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Optimal Policies and the Informal Sector¹

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Abstract

This paper characterizes optimal policies in the presence of tax evasion and undocumented workers. Equilibrium can be characterized as segmented or non-segmented, depending on whether domestic workers work exclusively in the formal sector (segmented) or also in the informal sector (non-segmented). Surprisingly, in equilibrium, wages are always equalized between domestic and undocumented workers, even if they do not work in the same sectors of the economy. This is driven by the interaction of firm level decisions with optimal government policy. We also find that enforcement may not always be decreasing in its cost, and that governments will optimally enforce segmentation if enforcement costs are not too high.

Key Words: Informal Labour Market; Enforcement; Undocumented Workers; Public Good Provision

JEL: H32, H26, K42

1 Introduction

The informal economy affects not only the size and scale of productive output, but also optimal government policy. This sector arises for a variety of reasons: perhaps primarily, as a source of employment for undocumented workers and as a method of evading taxes for employers. These two motivations have been studied independently; in this paper, we look at them jointly and find that illegal immigration has a large impact on the nature of optimal tax and enforcement policy, and interacts with standard tax evasion incentives, playing an important role not only in the determination of equilibrium wages, but also in the organization of production across the formal and informal sectors.

In developed countries, illegal immigration, tax evasion, and the informal economy are of sufficient importance to impact on the performance of the economy. Even conservative estimates suggest these phenomena are large and economically significant. According to a report of the Pew Hispanic Center, the number of illegal immigrants living in the United States was 11.9 million in March 2008, of which 8.3 million participated in the U.S. labor force (Passel and Cohn, 2009). These numbers imply that unauthorized immigrants are 4% of the U.S. population and no less than 5.4% of its workforce. Estimates of the number of illegal immigrants in Canada by police and immigration personnel range between 50,000 and 200,000 according to the Canadian Encyclopedia.¹ Given estimates of this size, it is not surprising that immigration policy is the focus of much public debate. Tax evasion by individuals is also an important phenomenon.² For example, Slemrod and Yitzhaki (2002) report that in the United States, according to the Internal Revenue Service, 17% of personal income tax liabilities were simply not paid in 1992. Finally, while measuring the size of the informal sector is notoriously difficult, Schneider and Enste (2000) provide estimates for a large number of countries. According to their estimates for the early nineties, the smallest informal sectors (8-10% of the economy) were in Austria, Switzerland, and the United States. At the other extreme were some developing countries where the informal sector represented 68-76% of the economy (e.g. Egypt, Nigeria, Thailand, Tunisia). As for Canada, its informal sector ranges between 10-13.5% of its economy.

The theoretical literature on each of the above phenomena is large but somewhat segmented in that it tends to address each of them separately. For example, the tax

¹See the article on *Immigration Policy* at www.thecanadianencyclopedia.com.

²The evidence of tax evasion by firms is very limited.

evasion literature is not really concerned with the informal sector or illegal immigration. Initiated by Allingham and Sandmo (1972) and surveyed by Slemrod and Yitzhaki (2002), the tax evasion literature mainly focuses on the decision by individuals, otherwise perfectly honest, to conceal a portion of their income from the tax authorities. Following Reinganum and Wilde (1985), an important secondary strand of that literature characterizes the optimal auditing policies of a tax authority facing individuals behaving à la Allingham and Sandmo (1972). As for the literature on illegal immigration, Ethier (1986) initiated it by studying the impact of illegal immigration on the host country, while Bond and Chen (1987) enriched Ethier's model by adding a second country and capital mobility to examine the welfare effects of firm level enforcement. It is probably fair to say that a significant portion of the literature that followed these papers focuses on the impact of illegal immigrants on the well-being of domestic workers,³ and that tax evasion was not a primary issue of concern for those working in this area. Finally, there is a theoretical literature on the informal sector. For example, Rauch (1991), Fortin *et al.* (1997), Fugazza and Jacques (2003), and de Paula and Scheinkman (2007), all model the choice of entrepreneurs to operate in the legal or the informal sector, based on factors like scale economies, wage regulations and taxes. However, this literature is not concerned with the presence of illegal immigrants despite the fact that by their very presence, they may affect this choice.

Few models have integrated the above three phenomena despite the fact that there are obvious connections between them, and no paper that we are aware of has looked at optimal policy in this context. The presence of undocumented workers reduces the cost to firms of entering the informal sector, relative wages affect the incentives of documented or domestic workers to work in either sector, and the willingness of firms to move into the informal sector reduces the capacity of the state to raise tax revenue and fund public goods. In this paper, we allow for all of these channels. The starkest result that we find is that wages are always equalized across the formal and informal sectors (except of course in the presence of a binding minimum wage in the formal sector). This is even the case when the labour market equilibrium is characterized as being segmented, where domestic workers only work in the formal sector and undocumented workers work in the informal sector.⁴ This is due to the

³To date, there is no consensus on the empirical impact of immigration (legal/illegal) on the native population (employment, wages). See Borjas (1999) and Borjas, Grogger and Hanson (2008).

⁴Djajic (1997) has a related model and considers a segmented equilibrium, but in that model, wages of the two sectors are disconnected in a segmented equilibrium.

fact that in a segmented equilibrium wages are determined by the combination of firm decisions and optimal policy. For this reason, in our model, domestic workers always prefer to have fewer undocumented workers. However, total welfare is increasing in the number of undocumented workers. We find that the public good is under-provided relative to the first best, since enforcement is costly and thereby distorts public good provision.

Enforcement and taxes interact in somewhat subtle ways. For example, in contrast to standard findings in the literatures on crime and tax evasion,⁵ optimal enforcement may not be always be decreasing in its cost. In a segmented equilibrium, optimal enforcement and taxes are complementary policies and since enforcement is costly in terms of the public good, when the marginal value of the public good is high, increasing the cost of the enforcement may actually lead the government to want to increase enforcement to maintain public good provision. When the marginal benefit is small, increasing the cost of enforcement leads to a reduction in optimal enforcement. We also find that increasing the number of undocumented workers increases the cost of public good provision. This is because more undocumented workers reduce informal wages, making it more difficult to raise tax revenue in the legal sector. As a consequence, if society places insufficient weight on these undocumented workers' consumption of the public good, optimal public good provision will fall as the total population increases.

In a non-segmented equilibrium when domestic workers choose to work in both the formal and informal sectors, the size of the formal sector is decreasing in the number of informal workers. Although, market segmentation maximizes the size of the tax base, if the cost of enforcement is too high, it will not be socially optimal to ensure all domestic workers stay in the legal sector. For these cost ranges, optimal policies will enforce a non-segmented equilibrium.

We extend our base model in several important ways. We consider what happens when the number of undocumented workers or the level of illegal immigration is endogenous. We find similar results, but also an additional interesting feature, the government can use the level of public provision directly as a way of altering the number of illegal migrants and as a lever on the illegal wage. This is due to the fact that the migration decision results from comparing source country utility with the destination country utility. Increasing the public good provided makes a destination country more attractive, which encourages migration. The arbitrage condition then

⁵See, for example, Kaplow (1990) and Becker (1968).

implies that informal sector wages must fall. We are unaware of other papers that formally model this mechanism – that reducing the public good can substitute for increased enforcement.

We also consider what happens when firms in the formal sector are obligated to pay a minimum wage to workers. A minimum wage breaks the arbitrage condition linking formal and informal sector wages. It also increases the cost of operating in the formal sector, and so it can reduce the ability of the government to collect taxes. Consequently, the presence of a minimum wage strengthens the need for enforcement, and makes a segmented equilibrium suboptimal. The existence of the informal sector reduces the unemployment cost caused by job rationing, and the presence of undocumented workers may increase the responsiveness of sector size to changes in enforcement. We also consider amnesties for undocumented workers, and find that at the margin, they are socially beneficial. Lastly, we endogenize output price and allow for taxes to be levied on workers to ensure that our findings are robust.

To our knowledge, there are only two papers similar to ours in the literature. In Djajic (1997), the incentive of firms to hire illegal immigrants arises because of a wage differential between the legal and the informal sectors, but not from the obligation to pay taxes when hiring domestic workers in the legal sector. In our model, firms may be tempted by the informal sector because of a wage differential but also because they want to evade taxes.⁶ Also, Djajic provides a positive analysis of the impact of government policies (enforcement, increase in the stock of illegal workers) but he does not characterize optimal policies. Epstein and Heizler (2007) construct a partial equilibrium model in which a representative firm may hire domestic workers and/or illegal immigrants. This is in contrast with our general equilibrium model in which firms cannot simultaneously hire both types of workers. Epstein and Heizler (2007) also do not tax firms to provide a public good; while in our model, tax evasion incentives play a central role. Like us, they perform an analysis of optimal enforcement policies.

The paper is organized as follows: Section 2 presents the basic version of the model, Section 3 characterizes optimal policies, and Section 4 examines the extensions by incorporating the minimum wage, endogenous migration, amnesties, endogenous price, and worker taxation. Lastly, Section 5 concludes.

