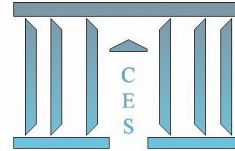




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**An inter-temporal optimization of flexible nuclear plants  
operation in market based electricity systems : The case of  
competition with reservoir**

Maria LYKIDI, Jean-Michel GLACHANT, Pascal GOURDEL

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# An inter-temporal optimization of flexible nuclear plants operation in market based electricity systems: The case of competition with reservoir.

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## Abstract

In electricity markets where competition has been established for a long time, a nuclear operator familiar with the operation of such markets could be interested in the optimal long-term management of a flexible nuclear set (like the French) in a competitive market. To obtain a long vision of the optimal management of a nuclear set, we realize a full inter-temporal optimization of the production which results from the maximization of the value of generation over the whole game. Our model takes into consideration the periodical shut-down of nuclear units to reload their fuel, which permits to analyze the nuclear fuel as a stock behaving like a reservoir. A flexible nuclear reservoir permits different allocations of the nuclear fuel during the different demand seasons of the year. Our analysis is realized within a general deterministic dynamic framework where perfect competition is assumed and two flexible types of generation exist: nuclear and thermal non-nuclear.

The marginal cost of nuclear production is (significantly) lower than the one of non-thermal production, which induces a discontinuity of producers' profit. In view of this price discontinuity, a "regularization" of the merit order price is achieved within our numerical model which leads to an alternative optimization problem (regularized problem) that constitutes a good approximation of our initial problem. We also prove that in the absence of binding productions constraints, solutions are fully characterized by a constant nuclear production. However, such solutions do not exist within our numerical model because of production constraints that are active at the optimum.

Finally, we study the maximization of social welfare in an identical framework. Similarly, we demonstrate that in the absence of binding production constraints a constant non-nuclear thermal production is a characteristic property of solutions of the social welfare maximization problem.

**Key words:** Electricity market, nuclear generation, inter-temporal optimal reservoir operation, competition with reservoir, price discontinuity, social welfare.

**JEL code numbers:** C61, C63, D24, D41, L11.

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## Résumé

Dans les marchés de l'électricité où la concurrence a été établie depuis longtemps, un exploitant nucléaire familiarisé avec le fonctionnement de marchés concurrentiels pourrait être intéressé par la gestion optimale d'un parc nucléaire flexible (comme le français) à long terme. Pour obtenir une vision longue de la gestion optimale d'un parc nucléaire, nous réalisons une optimisation inter-temporelle de la production qui résulte de la maximisation de la valeur de la production sur le jeu entier. Notre modèle prend en considération les arrêts périodiques des unités nucléaires pour recharger leur combustible, ce qui permet d'analyser le combustible nucléaire comme un stock que l'on gère comme un réservoir. Un réservoir nucléaire flexible permet des allocations différentes du combustible nucléaire pendant les différentes saisons de demande de l'année. Notre analyse est réalisée dans un cadre dynamique déterministe général, où la concurrence est supposée parfaite et deux types de production flexibles existent: le nucléaire et le non-nucléaire thermique.

Le coût marginal de production nucléaire est (significativement) inférieur à celui de production non-thermique, ce qui induit une discontinuité de profit des producteurs. Compte tenu de cette discontinuité des prix, une "régularisation" du prix merit order est réalisée au sein de notre modèle numérique qui conduit à un problème d'optimisation alternative (problème régularisé) qui constitue une bonne approximation de notre problème initial. Nous montrons aussi que quand les contraintes de production ne sont pas saturées, les solutions sont entièrement caractérisées par une production nucléaire constante. Toutefois, de telles solutions n'existent pas au sein de notre modèle numérique en raison des contraintes de production qui sont actives à l'optimum.

Enfin, nous étudions la maximisation du bien-être social dans un cadre identique. De la même façon, nous démontrons que en l'absence de saturation des contraintes de production, une production non-nucléaire thermique constante est une propriété caractéristique des solutions du problème de maximisation du bien-être social.

**Mots clés:** Electricité, technologie nucléaire, "réservoir" de combustible nucléaire, gestion intertemporelle optimale des réservoirs, compétition parfaite avec réservoir, suivi de charge, demande saisonnière, discontinuité des prix, bien-être social.

**JEL :** C61, C63, D24, D41, L11.

# 1 Introduction

In electricity markets where competition has been established for a long time, a nuclear operator familiar with the operation of such markets could be interested in the optimal long-term management of a flexible nuclear set (like the French) in a competitive market. In our previous paper (see Ref. [23]), we assumed that the nuclear managers are just quitting the former vertically integrated monopoly organization and discovering how competitive wholesale markets work. In view of this assumption, we modeled the optimal behaviour of flexible nuclear plants month by month by taking into consideration the production constraints imposed by generation capacity and nuclear fuel storage as well as the supply-demand equilibrium constraints. This could be explained by the fact that they cannot immediately manage all the factors affecting the market equilibrium in the medium term as well as the significant number of constraints determining a feasible production vector, hence they reduce their management horizon to that proportion of the market that is easier to apprehend: the monthly horizon. This analysis constituted a reasonable starting step to determine what the optimal management of a flexible nuclear set should be in a perfectly competitive electricity market. In this paper, we will assume that a nuclear producer knows how to behave in a market based operation framework. In addition to this, it has a better knowledge of the management of a flexible nuclear set on a monthly basis operation thanks to the study of its optimal per month production behaviour realized in our previous paper. Consequently, under these assumptions, a reasonable next step for a nuclear manager could be the determination of the optimal long-term management of a flexible nuclear set (like the French) in this competitive setting. In order to obtain a long-term vision of the optimal management of flexible nuclear plants, we realize a full inter-temporal optimization of the production which results from the maximization of the value of generation over the whole game. Market based management of flexible nuclear plants would then proceed with the determination of the global optimum of the optimal inter-temporal production problem. Our model takes into consideration the periodical shut-down of nuclear units to reload their fuel, which permits to analyze the nuclear fuel as a stock behaving like a reservoir. A flexible nuclear reservoir permits different allocations of the nuclear fuel during the different demand seasons of the year. This analysis makes sense in a theoretical level however, it does not exist an economic literature which permits to have knowledge on this particular topic.

A key characteristic of the nuclear electricity generation technology compared to other generation technologies consists in the conjunction of very high fixed costs and relatively low production costs. This leads to a differentiation of the nuclear generation technology from the fossil fuel generation technologies (e. g. coal, gas, fuel oil) with respect to the mode of operation. In view of the existing economics of nuclear (see Ref. [19]), the nuclear plants should run all year at maximum capacity since that helps covering its fixed costs. On the contrary, gas or coal power plants realize a load follow-up by adjusting their production to load variations during a year. This mode of operation of nuclear could be “easily” applied in electricity systems organized as vertically integrated monopolies (see Ref. [8]).

In the French case, however, the operation of the nuclear generation set is not of that kind because of the significantly high share of nuclear generation in the energy mix (78% of nuclear in total domestic electricity generation). Consequently, the french nuclear set needs to be “flexible”: nuclear units should be operated to follow a part of the demand variations instead of behaving as rigid base load units (see Ref. [14]). Indeed, according to the Report of monitoring of the Commission of Regulation of Energy (CRE) in 2007, the nuclear set was used to meet not only the “base” demand (share of constant consumption throughout the year), but also a part of the “semi base” demand (share of variable consumption). In particular, the

majority of the stops of the nuclear plants to reload their fuel was programmed in summer (low demand season in continental Europe) which made it possible to release the main part of the nuclear capacities in winter (high demand season) (see Ref. [15]). Thus, the nuclear production contributed to respond to the daily and seasonal variations of the demand. Since the vertically integrated monopoly regime of electricity systems has been replaced by an operation constrained by competitive markets (see Ref. [6]), we could assume that nuclear operators have learned how to behave within these markets. In such a competitive framework, one can ask what is the optimal management of a flexible nuclear set in the long-term?

We focus on the medium-term horizon as we did in our previous paper in which a nuclear manager has to allocate its output according to the different seasons of the year. Once again, we emphasize two time seasons differentiated by their demand levels: a “high demand” season (winter) and a “low” demand season (summer). A main characteristic of the optimal operation of a flexible competitive nuclear fleet is that the fuel of the nuclear reactor can be viewed as a “reservoir” of energy partly similar to the water reservoir of hydro storage stations. As we mentioned in our previous paper, nuclear units stop periodically in order to reload their fuel. Then, producers set the amount of the reloaded nuclear fuel which is limited and exhaustible according to the forecasted demand levels during a year. In view of the different demands and pricing characteristics of seasons, there are different profiles of nuclear fuel uses. In this paper, a nuclear generator being familiar with the operation of a competitive market and of a nuclear fuel reservoir is interested to obtain a long vision of the optimal operation of the nuclear reservoir throughout the entire time horizon of the game. Thus, it realizes an inter-temporal optimization based on the direct optimization of the nuclear fuel use over the entire period of the game. This mode of inter-temporal optimization will then be contrasted with the results of the per month optimization.

Finally, in view of the progressive augmentation of the electricity price for consumers induced by the growing demand, the recent increase of the price of nuclear per MWh sold by the French historical operator to alternative producers, the massive investment in renewable, etc, we are interested in determining production vectors (nuclear, thermal non-nuclear) that maximize the utility of both producers and consumers. Thus, we study the social welfare maximization problem in a competitive electricity market where producers dispose both nuclear and thermal non-nuclear generation technologies. Our goal is to determine the optimal production and storage levels that maximize social welfare by taking into account constraints imposed by nuclear fuel storage and generation capacity.

In section 2, we start with the modelling of the inter-temporal management of “market based” flexible nuclear reservoirs in a perfect competitive setting. In section 3, we study the symmetry of equilibrium of the optimal inter-temporal production problem while in section 4, we proceed with the optimization of this problem. In section 5, we run numerical tests of our model with the set of data described in subsection 5.1 and by using Scilab. Finally, in section 6, we focus on the social welfare maximization problem. Section 7 concludes.

## 2 Model: Perfect competitive case

In this section, we present our general deterministic model of a perfectly competitive electricity market where the producers operate both with nuclear and thermal non-nuclear plants. The hypothesis considered in this model are similar to those assumed in the model presented in our previous paper. More precisely, a perfect competition is assumed meaning that firms are price-

takers<sup>1</sup>: they treat price as a parameter and not as a choice variable. We also assume perfect equilibrium between supply and demand and perfect information among producers. First, our modelling aims at determining the inter-temporal optimal management of the nuclear generation set in that competitive regime. Once again, we focus to the medium-term horizon which is characterized by the seasonal variation of demand between winter and summer. Second, we take into account generation capacity and fuel storage constraints that are decisive for the determination of the equilibrium outcomes in this wholesale electricity market. Finally, we look at the social welfare maximization problem by taking into consideration these production constraints. For simplicity reasons and in the absence of access to detailed data the electricity importations/exportations are not taken into account within our model. However, in the hypothetical case that electricity importations/exportations were part of our modelling, they could be considered either exogenous or endogenous to our model. If they were exogenous then the demand would be translated by the production that is imported/exported which would not modify our modeling. On the contrary, if they were endogenous, the complexity of the modelling would increase since several new parameters have to be taken into account in our model e.g. technical constraints imposed by the transmission power lines, the price elasticity of foreign demand, etc.

## 2.1 Modelling the demand

We make the assumption that the demand, being exogenous, is perfectly inelastic. As we mentioned in our previous paper, this assumption is obviously a simplification motivated by some arguments: in short-term to medium-term the demand is already determined by previous investments in electrical devices and ways of life whose evolutions require time, thus it is less sensitive to price. Let us also remind that electricity is sold to consumers by retailing companies which pay the wholesale spot prices directly to the producers. There is no bilateral contracting regime between retailers and producers.

Furthermore, the monthly demand is translated by the aggregate monthly hydro production coming from the run-of-river hydro units. The run-of-river hydro units have little or no capacity for energy storage, hence they can not co-ordinate the output of electricity generation to match consumer demand. Consequently, they serve as base load power plants. Since the hydro technology with no reservoir (run-of-river) is a base load generation technology which is presumably never marginal, it is necessary to call up nuclear to cover the different levels of demand. We do this translation in order to remove the part of the base load demand served by the run-of-river hydro units and thus to obtain a more clear vision of the demand which will be served by the nuclear and non-nuclear thermal units. The seasonal variations of hydro production due to precipitation and snow melting are not taken into consideration, thereby we assume that the aggregate monthly hydro-production is constant through the entire time horizon of our model. We do not take into account the capacity coming from hydro units with possibility of storage (peaking power plants) because of the additional capacity and storage constraints which would increase the complexity of the model.

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<sup>1</sup>Let us recall that price taking firms guarantee that when firms maximize their profits (by choosing the quantity they wish to produce and the technology of generation to produce it with) the market price will be equal to marginal cost.

## 2.2 Modelling the time horizon

The time horizon of the model is  $T = 36$  months<sup>2</sup> beginning by the month of January. We choose a time horizon of 36 months for our modelling because we need a sufficiently long time horizon to follow up the evolution of the optimal production and storage fuel levels as well as the variations of price and profit. A French nuclear producer has two main options with respect to the scheduling of fuel reloading: (i) 1/3 of fuel reservoir that corresponds to 18 months of campaign<sup>3</sup> and 396 days equivalent to full capacity for a unit of 1300 MW, (ii) 1/4 of fuel reservoir that corresponds to 12 months of campaign and 258 days equivalent to full capacity for a unit of 1500 MW (Source: EDF, CEA, see Ref. [18]). Note that both options result from the operational schema of EDF (Electricité de France). The scheduling of fuel reloading is entirely exogenous within our model because the regular length of a campaign depends on many factors (technical specificities of the reactor, size, age, management decision to reload the reactor's heart per third or quarter of its full capacity, type of nuclear fuel put into the fuel bars, forecasted average rate of use of the reactor, regulatory constraints issued by safety inspectors...) which are difficult to control in order to determine endogenously the duration of the campaign within our model. In view of the exogenous scheduling of the fuel reloading (per third, per quarter of fuel reservoir), we consider that the quantity of the fuel reloaded in the reservoir is also exogenous and is decided exclusively by EDF. In order to get a tractable model, we need a cyclic model for the modelling of the campaign. We exclude the case of having both a campaign of 12 and of 18 months to avoid complicate our model. We do not retain the first modelling, hence a campaign of 18 months because it is not consistent with the "good" seasonal allocation of shutdowns of the nuclear plants. Indeed, if a nuclear operator reloads fuel in summer when the demand is low the date of the next reloading will be then in winter when the demand is high. Consequently, we retain a modelling close to the second modelling, thus a duration of campaign equivalent to 12 months to get a cyclic model with a periodicity of one year. The one year period can be then decomposed into 11 months being the period of production and 1 month corresponding to the month of reloading of the fuel. We also assume that the value of profit is not discounted during the period of 36 months; the discounting rate would have an important impact on the value of profit in the case of a longer time horizon (e.g. 84 months). We do not deal with the question of the optimal allocation of the shutdowns in this thesis for several reasons: (i) lack of operational data for confidentiality reasons, (ii) for operational reasons and because of the intervention of many qualified persons external to the nuclear producer for the reloading of a reactor, the length of the campaign is determined in advance in order to get a general scheduling of reloading, (iii) lack of information with regard to the periodical inspections of nuclear reactors and the inspections imposed by the Nuclear Safety Authority. Hence, we assume that the refueling dates are exogenous given via a programming realized by EDF (model ORION) which determines the optimal allocation of the shutdowns of nuclear units for reloading.

