-capita output can be represented as:

\[
y_t = z_t \left( k^d_t \right)^\alpha \left[ n_1 t h(1 + u)^t \right]^\gamma \left( b^d_t \right)^\phi \left( l^d_t \right)^{1-\alpha-\gamma-\phi}
\]
where:

\[ \log(z_{t+1}) = \rho_0 + \rho_1 \log(z_t) + \varepsilon_{t+1}, \quad \varepsilon_t \sim N(0, \sigma^2_\varepsilon) \]

The loans-in-the-production function assumption articulates the credit channel of the economy. The motivation for this loan-in-the-production function assumption is that firms usually need to pay for some intermediate inputs (or labor services) in advance of production and must rely on liquid funds provided by banks to do so. Without these liquid external funds, firms cannot operate their technologies. In this sense, loans can be understood as a different input of production.

In fact, a model with a loans-in-the-production function assumption is isomorphic to a model with a cash-in-advance (CIA) constraint on the intermediate input bill or wage bill of the final good producing firm. Consider, for instance, a variation of the CIA model suggested by Carlstrom and Fuerst (1995). The final good producing firm needs capital, labor, land and intermediate inputs to operate its Cobb-Douglas technology. Let \( m^d \) represent the per-capita volume of intermediate inputs demanded by the firm. In this case, the firm’s per-capita output is given by:

\[
y_t = Z_t \left( k_t^d \right)^{\alpha} \left[ n_t h (1 + u)^{1/\gamma} \left( m_t^d \right)^{\phi} \right]^{1 - \alpha - \gamma - \phi}
\]

where \( Z \) is a productivity shock. Suppose that intermediate inputs are provided by foreign agents at exogenous price \( p \). However, the firm must pay the suppliers of intermediate inputs in advance of production. This means that the final good producing firm is subject to a CIA constraint on the intermediate input bill \( pm^d \). The central assumption is that the current intermediate input bill is financed with a bank loan. These loans are repaid (without default) at the end of the period and the bank charges interest rate \( \rho \) for them. Hence, the final good producing firm solves the following problem in each period:
\[
\begin{align*}
\text{Max} & \{k_t^d, n_{1t}, m_t^d, q_t^d, b_t^d\} & \quad & y_t + b_t^d - p_t m_t^d - r_t k_t^d - w_t n_{1t} - s_t l_t^d - (1 + \rho_t) b_t^d \nonumber \\
\text{s.t.} & \quad & y_t = Z_t (k_t^d)^\alpha [n_{1t} h(1 + u)^t]^\gamma (m_t^d)^\phi (l_t^d)^{1 - \alpha - \gamma - \phi} \nonumber \\
& \quad & b_t^d = p_t m_t^d \nonumber 
\end{align*}
\]

Rearranging implies that the firm solves:

\[
\begin{align*}
\text{Max} & \{k_t^d, n_{1t}, b_t^d, l_t^d\} & \quad & z_t (k_t^d)^\alpha [n_{1t} h(1 + u)^t]^\gamma (b_t^d)^\phi (l_t^d)^{1 - \alpha - \gamma - \phi} \nonumber \\
& \quad & -r_t k_t^d - w_t n_{1t} - s_t l_t^d - (1 + \rho_t) b_t^d \nonumber 
\end{align*}
\]

where \(z = \left(\frac{Z}{p}\right)\) (i.e. the exogenously determined price of intermediate inputs becomes part of the firm’s overall productivity). Hence, the model with a CIA constraint on the firm’s intermediate input bill is isomorphic to a model with loans-in-the production function. At the end of the day both treatments highlight the role of liquidity or working capital provided by financial intermediaries as essential to production processes. Due to its simplicity, the loans-in-the production function assumption is used hereafter to articulate a credit channel in the economy.

2.3 Bank

In order to study the macroeconomic impact of productivity fluctuations in financial intermediation, the model must employ an appropriate representation of the banking technology through which resources are intermediated. Indeed, the model suggested here employs a technological specification for banks that follows the “intermediation approach” of Sealey and Lindley (1977). Under this approach all deposits and funds borrowed from financial markets are considered inputs of production [Freixas and Ro-
chet (1998)]. Alternative approaches are the “production approach” and the “user cost approach” which treat depositor services as part of a financial intermediary’s final output [e.g.: Tirtiroglu, Daniels and Tirtiroglu (1998)].

Consider a setup where banks combine deposits and labor in a Cobb Douglas technology to produce the intra-period safe loans that the final good producing firm requires each period. Let \( d_t \) and \( n_t \) represent the per-capita volume of deposits and employees demanded by the bank. Thus, the banks’s output, in per-capita terms, can be represented as:

\[
b_t = x_t d_t^n \left[ n_t h(1 + v)^t \right]^{1-\theta}
\]

where:

\(^6\)The idea behind this approach is that all liabilities in the bank’s balance sheet (core deposits and purchased funds) plus financial equity capital provide funds and are considered to be inputs since they generate costs [Berger and Mester (2001)]. On the other hand, all assets (loans and investments outstanding) use bank funds to generate revenues and are considered outputs [Freixas and Rochet (1998)]. Note that following this approach implies interpreting depositor services as payments to financial inputs that do not receive interest remuneration (like demand deposits) [see Berger and Mester (2001) pp. 16].

\(^7\)According to Berger and Mester (2001), pp. 16: “...the asset approach -which treats deposits as an input- is most compatible with the profit maximization concepts ... because deposits and other liabilities by themselves generate negative cash flows and reduce profits, whereas loans and other assets generate positive cash flows and profits. As a practical matter, it would also be difficult to specify a positive output price for depositor services under the other approaches, since most of these services are not explicitly priced.”

\(^8\)Actually, any crs technology in the banking sector can be used. In fact, alternative functional forms for the bank’s production function like Leontief or Leontief with adjustment costs in employment were also studied (adjustment costs capture the idea that banks pay certain cost when they change their employees due to specific information or knowledge that the employees have about the bank’s clients). Here the Cobb-Douglas case is presented due to its analytical tractability.
\[
\log(x_{t+1}) = \varphi_0 + \varphi_1 \log(x_t) + v_{t+1}, \quad v_t \sim N(0, \sigma_v^2)
\]

Note that the intermediation technology is costly. In fact, \( wn_2h \geq 0 \) captures all the resources used in the intermediation process. This formalizes the idea that in order to intermediate deposits into loans, banks must carry out a variety of costly activities like evaluating creditors, managing deposits, renting buildings, maintaining ATMs, etc. [Edwards and Vegh (1997)].\(^9\)

The technological specification suggested here for banks is similar to the one used by Cole and Ohanian (2000). In their paper the intermediation technology is \( G(D, Z) \) where \( D \) is uninstalled physical capital, \( Z \) is intermediation capital (in fixed supply), \( G(\cdot) \) exhibits crs and \( D - G(D, Z) \geq 0 \) represents resources used in the intermediation process. Under the technology specified here deposits \((d)\) are the natural counterpart to their uninstalled physical capital \((D)\) and productivity parameter \(x\) in combination with labor \((n_2h)\) play an analogous role to that of their intermediation capital \((Z)\).

With crs in the intermediation technology it is possible to assume an atomistic structure in the banking industry. This assumption is also consistent with the fact that firms of many sizes coexist in the financial sector. Under this environment banks behave competitively and are price takers. Formally, in every period banks solve the following problem:

\[
\max_{d_t, n_2t} \quad (1 + \rho_t)dx_td_t^\theta[n_2th(1 + v)^t]^{1-\theta} - (1 + R)d_t - w_tn_2t
\]

Free entry and exit will drive profits to zero so that in equilibrium banks produce where the relative price of their output \((1 + \rho_t)\) equals marginal cost.

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\(^9\)For instance, in the year 2000, 63.27\% of all U.S. commercial banking employees had office and administrative support occupations [e.g. tellers (23.04\%)]. Only 14.86\% of all employees carried out business and financial operations occupations [e.g. loan officers (4.86\%), financial analysts (1.05\%), credit analysts (0.91\%), personal financial advisors (0.93\%)]. Source: BLS.
2.4 Credit Constraint

To introduce a credit constraint into the model an environment similar to the one suggested by Kocherlakota (2000) will be assumed. Suppose the bank is owned by the international depositors. Funding the firm’s working capital is risky for the bank. The reason is that at the end of every period the owner of the firm -the household- can run away with the proceeds from the firm plus a fraction $\xi \in [0, 1]$ of his/her total assets (i.e. land plus the undepreciated stock of capital), without paying back the loan to the bank. Assume also that default is not penalized with market exclusion. The bank is aware of the risk involved in lending to the firm. As a result, the bank takes care not to let the firm borrow beyond the amount that would make it worthwhile for the owner to run away without repaying the loan. The next proposition reveals the incentive-compatibility condition imposed by the bank so that it is optimal for the firm never to default in equilibrium.

**Proposition 2.1.** Under the following credit constraint:

$$b^d \leq \frac{(1 - \xi)[q^d + (1 - \delta)k^d]}{(1 + \rho)}$$

it is optimal for the firm to repay the loan.

*Proof.* See Technical Appendix.

The previous proposition shows that, in order to eliminate the risk of default, the bank imposes a natural credit constraint on the firm: the outstanding value of the firm’s debt at the end of the period $[(1 + \rho)b^d]$ can never exceed the value of the owner’s seizable/collateralizable resources at the end of the period $((1 - \xi)[q^d + (1 - \delta)k^d])$. As in other credit limit models, borrowing is so tightly constrained by the volume of collateral that default never occurs in equilibrium. Note also that the credit constraint is a decreasing function of the gross loan rate. This captures the idea that any rise in
the interest rate melts down collateral by reducing the volume of principal associated to any given volume of outstanding debt at the end of the period.

It is important to highlight that the credit constraint on the firm does not feed back into the bank’s problem. As in the absence of the constraint, the bank is simply trying to use labor and deposits optimally to provide that volume of credit that the firm plans (and is allowed) to use. In fact, if the credit constraint was to be imposed not only on the firm but also on the bank (as a limit or upper bound on its volume of output), the bank’s profits would be given by $\mu(1 + \rho)b$ where $\mu$ is the Kuhn-Tucker multiplier associated to the constraint. Note that if the credit constraint binds ($\mu > 0$), it would seem as if the bank was obtaining positive profits. However, there cannot be positive profits in equilibrium. \(^\text{10}\) Thus, the bank’s problem remains intact.

### 2.5 Balanced Growth Path

Along the balanced growth path of the economy the per-capita stock of capital, per-capita consumption, per-capita final good output, per-capita deposits, the wage and the rental rate of land grow at the same rate. Let $g$ be this rate. On the other hand, employment allocated to each sector (i.e. bank and firm), the per-capita stock of land and the rental rate of capital remain constant. It can be shown that:

\[
g = \left[ (1 + u)^\gamma (1 + v)^\phi (1 - \theta) \right]^{\frac{1}{1 - \alpha - \phi}}
\]

\(^{10}\)Suppose a bank is operating in the economy giving the firm a volume of credit and a loan rate as dictated by the constraint. If the bank obtains a positive profit from this, another bank would immediately come in and offer the firm a bigger volume of credit at a lesser loan rate [-recall that the credit constraint is on $(1 + \rho)b$-]. This implies the same amount of revenue for the bank but higher input costs (recall that the bank takes the wage and the deposit rate as given). Thus, the entering firm would basically drive down the industry’s profits. But then some other bank would come in and follow the same strategy. Consequently, in equilibrium, profits go to zero. This also means that $\mu = 0$ both when the credit constraint binds and when it is slack.
To simplify notation, from now on all variables (which are already in per-capita terms) are also in growth-detrended terms.

### 2.6 Recursive Competitive Equilibrium

The aggregate state of the economy is given by the two stochastic shocks and the aggregate stock of capital: \((z, x, K)\). At an individual level, the state is given by the individual capital stock and individual landholdings \((k, l)\). The following definitions formalize the recursive competitive equilibrium of the per-capita economy along its balanced growth path, in terms of growth-detrended variables.