⁶There are taxes in Djajic (1997), but they are paid by employees. We discuss the implications of employee taxes in our model in the extension section.

2 The Model

We model a simple economy in which firms may choose to operate in either an informal sector to evade taxes, or to operate in a formal and regulated sector. There are M domestic workers who can work either in the formal (legal) sector (M_L) or the informal sector (M_I) where $M_L + M_I = M$. There are also U undocumented workers who can only work in the informal sector where $M > U$. Each domestic and undocumented worker, if hired, supplies one unit of labour inelastically. Domestic workers choose to work in the sector offering the highest wage.

The economy has N entrepreneurs with varying productivity θ . For simplicity, we assume that productivity is uniformly distributed on $[0, 1]$. Each entrepreneur has an income endowment k that can be consumed or invested to start-up a firm. The number of firms in the economy is endogenous and depends on government policies. Entrepreneurs can choose to not operate a firm (N_0 make that choice), to start-up a firm in the formal sector (N_L make that choice) or to start-up a firm in the informal sector (N_I make that choice) so $N_0 + N_L + N_I = N$. Both informal and formal sector firms produce the same good X , which is sold at an exogenous price P , which we normalize to one.⁷ To produce output, entrepreneurs need to hire one worker and, to guarantee full employment, we assume that $N > M + U$.⁸

All $M + N + U$ individuals have the identical utility function $x + v(G)$, with $v' > 0 > v''$ and $v'(0) \rightarrow \infty$, where x is consumption of a private good and G is the amount of a public good provided by the government.

An entrepreneur who does not start a firm can consume his endowment and obtain $EU_0(\theta) = k + v(G)$. Operating in the formal sector requires k to be invested.⁹ A formal firm produces θ units of the domestic good X , and pays the formal wage w_L to its worker and the tax t to the government.¹⁰ This yields expected utility in the

⁷Later, we discuss the implications of making the price endogenous.

⁸This assumption is for simplicity and without consequences if we interpret “one worker” as a mass of workers of a size larger than one, and M and U as the number of such masses of workers. For example, if a mass of workers was of size equal to 10, then the inequality would be satisfied with $N = 100$ and $M + U = 90$, and the economy would be populated with 100 entrepreneurs and 900 workers.

⁹The start-up investment costs could also be some positive fraction of k — with the remainder of k being consumed by the entrepreneur operating the firm — without changing our results.

¹⁰We assume that θ is unobservable. If it were observable, the government would use this infor-

formal sector of

$$EU_L(\theta) = P\theta - w_L - t + v(G). \quad (1)$$

An informal firm has the same investment costs and output as a legal firm but must pay the informal market wage w_I , and incur any sanction imposed in expectation $e\theta S$.¹¹ This expected sanction can be decomposed into two parts, the probability of detection $e\theta$, where e is the amount of public enforcement and where we have assumed that larger firms are more likely to be detected, and a non-monetary positive sanction S . This yields expected utility in the informal sector of

$$EU_I(\theta) = P\theta - w_I - e\theta S. \quad (2)$$

The government levies taxes on the formal sector to finance the provision of a pure public good G available to all residents and it may invest in costly enforcement to reduce tax evasion. The cost of a unit of the public good is unity. The cost of enforcement is simply given by $C(e) = c \cdot e$. Therefore, the government budget constraint is

$$G + ce = N_L t. \quad (3)$$

where N_L is the number of firms (entrepreneurs) operating in the formal sector and paying taxes (endogenized below).

Because sanctions are assumed to be non-monetary, they do not add to government revenue. Alternatively, sanctions could have been assumed to be monetary, but costly to collect. In the current case, monetary sanctions would be fully dissipated by administrative costs.¹² Further, note that sanctions are assumed exogenous and finite — non-maximal — so that some entrepreneurs (and workers) choose to be active in the informal sector. Obviously, if sanctions were increased without bounds, then it would be possible to completely eliminate the informal sector as shown in Becker (1968).

We have assumed that by incurring total costs $C(e) = c \cdot e$, the government can detect each firm, with a given θ , operating in the informal sector with probability $e\theta$. Such an assumption is consistent with an audit environment in which the government

mation to directly target lower taxes to firms on the formal-informal sector margin.

¹¹Informal firms could also be less efficient producers as in Fortin *et al.* (1997). This would affect the marginal decision between the sectors, as expected, but our main results would not be affected.

¹²Costly collection of monetary sanctions has long been recognized in the literature. For example, see Polinsky and Shavell (2007). Note that Djajic (2007) makes an assumption equivalent to ours.

cannot distinguish between the N entrepreneurs, and so audits all entrepreneurs. These audits are of course without consequences for those who have either chosen to operate in the formal sector or to remain idle. We could have also assumed that the cost of public enforcement is increasing in the size of the population being audited. For example, $C(e) = \bar{c}(N) \cdot e$ with $\bar{c}'(N) > 0$. With N fixed, our framework is consistent with such an audit environment.¹³

2.1 Entrepreneurs' Decision

Now, consider the optimal decisions of entrepreneurs. Given wages and government policies, entrepreneurs will decide whether or not to start a firm, and if they start a firm, which type. We restrict attention to the case where at least some entrepreneurs want to start formal firms.¹⁴ Note that the slope of the expected utility function (with respect to entrepreneur's productive ability) is 1 in the formal sector, and $1 - eS$ in the informal sector. The ability level $\hat{\theta}$ that makes an entrepreneur indifferent between starting a firm in the formal sector and starting a firm in the informal sector is determined from the intersection of the two expected utility functions. Because the relative cost of operating in the informal sector is increasing in θ , all entrepreneurs with $\theta > \hat{\theta}$ prefer to operate in the formal sector, while all entrepreneurs with $\theta < \hat{\theta}$ prefer operating in the informal sector where

$$\hat{\theta} = \frac{w_L - w_I + t}{eS}. \quad (4)$$

Analogously, we define $\bar{\theta}$ as the ability that makes an entrepreneur indifferent between starting a firm in the informal sector and not starting a firm at all. Since utilities are increasing in productivity, all entrepreneurs with $\theta > \bar{\theta}$ prefer starting a firm, while entrepreneurs with $\theta < \bar{\theta}$ prefer to not start a firm where

$$\bar{\theta} = \frac{w_I + k}{1 - eS}. \quad (5)$$

Since $k > 0$, the least productive entrepreneur ($\theta = 0$) never starts a firm. Consequently, $\bar{\theta} > 0$.

¹³There are of course analyses in which the probability of detection depends directly on the size of the informal/criminal sector or on the number of criminals — e.g. Freeman *et al.* (1996). Generally such an assumption leads to multiple equilibria, a situation we have chosen to avoid.

¹⁴Given our restrictions on $v(G)$, it will never be optimal for government policy to foreclose this sector completely.

With $\hat{\theta} > \bar{\theta}$ (as in Figure 1), there will be an informal sector. Entrepreneurs below $\bar{\theta}$ do not start-up a firm, those between $\bar{\theta}$ and $\hat{\theta}$ start a firm in the informal sector, and those above $\hat{\theta}$ start a firm in the formal sector. In this situation, formal sector labour demand will be given by $N_L = N(1 - \hat{\theta})$ and informal sector labour demand will be given by $N_I = N(\hat{\theta} - \bar{\theta})$.

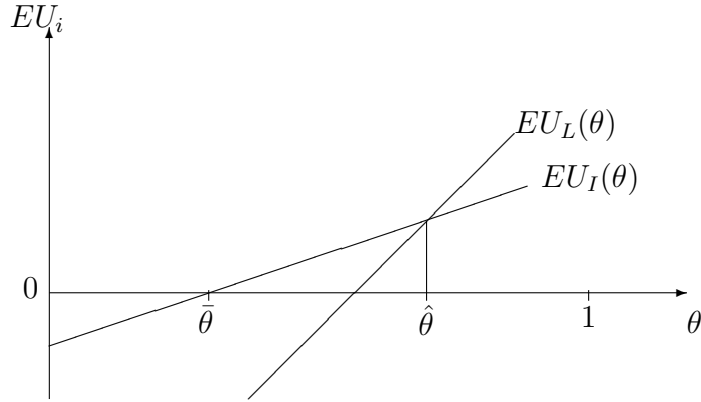


Figure 1: Expected Utility in the Formal and Informal Sectors ($\hat{\theta} > \bar{\theta}$).

2.2 Equilibrium

To close the model, we need: (1) labour demand to equal labour supply in both sectors, and (2) a wage arbitrage condition equating wages across the sectors to hold when some domestic workers work in the informal sector. When only undocumented workers work informally such a condition does not need to be satisfied, as undocumented workers cannot work in the formal sector. The market will clear in this case as long as the formal wage is at least as large as the informal wage so that all domestic workers prefer to work in the formal sector. Consequently, the equilibrium will take one of two forms.

