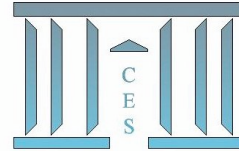




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## Insider Trading in a Two-Tier Real Market Structure Model

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# Insider Trading In A Two-Tier Real Market Structure Model

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## Abstract

In this paper we study the real and financial effects of insider trading in the spirit of Jain and Mirman (1999). Unlike the previous works that address this issue, we suppose that the production of the real good is costly and depends mainly of the price of an intermediate good produced locally by a privately-owned firm. We show that the real output of the final good chosen by the insider as well as the price of the intermediate good set by the privately-owned firm are both greater than it would be in the absence of insider trading. Furthermore, the parameters of both real markets affect the stock price and the stock pricing rule. Besides, when compared to Jain and Mirman (2000) and (2002), this two-tier real market structure does not alter the amount of information disseminated in the stock price or the level of insider trading. Next, we add a second insider to the model. We show that competition in the financial sector decreases the level of output produced by firm 1 and the price of the intermediate good with respect to initial model. Moreover, it affects the insiders' trades and increases the amount of information revealed in the stock price. <sup>1</sup>

*JEL classification:* G14, D82

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*Keywords:* Insider Trading, Two-Tier Market, Stackelberg, Correlated signals, Kyle model

## 1 Introduction

It is not a coincidence that corporate executives seem to always buy and sell their company's securities at the right times. After all, they have access to every bit of company information, due to their participation to real activities underlying the financial assets that are the subject of insider trading. Because real and financial decisions are inevitably intertwined, the theoretical research on insider trading has started to extend the pure financial models in the spirit of Kyle (1985) to include the real aspects of the firm (Jain and Mirman, 2000 and 2002; Daher and Mirman, 2006 and 2007). The insider is modeled as the manager of the firm who chooses how much stock to buy and how much output to produce in the real market, thus affecting the profitability of the firm. Sometimes, the publicly-owned firm is a quantity-setting monopolist in the real market (Jain and Mirman, 2000; Daher and Mirman, 2007), and sometimes it is competing with another privately-owned firm in a Cournot way to determine the quantity produced of the homogeneous good (Jain and Mirman, 2002; Daher and Mirman, 2006). In both cases and to simplify the analysis, the real good was produced at a no cost.

However, the production process involves the use of primary production factors such as labor, capital, and land, as well as intermediate goods. It is a characteristic feature of industrial economies that commodities are produced by means of commodities. Data from the OECD input-output tables (OECD, 2004) show that the share of intermediate goods in production ranges from 19% to 82% across different sectors. Any change in the cost of these intermediate goods will ripple throughout a market economy, affecting the market of the final good. In other words, buying and selling relationships link firms vertically, and through these links firms engage in market interactions while performing different functions in the value chain.

In this paper, we address this issue by assuming that the production of the final good by the publicly-owned firm (firm 1) involves constant labor costs as well as costs of intermediate goods. Those are locally produced by a privately-owned firm (firm 2). Specifically, we analyze insider trading in a static model in the spirit of Kyle (1985), where the insider of firm 1 is also the manager of that firm and thus makes both real and financial decisions.

However, in our model, the choice of the real variables will affect the market of the intermediate good, and vice versa. A sketchy analysis suggests that an increase in the price of the intermediate good is able to decrease the profit of firm 1, *ceteris paribus*. A lower profit induces the market maker to set a lower stock price. Inversely, any negative demand shock in the market of the final good is able to affect the demand for the intermediate good and thus its price.

In our model, firm 1, managed by the insider, is assumed to be a monopolist. We follow Kyle (1985) in modeling the financial market environment and thus study a linear-normal equilibrium. Further, we assume that the stock orders are submitted by an insider as well as noise traders. A market maker sets stock prices competitively. The insider knows the true value of a random shock to the value of the firm whereas the market maker knows only the distribution of this shock. However, following Jain and Mirman (1999), the market maker observes the total stock order as well as the noisy market price of the real good before setting the price of the stock. The insider chooses the real output of the final good and the stock to be traded simultaneously. Firm 1 buys the necessary quantity of the intermediate good from firm 2, which also holds a monopoly power on the market of the intermediate good.

We show that insider trading affects the markets of both the final and the intermediate goods, and that the financial market variables change due to the insider's real decisions. For instance, the real output chosen by the insider or manager of firm 1 as well as the price of the intermediate good set by firm 2 are both greater than it would be in the absence of insider trading. Furthermore, the parameters of both real markets affect the stock price and the stock pricing rule. Finally, we show that, when compared to Jain and Mirman (2000) and (2002), the two-tier real market structure does not alter the amount of information disseminated in the stock price or the level of insider trading.

Next, following Daher and Mirman (2006) and (2007), we add a second insider, the owner of firm 1, and assume that he has no managerial responsibilities. Then, we carry out a comparative static analysis between model II (duopoly in the financial market) and model I (monopoly in the financial market). We show that competition in the financial sector affects the stock price coefficients as well as real variables. In particular, the level of output produced by firm 1 and the price of the intermediate good both decrease with respect to model I. Hence, the addition of another informed trader influences the production decision of firm 1, as well as the decision of firm 2. Moreover, competition in the stock sector affects the insiders' trades. Finally,

the results show that Cournot competition in the stock market increases the amount of information revealed in the stock price.

