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"I am a great believer in luck,
and I find the harder I work,
the more I have of it"
Thomas Jefferson

Abstract

This paper suggests that people can learn to behave in a way which makes them unlucky or lucky. Learning from experience will lead them to make choices which may lead to "luckier" outcomes than others. By so doing they may reinforce the choices of those who find themselves with unlucky outcomes. In this situation, people have reasonably learned to behave as they do and their behaviour is consistent with their experience. The lucky ones were not "born under a lucky star" they learned to be lucky.

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1 Introduction

There are many attitudes to success. For some it is jealousy, based on the belief that the successful are just lucky and that they do not merit what they have obtained, for others it is admiration for the effort or skill that must have led to the success. What is interesting is that there does not seem to be a clear and commonly accepted definition of luck. The Oxford Dictionary provides us with the following definition: "success or failure apparently brought by chance rather than by one's own actions."

The idea that "luck" is exogenous to one's own actions has led to a debate on the importance of "luck" versus "action" in determining success. Discussions such as "luck" versus "skill" in the management and finance literature (e.g., Hartzmark, 1991; Gompers et al., 2006; Cuthbertson et al., 2008), and "luck" versus "policy" in the macroeconomic literature (e.g., Ahmed et al., 2004; Leung et al., 2004) are a few examples.

People's perception of the importance of "luck" versus "effort" in determining ones' success can have a strong effect on the degree of government intervention in a society (Piketty, 1995; Alesina and Angeletos, 2005) or severity of punishment for criminals (Di Tella and Dubra, 2008). These authors argue that in a society where people believe that individual efforts determine success, a lower degree of government intervention or a severer punishment for criminals will be chosen. On the other hand, the degree of government intervention will be higher in a society where people believe that luck, not individual efforts, determines success.

Regardless of the belief in a society about the importance of luck in determining one's own success, we have many expressions such as "fortune favours the brave" or "nothing venture nothing gain" which suggest that behaviour and attitudes can contribute to apparently lucky favourable outcomes.

Take the simple example of car insurance. Here two factors are in play. Firstly, there is the idea that certain events are due purely to "bad luck" and that the consequences of these should be mutualised and then there is the problem of adverse selection. Those who insure themselves are more likely to be those to whom "unlucky" events may happen. Bad drivers do not have the same probabilities of an accident as good drivers almost by definition. Insurance policies with their "bonus" and "malus" are structured to separate out these two problems.

But this means that, since it is possible to improve one's driving by learning, one can learn to become "lucky" or, put alternatively, to experience fewer negative shocks than others. An alternative view, that of a psychologist Richard Wiseman (2003), suggests that "lucky" people are more aware of the opportunities that arise and hence more likely to profit from them. Thus it is not enough to study the process of arrival of “good” or “bad” luck but rather the attitude of the actors themselves. Others have argued that more optimistic people tend to be luckier and some psychologists such as Martin Seligman (1991) suggest that one can learn to be optimistic.

It is actually quite difficult to cite examples of “pure luck”. Winning a large sum in a National Lottery might seem like an obvious example. However, you can learn how to increase the expected value of the outcome, at least in some countries, by studying the numbers that people choose. Some are very popular and some are hardly chosen at all. Therefore, armed with this information, one should choose an unpopular number since, if one wins, one has to share her gains with fewer people.

This remark points to another important feature of certain lucky outcomes which is that they are part of a constant sum game. What one individual gains deprives the others of that opportunity. Thus, good luck for some implies, in this framework, bad luck for others. This means that if one person learns how to seize lucky occasions he may deprive others and thereby make them “unlucky”.

What we will suggest in this paper is that people can, in some cases, learn to behave in a way which makes them unlucky or lucky. Learning from experience will lead them to make choices which may lead to “luckier” outcomes than others. By so doing they may reinforce the choices of those who find themselves with unlucky outcomes. Our simple example is based on the everyday experience of one of us in parking his car. There are many parking spots in the small streets leading to the Vieille Charité in Marseille, where the group to which one of us belongs, is based. Having taken a spot, one then walks the rest of the way and this takes time and effort. What one observes, is that some people are systematically parked close to the office while others are always far away. Excluding *ex-ante* heterogeneity of agents in their access to the parking, the natural question is: are the former just lucky or is something else at work? We wish to suggest that people have reasonably learned to behave as they do and that their behavior is consistent with their experience. The lucky ones were not “born under a lucky star”, they learned to be lucky.[1] The second important issue concerns those aspects in the environment of agents which favors asymmetric outcomes with individuals being persistently lucky (or unlucky). We find that such state of affairs to prevails depending both on the way agents learn,

[1]Note that, in this paper, we are not trying to explain the reason for which people tend to attribute success to their own skills and failures to bad luck. Consideration for such “self-serving” bias can be found, for example, in Compte and Postlewaite (2004); van den Steen (2002) and the literature cited therein.

on payoffs and on the timing of the search process. Specifically, an high degree of synchronization in the access to parking, with most agents going to and leaving work at roughly the same time (in the morning and in the afternoon, respectively), favors asymmetric outcomes whereas asynchronous access to resources favors symmetric outcomes. Interestingly, increasing the cost agents incur if they fail to find a parking, favors asymmetric outcomes only in an intermediate range.

Although the model will be presented with a particular example of parking along the one way street, the same idea can be applied to other situations in which agents search for better deals. For example, in a market place, buyers search for cheaper prices by visiting several sellers.[2] What buyers do not know in this case is the prices offered by various sellers in the market. In the standard model of search, buyers have some beliefs about the underlying distribution of prices offered by the sellers in the market, and decide to accept the proposed price by comparing the cost and the expected benefit of continuing their search for better offers.

There is a literature on the optimal strategy to adopt when one is faced with a series of prospects but with no possibility of revisiting previous candidates. This is studied by Krishnan (2006) who likens the problem to that of choosing an exit from a highway. Once one has taken a particular exit one cannot, in general, go back to previous exits nor profit from later ones. However, the fact that the behaviour of others influences the choices available is not taken into account.

Our problem also is related to the famous "secretary problem" originally posed by Gleason in 1955 according to Gilbert and Mosteller (1966). In this problem surveyed by Freeman (1983) and Chow et al. (1991), one is faced with a string of secretaries and has to choose one. In the classical version the number of secretaries is finite and if one gets to the last candidate that candidate has to be selected. The problem is when to stop and choose the current candidate. As in our case there is no going back. Two features are different from those of our problem. Firstly, as in the previous case the interference from other secretary hirers is not taken into account. Secondly, in our problem the value of each successive slot is increasing but only if it is free. Otherwise it provides no value to the person searching. Gilbert and Mosteller (1966) did, in fact, treat a strategic version of the problem since the selector is faced with an opponent who can decide in which order to present the

[2]Literature on learning by buyers in the market mostly focuses on buyers learning which seller to visit. See, among others, Weisbuch et al. (2000); Hopkins (2007), which also reports evidence of consistent heterogeneity in prices across buyers. Of course, it is not only buyers who learn in the market but also sellers learn about the best price to charge. See Hopkins and Seymour (2002) for an interesting analysis for such cases.

candidates.

Steblay et al. (2001) look at the problem of a police line-up in which the suspects are presented sequentially but the person who has to identify the criminal has to identify him on the spot and cannot go back to the previous suspects. Again, there is no competition for the suspects amongst those identifying.

Lee et al. (2004) examine the capacity of people to choose in this sort of problem in an experimental setting. Their interest was in the psychological aspect of the choice making which is, of course, related to our idea that luck is not random but due to other factors.

The question the standard search literature does not address, in general, is how buyers form their beliefs about the distribution of opportunities (see, however, Rothschild, 1974). Our working hypothesis is that agents learn what to accept through trials and errors, although they may not necessarily learn the underlying distribution of opportunities. In other words, in our paper, the agents are not learning to solve this problem, which is why they can converge to different solutions in spite of the fact that they face the same problem. Notice, furthermore, that in our context the underlying distribution of the opportunities available to an agent is determined endogenously, as it depends on what other agents have learned to accept.

While our emphasis is on learning, we shall first propose an analysis of some static results, for large populations, in two extreme cases. These two cases bring polar predictions on the emergence of lucky and unlucky individuals. We shall then proceed to analyze the learning dynamics. However, a simple theoretical analysis can be done only in the case of two agents. The general case of a large population of agents will be analyzed through numerical simulations. What we find, confirms both the qualitative features disclosed by the analysis of learning in the case of two agent, and the polar predictions of our static analysis.

2 The Model

Consider a hypothetical city where there is a one way street leading to the city center. This is the unique street in the city where people can park their cars. Suppose there are S parking spots that are distributed evenly along the street. Parking spots are indexed by their distance from the city center, so that spot $s \in \{1, 2, ..., S\}$ is at distance $s - 1$ from the center.

There are $N \geq S$ agents living around the city.[3] At the beginning of each period

[3]The case of $N < S$ is equivalent to considering the case with N parking spots although the

$t \in \{0, 1, 2, ..., \}$, each agent can be either in the city, already parked, or outside the city. If an agent is outside the city, there is a probability p_e that she comes into the city and looks for a parking spot along the one way street. When an agent looks for a parking spot, she chooses a strategy $k \in \{1, .., S\}$ which means that she will park at the first empty spot, s, that is closer to the center than spot $k+1$ (i.e, $s \leq k$). When she parks at spot s, she receives the payoff $\pi(s)$. The payoff is normalized between zero ($\pi(S) = 0$) and one (i.e. $\pi(1) = 1$), and is decreasing in s, i.e, the closer the spot is to the center, the higher the payoff is. As a specific case, we shall consider the payoff for parking at spot s as being given by

$$\pi(s) = 1 - \left(\frac{s-1}{S-1}\right)^{\alpha} \tag{1}$$

where $\alpha > 0$ is a parameter that determines the curvature of the payoff function.

If she did (could) not park before reaching the city center, she has to park at a pay parking lot. Because of the fee this entails, she receives payoff $-L$ (where $L \geq 0$) when she fails to park before the city center, i.e., the spot 1.

After all the players who looked for a parking spot made their decisions (some have parked along the street and others may have failed to park before the city center and parked in pay parking lots) and received their payoffs, each parked agent, including the ones who found a spot in this period, leaves the city with probability p_l. If an agent leaves the city, she will be outside the city in the beginning of the next period, while if she does not leave the city, she enters next period "in the city already parked."

3 Equilibrium

In an equilibrium setting, the existence of lucky and unlucky individuals translates into the presence of asymmetric Nash equilibria where agents play different strategies, some leading to better outcomes than others.