In the first type of equilibrium, **labour markets are segmented** and no domestic workers are employed in the informal sector so $M_L = M$ and $M_I = 0$. In the second type of equilibrium, **labour markets are not segmented** and some domestic

workers choose to work in the informal sector so $M_I > 0$. The type of equilibrium obtained depends on government policy and so will be endogenous.

With a flexible informal wage, the supply of workers in the informal sector must equal the demand for workers by informal firms:

$$M_I + U = N(\hat{\theta} - \bar{\theta}). \quad (6)$$

With a flexible formal wage, the supply of domestic workers must equal the demand for workers by formal firms:

$$M_L = N(1 - \hat{\theta}). \quad (7)$$

From (6) and (7), and using $M = M_I + M_L$ we find that $\bar{\theta}^* = 1 - m - u$, where $m = M/N$ and $u = U/N$. Therefore, in any equilibrium there is full employment of undocumented and domestic workers.

Using this full employment condition, together with the definition of $\bar{\theta}$ given by (5) we can solve for the wage in the informal sector as a function of government policies. In any equilibrium, the wage in the informal sector is given by

$$w_I = (1 - eS)(1 - m - u) - k, \quad (8)$$

and is decreasing in the amount of enforcement.

The full employment condition also implies that all entrepreneurs with productivity θ at least as large as $\bar{\theta}^* = 1 - m - u$ will start up a firm in either the formal or informal sector, and produce θ . Therefore, total output in the economy is given by

$$N \int_{1-m-u}^1 \theta d\theta, \quad (9)$$

and is independent of taxes and enforcement. In other words, the size of the economy is fixed for given populations of domestic and undocumented workers. Therefore, the government's problem is a distributional one. Its policy choices will only affect the distribution of total output in the economy between public and private goods, and across different individuals in the economy (entrepreneurs, domestic workers and undocumented workers).

2.3 Welfare

The government has a utilitarian objective and cares about all individuals in the economy, including, to a perhaps lesser degree, undocumented workers assigning them

a welfare weight $\alpha \in [0, 1]$. For the time being, we will restrict government policy to be of only three dimensions: a tax on firms, a level of public good, and a level of enforcement. One could also imagine that the government may have some choice over U , perhaps through a choice of border policy. In the extension section, we consider in a very simple way how optimal policy changes when U is endogenous and briefly discuss the effects of other government instruments such as a tax on workers. We begin by defining the government's objective function.

Definition 1 *Total weighted welfare is given by:*

$$N \int_0^{1-m-u} EU_0(\theta)d\theta + N \int_{1-m-u}^{\hat{\theta}} EU_I(\theta)d\theta + N \int_{\hat{\theta}}^1 EU_L(\theta)d\theta \\ + M_L [w_L + v(G)] + (M_I + \alpha U) [w_I + v(G)].$$

The first three terms are the sum of utilities of the entrepreneurs who do not start-up firms, start up firms in the informal sector and start-up firms in the formal sector, respectively. The second last term is total utility for domestic workers employed in the formal sector and the last term is total utility for workers (with weight $\alpha \in [0, 1]$ on undocumented workers) employed in the informal sector. Using the expressions for entrepreneurs' expected utilities, the informal sector wage, and the labour market clearing conditions, total weighted welfare can be written as a function of government policies, $\Omega(t, e, G; \alpha)$, where¹⁵

$$\Omega(t, e, G; \alpha) = N \int_{1-m-u}^1 \theta d\theta + [N - M - U]k - N \int_{1-m-u}^{\hat{\theta}} e\theta S d\theta - N \int_{\hat{\theta}}^1 t d\theta \\ - (1 - \alpha)U [(1 - eS)(1 - m - u) - k] + (M + N + \alpha U)v(G).$$

Any wage paid by entrepreneurs is received by workers. Consequently, terms involving wages have no net effect on total welfare if all workers are counted equally. If the welfare of undocumented workers is discounted ($\alpha < 1$), then the welfare loss to entrepreneurs having to pay the informal wage is greater than the welfare gain to undocumented workers from receiving the informal wages. Consequently, there will be a welfare effect of enforcement policies through changes in the informal wage if $\alpha \neq 1$.

¹⁵Detailed derivation of this expression is given in the Appendix.

3 Optimal Policies

We first characterize optimal policies in the segmented equilibrium and then turn to the non-segmented equilibrium. All proofs are gathered in the Appendix.

3.1 Segmented Equilibrium

In a segmented equilibrium, all domestic workers choose to work in the formal sector, so $M_L = M$ and from (7), we obtain $\hat{\theta}^* = 1 - m$. Using the definition of $\hat{\theta}$ given by (4), we can solve for the equilibrium wage in the formal sector. In a segmented equilibrium, the formal wage is given by

$$w_L = (1 - m - u) + ueS - k - t, \quad (10)$$

and is increasing in public enforcement and decreasing in the tax rate.

To guarantee this equilibrium sorting of domestic workers, it must be that $w_L \geq w_I$, or using (8) and (10),

$$e \geq \frac{t}{(1 - m)S}. \quad (11)$$

The government maximizes its weighted utilitarian welfare function $\Omega(t, e, G; \alpha)$ subject to its budget constraint (3) and the constraint (11) where δ is the Lagrange multiplier on the latter constraint. The first order conditions on t and e are

$$[M + N + \alpha U]v'(G)M - M - \delta \frac{1}{(1 - m)S} = 0, \quad (12)$$

$$-[M + N + \alpha U]v'(G)c - SN \int_{1-m-u}^{1-m} \theta d\theta + (1 - \alpha)US(1 - m - u) + \delta = 0, \quad (13)$$

where $G = Mt - ce$ since in a segmented equilibrium, all domestic workers are employed in the formal sector ($N_L = M$).

Public enforcement is socially costly for two reasons. First, when $c > 0$ monitoring uses up government resources and reduces the amount of public good that can be provided. Second, firms operating in the informal sector face some expected sanction which is socially costly. When $\alpha < 1$ there is also a social benefit of enforcement. Increased enforcement reduces the informal wage which, when $\alpha < 1$, results in a net social benefit since the social gain to informal firms having to pay lower wages

is greater than the social loss of lower wages to the undocumented workers. It turns out that for any α , the social cost of informal firms of having to incur expected sanctions will always be greater than the potential social benefit through changes in the informal wage.¹⁶ This implies that the constraint (11) will be binding and $\delta^* > 0$. Optimal policies will be chosen such that $t = (1 - m)eS$.

We now have our first Proposition.

Proposition 1 *In a segmented equilibrium, the wages in both the formal and informal sectors are the same and given by $w_L^* = w_I^* = (1 - e^*S)(1 - m - u) - k$ where e^* is the optimal level of public enforcement.*

Equilibrium wages are decreasing in public enforcement, the number of domestic workers, and the number of undocumented workers (all else equal). If public enforcement was held fixed, as in Djajic (1997), then formal wages would be decreasing in the number of undocumented workers as in his paper. Here, policies are chosen by the government and the optimal policies will depend on the number of undocumented workers. Further, the government by its choice of enforcement removes any incentive for firms operating in the formal sector to switch to the informal sector in order to pay a lower wage. Optimal enforcement policies ensure that wages are equated across the two sectors. Consequently, firms who switch to the informal sector would do so solely to evade taxes. This is in contrast to Djajic (1997) in which there may be a strictly positive net wage differential between the two sectors in a segmented equilibrium and thus firms may be attracted to the informal sector hoping to reduce their wage bill.

We now turn to the determination of optimal policies.

Using the two first-order conditions and the binding constraint, the optimal provision of public good is determined by the following modified Samuelson condition:

$$[N + M + \alpha U]v'(G) = \left[\frac{1 + \frac{1}{M(1-m)} \left(N \int_{1-m-u}^{1-m} \theta d\theta - (1 - \alpha)U(1 - m - u) \right)}{1 - \frac{1}{M(1-m)} \frac{c}{S}} \right]. \quad (14)$$

From this modified Samuelson condition, we have the following Proposition:

¹⁶This follows from noting that the sum of the second and third term in (13) is always negative for any value of α . See Appendix for details.

Proposition 2 *In a segmented equilibrium, the optimal public good provision is a) lower than in a first-best outcome, b) decreasing in the marginal enforcement cost, and c) decreasing in the number of undocumented workers when $\alpha = 0$.*

Efficiency requires that the sum of the marginal benefits from consumption of the public good equals the marginal cost of provision. Lump-sum taxes and costless enforcement would guarantee efficiency as the sum of the marginal benefits would be equated to 1. With costly public enforcement, $c > 0$, the denominator of (14) is less than one and given enforcement imposes a net social cost, the numerator is greater than one. Therefore, the marginal social cost of providing the public good will be greater than 1 and public good provision in the segmented equilibrium is therefore lower than in the first-best. Further, an increase in the marginal enforcement costs c increases the social marginal cost of providing the public good. Therefore, for the modified Samuelson condition (14) to continue to hold when public enforcement becomes more costly, the optimal amount of G being provided must go down.