## 2.3 Modelling the generating units

We study a competitive electricity market with  $N \geq 2$  producers who manage both nuclear and non-nuclear thermal generation resources. We define 12 different types of nuclear generating units. A producer  $n = 1, \dots, N$  operates with all types of nuclear generating units. It holds

<sup>2</sup>The time horizon of the model is a multiplicative of twelve, being expressed in months. Therefore it could be modified.

<sup>3</sup>The length of a campaign (of production) is determined by the maximum number of days during which a nuclear unit produces until exhaustion of its fuel of reloading.

a certain level of capacity from each type of nuclear unit. The nuclear units also have their own single cost function. However, we assume that these units differ by the available nuclear capacity that each of them holds as by the month of their fuel reloading. We can then define our twelve “types” of nuclear units. Each type indexed by  $j = 1, \dots, 12$  corresponds to a different month of reloading of the nuclear unit. Then, a unit which belongs to the type of unit  $j = 1$  (respectively  $j = 2, \dots, j = 12$ ) shuts down in the month of January (respectively February,  $\dots$ , December).

The level of the nuclear production during the month  $t = 1, \dots, T$  for the unit  $j$  of producer  $n$  will be denoted by  $q_{njt}^{nuc}$ . Moreover, the maximum nuclear production that can be realized by the unit  $j$  of producer  $n$  during a month is given by the parameter  $Q_{max}^{n,j,nuc}$ , while the minimum nuclear production is equal to  $Q_{min}^{n,j,nuc}$ .

Symmetrically, each producer disposes a certain amount of non-nuclear thermal capacity. The level of the non-nuclear thermal production during the month  $t = 1, \dots, T$  for the producer  $n$  will be denoted by  $q_{nt}^{th}$ . Furthermore the maximum non-nuclear thermal production during a month for the producer  $n$  is given by the parameter  $Q_{max}^{n,th}$  and corresponds to the nominal non-nuclear thermal capacity, while there is no minimum for non-nuclear thermal production  $Q_{min}^{n,th} = 0$ .

The cost functions of both nuclear and non-nuclear thermal production are common among the different producers.

## 2.4 Modelling the production costs

The nuclear cost function is made of a fixed part determined by the cost of investment, the fixed cost of exploitation and taxes and a variable part which corresponds to the variable cost of exploitation and the fuel cost. We assume that the cost function<sup>4</sup>  $C_{n,j}^{nuc}(\cdot)$  of the nuclear production is affine and defined as

$$C_{n,j}^{nuc}(q_{njt}^{nuc}) = a_{nuc}^{n,j} + b_{nuc} q_{njt}^{nuc}.$$

The non-nuclear thermal cost function is also made of a fixed part which corresponds to the cost of investment, the fixed cost of exploitation and taxes and a variable part covering the variable cost of exploitation, the fuel cost, the cost of CO<sub>2</sub> as well as the taxes on the gas fuel. We assume that the non-nuclear production has a quadratic cost function  $C_n^{th}(\cdot)$  which is the following

$$C_n^{th}(q_{nt}^{th}) = a_{th}^n + b_{th} q_{nt}^{th} + c_{th}^n q_{nt}^{th^2}.$$

**Proposition 2.1** *The fixed part  $a_{th}^n$  of the non-nuclear thermal cost function is proportional to the capacity  $Q_{max}^{n,th}$  while the coefficients  $b_{th}, c_{th}^n$  of the variable part of the non-nuclear thermal cost function are such that : (i)  $b_{th}$  does not depend on the capacity  $Q_{max}^{n,th}$ , (ii)  $c_{th}^n$  is inversely proportional to the capacity  $Q_{max}^{n,th}$ .*

### Proof

In order to understand this dependency of the coefficients  $a_{th}^n, b_{th}, c_{th}^n$ , let us consider a particular case. Let us assume that the non-nuclear thermal capacity of producer 2 is twice the non-nuclear thermal capacity of producer 1 :  $Q_{max}^{2,th} = 2Q_{max}^{1,th}$ . Therefore, producer 2 can be seen as the aggregation of the identical copies of producer 1. Then, the “total” cost  $C_2^{th}(q_{2t}^{th})$  of

<sup>4</sup>The coefficient  $a_{nuc}^{n,j}$  is proportional to the capacity  $Q_{max}^{n,j,nuc}$  since it corresponds to the fixed part of the nuclear cost function.



production (obtained by the capacity  $Q_{max}^{2,th}$ ) is equal to the minimum of the sum of “individual” costs. More precisely, we have

$$C_2^{th}(q_{2t}^{th}) = \min_{q_{1t}^{th}, \bar{q}_{1t}^{th}} (C_1^{th}(q_{1t}^{th}) + C_1^{th}(\bar{q}_{1t}^{th}))$$

subject to the constraints

$$0 \leq q_{1t}^{th} \leq Q_{max}^{1,th}, \quad \text{for all } t$$

$$0 \leq \bar{q}_{1t}^{th} \leq Q_{max}^{1,th}, \quad \text{for all } t$$

$$q_{1t}^{th} + \bar{q}_{1t}^{th} = q_{2t}^{th}, \quad \text{for all } t$$

The resolution of this convex (and polynomial) optimization problem is very simple and the solution is symmetric :  $q_{1t}^{th} = \bar{q}_{1t}^{th} = \frac{q_{2t}^{th}}{2}$ . Hence, the cost function will be

$$C_2^{th}(q_{2t}^{th}) = 2a_{th}^1 + b_{th}q_{2t}^{th} + \frac{c_{th}^1}{2}q_{2t}^{th2}.$$

Consequently, we conclude that when the capacity doubles, the coefficient  $a_{th}^1$  is multiplied by two, the coefficient  $b_{th}$  is unchanged and the coefficient  $c_{th}^1$  is divided by two.

We showed the dependency of the coefficients of the non-nuclear thermal cost  $C_n^{th}$  from the capacity  $Q_{max}^{n,th}$  in the particular case that  $Q_{max}^{2,th} = \nu Q_{max}^{1,th}$ , where  $\nu \in \mathbb{N}^*$ . Symmetrically, this dependency is also verified in the case that  $Q_{max}^{2,th} = \frac{1}{\nu} Q_{max}^{1,th}$ . Thus, one deduces that a similar result is obtained when<sup>5</sup>  $\nu \in \mathbb{Q}_+^*$  (we remind that a positive rational number is expressed as a fraction  $\frac{a}{b}$ , where  $a, b \in \mathbb{N}^*$ ). However, the set of rational numbers  $\mathbb{Q}$  is dense in  $\mathbb{R}$  which means that all real numbers can be approximated by rational numbers (see Ref. [25]). Hence, this dependency still holds in the general case that<sup>6</sup>  $\nu \in \mathbb{R}_+^*$ . □

The nuclear and non-nuclear cost functions are monotone increasing and convex functions of  $q_{njt}^{nuc}$  and  $q_{nt}^{th}$  respectively. As we explained in our previous paper, we choose a quadratic cost function in the case of non-nuclear thermal because of the increasing marginal cost of the non-nuclear thermal production since it results from different fossil fuel generation technologies (e.g. coal, gas -combined cycle or not-, fuel oil). Moreover, the non-nuclear production needed a non constant marginal cost function in order to recover its fixed costs. Indeed, if we assume a constant marginal cost function for non-nuclear thermal then the value of the non-nuclear thermal production when non-nuclear is the marginal technology does not permit to recover its fixed costs. So, we assume that the marginal cost of nuclear  $mc^{nuc}(q_{njt}^{nuc})$  is a constant function of  $q_{njt}^{nuc}$  while that of non-nuclear thermal  $mc_n^{th}(q_{nt}^{th})$  is an increasing function of  $q_{nt}^{th}$ .

## 2.5 Modelling the nuclear fuel stock

Let us denote  $S_{reload}^{n,j}$ , the nuclear fuel stock of reloading available to the unit  $j$  of producer  $n$ . The variable  $S_t^{n,j} \geq 0$ , which represents the quantity of fuel stored in the nuclear reservoir and available to the unit  $j$  of producer  $n$  at the beginning of the month  $t$ , is the potential energy that can be produced with this stock. The evolution of the nuclear fuel stock is then determined by the following rules

<sup>5</sup>The set of positive rational numbers is usually denoted as  $\mathbb{Q}_+^*$ .

<sup>6</sup>The set of positive real numbers is usually denoted as  $\mathbb{R}_+^*$ .

$$S_1^{n,j} \text{ given, } S_{t+1}^{n,j} = \begin{cases} S_t^{n,j} - q_{njt}^{nuc}, & \text{if no reload during month } t \text{ for unit } j \\ S_{reload}^{n,j}, & \text{if unit } j \text{ reloads during month } t \end{cases} \quad (1)$$

The relationship (1) traces the evolution of the stock given the flow of the nuclear production. In the case that  $t$  is the month during which the producer reloads the fuel of the reactor, the stock at the beginning of the following month (beginning of the campaign) is equal to  $S_{reload}^{n,j}$ .

Moreover, we impose

$$S_{T+1}^{n,j} \geq S_1^{n,j} \quad (2)$$

The constraint (2) implies that the producer  $n$  must keep his nuclear units at the end of the game in the same storage condition as the initial one. It means that each nuclear producer has to finish the period  $T$  at least with the same quantity of nuclear fuel as the initial one. In this way each producer has to “spare” its nuclear fuel during the production period. The absence of this constraint could lead to an “over-consumption” of the nuclear fuel stock in order to produce the maximum; this could generate some negative effects (e.g. insufficient level of stock to reach at least the minimum nuclear production level during some months (excluding the month of reloading)). Furthermore, the constraint (2) guarantees that the producer will start a new cycle of this game with a quantity of stock equal to  $S_1^{n,j}$  at the beginning of the game.

Let us notice that the producer  $n$  spends all its nuclear fuel stock of reloading  $S_{reload}^{n,j}$  during a campaign (11 months), thus it disposes a quantity of nuclear fuel stock equal to zero at the end of the campaign (beginning of the month of reloading). Several reasons lead us to this ascertainment:

- The technical aspect related with the way that the length of a campaign of nuclear units is determined (cf. Footnote 3, page 6).
- The evaluation of the variable part ( $b_{nuc}$ ) of the nuclear cost function which partially corresponds to the fuel cost is based on the fact that a producer uses all the available nuclear fuel stock: if a nuclear producer keeps paying in order to obtain the fuel stock  $S_{reload}^{n,j}$  even in the case that it does not consume all the stock during a campaign, then this cost can be regarded as a fixed cost which is paid at the beginning of each campaign. Consequently, the fuel cost should be integrated into the fixed part of the nuclear cost function, which means that the coefficient  $a_{nuc}^{n,j}$  and thus the nuclear cost would be modified.
- The cost that a nuclear producer undergoes to get rid of the unused nuclear fuel at the end of the campaigns (cost related to the reprocessing of nuclear fuel).

For the same reasons, the constraint (2) can not hold as inequality constraint ( $S_{T+1}^{n,j} > S_1^{n,j}$ ) which means that the surplus of stock at the end of the game is zero. Note that there exists an obvious analogy with Walras' Law. Consequently, the constraint (2) will take the form

$$S_{T+1}^{n,j} = S_1^{n,j} \quad (3)$$

We proceed now with Proposition 2.2 in order to define the nuclear fuel constraints for the unit  $j$  of producer  $n$ .

**Proposition 2.2** *If the evolution of the stock is determined by the relationship (1) and the constraint (3) is imposed, then the nuclear fuel constraints for the nuclear unit  $j$  of producer  $n$  are defined as following :*

$$\left\{ \begin{array}{ll} \sum_{t=2}^{12} q_{n1t}^{nuc} = S_{reload}^{n,1}, & \text{so that unit 1 uses stock reloaded during month 1} \\ \sum_{t=14}^{24} q_{n1t}^{nuc} = S_{reload}^{n,1}, & \text{so that unit 1 uses stock reloaded during month 13} \\ \sum_{t=26}^T q_{n1t}^{nuc} = S_{reload}^{n,1}, & \text{so that unit 1 uses stock reloaded during month 25} \end{array} \right. \quad (4)$$

$$\left\{ \begin{array}{ll} \sum_{t=1}^{j-1} q_{njt}^{nuc} = S_1^{n,j}, & \text{so that unit } j \text{ uses stock available in month 1} \\ \sum_{t=j+1}^{j+12-1} q_{njt}^{nuc} = S_{reload}^{n,j}, & \text{so that unit } j \text{ uses stock reloaded during month } j \\ \sum_{t=j+12+1}^{j+2*12-1} q_{njt}^{nuc} = S_{reload}^{n,j}, & \text{so that unit } j \text{ uses stock reloaded during month} \\ & \text{ } j + 12 \\ \sum_{t=j+2*12+1}^T q_{njt}^{nuc} = S_{reload}^{n,j} - S_1^{n,j}, & \text{so that unit } j \text{ uses stock reloaded during month} \\ & \text{ } j + 24 \text{ until the end of the game} \end{array} \right. \quad (5)$$

$$\left\{ \begin{array}{ll} \sum_{t=1}^{11} q_{n12t}^{nuc} = S_{reload}^{n,12}, & \text{so that unit 12 uses stock of reloading from month 1} \\ \sum_{t=13}^{23} q_{n12t}^{nuc} = S_{reload}^{n,12}, & \text{so that unit 12 uses stock reloaded during month 12} \\ \sum_{t=25}^{T-1} q_{n12t}^{nuc} = S_{reload}^{n,12}, & \text{so that unit 12 uses stock reloaded during month 24} \end{array} \right. \quad (6)$$

### Proof

#### • $j=1$

Let us start our proof by the simplest case: the nuclear units of type  $j = 1$ . They reload their nuclear fuel in the month of January. The producer  $n$  finishes the campaigns with a quantity of stock equal to zero.