Our purpose here is not to provide a thorough characterization of Nash equilibria in this game but rather to show that asymmetric equilibria, where some agents are luckier than others, can be sustained as equilibria for particular values of parameters where symmetric equilibria do not exist. Likewise, symmetric equilibria where agents adopt the same strategy, happen to be stable in a different region of parameter space.

We first consider the case $p_e, p_l \ll 1/N$, where the distinction between different

payoffs have to be normalized accordingly.

periods becomes immaterial, and then the case $p_e = p_l = 1$, where each period can be considered as a different game. In the former case ($p_e, p_l \ll 1/N$), we find that symmetric equilibria prevail in the sense that they exist in a broad region of parameter space whereas only in a rather limited region asymmetric equilibria exist. In the latter case ($p_e = p_l = 1$), we shall see that the converse is true. Symmetric Nash equilibria only exist for rather specific choices of parameters. The intermediate region between these two extremes should reveal a qualitative change between the two polar behaviors. This will be explored with our numerical simulations of learning dynamics later in the paper.

3.1 The case $p_e, p_l \ll 1/N$

Let us consider case where $S = N$ and the probability of agents entering and leaving the city in any round is vanishingly small. More precisely, we take $p_e, p_l \to 0$ but we keep ratio $p_e/p_l = \eta$ finite. This limit allows us to treat the turnover of agents in the city as a continuous time process, where agents leave the city at unit rate and enter the city at rate η.[4] Take a particular agent i. After a transient time, her probability to be in the city will converge to a stationary value. The latter should be such that the probability per unit time of her going into town, given that she was not in town, equals the probability per unit time of her leaving the city. This condition – generally referred to as *detailed balance* – between agents entering and leaving the city in a time interval, implies that any particular agent, in the stationary state, will be in the city with probability

$$1 - h = \frac{\eta}{1 + \eta} \tag{2}$$

Since arrival and departures are independent events, the number C of cars currently parked, in the stationary state, has a binomial distribution

$$P\{C = c\} = \binom{N}{c} (1 - h)^c h^{N-c} \tag{3}$$

Agents adopt threshold strategies labeled by an integer k: They will go to position k and then park in the first free parking $s \leq k$ which they find. If there is no free parking slot, they incur the cost L. Call $\sigma_{i,k}$ a mixed strategy for agent i, which

[4]In this section, time t will be considered a continuous variable. One period of the game corresponds to a vanishingly small time interval $dt = p_l$. In this time variable, the probability that agent i enters (leaves) the city in time interval $[t, t + dt)$ is ηdt (dt) for all t and for all agents i out of (in) the city.

is the probability that i picks strategy k. We want to investigate Nash equilibria of this N player game. Since equilibria are not the main subject of this paper, we relegate the derivation to the appendix, for the sake of clarity of exposition. Here we shall confine our discussion to the main results and a basic intuition.

In order to discuss asymmetric Nash equilibria, consider the set of pure strategies

$$\{\sigma_{i,k}\} = \delta_{k,k_i}, \qquad (k_1, \ldots, k_N) \text{ a permutation of } (1, \ldots, N) \tag{4}$$

where $\delta_{i,j} = 0$ if $i \neq j$ and $\delta_{i,i} = 1$. We find that:

Proposition 1: *The strategy profile (4) is a Nash equilibrium, provided:*

$$L \geq \frac{1}{1+\eta} \sum_{k=1}^{N} \left(1 + \frac{1}{\eta}\right)^k \pi(k). \tag{5}$$

This implies that, for any finite L and η, the strategy profile in Eq. (4) is a Nash equilibrium only for N small enough. For finite η and large N, the right hand side of Eq. (5) is exponentially large with N, i.e. a huge cost L is required in order for the asymmetric Nash Equilibrium to exists. Conversely, when $\eta \sim N$ is also very large, the condition (4) is satisfied even for relatively small L.

The intuition behind this result is simple: In order for the asymmetric state to be stable it must be the case that the worst off agent, who is parking at spot N, has no incentive to deviate, i.e., her expected payoff from deviating to other strategies be negative. Only if there is no free spot, which occurs with exponentially small probability $(1-h)^{N-1}$, will she have to pay the cost L. So the penalty is irrelevant unless $L(1-h)^{N-1}$ is of order one. A formal derivation, which is given in the appendix, leads to the specific prediction of Proposition 1.

Let us now consider a symmetric Nash equilibrium where every agent uses the same strategy σ_k. In general, a mixed strategy is a distribution on k conditional on the information available to agent i. This in particular includes the information on the spots in which she has parked in the past. For the present discussion, we assume agents to be naïve, i.e., to disregard this latter piece of information. Accordingly, we shall speak of *equilibria with naïve agents*. This can be justified for large population sizes ($N \gg 1$), in view of the random arrival of agents in the city.

Proposition 2: *A symmetric equilibrium with naïve agents exists in pure strategies, i.e.* $\sigma_k = \delta_{k,k^*}$ *for any value of* η *and* L.

As shown in the appendix, in the limit as N becomes large, this equilibrium has the following properties:

- the fraction $z^* = k^*/N$ of potentially occupied parking slots is less than the fraction $1 - h$ of agents wishing to park. In other words, a positive fraction $1 - h - z^*$ of agents have to pay the cost L

- the threshold $z^* = k^*/N$ increases with L and with η (i.e. with the average number of agents trying to park).

- the probability e_s that a parking spot is free decreases with s, i.e., the more profitable a parking spot is, the more likely it is that it is unoccupied.

This equilibrium is not particularly efficient, first because a finite fraction of the agents have to pay the cost L, secondly because the best parking slots are those which are more likely to be unoccupied. Indeed all parking slots in the asymmetric equilibrium are occupied with the same probability and the average payoff of agents is higher than in the symmetric equilibrium. Hence, the asymmetric equilibrium is more efficient than the symmetric one, both in terms of better exploitation of resources and of the average payoff of the agents.

In summary, the analysis of the (naïve) Nash equilibria of the N players game, for $p_e, p_l \ll 1$, suggests that both symmetric and asymmetric equilibria can arise. The latter, when N is large, arises either when the cost L is extraordinarily high, or when the number of free spots is small.

3.2 The case $p_e = p_l = 1$

Next we turn to the case where all the parked agents leave the city at the end of the period ($p_l = 1$) and all the agents who are outside the city at the beginning of the period come into the city ($p_e = 1$). To make our analysis simple, we consider the situation where the number of agents wishing to park in the city are the same as the number of spots available, $N = S$.

We again consider two outcomes, the symmetric one and the completely asymmetric one, and see whether they can be Nash equilibria of the game.[5] The symmetric outcome is where everyone chooses the same strategy (i.e., $k^i = k$ for all i) while the completely asymmetric outcome is one where everyone chooses different strategies (i.e., without loss of generality, $k^i = i$ for all i). The results can be summarized in the following two propositions:

[5]We confine our discussion to Nash equilibria in pure strategies.

Proposition 3: *(i) Symmetric outcomes, where everyone chooses the same strategy, cannot be a Nash equilibrium of the game when* $N - (N-1)^{1+\alpha} < 0$. *(ii) If* $0 \leq L \leq \frac{N}{(N-1)^{1+\alpha}} - 1$, *then, the symmetric outcome where everyone chooses strategy 1, i.e,* $k^i = 1$ *for all* i *is a Nash equilibrium of the game.*

Proposition 4: *The completely asymmetric outcome where everyone chooses a different strategy is a Nash equilibrium of the game when*

$$L > \max_j \left((N-j+1) \sum_{l=1}^{N-j} \frac{1}{(l+1)l} \pi(N-j+1-l) \right). \tag{6}$$

It is worth remarking that the stability of the symmetric Nash equilibrium, for large N, requires that α and L lie within a very narrow range. With a little algebra, the condition in Proposition 3, can be rewritten as $\alpha < -\log(1-1/N)/\log(N-1) \simeq 1/(N \log N)$. When this condition is satisfied, the value of L also needs to be very small. Indeed we have $L < \frac{N}{(N-1)^{1+\alpha}} - 1 < 1/(N-1)$, which implies that both α and L should vanish as $N \to \infty$, for the symmetric Nash equilibrium to exist. We will now prove the two propositions.

To check whether the symmetric outcomes where $k^i = k$ for all i, that is when all individuals choose the same strategy, can be a Nash equilibrium, we need to check whether a unilateral deviation to $\hat{k} \neq k$ is profitable.

Since, each period, agents arrive at the street in a random order, the expected payoff for an agent when everyone is choosing strategy k depends on how many agents have already arrived at the street before her. For example, if i is the first to arrive which happens with probability $1/N$, then she will park at the spot k and obtain $\pi(k)$. If she is the second, which again happens with probability $1/N$, then she will park at spot $k-1$ and obtain $\pi(k-1)$, and so on. That is, if she is the $j(\leq k)$-th agent to arrive at the street in that period, she will park at spot $k-(j-1)$ and obtain $\pi(k-(j-1))$. But if there are more than k agents before her (i.e., she is the $j(> k)$-th agent to arrive), then, she will not be able to park before the center and will obtain $-L$.

Therefore, the expected payoff for an agent when everyone is choosing strategy k is

$$E(\Pi^i(k|k^{-i} = k)) = \frac{1}{N} \left(\sum_{l=1}^{k} \pi(l) - (N-k)L \right) \tag{7}$$

Now consider the expected payoff from a unilateral deviation. An agent, say i, can deviate and choose strategy $\hat{k} = k+1$ or $\hat{k} < k$, in particular, $\hat{k} = 1$. The expected payoff for agent i who deviate to $\hat{k} = k+1$ while everyone else is choosing strategy k can be easily obtained because i can park at spot $k+1$ for sure. That is,

$$E(\Pi^i(k+1|k^{-i} = k)) = \pi(k+1) \tag{8}$$

Now the expected payoff for agent i who deviates to $\hat{k} = 1$ when all the other agents are choosing strategy $k \geq 2$ can be derived as

$$E(\Pi^i(1|k^{-i} = k, k \geq 2)) = \frac{1}{N}\left(k\pi(1) - (N-k)L\right) \tag{9}$$

Intuitively, when all the other players are choosing strategy k, unless there are k other agents in the street before agent i, i can park at spot 1. If there are more than k agents before i, i fails to park and obtains $-L$.

Now, one can check if $k^i = k$ for all i can be an equilibrium. We do this by considering first $k = 1$ and then $k > 1$.