When there is an increase in the number of undocumented workers in a segmented equilibrium more entrepreneurs will necessarily operate in the informal sector. Consequently more firms will face the expected sanction. Therefore, the social cost of enforcement goes up. Given the constraint is optimally binding, any increase in taxes (public good) will necessarily be accompanied by an increase in enforcement. Therefore this increase in the social cost of enforcement also increases the social cost of providing the public good. When there is zero welfare weight on undocumented workers, the increase in the number of undocumented workers has no effect on the sum of the social marginal benefit of the public good. Consequently, the amount of public good being provided will go down as U increases when $\alpha = 0$. When $\alpha > 0$, then there is also a social benefit to an increase in the number of undocumented workers and it is not clear what will happen to public good provision in this case. Likewise, an increase in α increases the social cost of providing the public good but also increases the marginal benefit of the public good. Therefore, the effect of α on optimal policies will be ambiguous.

Proposition 2, together with the government's budget constraint and the binding constraint, $G^* = t^*M - ce^* = [M(1 - m)S - c]e^*$, implies that when $\alpha = 0$ an increase in the number of undocumented workers will also *reduce* the optimal amount of enforcement. Proposition 2, however, does not necessarily imply that optimal enforcement decreases with the marginal cost of public enforcement c . From Proposition

2, we have

$$\frac{dG^*}{dc} = [M(1 - m)S - c] \frac{\partial e^*}{\partial c} - e^* < 0. \quad (15)$$

Therefore, $\partial e^*/\partial c$ could be positive and (15) could still be satisfied. Intuitively, the government could compensate for the diminution of available resources by increasing taxes and enforcement, but overall the direct strain on resources must dominate so public good provision must go down. We have the following Proposition:

Proposition 3 *In a segmented equilibrium, optimal enforcement is a) increasing (decreasing) in the marginal enforcement cost c when the absolute value of the elasticity of the marginal benefit of the public good is greater than (less than) unity, and b) decreasing in the number of undocumented workers when $\alpha = 0$.*

With an increase in the marginal cost of enforcement, the government has an incentive to reduce the amount of enforcement. At the same time, for any given enforcement level this increase in cost uses up more resources and so reduces the amount of public good the government can provide. This implies that the marginal benefit of providing the public good is also higher, and one way to increase public good provision is to increase compliance by increasing enforcement. If the absolute value of the elasticity of the marginal benefit is large, then a small decrease in G (via an increase in c) results in a large reduction in the marginal benefit of the public good. Consequently, the government will optimally want to increase enforcement to push back up the amount of public good provided. The converse is then true.

To our knowledge, this result that optimal public enforcement may increase with an increase in the marginal cost of enforcement is original. Other papers have considered a framework with tax evasion and public good provision (e.g. Kaplow 1990, Cremer and Gahvari 2000), but their focus was different and the phenomenon we have identified was not uncovered.¹⁷

To summarize, the government will optimally choose taxes and public enforcement such that the constraint ensuring domestic workers are at least as well off working

¹⁷Kaplow (1990) is more interested in optimal tax formulae (à la Ramsey) in the presence of tax evasion and enforcement, so his focus is on the relative size of various consumption taxes rather than on the tradeoff between public enforcement and the overall level of taxes. As for Cremer and Gahvari (2000), they are interested in tax competition and fiscal harmonization in the presence of tax evasion and do not focus on the characterization of optimal enforcement in a simple one country/government model.

in the formal than in the informal sector is just binding. Consequently, equilibrium wages in the formal and informal sectors will be the same. Second, any change in enforcement policy will optimally be accompanied by a change in tax policy in the same direction, that is, taxes and public enforcement are complementary policies in a segmented equilibrium. Third, the size of the formal sector in the segmented equilibrium will be fixed. Consequently, the ratio of tax rate to public enforcement will be independent of marginal enforcement costs, the welfare weight on the undocumented workers and the number of undocumented workers. It also follows from the government's budget constraint that the ratio of public good provision to public enforcement will also be independent of α and U , but that an increase in the marginal enforcement costs reduces public good provision relative to enforcement. We are also able to say something about what happens to equilibrium wages only under some circumstances.¹⁸ Of course what happens to workers' utility will also depend on changes in optimal public good provision which can move in the opposite direction.

3.2 Non-Segmented Equilibrium

In a non-segmented labour market, some domestic workers choose to work in the informal sector, $M_I = M - M_L > 0$. Formal and informal wages must be equal in equilibrium as domestic workers are willing to take jobs in either sector. Equilibrium wages are pinned down by the least profitable firm in the informal sector and from (8) are given by:

$$w_I^* = w_L^* = [1 - eS](1 - m - u) - k. \quad (16)$$

Equilibrium wages in both sector are independent of the tax rate, but are a decreasing function of public enforcement. These wages are the same as in the segmented equilibrium.

Given the equality of wages in both sectors, it follows from the definition of $\hat{\theta}$ given by (4), that in equilibrium

$$\hat{\theta}^* = \frac{t}{eS}. \quad (17)$$

Differentiating (17), we obtain

¹⁸For example, equilibrium wages will be decreasing (increasing) in the marginal enforcement cost if optimal enforcement is increasing (decreasing) in c .

$$\frac{\partial \hat{\theta}^*}{\partial t} = \frac{1}{eS} > 0, \quad \frac{\partial \hat{\theta}^*}{\partial e} = -\frac{t}{e^2 S} < 0. \quad (18)$$

Recall there is full employment and $\bar{\theta}^* = 1 - m - u$.

The government maximizes the weighted utilitarian welfare function $\Omega(t, e, G; \alpha)$ subject to its budget constraint (3) and to a constraint that ensures that the number of firms in the legal sector is no larger than the number of domestic workers, that is, $\hat{\theta}^* \geq 1 - m$ or using (17)

$$e \leq \frac{t}{(1 - m)S}. \quad (19)$$

The above constraint will provide a simple way to differentiate between the two types of equilibria. When it is binding, the equilibrium is segmented as can be seen from the discussion above, and if it is slack, the equilibrium is non-segmented. Let ϕ be the Lagrange multiplier on the constraint, the first order conditions on t and e are given by:

$$[M + N + \alpha U]v'(G) \left(N(1 - \hat{\theta}^*) - Nt \frac{\partial \hat{\theta}^*}{\partial t} \right) - N(1 - \hat{\theta}^*) - \phi \frac{1}{(1 - m)S} = 0, \quad (20)$$

$$[M + N + \alpha U]v'(G) \left[-Nt \frac{\partial \hat{\theta}^*}{\partial e} - c \right] - SN \int_{\bar{\theta}^*}^{\hat{\theta}^*} \theta d\theta + (1 - \alpha)US(1 - m - u) + \phi = 0. \quad (21)$$

To properly analyze these first order conditions, two separate issues need to be investigated. First we need to know if the constraint is binding or not. Intuitively, we need to know if the government optimally chooses policies so that the labour markets are segmented or not segmented. The second issue is to describe what the tax and enforcement policies look like given we are in a non-segmented case. We begin with this latter issue and assume that $\phi = 0$.¹⁹ Given this, we have the following Proposition:

Proposition 4 *In a non-segmented equilibrium, the optimal size of the formal sector is decreasing in the number of undocumented workers, in the marginal cost of enforcement, and in the welfare weight put on undocumented workers.*

¹⁹Note that we assume that the term $N(1 - \hat{\theta}^*) - Nt \partial \hat{\theta}^* / \partial t$ in equation (20) is strictly positive, reflecting the fact that the government chooses a tax rate on the left-hand side of the Laffer curve.

As the number of undocumented workers increases, the net social cost of using public enforcement increases. There are more firms in the informal sector and they are all paying the expected sanction. There is a social gain via the reduction in the informal wage but this gain is always less than the social cost of informal firms facing an expected sanction. Therefore, an increase in U gives the government an incentive to reduce public enforcement (relative to taxes) so $\hat{\theta}^*$ moves up and the size of the formal sector shrinks. An increase in c increases the resource cost of using public enforcement to keep firms in the formal sector. Consequently, the government will optimally reduce enforcement relative to taxes when the marginal cost of enforcement increases. Finally, an increase in the welfare weight on undocumented workers reduces the social gain from using public enforcement via the reduction in the informal wage. Once again, the government has an incentive to reduce enforcement relative to taxes and the size of the formal sector shrinks with an increase in α .

Unlike in a segmented equilibrium, the ratio of taxes to public enforcement is not pinned down. Consequently, optimal public enforcement and taxes are no longer complementary policies. Further, an increase in the number of undocumented workers increases not just the size of the informal sector (as in a segmented equilibrium) but also the size of the formal sector through its effect on optimal policies. The ratio of public good provision to enforcement now depends on the welfare weight put on undocumented workers and the number of undocumented workers. To see this, we can write the government's budget constraint using (17) as $G^* = N(1 - \hat{\theta}^*)t^* - ce^* = [N(1 - \hat{\theta}^*)S\hat{\theta}^* - c]e^*$ which implies that G^*/e^* is increasing in $\hat{\theta}^*$ and from Proposition 3 it follows that this ratio is increasing in α and U .