## 2 Model I: The Monopoly Case

Let  $(\Omega, \mathcal{F}, \mathbb{P})$  be a probability space, all the random variables are defined with respect to this probability space. Consider an economy characterized by a financial sector and a two-tier real market structure in which the production of a final good requires an intermediate good (or service). More specifically, the real market is characterized by two firms, firm 1 holding a monopoly in producing the final good and firm 2 also holds a monopoly in the production of the intermediate good<sup>2</sup>.

The structure of the real market draws on existing models that explore the effect of foreign direct investment on backward linkages<sup>3</sup>. More specifically, our model is a variant of the Cournot model developed in Lin and Saggi (2007), which in turn is based on Salinger (1988). For producing one unit of the final good, firm 1 requires  $l$  units of labor and  $c$  units of the intermediate good. The marginal cost of producing one unit of the final good is then equal to the sum of the labor cost and the cost of intermediate goods. For simplicity, the wage rate is normalized to 1. The price of the intermediate good  $w$  is set by firm 2. The inverse demand function for the final good is assumed to be linear and stochastic, i.e.,

$$q' = (a - y)\tilde{z}$$

and the unitary profit of firm 1 is also stochastic and given by,

$$\varpi' = (a - y - l - cw)\tilde{z}$$

where  $a$  is positive constant and  $\tilde{z}$  is a random variable, normally distributed with mean  $\bar{z}$  (assumed positive) and variance  $\sigma_z^2$ .

Firm 2 produces the intermediate good at constant marginal cost  $\tau$ . Firm 1 and firm 2 interact in a two-stages process. First, firm 2 chooses the quantity of the intermediate good to supply, and accordingly, its unitary price  $w$ . Then, the manager of firm 1 decides the amount of the final good  $y$  to

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<sup>2</sup>Note that the monopoly structure adopted in the real market can be easily extended to Cournot oligopoly in the market of both the final and the intermediate goods.

<sup>3</sup>See for example Batra et al. (2003) and Lin and Saggi (2007).

produce given the price of the intermediate good  $w$ .

On the other hand, we consider a financial market where firm 2 is privately owned and firm 1 is publicly owned. The stock of firm 1 is publicly traded in a competitive financial market. The value of the stock is the net profits of the firm per share. We assume, like in Kyle (1985), that there are three kinds of agents in the market. First, there is a risk-neutral rational trader, the manager of firm 1, who knows the realization  $z$  of  $\tilde{z}$ , the value of the stock. Second, there are non-rational noise traders, representing small investors with no information on  $z$ . The aggregate noise trade is assumed to be a random variable  $\tilde{u}$ , which is normally distributed with mean zero and variance  $\sigma_u^2$ . Finally, there are  $K$  ( $K \geq 2$ ) risk-neutral market makers who act like Bertrand competitors.

We follow Jain and Mirman (2000) by allowing the market makers to observe two signals. The first one, from the real market, is denoted by  $\tilde{q} = (a - y - l - cw)(\tilde{z} + \tilde{\varepsilon})$  where  $\tilde{\varepsilon}$  is normally distributed with mean 0 and variance  $\sigma_\varepsilon^2$ . The second one is the total order flow, i.e.,  $\tilde{r} = \tilde{x} + \tilde{u}$  where  $\tilde{x}$  is the manager's trading order. We assume that  $\tilde{z}$ ,  $\tilde{u}$  and  $\tilde{\varepsilon}$  are pairwise independent.

Following Kyle (1985), the financial trading mechanism is organized in two steps. In step one, a linear pricing rule and optimal order rule are determined by the market makers and the insider, respectively, as a Bayesian equilibrium. The market makers determine a (linear) pricing rule  $p$ , based on their a priori beliefs, where  $p$  is a measurable function  $p : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ . The insider chooses a stock trade function  $\tilde{x} = x(\tilde{z})$ , where  $x : \mathbb{R} \rightarrow \mathbb{R}$  is a measurable function. In the second step, the insiders observe the realization  $z$  of  $\tilde{z}$ ,<sup>4</sup> and submit their stock order to the market makers based on the equilibrium stock trade functions. The market makers also receive orders from the noise traders, all these orders arrive as a total order flow signal  $\tilde{r} = x(\tilde{z}) + \tilde{u}$ . The two signals are used by the market makers to set the price  $\tilde{p} = p(\tilde{q}, \tilde{r})$ , based on the equilibrium price function, to clear the market.

The insider only knows the realization  $z$  of  $\tilde{z}$  and does not know the values of  $\tilde{u}$ ,  $\tilde{\varepsilon}$ ,  $\tilde{r}$ ,  $\tilde{z} + \tilde{\varepsilon}$  before his order flow decisions are made. Moreover, neither the market maker nor firm 2 know the realization  $z$  of  $\tilde{z}$  but both know its distribution. Finally, the market makers cannot observe either  $x$ ,  $u$  or  $\varepsilon$ .

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<sup>4</sup>Random variables are denoted with a tilde. Realized values lack the tilde. The mean of the random variable is denoted with bar.