**Definition 2.2.** \(P1\) is the following dynamic programming problem for the household:

\[
V(z, x, K, k, l) = \max \left\{ \log[w(z, x, K)nh + [r(z, x, K) + (1 - \delta)]k + [s(z, x, K) + q(z, x, K)]l - (1 + \eta)(1 + g)k'] - q(z, x, K)l' + n \log(1 - h) + \tilde{\beta}EV(z', x', K', k', l') \right\}
\]

s.t.
\[
K' = H(z, x, K) \\
\log(z') = \rho_0 + \rho_1 \log(z) + \varepsilon', \quad \varepsilon' \sim N(0, \sigma^2_\varepsilon) \\
\log(x') = \varphi_0 + \varphi_1 \log(x) + \upsilon', \quad \upsilon' \sim N(0, \sigma^2_\upsilon) \\
\text{cov}(\varepsilon, \upsilon) = 0
\]

**Definition 2.3.** If there are no credit constraints, \(P2\) is the following static problem for the final good producing firm:

\[
\max \left\{ y - w(z, x, K)n_1h - r(z, x, K)kd - s(z, x, K)ld - [1 + \rho(z, x, K)]ld \right\}
\]

s.t.
\[
y = z(kd)^\alpha (n_1h)^\gamma (bd)^\phi (ld)^{1-a-\gamma-\phi}
\]

If there is a credit constraint, \(P2\) is the following static problem for the final good producing firm:
\[
\begin{align*}
\text{Max}_{\{n_1, k^d, l^d, b^d\}} & \quad y - w(z, x, K)n_1h - r(z, x, K)k^d - s(z, x, K)b^d \\
& \quad -[1 + \rho(z, x, K)]b^d \\
\text{s.t.} & \quad y = z \left( k^d \right)^\alpha \left( n_1h \right)^\gamma \left( b^d \right)^\phi \left( l^d \right)^{1-\alpha-\gamma-\phi} \\
& \quad b^d \leq \frac{(1-\xi)[q(z, x, K)l^d + (1-\delta)k^d]}{[1+\rho(z, x, K)]}
\end{align*}
\]

**Definition 2.4.** \( P3 \) is the following static problem for the bank:

\[
\begin{align*}
\text{Max}_{\{n_2, d\}} & \quad [1 + \rho(z, x, K)]b - (1 + R)d - w(z, x, K)n_2h \\
\text{s.t.} & \quad b = xd^d(n_2h)^{1-\theta}
\end{align*}
\]

**Definition 2.5.** A recursive competitive equilibrium (RCE) is

1. A value function: \( V(z, x, K, k, l) \).
2. A set of individual decision rules: \( k'(z, x, K, k, l), l'(z, x, K, k, l) \) and \( n(z, x, K, k, l) \).
3. A set of demands by the final good producing firm: \( k^d(z, x, K), n_1(z, x, K), b^d(z, x, K) \) and \( l^d(z, x, K) \).
4. A set of demands by the bank: \( d(z, x, K) \) and \( n_2(z, x, K) \)
5. A set of pricing functions: \( w(z, x, K), r(z, x, K), s(z, x, K), q(z, x, K) \) and \( \rho(z, x, K) \).
6. An aggregate decision rule: \( H(z, x, K) \).

such that:

- Given (5) and (6), (1) and (2) solve \( P1 \).
- Given (5), (3) solves \( P2 \).
- Given (5), (4) solves \( P3 \).
• Markets clear:

1. \( n_1(z, x, K) + n_2(z, x, K) = n(z, x, K, 1) \)
2. \( k^d(z, x, K) = K \)
3. \( l^d(z, x, K) = 1 \)
4. \( l'(z, x, K, 1) = 1 \)
5. \( b^d(z, x, K) = b(z, x, K) = xd(z, x, K)^\theta [n_2(z, x, K)h]^{1-\theta} \)

• Aggregate Consistency: \( k'(z, x, K, 1) = H(z, x, K) \).

2.7 Unconstrained Economy

Suppose there is no credit constraint. The clearing of the five markets in the RCE definition implies, by Walras’ Law, that the final good market also clears:

\[
c = y + (1 - \delta)k - (1 + R)d - (1 + g)(1 + \eta)k'
\]

where:

\[
y = zk^\alpha b^\phi (n_1 h)^\gamma = (xz^\phi) k^\alpha d^\phi (n_1 h)^\gamma (n_2 h)^{(1-\theta)\phi}
\]

Equation (1) is simply the resource constraint for the economy.

Optimality conditions for the household are:

\[
\frac{1 + g}{c} = \beta E \left[ \frac{1}{c'} [r' + (1 - \delta)] \right]
\]

\[
\frac{q}{c} = \tilde{\beta} E \left[ \frac{1}{c'} (s' + q') \right]
\]

16
\[
\frac{1}{c} \log(1 - h) = 0
\] (4)

Equations (2) and (3) are the standard Euler equations governing the optimal consumption/capital accumulation and consumption/land accumulation decisions of the household. Equation (4) simply equates the marginal rate of substitution between leisure and consumption [i.e. \(-\log(1 - h)c\)] to the marginal opportunity cost of leisure in terms of consumption [i.e. \(wh\)].

Optimality conditions for the bank are:

\[
\theta(1 + \rho)b = d(1 + R)
\] (5)

\[
(1 - \theta)(1 + \rho)b = w_n h
\] (6)

(5) shows that it is optimal for the bank to demand deposits so that the marginal cost of deposits [i.e. \((1 + R)\)] is equated to the value of the marginal productivity of deposits [i.e. \((1 + \rho)\theta b\)]. Equation (6) shows the same thing for employment demand by the bank. Both equations imply zero profit for the bank, which is a natural result of its crs technology.

Optimality conditions for the firm are:

\[
\phi y = (1 + \rho)b_d
\] (7)

\[
\gamma y = w_n h
\] (8)

\[
\alpha y = r k_d
\] (9)

\[
(1 - \alpha - \gamma - \phi)y = s_l d
\] (10)
(7)-(10) indicate that it is optimal for the firm to produce in that point where the marginal productivity and cost of each input are equated. They also imply zero profits for the firm, as expected from its crs technology. (6) and (8) show that, in equilibrium, the marginal productivity of labor is equated across both sectors of the economy.

The next proposition establishes the price of land in equilibrium:

**Proposition 2.6.** *The equilibrium price of land in period* \( t \) *is given by:*

\[
q_t = E_t \left[ \sum_{j=1}^{\infty} \tilde{\beta}^j \frac{c_t}{c_{t+j}} \Delta_{t+j} \right]
\]

*where:*

\[
\Delta_t = (1 - \alpha - \gamma - \phi) y_t
\]

*Proof.* See technical appendix.

The previous proposition shows that the price of land is given by the expected present discounted value of its forever flow of future rental payments. Rental payments to land are given by its marginal productivity and, as is standard in this type of models, the asset pricing kernel is the stochastic discount factor.

### 2.8 Credit Constrained Economy

Suppose now that there is a credit constraint. Given that (P1) and (P3) do not change, equations (1)-(6) remain intact. Let \( \lambda \geq 0 \) denote the Kuhn-Tucker multiplier associated to the credit constraint in the firm’s static problem (P2). This multiplier determines the shadow price of collateral. Optimality conditions for the final good producing firm are given by:

\[
\lambda = \frac{\phi y}{b^d} - (1 + \rho) \geq 0 \quad (7^{cc})
\]

\[
\gamma y = wn_1 h \quad (8^{cc})
\]
\[ r = \frac{\alpha y_k}{b^d} + \left[ \frac{\phi y_{b^d} - (1 + \rho)}{b^d} \right] \frac{(1 - \xi)(1 - \delta)}{(1 + \rho)} \tag{9cc} \]

\[ s = \frac{(1 - \alpha - \gamma - \phi)y}{b^d} + \left[ \frac{\phi y_{b^d} - (1 + \rho)}{b^d} \right] \frac{(1 - \xi)q}{(1 + \rho)} \tag{10cc} \]

Equation \((7cc)\) indicates that the shadow price of collateral is given by the gap between the marginal product and the marginal cost of one unit of credit. Note that in the non-constrained environment optimality for the firm dictates that the marginal productivity of credit must always be equated to the gross loan rate or marginal cost of credit [see equation \((7)\)]. However, with a credit constraint the optimal level of loans may exceed the credit limit. In fact, if the credit constraint binds (i.e. \(\lambda > 0\)), equation \((7cc)\) shows that the marginal product of credit will exceed the gross loan rate (or marginal cost of credit) and the shadow price of collateral will be positive given that collateral plays the role of a scarce resource. Moreover, an output loss or inefficiency will be observed in the economy. Contrarily, if the credit constraint does not bind (i.e. \(\lambda = 0\)), equation \((7cc)\) becomes equation \((7)\), the marginal product and cost of credit will be equated and the shadow price of collateral will be zero given that there is no need or demand for collateral. No output loss whatsoever will be observed in the economy.

Note that equation \((8cc)\) is identical to equation \((8)\). This indicates that, regardless of whether the credit constraint binds or not, it is optimal for the firm to demand labor so its marginal product and cost are equated. Moreover, \((6)\) and \((8cc)\) show that, as in the unconstrained setup, the equilibrium wage is given by the marginal product of labor in either sector of the economy. In other words, marginal productivities of labor are always equated across both sectors of the economy. This is independent of whether there is a binding or non-binding credit constraint or no credit constraint at all.

Note from \((6), (7)\) and \((8)\) that under the unconstrained environment \( \frac{z_1}{n_1} = \frac{\phi y}{n_2} \). This means that the marginal contribution of employment to final good output does
not depend on whether the employment is allocated to the final good sector itself or to its intermediate input (i.e. the bank). However, when there is a credit constraint it can be seen from (6), (7) and (8) that \( \frac{n_1}{n_2} \leq \frac{\phi(1-\theta)}{n_2} \). Thus, if the credit constraint binds, the marginal contribution to final good output of employment in the bank exceeds that of employment in the final good sector itself.

Equation (9) shows that the rental rate of capital has two components. The first one, which is the marginal product of capital, is standard and simply captures the direct contribution of capital to output as an input of production. The second component, which is the second term on the right-hand side of (9), captures the indirect contribution of capital to output due to the role that capital plays as collateral. Indeed, one additional unit of capital releases \((1-\xi)(1-\delta)/(1+\rho)\) additional units of credit. Each of these additional units of available credit generate \(\left[\frac{\phi y}{b} - (1 + \rho)\right]\) additional units of net output gain. Note, however, that the second component of the rental rate of capital is only relevant if the credit constraint binds [i.e. if \(\lambda > 0 \iff \frac{\phi y}{b} > (1 + \rho)\)]. In fact, if the credit constraint does not bind [i.e. if \(\lambda = 0 \iff \frac{\phi y}{b} = (1 + \rho)\)], then (9) becomes (9).

Equation (10) shows that in a credit constrained environment the rental rate of land also has two components. The first one, which is the marginal product of land, simply represents the direct contribution of land to output as an input of production. The second component, given by the second term on the right-hand side of (10), captures the indirect contribution of land to output due to the role that land plays as collateral. In fact, one additional unit of land releases \((1-\xi)q/(1+\rho)\) additional units of credit. Each of these additional units of available credit generate \(\left[\frac{\phi y}{b} - (1 + \rho)\right]\) additional units of net output gain. Again, note that the second component of the rental rate of land only kicks in if the credit constraint binds [i.e. if \(\lambda > 0 \iff \frac{\phi y}{b} > (1 + \rho)\)]. In fact, if the credit constraint does not bind [i.e. if \(\lambda = 0 \iff \frac{\phi y}{b} = (1 + \rho)\)], then (10) becomes (10).
(9cc) and (10cc) basically show that, as long as the credit constraint binds, the collateral properties of land and capital enhance their marginal contribution to output. This result is important in itself because it alters the definition of the equilibrium rental rates of land and capital with respect to the unconstrained environment. This result is also important because the rental rate of capital and the price of land—which depends on the rental rate of land—play a central role in the articulation of the dynamic credit multiplier.

Using (7cc)-(10cc) the following result holds:

\[(1 + \rho)b^d = \phi y - \lambda b^d \leq \phi y \tag{11}\]

\[rk^d + sl^d = \alpha y + (1 - \alpha - \gamma - \phi)y + \lambda b^d \quad if \quad \lambda > 0\]
\[= \alpha y + (1 - \alpha - \gamma - \phi)y \quad if \quad \lambda = 0\]

\[w(n_t h) = \gamma y\]

(11) shows that whenever the credit constraint binds (i.e. \(\lambda > 0\)), the share of credit in output falls short of its natural share \(\phi\). As a result, the shares of capital and land in output exceed their natural shares \(\alpha\) and \((1 - \alpha - \gamma - \phi)\). However, labor share in the final good producing firm is still \(\gamma\) and the zero profit condition still holds. If the credit constraint does not bind (i.e. \(\lambda = 0\)), the fraction of output paid to every input coincides with its natural share. These results play a key role for the calibration of the parameters of this economy when the credit constraint binds.