We now assess the condition under which it is optimal for the government to choose a segmented equilibrium or, equivalently, ensure that the constraint (19) is binding.²⁰ To do so we will examine the left hand side of first order condition on e just as it is about to become binding, i.e. as $t/(eS) \rightarrow 1 - m$. Since the constraint is not yet binding $\phi = 0$, but $\hat{\theta}^* \rightarrow 1 - m$. The left hand side of first order condition (21) becomes

$$[M + N + \alpha U]v'(G) [N(1 - m)^2 S - c] - SN \int_{1-m-u}^{1-m} \theta d\theta + (1 - \alpha)US(1 - m - u). \quad (22)$$

As a benchmark, consider the case in which there are no undocumented workers,

²⁰Although Djajic (1997) identifies the possibility of these two types of equilibria, there is no discussion in his paper about the optimality of either type of equilibrium nor conditions under which one type of equilibrium is socially preferred to the other.

that is, $U = 0$. In this case, all firms operating in a segmented equilibrium are operating in the formal sector. To support this type of equilibrium, the government needs to spend resources on monitoring and auditing firms. But, because there are no firms operating in the informal sector, no sanctioning is imposed and there is no social loss from enforcement. It follows from (22) that the government will optimally enforce a fully segmented labour market if and only if $c < N(1 - m)^2S$. Not surprisingly, if enforcement is too costly, it is preferable to leave some domestic workers in the informal sector. The higher the sanction, the more effective enforcement is for a given level of public enforcement e , and so a larger range of marginal enforcement costs can support a segmented equilibrium.

Now, with undocumented workers some firms operate in the informal sector even in the segmented equilibrium, and these firms face costly expected sanctions. This reduces the attractiveness of using public enforcement to maintain a segmented equilibrium. Therefore, the range of marginal enforcement costs that can support the optimality of a segmented equilibrium is smaller than when $U = 0$.

Another way to see this is to note that for a non-segmented equilibrium to occur it must be that $\hat{\theta} > 1 - m$. It also follows from (21), (16), and (18) that in a non-segmented equilibrium $\hat{\theta}^* > [c/NS]^{1/2}$ for any value of U and α . Together, these expressions imply that it is possible to have a non-segmented equilibrium for some $c < N(1 - m)^2S$. Further, when $c > N(1 - m)^2S$ a non-segmented equilibrium will necessarily be obtained. Therefore, we have the following Proposition.

Proposition 5 *A necessary condition for the government to optimally enforce a segmented equilibrium is $c < N(1 - m)^2S$, and a sufficient condition for the government to optimally enforce a non-segmented equilibrium is $c > N(1 - m)^2S$.*

4 Extensions

4.1 Amnesties

In this section, we consider the consequences of legalizing the status of some of the undocumented workers. To do this, we conduct the following exercise: marginally increase M while reducing U by the same amount. As we are considering only marginal changes in population, it is natural to assume that the economy remains in the same

type of equilibria.

First, we look at what happens to total welfare as U decreases marginally. Differentiating the maximized total welfare $\Omega^*(t, e, G; \alpha)$ with respect to U , and applying the Envelope Theorem, we obtain

$$\frac{d\Omega^*}{dU} = \alpha [w_I^* + v(G^*)] + (1 - e^*S)(1 - \alpha)u > 0. \quad (23)$$

According to the first term in (23), fewer undocumented workers directly reduces welfare, unless the welfare of undocumented workers is not valued, as it would be the case if $\alpha = 0$. A reduction in the supply of undocumented workers also has a welfare effect by affecting the informal wage. A decrease in the supply of undocumented workers leads to a higher informal wage, and because undocumented workers are weighted less than entrepreneurs (unless $\alpha = 1$), the higher informal wage is welfare reducing. This effect is captured by the second term. Overall, the above expression holds regardless of whether the economy is in a non-segmented or segmented equilibrium but of course the value of the expression will depend on the optimal policies.

Consider now the effect on total welfare of an increase in M . Since the effect of a change in M depends on whether the economy is in a segmented or in a non-segmented equilibrium, both forms of equilibria will be analyzed separately. Differentiating total welfare in the case of a non-segmented equilibrium, and applying the Envelope Theorem, we obtain

$$\frac{d\Omega^*}{dM} = [w_L^* + v(G^*)] + (1 - e^*S)(1 - \alpha)u. \quad (24)$$

The interpretation of this expression is similar to the one given above. The first term is the direct gain in welfare from one more domestic worker. The increase in the number of domestic workers also leads to a lower informal wage. For any $\alpha < 1$, entrepreneurs are valued more than undocumented workers and a reduction in the wage paid to these workers is beneficial. This effect is captured by the second term. In the case of a segmented equilibrium, there will be an additional positive term in (24) reflecting the fact that domestic workers are fully employed in the legal sector. Changing the status of an undocumented worker employed in the informal sector to a domestic worker employed in the formal sector necessarily increases public good provision by increasing the tax base and total welfare increases by an additional $[N + M + \alpha U]v'(G^*)t^*$.

To determine whether an amnesty is optimal in either type of equilibrium, we simply look at the difference between the two expressions in (23) and (24) given

$dU = -dM < 0$ and note that in either type of equilibrium the informal and formal wages will be the same. In the case of a non-segmented equilibrium, we find

$$\frac{d\Omega}{dM} - \frac{d\Omega}{dU} = (1 - \alpha)[w_L^* + v(G^*)]. \quad (25)$$

Therefore, in a non-segmented equilibrium, legalizing the status of an undocumented worker will be welfare-improving when $\alpha \neq 1$ and will be welfare-neutral when $\alpha = 1$. Because in a segmented equilibrium replacing an undocumented worker by a documented worker necessarily implies that one more firm will pay taxes, legalizing the status of an undocumented worker will be welfare-improving for all values of α . This exercise, of course, is only valid for marginal changes. Legalizing the status of a large number of undocumented workers would necessarily change the optimal policies and possibly the type of equilibria in which the economy rests.

In the positive analysis of Djajic (1997), an amnesty had no impact in a non-segmented equilibrium and decreased the wage of legal unskilled workers in a segmented equilibrium. We obtain different results for two reasons. First, because we allow for undocumented workers to be valued less, an amnesty may be beneficial even in a non-segmented equilibrium. As in Djajic (1997), granting an amnesty to undocumented workers in a non-segmented equilibrium does not change the number of legal/illegal workers. The newly documented worker will simply displace another worker in the legal sector. However, since the newly documented worker receives full weight (instead of a weight of $\alpha < 1$), welfare may increase. We admit that the difference is due to the way we constructed our welfare function, and consequently is not very surprising. The second difference is more fundamental. In Djajic (1997) where enforcement policy is exogenous, wages in the formal and the informal sector may differ in a segmented equilibrium. In this paper, we take into account how optimal policies change with the number of both domestic and undocumented workers. In equilibrium, as stated by Proposition 1, both wages are equalized, and given by $w^* = (P - e^*S)(1 - m - u) - k$. A marginal increase in M , combine with a marginal reduction in U such that $dM = -dU$ would have no effect on equilibrium wages in either type of equilibria. Note that this result would not likely hold for a “large amnesty” where the optimal policies change.

4.2 Minimum Wage

Firms, in addition to evading taxes, may also avoid other (enforced) labour market regulations such as minimum wage legislation when operating in the informal sector. To investigate the impact of this additional potential incentive to operate outside of the formal sector, we now introduce an exogenous minimum wage denoted by \bar{w} , that must be paid to all individuals working in the formal sector. Further, we assume that the penalty for evading the minimum wage and/or taxes are equivalent.²¹ We first determine how a fixed minimum wage in the formal sector affects optimal enforcement and optimal public good provision and then discuss how a minimum wage affects the condition determining whether the government will optimally enforce a segmented equilibrium.

In our framework, we assume that any documented workers who cannot find a job in the formal sector can (and will) work in the informal sector instead.²² Consequently, there will be full employment of all workers so $\bar{\theta}^* = 1 - m - u$. This implies the following: First, total production will continue to be given by (9) and will be independent of all government policies including the minimum wage. Second, the informal wage will continue to be given by (8).

With a minimum wage, the wage in the formal sector is no longer flexible and is equal to the minimum wage. Consequently, the formal sector cutoff will now be

$$\hat{\theta}^* = \frac{\bar{w} - w_I^* + t}{eS}. \quad (26)$$

Taxes and the minimum wage affect the size of the formal sector in the same manner. As taxes (or the minimum wage) increase, more entrepreneurs choose the informal sector. An increase in enforcement has the opposite effect on the size of the formal sector. Finally, the government's objective function given the minimum wage is the same as Definition 1 except with $w_L = \bar{w}$ which we denote by $\Omega(t, e, G; \alpha, \bar{w})$. We now discuss the optimal policies given the minimum wage in the two types of equilibria, starting with the segmented equilibrium.