When all the agents choose $k = 1$, unilateral deviation to $\hat{k} = 2$ is profitable if $E(\Pi^i(1|k^{-i} = 1)) < E(\Pi^i(2|k^{-i} = 1))$. That is, from equations (7) and (8),

$$\frac{1}{N}\left(\pi(1) - (N-1)L\right) < \pi(2)$$

or

$$L > \frac{1}{N-1}\left(\pi(1) - N\pi(2)\right) \tag{10}$$

With $\pi(s) = 1 - \left(\frac{s-1}{S-1}\right)^{\alpha}$ and $N = S$, we have

$$L > \frac{N}{(N-1)^{1+\alpha}} - 1 \tag{11}$$

But note that if $N-(N-1)^{1+\alpha} < 0$, this condition is always satisfied because $L \geq 0$. Thus, $k^i = 1$ for all i cannot be a Nash equilibrium when $N-(N-1)^{1+\alpha} < 0$.

Let us now move to the case where $k > 1$. From equations (7) and (9), unilateral deviation to $\hat{k} = 1$ is profitable if

$$\frac{1}{N}\left(\sum_{l=1}^{k}\pi(l) - (N-k)L\right) < \frac{1}{N}\left(k\pi(1) - (N-k)L\right)$$

that is, if

$$\sum_{l=1}^{k}\pi(l) < k\pi(1) \tag{12}$$

which is always true for all $k > 1$ given the definition of $\pi(s)$. Thus $k^i = k(> 1)$ for all i cannot be a Nash equilibrium.

From equations (11) and (12), the symmetric outcomes cannot be a Nash equilibrium of the game if $N - (N-1)^{1+\alpha} < 0$. On the other hand, if $0 \le L \le \frac{N}{(N-1)^{1+\alpha}} - 1$, then $k^i = 1$ for all i is a Nash equilibrium of the game, as stated in Proposition 3. For example, in the case of a linear payoff ($\alpha = 1.0$), a symmetric outcome is not a Nash equilibrium for $N > 2$.

Now let us consider whether the completely asymmetric outcome can be a Nash equilibrium. Without loss of generality, let us consider an outcome such that agent i chooses strategy i, i.e, $k^i = i$ for all i. Since the payoff is the lowest for $i = N$ in this particular case, agent N has the highest incentive to deviate. We consider the possible payoffs that $i = N$ could obtain by deviating from $k^N = N$ to another strategy $k^N = k < N$.

The expected payoff for agent N of a deviation to strategy $k^N = N - 1$ while other agents are using $k^i = i$ for all $i < N$, $\Pi^N(N-1|k^i = i, \forall i \neq N)$, can be computed for generic N:

$$E(\Pi^N(N-1|k^i = i, \forall i \neq N)) = \sum_{l=1}^{N-1} \frac{1}{(l+1)l}\pi(N-l) - \frac{L}{N} \tag{13}$$

In general, the expected payoff from a deviation to strategy $k^N = N - j(j \ge 1)$ is given by

$$E(\Pi^N(N-j|k^i = i, \forall i \neq N)) = \sum_{l=1}^{N-j} \frac{1}{(l+1)l}\pi(N-j+1-l) - \frac{L}{N-j+1} \tag{14}$$

Since $\pi(N) = 0$, the condition for $k^i = i$ for all i to be an equilibrium is, therefore, given by Eq. (6) as stated in Proposition 4. Note that the critical L increases with N. The larger is the number of agents (and parking spots), the higher is the cost required to sustain this equilibrium.[6]

Summarizing, when everyone searches for parking spots every period, i.e, $p_e = p_l = 1$, we find that symmetric Nash equilibria exist only for α and L being very small. We have also shown that unless the loss of not parking before the center, L, is large enough,[7] the completely asymmetric outcome is not a Nash equilibrium.

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[6]It should be noted that deviating to $N - 1$ does not necessarily generates the highest expected payoff for player N.

[7]In the case of a linear payoff ($\alpha = 1.0$), the critical L for having the completely asymmetric Nash equilibrium is larger than the maximum gain from trying to parking at free spots along the

For intermediate values of α and L we do not have a characterization of Nash equilibria for general N. For $\alpha = 1$ and $L = 0$ we found the following Nash equilibria for small N

$$\begin{aligned} N = 2: &\quad \mathbf{k} = \{1, 1\} \\ N = 3: &\quad \mathbf{k} = \{1, 1, 2\} \\ N = 4: &\quad \mathbf{k} = \{1, 1, 2, 2\} \\ N = 5: &\quad \mathbf{k} = \{1, 1, 2, 2, 3\} \\ N = 6: &\quad \mathbf{k} = \{1, 1, 2, 2, 3, 4\} \\ \dots \end{aligned}$$

which suggests that as N increases Nash equilibria exhibit an increasing degree of asymmetry. For $L > 0$ we expect an even higher degree of asymmetry.

4 Learning dynamics

What we are interested in this paper, however, is not Nash equilibria per se. We are interested in whether agents learn to behave in such a way that some agents choose strategies with small k (lucky ones) while others chooses strategies with large k (unlucky ones). In order to address this point, we need to introduce the learning processes.

As stated above, every time agent i searches for a parking spot, she chooses a strategy k that involves her parking at the first empty spot that is no worse than (as close to the center as) spot k (i.e., $s \leq k$). The choice of strategy in period t depends on her past experiences from choosing (and not choosing) various strategies. The past experiences are summarized by agent i's attractions to each strategy, $A_k^i(t)$, at the beginning of the period.

The probability with which she chooses strategy k in period t is

$$P_k^i(t) = \frac{\exp(\lambda A_k^i(t))}{\sum_{j=1}^{S} \exp(\lambda A_j^i(t))} \tag{15}$$

where λ is a parameter of the model that determines the sensitivity of strategy choice to attractions. If $\lambda = 0$, all the strategies are equally likely to be chosen regardless

street, $L \geq \pi(1) - \pi(S) = 1$ for all $N \geq 2$.

of their attractions. As λ becomes larger, the strategies with higher attractions become disproportionally more likely to be chosen and, if $\lambda = \infty$, the strategy with the highest attraction will be chosen with probability one. As shown by Weisbuch et al. (2000), λ controls the balance between exploration and exploitation. The logistic transformation introduced here is common in the literature on learning (see, for example, Erev and Roth, 1998; Camerer, 2003; Brock and Hommes, 1997, 1998).[8] We assume that all the agents have the same attractions for all the strategies at the beginning of period zero, and that this is equal to the average payoff of parking along the street, i.e., $A_k^i(0) = \frac{1}{S}\sum_{s=1}^{S}\pi(s)$ for all k and i.

After searching for a parking spot in period t, the attraction for strategy k evolves as follows:

$$A_k^i(t+1) = \omega A_k^i(t) + (1-\omega)R_k^i(t) \tag{16}$$

where ω is the weight put on the past value of attraction. Here the reinforcement $R_k^i(t)$ is equal to the payoff i has received in period t for all the parking spots where agent i has actively searched for a spot, i.e.,

$$R_k^i(t) = \begin{cases} \pi[s_i(t)] & \text{if} \quad s^i(t) \leq k \leq k^i(t) \\ 0 & \text{else} \end{cases} \tag{17}$$

Here $k^i(t)$ is the strategy i has chosen in t and $s^i(t)$ is the spot i has parked in that period, i.e., the first empty spot i found such that $s \leq k^i(t)$. We set the convention that $s^i(t) = 0$ if i failed to find an empty spot, and we set accordingly $\pi(0) = -L$. Outside the searched interval, i.e. for $k \notin [s^i(t), k^i(t)]$, we set $R_k^i(t) = 0$, and the attractions depreciate. Note that attractions are not updated and remain constant when the agent i does not search.

The fact that agents do not update attractions for strategies outside the interval between where they start searching and where they actually park is reasonable in a volatile context such as the one we are interested here. Opportunities may have a short lifetime and be picked up by other agents. Counterfactually updating the attractions of strategies which were not really played may be unfeasible and/or unrealistic.

[8] Erev and Roth (1998); Camerer (2003) use the logistic formulation to better explain the behavior of experimental subjects in the laboratory experiments. Motivated by their experimental results, McKelvey and Palfrey (1995) use this idea to develop the "Quantal Response Equilibrium", which they consider to be a better solution concept because it allows for noisy action choices by players. Brock and Hommes (1997, 1998) use this logistic function in models where players learn about performance of various price forecasting strategies in market setting and decide which strategies to choose.

In the model considered here, all the agents are *ex ante* identical. In particular, we assume there is no heterogeneity in risk preferences among agents in order to understand the role of learning in generating heterogeneous behaviors. However, one interpretation of our results is that agents learn which level of risk aversion to have. Thus the “unlucky” agents have learned to be more risk averse than the “lucky” ones.

As discussed in the introduction, the model has a wider applicability than the particular example of parking along the one way street. In general, it describes a situation where agents learn what to accept, while they form their beliefs about the distribution of opportunities. The underlying distribution of the available opportunities, in its turn, is determined endogenously, as it depends on what other agents have learned to accept.

4.1 Learning in the $N = S = 2$ case

Let us consider the simple example of two agents and two parking spots with $p_e = p_l = 1$. As discussed above, player i's choice of strategy in period t depends on her attraction for strategy k, A^i_k, asshown in Eq. (15). Let

$$\mathcal{A}(t) = \begin{pmatrix} A^1_1(t) & A^1_2(t) \\ A^2_1(t) & A^2_2(t) \end{pmatrix}$$

be the matrix of attractions for two agents in period t. Since the attractions evolves as in Eq. (16), we have

$$\mathcal{A}(t+1) = \omega \mathcal{A}(t) + (1-\omega)\mathcal{R}(t)$$

where $\mathcal{R}(t)$ is the matrix of stimulus agents receive for two strategies in period t, i.e.,

$$\mathcal{R}(t) = \begin{pmatrix} \left[\frac{1+L}{2}k^2(t) - L\right][2-k^1(t)] + \frac{1}{2}[k^2(t)-1][k^1(t)-1] & \frac{1}{2}[k^2(t)-1][k^1(t)-1] \\ \left[\frac{1+L}{2}k^1(t) - L\right][2-k^2(t)] + \frac{1}{2}[k^1(t)-1][k^2(t)-1] & \frac{1}{2}[k^1(t)-1][k^2(t)-1] \end{pmatrix}$$

where the entry in row i and column k corresponds to the stimulus that agent i receives for strategy $k^i = k$, given that the other player plays $k^{-i}(t) \in \{1,2\}$, where $-i$ denotes agent 2 if $i = 1$ and 1 if $i = 2$. The probability that agent i choses strategy $k = 1$ can be written as

$$\text{Prob}\{k^i = 1|t\} = P_1^i(t) = \frac{e^{\lambda A_1^i(t)}}{e^{\lambda A_1^i(t)} + e^{\lambda A_2^i(t)}} = \frac{1}{1 + e^{-q^i(t)}} \tag{18}$$

where

$$q^i(t) = \lambda[A_1^i(t) - A_2^i(t)].$$

This means that choice behavior depends only on $q^i(t)$. The learning dynamics can be derived for this quantity, taking the difference of the equations for $A_1^i(t)$ and $A_2^i(t)$. With a little algebra, one finds

$$q^i(t) = \omega q^i(t-1) + (1-\omega)\lambda(2 - k^i(t-1)) \left[\frac{1+L}{2} k^{-i}(t-1) - L\right] \tag{19}$$

and similarly for agent 2. Notice that $q^i(t)$ is a stochastic variable which depends on the choice $k^{-i}(t-1) \in \{1, 2\}$ of the other agent, which in turn is drawn from a distribution which depends on $q^{-i}(t-1)$.