The value per share of the firm 1 is the net profit of the firm per share, i.e.,

$$v' = (a - y - l - cw)y\tilde{z} \quad a, b > 0 \quad (1)$$

and the profit of the manager is given by,

$$\psi_1 = (v' - A - \tilde{p})\tilde{x}_1 + A\tilde{x}_1 \quad (2)$$

where  $A$  is the compensation scheme received by the manager for his managerial tasks<sup>5</sup>. When the manager receives his compensation, the value of the firm becomes:

$$v = v' - A.$$

Firm 2 chooses the quantity of the intermediate good  $y_I$  that maximizes its profits, given the quantity of the final good  $y$  produced by firm 1, and sets accordingly the price  $w$  of the intermediate product. We assume that the intermediate product is produced at a constant marginal cost  $\tau$ . Hence, firm 2's profits are given by,

$$\psi_2 = (w - \tau)y_I$$

such that  $w = f(y_I)$  is the inverse demand function for the intermediate good and  $cy = y_I$ .

## 2.1 The Equilibrium Concept

This is a game of incomplete information because the market makers, unlike the insider, do not know the realization of  $\tilde{z}$ . Hence, we seek a Bayesian-Stackelberg equilibrium. A Bayesian-Stackelberg equilibrium is a vector of four functions  $[x(\cdot), y(\cdot), y_I(\cdot), p(\cdot, \cdot)]$  such that:

- (a) Profit maximization of the manager of firm 1,

$$\begin{aligned} E((a - y - l - cw)y\tilde{z} - p((a - y - l - cw)(\tilde{z} + \tilde{\varepsilon}), \tilde{x} + \tilde{u}))\tilde{x} \geq \\ E((a - y' - l - cw)y'\tilde{z} - p((a - y' - l - cw)(\tilde{z} + \tilde{\varepsilon}), \tilde{x}' + \tilde{u}))\tilde{x}' \end{aligned} \quad (3)$$

for any level of trading order  $\tilde{x}'$  and level of output  $y'$  decided by the manager of firm 1 .

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<sup>5</sup>Following Jain and Mirman (2000), the existence of such compensation scheme is to ensure the second order condition of the manager's maximization problem. For more details, see Jain and Mirman (2000) and the extended works thereafter.

(b) Profit maximization of firm 2,

$$(w(y_I(y)) - \tau)y_I(y) \geq (w(y'_I(y))) - \tau)y'_I(y) \quad (4)$$

for any level of quantity  $y'_I$  of the intermediate good decided by firm 2 such that  $cy = y_I$  and  $w$  is the inverse demand function for the intermediate good.

(c) Semi-Strong Market Efficiency: The pricing rule  $p(\cdot, \cdot)$  satisfies,

$$p(\tilde{q}, \tilde{r}) = E[\tilde{v}|\tilde{q}, \tilde{r}]. \quad (5)$$

An equilibrium is linear if there exists constants  $\mu_0, \mu_1, \mu_2$  such that,

$$\forall q, r, \quad p(q, r) = \mu_0 + \mu_1 q + \mu_2 r. \quad (6)$$

Note that condition (3) defines the optimal strategy of the insider while (4) reflects the Stackelberg structure of the real market. Since firm 2 is the leader (firm 1 is the follower) so it takes the firm's 1 output decision into its maximization problem. Condition (5) guarantees the zero expected profits for the market makers. The stock price, set by the market makers, is equal to the conditional expectation of the asset given the available information. We restrict our study to linear equilibrium. The normal distributions of the exogenous random variables, together with the particular expression of the demand, enable us to derive and to prove the existence of a unique linear equilibrium.

## 2.2 Optimality Results

We have a game of incomplete information because the market makers, unlike the insider, do not know the realization of  $\tilde{z}$ . We solve backward the maximization problem of the insider in stage 2, given the assumption of linear equilibrium. The manager chooses  $(y, x)$  to maximize his expected profit, given  $z$ . So,

$$\text{Max}_{(y,x)} G$$

where  $G = E[v - p]x + A.x = E[v' - p]x$ . This is equivalent to,

$$\text{Max}_{(y,x)} [(a - y - l - cw)yz - \mu_0 - \mu_1(a - y - l - cw)z - \mu_2 x]x.$$

The first order necessary conditions are,

$$x(z) = \frac{(a - y - l - cw)(y - \mu_1)z - \mu_0}{2\mu_2}, \quad (7)$$



and

$$y = \frac{(a - l - cw + \mu_1)}{2}. \quad (8)$$

From equation 8, the inverse demand for the intermediate goods equals:

$$w = \frac{(a - l - 2y + \mu_1)}{c}. \quad (9)$$

In stage 1, firm 2 maximizes its profits, given the quantity of the final good produced by firm 1. Plugging the expression of  $w$  in equation 9, the maximization problem of firm 2 becomes,

$$\text{Max}_{y_I} \left( \left( \frac{a - l - 2y_I/c + \mu_1}{c} - \tau \right) y_I \right)$$

The first order condition implies

$$y_I = \frac{(a - l + \mu_1)c - c^2\tau}{4} \quad (10)$$

$$w = \frac{(a - l + \mu_1 + c\tau)}{2c} \quad (11)$$

Now, we determine the price function coefficients. First, we have,

**Lemma 1** *The coefficients of the price function are,*

(i)

$$\mu_1 = \frac{(a - l - c\tau)\sigma_z^2}{3\sigma_z^2 + 8\sigma_\varepsilon^2} = Fk,$$

(ii)

$$\mu_2 = \frac{F^2\sigma_\varepsilon(1 - 3k)\sqrt{(1 - 3k)k}}{8\sqrt{2}\sigma_u},$$

where  $k = \frac{\sigma_z^2}{3\sigma_z^2 + 8\sigma_\varepsilon^2}$  and  $F = (a - l - c\tau)$  with  $F > 0$

*Proof:* See the Appendix.