²¹Consequently, no firm would ever choose to evade only one of these regulations. All firms in the formal sector will respect both, and all firms in the informal sector will not.

²²We have worked out the equilibria and optimal policies for the case in which workers who would like to work in the legal sector and are unlucky cannot switch *ex post* to an informal sector job. This is the case in which an excess supply in the legal sector corresponds to involuntary unemployment. It turns out that the algebra for that case is more involved but that the results are fairly similar.

In general, the introduction of a minimum wage creates excess formal labour supply, but in a segmented equilibrium, all domestic workers must be employed in the formal sector. How is this possible? The answer resides in the government having additional policy instruments that can affect demand for labour in the formal sector. The minimum wage discourages firms from operating legally, but at the same time the expected sanction discourages firms from operating informally. If e is sufficiently large relative to \bar{w} and t , excess supply can be eliminated so that $M_L = M$, and $\hat{\theta}^* = 1 - m$. A government who wants all domestic workers to be employed in the formal sector can therefore obtain the desired result by ensuring that

$$e \geq \frac{\bar{w} - w_I^* + t}{(1 - m)S}. \quad (27)$$

The government maximizes $\Omega(t, e, G; \alpha, \bar{w})$ subject its budget constraint (3) and the constraint (27). The first order conditions on t and e are given by

$$[N + M + \alpha U]v'(G) \left[(1 - \hat{\theta}^*) - t \frac{\partial \hat{\theta}^*}{\partial t} \right] - (1 - \hat{\theta}^*) - (\bar{w} - w_I^*) \frac{\partial \hat{\theta}^*}{\partial t} - \bar{\phi} \frac{1}{N(1 - m)S} = 0, \quad (28)$$

$$\begin{aligned} & [N + M + \alpha U]v'(G) \left[-t \frac{\partial \hat{\theta}^*}{\partial e} - \frac{c}{N} \right] - (\bar{w} - w_I^*) \frac{\partial \hat{\theta}^*}{\partial e} \\ & - S \left[\int_{1-m-u}^{1-m} \theta d\theta + (1 - \alpha)u(1 - m - u) \right] + \bar{\phi} \frac{1}{N} = 0. \end{aligned} \quad (29)$$

With the Lagrange multiplier $\bar{\phi}$ on the constraint (27), these first order conditions are similar to the ones obtained without a minimum wage. However, an important difference is worth highlighting. Equations (28) and (29) both contain the additional term $-(\bar{w} - w_I^*)$ multiplied by the derivative of $\hat{\theta}^*$ with respect to t and e , respectively. Without a minimum wage, the decision to operate legally is based solely on the difference between taxes and expected punishment, implying that for the marginal entrepreneur, the gain from evading taxes must equal the expected sanction.

It is clear that the constraint (27) will bind at the optimum. Intuitively, if it were slack and given enforcement is costly, the government could always reduce e and maintain segmentation. Given the binding constraint, the modified Samuelson condition describing the optimal level of the public good without a minimum wage given by (14) also applies when a minimum wage is present. As this condition depends only on exogenous parameters and G , it follows that the level of public good in a

segmented equilibrium will be the same with and without a minimum wage. Interestingly however, providing this same amount of public good given the minimum wage will require higher taxes and more enforcement as condition (27) is more restrictive in the presence of a minimum wage. It follows that the informal wage will be lower with a minimum wage than without a minimum wage in the segmented equilibrium. Finally, given (14) still holds with a minimum wage it follows that Propositions 2 and 3 also hold.²³

In a non-segmented equilibrium with a minimum wage in place, the formal labour market no longer clears; some workers who want minimum wage jobs may not be able to find them. However, relative to a situation with no informal sector, this excess supply will not translate into unemployment. However, some domestic workers will be forced to work in the informal sector because of the excess supply in the legal sector. Once again the government maximizes the weighted welfare function $\Omega(t, e, G; \alpha, \bar{w})$ subject to its budget constraint (3) and to a constraint that ensures some domestic workers are in the informal sector, that is, $\hat{\theta} \geq 1 - m$ or using (4) with $w_L = \bar{w}$,

$$e \leq \frac{\bar{w} - w_I^* + t}{(1 - m)S}. \quad (30)$$

The above constraint will again provide a simple way to differentiate between a segmented and a non-segmented equilibria. When it is binding, the equilibrium is segmented as discussed above, while if it is slack the equilibrium is non-segmented. The first order conditions on t and e are given by (28) and (29), subject its budget constraint (3) with $\bar{\phi} = 0$.

In a non-segmented equilibrium, $\hat{\theta}^* > 1 - m$ and any marginal change in $\hat{\theta}^*$ will impact public good provision through tax revenues, but will not affect total welfare otherwise. The presence of a minimum wage gives entrepreneurs an additional reason to participate in the informal sector. Consequently, for the marginal entrepreneur the expected sanction is strictly larger than the gain from evading taxes.

A minimum wage affects the provision of the public good because it affects the cost of using both instruments. On the one hand, a higher level of public good can be achieved by increasing taxes. An increase in taxes pushes more firms to operate informally, and this is costly because more firms are exposed to the expected sanction instead of paying taxes and, with a minimum wage, the expected sanction

²³Obviously wages cannot be the same across sectors given a minimum wage, and Proposition 1 does not hold.

is strictly larger than taxes paid. On the other hand, a higher level of the public good can be achieved by increasing enforcement instead. With higher enforcement, fewer firms operate informally. Consequently, we cannot say if the presence of a minimum wage implies higher or lower levels of the public good in a non-segmented equilibrium. However, we can argue that the presence of a minimum wage favours the use of enforcement versus taxes in a non-segmented equilibrium. Intuitively, a minimum wage makes the informal sector more attractive, so the government reacts by monitoring more and taxing less.

Finally, we need to assess when it is optimal for the government to choose a segmented equilibrium. To do so, we examine the left hand side of the first order condition on e *just* as it is about to become binding. Since the constraint is not yet binding $\bar{\phi} = 0$, but $\hat{\theta}^* \rightarrow 1 - m$. The left hand side of first order condition (29) becomes

$$[M + N + \alpha U]v'(G) \left[N(1 - m)uS \frac{t}{t + \bar{w} - w_I^*} - c \right] - SN \left[\int_{1-m-u}^{1-m} \theta d\theta - (1 - \alpha)u(1 - m - u) \right] - (\bar{w} - w_I^*) \frac{\partial \hat{\theta}^*}{\partial e}. \quad (31)$$

Thus, when there are no undocumented workers and a minimum wage, a government maximizing total welfare will never enforce a segmented equilibrium. Because of the additional benefit of being in the informal sector, enforcement is not as effective. In fact, without undocumented workers, as $\hat{\theta}^*$ approaches $(1 - m)$, it becomes totally unresponsive to changes in e . The presence of undocumented workers may in fact increase the range of marginal enforcement costs that can support a segmented equilibrium because it increases the responsiveness of $\hat{\theta}^*$ to changes in monitoring.

4.3 Endogenous Undocumented Immigrants

So far, we considered the number of undocumented immigrants as an exogenous variable, however it is perhaps more natural to assume that the level of undocumented migration is a function of the opportunities available in the host country. We now assume that undocumented workers enter the country as long as jobs paying some fixed reservation utility w_R exist. This reservation utility can be thought of as simply the wage in the source country, or as that wage augmented by any moving or migration

costs, plus any possible utility from public good provision in the source country.²⁴ In this way, we could imagine that captured within these costs is some border control policy. Varying this reservation utility can be thought of as a very reduced form way of capturing the impact of border policy.

Without affecting the general structure, the addition of endogenous immigration choices introduces new trade offs in the design of optimal policies. On the one hand, most of the important results still apply, including Proposition 1 stating that wages are equalized across the formal and informal sectors in both types of equilibrium. On the other hand, a government who maximizes total welfare may find it advantageous to expand public spending in order to attract cheap labour from abroad, and therefore border enforcement may be less desirable than firm-level enforcement.