Then, from the relationship (1), we have that

$$0 = S_{13}^{n,1} = S_{12}^{n,1} - q_{n,1,12}^{nuc} = \dots = S_2^{n,1} - \sum_{t=2}^{12} q_{n,1,t}^{nuc} = S_{reload}^{n,1} - \sum_{t=2}^{12} q_{n,1,t}^{nuc} \Leftrightarrow \sum_{t=2}^{12} q_{n1t}^{nuc} = S_{reload}^{n,1}$$

Thus, the nuclear production realized by the unit 1 of producer  $n$  during its first campaign is equal to the nuclear fuel stock of reloading that is available to the unit 1 at the beginning of the campaign.

Symmetrically, the nuclear production realized by the unit 1 of producer  $n$  during its second respectively third campaign is determined by the following constraints

$$0 = S_{25}^{n,1} = S_{24}^{n,1} - q_{n,1,24}^{nuc} = \dots = S_{14}^{n,1} - \sum_{t=14}^{24} q_{n1t}^{nuc} = S_{reload}^{n,1} - \sum_{t=14}^{24} q_{n1t}^{nuc} \Leftrightarrow \sum_{t=14}^{24} q_{n,1,t}^{nuc} = S_{reload}^{n,1}$$

respectively

$$0 = S_1^{n,1} = S_{36}^{n,1} - q_{n,1,36}^{nuc} = \dots = S_{26}^{n,1} - \sum_{t=26}^{36} q_{n1t}^{nuc} = S_{reload}^{n,1} - \sum_{t=26}^{36} q_{n1t}^{nuc} \Leftrightarrow \sum_{t=26}^{36} q_{n,1,t}^{nuc} = S_{reload}^{n,1}$$

- **j=2**

We continue our proof with the nuclear units of type 2. They reload their nuclear fuel in the month of February.

According to the relationship (1), given the quantity of nuclear fuel available to the unit 2 of producer  $n$  at the beginning of the time horizon of the model,  $S_1^{n,2} \geq 0$ , the nuclear fuel constraints are determined as following

$$0 = S_2^{n,2} = S_1^{n,2} - q_{n,2,1}^{nuc} \Leftrightarrow q_{n,2,1}^{nuc} = S_1^{n,2}$$

$$0 = S_{14}^{n,2} = S_{13}^{n,2} - q_{n,2,13}^{nuc} = \dots = S_3^{n,2} - \sum_{t=3}^{13} q_{n,2,t}^{nuc} = S_{reload}^{n,2} - \sum_{t=3}^{13} q_{n,2,t}^{nuc} \Leftrightarrow \sum_{t=3}^{13} q_{n,2,t}^{nuc} = S_{reload}^{n,2}$$

$$0 = S_{26}^{n,2} = S_{25}^{n,2} - q_{n,2,25}^{nuc} = \dots = S_{15}^{n,2} - \sum_{t=15}^{25} q_{n,2,t}^{nuc} = S_{reload}^{n,2} - \sum_{t=15}^{25} q_{n,2,t}^{nuc} \Leftrightarrow \sum_{t=15}^{25} q_{n,2,t}^{nuc} = S_{reload}^{n,2}$$

$$S_{T+1}^{n,2} = S_T^{n,2} - q_{n,2,T}^{nuc} = \dots = S_{27}^{n,2} - \sum_{t=27}^T q_{n,2,t}^{nuc} = S_{reload}^{n,2} - \sum_{t=27}^T q_{n,2,t}^{nuc} = S_1^{n,2} \Leftrightarrow \sum_{t=27}^T q_{n,2,t}^{nuc} = S_{reload}^{n,2} - S_1^{n,2}$$

The last constraint results from the evolution of the stock and from the obligation of producer  $n$  to keep its nuclear units at the end of the period  $T$  in the same storage condition as the initial one (constraint (3)).

- **j ∈ {3, ..., 11}**

The nuclear units of type {3, ..., 11} reload their fuel in the month of {March, ..., November}. The proof applied in this case is symmetrical with the one provided for the nuclear units of type 2.

- **j=12**

Finally, we finish our proof with the nuclear units of type 12. They reload their nuclear fuel in the month of December. The producer  $n$  finishes the campaigns with a quantity of stock equal to zero. The proof is similar to the one given for the nuclear units of type 1. □

Later, the length of a campaign will also correspond to the maximum number of days that a nuclear unit produces until the “available to the unit” nuclear fuel stock is exhausted.

## 2.6 The modelling of the optimal inter-temporal production behaviour

The optimal inter-temporal production problem that producer  $n$  resolves is the following:

$$\max_{((q_{njt}^{nuc})_{j=1}^J, (q_{nt}^{th})_{t=1}^T)} \sum_{t=1}^T (p_t (\sum_{j=1}^J q_{njt}^{nuc} + q_{nt}^{th}) - \sum_{j=1}^J C_{nj}^{nuc}(q_{njt}^{nuc}) - C_n^{th}(q_{nt}^{th})) \quad (7)$$

subject to the nuclear fuel storage constraints (4), (5), (6) as well as

$$\begin{cases} Q_{min}^{n,j,nuc} \leq q_{njt}^{nuc} \leq Q_{max}^{n,j,nuc}, & \text{if no reload during month } t \text{ for unit } j \\ q_{njt}^{nuc} = 0, & \text{if unit } j \text{ reloads during month } t \end{cases} \quad (8)$$

$$0 \leq q_{nt}^{th} \leq Q_{max}^{n,th}, \quad \text{for all } t \quad (9)$$

where  $J$  is the total number of units ( $J = 12$ ) and the price  $p_t$  is given (perfect competition) by the equality between supply and demand.

The constraint (8) shows that the nuclear production of each month is bound by the minimum/maximum quantity of nuclear production which can be obtained during a month. The non-nuclear thermal production is a non negative quantity which is also bound by the maximum non-nuclear thermal production (constraint (9)). The producer can use the non-nuclear thermal resources to produce electricity until it reaches the level of demand of the corresponding month without however violating the constraint (9).

### 3 Symmetry of equilibrium of the optimal inter-temporal production problem

In this section, we provide an economical property of producer's optimal behaviour. Under the assumption that each producer disposes the same level of nuclear and non-nuclear thermal capacity, we show that an equilibrium of the inter-temporal profit maximization problem (7) is "almost" symmetric.

#### 3.1 The notion of equilibrium

Let us give a definition of equilibrium with respect to a system of prices  $p \in \mathbb{R}^T$

**Definition 3.1** The production vector  $(((\bar{q}_{1jt}^{nuc})_{j=1}^J, \bar{q}_{1t}^{th})^T, \dots, ((\bar{q}_{Njt}^{nuc})_{j=1}^J, \bar{q}_{Nt}^{th})^T)$  is an equilibrium with respect to a system of prices  $p \in \mathbb{R}^T$  if:

- (i) it maximizes the profit of producer  $n$  on the set of feasible solutions, for all  $n$ .
- (ii) at each date  $t$ , it respects the equality between supply and demand.

According to the subsection 2.1, the monthly demand which is considered in this model results from the difference between the level of demand  $D_t$  observed in month  $t$  and the aggregate hydro production  $Q_{Tot,t}^{hyd}$  provided during the month  $t$ . At each date  $t$ , the equality between supply and demand is defined as

$$\sum_{n=1}^N \left( \sum_{j=1}^J q_{njt}^{nuc} + q_{nt}^{th} \right) = D_t - Q_{Tot,t}^{hyd} \quad (10)$$

#### 3.2 The notion of symmetrisability

In view of the assumption of symmetry of the nuclear and non-nuclear thermal capacity among producers, the exogenous variables  $(Q_{max}^{n,j,nuc}, Q_{min}^{n,j,nuc}, Q_{max}^{n,th}, S_{reload}^{n,j}, S_1^{n,j})$  of the optimization problem (7) as well as the production cost functions  $(C_{n,j}^{nuc}, C_n^{th})$  and thus, the marginal non-nuclear thermal cost function  $(mc_n^{th})$  no longer depend on  $n$ . If  $(((\bar{q}_{1jt}^{nuc})_{j=1}^J, \bar{q}_{1t}^{th})^T, \dots, ((\bar{q}_{Njt}^{nuc})_{j=1}^J, \bar{q}_{Nt}^{th})^T)$  is an equilibrium of this problem, we show that the non-nuclear thermal component of the equilibrium is symmetric while the nuclear component of the equilibrium is symmetrisable. Let us define the notion of a symmetrisable equilibrium as following:

**Definition 3.2** Let  $(q_1, q_2, \dots, q_N)$  be an equilibrium. This equilibrium is called symmetrisable if there exists a symmetric allocation  $(\tilde{q}_1, \tilde{q}_2, \dots, \tilde{q}_N)$ , which is an equilibrium and “leads” to the same prices as the initial allocation  $(q_n)_{n=1}^N$ .

Notice that the notion of symmetrisability of an equilibrium provides us with an interesting economical feature: the profit of a symmetrisable equilibrium  $(\pi_1, \pi_2, \dots, \pi_N)$  is symmetric. This means that the production levels included in a symmetrisable equilibrium are equivalently profitable for all producers. This arises from the fact that the profit resulting from a symmetrisable equilibrium is equal to the profit which results from the deduced symmetric equilibrium since the price induced by the symmetrisable equilibrium is equal to the price induced by the symmetric equilibrium. For a symmetric equilibrium, the value of profit is equal among the different producers at the equilibrium state, since the price  $p_t$  as well as the production level are identical for all players and the production cost is symmetric for both technologies.

### 3.3 Symmetry of the non-nuclear thermal component and symmetrisability of the nuclear component of equilibrium

Before we proceed with Proposition 3.1, let us provide a useful remark:

**Remark 3.1** *The optimal inter-temporal production problem (7) is additively separable with respect to the couple  $(q_{njt}^{nuc}, q_{nt}^{th})$  (see Ref. [30]). This results from the fact that the inter-temporal profit function is an additively separable function since it can be written as the sum of the inter-temporal profit resulting from the nuclear production and the inter-temporal profit coming from the non-nuclear thermal production and because there is no constraints linking the production variables  $(q_{njt}^{nuc})$  and  $(q_{nt}^{th})$ . Consequently, the two inter-temporal profit maximization problems (nuclear and non-nuclear thermal) can be studied independently. First, let us study the non-nuclear thermal inter-temporal profit maximization problem of producer  $n$ . By the same argument, it is additively separable with respect to the time index  $t$  since the inter-temporal profit is additively separable with respect to  $t$  and the minimum and maximum non-nuclear thermal production constraints are mutually “independent” on  $t$ . Then, we look at the nuclear inter-temporal profit maximization problem of producer  $n$ . It is additively separable with respect to the unit index  $j$  because the profit is additively separable with respect to  $j$  and the nuclear production constraints are mutually “independent” on  $j$ . However, this is not the case for the time index  $t$  because the nuclear production constraints and more precisely, the nuclear fuel constraints link the nuclear production variables  $(q_{njt}^{nuc})$  on  $t$  (inside a campaign). Hence, the nuclear optimal production problem of producer  $n$  is not separable with respect to  $t$ .*

We are ready now to state the following proposition.

**Proposition 3.1** *Let be  $((q_{1jt}^{nuc})_{j=1}^J, q_{1t}^{th})_{t=1}^T, \dots, ((q_{Njt}^{nuc})_{j=1}^J, q_{Nt}^{th})_{t=1}^T$  an equilibrium of the optimal inter-temporal production problem (7). If the nuclear and the non-nuclear thermal capacity are symmetric among producers then the non-nuclear thermal component of the equilibrium is symmetric while the nuclear component of the equilibrium is symmetrisable.*

#### Proof

First, we show that the non-nuclear thermal component of the equilibrium  $(q_{1t}^{th}, q_{2t}^{th}, \dots, q_{Nt}^{th})$  is symmetric. In view of the Remark 3.3.1, it follows that, for all  $t$ , the non-nuclear thermal production  $q_{nt}^{th}$  is a solution of

$$\max_{\psi_{nt}^{th}} p_t \psi_{nt}^{th} - C^{th}(\psi_{nt}^{th})$$

subject to the constraint

$$0 \leq \psi_{nt}^{th} \leq Q_{max}^{th}, \text{ for all } t$$

where  $n \in \{1, \dots, N\}$ .

We want to show that  $q_{1t}^{th} = q_{2t}^{th} = \dots = q_{Nt}^{th}$ . One has  $q_{1t}^{th}$  (respectively  $q_{2t}^{th}, \dots, q_{Nt}^{th}$ ) a solution of

$$\max_{\psi_{1t}^{th}} p_t \psi_{1t}^{th} - C^{th}(\psi_{1t}^{th}) \tag{11}$$

subject to the constraint

$$0 \leq \psi_{1t}^{th} \leq Q_{max}^{th}$$

In view of the strict concavity of the cost function  $C^{th}$ , the problem (11) has a unique solution. Since  $q_{1t}^{th}, q_{2t}^{th}, \dots, q_{Nt}^{th}$  are all solutions of problem (11), they are equal and the symmetry is proved.

**Remark 3.2** *Note that when the non-nuclear thermal production is positive, the price is determined by the marginal cost of the non-nuclear thermal production ( $p_t = mc^{th}(q_t^{th})$ ).*

Let us now focus on the nuclear component of the equilibrium. We prove that the nuclear component of the equilibrium  $(q_{1jt}^{nuc}, q_{2jt}^{nuc}, \dots, q_{Njt}^{nuc})$  is symmetrisable. By analogy with the case of the non-nuclear thermal technology, for all  $j$ , the nuclear production  $q_{njt}^{nuc}$  of producer  $n$  is a solution of

$$\max_{\psi_{njt}^{nuc}} \sum_{t=1}^T (p_t \psi_{njt}^{nuc} - C_j^{nuc}(\psi_{njt}^{nuc})) \tag{12}$$

subject to the nuclear fuel storage constraints and the minimum/maximum nuclear production constraints.