In the limit $\omega \simeq 1$ the second term is small compared to the first, which means that if $q^i(t)$ reaches a fixed point value, the stochastic deviations from this will be small. In other words, it is reasonable to approximate $k^{-i}(t-1)$ by its expected value $E[k^{-i}(t-1)] = 2 - \text{Prob}\{k^{-i}(t-1) = 1\}$ using Eq. (18). This leaves us with a dynamical system for the two variables (q^1, q^2) which has the form

$$q^i(t) = \omega q^i(t-1) + (1-\omega)\lambda \frac{1}{1 + e^{-q^i(t-1)}} \left[1 - \frac{1+L}{2} \frac{1}{1 + e^{-q^{-i}(t-1)}}\right]$$

Fixed points of these equations are of the general form $q^1 = q + z$ and $q^2 = q - z$ where q and z, after some algebraic manipulations, are solutions of the following equations:

$$q = \frac{\lambda}{2} \left[\frac{(1-L)e^q/2 + \cosh z}{\cosh q + \cosh z}\right] \tag{20}$$

$$z = \frac{\lambda}{2} \left[\frac{\sinh z}{\cosh q + \cosh z}\right] \tag{21}$$

The second of these equations is always satisfied if one sets $z = 0$. This corresponds to a symmetric equilibrium where agents behave in the same way ($q^1 = q^2$).

In order to study the stability of the solutions, we can introduce the variable

$\zeta(t) = [q^1(t) - q^2(t)]/2$. The same algebra as above, leads to

$$\zeta(t+1) = \omega\zeta(t) + (1-\omega)\frac{\lambda}{2}\frac{\sinh\zeta(t)}{\cosh\chi(t) + \cosh\zeta(t)} \tag{22}$$

where $\chi(t) = [q^1(t) + q^2(t)]/2$. Now, the linear stability of the equilibrium with $\chi = q$ and $\zeta = z = 0$ can be studied through the linearization of Eq. (22) around $\chi = q$ and $\zeta = 0$. This yields

$$\zeta(t+1) \cong \left[\omega + (1-\omega)\frac{\lambda}{2}\frac{1}{1+\cosh q}\right]\zeta(t) + O(\zeta^2). \tag{23}$$

Therefore the solution $z = 0$ is stable as long as

$$\frac{\lambda}{2}\frac{1}{1+\cosh q} \leq 1. \tag{24}$$

The region of stability of the symmetric state, in the plane (λ, L), can be obtained in parametric form (i.e. varying q) combining Eqs. (20) with $z = 0$ and (24). The result is shown in Fig. 1. The increase of λ always drives the system from the symmetric to the asymmetric state. The behavior of the system has however, a non-trivial dependence on L: For a fixed $\lambda > 4$, the symmetric state is stable either for small L or for large L. For small L, the symmetric state where both players choose strategy 1 is stable, while for large L, the state in which both players choose strategy 2 is stable.

5 Simulation results

The cases with larger Ns are not analytically tractable so that we employ numerical simulations to analyze the learning dynamics. The model presented in the paper has eight parameters: the number of agent N, the number of parking spots S, the probability of each agent coming into the city if she is outside the city, p_e, and leaving the city if she is parked in the city, p_l. The parameter α determines the shape of the payoff function, L gives the size of loss in case of failing to park before the city center, λ governs the sensitivity of parking decisions to attractions to park and not to park. ω represents the importance of past experiences in the evolution of attraction.

Initially, we randomly assign agents to parking spots, and let each agent leave the spot with probability p_l. To ensure that the number of agents who look for a parking

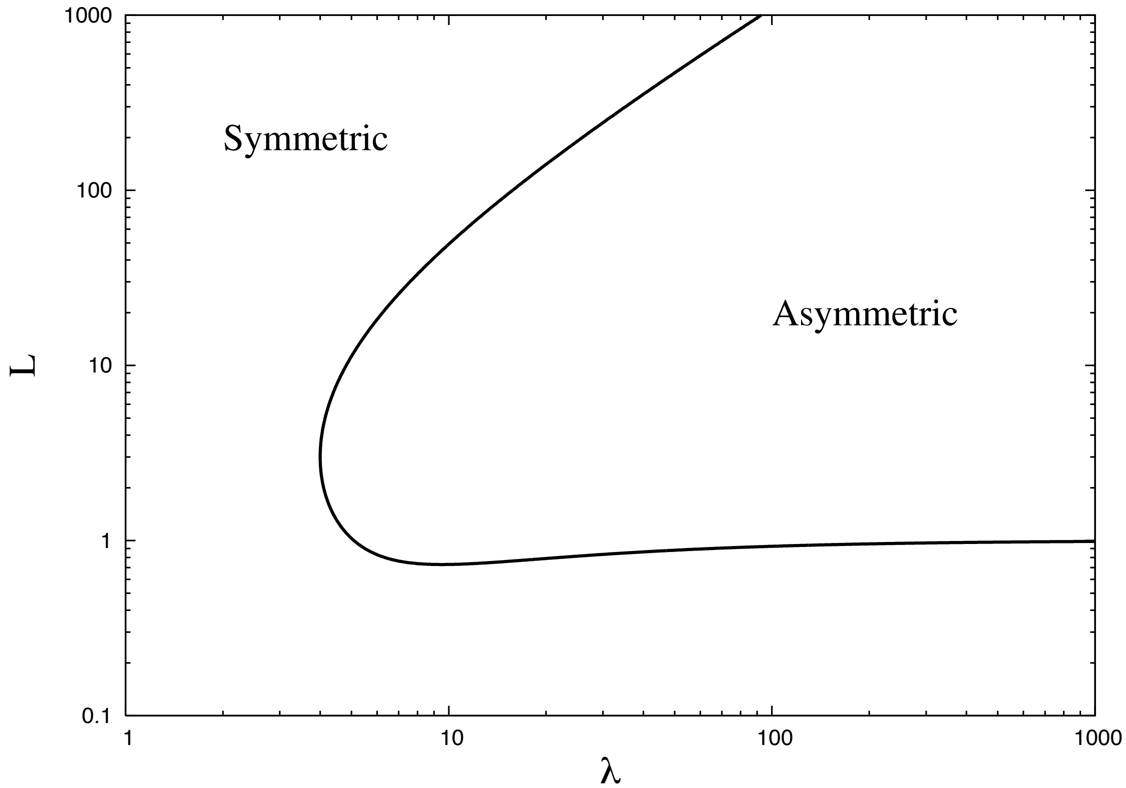

Figure 1: Region of stability of the symmetric and asymmetric solutions of the learning dynamics for the $N = 2$ case.

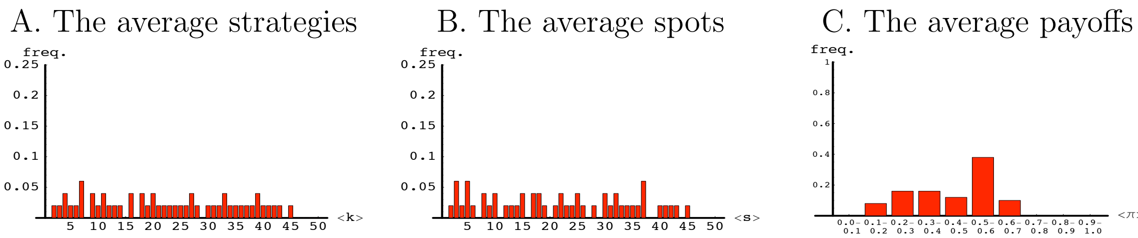

Figure 2: The distribution of (A) the average strategies, (B) the average spots parked, and (C) the average payoffs for each agent. Note that $s = 1$ is the city center and the larger the s is, the further the spot is from the center. $N = S = 50$, $p_l = 1.0$, $L = 0.5$, $\alpha = 1.0$, $\lambda = 100.0$, $\omega = 0.9$. The mean (the standard deviation) of the distributions of average strategies, average spots, and average payoffs are 22.2 (12.7), 21.4 (12.8), and 0.4 (0.2), respectively.

spot in one period and the number of available parking spots at the beginning of the period is the same on average[9], we set $p_e = \frac{p_l S}{N-(1-p_l)S}$. Thus, effectively, we have one less parameter for the model. In addition, we first fix the number of agents N to the same as the number of parking spot S such that $N = S = 50$.[10]

5.1 Asymmetric outcomes

Figure 2 shows three distributions: (A) the distribution of the average strategies chosen, (B) the distribution of the average spots each agent parked[11] (average parked spot), and (C) the distribution of the average payoffs each agent received per search[12] from a single simulation run.

The parameters are set so that all the parked agents leave the city at the end of the period, $p_l = 1.0$, the loss of not finding a parking spot before the city center L is 0.5, the payoff is linear with respect to the relative distance of the spot from the center, $\alpha = 1.0$, players have quite a long memory of past experiences $\omega = 0.9$, and are sensitive to the attractions in choosing their strategies, $\lambda = 100.0$.

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[9] An estimate of the number of available parking spots is obtained assuming that all parking spots were occupied in the previous period and that each of them was made available with probability p_l, before the present period. This yields Sp_l free parking spots, and $(1 - p_l)S$ agents in the city parked. Thus, the number of agents outside the city at the beginning of the period is $N - (1 - p_l)S$ and therefore, $p_e(N - (1 - p_l)S)$ agents will enter the city.

[10] Note that when $N = S$, we have $p_e = 1$.