The next proposition establishes the price of land in equilibrium:

**Proposition 2.7.** The equilibrium price of land in period \(t\) is given by:

\[q_t = E_t \left\{ \sum_{j=1}^{\infty} \tilde{\beta}^j \frac{c_t}{c_{t+j}} \left[ \Delta_{t+j} \prod_{i=1}^{j-1} \Omega_{t+i} \right] \right\}\]
where:

\[ \Delta_t = (1 - \alpha - \gamma - \phi)y_t \]

and:

\[ \Omega_t = (1 - \xi) \left( \frac{\phi y_t / b_t}{1 + \rho_t} \right) + \xi \]

**Proof.** See technical appendix.

Proposition 7 shows that the price of land is given by the expected present discounted value of its forever flow of future rental payments. Discounting is done with the stochastic discount factor. Note also that future rental payments to land are an increasing function of the future marginal productivity of land and credit. Again, this captures the idea that future rental payments to land include not only the future direct contribution of land to output as an input of production \([(1 - \alpha - \gamma - \phi)y_{t+1}]\), but also the future cumulated indirect contribution of land to output as a collateralizable asset or credit constraint loosener \(\left( \prod_{i=1}^{j-1} \Omega_{t+i} \right)\). This is a nice result because it shows that credit constrained agents value assets not only because of their future direct contribution to output, but also because assets operate as collateral. Simply put, accumulating an additional unit of land is valuable not only because it increases future production directly, but also because it rises the future volume of collateral, loosens the future credit constraint and increases the future availability of external funds, thus expanding future output indirectly.\(^{11}\)

\(^{11}\)This feature of the asset is not present in the Kocherlakota (2000) model. In his setup loans are repaid in the next period and enter linearly into the production function. Thus, the marginal productivity of credit is 1 while the present value marginal cost of credit is \((1 + \rho)/(1 + \rho) = 1\). Consequently, the gap between the marginal productivity and cost of credit is 1 and the \(\left( \prod_{i=1}^{j-1} \Omega_{t+i} \right)\) term disappears from the asset pricing equation.
2.9 Credit Multipliers

The credit constrained economy displays a static and a dynamic credit multiplier [see KM (1997)]. The mechanics are very simple. Suppose that the credit constraint binds and consider an adverse TFP shock to the bank. This induces a contemporaneous hike in the loan rate charged by the bank in equilibrium. The jump in the loan rate immediately melts down the volume of collateral and tightens the credit limit of the firm. Therefore, the firm suffers a crunch in the volume of available external funds or working capital. Naturally, the firm’s ability to finance its production plan is reduced with this credit crunch. As the firm’s output falls, its demand for labor services, capital services and land services goes down. Thus, there is a reduction in the wage, the rental rate of capital and the rental rate of land. This immediately drives down the income of the household. In an attempt to smooth out consumption, the household instantaneously reduces its accumulation of capital and land.

Given that land is in fixed supply, the fall in demand for this asset reduces its price. The fall in the price of land drives down the current value of the firm’s collateralizable assets and, hence, tightens even further the credit constraint faced by the firm. The volume of available external funds or working capital is further reduced and, hence, the fall in income is magnified. This is the static multiplier.

Now, recall that the price of land is given by the present discounted value of its forever flow of future rental payments. As seen above, these rental payments are an increasing function of the marginal productivity of land and credit. Since the capital stock is complementary to both land and credit, the reduction in capital accumulation during the period of the shock implies a fall in the future marginal productivity of land and credit. Consequently, there is a fall in the future flow of rental payments to land. This drives down the price of land in the period of the shock. This reduces even more the value of land on impact and, hence, tightens even further the credit constraint in the period of the shock. The credit crunch is enhanced and the fall in income is
magnified. This is the dynamic multiplier.

While the multipliers amplify the shock, a different process propagates it. The reduction in capital accumulation reduces, by definition, the volume of collateral available for next period. Thus, the borrowing constraint of next period is also tightened even if the shock has vanished and the lending rate has returned to its normal level. Furthermore, recall that the rental rates of land and capital are a function not only of the marginal productivity of land and capital, but also of the marginal productivity of credit. Thus, the reduction in land and capital accumulation in the period of the shock pushes down the following period’s rental rates of land and capital (by reducing the following period’s marginal productivity of credit). This prevents land and capital accumulation and land’s price from picking up faster in the period after the shock. Thus, not only does the household have less collateral in the period after the shock (even if the shock has vanished completely), but it also has fewer incentives to start accumulating land and capital once again. All this propagates the credit crunch into the subsequent periods. The economy takes longer to converge back to the steady state than in a financially frictionless setup.

3 CALIBRATION

Parameters were calibrated to a quarterly frequency using U.S. data for the period 1959-1999 (see calibration appendix). Specifically, parameter values were chosen so that the model (with and without a binding credit constraint), in stationary state, replicates the following 1959-1999 averages observed in the U.S.:

It is important to highlight that, for calibration purposes, credit was measured as commercial and industrial loans from commercial banks. Recall that in the theoretical model credit is interpreted as intra-period working capital or liquidity that firms need in order to pay wages or other intermediate inputs and, thereby, operate their technolo-
Table 2:

<table>
<thead>
<tr>
<th></th>
<th>c/y</th>
<th>i/y</th>
<th>d/y</th>
<th>k/y</th>
<th>n</th>
<th>n1</th>
<th>n2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.5914</td>
<td>0.3422</td>
<td>0.0661</td>
<td>10.1740</td>
<td>0.9399</td>
<td>0.9299</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 3:

<table>
<thead>
<tr>
<th></th>
<th>labor share</th>
<th>land share</th>
<th>capital share</th>
<th>deposit share</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.5394</td>
<td>0.0130</td>
<td>0.3811</td>
<td>0.0664</td>
</tr>
</tbody>
</table>

Credit in the model is not used as an explicit source of finance for consumption or investment. If it is plausible to assume that firms rely mainly on commercial banks for working capital credit lines, commercial and industrial loans from commercial banks seem the proper empirical counterpart of credit in the model. Hence, for calibration purposes, other credit sources (e.g., credit from investment banks, bond issuing, credit from S&L or thrift institutions, etc.) and other credit uses (e.g., consumption loans, real estate loans, loans to the government, etc.) do not belong in the empirical counterpart of the model’s credit definition.\(^\text{12}\)

Table (4) illustrate the calibrated parameter values if the credit constraint is left out (see calibration appendix):

Table (5) illustrate the calibrated parameter values when the credit constraint is introduced and binds (see calibration appendix). These are the parameter values of the model if it were true that the productive apparatus of the U.S. economy has faced

\(^{12}\)Commercial and industrial loans from commercial banks over-estimate credit (as defined in the model) because some of these commercial and industrial loans are used to finance investment projects and not only working capital. However, by excluding the other sources of finance available to a firm, the measure of credit is also being underestimated because some of the other types of financial institutions also issue temporary lines of credit that firms may use for working capital.
credit constraints during the 1959-1999 period:

Except for $\alpha$ and $\phi$, all parameters keep the same value under both setups. This is a natural result. Recall that, with a binding credit constraint, the share of credit in output has to lie below its natural share $\phi$. On the other hand, under a credit constrained environment, the shares of capital and land in output must exceed their natural shares $\alpha$ and $(1 - \alpha - \gamma - \phi)$. In consequence, with a binding credit constraint, the calibrated values for $\alpha = 0.375$ and $(1 - \alpha - \gamma - \phi) = 0.0110$ must fall short of the capital and land shares measured in the data and replicated by the model ($0.3811$ and $0.0130$). Additionally, the calibrated values for $\phi \theta = 0.0736$ and $\gamma + \phi(1 - \theta) = 0.5401$ must lie above the deposit and labor shares measured in the data and replicated by the model ($0.0664$ and $0.5394$). Given that the calibrated values for $\theta$ and $\gamma$ do not change with respect to the financially frictionless economy, the previous condition implies that, in the credit constrained economy, the calibrated value for $\phi$ must exceed the value calibrated for the unconstrained economy.

If the model with the binding credit constraint is taken literally, the calibrated value for $\xi$ implies that only 0.54% of household’s/firm’s assets in the U.S. economy are collateralizable for bank loans (i.e. seizable by banks). This seems a very low
number. Yet, a different value for $\xi$ does not allow the model to replicate the 1959-1999 U.S. empirical regularities under a binding credit constraint. Hence, this result for $\xi$ might shed some doubt on the empirical validity of an overall credit constraint on the productive apparatus of the U.S. economy during the 1959-1999 period.

The calibration seems reasonable except for two features. First, the model’s share of commercial banks in output is high (7.2%) considering that the gross product attributed to commercial banking activities as a percentage of total GDP has fluctuated between 1.1% and 2.7% between 1947 and 1987. However, the model replicates with exactitude the share of deposits in income (6.64%). The second uncomfortable result of the calibration is that the fraction of time that agents spend in market activities ($nh = 0.58$) is high considering that this number has been estimated to be around 0.31. An alternative would be to calibrate the model so that it replicates this number. The problem with this calibration procedure is that it would not replicate exactly labor’s share or land’s share of output.

The steady state of the model implies that $ql/y = 3.4430$ and $b/y = 0.0519$. These ratios were not targeted with the calibration strategy. According to land market value data from the discontinued C.9 release of the Federal Reserve Board of Governors, the average land to output ratio was 0.3531 during the period 1959 – 1994. According to data on commercial and industrial loans from commercial banks the average loans to output ratio is 0.0801 for the period 1959 – 1999. Note then that the model does not replicate the land to output ratio and the loans to output ratio observed in the data. The mismatch is especially severe in the land to output ratio. However, land market value data is not very reliable; in fact, it is discontinued. Moreover, this data is not needed for the calibration strategy. Now, the data-model mismatch of the loans

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13Source: Dept. of Commerce, BEA. Data for the period 1987-2001 is only available for the aggregate of all depository institutions. Between 1987 and 2001 the gross product of all depository institutions as a percentage of total GDP has fluctuated between 2.8% and 3.7%.
to output ratio is not too serious considering that the model still replicates accurately the observed deposit to output ratio. Recall that deposits are, ultimately, the relevant intermediate financial input into final good production.\textsuperscript{14}

Now, recall the parameters of the stochastic processes governing $\log(z)$ and $\log(x)$:

$$\log(z_{t+1}) = \rho_0 + \rho_1 \log(z_t) + \varepsilon_{t+1}, \quad \varepsilon_t \sim N(0, \sigma^2_\varepsilon)$$

$$\log(x_{t+1}) = \varphi_0 + \varphi_1 \log(x_t) + \upsilon_{t+1}, \quad \upsilon_t \sim N(0, \sigma^2_\upsilon)$$

The way in which $z$ and $x$ can be constructed from data is detailed in the shock identification appendix. U.S. quarterly data was used to construct series for $z$ and $x$. The parameters of the AR(1) processes governing $\log(z)$ and $\log(x)$ were estimated with simple ordinary least squares techniques. The resulting estimates depend on whether data for deposits is used or not (see shock identification appendix). The following table reports the results of the estimation using the complete data set (i.e. including deposits) for the period 1959 — 1999. Recall that this is the period to which the other parameters of the model were calibrated.\textsuperscript{15}

Once the parameters of the model ($\beta, \delta, \alpha, \gamma, \phi, \theta, h, g, \eta, R$) are calibrated and once the parameters of the stochastic processes governing $\log(z)$ and $\log(x)$ ($\rho_0, \rho_1, \sigma_\varepsilon, \varphi_0, \varphi_1, \sigma_\upsilon, \sigma_{\varepsilon\upsilon}$) are estimated, the recursive competitive equilibrium is solved with the linear-quadratic method [see Cooley and Hansen (1995) or Ljunqvist and Sargent (2000), chapter 4]. See the solution and graph appendix for a check on the accuracy and robustness of the

\textsuperscript{14}An alternative calibration strategy is to target the observed $b/y$ instead of the observed $d/y$. Following such strategy requires the use of loan interest rate data while targeting the deposit to output ratio requires the use of deposit interest rate data. Given the heterogeneity implicit in the different loan interest rate series available, the choice of a representative or average loan interest rate is a difficult choice that can be avoided by choosing to target $d/y$ instead of $b/y$.