We assume that any informal wage satisfying $w_I + v(G) > w_R$ will induce undocumented immigrants to migrate until $w_I = w_R - v(G)$. We can use this expression to determine the number of undocumented workers as a function of government policies by first solving for

$$\bar{\theta} = \frac{w_R - v(G) + k}{1 - eS}, \quad (32)$$

which together for the expression $\bar{\theta}^* = 1 - m - u$,²⁵ and recalling that the level of the public good is given by $G = N(1 - \hat{\theta})t - ce$, implies that the ratio of undocumented workers to entrepreneurs as a function of government policies is given by

$$\tilde{u} = \tilde{U}/N = (1 - m) - \frac{w_R - v(N(1 - \hat{\theta})t - ce) + k}{1 - eS}. \quad (33)$$

The properties of this ratio of undocumented workers to entrepreneurs are quite intuitive. The ratio is decreasing in the reservation utility and in the number of domestic workers, and increasing in the level of public good provision. Similarly to Definition 1, we can construct Definition 2:

Definition 2 *Total weighted welfare $\Omega(t, e, G; \alpha, w_R)$ is given by:*

$$\begin{aligned} \Omega(t, e, G; \alpha, w_R) &= N \int_{1-m-\tilde{u}}^1 \theta d\theta + [N - M - \tilde{U}]k - N(eS) \int_{1-m-\tilde{u}}^{\hat{\theta}^*} \theta d\theta - N \int_{\hat{\theta}^*}^1 t d\theta \\ &\quad - (1 - \alpha)\tilde{U}[w_R - v(G)] + [N + M + \alpha\tilde{U}]v(tN(1 - \hat{\theta}^*) - ce). \end{aligned}$$

²⁴Our model is related to that of Harris and Todaro (1970) on rural/urban migration. In Harris-Todaro model, agents migrate until expected income between locations is equalized. In ours, agents migrate until expected utility, including that from the public good, is equated across locations.

²⁵There will still be full employment because the labour markets clear in both sectors.

Possible equilibrium still take one of two forms: it can either be segmented, potentially allowing for a formal wage higher than the informal wage, or it can be non-segmented in which case the formal wage will be driven down to the informal wage (reservation utility w_R less utility of public good provision). We consider these two cases in turn.

In a non-segmented equilibrium, the labour market clearing condition requires that $w_L = w_I = w_R - v(G)$. Therefore, from the definition of $\hat{\theta}$ given by (4) we have $\hat{\theta}^* = t/(eS)$. From (33), it follows that increasing enforcement e shrinks the informal sector and reduces the number of undocumented immigrants on both margins. If the reservation utility is loosely taken to proxy for some border enforcement policy, then increasing border enforcement (increasing w_R) discourages entry and reduces the size of the informal sector given other government policies. Public good provision on the other hand, increases the size of the illegal sector by encouraging migration of illegal immigrants. Naturally, it requires that this public good is perfectly non-excludable. Note that in this environment, entrepreneurs will have a direct benefit from the public good as well as an indirect benefit from reduced wages. Workers will have the opposite.

The government chooses taxes using exactly the same tradeoffs as before, except now there is an additional benefit of higher taxes. By raising taxes and increasing public good provision, the government lowers the informal wage and expands the size of the informal sector. Expanding the informal sector has both a positive and a negative consequence. Because more firms operate in the informal sector, more wasteful punishments are levied. At the same time, total production – including informal production – increases. Lower wages also redistributes income from undocumented workers to entrepreneurs. If the welfare weight on undocumented workers is less than one, this is socially beneficial. If $\alpha = 1$ there is no additional benefit. Notice however, that it also redistributes income away from domestic workers since their wages are tied to those in the informal sector. Enforcement has the complete reverse effect. When e increases, it causes the size of the informal sector to shrink and total production to fall as $\bar{\theta}^*$ increases.

It is also interesting to see what happen when w_R falls, as if border enforcement were to be relaxed. Lower reservation wage w_R expands the size of the informal sector, but only along one margin. It increases the number of firms operating in the informal sector, without changing the number of firms operating in the formal sector. Consequently, relaxing border enforcement is more attractive relative to relaxing

enforcement on firms. Obviously, this is only true if domestic workers and firms are weighted equally; if workers are valued more, increasing border enforcement may be more desirable since it increases wages.

In a segmented equilibrium, the labour market clearing condition is given by $w_L \geq w_I = w_R - v(G)$. With a flexible formal wage, the supply of workers in this sector must equal the demand for workers by formal firms so $M = N(1 - \hat{\theta})$ which yields $\hat{\theta}^* = 1 - m$ and the equilibrium formal wage: $w_L^* = (1 - m)eS + w_R - t$. Even with endogenous illegal immigration, Proposition 1 applies. Legal and illegal wages must be equalized, $w_L^* = w_I^*$. Again, there is no reason to increase enforcement above what is necessary to ensure that all domestic workers are in the formal sector. Illegal immigration increases output. Obviously, if workers were to be more valued than entrepreneurs the situation could be different as illegal immigration lowers wages.

When considering the optimal policy in the segmented equilibrium, much of the same intuition will carry through as discussed for the case of the non-segmented equilibrium. The government can expand production by reducing the cost of setting up a firm in the informal sector by decreasing e or increasing G . It can strategically manipulate the amount of the public good to reduce informal sector wages and redistribute the surplus toward entrepreneurs operating in the informal sector. It can also reduce border enforcement to expand output.

4.4 Endogenous Price

So far, we have also taken the price of the good produced in the economy as exogenous and have shown that regardless of how the government weights the welfare of undocumented workers maximized total welfare is increasing in the number of undocumented workers. But by holding price fixed, we are ignoring the potential impact an increase in the supply of undocumented workers may have on the equilibrium price. Empirically it has been shown that an increase in the size of the low-skilled immigrant population can put downward pressure on low-skilled wages and thereby depress prices of the goods and services produced by this labour (Cortes, 2008). How would allowing for an endogenous price affect our conclusions?

Consider the case of an increase in U . With a fixed price, this increase has two effects. First, it attracts entrepreneurs into the informal sector who were previously not operating firms. These entrepreneurs are now better off and consequently, total

welfare increases. Total production also increases. Second, there are now more individuals benefiting from the public good which also increases total welfare. These two effects are present when the price is endogenous but there is also an additional effect. The increased production puts downward pressure on the price which reduces the value of output produced by all firms in the economy and consequently, the welfare of all entrepreneurs. At the same time, everyone benefits from the lower price. It is possible to show that the net effect of an increase in U on total welfare remains positive even when the price is endogenous.²⁶ It is also the case that having price endogenous does not change the relative trade-offs between government policies.

4.5 Taxes on Workers

In this paper, we restrict attention to taxing firms. If the government could also tax workers in the formal sector, our model would generate qualitative the same insights. Informal workers would face some probability of detection, say ρ , and some sanction F . Sanctions on workers would also impose a social cost. Formal workers would pay a wage tax, τ , used to finance the public good. The government would now have at its disposal potentially two additional instruments, ρ and τ .²⁷

To support a segmented equilibrium in this framework, it must be that $w_L - \tau \geq w_I - \rho F$ or using the wage expressions (8) and (10), $eS(1 - m) + \rho F \geq t + \tau$. This constraint will still bind, and consequently, wages (net of taxes and expected penalties) will be equalized across the formal and informal sectors in both types of equilibria, segmented and non-segmented.

In a segmented equilibrium, the government would generate total tax revenue of

²⁶For the price of output produced in the economy to be endogenous, an additional good with a fixed price must be incorporated into the framework. The above conclusion is obtained assuming standard Cobb-Douglas preferences over the two goods.

²⁷One issue to consider is whether ρ is correlated with e at the firm level. The simplest case would involve firms and workers being audited independently by two separate agencies without information sharing. Then all workers would face the same audit probability ρ independent of e . With information sharing, solving for the equilibrium could be more challenging. The probability a given worker is audited would depend on the particular θ of the firm that he or she is matched with. As long as θ is not observable however, every worker would face the same expected probability of audit, which would be endogenously determined in equilibrium. In what follows, we refer to the simple case with no information sharing.

$(t + \tau)N$. Because both forms of taxes share the same fixed tax base, and generate the same marginal social cost (as shown in the last paragraph), they are perfect substitutes. Enforcement on firms and workers however, may behave differently. The first-order conditions on firms' enforcement would be unaffected, while a similar first-order condition on workers' enforcement would be added. Since the two forms of enforcement may impose different social costs, their relative merit would depend on F versus S .

In a non-segmented equilibrium, the productivity cut-off between formal and informal firms will be $(t + \tau - \rho F)/eS$. Again, the tax bases are the same. Moreover, those tax bases are affected the same way by variations in t , and in τ . Consequently, both forms of taxation are again perfect substitutes, while enforcement may behave differently.

Similarly, we could allow the government to choose to tax formal consumption. If formal and informal consumption goods are perfect substitutes, they would have to be available at the same price. Enforcement would have to be sufficiently high to prevent the legal sector from being foreclosed. This model would be different in a subtle way since the benefit of avoiding taxes would be proportional to productivity, which is not the case in our basic model. But regardless, the same qualitative results would obtain.

5 Conclusion

In this paper, we construct a simple model of tax evasion with an informal sector, and consider the role of undocumented workers on optimal tax and enforcement policy. We find that optimal policies play a crucial role in the wage determination process and lead wages to be equalized even when domestic and undocumented workers are not competing against each other in the same sector. This result does not arise in previous papers that have considered illegal immigration.