The constraints of the reduced optimization problem (12) determine its domain of definition. It is clear that it is a convex set, since it is defined by affine constraints. In addition, the objective function of the reduced profit maximization problem (12) is affine. Thus, it is a concave function. In view of the convexity of the domain of definition and the concavity of the objective function of the problem (12), we conclude that the set of solutions of the optimization problem (12) is convex (see Ref. [20]). Therefore, the allocation  $(\tilde{q}_{1jt}^{nuc}, \tilde{q}_{2jt}^{nuc}, \dots, \tilde{q}_{Njt}^{nuc})$ , where  $(\tilde{q}_{1jt}^{nuc} = \tilde{q}_{2jt}^{nuc} = \dots = \tilde{q}_{Njt}^{nuc} = \sum_{n=1}^N \frac{q_{njt}^{nuc}}{N}$  for all  $j, t$ ) is also a solution of this problem. In addition, at each date  $t$ , it respects the equality between supply and demand. Consequently, the symmetric allocation  $(\tilde{q}_{1jt}^{nuc}, \tilde{q}_{2jt}^{nuc}, \dots, \tilde{q}_{Njt}^{nuc})$  is an equilibrium. In order to show that it “leads” to the same price as the initial equilibrium  $(q_{1jt}^{nuc}, q_{2jt}^{nuc}, \dots, q_{Njt}^{nuc})$ , we proceed with Lemma 3.1.

**Lemma 3.1** *The price induced by  $((q_{1jt}^{nuc})_{j=1}^J, q_{1t}^{th}), \dots, ((q_{Njt}^{nuc})_{j=1}^J, q_{Nt}^{th})$  is equal to the price induced by  $((\tilde{q}_{1jt}^{nuc})_{j=1}^J, q_{1t}^{th}), \dots, ((\tilde{q}_{Njt}^{nuc})_{j=1}^J, q_{Nt}^{th})$  for all  $t$ .*

**Proof**

Let be  $P^n = \{t \in \{1, \dots, T\} \text{ s.t. } q_{nt}^{th} = 0\}$  for each  $n \in \{1, \dots, N\}$ . Note that in view of the first part of the proof, the set  $P^n$  does not depend on  $n$ . If  $t \notin P$ , there is nothing to prove since the price is determined by the marginal cost of non-nuclear thermal marginal production which is unchanged. If  $t \in P$ , in both cases, the price  $p_t$  is determined by the marginal cost of

the nuclear production which is constant ( $p_t = mc^{nuc}(q_{njt}^{nuc}) = b_{nuc}$  for all  $t \in P$ ). Consequently, it is obvious that both prices are equal for all  $t$ .  $\square$

Hence, in view of Lemma 3.1, the equilibrium  $(\bar{q}_{1jt}^{nuc}, \bar{q}_{2jt}^{nuc}, \dots, \bar{q}_{Njt}^{nuc})$  “leads” to the same price as the initial one. So, according to the Definition 3.2, the nuclear component of the equilibrium  $(q_{1jt}^{nuc}, q_{2jt}^{nuc}, \dots, q_{Njt}^{nuc})$  is symmetrisable.

We conclude that the equilibrium  $((q_{1jt}^{nuc})_{j=1}^J, (q_{1t}^{th})_{t=1}^T, \dots, ((q_{Njt}^{nuc})_{j=1}^J, (q_{Nt}^{th})_{t=1}^T)$  is characterized by a symmetric non-nuclear thermal component and a symmetrisable nuclear component.  $\square$

It should be noticed that the nuclear component of the equilibrium  $(q_{1jt}^{nuc}, q_{2jt}^{nuc}, \dots, q_{Njt}^{nuc})$  is potentially asymmetric. In order to understand this asymmetry, let us give an example in the case of two producers ( $N = 2$ ). Let  $(\hat{q}_{1jt}^{nuc}, \hat{q}_{2jt}^{nuc})$  be a symmetric equilibrium such that the price is the same during the period 1 and the period 2 (i.e.  $p_1 = p_2$ ). This occurs in particular, if nuclear is the marginal technology in periods 1 and 2. Then, any feasible production realized by the unit 3 of producer 1 (respectively 2) in periods 1, 2 that means a solution of the following system:

$$\left\{ \begin{array}{ll} q_{131} + q_{231} = \hat{q}_{131} + \hat{q}_{231} = D_1 - Q_{Tot,1}^{hyd}, & \text{supply - demand equilibrium constraint} \\ & \text{in month 1} \\ q_{132} + q_{232} = \hat{q}_{132} + \hat{q}_{232} = D_2 - Q_{Tot,2}^{hyd}, & \text{supply - demand equilibrium constraint} \\ & \text{in month 2} \\ q_{131} + q_{132} = \hat{q}_{131} + \hat{q}_{132} = S_1^3, & \text{nuclear fuel constraint for unit 3 of producer 1} \\ q_{231} + q_{232} = \hat{q}_{231} + \hat{q}_{232} = S_2^3, & \text{nuclear fuel constraint for unit 3 of producer 2} \end{array} \right.$$

and unchanged during the remaining periods ( $q_{njt}^{nuc} = \hat{q}_{njt}^{nuc}$ , for all  $n \in \{1, 2\}$ , for all  $j \neq 3$ , and for  $t \geq 3$ ) will be still an equilibrium. Consequently, there exists at least one asymmetric equilibrium of the optimal production problem (12), where for example unit 3 produces more for producer 1 than for producer 2 in period 1 (and the opposite in period 2 in order to compensate).

In view of Proposition 3.1, we will only focus on the symmetric solution, thus it does not depend on  $n$  any longer. This leads to a decrease of the number of the optimization variables of the optimal inter-temporal production problem (7) which simplifies its resolution.

## 4 Optimization of the inter-temporal production

In this section, we study the optimization of the inter-temporal production. First, we show that under some assumptions the inter-temporal profit decreases for all the production vectors with zero non-nuclear thermal production. Therefore, we search for a solution of the optimal inter-temporal production problem (7) among the feasible solutions which are determined by strictly positive non-nuclear thermal production levels. Then, we continue our study with the proof of a property that characterizes the “interior” optimal solutions of problem (7). We also prove for a production vector with strictly positive non-nuclear thermal production levels that if it has this property then it constitutes a solution of the inter-temporal optimal production problem (7).



## 4.1 Producers' inter-temporal profit decrease in the absence of non-nuclear thermal production

We define the set of feasible solutions of the optimal production problem (7) as

$$C = \left\{ q \in M \text{ s.t. } \begin{array}{l} Q_{min}^{j,nuc} \leq q_{jt}^{nuc} \leq Q_{max}^{j,nuc}, \text{ for all } j, t \\ 0 \leq q_t^{th} \leq Q_{max}^{th}, \text{ for all } t \end{array} \right\}$$

where  $M$  is defined by all the production vectors of the form  $q = ((q_{j1}^{nuc})_{j=1}^J, \dots, (q_{jT}^{nuc})_{j=1}^J, q_1^{th}, \dots, q_T^{th})$  that respect the nuclear fuel constraints ((4), (5), (6)) as well as the supply-demand equilibrium constraint (10) for all  $t$ . The set  $M$  is affine and the set  $C$  is compact and convex.

Moreover, we define  $F$  as the relative interior<sup>7</sup> of  $C$  ( $F = ri(C)$ ). It has the following form

$$F = \left\{ q \in M \text{ s.t. } \begin{array}{l} Q_{min}^{j,nuc} < q_{jt}^{nuc} < Q_{max}^{j,nuc}, \text{ for all } j, t \\ 0 < q_t^{th} < Q_{max}^{th}, \text{ for all } t \end{array} \right\}$$

Let us focus on the set  $F^{th}$  defined as

$$F^{th} = \left\{ q \in M \text{ s.t. } \begin{array}{l} Q_{min}^{j,nuc} \leq q_{jt}^{nuc} \leq Q_{max}^{j,nuc}, \text{ for all } j, t \\ 0 < q_t^{th} \leq Q_{max}^{th}, \text{ for all } t \end{array} \right\}$$

**Remark 4.1**  $F^{th}$  is containing  $F$  and is contained in  $C$  and  $C$  is contained in  $M$  ( $F \subset F^{th} \subset C \subset M$ ).

Since the marginal technology is the non-nuclear thermal on  $F^{th}$ , the price is determined by the non-nuclear thermal production. We now proceed with Proposition 4.1.

**Proposition 4.1** *If  $F^{th}$  is a non-empty set, then  $\overline{F^{th}} = C$ .*

### Proof

First, we show that  $\overline{F^{th}} \subset C$ . Since  $F^{th}$  is contained to  $C$  (see Remark 3.4.1) and  $C$  is a compact set, we have that

$$F^{th} \subset C \Rightarrow \overline{F^{th}} \subset \overline{C} = C.$$

Secondly, we prove that  $\overline{F^{th}} \supset C$ . Let  $q_1 \in C$  and  $q_0 \in F^{th}$ . For all  $m \in \mathbb{N}$ , there exists a sequence  $z_m = (1 - \frac{1}{m+1})q_1 + \frac{1}{m+1}q_0$  belonging to  $F^{th}$  such that  $\lim_{m \rightarrow \infty} z_m = \lim_{m \rightarrow \infty} (1 - \frac{1}{m+1})q_1 + \frac{1}{m+1}q_0 = q_1$ . Hence,  $q_1 \in \overline{F^{th}}$  and the inclusion is proved.

From the first and the second part of the proof, we conclude that  $\overline{F^{th}} = C$ . □

Note that in section 6.2, we show that  $F$  is a non-empty set for our numerical data, thus the assumption of Proposition 4.1 is satisfied.

From a geometrical point of view one deduces from Proposition 4.1 that all the points of the set  $C$  and thus those which belong to  $C \setminus F^{th}$  can be approached by points that belong to  $F^{th}$ . This result is fundamental in order to show in the next proposition the discontinuity and more specifically the decrease of the inter-temporal profit on these particular points which results from a decrease of the price (see Ref. [31]).

**Proposition 4.2** *If  $b_{nuc} < b_{th}$  and  $\bar{q} \in C \setminus F^{th}$ , there exists a sequence  $(q_r)_{r \in \mathbb{N}} \in F^{th}$  with  $\lim_{r \rightarrow \infty} q_r = \bar{q}$  such that  $\lim_{r \rightarrow \infty} \pi(q_r) > \pi(\bar{q})$ .*

<sup>7</sup>It is important to emphasize that the usual interior of  $C$  is empty since  $M$  is an affine set that is not equal to  $\mathbb{R}^n$ . Consequently, we focus on a generalization called relative interior (see Ref. [21], Ref. [30]).

**Proof**

According to the assumptions,  $\bar{q}$  is a production vector which belongs to  $C \setminus F^{th} \subset C$ . It follows that the set  $S = \{t \in \{1, \dots, T\} \text{ s.t. } \bar{q}_t^{th} = 0\}$  is a non-empty set. Let us denote  $\bar{q} = ((\bar{q}_{j1}^{nuc}, \dots, \bar{q}_{jT}^{nuc})_{j=1}^J, \bar{q}_1^{th}, \dots, \bar{q}_T^{th})$ .

Profit's function  $\pi : C \rightarrow \mathbb{R}$  is defined as

$$\pi(q) = \sum_{t=1}^T (p_t (\sum_{j=1}^J q_{jt}^{nuc} + q_t^{th}) - \sum_{j=1}^J C_j^{nuc}(q_{jt}^{nuc}) - C^{th}(q_t^{th}))$$

In view of Proposition 4.1, the production vector  $\bar{q} \in \overline{F^{th}}$ . It follows that there exists a sequence  $(q_r)_{r \in \mathbb{N}}$  such that  $q_r \in F^{th}$  and  $\lim_{r \rightarrow \infty} q_r = \bar{q}$ . Let us denote  $q_r = ((q_{j1r}^{nuc}, \dots, q_{jTr}^{nuc})_{j=1}^J, q_{1r}^{th}, \dots, q_{Tr}^{th})$ . For all  $r$ , we can compute the associated merit order price  $p_r = (p_{1r}, \dots, p_{Tr}) = (mc^{th}(q_{1r}^{th}), \dots, mc^{th}(q_{Tr}^{th}))$ . The price vector  $\bar{p} = (\bar{p}_1, \dots, \dots, \bar{p}_T)$  represents the merit order price associated with the production vector  $\bar{q}$ . Since at the limit, the value of  $q_r$  is equal to  $\bar{q}$ , we deduce that the nuclear is the marginal technology during the period  $t$  of the game, for all  $t \in S$ . Thus, the price vector  $\bar{p}$  is such that the price  $\bar{p}_t$  in period  $t \in S$  is determined by the nuclear marginal cost  $b_{nuc}$ . For all  $t \in S$ , it follows that

$$\lim_{r \rightarrow \infty} p_{tr} = \lim_{r \rightarrow \infty} mc^{th}(q_{tr}^{th}) = mc^{th}(\bar{q}_t^{th}) = mc^{th}(0) = b_{th} > b_{nuc} = \bar{p}_t. \quad (13)$$

For all  $t \notin S$ , the non-nuclear thermal production is strictly positive, thus the price  $\bar{p}_t$  is determined by the non-nuclear thermal marginal cost ( $\bar{p}_t = mc^{th}(\bar{q}_t^{th})$ ). So, we have

$$\lim_{r \rightarrow \infty} p_{tr} = \lim_{r \rightarrow \infty} mc^{th}(q_{tr}^{th}) = mc^{th}(\bar{q}_t^{th}) = \bar{p}_t. \quad (14)$$

At the limit, we obtain

$$\begin{aligned} & \lim_{r \rightarrow \infty} \pi(q_r) - \pi(\bar{q}) = \\ & \sum_{t \notin S} \lim_{r \rightarrow \infty} (p_{tr} (\sum_{j=1}^J q_{jtr}^{nuc} + q_{tr}^{th}) - \bar{p}_t (\sum_{j=1}^J \bar{q}_{jt}^{nuc} + \bar{q}_t^{th})) + \\ & \sum_{t \in S} \lim_{r \rightarrow \infty} (p_{tr} (\sum_{j=1}^J q_{jtr}^{nuc} + q_{tr}^{th}) - \bar{p}_t (\sum_{j=1}^J \bar{q}_{jt}^{nuc} + \bar{q}_t^{th})) + \\ & \sum_{t=1}^T \lim_{r \rightarrow \infty} ((\sum_{j=1}^J C_j^{nuc}(q_{jtr}^{nuc}) - C^{th}(q_{tr}^{th})) - (\sum_{j=1}^J C_j^{nuc}(\bar{q}_{jt}^{nuc}) - C^{th}(\bar{q}_t^{th}))) \end{aligned}$$

Since  $\lim_{r \rightarrow \infty} q_r = \bar{q}$  and the price is continuous on the set  $T \setminus S$  (see relationship (14)) the first term converges to zero. However, in view of the non-emptiness of  $S$ , of the relationship (13) and of the strictly positive nuclear production ( $\sum_{j=1}^J \bar{q}_{jt}^{nuc} \geq \sum_{j=1}^J Q_{min}^{j,nuc} > 0$ ) the second term is strictly positive. Finally, the third term converges to zero because of the continuity of the production cost functions.