\textsuperscript{15}The estimates do not change significantly when deposits are removed from the data set to construct $z$ and $x$ (see shock identification appendix).
solution method.

4 EXPERIMENT

The experiment consists of setting the economy at its non-stochastic, stationary state and then hitting it with a positive, one standard deviation shock to $\varepsilon$ and to $\upsilon$. Each shock is done separately both under the financially frictionless environment and under the credit constrained environment.

The solution and graph appendix illustrates the response of ($y, c, i, q, d, b, n_1, n_2, w, r, s, \rho$) to each shock. The thin line of each graph traces the response to the $\upsilon$ shock while the thick line traces the response to the $\varepsilon$ shock. Graphs 1-12 refer to the case when there is no credit constraint while graphs 13-24 refer to the binding credit constraint scenario. First of all note that the propagation and amplification mechanisms of the credit constraint are not strong enough to alter in a significant way the qualitative response of the economy.\footnote{The only variable whose response is different under a binding credit constraint is the rental rate of land. Note that when there is no constraint the response of $s$ to the $\sigma_\varepsilon$ shock displays a hump. With a binding credit constraint the hump disappears. The reason is that with a binding credit constraint the rental rate of land depends not only on the marginal productivity of land but also on $q$ [recall equation (10)]. Hence, with a binding credit constraint the behavior of $q$ dampens the hump-shaped response of $s$ under the financially frictionless environment.}

The following tables display, for several time windows, the cumulated response of per capita output, consumption, investment and credit to each of the shocks in terms
Table 7: $\varepsilon$ shock

<table>
<thead>
<tr>
<th>$j \downarrow, a \rightarrow$</th>
<th>$y$</th>
<th>$c$</th>
<th>$i$</th>
<th>$b$</th>
<th>$y^{cc}$</th>
<th>$\varepsilon^{cc}$</th>
<th>$i^{cc}$</th>
<th>$b^{cc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>4.24</td>
<td>1.36</td>
<td>2.60</td>
<td>0.21</td>
<td>3.88</td>
<td>1.29</td>
<td>2.53</td>
<td>0.05</td>
</tr>
<tr>
<td>2 years</td>
<td>8.62</td>
<td>3.01</td>
<td>5.03</td>
<td>0.42</td>
<td>7.96</td>
<td>2.89</td>
<td>4.91</td>
<td>0.13</td>
</tr>
<tr>
<td>5 years</td>
<td>22.22</td>
<td>9.20</td>
<td>11.55</td>
<td>1.08</td>
<td>20.98</td>
<td>8.93</td>
<td>11.34</td>
<td>0.54</td>
</tr>
<tr>
<td>10 years</td>
<td>45.49</td>
<td>21.62</td>
<td>20.86</td>
<td>2.20</td>
<td>43.48</td>
<td>21.21</td>
<td>20.63</td>
<td>1.49</td>
</tr>
<tr>
<td>20 years</td>
<td>90.94</td>
<td>47.98</td>
<td>36.94</td>
<td>4.36</td>
<td>89.30</td>
<td>47.56</td>
<td>36.77</td>
<td>3.65</td>
</tr>
</tbody>
</table>

Table 8: $\upsilon$ shock

<table>
<thead>
<tr>
<th>$j \downarrow, a \rightarrow$</th>
<th>$y$</th>
<th>$c$</th>
<th>$i$</th>
<th>$b$</th>
<th>$y^{cc}$</th>
<th>$\upsilon^{cc}$</th>
<th>$i^{cc}$</th>
<th>$b^{cc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>0.81</td>
<td>0.17</td>
<td>0.58</td>
<td>0.36</td>
<td>0.80</td>
<td>0.18</td>
<td>0.61</td>
<td>0.33</td>
</tr>
<tr>
<td>2 years</td>
<td>1.57</td>
<td>0.40</td>
<td>1.07</td>
<td>0.69</td>
<td>1.58</td>
<td>0.42</td>
<td>1.13</td>
<td>0.64</td>
</tr>
<tr>
<td>5 years</td>
<td>3.54</td>
<td>1.21</td>
<td>2.10</td>
<td>1.48</td>
<td>3.67</td>
<td>1.29</td>
<td>2.24</td>
<td>1.42</td>
</tr>
<tr>
<td>10 years</td>
<td>5.92</td>
<td>2.53</td>
<td>2.99</td>
<td>2.34</td>
<td>6.29</td>
<td>2.75</td>
<td>3.23</td>
<td>2.31</td>
</tr>
<tr>
<td>20 years</td>
<td>8.30</td>
<td>4.16</td>
<td>3.58</td>
<td>3.13</td>
<td>9.04</td>
<td>4.57</td>
<td>3.89</td>
<td>3.18</td>
</tr>
</tbody>
</table>

of percentage points of steady state final good output:

$$\sum_{i=1}^{j} \frac{\Delta a_{i}(t)}{y_{ss}} \times 100 \quad for \quad a = y, c, i, b; \quad i = \varepsilon, \upsilon$$

Columns two through five ($y, c, i, b$) refer to the results from the model with no credit constraint while columns six through nine ($y^{cc}, c^{cc}, i^{cc}, b^{cc}$) portray the results from the model with a binding credit constraint.

Note from table 1 that in the case of the non-banking TFP shock ($\varepsilon$) the effects under the credit constrained environment are quantitatively lower than under the fi-
nancially frictionless environment! Table 2 shows that in the case of the banking TFP shock ($\nu$) the opposite is true. But with either shock the quantitative difference between the effects under financially constrained and unconstrained scenarios is very small (as a percentage of stationary output). This is in spite of the static and dynamic credit multipliers of the credit constrained economy which are supposed to amplify and propagate the shocks.

How robust are these results to alternative banking technologies? For instance, it has been argued that bank employees possess valuable information about the bank’s clients. In other words, certain information capital is embedded in bank employees. If so, releasing employees is costly due to the information capital lost by the bank. Hiring new employees is also costly due to the information capital that must be accumulated before these new employees become fully productive. According to this view, a bank faces adjustment costs in employment. Alternatively, it might be that the elasticity of substitution between deposits and labor in the intermediation technology is very low. Suppose all this is true. A negative banking shock should certainly induce a higher response of the loan rate than with the Cobb-Douglas banking technology suggested here. In consequence, there might be more action coming from the financial friction. Indeed, the financial acceleration mechanism might exhibit quantitatively significant effects.

To address this issue the model was recalibrated with a Leontief production function in the bank and the same experiment as above was carried out. Under the unconstrained environment the qualitative and quantitative results do not change significantly. The only significant difference is that the response of credit to the banking shock falls considerably. This is a natural result because with zero elasticity of substitution between deposits and labor all the adjustment must come from the loan rate. However, the model’s implications for the stationary value of the loan rate are at absurd variance with the data. Under a credit constrained environment (with or without ad-
justment costs in employment) the qualitative and quantitative results of the Leontief experiment might change significantly but, again, the implications for the stationary value of the loan rate are at great variance with the data.

5 CONCLUSIONS

The KM framework features credit constraints as the essential factor that propagates and amplifies the effects of macroeconomic shocks. However, the quantitative significance of the macroeconomic effects coming from the credit constraint in the KM setup is an open question. As I have shown in this paper, under a plausible calibration of the KM framework the quantitative significance of the macroeconomic effects coming from the credit constraint in the KM setup is very low.

Theoretically, Kocherlakota (2000) showed that such a result for the KM setup was possible. Empirically, the result should come as no surprise either given the long-run behavior of loans relative to GNP or relative to the capital stock in the U.S. (see figure 1) The graph on the top traces the evolution of the change in the U.S. stock of commercial and industrial loans as a fraction of GNP. The graph on the bottom traces the evolution of the ratio between the U.S. stock of commercial and industrial loans and the stock of private total capital and private non-residential capital.

Note that the weight of commercial and industrial loans in GNP or in the capital stock has been low. Hence, any plausible calibration of the KM framework replicating such low weight will not produce significant responses in investment value attributable to a credit constraint. Simply put, loans are not quantitatively that important in U.S. economic activity so as to generate plausibly calibrated parameter values that induce quantitatively significant propagation/amplification effects imputable to the credit constraint in the KM setup. In fact, according to my calibration strategy (see above), only 0.54% of assets in the U.S. economy are employed as collateral for commercial and in-
Undoubtedly, to find the missing credit constraint action in U.S. macro data we need a more stylized macroeconomic model. Needless to say, it must be one that does at least as well as the standard RBC model in replicating basic empirical regularities. On the other hand, it might be the case that the missing credit multiplier action is buried in firm level data that vanishes in the aggregate over all sectors of the economy. If so, what is the macroeconomic relevance of these financial accelerator/credit multiplier models? Are they plausible theoretical structures for macro policy recommendations? These questions are an interesting avenue of future research.
Figure 1:

Change in C and I LOANS / GNP(level)

C and I LOANS / PRIVATE CAPITAL STOCK
6 BIBLIOGRAPHY


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7 TECHNICAL APPENDIX

This appendix displays the proofs to the propositions in the paper.

7.1 Proof of Proposition 1

The following proof applies to proposition 1.

Proof. Note that if the firm defaults on the bank loan, the bank may not have sufficient resources to pay the labor it employs. If so, the bank will totally or partially default on payments to labor services provided by the household and the labor supply decision will be affected by the decision to default or not default. This does not happen if the bank’s labor costs do not exceed the value of the seized assets: 

\[ w(n_2h) \leq (1 - \xi)[ql + (1 - \delta)k]. \]

Let the latter condition be called the bank’s full wage payment condition (fwp). From \((P3)\) in definition 4 optimal labor demand by the bank implies 

\[ w(n_2h) = (1 - \theta)(1 + \rho)b. \]

With this result (fwp) becomes:

\[ b \leq \frac{(1 - \xi)[ql + (1 - \delta)k]}{(1 - \theta)(1 + \rho)} \] (fwp)

Conjecture: (fwp) holds.

Suppose that at the beginning of any given period the bank extends a loan \(b\) to the firm to be repaid at the end of the period. The household has rented all of its capital stock and all of its land to the firm. After observing the shocks the household has also supplied an amount of labor that is not affected by the possibility of default on the bank. Hence, with or without default the output of the firm and the bank are the same.

Suppose the firm’s owner (i.e. household) defaults on the bank loan in any given period. Under this scenario the household only gets to keep \(\xi[ql + (1 - \delta)k]\) of its total assets but appropriates 100% of the firm’s output and is paid \((1 - \theta)\) of the bank’s
output. The household’s instantaneous payoff under a default-strategy ($\pi^D$) is:

$$\pi^D = (1 - \theta)(1 + \rho)b + y + \xi[ql + (1 - \delta)k]$$

Now suppose that the firm’s owner or household does not default on the bank loan. If so, the household gets to keep 100% of its assets and is paid $(1 - \theta)$ of the bank’s output but does not appropriate the full share of the firm’s output. Under a no-default strategy the instantaneous payoff ($\pi^{ND}$) to the household is:

$$\pi^{ND} = (1 - \theta)(1 + \rho)b + y - (1 + \rho)b + ql + (1 - \delta)k$$

Given that default is not penalized with market exclusion, a firm will not default on a loan only if $\pi^D \leq \pi^{ND}$. This implies:

$$(1 - \theta)(1 + \rho)b + y + \xi[ql + (1 - \delta)x] \leq y - \theta(1 + \rho)b + ql + (1 - \delta)k$$

or:

$$b \leq \frac{(1 - \xi)[ql + (1 - \delta)k]}{(1 + \rho)}$$

and the conjecture holds. Given that in equilibrium $l^d = l$, $k^d = k$ and $b^d = b$, the previous condition can also be rewritten as:

$$b^d \leq \frac{(1 - \xi)[ql^d + (1 - \delta)k^d]}{(1 + \rho)}$$

Q.E.D.

7.2 Proof of Proposition 6

The following proof applies to proposition 6.