[11] Let $n^i(s)$ be the number of times agent i has parked at spot s in the periods under consideration, and let $f^i(s) = \frac{n^i(s)}{\sum_j n^i(j)}$ be the relative frequency at which agent i parked at s. Then the average spot agent i parked is $< s^i >= \sum_s sf^i(s)$.

[12] Although the average parked spots do not take into the account agents failing to park before the center, such cases are included in calculating the average payoff per search.

The data are taken from 500 periods, after letting the simulation run for 5000 periods, i.e., from $5001 \leq t \leq 5500$. Given the parameter values, agents search for a parking spot 500 times during these 500 periods.[13]

The panel (A) and (B) of Fig. 2 show substantial variations in the strategies and spots chosen by agents. The standard deviations of the distributions the average strategies chosen and average spots parked are 12.8 and 12.7, respectively. For a comparison, one should note that the size of standard deviation under the maximum heterogeneity is 14.58 for both distributions.[14] One can see from the figures that while there are agents who have chosen strategies such that they park at empty spots far away from the center (large k), there are agents who have chosen strategies to park very close to the center (small k). Some learned to obtain luckier outcomes while others did not. Correspondingly, their average payoffs vary.

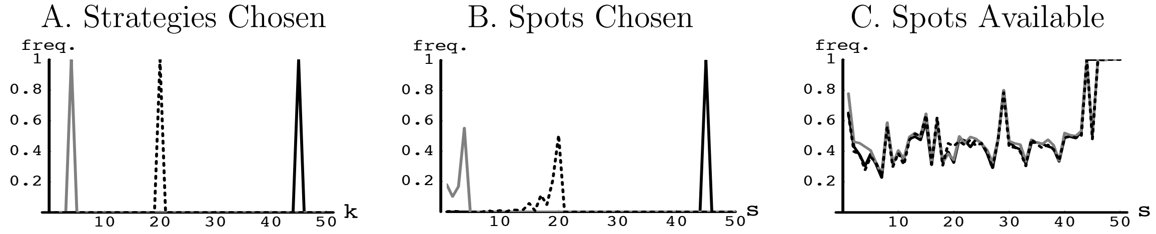
While the three distributions shown in Fig. 2 demonstrate the heterogeneity in the average behaviors of agents, we would also like to know how each agent has behaved. For example, where has an agent who parked on average at spot 25 parked? Has she parked in a wide range of spots with similar frequencies or in a few spots around 25 with very high frequencies and not elsewhere? To see this, we have plotted the distribution of strategies chosen (panel A) and the distribution of parked spots (panel B) for three agents who obtained the lowest (solid black), median (dashed black), and the highest (solid gray) average payoff per search in Figure 3. The figure also shows, in panel C, the distribution of spots that were available when these three agents searched. Also presented in the figure (panel D) are the basic statistics, such as the average spot parked and the average strategies chosen, for three agents.

The average payoff per search for the agents with the lowest, median, and the highest payoffs were 0.10, 0.48, and 0.63, respectively. The figure shows that agents do not park uniformly across the spots. The distributions of the strategies chosen by three agents differ substantially from each other. The agent with the lowest average payoff has chosen strategies leading her to park in the first empty spot and, therefore, far away from the center (the average strategy was 45). The agent with the highest payoff, on the other hand, has chosen strategies 4.0, i.e., to park very close to center, although not right at the center. The agent who received the median payoff has chosen to park in the first empty spot beyond the half way to the center (average strategy was 20.0.).

As noted above, the highest average payoff per search was 0.63 for this particular

[13]The simulation results show that, given the parameter values, the system reaches stationary states after 5000 periods. See the Figure 11 in the Appendix.

[14]The maximum heterogeneity can be achieved when both distributions are uniform.



(D) Basic Statistics for Three Agents

	Ave. Payoff	Ave. Spots	Ave. Strategy	Success Rates	No. of Search
Highest	0.63	3.1	4.0	0.78	500
Lowest	0.10	45.0	45.0	1.00	500
Median	0.48	18.4	20.0	0.86	500

Figure 3: The distribution of (A) strategies chosen by agents, (B) spots chosen by agents, and (C) available spots at the time of search for agents with the lowest (solid black), median (dashed black), and the highest (solid gray) average payoff per search. The basic statistics for these three agents are shown in (D). $N = S = 50$, $p_l = 1.0$, $L = 0.5$, $\alpha = 1.0$, $\lambda = 100.0$, $\omega = 0.9$. For $5001 \leq t \leq 5500$

simulation. But this seems to be too low considering that the agent with the highest average payoff parked on average around spot 3, which should generate an average payoff close to 0.94. Why is her payoff so low? The reason is because such agents sometimes fail to park before reaching the center, and obtain the loss $-L$. The success rate, the probability of successfully parking before the center, for the agent with the highest payoff was 0.78. While those for the agent with lowest and median payoff were 1.0 and 0.86, respectively. These numbers show that agents who parked closer to the center often succeeded in doing so, but sometimes failed. A quick calculation verifies the seemingly low payoff. The agent with the highest payoff obtained, on average, payoff about 0.94 at success rate 0.78, but failed in about 0.22 of the times and lost 0.5, yielding an average payoff of about 0.63. The remarkable point is that, despite of such costly failures, the agent with the highest payoff held on to strategies inducing parking close to, and did not use the ones leading to parking far away from, the center.

The large differences in the strategies and spots chosen by these agents, as well as their payoffs, cannot be explained by the differences in available opportunities. The panel C of the Figure 3 shows that, at the time of search, there is no major differences, at least on average, in the available parking spots. Thus, one can conclude that agents learned to behave in different ways and that this resulted in substantial variations in the outcomes, although they were facing the similar opportunities.

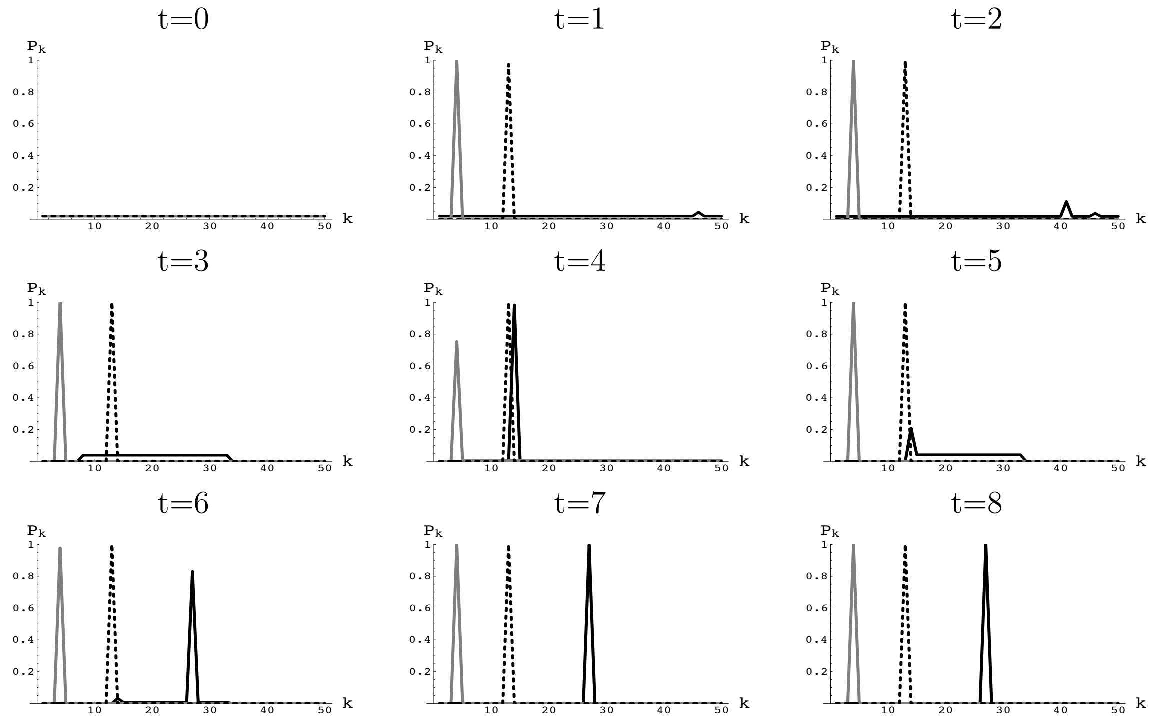


Figure 4: Initial dynamics of probabilities of choosing each strategy for 3 players shown in Fig 3. The colors correspond to that in Fig 3 as well. Note that for $t = 0$, all the players are identical.

To better understand the dynamics of individual behavior, the initial dynamics of the probabilities with which these three agents choose their strategies, $P_k^i(t)$, are shown in Figure 4. As noted above, all these agents are identical at the very beginning of the simulation, $t = 0$. Thus, they all have the identical probability distribution over strategies. As shown in the figure, agents first choose their strategies randomly. Based on the obtained payoffs and chosen strategies, they change their strategy choices. For example, the agent with the highest average payoff during $5001 \leq t \leq 5500$ (shown in gray) has chosen $k = 4$ in period 0 and obtained a high payoff. As a result, he learned to play $k = 4$ with a probability close to one from $t = 1$ on. But the probability of this agent choosing strategy 4 becomes a bit lower in $t = 4$. This is because the agent failed to find a spot in period 2 and 3. But the agent tries with strategy 4 again in period 4 and was successful in parking at spot 3. As a result, the probability of her choosing strategy 4 rose again. Although she failed to park once more in the following period, her probability of choosing strategy 4 remained high. What we can observe here is that once an agent learns to play a strategy with small k, unless she failed to park before the center too frequently (or the loss of doing this is too large), she continues to play such strategy.

The results for the agent with the lowest average payoff during $5001 \leq t \leq 5500$ (shown in solid black) is more complex. The agent has first chosen strategy 46 and parked in the spot 46. As a result, her probability of choosing the same strategy became a little bit higher than the other strategies (see $t = 1$). Next, she has chosen strategy 41 and again successfully parked at spot 41 (t=2). In the next period, she has chosen strategy 33 and parked at spot 8. This results in putting higher probabilities on choosing strategies $k \in [8, 33]$ (t=3). Among which, she chose strategy 14 in the following period and parked at spot 14 resulting in placing a very high weight in choosing strategy 14 (t=4). In the following period, he chose strategy 14 but failed to park. As a result, the relative weights on strategies $k \in [1, 14]$ become lower while those for $k \in [15, 33]$ become higher (t=5). He then chose strategy 27 and parked at spot 27 (t=6). The agent continues to choose strategy 27 and park at the spot 27 for a while. But as one can see from the result shown in Figure 3, she learned to park even further away in later periods.