Consequently, we deduce that  $\lim_{r \rightarrow \infty} \pi(q_r) - \pi(\bar{q}) > 0 \Leftrightarrow \lim_{r \rightarrow \infty} \pi(q_r) > \pi(\bar{q})$  and the proposition is proved.  $\square$

**Corollary 4.1** *The inter-temporal profit maximization problem determined on  $C$  is equivalent to the inter-temporal profit maximization problem determined on  $F^{th}$  (same set of solutions and same value<sup>8</sup>).*

**Proof**

This corollary is an obvious consequence of Proposition 4.2. □

It should be noticed that if the inter-temporal profit maximization problem is determined on  $C$  which is a compact set, the objective function is not continuous in view of Proposition 4.2. If the inter-temporal profit maximization problem is determined on  $F^{th}$ , the objective function is continuous according to Proposition 4.2 while  $F^{th}$  is not a compact set. Thus, it is not possible to conclude on the existence of solutions of this problem.

## 4.2 A property when the optimal solution is “interior”.

In view of our data,  $b_{nuc} < b_{th}$  holds, hence according to Proposition 4.2, the inter-temporal profit of a producer decreases for all productions vectors that belong to the subset  $C \setminus F^{th}$  of  $C$ . This leads a producer to search for a solution that maximizes the inter-temporal profit on  $F^{th}$ . The next proposition gives a property when in addition the solution belongs to the set  $F$ .

**Proposition 4.3** *If there exists an equilibrium  $((\bar{q}_{jt}^{nuc})_{j=1}^J, \bar{q}_t^{th})_{t=1}^T \in F$  such that the inter-temporal profit of a producer is maximum on  $C$  and  $(\bar{q}_t^{nuc})_{t=1}^T$  is the corresponding monthly nuclear production vector then  $\bar{q}_1^{nuc} = \bar{q}_2^{nuc} = \dots = \bar{q}_T^{nuc}$ .*

**Proof**

In order to determine a global solution of the inter-temporal profit maximization problem on  $C$ , we choose to apply the Karush - Kuhn - Tucker (KKT) conditions (see Ref. [20]). However, KKT conditions can not be applied because the objective function (profit function) is not continuous within this set (see Proposition 4.2). In view of Corollary 4.1,  $\bar{q} = ((\bar{q}_{jt}^{nuc})_{j=1}^J, \bar{q}_t^{th})_{t=1}^T$  solves the following optimal inter-temporal production problem

$$\max_{((q_{jt}^{nuc})_{j=1}^J, q_t^{th})_{t=1}^T \in F^{th}} \sum_{t=1}^T (p_t (\sum_{j=1}^J q_{jt}^{nuc} + q_t^{th}) - \sum_{j=1}^J C_j^{nuc}(q_{jt}^{nuc}) - C^{th}(q_t^{th})) \tag{15}$$

The objective function being continuous within  $F^{th}$  permits the application of KKT conditions in order to determine  $\bar{q} = ((\bar{q}_{jt}^{nuc})_{j=1}^J, \bar{q}_t^{th})_{t=1}^T$ . Since  $M$  is an affine set, Slater’s condition is satisfied (see Ref. [28], Ref. [29]). Therefore, there exists  $(\mu_t)_{t=1}^T \in \mathbb{R}^T$  and  $(\lambda_j^k)_{j=1}^J \in \mathbb{R}^{(J-2)4+6}$  such that the KKT conditions are met, where  $\mu_t$  is the Lagrange multiplier associated with the supply-demand equilibrium constraint at each month  $t$  and  $\lambda_j^k$  is the Lagrange multiplier for the nuclear fuel constraint of the unit  $j$  during the campaign  $k$  which is defined as

$$\begin{cases} k \in \{1, 2, 3\}, & \text{for } j \in \{1, 12\} \\ k \in \{1, 2, 3, 4\}, & \text{for } j \in \{2, \dots, 11\} \end{cases}$$

In view of KKT conditions of complementarity and since  $\bar{q} \in F$ , all the Lagrange multipliers associated with the min/max production constraints are equal to zero and they will be omitted in the Lagrangien function of this problem.

Let us call  $L$  the reduced Lagrangien of the optimal production problem (15)

<sup>8</sup>The value of an optimization problem is defined as the upper bound of the set  $\{f(x)|x \in C\}$ , where  $f$  is the objective function and  $C$  is the set of feasible solutions. The value always exists (in the extended real line) even if the set of solutions is empty.

$$L(q) = \pi - \sigma \cdot (Dq - E)$$

where  $\pi$  is the inter-temporal profit of a producer,  $\sigma = (\lambda_j^k, \mu_t)$  is the vector of the Lagrange multipliers,  $q = ((q_{jt}^{nuc})_{j=1}^J, q_t^{th})_{t=1}^T$  is a production vector of  $F^{th}$ ,  $E = (E_j)_{j=1}^J$  is the vector that contains the nuclear fuel stock of the unit  $j$  during a campaign and  $D$  is the matrix so that the set  $M$  defined in subsection 4.1 is equal to  $\{q' \text{ s.t. } Dq' - E = 0\}$ .

The inter-temporal profit  $\pi$  of a producer is

$$\sum_{t=1}^T (p_t (\sum_{j=1}^J q_{jt}^{nuc} + q_t^{th}) - \sum_{j=1}^J C_j^{nuc}(q_{jt}^{nuc}) - C^{th}(q_t^{th}))$$

We remind that the price  $p_t$  is given by the marginal cost of the non-nuclear thermal production on the set  $F^{th}$ . Following some calculations and by using the supply-demand equilibrium constraints, we deduce that producer's inter-temporal profit is a quadratic function of the non-nuclear thermal production  $q_t^{th}$

$$-c_{th} \sum_{t=1}^T (q_t^{th})^2 + 2c_{th} \sum_{t=1}^T (D_t - Q_{Tot,t}^{hyd}) q_t^{th} + c \quad (16)$$

where  $c = (-\sum_{t=1}^T a_{th} - \sum_{t=1}^T \sum_{j=1}^J a_{nuc}^j + (b_{th} - b_{nuc})S_{reload})$  is the constant part of the profit function. The quantity  $S_{reload}$  denotes the total nuclear fuel stock of reloading that is available over the entire time horizon  $T$ , hence  $\sum_{t=1}^T \sum_{j=1}^J q_{jt}^{nuc} = S_{reload}$ .

According to the KKT conditions, one has

$$\frac{\partial L}{\partial q_{jt}^{nuc}}(\bar{q}) = 0, \text{ for all } j, t$$

and

$$\frac{\partial L}{\partial q_t^{th}}(\bar{q}) = 0, \text{ for all } t.$$

The derivative of Lagrangien with respect to the thermal production  $q_1^{th}$  at month 1 is

$$\frac{\partial L}{\partial q_1^{th}}(\bar{q}) = 2c_{th}(D_1 - Q_{Tot,1}^{hyd}) - 2c_{th}\bar{q}_1^{th} - \mu_1 = 0 \Leftrightarrow (D_1 - Q_{Tot,1}^{hyd}) - \bar{q}_1^{th} = \frac{\mu_1}{2c_{th}} \quad (17)$$

By a symmetric argument, the derivative of Lagrangien with respect to the thermal production  $q_2^{th}$  at month 2 is

$$\frac{\partial L}{\partial q_2^{th}}(\bar{q}) = 2c_{th}(D_2 - Q_{Tot,2}^{hyd}) - 2c_{th}\bar{q}_2^{th} - \mu_2 = 0 \Leftrightarrow (D_2 - Q_{Tot,2}^{hyd}) - \bar{q}_2^{th} = \frac{\mu_2}{2c_{th}} \quad (18)$$

In order to compare  $\mu_1$  and  $\mu_2$ , let us focus on a unit which is active in both month 1 and 2. The derivative of Lagrangien with respect to the nuclear production  $q_{31}^{nuc}$  of unit 3 at month 1 is

$$\frac{\partial L}{\partial q_{31}^{nuc}}(\bar{q}) = -\mu_1 - \lambda_3^1 = 0 \Leftrightarrow \mu_1 = -\lambda_3^1 \quad (19)$$

The derivative of Lagrangien with respect to the nuclear production  $q_{32}^{nuc}$  of unit 3 at month 2 is

$$\frac{\partial L}{\partial q_{32}^{nuc}}(\bar{q}) = -\mu_2 - \lambda_3^1 = 0 \Leftrightarrow \mu_2 = -\lambda_3^1 \quad (20)$$

From the last two equations, we deduce that  $\mu_1 = \mu_2$ . In view of the equality between supply and demand, this means that

$$(D_1 - Q_{Tot,1}^{hyd}) - \bar{q}_1^{th} = (D_2 - Q_{Tot,2}^{hyd}) - \bar{q}_2^{th} \Leftrightarrow \bar{q}_1^{nuc} = \bar{q}_2^{nuc}$$

By using a unit available at month  $t$  and  $t + 1$ , we obtain  $\mu_t = \mu_{t+1}$ , which implies that  $\bar{q}_t^{nuc} = \bar{q}_{t+1}^{nuc}$ . Consequently,

$$\bar{q}_1^{nuc} = \bar{q}_2^{nuc} = \dots = \bar{q}_T^{nuc}.$$

We conclude that an equilibrium of the optimal production problem (15) is characterized by a constant monthly nuclear production and a variable non-nuclear thermal production which follows demand's seasonal variations. □

In view of the equality between supply and demand and since  $\bar{q} \in F$ , we deduce from equation (17) (respectively (18)) that the sign of the multiplier  $\mu_1$  (respectively  $\mu_2$ ) is strictly positive. By symmetry, the Lagrange multiplier  $\mu_t$  is strictly positive ( $\mu_t > 0$ ) for all  $t$ . Consequently, in view of equations (19) and (20), the multiplier  $\lambda_3^1$  (respectively  $\lambda_j^k$ ) has a strictly negative sign. Indeed, if an additional unit of nuclear fuel became available for the unit  $j$  during the campaign  $k$ , the non-nuclear thermal production would decrease which would lead to a lower market price and thus to a lower production value and the nuclear production cost would increase while the non-nuclear thermal production cost would decrease. However, the first effect that concerns the decrease of the production value is the most important. Consequently, the “additional” profit resulting from an additional nuclear fuel unit and thus the value of the multiplier  $\lambda_j^k$  should be negative. The multiplier  $\lambda_j^k$  indicates the “marginal value of nuclear fuel stock”, i.e. the additional profit  $|\lambda_j^k|$  unit  $j$  would get if the nuclear fuel stock decreased by one unit during the campaign  $k$ .

Let us notice that  $F$  is not a compact set, thus the inter-temporal profit maximization problem may not have a solution on  $F$ . Consequently, the existence of a solution of the problem (15) on  $F$  takes the form of an assumption in Proposition 4.3.

We now proceed by showing that a constant monthly nuclear production constitutes a sufficient condition for optimality on  $C$ . Let us state the following proposition.

**Proposition 4.4** *If  $((\bar{q}_{jt}^{nuc})_{j=1}^J, \bar{q}_t^{th})_{t=1}^T$  is a production vector belonging to  $F^{th}$  such that  $\bar{q}_1^{nuc} = \bar{q}_2^{nuc} = \dots = \bar{q}_T^{nuc}$ , where  $(\bar{q}_t^{nuc})_{t=1}^T$  is the corresponding monthly nuclear production vector then  $((\bar{q}_{jt}^{nuc})_{j=1}^J, \bar{q}_t^{th})_{t=1}^T$  is a solution of the inter-temporal profit maximization problem on  $C$ .*

### Proof

In view of Corollary 4.1, it suffices to show that  $\bar{q} = ((\bar{q}_{jt}^{nuc})_{j=1}^J, \bar{q}_t^{th})_{t=1}^T$  is a solution of the inter-temporal profit maximization problem on  $F^{th}$ . More precisely, it is sufficient to prove that there exist Lagrange multipliers such that the KKT conditions associated with this optimization problem are satisfied at  $\bar{q}$  given that the inter-temporal profit function is concave. In order to show the concavity of this function, we proceed with the following Lemma.

**Lemma 4.1** *The profit function of the inter-temporal profit maximization problem on  $F^{th}$  is concave.*

### Proof of Lemma 4.1

The inter-temporal profit  $\pi$  is a quadratic function of the non-nuclear thermal production  $q_t^{th}$  on  $F^{th}$  given by the function (16)

$$-c_{th} \sum_{t=1}^T (q_t^{th})^2 + 2c_{th} \sum_{t=1}^T (D_t - Q_{Tot,t}^{hyd}) q_t^{th} + c$$

(see proof of Proposition 4.3). We notice that this is a quadratic function of the form  $f(u) = a(u^T \cdot u) + bu + c$ , where  $a = -c_{th}$  and  $b = 2c_{th} \sum_{t=1}^T (D_t - Q_{Tot,t}^{hyd})$ . Since  $a < 0$ , the function  $f(u)$  is concave. It is also a strictly concave function. Thus, taking into account the other variables, we conclude that the inter-temporal profit function (16) is concave.  $\square$

Let us now continue our proof.