Proof. The Euler equation that governs the consumption-land accumulation decision by the household is given by:

$$\frac{q_t}{c_t} = \beta E_t \left[ \frac{1}{c_{t+1}} (s_{t+1} + q_{t+1}) \right]$$

(3)
Using (10) and the fact that in equilibrium \( l^d = 1 \), (3) becomes:

\[
\frac{q_t}{c_t} = \bar{\beta} E_t \left[ \frac{1}{c_{t+1}} \left[(1 - \alpha - \gamma - \phi) y_{t+1} + q_{t+1}\right] \right]
\]

Let:

\[
\Delta_t = (1 - \alpha - \gamma - \phi) y_t
\]

Then the following holds:

\[
\frac{q_t}{c_t} = \bar{\beta} E_t \left[ \frac{1}{c_{t+1}} \left(\Delta_{t+1} + q_{t+1}\right) \right]
\]

or:

\[
q_t = \bar{\beta} E_t \left( \frac{c_t}{c_{t+1}} \Delta_{t+1} \right) + \bar{\beta} E_t \left( \frac{c_t}{c_{t+1}} q_{t+1} \right)
\]

(TA1)

Using (TA1) to substitute for \( q_{t+1} \) in (TA1) implies:

\[
q_t = \bar{\beta} E_t \left( \frac{c_t}{c_{t+1}} \Delta_{t+1} \right) + \bar{\beta} E_t \left( \frac{c_t}{c_{t+1}} \bar{\beta} E_{t+1} \left( \frac{c_{t+1}}{c_{t+2}} \Delta_{t+2} \right) + \bar{\beta} E_{t+1} \left( \frac{c_{t+1}}{c_{t+2}} q_{t+2} \right) \right)
\]

(TA2)

Using the law of iterated expectations (TA2) becomes:

\[
q_t = \bar{\beta} E_t \left( \frac{c_t}{c_{t+1}} \Delta_{t+1} \right) + \bar{\beta}^2 E_t \left( \frac{c_t}{c_{t+2}} \Delta_{t+2} \right) + \bar{\beta}^2 E_t \left( \frac{c_t}{c_{t+2}} q_{t+2} \right)
\]

(TA3)

Using (TA1) to substitute for \( q_{t+2} \) in (TA3) implies:

\[
q_t = \bar{\beta} E_t \left( \frac{c_t}{c_{t+1}} \Delta_{t+1} \right) + \bar{\beta}^2 E_t \left( \frac{c_t}{c_{t+2}} \Delta_{t+2} \right) + \bar{\beta}^2 E_t \left( \frac{c_t}{c_{t+2}} \bar{\beta} E_{t+2} \left( \frac{c_{t+2}}{c_{t+3}} \Delta_{t+3} \right) + \bar{\beta} E_{t+2} \left( \frac{c_{t+2}}{c_{t+3}} q_{t+3} \right) \right)
\]

(TA4)

Using the law of iterated expectations (TA4) becomes:
\[ q_t = \tilde{\beta} E_t \left[ \frac{c_t}{c_{t+1}} \Delta_{t+1} \right] + \tilde{\beta}^2 E_t \left[ \frac{c_t}{c_{t+2}} \Delta_{t+2} \right] + \tilde{\beta}^3 E_t \left[ \frac{c_t}{c_{t+3}} \Delta_{t+3} \right] + \tilde{\beta}^3 E_t \left[ \frac{c_t}{c_{t+3}} q_{t+3} \right] \] (TA5)

After additional iterations on (TA5) and imposing the no bubble condition:

\[ \lim_{s \to \infty} E_t \left[ \beta^s \frac{c_t}{c_{t+s}} q_{t+s} \right] = 0 \] (TA6)

the following is obtained:

\[ q_t = E_t \left[ \sum_{j=1}^{\infty} \tilde{\beta}^j \frac{c_t}{c_{t+j}} \Delta_{t+j} \right] \]

where:

\[ \Delta_t = (1 - \alpha - \gamma - \phi) y_t \]

Q.E.D.

7.3 Proof of Proposition 7

The following proof applies to proposition 7.

Proof. The Euler equation that governs the consumption-land accumulation decision by the household is given by:

\[ \frac{q_t}{c_t} = \tilde{\beta} E_t \left[ \frac{1}{c_{t+1}} (s_{t+1} + q_{t+1}) \right] \] (3)

Using (10\textsuperscript{c}) and the fact that in equilibrium \( l^d = 1 \), (3) becomes:

\[ \frac{q_t}{c_t} = \tilde{\beta} E_t \left( \frac{1}{c_{t+1}} [(1 - \alpha - \gamma - \phi) y_{t+1} + \left( \frac{\phi y_{t+1}}{b_{t+1}} - (1 + \rho_{t+1}) \right) \frac{(1 - \xi) q_{t+1}}{(1 + \rho_{t+1}) + q_{t+1}}] \right) \]

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Rearranging:

\[
\frac{q_t}{c_t} = \tilde{\beta} E_t \left\{ \frac{1}{c_{t+1}} [(1 - \alpha - \gamma - \phi) y_{t+1} + \left( (1 - \xi) \frac{(\phi y_{t+1}/b_{t+1})}{(1 + \rho_{t+1})} - (1 - \xi) \right) q_{t+1} + q_{t+1}] \right\}
\]

Simplifying some more implies:

\[
\frac{q_t}{c_t} = \tilde{\beta} E_t \left\{ \frac{1}{c_{t+1}} \left[ (1 - \alpha - \gamma - \phi) y_{t+1} + \left( (1 - \xi) \frac{(\phi y_{t+1}/b_{t+1})}{(1 + \rho_{t+1})} + \xi \right) q_{t+1} \right] \right\}
\]

Let:

\[
\Delta_t = (1 - \alpha - \gamma - \phi) y_t
\]

and:

\[
\Omega_t = (1 - \xi) \frac{(\phi y_t/b_t)}{(1 + \rho_t)} + \xi
\]

Then the following holds:

\[
\frac{q_t}{c_t} = \tilde{\beta} E_t \left[ \frac{1}{c_{t+1}} (\Delta_{t+1} + \Omega_{t+1} q_{t+1}) \right]
\]

or:

\[
q_t = \tilde{\beta} E_t \left[ \frac{c_t}{c_{t+1}} \Delta_{t+1} \right] + \tilde{\beta} E_t \left[ \frac{c_t}{c_{t+1}} \Omega_{t+1} q_{t+1} \right] \quad (TA1^{cc})
\]

Using (TA1^{cc}) to substitute for \(q_{t+1}\) in (TA1^{cc}) implies:

\[
q_t = \tilde{\beta} E_t \left[ \frac{c_t}{c_{t+1}} \Delta_{t+1} \right] + \tilde{\beta} E_t \left[ \frac{c_t}{c_{t+1}} \Omega_{t+1} \left( \frac{c_{t+1}}{c_{t+2}} \Delta_{t+2} \right) \right] + \tilde{\beta} E_t \left[ \frac{c_t}{c_{t+1}} \Omega_{t+1} \left( \frac{c_{t+1}}{c_{t+2}} \Omega_{t+2} q_{t+2} \right) \right] \quad (TA2^{cc})
\]
Using the law of iterated expectations (TA2\textsuperscript{cc}) becomes:

$$q_t = \tilde{\beta} E_t \left[ \frac{c_t}{c_{t+1}} \Delta_{t+1} \right]$$

$$+ \tilde{\beta}^2 E_t \left[ \frac{c_t}{c_{t+2}} (\Omega_{t+1}\Delta_{t+2}) \right] + \tilde{\beta}^2 E_t \left[ \frac{c_t}{c_{t+2}} \Omega_{t+1}\Omega_{t+2}q_{t+2} \right]$$  \hspace{1cm} (TA3\textsuperscript{cc})

Using (TA1\textsuperscript{cc}) to substitute for \(q_{t+2}\) in (TA3\textsuperscript{cc}) implies:

$$q_t = \tilde{\beta} E_t \left[ \frac{c_t}{c_{t+2}} (\Omega_{t+1}\Delta_{t+2}) \right]$$

$$+ \tilde{\beta}^2 E_t \left\{ \frac{c_t}{c_{t+2}} \Omega_{t+1}\Omega_{t+2} \left[ \tilde{\beta} E_{t+2} \left( \frac{c_{t+2}}{c_{t+3}} \Delta_{t+3} \right) + \tilde{\beta}^2 E_{t+2} \left( \frac{c_{t+2}}{c_{t+3}} \Omega_{t+3}q_{t+3} \right) \right] \right\}$$ \hspace{1cm} (TA4\textsuperscript{cc})

Using the law of iterated expectations (TA4\textsuperscript{cc}) becomes:

$$q_t = \tilde{\beta} E_t \left[ \frac{c_t}{c_{t+1}} \Delta_{t+1} \right] + \tilde{\beta}^2 E_t \left[ \frac{c_t}{c_{t+2}} (\Omega_{t+1}\Delta_{t+2}) \right]$$

$$+ \tilde{\beta}^3 E_t \left[ \frac{c_t}{c_{t+3}} (\Omega_{t+1}\Omega_{t+2}\Delta_{t+3}) \right] + \tilde{\beta}^3 E_t \left[ \frac{c_t}{c_{t+3}} (\Omega_{t+1}\Omega_{t+2}\Omega_{t+3}q_{t+3}) \right]$$ \hspace{1cm} (TA5\textsuperscript{cc})

After additional iterations on (TA5\textsuperscript{cc}) and imposing the no bubble condition:

$$\lim_{s \to \infty} E_t \left[ \frac{c_t}{c_{t+s}} q_{t+s} \prod_{i=1}^{s} \tilde{\beta}\Omega_{t+i} \right] = 0$$ \hspace{1cm} (TA6\textsuperscript{cc})

the following is obtained:

$$q_t = E_t \left\{ \sum_{j=1}^{\infty} \tilde{\beta}^j \frac{c_t}{c_{t+j}} \left[ \Delta_{t+j} \prod_{i=1}^{j-1} \Omega_{t+i} \right] \right\}$$

where:

$$\Delta_t = (1 - \alpha - \gamma - \phi) y_t$$

and:

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\[ \Omega_t = (1 - \xi) \frac{(\phi y_t/b_t)}{(1 + \rho_t)} + \xi \]

Q.E.D. \[ \square \]

The next corollary establishes a sufficient condition to rule out the bubble component of the price of land.

**Corollary 7.1.** If \( \tilde{\beta} \left( \frac{\phi y_t}{b_t} \right) < (1 + \rho_t) \quad \forall t \) then the no-bubble condition holds.

**Proof.** A sufficient condition for (TA6\textsuperscript{cc}) to hold is:

\[ \tilde{\beta} \Omega_t < 1 \quad \forall t \]

This is:

\[ \tilde{\beta} \left[ (1 - \xi) \frac{(\phi y_t/b_t)}{(1 + \rho_t)} + \xi \right] < 1 \quad \forall t \]

or:

\[ \tilde{\beta} \left\{ \frac{\phi y_t}{b_t} - \xi \left[ \frac{\phi y_t}{b_t} - (1 + \rho_t) \right] \right\} < (1 + \rho_t) \quad \forall t \quad \text{(TA7\textsuperscript{cc})} \]

Now, recall that \( \lambda_t \) is the Kuhn-Tucker (KT) multiplier associated to the borrowing constraint and is defined by equation (7\textsuperscript{cc}) of the paper:

\[ \lambda_t = \frac{\phi y_t}{b_t} - (1 + \rho_t) \quad \geq 0 \quad \text{(7\textsuperscript{cc})} \]

The KT theorem establishes that \( \lambda_t \geq 0 \quad \forall t \). From the definition of \( \lambda_t \) in equation (7\textsuperscript{cc}) it can be seen that the KT theorem also implies that \( (\phi y_t/b_t) - (1 + \rho_t) \geq 0 \quad \forall t \). Hence, \( \tilde{\beta} \xi [(\phi y_t/b_t) - (1 + \rho_t)] \geq 0 \quad \forall t \). This last result implies that

\[ \tilde{\beta} \{ (\phi y_t/b_t) - \xi [(\phi y_t/b_t) - (1 + \rho_t)] \} \leq \tilde{\beta} (\phi y_t/b_t) \quad \forall t \]. Thus, a sufficient yet stronger condition for (TA7\textsuperscript{cc}) to hold is:

\[ \tilde{\beta} \frac{\phi y_t}{b_t} < (1 + \rho_t) \quad \forall t \]

Q.E.D. \[ \square \]
8 CALIBRATION APPENDIX

Parameter values were chosen so that the model, in stationary state, mimics some long-run empirical regularities observed in the U.S. Specifically, the parameters were calibrated to a quarterly frequency using U.S. data for the period 1959-1999.