Second, as in Jain-Mirman (2000), the compensation scheme of the insider is needed to ensure the second order condition. Indeed, combining the second order condition

$$2x(\tilde{z})\tilde{z} > 0 \quad \text{and} \quad \mu_2 > 0, \quad (12)$$

with expression of the  $\mu_0$  given by

$$\mu_0 = \bar{v} - \mu_1\bar{q} - \mu_2\bar{r} = (a - y - l - cw)(y - \mu_1)\bar{z} - A$$

In proposition 1, we provide a sufficient and necessary condition to ensure the existence of linear equilibrium.

**Proposition 1** *A linear equilibrium exists if and only if,*

$$\mu_0 = 0 \Leftrightarrow A = \frac{(a - y - l - cw)(y - \mu_1)}{2}\bar{z}.$$

*Proof:* See Jain and Mirman (2000).

We turn now to the characterization of the equilibrium. Proposition 2 whose proof is relegated in appendix B, characterizes the amount of information revealed in the equilibrium stock price as well as the insider's conditional profits.

**Proposition 2** *The information revelation and the insider's conditional profits are:*

$$Var(\tilde{z}|\tilde{q}, \tilde{r}) = \frac{\sigma_\varepsilon^2}{\sigma_z^2 + 2\sigma_\varepsilon^2}\sigma_z^2 \quad (13)$$

$$\pi(\tilde{z}) = \frac{2(a - l - c\tau)^2\sigma_u\sigma_\varepsilon^4\tilde{z}^2}{(3\sigma_z^2 + 8\sigma_\varepsilon^2)^2\sigma_z} \quad (14)$$

## 2.3 Discussion of the equilibrium

There are several properties of this equilibrium that are worth emphasizing. First, note that the total output produced by firm 1 and the price of the intermediate good charged by firm 2 are both more than what is expected in a two-tier real market structure without insider trading. Indeed, equations (8) and (11) given by

$$y = \frac{(a - l - cw + \mu_1)}{2} \quad w = \frac{(a - l + \mu_1 + c\tau)}{2c} \quad (15)$$

show that both the total output  $y$  and the intermediate good price  $w$  are increasing functions of  $\mu_1$ , the market maker response to the real signal. Thus insider trading increases the production of the final good, thereby lowering the price. This happens since an increased output has the effect of lowering the real output price. A lower real output price conveys, *ceteris paribus*, a lower realization (on average) of  $z$  to the market maker and thus induces him to set a lower stock price.

Moreover, insider trading increases the price of the intermediate good as well. Indeed, a greater production of the final output increases the demand of the intermediate good addressed to firm 2, which in turn, profits to charge a higher price. Inversely, any increase in the price of the intermediary good is supposed to decrease the quantity demanded of that good by firm 1, and thus decrease the production of the final good. The market price of the final good will then rise to balance the market. If the real output price increases more than the price of the intermediary good, the market maker observes a higher realization (on average) of  $z$  and thus sets a higher stock price.

Recall that our model adds a two-tier real market structure to the financial market of Jain and Mirman (1999) with one insider and two signals observed by the market maker. The effect of real market on Jain and Mirman (1999) has also been explored in Jain and Mirman (2000) and (2002). Indeed, Jain and Mirman (2000) investigated the effect of the monopoly structure in the real market on the model of Jain and Mirman (1999). Jain and Mirman (2002) added Cournot competition in the real market to Jain and Mirman (2000). Both papers also showed that insider trading increases the production of the real good. One of the characteristics of the two-tier real market structure presented in this paper is that firm 1, that acts as a follower to firm 2 in the two-tier real market structure, is influenced by the decision of firm 2 which charges the price of the intermediate good. Thus, the insider is thus less able to influence the market price of the final good (and the signal available to the market maker) by increasing his firm's output. Indeed, the signal observed by the market maker now depends on both the price of final good and the price of the intermediate good.

Second, note that the two-tier real market structure does not alter the amount of information disseminated in the stock price. The conditional variance in this model is the same found in Jain and Mirman (2000) and (2002) (see equation 13), implying that the real market structure has no effect on information revelation. This is due to the fact that to the deterministic nature of the real market variables.

Third, the level of insider trading remains the same as in the previous works. Intuitively, this is due to the fact that the effect of any change in output on the two signals is predictable for the market maker (since the outputs are deterministic) and thus is correctly incorporated in the stock price function (through lower weights on the two signals). Thus the insider's optimal response remains unchanged.

In the next section, we extend our model to two insiders. In other words, we introduce Cournot competition among the insiders in the financial sector. Formally,

### 3 Model II: The Duopoly Case

In this model, we extend model I to incorporate Cournot competition between two insiders in the financial market: the manager of the firm and the owner who has no managerial responsibilities<sup>6</sup>. We assume that the two insiders know the realization  $z$  of  $\tilde{z}$  and trade based on their private information. Introducing Cournot competition in the financial market allows us to study the effect of the strategic competition in the financial market on the equilibrium outcomes.

Under Cournot competition in the financial market, a Bayesian-Stackelberg equilibrium is a vector of five functions  $[x_1(\cdot), x_2(\cdot), y(\cdot), y_I(\cdot), p(\cdot, \cdot)]$  such that:

- (a) Profit maximization of the manager of firm 1,

$$E((a - y - l - cw)y\tilde{z} - p((a - y - l - cw)(\tilde{z} + \tilde{\varepsilon}), \tilde{x}_1 + \tilde{x}_2 + \tilde{u}))\tilde{x}_1 \geq E((a - y' - l - cw)y'\tilde{z} - p((a - y' - l - cw)(\tilde{z} + \tilde{\varepsilon}), \tilde{x}'_1 + \tilde{x}_2 + \tilde{u}))\tilde{x}'_1 \quad (16)$$

for any level of trading order  $\tilde{x}'_1$  and level of output  $y'$  decided by the manager of firm 1 .