First, we set the Lagrange multipliers associated with the min/max nuclear production constraints and with the min/max non-nuclear thermal production constraints equal to zero. Hence, given that the nuclear fuel constraints as well as the supply-demand equilibrium constraints are pure equalities, KKT complementary conditions are satisfied at  $((\bar{q}_{jt}^{nuc})_{j=1}^J, \bar{q}_t^{th})_{t=1}^T$ . Then, we look at the Lagrange multipliers associated with the supply-demand equilibrium constraint in month  $t$  ( $\tilde{\mu}_t$ ) and the nuclear fuel constraints of the unit  $j$  during the campaign  $k$  ( $\tilde{\lambda}_j^k$ ). We set

$$\left\{ \begin{array}{l} \tilde{\mu}_t = 2c_{th}((D_t - Q_{Tot,t}^{hyd}) - \bar{q}_t^{th}) = 2c_{th}\bar{q}_t^{nuc}, \quad \text{for all } t \end{array} \right.$$

Since  $\bar{q}_1^{nuc} = \bar{q}_2^{nuc} = \dots = \bar{q}_T^{nuc}$ , we deduce that  $\tilde{\mu}_1 = \tilde{\mu}_2 = \dots = \tilde{\mu}_T$ . We call  $\tilde{\mu}$  their common value and we set

$$\left\{ \begin{array}{l} \tilde{\lambda}_j^k = -\tilde{\mu}, \quad \text{for all } j, k \end{array} \right.$$

For those multipliers the Lagrangien function of the optimal production problem is determined on  $F^{th}$  as following

$$H(q) = \pi - \tilde{\sigma} \cdot (Dq - E)$$

where  $\tilde{\sigma} = (\tilde{\mu}_t, \tilde{\lambda}_j^k)$ . We recognize then the reduced Lagrangien function  $L$  of the optimal production problem (15). By taking the analysis of (17) respectively (19) and by symmetry, we realize that

$$\frac{\partial L}{\partial q_t^{th}}(\bar{q}) = 0, \quad \text{for all } t$$

respectively

$$\frac{\partial L}{\partial q_{jt}^{nuc}}(\bar{q}) = 0, \quad \text{for all } j, t.$$

Consequently, the production vector  $((\bar{q}_{jt}^{nuc})_{j=1}^J, \bar{q}_t^{th})_{t=1}^T$  is also a solution of the KKT conditions associated with the inter-temporal profit maximization problem determined on the set  $F^{th}$ . However, KKT conditions are sufficient for optimality since the objective function (profit function) is concave. Thus,  $((\bar{q}_{jt}^{nuc})_{j=1}^J, \bar{q}_t^{th})_{t=1}^T$  is a solution of the inter-temporal profit maximization problem on  $F^{th}$  and the proposition is proved.  $\square$

In view of Propositions 4.3 and 4.4, we conclude that in the absence of binding productions constraints, the solutions of the optimal inter-temporal production problem are fully characterized by a constant nuclear production. Consequently, from a theoretical point of view, a

producer maximizes its inter-temporal profit by running its nuclear units in a constant way and by using its non-nuclear thermal units to follow-up load in order to cover the residual demand each month. However, since nuclear does not follow-up load and the nuclear fuel stock that is reloaded in the reservoir corresponds to 258 days equivalent to full capacity, nuclear plants never operate at maximum capacity during the year.

## 5 Numerical Illustration

### 5.1 Data

The data used in our numerical dynamic model has been already used in our previous paper. It is French and of different years due to the difficulty of collection: \* level of French demand during the year 2006 – 2007, \* generation capacity of hydro (run-of-river), nuclear and non-nuclear thermal and \* nuclear fuel stock of reloading, \* fixed and variable costs of nuclear, coal and gas generation. The French Transmission & System Operator (named RTE) gives the daily consumption in MWh for the entire year 2007 with which we determine the monthly consumption. RTE also provides the annual capacity of nuclear as well as the annual capacity of gas and coal for the year 2009. In addition, the nuclear fuel stock of reloading as well as the annual capacity and production of hydro have been provided by (Electricité de France). The costs of production come from the official report “Reference Costs of Electricity Production” issued by the ministry of industry (General Direction of Energy and Raw Materials -DIDEME-) (See Ref. [16]) in 2003. It gives the total cost for each technology (nuclear, coal, gas, fuel) as follows: cost of investment, variable and fixed cost of exploitation, fuel cost, taxes, *R&D* costs for the nuclear and cost of CO<sub>2</sub> per ton in the case of coal and gas for a base load (8760h) and semi-base (3000h) operation. These costs are estimated for the year 2007 and 2015 with different discount rate (3%, 5%, 8%, 11%) taking into account the influence of exchange rate on the production cost.

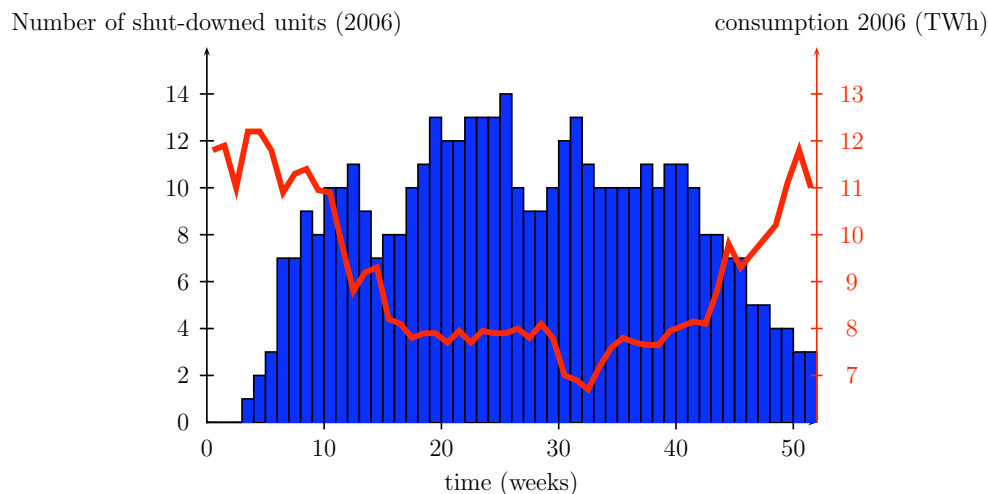


Figure 1: Availability of nuclear units.

Our modelling is based on the scenario which is also considered by the optimization per month production behavior modelling. According to this scenario, one dollar is equal to one euro, the discount rate is 8%, the cost of CO<sub>2</sub> per ton reaches 20 euros, the price of coal is 30 dollars per ton and the price of gas is 3.3 dollars per MBtu (1 MBtu=293.1 KWh). The

choice of this particular scenario is mainly based on the scenario considered by DGEMP & DIDEME for the estimation of costs of the different types of generation technologies. The value of the coefficient  $a_{th}$  involved in the non-nuclear thermal cost function corresponds to the fixed cost provided by the data (investment cost, fixed exploitation cost, taxes), while the other coefficients have been determined by interpolation in order to meet the variable cost of coal and gas provided by our data base (fuel cost, variable exploitation cost, CO<sub>2</sub> cost, taxes on the gas fuel). The consideration of the fixed costs in the production cost of both technologies (nuclear, non-nuclear thermal) permits to obtain a more realistic vision of the value of profit within our medium-term horizon. The capacity of each nuclear unit has been simulated<sup>9</sup> in order to approximate the graph of figure<sup>10</sup> 1, which shows the availability of nuclear units per week. For example, the capacity of the nuclear unit  $j = 1$  (respectively  $j = 2, \dots, j = 12$ ) corresponds to the sum of capacities of shut-downed units in December (respectively January,  $\dots$ , November). Moreover, the initial value of the nuclear fuel stock has been set by simulating the nuclear fuel stock of each unit available at the beginning of the time horizon of the model. We also take into account the electricity losses of the network, as estimated by RTE. Finally, as we mentioned in the first chapter, an EPR reactor can maneuver between 25% of nominal capacity and 100% of nominal capacity in order to follow-up load. We take into account these levels of maneuverability within our numerical model to determine the minimum/maximum nuclear production constraints (see Table 2, section 9).

A more complete analysis of this data can be found in our previous chapter.

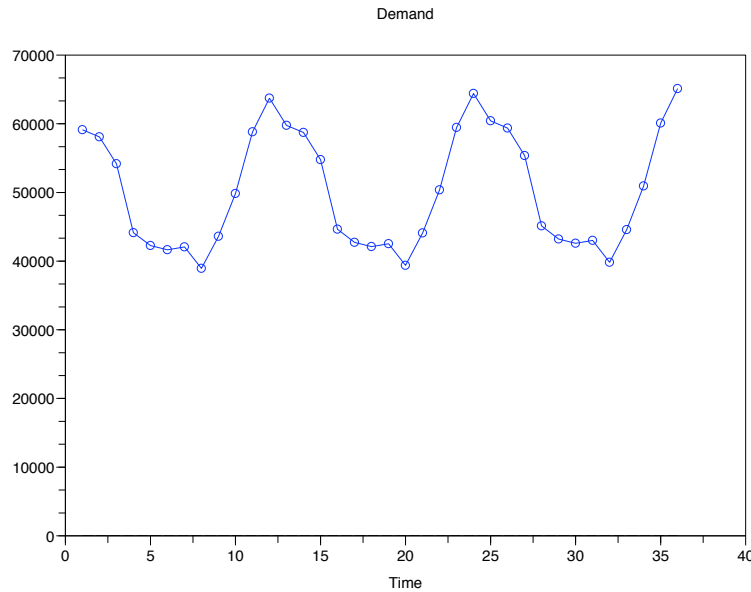


Figure 2: Simulated demand (in MW)

<sup>9</sup>Access to detailed nuclear capacity data for each short period of time is not possible due to the confidentiality of such data.

<sup>10</sup>Each blue bar shows the number of shut-downed units during a week and the red line shows the evolution of the consumption over time. The different levels of consumption are measured in the right axis while the number of shut-downed units is reflected in the left axis.



The levels of monthly demand<sup>11</sup> obtained with the time scale of our model (January 2007 – December 2009) are presented in the figure 2 (we suppose an augmentation of the demand by a rate of 1% per year). Once again, we can see the seasonal variation of the demand level between winter (high demand) and summer (low demand). In particular, we observe high levels of demand during winter (November – February) with demand peaks in December. The demand decreases considerably during spring as well as during summer (May – August). Let us recall that no demand peaks are observed during summer period which implies the absence of significant extremes of temperature.

## 5.2 Simulation results

In this section, we proceed with an analysis of the numerical results for the nuclear and non-nuclear thermal production levels as well as the storage levels obtained via a simple numerical model.

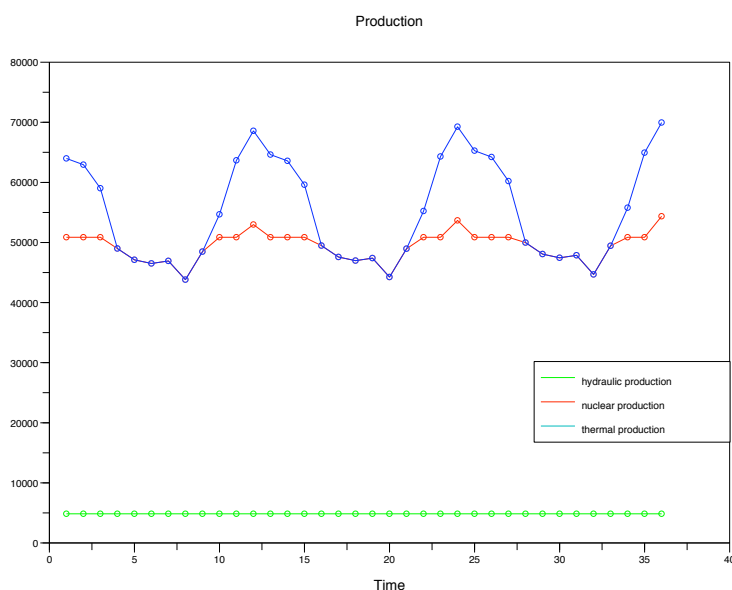


Figure 3: Simulated hydro/nuclear/non-nuclear thermal production (in MW)

<sup>11</sup>Note that there is a rescaling on these data in order to take into account the diversity on the length of the months.

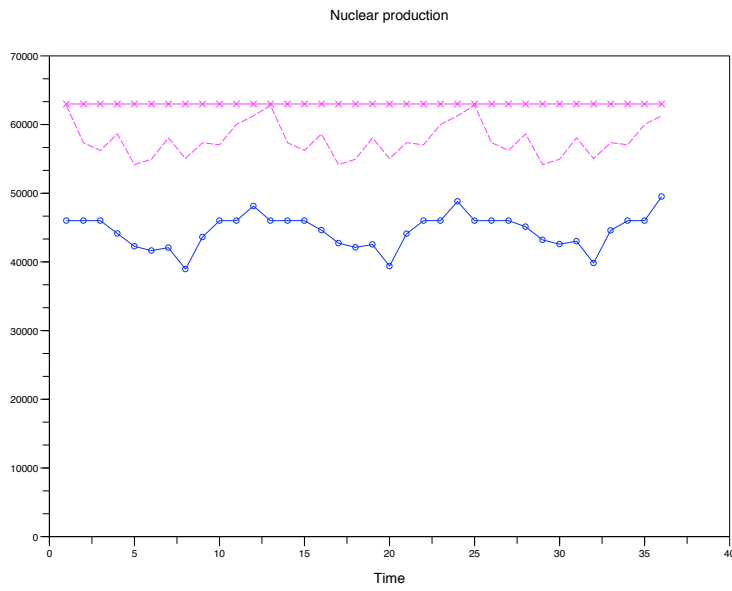


Figure 4: Simulated nuclear production (in MW)

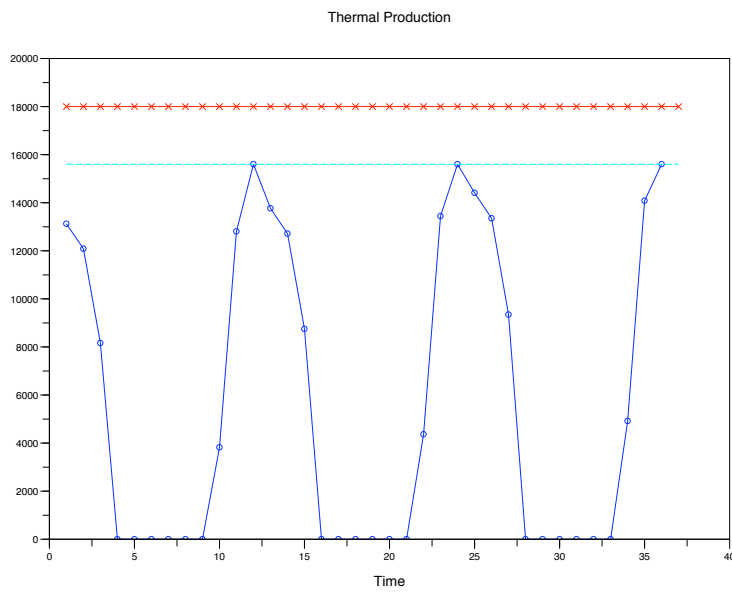


Figure 5: Simulated non-nuclear thermal production (in MW)