8.1 No Credit Constraint

From (1)-(10) it can be shown that the following system of equations characterizes the steady state variables of the no-constraint economy relative to output (recall \( l = 1 \)):

\[
(1 + g) \frac{k}{y} = \beta \left[ \frac{r k}{y} + (1 - \delta) \frac{k}{y} \right] \tag{1ss}
\]

\[
\frac{q}{y} = \left( \frac{\beta}{1 - \beta} \right) \frac{s}{y} \tag{2ss}
\]

\[
\frac{wh}{y} = -\log(1-h) \frac{c}{y} \tag{3ss}
\]

\[
\frac{wn_1 h}{y} = \gamma \tag{4ss}
\]

\[
\frac{r k}{y} = \alpha \tag{5ss}
\]

\[
\frac{s}{y} = 1 - \alpha - \gamma - \phi \tag{6ss}
\]

\[
\frac{b(1 + \rho)}{y} = \phi \tag{7ss}
\]

\[
(1 + R) \frac{d}{y} = \theta \frac{b(1 + \rho)}{y} \tag{8ss}
\]

\[
\frac{wn_2 h}{y} = (1 - \theta) \frac{b(1 + \rho)}{y} \tag{9ss}
\]

\[
\frac{c}{y} = 1 - (\eta + g + \eta g + \delta) \frac{k}{y} - (1 + R) \frac{d}{y} \tag{10ss}
\]

\[
n = n_1 + n_2 \tag{11ss}
\]
From (10ₚₚ):

\[ \delta = 1 - (\eta + g + g\eta)\frac{k}{y} - \frac{c}{y} - \text{deposit share} \]  

(1ₚₚ)

From (1ₚₚ):

\[ \beta = \frac{(1 + g)\frac{k}{y}}{\text{capital share} + (1 - \delta)\frac{k}{y}} \]  

(2ₚₚ)

From (3ₚₚ):

\[ h = 1 - \exp \left[ -\text{labor share} \left( \frac{n_1}{n} \right) \right] \]  

(3ₚₚ)

From (4ₚₚ):

\[ \gamma = \text{labor share} \left( \frac{n_1}{n} \right) \]  

(4ₚₚ)

From (7ₚₚ) and (8ₚₚ):

\[ \theta \phi = \text{deposit share} \]

From (4ₚₚ), (7ₚₚ), (9ₚₚ) and (11ₚₚ):

\[ \text{labor share} = \gamma + (1 - \theta)\phi = \gamma + \phi - \theta \phi \]

Hence:

\[ \text{labor share} = \gamma + \phi - \text{deposit share} \]

Rearranging:

\[ \phi = \text{labor share} + \text{deposit share} - \gamma \]  

(5ₚₚ)
From (7_ss), (8_ss) and (5_cal):

\[
\theta = \frac{\text{deposit share}}{\text{labor share + deposit share} - \gamma}
\]  

(6_cal)

From (5_ss):

\[
\alpha = \text{capital share}
\]  

(7_cal)

Note then that given:

\[
\Theta = \left[ k, \frac{c}{y}, n_1, n, \text{capital share, labor share, deposit share, } \eta, g \right]
\]

equations (1_cal)-(7_cal) determine:

\[
\Psi = (\delta, \beta, h, \gamma, \phi, \theta, \alpha)
\]

8.2 Binding Credit Constraint

Let

\[
\Gamma = \frac{b(1 + \rho)}{y}
\]

Suppose that the credit constraint binds. Hence, from (1)-(6) and (7^cc)-(10^cc) it can be shown that the following system of equations characterizes the steady state variables of the credit constrained economy relative to output (recall \(l = 1\)):

\[
(1 + g) \frac{k}{y} = \beta \left[ \frac{rk}{y} + (1 - \delta) \frac{k}{y} \right]
\]  

(1^ss)

\[
\frac{q}{y} = \left( \frac{\tilde{\beta}}{1 - \beta} \right) \frac{s}{y}
\]  

(2^ss)
\[
\frac{wh}{y} = - \log(1 - h) \frac{c}{y} \quad (3_{ss})
\]

\[
\frac{wn_1 h}{y} = \gamma \quad (4_{ss})
\]

\[
\frac{r k}{y} = \alpha + \left(\frac{\phi}{\Gamma} - 1\right)(1 - \xi)(1 - \delta) \frac{k}{y} \quad (5_{ss})
\]

\[
\frac{s}{y} = (1 - \alpha - \gamma - \phi) + \left(\frac{\phi}{\Gamma} - 1\right)(1 - \xi) \frac{q}{y} \quad (6_{ss})
\]

\[
\Gamma = (1 - \xi) \left[ \frac{q}{y} + (1 - \delta) \frac{k}{y} \right] \quad (7_{ss})
\]

\[
\frac{d(1 + R)}{y} = \theta \Gamma \quad (8_{ss})
\]

\[
\frac{wn_2 h}{y} = (1 - \theta) \Gamma \quad (9_{ss})
\]

\[
\frac{c}{y} = 1 - (\delta + \eta + g + g\eta) \frac{k}{y} - (1 + R) \frac{d}{y} \quad (10_{ss})
\]

\[
n_1 + n_2 = n \quad (11_{ss})
\]

From (10_{ss}):

\[
\delta = \frac{1 - (\eta + g + g\eta) \frac{k}{y} - \xi}{\frac{k}{y}} \quad \text{deposit share} \quad (1_{cal})
\]

From (1_{ss}):

\[
\beta = \frac{(1 + g) \frac{k}{y}}{\text{capital share} + (1 - \delta) \frac{k}{y}} \quad (2_{cal})
\]
From (3:ss):

\[ h = 1 - \exp \left( -\text{labor share} \cdot \frac{n}{y} \right) \tag{3_{\text{cal}}} \]

From (4:ss):

\[ \gamma = \text{labor share} \cdot \frac{n_1}{n} \tag{4_{\text{cal}}} \]

From (8:ss):

\[ \theta = \frac{\text{deposit share}}{\Gamma} \]

From (9:ss):

\[ \Gamma = \left( \frac{wn_2h}{y} \right) \frac{1}{(1 - \theta)} \]

Hence:

\[ \theta = \frac{\text{deposit share}}{\frac{wn_2h}{y}} (1 - \theta) \]

Using (11:ss) this is:

\[ \theta = \frac{\text{deposit share}}{\frac{wnh}{y} - \frac{wn_1h}{y}} (1 - \theta) = \frac{\text{deposit share}}{\text{labor share} - \frac{wn_1h}{y}} (1 - \theta) \]

Using (4:ss) this is:

\[ \theta = \frac{\text{deposit share}}{\text{labor share} - \gamma} (1 - \theta) \]

Rearranging implies:

\[ \theta = \frac{\text{deposit share}}{\text{labor share} + \text{deposit share} - \gamma} \tag{5_{\text{cal}}} \]
From (2\textsubscript{ss}) in (7\textsubscript{ss}):

\[ \Gamma = (1 - \xi) \left[ \left( \frac{\beta}{1 - \beta} \right) \text{land share} + (1 - \delta) \frac{k}{y} \right] \]

using (8\textsubscript{ss}) this is:

\[ \text{deposit share} \theta = (1 - \xi) \left[ \left( \frac{\beta}{1 - \beta} \right) \text{land share} + (1 - \delta) \frac{k}{y} \right] \]

Rearranging implies:

\[ \xi = 1 - \frac{\text{deposit share}}{\theta} \left[ \left( \frac{\beta}{1 - \beta} \right) \text{land share} + (1 - \delta) \frac{k}{y} \right] \]

(6\textsubscript{cal})

At this point the only parameters pending for calibration are \( \phi \) and \( \alpha \). The only two equations that have not been used are (5\textsubscript{ss}) and (6\textsubscript{ss}). Apparently, both of them constitute a system of two equations in \( \phi \) and \( \alpha \). However, note that adding up (5\textsubscript{ss}) and (6\textsubscript{ss}) implies:

\[ \text{capital share} + \text{land share} = 1 - \gamma - \phi + \left( \frac{\phi}{\Gamma} - 1 \right) (1 - \xi) \left[ \frac{q}{y} + (1 - \delta) \frac{k}{y} \right] \]

Using (7\textsubscript{ss}) this is:

\[ \text{capital share} + \text{land share} = 1 - \gamma - \phi + \left( \frac{\phi}{\Gamma} - 1 \right) \Gamma \]

or:

\[ \text{capital share} + \text{land share} + \gamma + \Gamma = 1 \]

The latter equation holds by construction given that \( \Gamma \) is credit share and that there are constant returns to scale. Thus, (5\textsubscript{ss}) and (6\textsubscript{ss}) are an underidentified system to pin down \( \phi \) and \( \alpha \) in terms of observables. Consequently, one of the two parameters is
free. The calibration strategy is to pick $\phi > \Gamma = \frac{(deposit \ share)}{\theta}$ (so that the credit constraint binds)$^{17}$ and then obtain $\alpha$ from (5$^{ss}$) and (8$^{ss}$) in the following way:

$$\alpha = capital\ share - \left( \frac{\phi\theta}{deposit\ share} - 1 \right) (1 - \xi)(1 - \delta)\frac{k}{y} \quad (7^{cc})$$

Equation (6$^{ss}$) must hold by construction.

Note then that given:

$$\Theta' = \left[ \frac{k}{y}, \frac{c}{y}, n_1, n, capital\ share, labor\ share, land\ share, deposit\ share, \eta, g \right]$$

equations (1$^{cc}$)-(7$^{cc}$) determine:

$$\Psi' = (\delta, \beta, h, \gamma, \theta, \xi, \alpha)$$

and $\phi$ is chosen freely.

Note that (1$^{cc}$)-(4$^{cc}$) are identical to (1$^{cal}$)-(4$^{cal}$) and that (5$^{cc}$) is identical to (6$^{cal}$). Hence, regardless of whether the credit constraint binds or not, the calibration for $\delta$, $\beta$, $h$, $\gamma$ and $\theta$ is always the same. However, the equations that calibrate $\alpha$ and $\phi$ change when the credit constraint binds [i.e. $\phi > \Gamma = \frac{(deposit\ share)}{\theta}$ and (7$^{cc}$) differ from (5$^{cal}$) and (7$^{cal}$)]. Note, however, that both sets of equations are identical if $\phi = \Gamma = \frac{(deposit\ share)}{\theta}$, which means that the credit constraint does not bind.$^{18}$

### 8.3 Data to Measure $\Theta$ and $\Theta'$

To construct the empirical counterparts of $\Theta$ and $\Theta'$ the following U.S. data was used:

$^{17}$Recall that the credit constraint binds if $\lambda = \frac{\phi y}{b} - (1 + \rho) > 0$. This condition is equivalent to: $\phi > \frac{b(1 + \rho)}{y} = \Gamma$

$^{18}$Recall that the credit constraint does not bind if $\lambda = \frac{\phi y}{b} - (1 + \rho) = 0$. This condition is equivalent to: $\phi = \frac{b(1 + \rho)}{y} = \Gamma$


- **rₜ(2)**: National monthly median cost of funds ratio to SAIF-insured institutions. It is defined as interest (dividends) paid or accrued on deposits, on FHLB advances and on other borrowed money during a month as a percent of balances of deposits at month end. It reflects rates on all funds, not just new funds. This ratio is annualized by multiplying by 12 and adjusted for variation in length of month. Source: OTS. Original frequency: Monthly. Units: Percent. Sample: 1979.05-2001.11.


• **k_P**: Current cost, net stock of private total fixed assets (equipment and software plus structures plus residential assets, including owner-occupied housing) located in the US that are owned by private business or nonprofit institutions. Current cost valuation measures the value of these assets in the prices of the given period, which are end of year for net stocks and annual averages for depreciation. Source: U.S. Dept. of Commerce, Bureau of Economic Analysis. Original frequency: Annual. Units: Billions of dollars. Sample: 1925-2000.

• **k_G**: Current cost, net stock of government’s total fixed assets (equipment and software plus structures plus residential assets). Current cost valuation measures the value of these assets in the prices of the given period, which are end of year for net stocks and annual averages for depreciation. Source: U.S. Dept. of Commerce, Bureau of Economic Analysis. Original frequency: Annual. Units: Billions of dollars. Sample: 1925-2000.


• \( \text{GNP} \): Gross National Product (seasonally adjusted annual rates). Source: U.S.