- (b) Profit maximization of the owner,

$$E((a - y - l - cw)y\tilde{z} - B - p((a - y - l - cw)(\tilde{z} + \tilde{\varepsilon}), \tilde{x}_1 + \tilde{x}_2 + \tilde{u}))\tilde{x}_2 \geq E((a - y - l - cw)y\tilde{z} - p((a - y' - l - cw)(\tilde{z} + \tilde{\varepsilon}), \tilde{x}_1 + \tilde{x}'_2 + \tilde{u}))\tilde{x}'_2 \quad (17)$$

for any level of trading order  $\tilde{x}'_2$  decided by the owner of firm 1 .

<sup>6</sup>It should be pointed out that the results can be automatically generalized to  $n$  insiders, but for the sake of comparison we restrict our attention to the two-insiders case.

(c) Profit maximization of firm 2,

$$(w(y_I(y)) - \tau)y_I(y) \geq (w(y'_I(y))) - \tau)y'_I(y) \quad (18)$$

for any level of quantity  $y'_I$  of the intermediate good decided by firm 2 such that  $cy = y_I$  and  $w$  is the inverse demand function for the intermediate good.

(d) Semi-Strong Market Efficiency: The pricing rule  $p(\cdot, \cdot)$  satisfies,

$$p(\tilde{q}, \tilde{r}) = E[\tilde{v}|\tilde{q}, \tilde{r}]. \quad (19)$$

Proposition 3, we characterize the unique linear equilibrium.

**Proposition 3** *Under Cournot competition in the financial market, a linear equilibrium exists and is unique. It is characterized by*

$$\tilde{x}_1 = \frac{(a - y - l - cw)(y - \mu_1)\tilde{z}}{3\mu_2} \quad \tilde{x}_2 = \frac{(a - y - l - cw)(y - \mu_1)(\tilde{z} - \bar{z})}{3\mu_2} \quad (20)$$

$$y = \frac{(a - l - cw + \mu_1)}{2} \quad w = \frac{(a - l + \mu_1 + c\tau)}{2c} \quad (21)$$

$$\mu_0 = B = \frac{(a - y - l - cw)(y - \mu_1)\bar{z}}{3}, \quad \mu_1 = \frac{(a - l - c\tau)\sigma_z^2}{3\sigma_z^2 + 12\sigma_\varepsilon^2} = Fk_1, \quad (22)$$

$$\mu_2 = \frac{F^2\sigma_\varepsilon(1 - 3k_1)\sqrt{(1 - 3k_1)k_1}}{4\sqrt{6}\sigma_u}, \quad Var(\tilde{z}|\tilde{q}, \tilde{r}) = \frac{\sigma_\varepsilon^2}{\sigma_z^2 + 3\sigma_\varepsilon^2}\sigma_z^2 \quad (23)$$

and

$$\pi_1 = \frac{9(a - l - c\tau)^2\sigma_u\sigma_\varepsilon^4\tilde{z}^2}{2\sqrt{2}(3\sigma_z^2 + 12\sigma_\varepsilon^2)^2\sigma_z} \quad \pi_2 = \frac{9(a - l - c\tau)^2\sigma_u\sigma_\varepsilon^4(\tilde{z} - \bar{z})^2}{2\sqrt{2}(3\sigma_z^2 + 12\sigma_\varepsilon^2)^2\sigma_z} \quad (24)$$

where  $k_1 = \frac{\sigma_z^2}{3\sigma_z^2 + 12\sigma_\varepsilon^2}$  and  $F = (a - l - c\tau)$

## 4 Comparative statics

In this section, we present several comparisons between the results of models I and II. Note that these comparisons highlight the effects of competition in the financial sector on both the real and financial sectors as well as the decisions of the insiders. The effect of competition in the financial sector on financial and real variables has been studied in several works before: For instance, Daher and Mirman (2007) investigated the effect of Cournot competition in the financial sector with a monopoly structure in the real market, on the model of Jain and Mirman (2000), characterized by a monopoly structure in both the real and financial markets. Moreover, Daher and Mirman (2006) introduced Cournot competition in the financial sector to the model of Jain and Mirman (2002) characterized by a monopoly structure in the financial market and Cournot competition in the real market.

Introducing Cournot competition to model I has two effects. On the one hand, it enables us to study the effect of Cournot competition between the insiders as compared to the monopoly case studied in the first model of this paper. On the other hand, it allows us to capture the effect of the two-tier real market structure on the equilibrium variables, as compared with the monopoly and duopoly real market structure existing in previous works.

### 4.1 Equilibrium outcomes

Lemma 2 provides a comparative static analysis of equilibrium outcomes between the Cournot competition model (model II) and the previous model (model I). It shows that the stock price coefficients are affected by Cournot competition in the financial sector. Moreover, the production of firm 1 and the price charged by firm 2 decrease with respect to model I. Finally, we can notice that the manager trades less and earns less than in model 1. Formally

**Lemma 2** *The effects of Cournot competition on equilibrium outcomes relative to the monopoly case are given by<sup>7</sup>,*

$$\mu_0 > \mu_0^{M1} \quad \mu_1 < \mu_1^{M1} \quad \mu_2 < \mu_2^{M1} \quad (25)$$

$$w < w^{M1} \quad y < y^{M1} \quad \tilde{x}_1 < \tilde{x}^{M1} \quad (26)$$

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<sup>7</sup>The superscript M1 refers to model I.