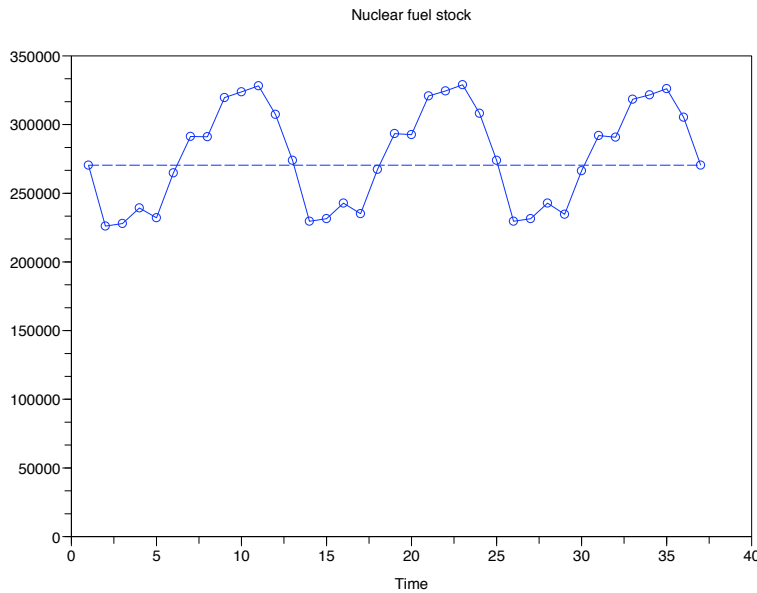


Figure 6: Simulated nuclear fuel stock (in MW)

Since the hypothesis of Proposition 4.2 holds within our data, the discontinuity (more precisely the decrease) of the price at production vectors characterized by zero non-nuclear thermal production induces a decrease of the value of profit. Let us emphasize that it does not exist an algorithm that maximizes a discontinuous function. In an alternative model, we give to the price the value of the non-nuclear thermal marginal cost ( $mc^{th}(0) = b_{th}$ ) instead of ( $b_{nuc}$ ) during periods when nuclear is the marginal technology. Thus, a nuclear producer pays at least  $b_{th}$  in marginal nuclear. In view of this “regularization” of the merit order price, the inter-temporal profit is maximized on the entire set of feasible solutions  $C$  within our numerical model. However, the “regularized” problem differs from the economical problem described in subsection 2.6 with respect to the objective function. More precisely, the inter-temporal profit of this problem is greater than the inter-temporal profit of the economical problem since the value of  $b_{th}$  (26.24 Euro/MWh) is greater than the value of  $b_{nuc}$  (5.01 Euro/MWh). Nevertheless, the value of the “regularized” problem and the value of the economical problem are identical (see Annex, Proposition 8.1), hence the “regularized” problem is a “good” approximation of our economical problem (see Ref. [30]). We also prove that the set of solutions of the economical problem is given by the intersection of the set of solutions of the “regularized” problem and the set  $F^{th}$  (see Annex, Proposition 8.6). The solution of the “regularized” problem whose graphs appear in this section does not belong to the set  $F^{th}$  which means that the set of solutions of the economical problem is empty (see Annex, Proposition 8.7). This numerical solution is only an “approximate” solution of our economical problem.

We recall that under some assumptions, Proposition 4.3 shows that the non-nuclear thermal production is adjusted on the seasonal variations of the demand, while the nuclear production remains constant during the entire time horizon of the model. However, in view of our data, a constant nuclear production is not a feasible solution in the case of a flexible nuclear set (like the French nuclear set) because it leads to the violation of both minimum et maximum non-nuclear thermal production constraints (see Figure 8). Consequently, Proposition 4.3 says that the inter-temporal profit maximization problem has no solutions on  $F$  within our numerical model.

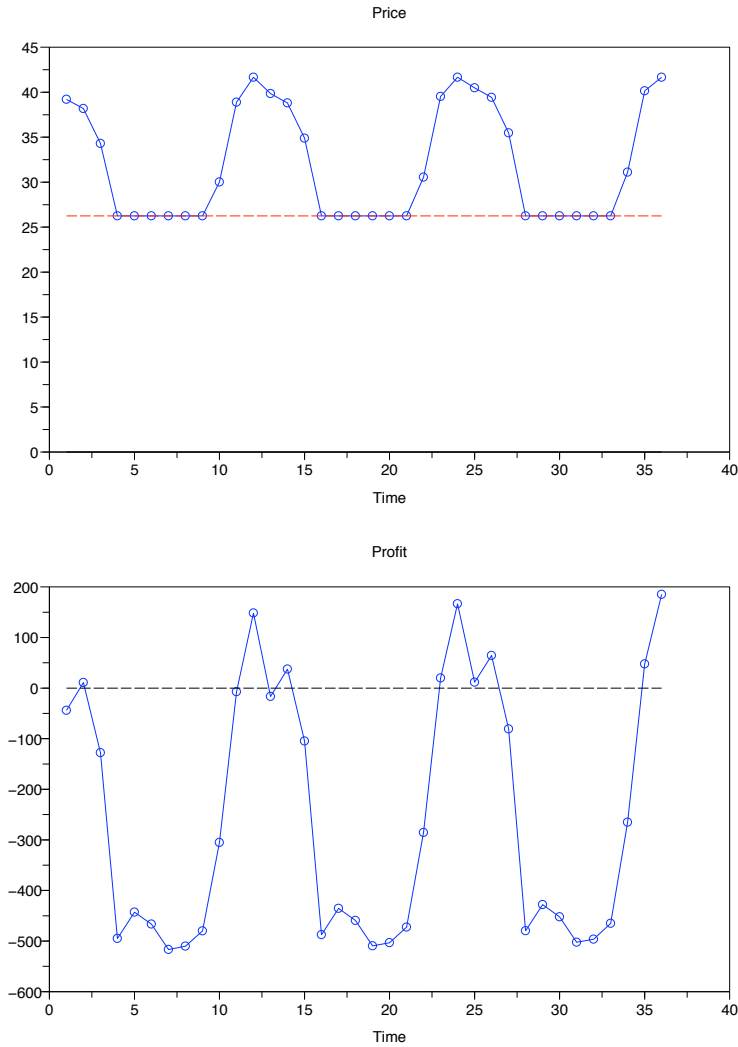


Figure 7: Simulated “regularized” price (in Euro/MWh)/Aggregated total “regularized” profit (in Euro (million))

Simulation results show that the non-nuclear thermal generation is marginal during the months of high demand in order to equilibrate supply and demand while the nuclear technology is marginal during the months of low demand. In particular, nuclear stays marginal during almost the entire period of spring and summer (April - September), while non-nuclear is marginal during autumn and winter (October - March) (see Figure 3, Figure 4, Figure 5).

In addition, one observes (both graphically and with a numerical test) that the non-nuclear thermal and the nuclear production increase (and respectively decrease) simultaneously during almost the entire time horizon of our model, which corresponds to the notion of “comonotonicity”<sup>12</sup> introduced by Yaari (1987) (see Figure 3). However, the monthly nuclear production never reaches its maximum value<sup>13</sup> whereas non-nuclear thermal production reaches its maximum value<sup>14</sup> during demand peaks in December (see Figure 4, Figure 5).

<sup>12</sup>The vector  $(X_t)_{t=1}^T$  is comonotonic to the vector  $(Y_t)_{t=1}^T$  if  $(X_{t'} - X_t)(Y_{t'} - Y_t) \geq 0$  holds for all  $t, t'$ .

<sup>13</sup>The maximum nuclear production during the month  $t$  given that some unit is inactive during this month (month of reloading) is represented by the purple dotted line. This quantity is obviously below the nominal capacity of the French nuclear set represented by the crossed purple line.

<sup>14</sup>The maximum non-nuclear thermal production during a month is represented by the white blue dotted line

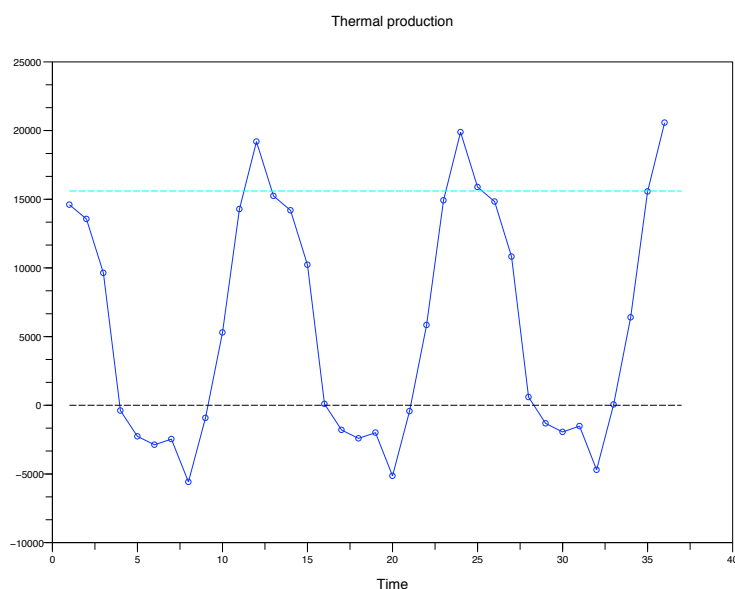


Figure 8: Simulated non-nuclear thermal production resulting from a constant nuclear production (in MW)

The nuclear production follows the seasonal variations of demand (high production during winter – low production during summer). This means “high” levels of nuclear fuel stock during summer and “low” levels of nuclear fuel stock during winter. Therefore, the periodical evolution of the nuclear production leads to a periodical evolution for the nuclear fuel stock too. Note that the trend of the stock oscillates around the “stock of reference”<sup>15</sup>. More precisely, the value of the stock exceeds the “stock of reference” during high demand seasons while it is lower than the “stock of reference” during low demand seasons (see Figure 4, Figure 6).

Since the non-nuclear thermal production is comonotonic to the nuclear production and obviously to itself and by taking into account the equilibrium between supply and demand, we deduce that the non-nuclear thermal production is comonotonic to the demand, hence it is high during winter (respectively low during summer) because of the high (respectively low) level of demand. In particular, non-nuclear thermal production is increasing during winter (beginning from October) until it reaches its peak value during the month of December. Afterwards, it decreases progressively until it takes its lowest value during summer which is a low demand season. However, it does not stay marginal during summer; nuclear is the marginal technology during the low demand seasons (April - September) through the entire time horizon of the model (see Figure 5). The main reason of the high duration of nuclear’s marginality is its profitability for the producers. Since the value of the price, when nuclear is the marginal technology, is given by the value of the non-nuclear thermal marginal cost ( $mc^{th}(0) = b_{th}$ ), the nuclear production is evaluated in a higher price. Consequently, producers are no longer penalized by producing in marginal nuclear.

Furthermore, we observe that the “regularized” price<sup>16</sup> is high during winter by taking its highest value in the month of December and relatively low during summer. Once more this

and corresponds to the nominal non-nuclear thermal capacity (including coal, gas, fuel, etc.) of the French set.

<sup>15</sup>The “stock of reference” is represented by the blue dotted line which shows the value of stock at the beginning, being also the value of stock at the end.

<sup>16</sup>The red dotted line indicates the level of the “regularized” price when nuclear is the marginal technology.

is explained by the fact that the “regularized” price (determined by the non-nuclear thermal marginal cost) is comonotonic to the non-nuclear thermal production which is comonotonic to the demand. Hence, it follows demand’s seasonal variations. Similarly, the aggregate total profit being comonotonic to the price is comonotonic to the demand which leads to high profits during winter and at the beginning of spring while lower profits are realized during summer and at the end of spring. In addition, we can see that its value can be decomposed in a cyclical component and a linear trend which is increasing (see Figure 7). However the reader should not focus on the precise amount of profit since its level depends on the too many approximations we did do (euro/dollar, oil prices, CO<sub>2</sub> cost, discounting rate, no mark-up rate, absence of profits coming from the hydro technology (run-of-river), etc) and because our modelling does not take into account the electricity importations/exportations or the production coming from renewable.

Let us also remark that varying the length of the model’s time horizon does not lead to different behaviour patterns since the periodical evolution of the nuclear and non-nuclear thermal production during the entire time horizon is the same (e.g. for  $T = 84$ , see Figure 9).

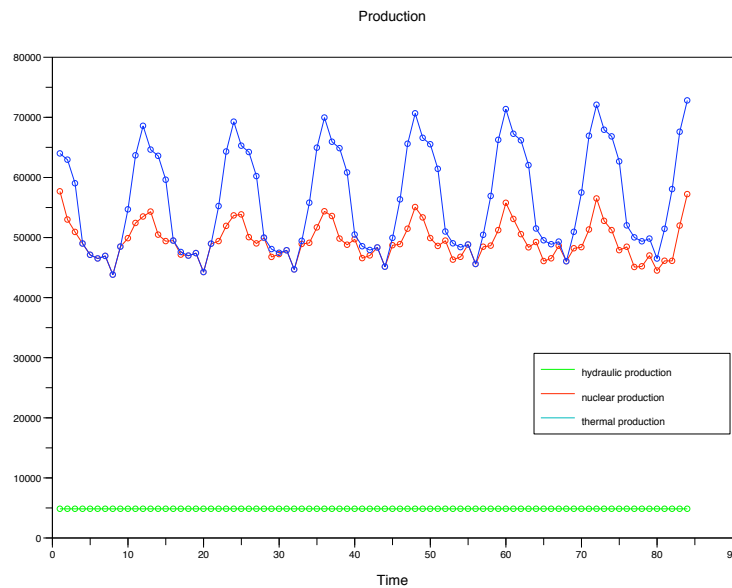


Figure 9: Simulated hydro/nuclear/non-nuclear thermal production (in MW) ( $T=84$ )

It should be mentioned that, within this time scale, the size of the French nuclear set does not seem to be considerably below the “optimal size”. Hence, we do not meet Spector’s conclusion according to which the French nuclear set is “sub-optimal”, which makes the owner of that set (the French state) recipient of a scarcity rent (see Ref. [9], Ref. [10]). In addition, we determine the average nuclear cost estimated here at 37.25 euros per MWh. This price is very close to the range of nuclear electricity prices appeared in the analysis of the Commission for Energy Regulation (CRE), which considered in 2010 a fair price (...) between 37.5 and 38.8 euros per MWh (see Ref. [22]). Furthermore, we determine the threshold of profitability of the non-nuclear thermal production realized by  $N$  producers during the month  $t$  ( $\theta_N$ ) by taking the profit resulting from the aggregated total monthly non-nuclear thermal production equal to zero:  $\theta_N = N \sqrt{\frac{a_{th}}{c_{th}}}$ . If the monthly non-nuclear thermal production level realized by the  $N$  producers is higher (lower) than  $\theta_N$ , then the profit is positive (negative). In view of

Proposition 2.1 and for a given total non-nuclear thermal capacity (independent of  $N$ ) divided in  $N$  identical producers, one has  $\theta_1 = \dots = \theta_N$  which means that the threshold of profitability of the non-nuclear thermal production  $\theta_N$  does not depend on  $N$ . The value of the threshold of profitability<sup>17</sup> provided by our numerical model is  $\theta \cong 18$  GW (or equivalently 13 TWh), hence it exceeds the level of the monthly non-nuclear thermal capacity  $Q_{max}^{th, Tot}$  (see Table 2, section 9) which leads to negative non-nuclear thermal profits through the entire period  $T$  (see Figure 5, Figure 10). Hence, if we take into consideration the too many approximations that we did do (in particular the absence of the mark-up rate), the total monthly non-nuclear thermal production cost is never covered.