• **Rental income:** Rental income of persons with capital consumption adjustment (seasonally adjusted annual rates). Source: U.S. Dept. of Commerce, Bureau of

Monthly frequencies are averaged out to quarterly frequencies. Quarterly frequencies are averaged out to annual frequencies. All nominal variables (including nominal interest rates) are transformed into real terms (billions of 1996 dollars) with the implicit chain-type GNP price deflator:


Some empirical issues regarding the “correct” empirical counterparts of \((R, d, k, c, i, y)\) must be discussed. First of all, note that interest rate data on cod’s \([r_T(1)]\) or the data on the cost of funds ratio of thrift institutions \([r_T(2)]\) overestimate the average interest rate paid by commercial banks. The reason is that commercial banks not only compete for time deposits but also provide checkable deposits paying an interest rate lower than \(r_T\) (zero during many years due to regulation Q). Thus, \(r_T\) has to be adjusted to obtain a more accurate measure of the cost of funds ratio of commercial banks. In fact, the cost of funds ratio of commercial banks can be represented as:

\[
R = \frac{r_T d_T + r_{CH} d_{CH}}{d_{TOT}}
\]

where \(r_{CH}\) represents the interest rate on checkable deposits and \(d_{CH}\) stands for total checkable deposits. It is sensible to assume \(r_{CH} = 0\). Under this assumption:

\[
R = r_T \left(\frac{d_T}{d_{TOT}}\right)
\]

Hence, \(r_T\) is adjusted with factor \(\frac{d_T}{d_{TOT}}\) in order to obtain a more accurate measurement of the cost of funds ratio of commercial banks.
The second important empirical issue is that not all deposits in commercial banks are used to finance commercial and industrial loans. Some of them are used to finance consumption loans, real estate loans, loans to the government, etc. Therefore, the data on total stock of deposits at commercial banks \( (d_{TOT}) \) must be adjusted so that the relevant stock of deposits used by commercial banks to finance commercial and industrial loans is captured. As a simple rule of thumb, the adjustment factor employed is the ratio of commercial and industrial loans at commercial banks to total loans and investments at commercial banks:

\[
d = d_{TOT} \left( \frac{b_{CI}}{b_{TOT}} \right)
\]

Note that this adjustment factor underestimates the amount of resources used by commercial banks to finance commercial and industrial loans. Indeed, net worth or liabilities and purchased funds different than deposits may also be used to finance commercial and industrial loans.

The third empirical issue that has to be discussed is the measurement of \((k, c, i, y)\). As is standard in the RBC literature, \((k, c, i)\) are measured with:

\[
k = k_P + k_G + k_D
\]

\[
c = c_{ND} + c_S + c_G
\]

\[
i = i_P + i_G + i_D + NX
\]

However, the “correct” empirical counterpart of \(y\) is not GNP because the latter only includes income imputable to land, labor and the private stock of capital \((k_P)\). It does not include income imputable to the government’s stock of capital \((k_G)\) or to the stock of consumer durables \((k_D)\) which are part of the empirical counterpart of \(k\).
GNP does not include income imputable to deposits either and in the model deposits operate as an input of production in addition to land, labor, and capital. Consequently, the proper empirical counterpart of $y$ is GNP adjusted to include income imputable to $k_G$, $k_D$ and $d$. To compute income imputable to $k_G$ and $k_D$ the methodology of Cooley and Prescott (1995) is followed. Let $y_P$ represent income imputable to $k_P$ and $\alpha_P$ represent $k_P$’s share of total income. Thus:

$$y_P = \alpha_P GNP = \text{Corporate profits} + \text{Net interest} +$$

$$\alpha_P \left( \text{Proprietor's income} + \text{NNP} - \text{National income} \right) +$$

Depreciation

Solving for $\alpha_P$ yields the following result:

$$\alpha_P = \frac{\text{Corporate profits} + \text{Net interest} + \text{Depreciation}}{GNP - (\text{Proprietor's income} + \text{NNP} - \text{National income})}$$

Given that there is data on all the elements of the right hand side of the previous equation, a series for $\alpha_P$ can be constructed. Now let $r$ stand for the rate of return on any type of capital (i.e. $k_P$, $k_G$ or $k_D$) and $\delta_P$ represent the depreciation rate of $k_P$. Hence, by definition:

$$y_P = \alpha_P GNP = (r + \delta_P)k_P$$

This is:

$$y_P = \alpha_P GNP = r k_P + \text{Depreciation}$$

Solving for $r$ implies:

$$r = \frac{\alpha_P GNP - \text{Depreciation}}{k_P}$$
Given that there is data on all the elements of the right hand side of the previous equation, a series for $r$ can be constructed. Now consider the law of motion of $k_G$:

$$k'_G = (1 - \delta_G)k_G + i_G$$

where $\delta_G$ represents the rate of depreciation of $k_G$. It is possible to solve for $\delta_G$:

$$\delta_G = 1 + \frac{i_G}{k_G} - \frac{k'_G}{k_G}$$

Given that there is data on all the elements of the right hand side of the previous equation, a series for $\delta_G$ can be constructed. Similarly, a series for $\delta_D$, the rate of depreciation of $k_D$, can be constructed:

$$\delta_D = 1 + \frac{i_D}{k_D} - \frac{k'_D}{k_D}$$

Now let $y_G$ and $y_D$ represent income imputable to $k_G$ and $k_D$. By definition:

$$y_G = (r + \delta_G)k_G \quad (***)$$

$$y_D = (r + \delta_D)k_D \quad (****)$$

Given that there is data on all the elements of the right hand side of the two previous equations, series for $y_G$ and $y_D$ can be constructed. Finally, the “correct” empirical counterpart of $y$ is constructed according to the following formula:

$$y = GNP + y_G + y_D + (1 + R)d$$

Specifically:

$$y = GNP + y_G + y_D + \left(1 + Avg_{1964-1999} \left[r_T(2) * \left(\frac{dT}{dT_{TOT}}\right)\right]\right) \left[\frac{dT_{TOT}}{b_{TOT}}\right]$$

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The average (as opposed to each observation) of \[ r_T(2) \times \left( \frac{dT}{d_TOT} \right) \] is used to construct \( y \) because data for such series is only available since 1964. If each observation of \[ r_T(2) \times \left( \frac{dT}{d_TOT} \right) \] is used to construct \( y \) then all the information of the other variables (i.e. \( GNP, y_G \) and \( y_D \)) prior to 1964 would be lost.

The measurement of the different elements of \( \Theta \) and \( \Theta' \) (which determine \( \Psi \) and \( \Psi' \)) is then given by:

- \( k_y \): \[ \text{Avg}_{1959-1999} \left( \frac{k_P + k_G + k_D}{y} \right) \times 4 \]
- \( c_y \): \[ \text{Avg}_{1959-1999} \left( \frac{c_{ND} + c_S + c_G}{y} \right) \]
- \( n_1 \): \[ \text{Avg}_{1959-1999}(n) - \text{Avg}_{1972-1999}(n_2). \]
- \( n \): \[ \text{Avg}_{1959-1999}(n). \]
- capital share: \[ \text{Avg}_{1959-1999} \left( \frac{y_P + y_G + y_D}{y} \right) \]
- land share: \[ \text{Avg}_{1959-1999} \left( \frac{\text{Rental income}}{y} \right) \]
- deposit share: \[ 1 + \text{Avg}_{1964-1999} \left[ r_T(2) \times \left( \frac{dT}{d_TOT} \right) \right] \times \text{Avg}_{1959-1999} \left[ d_{TOT} \left( \frac{b_{CH}}{b_{TOT}} \right) \right] \]
- labor share: \[ 1 - \text{capital share} - \text{deposit share} - \text{land share} \]
- \( \eta \): \[ \left( \frac{\text{pop}_{1999}}{\text{pop}_{1959}} \right)^{\frac{1}{1999-1959}} - 1 \]
- \( g \): \[ \left( \frac{y_{pop}}{y_{pop}} \right)^{\frac{1}{1999-1959}} - 1 \]

where \( y_P, y_G, y_D \) and \( y \) are defined and constructed according to (*), (**), (***) and (****).

Note that the model was calibrated with data for period 1959-1999. Even though the NIPA data goes until 2001, data for the capital stock only goes until 2000. Moreover, the last observation (2000) is lost in the calibration process due to the way \( \delta_G \)
and $\delta_D$ are constructed. On the other hand, even though the NIPA data starts earlier than 1959, data for deposits only starts in this year. The only two series which do not coincide with the calibration time period are that of the interest rate and that of employment in commercial banks which start in 1964 and 1972, respectively.
From (6)-(8):

\[ \gamma \frac{y}{n_1} = (1 - \theta)\phi \frac{y}{n_2} \]

This equation implies:

\[ n_2\gamma = n_1(1 - \theta)\phi \]

Given that \( n_2 = n - n_1 \) this equation implies:

\[ (n - n_1)\gamma = n_1(1 - \theta)\phi \]

or, rearranging:

\[ \gamma n = [\gamma + (1 - \theta)\phi] n_1 \]

This, in turn, implies:

\[ n_1 = \Omega n; \quad n_2 = (1 - \Omega)n \quad \text{(1sid)} \]

where:

\[ \Omega = \frac{\gamma}{\gamma + (1 - \theta)\phi} \]

Now recall the definition of per-capita final good output:

\[ y = (zx^\phi) k^\alpha d^{\theta\phi} \left[ (n_1h)\gamma (n_2h)^{(1-\theta)\phi} \right] \]

Using (1sid) this is:

\[ y = (zx^\phi) k^\alpha d^{\theta\phi} \left[ (\Omega nh)^\gamma ((1 - \Omega)nh)^{(1-\theta)\phi} \right] \]

Rearranging terms yields the following result for per-capita final good output:
\[ y = A \left( zx^\phi \right) k^\alpha d^\theta (nh)^{\gamma+\left(1-\theta\right)\phi} \]  

(2sid)

where:

\[ A_1 = \Omega^\gamma (1 - \Omega)^{\left(1-\theta\right)\phi} \]

From (2sid) final good total output is given by:

\[ Y = yN = A_1 \left( zx^\phi \right) K^\alpha D^{\theta\phi} (Nh)^{\gamma+\left(1-\theta\right)\phi} N^{1-\alpha-\gamma-\phi} \]  

(3sid)

\[ = A_1 \left( zx^\phi \right) K^\alpha D^{\theta\phi} (Nh)^{\gamma+\left(1-\theta\right)\phi} L^{1-\alpha-\gamma-\phi} \]

\[ = A_1 \left( zx^\phi \right) K^\alpha D^{\theta\phi} (H)^{\gamma+\left(1-\theta\right)\phi} L^{1-\alpha-\gamma-\phi} \]

where \( N \) stands for the total number of workers, \( H \) represents total work hours supplied by households and \( k \) and \( L \) the total stock of capital and land. Let \( e \) be \( (zx^\phi) \) or the composite shock, representing TFP at the aggregate level. Given that \( A_1 \) is a constant and that \( K \) and \( L \) are pretty stable across time, equation (3sid) shows that \( e \) can be measured with:

\[ e = \frac{Y}{H^{\gamma+\left(1-\theta\right)\phi} D^{\theta\phi}} \]  

(4sid)

or, if \( D \) is also sufficiently stable across time, then there is no need for data on deposits and \( e \) can be measured with:

\[ e = \frac{Y}{H^{\gamma+\left(1-\theta\right)\phi}} \]  

(4sid’)

The latter way of measuring \( e \) is the standard way of measuring aggregate TFP.

Now recall the definition of per-capita banking output:

\[ b = xd^\theta (n_2h)^{1-\theta} \]
Using (1sid) this is:

\[ b = x d^\theta [(1 - \Omega) nh]^{1 - \theta} \]

Rearranging terms yields the following result for per-capita banking output:

\[ b = A_2 x d^\theta (nh)^{1 - \theta} \] (5sid)

where:

\[ A_2 = (1 - \Omega)^{1 - \theta} \]

From (5sid) total banking output is given by:

\[ B = b N = A_2 x D^\theta (Nh)^{1 - \theta} \] (6sid)

\[ = A_2 x D^\theta H^{1 - \theta} \]

where \( H \) represents total work hours supplied by households. Given that \( A_2 \) is a constant, equation (6sid) shows that \( x \) can be measured with:

\[ x = \frac{B}{H^{1 - \theta} D^\theta} \] (7sid)

or, if \( D \) is sufficiently stable across time, then there is no need for data on deposits and \( x \) can be measured with:

\[ x = \frac{B}{H^{1 - \theta}} \] (7sid’)

By definition, \( z \) can be measured with:

\[ z = \frac{e}{x^\phi} = \frac{Y}{H^{1 + (1 - \theta) \phi} D^{\phi \phi}} = \frac{Y}{H^{1} B^{\phi}} \] (8sid)
U.S. quarterly data and equations (4sid), (7sid) and (8sid) [or, alternatively, equa-
tions (4sid'), (7sid') and (8sid) if no deposit data is to be employed] were used to
construct $e$, $x$ and $z$, respectively. Note that the construction of $z$ is not sensitive to
deposit data.