First, competition in the financial sector affects the stock price set by the market makers. Lemma 2, shows that the intercept coefficient of the stock price,  $\mu_0$ , is greater than in model I. Indeed, to satisfy the second order condition of the maximization problem of the manager of firm 1, and to guarantee the existence of a linear equilibrium, competition in the financial sector requires that the compensation scheme of the manager (which is positive) has the value  $\mu_0$ . In the absence of competition in the financial sector (model I),  $\mu_0$  must be equal to zero. However, in the presence of competition in the financial sector, the market makers set  $\mu_0$  different from zero in order to satisfy the zero expected profits.

The value of  $\mu_1$ , the response of the market makers to the real signal, is also less than in model I. The same results hold in Daher and Mirman (2007) who model Cournot competition in the financial market with a monopoly structure in the real market, when compared with Jain and Mirman (2000) where a monopoly structure characterizes both markets. They also hold in Daher and Mirman (2006) where Cournot duopoly exists in the financial and real markets, in comparison with Jain and Mirman (2002) where Cournot competition characterizes only the real market. In other words, the two-tier real market structure does not alter the previous findings concerning  $\mu_0$  and  $\mu_1$ . In order to understand this result recall the expression of  $\mu_1$  in model I and in our model, i.e.

$$\mu_1^{M1} = \frac{(a - l - c\tau)\sigma_z^2}{3\sigma_z^2 + 8\sigma_\varepsilon^2} \quad \text{and} \quad \mu_1 = \frac{(a - l - c\tau)\sigma_z^2}{3\sigma_z^2 + 12\sigma_\varepsilon^2}$$

which depends on the value of the firm, the noisy signal and the total order flow (through the coefficient of  $\sigma_\varepsilon^2$ ). Hence, as the number of insiders increases, the total order flow increases. Thus the coefficient of the noise in the denominator increases which lowers the value of  $\mu_1$ . This result reflects the fact that with two insiders there is more information in the order flow signal which gets more weights, and thus makes the value of  $\mu_1$  lower than in model I.

However, and contrary to what is found in the previous works, the response of the market makers to the total order flow signal,  $\mu_2$ , is always lower than in model I. This is an interesting result which shows that, in the presence of the two-tier real market structure, the value of  $\mu_2$  the relationship which exists between exogenous variances of the model, mainly  $\sigma_\varepsilon^2$  and  $\sigma_z^2$  does not affect the value of  $\mu_2$  like in Daher-Mirman (2006) and Daher-Mirman (2007).

Second, competition in the financial sector affects the real market variables. In fact, the level of output produced by firm 1 and the price of the intermediate good both decrease with respect to model I. To see this, recall the expressions of  $y$  and  $w$  in both models,

$$y = \frac{(a - l - cw + \mu_1)}{2} \quad w = \frac{(a - l + \mu_1 + c\tau)}{2c} \quad (27)$$

Hence,  $y$  and  $w$  are both a function of  $\mu_1$ , the coefficient of the real sector signal of the stock price, which is set by the market makers. Plugging the equation of  $w$  in the equation of  $y$  gives a positive relation between  $y$  and  $\mu_1$ . Lemma 2 shows that competition in the stock sector reduces the value of  $\mu_1$  and thus, the output of firm 1 as well as the price of the intermediate good with respect to model I. Hence, the addition of another informed trader influences the production decision of firm 1, as well as the decision of firm 2.

Moreover, it worth mentioning that regardless of the type of competition between the insiders, the introduction of the financial sector increases the quantity produced of the final good as well as the price of the intermediate good. Indeed, equation (27) reveals the positive correlation between the quantity produced  $y$  and  $\mu_1$  as well as between the price of the intermediate good  $w$  and  $\mu_1$ . Thus, the financial market has a positive effect on the two-tier real market variables.

Third, competition in the stock sector affects the insiders' trades. This result is consistent with the financial Kyle (1985) model with one signal (the total order flow), going from one to two insiders (Tighe, 1989). Indeed, Lemma 2(iv) shows that the manager's trading level is less than in model I. The same result holds in Daher and Mirman (2007) where Cournot competition in the financial market is introduced to the real market monopoly structure of Jain and Mirman (2000). This result is also true in Daher and Mirman (2006) who extend the real market duopoly structure of Jain and Mirman (2002) to incorporate Cournot competition between the insiders.

## 4.2 Informativeness of stock price

In this section we examine the extent to which the stock price reveals information. This issue was originally studied by Kyle (1985) as well as Rochet and Vila (1994). They found that the stock price reveals exactly half of the inside information, regardless of the parameter values of the model. Moreover, in Kyle (1985) with more than one informed trader, the amount of



information revealed increases in the number of traders, but is still constant and independent of the parameter values of the model. This independence result is due to the fact that the insider has only one option, i.e. either to buy or sell stock, and has no effect on the value of the firm. When the manager is able to affect the value of the firm, and with the availability of a second signal to the market maker, the results change. This was particularly the case of Jain and Mirman (2000) who show that, when the manager is a quantity-setting monopolist in the real market, the amount of information released by the insider is greater than half, and, depends on the parameters of the model. Daher and Mirman (2007) show that, when Cournot competition between the insiders exists, the stock price reveals even more information than Jain and Mirman (2000), since competition in the stock market makes the order flow more informative. The same is true in Daher and Mirman (2006) who extend the Cournot real market structure of Jain and Mirman (2002) to incorporate Cournot competition in the financial market.