5.2 Dependence on parameters

The previous subsection has shown, for a particular set of parameter values, that agents who are initially identical, learn to behave differently and sort themselves into parking at the different spots along the street. Some agents learned to park

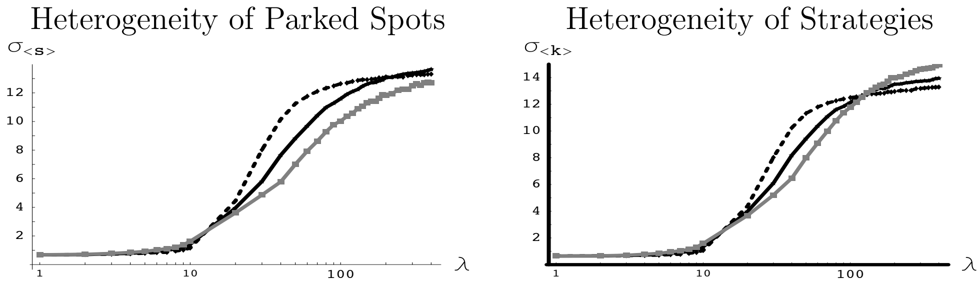


Figure 5: The effect of λ on the heterogeneity of parking behaviors. Dashed: $L = 0.5$, Black: $L = 1.5$, Gray: $L = 2.5$. The heterogeneity of behavior is measured by the standard deviation of the distribution of the average parked spots (left) and by the standard deviation of the distribution of the average strategies (right) . The statistics are based on taking average from 100 simulation runs. Error bars represent one standard deviation bound from the 100 simulations. $N = S = 50$, $p_l = 1.0$, $\alpha = 1.0$, $\omega = 0.9$.

far away from, while the others learned to park near to, the center. That is, some learned to be unlucky while others learned to be lucky. How do such results depend on the parameters of the model? This subsection analyzes the dependence of the results on the parameter values.

The simple 2 players case we have considered above shows that there is a critical value of λ beyond which we will see asymmetric outcomes. The analysis also shows that the result depends on L. Therefore, we first consider the dependence of the result on λ and L. As one can easily imagine, when $\lambda = 0$, agents always choose strategies randomly regardless of the values of propensities, and thus, we will not observe any heterogeneity in the behavior of agents. What happens for larger values of λ? Figure 5 shows the standard deviation of the distribution of the average parked spots, $\sigma_{<s>}$ (left) and the standard deviation of the distribution of the average strategies, $\sigma_{<k>}$ (right) for various values of λ. We consider three values of the loss from failing to park before the center, $L = 0.5$ (dashed black), $L = 1.5$ (solid black), and $L = 2.5$ (solid gray).[15] Each point in the figure is the average value from 100 simulation runs.

The figure shows that heterogeneity increases very sharply with λ for $10.0 \leq \lambda \leq 100$, and then it gradually reaches a plateau. This pattern can be observed for all three values of L. The figure also suggests that L may have an effect on the degree of heterogeneity for $\lambda \in [20, 100]$ but less when λ is large enough. To

[15]Other parameters are fixed as in the previous section (Namely, $N = S = 50$, $p_l = 1.0$, $\alpha = 1.0$, $\omega = 0.9$). The data are taken from the 500 periods after the first 5000 periods has past, i.e., from $5001 \leq t \leq 5500$.

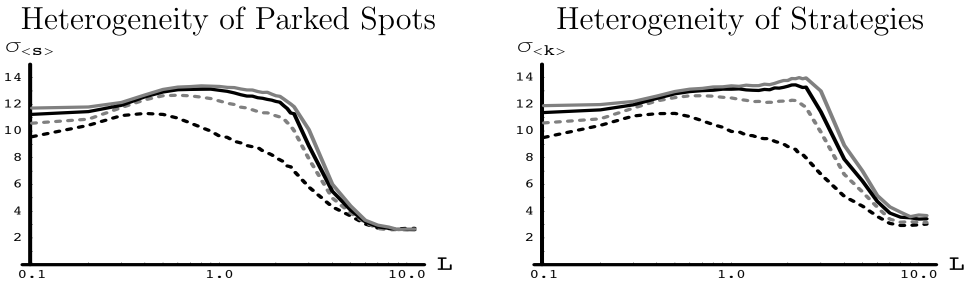


Figure 6: Effect of L on heterogeneity of parking behavior. Dashed Black: $\lambda = 50.0$, Dashed Gray: $\lambda = 100.0$, Black: $\lambda = 150.0$ Gray: $\lambda = 200.0$

see this differently, Figure 6 plots heterogeneity of parked spots and heterogeneity against various values of the loss, L, for four values of λ: $\lambda = 50.0$ (dashed black), $\lambda = 100.0$ (dashed gray), $\lambda = 150.0$ (solid black) and $\lambda = 200.0$ (solid gray). As in the previous figure, each point in the figure is the average value from 100 simulation runs.

The figure shows that for $\lambda = 150$ and $\lambda = 200$ the results are not distinguishable for all values of $L \in [0.0, 2.5]$. The basic pattern is that heterogeneity first increases slightly with L, and in the interval $0.5 \leq L \leq 2.0$, it either decreases gradually or, in some cases, stays constant (heterogeneity of strategies for $\lambda \in \{150, 200\}$). For larger values of L, the degree of heterogeneity declines quite drastically for all values of λ considered here. The lower degree of heterogeneity under large L is in line with the analytical results for $N = S = 2$ case shown above. The fact that heterogeneity is high even at $L = 0$ is in line with our equilibrium considerations for large N.[16] As noted in the previous section, when L is very high, agents learn to choose higher k strategies, and the rate at which agents fail to park before the center is lower.

These figures show that as long as λ large enough, and L not too high, we have very high degree of heterogeneity in the behavior as a result of learning.[17] This should be contrasted with the equilibrium analysis in previous section. There, the result showed that the larger the value of L is, the easier it is to sustain heterogeneous outcome.

[16] Recall that the equilibrium analysis for $p_l = p_e = 1$ suggests that for $N > 2$, even with $L = 0$ we should expect a high degree of heterogeneity.

[17] One should note that these results are from 500 periods. Because of all the strategies always have non-zero probabilities of being chosen, in a very long run, strategies chosen by agents may change. As shown in Figure 12 in the Appendix, it is indeed the case. If we consider much longer periods (say 25000 periods) over which to observe the behavior of agents and measure heterogeneity, then degree of heterogeneity will be smaller, although it is still much larger than for the $\lambda = 0.0$ case.

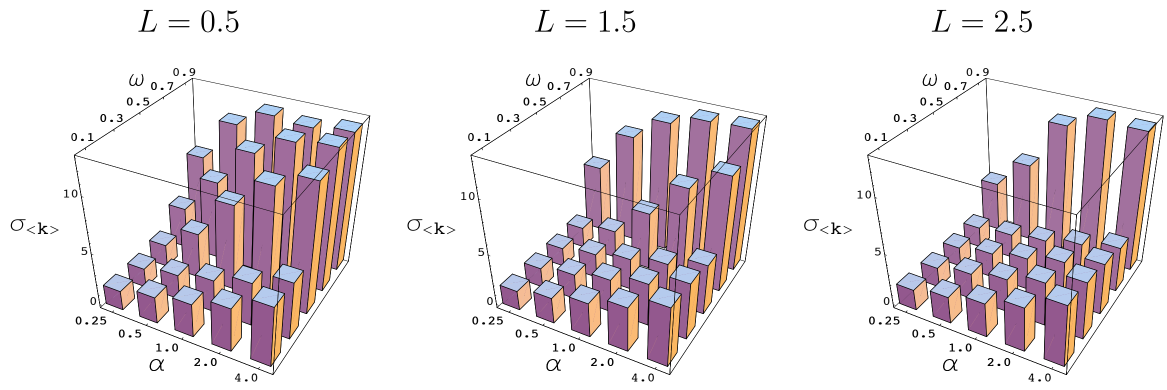


Figure 7: Standard deviations of the distribution of average strategies ($\sigma_{<k>}$) for various combinations of $\alpha \in \{0.25, 0.5, 1.0, 2.0, 4.0\}$ and $\omega \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ for three values of L: $L = 0.5$ (left), $L = 1.5$ (middle), $L = 2.5$ (right). $\sigma_{<k>}$ is the average from 100 simulation runs. For all the plots $\lambda = 100.0$, $N = S = 50$, $p_l = 1.0$, and data are taken from $5001 \leq t \leq 5500$.

How do the results depend on other parameters? Let us now consider ω and α. Figure 7 show the heterogeneity of parking behavior (measured by the standard deviation of the distribution of the average strategies chosen, $\sigma_{<k>}$) for various combination of α and ω. Three values of L, $L \in \{0.5, 1.5, 2.5\}$ are considered. What figure shows is that ω has to be large for there to be a high degree of heterogeneity in the behavior. The critical ω for which we obtain a high degree of heterogeneity increases with L.

The figure also shows that heterogeneity emerges more easily when α is high. This is reasonable since high α means that most of the spots are as good as parking near the center. On the contrary, when α is small, then most of the spots are as bad as parking at the furthest spots, thus, we see much lower degree of heterogeneity.

One of the reason for running extensive sets of simulations was to see how the probability at which those agents who are in the city leave, p_l, affects the resulting outcome. This was because the equilibrium analysis produced quite different results in the two extreme cases, namely the one where p_l and p_e are very small and the other where $p_e = p_l = 1$. In particular, the equilibrium analysis suggested that when $p_l = p_e = 1$ we should not expect to see homogeneous outcomes except for very special values of parameters, while when p_e and p_l are very small, homogeneous outcomes always exists.