The values of $\phi$, $\theta$ and $\gamma$ used for the construction of $e$, $x$ and $z$ were those resulting
from the calibration of the model under no credit constraints (the calibrated values
of $\theta$ and $\gamma$ do not depend on whether there is a constraint or not but the value of
$\phi$ does). Note also that the construction of $e$, $x$ and $z$ uses optimality conditions
for the unconstrained environment. In fact, if there is a binding credit constraint
$\gamma \frac{y_{n1}}{n_{1}} = (1 - \theta)\phi \frac{y_{n2}}{n_{2}}$ does not hold. Instead: $\gamma \frac{y_{n1}}{n_{1}} < (1 - \theta)\phi \frac{y_{n2}}{n_{2}}$. This does not allow
equation (1sid) to hold. However, an expression similar to (1sid) can be found where
$\Omega$ is not a constant but equal to $\gamma y / [(1 - \theta)(1 - \xi)(ql + (1 - \delta)k) + \gamma y]$. If $K$, $q$ and $L$
are stable across time, the construction of $e$, $x$ and $z$ under no credit constraints
should also apply for the binding credit constraint case.

The following list describes the data used to construct $e$, $x$ and $z$:

- $Y$: Real GNP (seasonally adjusted annual rates). Source: U.S. Dept. of Com-
merce, Bureau of Economic Analysis. Frequency: Quarterly. Units: Billions of
chained 1996 dollars. Sample: 1959.I-1999.IV if (4sid) is used to construct $e$ and
$z$; 1947.I-1999.IV if (4sid') is used to construct $e$ and $z$.

- $H$: Total annual hours in the private non-farm sector (seasonally adjusted).
Sample: Billions of hours. Sample: 1959.I-1999.IV if (4sid) and (7sid) are
used to construct $e$, $x$ and $z$; 1947.I-1999.IV if (4sid') and (7sid') are used to
construct $e$, $x$ and $z$.

- $D$: Total checkable deposits plus total time deposits at commercial banks, de-
flated by the GNP chain-type price index (seasonally adjusted stocks). Source:
H.6 Release. Federal Reserve Board of Governors. Frequency: Quarterly averages of monthly values. Units: Billions of chained 1996 dollars. Sample: 1959.I-1999.IV. **Note:** This series was not adjusted by the ratio of commercial and industrial loans at commercial banks to total loans and investments at commercial banks (as in the calibration process). Recall that this adjustment was necessary to account for the fact that not all deposits in commercial banks are used to finance commercial and industrial loans (which is the empirical counterpart of credit -see below-). However, adjusting deposits in the same way to construct a series for \( x \) and then estimate \( \log(x_{t+1}) = \varphi_0 + \varphi_1 \log(x_t) + \nu_{t+1} \) introduces additional noise coming from the fluctuations in the portfolio of commercial banks. Hence, not adjusting deposits for the construction of \( x \) is more appropriate as long as the total stock of deposits at commercial banks moves closely with that fraction of them that is used to finance commercial and industrial loans.

- **B:** Commercial and industrial loans at all commercial banks, deflated by the GNP chain-type price index (seasonally adjusted stocks). Source: H.8 Release. Federal Reserve Board of Governors. Frequency: Quarterly averages of monthly values. Units: Billions of chained 1996 dollars. Sample: 1959.I-1999.IV if (4sid) and (7sid) are used to construct \( e, x \) and \( z \); 1947.I-1999.IV if (4sid’) and (7sid’) are used to construct \( e, x \) and \( z \).

Next, the stochastic processes governing \( \log(e) \), \( \log(x) \) and \( \log(z) \) were estimated:

\[ \log(z_{t+1}) = \rho_0 + \rho_1 \log(z_t) + \varepsilon_{t+1}, \quad \varepsilon_t \sim N(0, \sigma^2_\varepsilon) \]

\[ \log(x_{t+1}) = \varphi_0 + \varphi_1 \log(x_t) + \nu_{t+1}, \quad \nu_t \sim N(0, \sigma^2_\nu) \]

\[ \log(e_{t+1}) = \pi_0 + \pi_1 \log(e_t) + \epsilon_{t+1}, \quad \epsilon_t \sim N(0, \sigma^2_\epsilon) \]
The estimates of the parameters in the AR(1) processes governing $\log(e)$, $\log(x)$ and $\log(z)$ depend on whether data for deposits is used or not [i.e. on whether equations (4sid), (7sid) and (8sid) or (4sid'), (7sid') and (8sid) are used to construct $e$, $x$ and $z$].

The resulting estimates are:

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<th>$\hat{\rho}_1$</th>
<th>$\hat{\sigma}_\varepsilon$</th>
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<tbody>
<tr>
<td>With deposit data (1959.I-1999.IV)</td>
<td>0.0153</td>
<td>0.9981</td>
<td>0.0069</td>
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<tr>
<td>Without deposit data (1947.I-1999.IV)</td>
<td>0.0158</td>
<td>0.9980</td>
<td>0.0073</td>
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<table>
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<th>$\hat{\sigma}_\nu$</th>
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<td>-0.0262</td>
<td>0.9728</td>
<td>0.0160</td>
</tr>
<tr>
<td>Without deposit data (1947.I-1999.IV)</td>
<td>0.0475</td>
<td>0.9934</td>
<td>0.0191</td>
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<table>
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<th>$\hat{\sigma}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>With deposit data (1959.I-1999.IV)</td>
<td>0.0193</td>
<td>0.9973</td>
<td>0.0067</td>
</tr>
<tr>
<td>Without deposit data (1947.I-1999.IV)</td>
<td>0.0206</td>
<td>0.9974</td>
<td>0.0073</td>
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<table>
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<tr>
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<th>$\hat{\sigma}_\nu$</th>
<th>$\hat{\sigma}_\varepsilon$</th>
</tr>
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<tr>
<td>With deposit data (1959.I-1999.IV)</td>
<td>$4.7 \times 10^{-5}$</td>
<td>2.3262</td>
<td>2.3998</td>
</tr>
<tr>
<td>Without deposit data (1947.I-1999.IV)</td>
<td>$5.3 \times 10^{-5}$</td>
<td>2.6120</td>
<td>2.6123</td>
</tr>
</tbody>
</table>
Once the parameters of the model \((\beta, \delta, \alpha, \gamma, \phi, \theta, h, g, \eta, R)\) are calibrated and once the parameters of the stochastic processes governing \(\log(z)\) and \(\log(x)\) \((\rho_0, \rho_1, \sigma_\varepsilon, \varphi_0, \varphi_1, \sigma_\psi, \sigma_\varepsilon\psi)\) are estimated, the recursive competitive equilibrium is solved with the linear-quadratic method [see Cooley and Hansen (1995) or Ljungqvist and Sargent (2000), chapter 4].

The following tables provide a check on the accuracy and robustness of the solution method. The first line presents the values of the model’s main macroeconomic aggregates under its non-stochastic, stationary version (SS) which, recall, replicates U.S. data averages for the period 1959 – 1999 (independently of whether there is a binding credit constraint or no constraint at all). Lines two and three show, for the same aggregates, their non-stochastic, stationary values as inferred from the optimal, linear decision rules under the no-constraint and binding credit constraint scenarios (RuleSS and RuleSS\textsuperscript{cc}, respectively). The last two lines present, for the no-constraint and binding credit constraint scenarios (Sim and Sim\textsuperscript{cc}, respectively), the aggregates’ means across 100 simulations of the economy of 264 periods (i.e. quarters) each. In each simulation the initial state is set at its non-stochastic, stationary value. The first 100 periods of each simulation are discarded so that each simulation represents 41 years (i.e. 1959-1999).

<table>
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<tr>
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<th>(d/y)</th>
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<tr>
<td>RuleSS\textsuperscript{cc}</td>
<td>0.5918</td>
<td>0.3420</td>
<td>0.0659</td>
<td>10.1662</td>
<td>0.9402</td>
<td>0.9292</td>
<td>0.0110</td>
</tr>
<tr>
<td>Sim</td>
<td>0.5935</td>
<td>0.3404</td>
<td>0.0658</td>
<td>10.1174</td>
<td>0.9401</td>
<td>0.9301</td>
<td>0.0100</td>
</tr>
<tr>
<td>Sim\textsuperscript{cc}</td>
<td>0.5938</td>
<td>0.3404</td>
<td>0.0655</td>
<td>10.1038</td>
<td>0.9406</td>
<td>0.9296</td>
<td>0.0110</td>
</tr>
</tbody>
</table>
Prices yielded by the model depend on whether there is a binding credit constraint or no constraint at all [see equations (7)-(10) and (7cc)-(10cc) and propositions 6 and 7]. The following table reports, for the model with no credit constraint, prices under its non-stochastic, stationary version, prices in stationary state as inferred from the optimal decision rules, and average prices across the simulations:

<table>
<thead>
<tr>
<th></th>
<th>labor share</th>
<th>land share</th>
<th>capital share</th>
<th>deposit share</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS</td>
<td>0.5394</td>
<td>0.0130</td>
<td>0.3811</td>
<td>0.0664</td>
</tr>
<tr>
<td>RuleSS</td>
<td>0.5394</td>
<td>0.0130</td>
<td>0.3811</td>
<td>0.0664</td>
</tr>
<tr>
<td>RuleSScc</td>
<td>0.5400</td>
<td>0.0131</td>
<td>0.3813</td>
<td>0.0662</td>
</tr>
<tr>
<td>Sim</td>
<td>0.5390</td>
<td>0.0130</td>
<td>0.3809</td>
<td>0.0661</td>
</tr>
<tr>
<td>Simcc</td>
<td>0.5396</td>
<td>0.0132</td>
<td>0.3814</td>
<td>0.0658</td>
</tr>
</tbody>
</table>

The following table reports, for the model with a binding credit constraint, prices under its non-stochastic, stationary version, prices in stationary state as inferred from the optimal decision rules, and average prices across the simulations:

<table>
<thead>
<tr>
<th></th>
<th>w</th>
<th>r</th>
<th>s</th>
<th>q</th>
<th>ρ</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS</td>
<td>1.8432</td>
<td>0.0375</td>
<td>0.0260</td>
<td>6.8680</td>
<td>0.3916</td>
</tr>
<tr>
<td>RuleSS</td>
<td>1.8432</td>
<td>0.0375</td>
<td>0.0260</td>
<td>6.8980</td>
<td>0.3916</td>
</tr>
<tr>
<td>Sim</td>
<td>1.8663</td>
<td>0.0377</td>
<td>0.0264</td>
<td>6.9212</td>
<td>0.4025</td>
</tr>
</tbody>
</table>

The following table reports, for the model with a binding credit constraint, prices under its non-stochastic, stationary version, prices in stationary state as inferred from the optimal decision rules, and average prices across the simulations:

<table>
<thead>
<tr>
<th></th>
<th>w</th>
<th>r</th>
<th>s</th>
<th>q</th>
<th>ρ</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS</td>
<td>1.7289</td>
<td>0.0375</td>
<td>0.0244</td>
<td>6.4421</td>
<td>0.3845</td>
</tr>
<tr>
<td>RuleSScc</td>
<td>1.7292</td>
<td>0.0375</td>
<td>0.0245</td>
<td>6.3784</td>
<td>0.3740</td>
</tr>
<tr>
<td>Simcc</td>
<td>1.7501</td>
<td>0.0378</td>
<td>0.0250</td>
<td>6.4243</td>
<td>0.3770</td>
</tr>
</tbody>
</table>

It can be seen from the previous tables that the solution method is robust to the analytical version of the model. Note that under both versions of the model the loan rate (ρ) is unrealistically high (40%).
Graph 11: Response of $s$

Graph 12: Response of $\rho$
Graph 19: Response of N1

Graph 20: Response of N2
Graph 23: Response of $s$

Graph 24: Response of $\rho$