Following Jain and Mirman (2000) as well as the subsequent works, we adopt the same measure of information i.e., the conditional variance of  $\tilde{z}$ , given the information of the market maker.

**Lemma 3** *The effect of Cournot competition on information revelation relative to the monopoly case is given by,*

$$Var(\tilde{z}|\tilde{q}, \tilde{r}) < Var(\tilde{z}|\tilde{q}, \tilde{r})^{M1} \quad (28)$$

Lemma 3 shows that there is more information revealed in model II than in model I. Cournot competition in the stock market increases the amount of information revealed in the stock price. This result is consistent with the findings of Daher and Mirman (2006) and (2007). Indeed, the expressions of the conditional variances measuring the amount of information revealed in the stock price in model I and in our model are:

$$Var(\tilde{z}|\tilde{q}, \tilde{r})^{M1} = \frac{\sigma_\varepsilon^2}{\sigma_z^2 + 2\sigma_\varepsilon^2}\sigma_z^2 \quad \text{and} \quad Var(\tilde{z}|\tilde{q}, \tilde{r}) = \frac{\sigma_\varepsilon^2}{\sigma_z^2 + 3\sigma_\varepsilon^2}\sigma_z^2$$

The key difference between these two conditional variances is the coefficient of  $\sigma_\varepsilon^2$  in the denominator. The greater the value of this coefficient, the greater the amount of information revealed in the stock price is. The origin of this value is the aggregate orders of the insiders. The aggregate order in the Cournot case is greater than in model I.<sup>8</sup>

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<sup>8</sup> $\tilde{x} = \tilde{x}_1 + \tilde{x}_2 = \frac{\sqrt{2}\sigma_u}{\sigma_z}(\tilde{z} - \bar{z}), \quad \tilde{x}^{M1} = \frac{\sigma_u}{\sigma_z}(\tilde{z} - \bar{z})$

### 4.3 Insider's profits

In this section, we compare the manager's profits with model I.

**Lemma 4** *The effect of Cournot competition on the manager's profits relative to the monopoly case is given by,*

$$\pi_1 < \pi^{M1} \quad \text{if } \sigma_z^2 < \frac{24(\sqrt{\sqrt{2}} - 1)}{9 - 6\sqrt{\sqrt{2}}}\sigma_\varepsilon^2 \quad \text{and} \quad \pi_1 > \pi^{M1} \quad \text{otherwise} \quad (29)$$

Adding a second informed trader to model I does not always lower the manager's profits relative to the monopoly case. Indeed, Lemma 4 shows that the profits of the manager in this model are greater than in model 1 when  $\sigma_\varepsilon^2$  is small relative to  $\sigma_z^2$ , and they are less than model I when  $\sigma_\varepsilon^2$  is large relative to  $\sigma_z^2$ . The intuition for this result is that when  $\sigma_\varepsilon^2$  is large relative to  $\sigma_z^2$ , the unconditional net profits of the firm is less in this model than in model I. In contrast, when  $\sigma_\varepsilon^2$  is small relative to  $\sigma_z^2$ , the opposite is true, i.e. the net profits of the firm in model I is lower.

Finally, it is worth noting that the profits of firm 2 decrease with respect to model I:

$$\psi_2 < \psi_2^{M1} \quad (30)$$

This result is straightforward from Lemma 2. Indeed, we showed that competition in the financial sector decreases the level of output produced by firm 1 and the price of the intermediate good charged by firm 2. The lower price for the intermediate good explains why the profits of Firm 2 decrease with respect to model I.

## Appendix A

We start this appendix by recalling the Theorem that we use to prove Lemma 1. Then we prove this Lemma.

**Theorem 1** *If the  $p \times 1$  vector  $Y$  is normally distributed with mean  $U$  and covariance  $V$  and if the vector  $Y$  is partitioned into two subvectors such that  $Y = \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}$  and if  $Y^* = \begin{pmatrix} Y_1 \\ Y_2^* \end{pmatrix}$*

$$U = \begin{pmatrix} U_1 \\ U_2 \end{pmatrix} \quad \text{and} \quad V = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix}$$

are the corresponding partitions of  $Y^*, U$  and  $V$ , then the conditional distribution of the  $m \times 1$  ( $m < p$ ) vector  $Y_1$  given the vector  $Y_2 = Y_2^*$  is the multivariate normal distribution with mean  $U_1 + V_{12}V_{22}^{-1}(Y_2^* - U_2)$  and covariance matrix  $V_{11} - V_{12}V_{22}^{-1}V_{21}$ .

*Proof:* See Graybill, Theorem 3.10 pp 63.