We also find that enforcement may not always be decreasing in its cost, and that governments will optimally enforce market segmentation if enforcement costs are not too high. We consider several extensions. First, we endogenize the amount of illegal migration, and find a novel role for the public good. Our public good can be used to depress domestic wages in the informal sector, and thereby redistribute income from undocumented workers to domestic entrepreneurs. In this environment as well, we

see that firm level enforcement is socially more desirable than border controls. Lastly, we introduce a minimum wage as a means of breaking the wage arbitrage condition and altering agents responsiveness to enforcement policy.

6 Appendix

Derivation of Government Objective Function

The $N\bar{\theta}$ entrepreneurs who don't start a firm collectively receive

$$N\bar{\theta} [k + v(G)].$$

The $N(\hat{\theta} - \bar{\theta})$ entrepreneurs who start a firm in the informal sector collectively receive

$$(1 - eS)N \int_{\bar{\theta}}^{\hat{\theta}} \theta d\theta - N[\hat{\theta} - \bar{\theta}]w_I + N[\hat{\theta} - \bar{\theta}]v(G),$$

while the $N(1 - \hat{\theta})$ of those who start a firm in the formal sector collectively receive

$$N \int_{\hat{\theta}}^1 \theta d\theta + N[1 - \hat{\theta}][w_L + t] + N[1 - \hat{\theta}]v(G).$$

Summing up the above expressions, total welfare for all entrepreneurs is given by:

$$\Omega_E = N \left[\int_{\bar{\theta}}^1 \theta d\theta - \int_{\bar{\theta}}^{\hat{\theta}} [e\theta S + w_I] d\theta - [1 - \hat{\theta}][w_L + t] + v(G) \right].$$

The M_L domestics workers in the formal sector collectively receive

$$M_L (w_L + v(G)),$$

while the $M_I + U$ workers in the informal sector collectively receive

$$(M_I + U) (w_I + v(G)).$$

Total welfare for all workers with weight $\alpha \in [0, 1]$ on the welfare of undocumented workers is given by:

$$\Omega_W = M_L w_L + (M_I + \alpha U) w_I + (M + \alpha U) v(G).$$

Summing up Ω_E and Ω_W , and using the labour market clearing conditions (6) and (7), and the expression for the informal wage given by (8), total weighted welfare is:

$$\begin{aligned}\Omega(t, e, G; \alpha) &= N \int_{\bar{\theta}}^1 \theta d\theta + [N - M - U]k - N \int_{\hat{\theta}}^1 t d\theta - N \int_{\bar{\theta}}^{\hat{\theta}} e\theta S d\theta \\ &\quad - (1 - \alpha)U [[1 - eS](1 - m - u) - k] + [M + N + \alpha U]v(G).\end{aligned}$$

Proof of Proposition 1

We proceed to two steps. First, we show that the constraint given by (11) is optimally binding. Second, we show that wages will be equalized across sectors.

Step 1. Rewriting (13), we have

$$\delta = [M + N + \alpha U]v'(G)c + SN \int_{1-m-u}^{1-m} \theta d\theta - (1 - \alpha)US(1 - m - u). \quad (34)$$

Given our assumption on v , the multiplier δ will be positive and the constraint will bind provided the sum of the last two terms on the right-hand side of (34) is positive.

We now show that this is the case.

$$SN \int_{1-m-u}^{1-m} \theta d\theta - (1 - \alpha)US(1 - m - u) = SN \left(\frac{u^2}{2} + \alpha u(1 - m - u) \right) > 0,$$

Step 2. Given the constraint is optimally binding, we have $t^* = (1 - m)Se^*$. Substituting this condition into the expression for the wage in the formal sector given by (10), the resulting expression together with (8) yields Proposition 1.

Proof of Proposition 2

In a segmented equilibrium, optimal public good provision is determined by

$$[N + M + \alpha U]v'(G^*) = \left[\frac{1 + \frac{1}{M(1-m)} \left(N \int_{1-m-u}^{1-m} \theta d\theta - (1 - \alpha)U(1 - m - u) \right)}{1 - \frac{1}{M(1-m)} \frac{c}{S}} \right]. \quad (35)$$

a) In the text.

b) An increase in c increases the right-hand side of (35) and therefore, $v'(G^*)$ must go up. Since $v'' < 0$, this means G^* must go down

c) When $\alpha = 0$, U only appears on the right-hand side of (35). Differentiating the right-hand side with respect to U with $\alpha = 0$, we obtain

$$\frac{S}{(M(1-m)S-c)}u > 0.$$

Therefore, with $\alpha = 0$ an increase in U increases $v'(G^*)$ and since $v'' < 0$, G^* must go down.

Proof of Proposition 3

Let

$$F(e, c) = [N + M + \alpha U]v'(G^*) - \left[\frac{1 + \frac{1}{M(1-m)} \left(N \int_{1-m-u}^{1-m} \theta d\theta - (1-\alpha)U(1-m-u) \right)}{1 - \frac{1}{M(1-m)} \frac{c}{S}} \right]$$

where $G^* = [M(1-m)S - c]e^*$.

a) It follows from (35) that the condition $F(e, c) = 0$ yields equilibrium enforcement e^* as a function of c . Totally differentiating F , we obtain

$$\frac{de^*}{dc} = -\frac{F_c}{F_e}$$

where

$$F_e = [N + M + \alpha U]v''(G)(M(1-m)S - c) < 0,$$

$$F_c = -[N + M + \alpha U]v'(G) \frac{1}{M(1-m)S - c} \left[1 + \frac{v''(G)}{v'(G)} G \right]$$

Therefore, optimal enforcement is increasing in c if $v''(G)G/v'(G) < -1$, decreasing in c if $v''(G)G/v'(G) > -1$ and independent of c if $v''(G)G/v'(G) = -1$.

b) From Proposition 2c, $dG^*/dU < 0$ when $\alpha = 0$. Since $G^* = [M(1-m)S - c]e^*$, optimal enforcement is also decreasing in U when $\alpha = 0$.

Proof of Proposition 4

Let

$$R(\hat{\theta}, c, \alpha, U) = \hat{\theta}^2 \frac{S}{2} - (1 - \hat{\theta}) \frac{c}{N} + \frac{S}{2} \bar{\theta}^2 + (1 - 2\hat{\theta})(1 - \alpha)uS\bar{\theta} - S\hat{\theta}\bar{\theta}^2.$$

Eliminating $v'(G)$ from the first-order conditions (20) and (21), and substituting in the expressions from (18) yields

$$\frac{(1 - 2\hat{\theta})}{\hat{\theta}^2 S - c/N} = \frac{(1 - \hat{\theta})}{\frac{S}{2}(\hat{\theta}^2 - \bar{\theta}^2) - (1 - \alpha)uS(1 - m - u)}. \quad (36)$$

Manipulating (36), we obtain

$$R(\hat{\theta}, c, \alpha, U) = 0$$

which yields the equilibrium value of $\hat{\theta}^*$ as a function of the marginal cost of enforcement, the welfare weight on the undocumented workers and the number of undocumented workers.

Totally differentiating R , we obtain

$$\frac{d\hat{\theta}^*}{dc} = -\frac{R_c}{R_{\hat{\theta}}}, \quad \frac{d\hat{\theta}^*}{d\alpha} = -\frac{R_\alpha}{R_{\hat{\theta}}}, \quad \frac{d\hat{\theta}^*}{dU} = -\frac{R_U}{R_{\hat{\theta}}}.$$

where

$$R_c = -\frac{1 - \hat{\theta}}{N} < 0$$

$$R_\alpha = -(1 - 2\hat{\theta})uS\bar{\theta} < 0$$

$$\begin{aligned} R_U &= \frac{(1 - 2\hat{\theta})(1 - \alpha)S\bar{\theta}}{N} + \left(S\bar{\theta}(1 - 2\hat{\theta}) + (1 - 2\hat{\theta})(1 - \alpha)uS \right) \frac{-1}{N} \\ &= -\frac{(1 - 2\hat{\theta})S}{N} (\alpha\bar{\theta} + (1 - \alpha)u) < 0 \end{aligned}$$

$$\begin{aligned} R_{\hat{\theta}} &= \hat{\theta}S + \frac{c}{N} - 2(1 - \alpha)uS\bar{\theta} - S\bar{\theta}^2 \\ &= \frac{c}{N} + S \left[\hat{\theta} - (1 - m + u - 2\alpha u)\bar{\theta} \right] \end{aligned}$$

In equilibrium, $\hat{\theta} > \bar{\theta}$. Therefore, if $(1 - m + u - 2\alpha u) < 1$ then $R_{\hat{\theta}}$ will be positive for all α . Note that $(1 - m + u - 2\alpha u)$ is monotonically decreasing in α and at $\alpha = 0$ the expression is $(1 - m + u) < 1$ since $m > u$. Therefore, for all α we have $(1 - m + u - 2\alpha u) < 1$ and $R_{\hat{\theta}} > 0$.

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