To see the effect of p_l, Figure 8 shows the heterogeneity of parked spots, σ_s (left), and chosen strategies σ_k (right), for various p_l and three values of $L \in \{0.5, 1.5, 2.5\}$

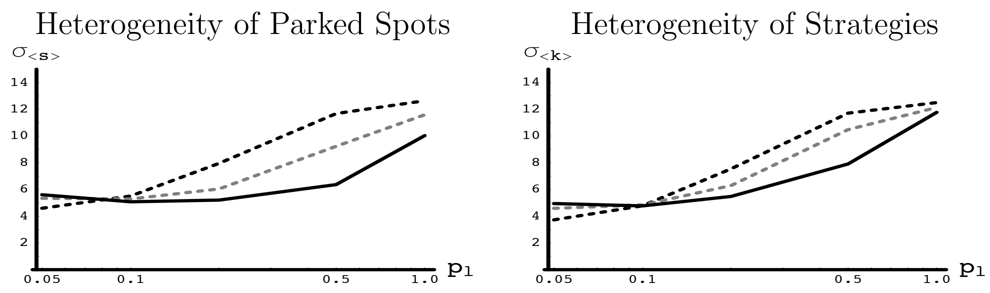


Figure 8: Effect of p_l on heterogeneity of parking behavior. Dashed Black: $L = 0.5$, Dashed Gray: $L = 1.5$, Black: $L = 2.5$. For all the simulation, $N = S = 50$, $\lambda = 100.0$, $\alpha = 1.0$, $\omega = 0.9$

(dashed black, dashed gray, and solid black). Other parameters are set so that $\lambda = 100$, $\alpha = 1.0$, and $\omega = 0.9$. It can be observed easily from the figure, there is an increasing relationship between p_l and the resultant heterogeneity. Namely, the higher the p_l is, the greater the σ_k (and thus, σ_s) becomes.[18]

5.3 Heterogeneous Agents

In the previous section, we considered cases where agents are *ex ante* identical, and showed that heterogeneity in the behavior emerges as agents learn. We have also shown that the emergence of heterogeneous behaviors depended, partially, on parameter values. For example, when λ is too low, we did not see differences in the behavior of agents. What happens if agents are not identical? In particular, what happens if there are agents with a low λ and a high λ? Since low λ agents behave quite randomly, such random behavior may interfere with how agents with the high λ learn to behave. We have experimented with half of the agents having $\lambda = 0.0$, who thus behave randomly, and the other half having $\lambda = 100.0$ for three values of α holding other parameter values the same as in Figure 2 and 3.

The results are shown in Figure 9 and 10. Figure 9 shows the distribution of the average strategies chosen and the average spots agents parked. As before these two looks quite similar. The spike around $< k >= 25$ and $< s >= 24$ is due to the existence of agents with $\lambda = 0.0$. Since these agents choose strategies randomly, on average, their chosen strategy will be close to 25. A part from the peak, the resulting distributions are still flat, suggesting there are lucky and unlucky agents.

[18]Note that when $p_l < 1$, agents do not search for parking every period. In order to make the number of searches, at least on average, as similar as possible across various p_l, we have taken data from $5001 * p_l^{-1} \leq t \leq 5500 * p_l^{-1}$.

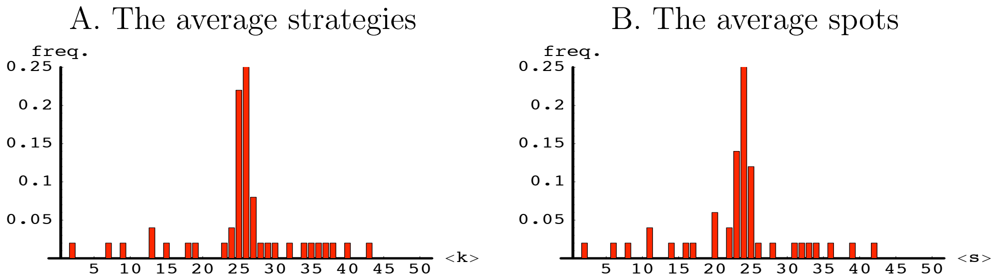


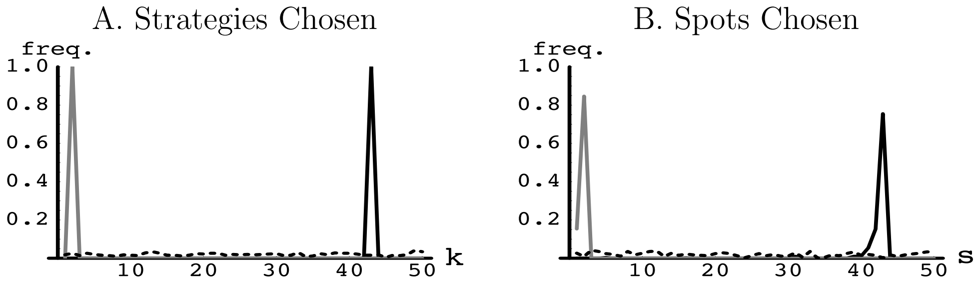
Figure 9: The distribution of (A) the average strategies chosen and (B) the average spots parked.Half of the agents have $\lambda = 0.0$ and the other half have $\lambda = 100.0$. $N = S = 50$, $p_l = 1.0$, $L = 0.5$, $\alpha = 1.0$, $\omega = 0.9$.

In particular, as Figure 10 shows, the agent with the highest (lowest) average payoff always parks close to (far away from) the center as in the case where all the agents had $\lambda = 100.0$. These two agents had $\lambda = 100.0$. One the other hand, the agent with the median payoff had $\lambda = 0.0$, and parked almost uniformly randomly across all the parking spots. The figure shows that the heterogeneity in the behaviors emerges even in the presence of randomly behaving agents.

6 Summary and Conclusion

In this paper we consider the idea that luck may not be totally exogenous. In a number of situations, otherwise identical individuals can find themselves in very different states. Those who are in the worse positions can be considered as "unlucky", yet if the choices available are unequal, then any allocation must treat people asymmetrically. It could well be the case that all people choose the same strategy and some by pure luck get the better opportunities and others do worse. What interested us, however, was whether it could be the case that some people choose, *ex ante*, to be in less favorable positions. Thus it is not the luck of the draw but the conscious choice of the individuals to be well or poorly treated.

Although we have chosen a simple parking problem as our basic example, there are many other cases in which individuals have to choose amongst a set of alternatives, whilst other are doing the same and there is no possibilty of going back to the ones that have been rejected. One has only to think of dating problems, to see that going back to the best alternative amongst those one has gone out with might be problematical. In a more serious vein it is not always possible to retake a job offer that one has previously refused and it may well be the case that other jobs have



(c) Basic Statistics for Three Agents

	Ave. Payoff	Ave. Spots	Ave. Strategy	Success Rates	No. of Search
Gray	0.72	1.85	2.0	0.82	500
Black	0.15	42.4	43.0	0.99	500
Dashed	0.43	24.2	26.5	0.91	500

Figure 10: The distribution of (A) the strategies chosen and (B) the spots parked for agents with the lowest (solid black), median (dashed black), and the highest (solid gray) average payoff per search. Half of the agents have $\lambda = 0.0$ and the other half have $\lambda = 100.0$. Note that $s = 1$ is the city center and the larger the s is, the further the spot is from the center. $N = S = 50$, $p_l = 1.0$, $L = 0.5$, $\alpha = 1.0$, $\omega = 0.9$.

already been taken.

Thus while our example is, of course, special it can be re-interpreted to cover a number of other economic situations. In our case, for some individuals to choose inferior options leaving the better ones for others can be an equilibrium of the associated game. Whilst this is interesting in the one-shot case, it is important to understand what happens in a repeated situation. The question we posed was whether, in such a case, the individuals can learn to be "unlucky" or "lucky". That is, we were interested in whether some individuals learn to systematically make the poorer choices, and thus always leaving the better opportunities open and letting others learn to be "lucky".

We showed analytically, in simple cases, that this can happen. Both symmetric and asymmetric equilibria can arise, depending on features of the environment where agents interact. One key parameter is the frequency (p_e, p_l) with which agents seek and leave parking spots, i.e. parking turnover. We identify two polar situations in the static analysis of Nash equilibria. When parking turnover is slow ($p_e, p_l \ll 1$), we find that asymmetric outcomes, when N is large, happen either when the cost L is extraordinarily large, or when the number of free spots is small, in the stationary state. At the opposite extreme, we also examine the fast turnover case, when everyone searches for parking spots every period ($p_e = p_l = 1$). In that case, we find that symmetric Nash equilibria exist only when α and L are very small. At the same time, we have also shown that unless the cost of not parking before the center, L, is large enough, the completely asymmetric outcome is not a Nash equilibrium. This implies that in the fast turnover case, some degree of heterogeneity is expected under generic conditions, and it identifies fast turnover as one of the conditions favoring the emergence of "luck" as a discriminating factor across agents. Interestingly, simulations show that this intuition also holds in more complicated cases.

A second interesting point of our analysis concerns the incidence of the cost L on the resulting outcome. The analysis of learning in the case of two agents shows that asymmetric outcomes only occur in an intermediate range of values of L. When L is low (high), the symmetric outcome where both agents choose strategy 1 (2) is stable. Thus, when L is low, the street parking spots are not fully utilized, while when L is higher, they are fully utilized either because the two players choose different strategies, or because both players choose the strategy to park further away. This result is also confirmed by numerical simulations for a larger number of agents. When the number of agent is large, we do not have symmetric outcomes for low L, instead we have high degree of heterogeneity even with $L = 0$. However, for large enough values of L, the degree of heterogeneity becomes low, and agents learn to choose strategies with higher k, i.e., to park further away from the center. That is, when L is very high, it is less likely that we observe "lucky" agents. This is in quite sharp contrast to the equilibrium result that shows the higher the L is, the more likely it is to have asymmetric outcomes. However, both in the equilibrium analysis and in the learning dynamics, high enough values of L lead to the efficient use of street parking.

One last remark is in order. An alternative explanation for the sort of phenomena that we have analysed is that people simply have different intrinsic aversions to risk. However our arguments suggest that these attitudes to risk may well be learned rather than inherent. This is why we claim that the idea that some people are "born under a lucky star" does not always stand up to scrutiny.

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A The asymmetric equilibrium for $p_e, p_l \ll 1$

In order to derive the stability condition (4), let us consider the case $k_i = i$. The agent with the largest incentive to deviate is clearly the worse off, $i = N$. Let $V(k) = E[u_N(s)|k_N = k]$ be the expected payoff of agent N if she deviates to pure strategy k. Then

$$V(k) = e_k \pi(k) + (1 - e_k)V(k - 1), \quad e_k = P\{k \text{ is free}\} \tag{25}$$

because if parking $s = k$ is occupied, agent N's situation is the same as if she had chosen strategy $k - 1$. Clearly $V(N) = \pi(N) = 0$ and $V(0) = -L$, because if there is no empty parking, agent N will have to pay cost L. In the Nash equilibrium we are studying, each position is occupied by a specific agent, so the probability $e_k = h$ that spot k is free is the probability that the corresponding agent is not in the city. The solution of Eq. (25) is easily obtained introducing generating functions for $V(k)$ and $\pi(k)$ and it reads

$$V(k) = h \sum_{j=1}^{k} (1 - h)^{k-j} \pi(j) - (1 - h)^k L \tag{26}$$

It is clear that this is largest for $k = N - 1$. The condition for the stability of the Nash equilibrium can be recast as a condition on the cost agent N incurs if there is no free space, which is Eq. (4).