**Proof of Lemma 1:**

We apply Theorem 1 to the normal random vector  $B = (\tilde{v}, \tilde{q}, \tilde{r})$ . First, in this case we have  $p = 3$  and  $m = 1$ . Second by identification, we have  $Y_1 = \tilde{v}$  and  $Y_2 = \begin{pmatrix} \tilde{q} \\ \tilde{r} \end{pmatrix}$ .  $U_1 = \bar{v}$ ,  $U_2 = \begin{pmatrix} \bar{q} \\ \bar{r} \end{pmatrix}$  and

$$V = \begin{pmatrix} \sigma_v^2 & \sigma_{vq} & \sigma_{vr} \\ \sigma_{vq} & \sigma_q^2 & \sigma_{qr} \\ \sigma_{vr} & \sigma_{qr} & \sigma_r^2 \end{pmatrix} = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix}$$

Where  $V_{11} = \sigma_v^2$ ,  $V_{12} = (\sigma_{vq}, \sigma_{vr})$ ,  $V_{21} = \begin{pmatrix} \sigma_{vq} \\ \sigma_{vr} \end{pmatrix}$  and  $V_{22} = \begin{pmatrix} \sigma_q^2 & \sigma_{qr} \\ \sigma_{qr} & \sigma_r^2 \end{pmatrix}$ .

Note that

$$V_{22}^{-1} = \frac{1}{D} \begin{pmatrix} \sigma_r^2 & -\sigma_{qr} \\ -\sigma_{qr} & \sigma_q^2 \end{pmatrix},$$

where  $D$  is the determinant of  $V_{22}$ , that is  $D = \sigma_q^2\sigma_r^2 - \sigma_{qr}^2$ .

So we obtain,

$$\mu_0 = \bar{v} - \mu_1\bar{q} - \mu_2\bar{r} \tag{31}$$

$$\mu_1 = \frac{\sigma_{vq}\sigma_r^2 - \sigma_{vr}\sigma_{qr}}{D} \tag{32}$$

$$\mu_2 = \frac{\sigma_{vr}\sigma_q^2 - \sigma_{vq}\sigma_{qr}}{D} \tag{33}$$

We start now by computing all the covariances in the last two expressions. Indeed, we have

$$\sigma_{vq} = (a - y - l - cw)^2 y \sigma_z^2$$

$$\sigma_r^2 = \frac{(a - y - l - cw)^2 (y - \mu_1)^2}{4\mu_2^2} \sigma_z^2 + \sigma_u^2$$

$$\sigma_{vr} = \frac{(a - y - l - cw)^2(y - \mu_1)y}{2\mu_2}\sigma_z^2$$

$$\sigma_q^2 = (a - y - l - cw)^2(\sigma_z^2 + \sigma_\varepsilon^2)$$

$$\sigma_{qr} = \frac{(a - y - l - cw)^2(y - \mu_1)}{2\mu_2}\sigma_z^2$$

Substituting for the variances and covariances in (32) and (33), we get

$$\mu_1 = \frac{(a - y - l - cw)^2 y \sigma_z^2 \sigma_u^2}{D}, \quad (34)$$

$$\mu_2 = \frac{(a - y - l - cw)^4 y (y - \mu_1) \sigma_z^2 \sigma_\varepsilon^2}{\mu_2 D}. \quad (35)$$

Computing (34) and (35), we obtain

$$\mu_2^2 = \frac{(a - y - l - cw)^2 \mu_1 (y - \mu_1) \sigma_\varepsilon^2}{2\sigma_u^2} \quad (36)$$

Calculating for the expression of  $D$ , we get

$$D = \frac{(a - y - l - cw)^4 (y - \mu_1)^2 \sigma_\varepsilon^2 \sigma_z^2}{4\mu_2^2} + (a - y - l - cw)^2 \sigma_u^2 (\sigma_\varepsilon^2 + \sigma_z^2).$$

Substituting the above expression of  $D$  in (35), we find

$$\mu_2^2 = \frac{(a - y - l - cw)^2 (y - \mu_1) (y + \mu_1) \sigma_\varepsilon^2 \sigma_z^2}{4\sigma_u^2 (\sigma_\varepsilon^2 + \sigma_z^2)}. \quad (37)$$

Solving (36) and (37), we get

$$2\mu_1(\sigma_\varepsilon^2 + \sigma_z^2) = \sigma_z^2(y + \mu_1).$$

Substituting for  $y$  to solve  $\mu_1$ , we obtain

$$\mu_1 = \frac{(a - l - c\tau)\sigma_z^2}{3\sigma_z^2 + 8\sigma_\varepsilon^2} \quad (38)$$

To solve for  $\mu_2$ , we substitute the above value of  $\mu_1$  in (36) and taking the positive root, we get

$$\mu_2 = \frac{F^2 \sigma_\varepsilon (1 - 3k) \sqrt{(1 - 3k)k}}{8\sqrt{2}\sigma_u}, \quad (39)$$

where  $k = \frac{\sigma_z^2}{3\sigma_z^2 + 8\sigma_\varepsilon^2}$  and  $F = (a - l - c\tau)$

## Appendix B: proof of Proposition 2

Applying Theorem 1 to the normal random vector  $(\tilde{z}, \tilde{q}, \tilde{r})$ , the conditional variance is given by

$$\begin{aligned} \text{Var}(\tilde{z}|\tilde{q}, \tilde{r}) &= \sigma_z^2 - \frac{\mu_1}{(a - y - l - cw)y} \sigma_{zq} - \frac{\mu_2}{(a - y - l - cw)y} \sigma_{zr} \\ &= \frac{(y - \mu_1)}{3y} \sigma_z^2 \\ &= \frac{\sigma_\varepsilon^2}{\sigma_z^2 + 2\sigma_\varepsilon^2} \sigma_z^2 \end{aligned}$$

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