B The symmetric equilibrium for $p_e, p_l \ll 1$

Let $x_s = 1$ if spot s is empty and $x_s = 0$ if it is empty. In the stationary state, the detailed balance condition reads

$$P\{x_s = 1\}W\{x_s = 1 \to 0\} = P\{x_s = 0\}W\{x_s = 0 \to 1\}$$

where W are the transition rates (probability per unit time). Now, $e_s = P\{x_s = 0\} = 1 - P\{x_s = 1\}$ and $W\{x_s = 1 \to 0\} = 1$, which is just the rate at which agents leave the city. The rate $W\{x_s = 0 \to 1\}$ requires more work. The number $W\{x_s = 0 \to 1\}dt$ of agents who reach site s in a time interval dt, is proportional to the number Nh of agents not in the city, times the probability ηdt that each of them enters the city in time interval dt, times the probability q_s that she actually reaches spot s (see below). In other words, $W\{x_s = 0 \to 1\} = N\eta h q_s$. Combining these relations we have

$$e_s = \frac{1}{1 + N(1-h)q_s} \tag{27}$$

In order to compute the probability q_s that an agent arrives at spot s we have to consider all those events where she aims at some $k \geq s$ and finds no empty place before s. If σ_k is the probability of choosing to start at site k, we have

$$\begin{aligned} q_s &= \sigma_s + (1-e_{s+1})\sigma_{s+1} + (1-e_{s+1})(1-e_{s+2})\sigma_{s+2} + \ldots + (1-e_{s+1})\cdots(1-e_N)\sigma_N \\ &= \sigma_s + (1-e_{s+1})q_{s+1} \end{aligned} \tag{28}$$

This allows us to derive a backward equation for e_s in terms of σ_k:

$$e_s = \left[1 + N(1-h)\sigma_s + \frac{(1-e_{s+1})^2}{e_{s+1}}\right]^{-1} \tag{29}$$

On the other hand, agents maximize their expected utility

$$V(\sigma) = \sum_{s=1}^{N} q_s(\sigma) e_s u(s) - L q_0(\sigma)$$

with the constraint $\sum_k \sigma_k = 1$. Taking into account that

$$\frac{\partial q_s}{\partial \sigma_k} = \theta_+(k-s) \prod_{j=s+1}^{k} (1-e_j)$$

where $\theta_+(k) = 0$ if $k < 0$ and $\theta_+(k) = 1$ for $k \geq 0$, the marginal utility of choosing to start at k is

$$\mu_k = \frac{\partial V}{\partial \sigma_k} = \sum_{s=1}^{k} u(s) e_s \prod_{j=s+1}^{k} (1-e_j) - L \prod_{j=1}^{k} (1-e_j), \quad k = 1, \ldots, N. \tag{30}$$

Notice that:

$$\mu_{k+1} - \mu_k = e_{k+1}\left[u(k+1) - \mu_k\right] \tag{31}$$

The optimality condition implies that $\sigma_k > 0$ on the set of values of k where μ_k is maximal. Let us analyze Eq. (31) in detail. At $k = 1$, we have $\mu_1 = u(1)e_1 - L(1-e_1) = u(1) - (1-e_1)[L + u(1)]$. So the curve μ_k vs k starts below the curve $u(k)$ for small k. As long as this is true, μ_k increases with k, as indicated by Eq. (31), whereas $u(k)$ decreases with k by hypothesis. There should be a point k^* where the two curves cross (i.e. $u(k^*) > \mu_{k^*-1}$ and $u(k^*+1) < \mu_{k^*}$) and this is where μ_k attains its maximum. Generically, for a given e_s, the maximum will be attained on a single point k^*. Therefore, the best response will be $\sigma_k = \delta_{k,k^*}$. The only problem which remains is that of characterizing the threshold k^* as a function of η, $u(k)$ and L. Much progress can be made by taking the limit $N \to \infty$. First we set

$$e_s = \frac{1}{N}\epsilon(z), \qquad z = s/N \tag{32}$$
$$\mu_k = m(z), \qquad z = k/N \tag{33}$$
$$u(k) = \nu(z) \tag{34}$$
$$k^* = Nz^* \tag{35}$$

Then Eqs. (29,31) can be cast in the form

$$\frac{d\epsilon}{dz} = -\epsilon^2, \qquad 0 \le z \le z^* \tag{36}$$
$$\frac{dm}{dz} = \epsilon(\nu - m) \tag{37}$$

The first is easily integrated with the initial condition $\epsilon(z^*) = 1/(1-h) = 1 + 1/\eta$ and it yields

$$\epsilon(z) = \frac{1+\eta}{\eta - (1+\eta)(z^* - z)} = \frac{1}{z_0 + z}, \qquad 0 \le z \le z^*. \tag{38}$$

with $z_0 = \eta/(1+\eta) - z^*$ The second is also easily integrated and it gives (note that $m(0) = -L$)

$$m(z) = \int_0^z due^{-\int_u^z dy\epsilon(y)}\nu(u)\epsilon(u) + m(0)e^{-\int_0^z dy\epsilon(y)} \tag{39}$$
$$= \frac{1}{z_0 + z}\left[\int_0^z dz'\nu(z') - Lz_0\right] \tag{40}$$

the condition for z^* is now $\dot{m}(z^*) = 0$ or $u(z^*) = m(z^*)$. This gives

$$\int_0^{z^*} dz'\left[\nu(z') + L\right] = \frac{\eta}{1+\eta}\left[\nu(z^*) + L\right] \tag{41}$$

It is easy to show that

$$z^* \le \frac{\eta}{1+\eta} = 1 - h \tag{42}$$

is less then the fraction of agents at work, and also the number of empty slots left

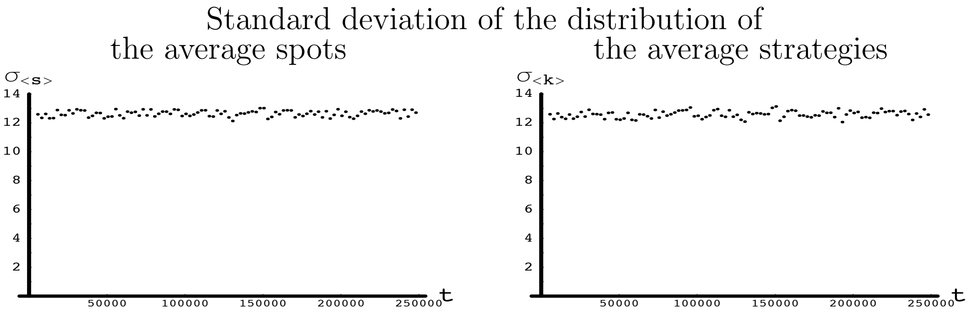

Figure 11: The time series of the measures of heterogeneity.

for $s \leq k^*$ is

$$\sum_{s=1}^{k^*} e_s = \int_0^{z^*} dz\epsilon(z) = \log\left[\frac{\eta}{\eta - (1+\eta)z^*}\right] \tag{43}$$

This means that a fraction $1 - h - z^*$ of the agents does not find an empty spot and has to pay the cost L. Therefore the expected utility of an agent in this Nash equilibrium is

$$E_s[u] = \int_0^{z^*} dz\nu(z) - (1 - h - z^*)L \tag{44}$$

where the first term is the utility of parked agents and the second is the cost of those who didn't find a place[19]

The same calculation for the asymmetric Nash equilibrium, when it is stable, gives

$$E_a[u] = (1-h)\int_0^1 dz\nu(z) \tag{45}$$

so for fixed L and large N, we know the asymmetric Nash equilibrium is stable only in the limit $h \to 0$ and in this limit it provides higher utility to agents.

C Stationarity of simulation result

The results shown in main text are all based on the statistics taken from $5001 \leq t \leq 5500$ (for $p_l = 1.0$ and adjusted appropriately for other values of p_l). One may question whether the results we are reporting is transient or not. As shown by Figure 11, our measure of heterogeneity is quite stable, and therefore, one can consider that initial 5000 periods were enough for the simulation to reach "steady

[19]To get this, pick an agent at random: with probability h, she's not in the city, and the contribution to the utility is zero. With probability $(1-h)(1-e_s) \simeq 1-h$ she is parked in spot $s \leq k^*$ and otherwise she is paying the cost L.

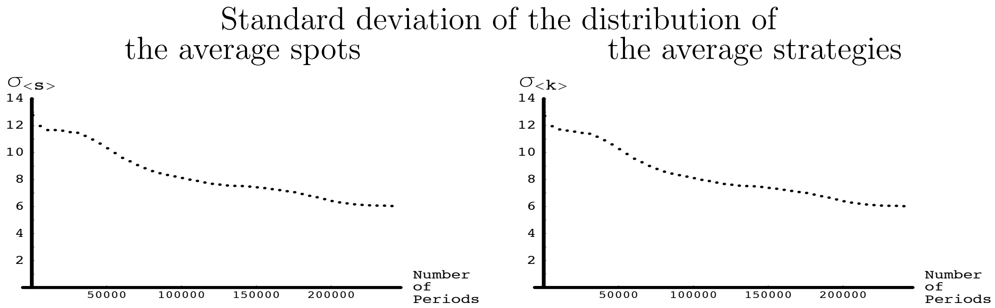

Figure 12: Long run outcomes. The standard deviation of the distribution of the average parked spots over many trials. $\lambda = 100$, $p_l = 1.0$, $L = 0.5$, $\alpha = 1.0$, $\omega = 0.9$.

state."

D Over longer periods

Figure 5 shows that for λ above certain value, we obtain heterogenous behaviors among agents. So far, the data were taken from the 500 periods after 5000 periods have past from the beginning of the simulation. What happens if we consider outcomes from longer intervals? Because of the randomness in the strategy choices, we expect that the longer the time horizon we consider, the less heterogeneous the behaviors of the agents become.

Figure 12 shows the $\sigma_{<s>}$ and $\sigma_{<k>}$ for various number of periods[20] up to 250000 periods for $\lambda = 100$. One can see that the longer the periods we consider, the lower the degree of heterogeneity becomes. But as noted, this is unavoidable because of the randomness in strategy choices.

[20] All after the initial 5000 periods have past.