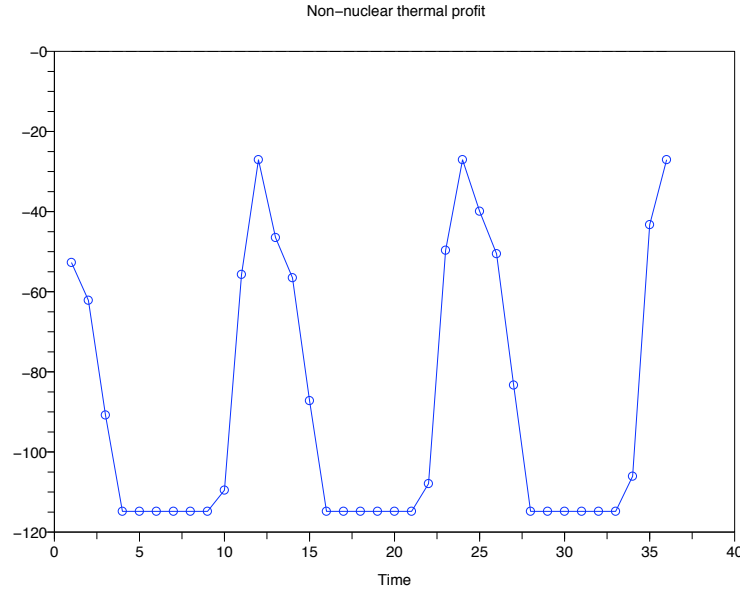


Figure 10: Aggregated total non-nuclear thermal profit (in Euro (million))

### 5.3 Optimal inter-temporal production problem VS Optimal per month production problem

In our previous paper, we modelled the effects of a flexible nuclear fuel reservoir operation via an optimal per month production problem, which consists of the maximization of the production value during a month given the production of the previous month. In this paper, we realize an inter-temporal optimization which is a mode of operation based on the direct optimization of the production over the whole game (36 months). In order to obtain an optimal per month production problem comparable with the optimal inter-temporal production problem treated by our numerical model (the “regularized” problem) with respect to the value of profit, nuclear is paid at price ( $b_{th}$ ), when it is the marginal technology, in both numerical models. This explains why figure 11 is not the one of our previous paper.

The optimal per month production problem provides us with a “local” optimum, thus with a solution that is optimal within a subset of feasible points. By contrast, the optimal inter-temporal production problem determines a global optimum which is the optimal solution

<sup>17</sup>In figure 5, the red crossed line represents the threshold of profitability of the non-nuclear thermal production.

among all possible solutions. Consequently, from a theoretical point of view, the optimal profit of production's inter-temporal optimization has to be greater than the optimal profit resulting from production's per month optimization. Indeed, our numerical results show that the value of the aggregate profit over the entire period  $T$  is more important when the value of generation is maximized over the whole time horizon of the model than in the case of a maximization on a monthly basis operation's horizon (see Table 4, section 9).

We also remark that the inter-temporal optimization approach provides us with higher profits during winter because of the relatively higher prices and lower profits during summer because of the relatively lower prices with respect to the optimization per month approach. Moreover, we observe that its value is decomposed in a cyclical component and a linear trend that increases progressively from one year to another in both cases (see Figure 7, Figure 11). However, we do not observe an “insufficiency” with respect to the size of the French nuclear set in both numerical exercises.

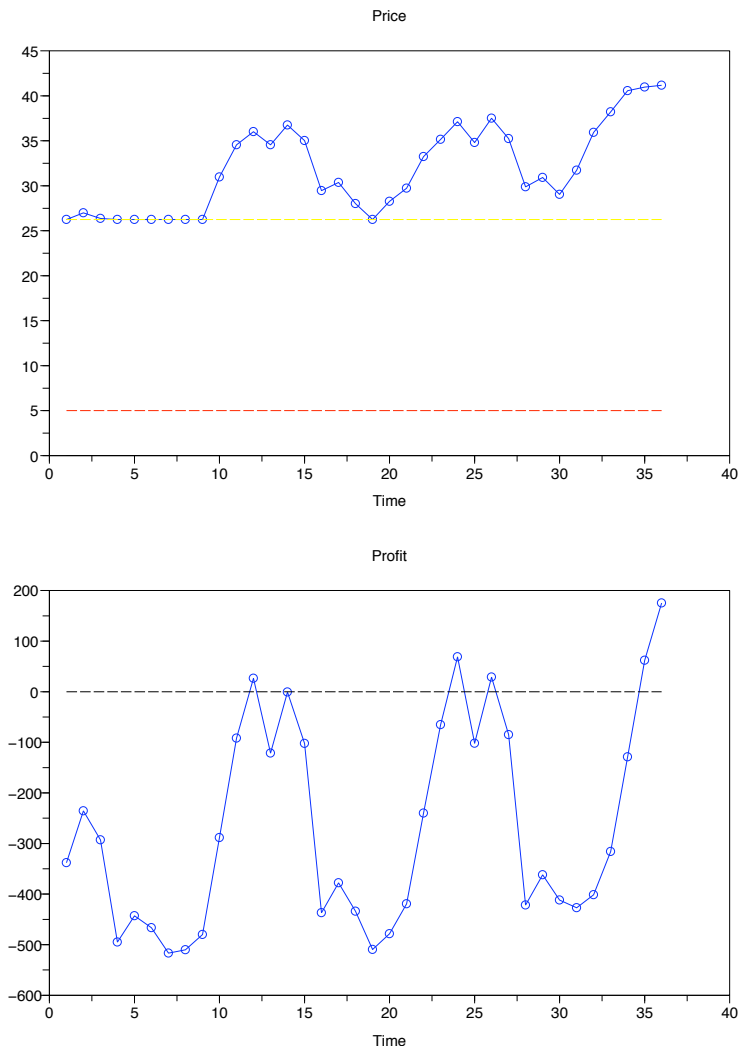


Figure 11: Simulated “regularized” price (in Euro/MWh)/ Aggregated total “regularized” profit (in Euro (million)) resulting from the optimal per month production problem

A difference that could be mentioned between the optimal inter-temporal production problem resolved within our numerical model (the “regularized” problem) and the optimal per month production problem concerns the duration of marginality of nuclear. In our numerical



model, nuclear is paid at price  $b_{th}$  which means that there is no penalty coming from the exclusive use of nuclear generation. Hence, being in marginal nuclear during periods of low demand (summer) is profitable for a producer. Indeed, we observe that the non-nuclear thermal technology is marginal only during the months of high demand in the inter-temporal optimization case while it remains marginal during most of the months of period  $T$  in the optimization per month case (see Figure 3, Figure 12).

It should be noticed that the discontinuity of the price observed at production vectors with zero levels of non-nuclear thermal production poses an “economical problem”. More precisely, a producer, who covers the monthly levels of demand during summer (low demand season) by running only its nuclear units, is penalized since its nuclear production is evaluated at a low price ( $b_{nuc}$ ). This price does not allow the amortization of the important fixed costs of nuclear. Hence, by realizing an infinitesimal nuclear capacity withholding, the non-nuclear thermal technology becomes the marginal technology that leads prices to a higher level (almost equal to  $b_{th}$ ) which justifies our “regularization” of the price merit order rule.

We remind that the production levels resulting from the “regularized” problem, constitute an “approximate” solution of our economical problem described in the subsection 2.6. Therefore, we do not proceed with an exhaustive comparison of the nuclear and the non-nuclear thermal production decisions as well as of the storage decisions obtained by the “regularized” problem and the optimal per month production problem.

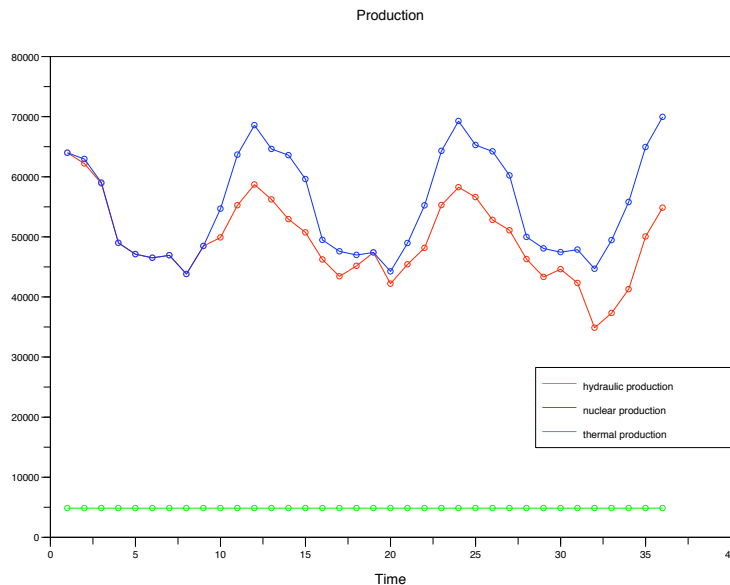


Figure 12: Simulated hydro/nuclear/non-nuclear thermal production (in MW) resulting from the optimal per month production problem

Finally, let us remark that a prolongation of the time horizon of the model  $T$  does not change producer’s behaviour in both cases (see Figure 9, Figure 13).

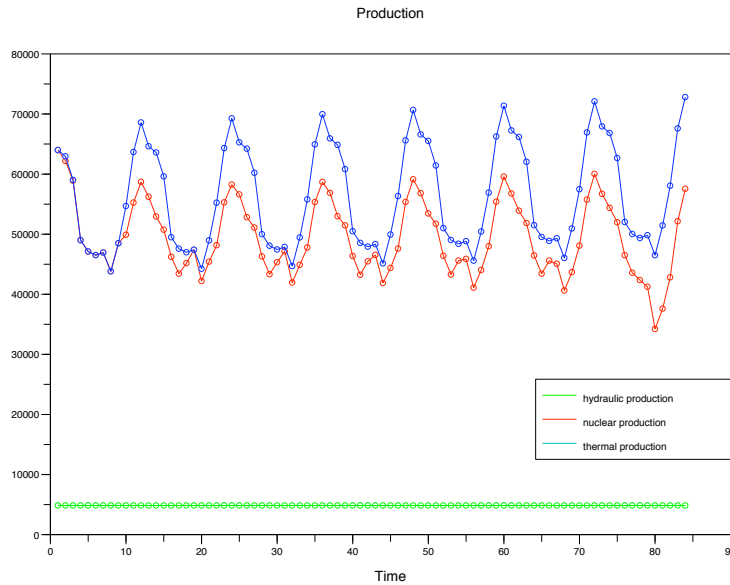


Figure 13: Simulated hydro/nuclear/non-nuclear thermal production (in MW) resulting from the optimal per month production problem ( $T=84$ )

## 6 Maximization of social welfare

In this section, we study the maximization of social welfare in a competitive electricity market in which there exist two types of generation: nuclear and non-nuclear thermal. We are interested in the decisions of the actors of the market (consumers and producers) that result from the maximization of the total utility of the society by taking into consideration the generation capacity constraints as well as the fuel storage constraints and the supply-demand equilibrium constraints within a medium-term horizon characterized by demand's seasonal variations.

### 6.1 A property of the “interior” solutions

The maximization of social welfare is an optimization problem which consists to the maximization of the total surplus. Total surplus is equal to consumer surplus (denoted by SC) associated with a given level of production plus producer surplus (denoted by SP). Consumer surplus is the difference between the total amount that consumers are willing and able to pay for electricity and the total amount that they actually do pay (electricity evaluated at the market price) (see Ref. [27]). The surplus of producer is equal to its revenue minus the variable costs or equivalently to the profit increased by the fixed costs (see Ref. [24], Ref. [27]). Without loss of generality, we may translate producers surplus by the fixed costs.

The social welfare maximization problem is

$$\max_{((q_{jt}^{nuc})_{j=1}^J, q_t^{th})_{t=1}^T \in C} \sum_{t=1}^T \left( SC \left( \sum_{j=1}^J q_{jt}^{nuc} + q_t^{th} \right) + SP \left( \sum_{j=1}^J q_{jt}^{nuc}, q_t^{th} \right) \right)$$

Thus, we have to solve

$$\max_{((q_{jt}^{nuc})_{j=1}^J, q_t^{th})_{t=1}^T \in C} \sum_{t=1}^T \left( \int_{p_t}^{\infty} D_t(p_t^*) dp_t^* \right) + [p_t (\sum_{j=1}^J q_{jt}^{nuc} + q_t^{th}) - \sum_{j=1}^J C_j^{nuc}(q_{jt}^{nuc}) - C^{th}(q_t^{th})]$$

or equivalently

$$\begin{aligned} & \max_{((q_{jt}^{nuc})_{j=1}^J, q_t^{th})_{t=1}^T \in C} \sum_{t=1}^T \left( \int_0^{\infty} D_t(p_t^*) dp_t^* - p_t (\sum_{j=1}^J q_{jt}^{nuc} + q_t^{th}) \right) \\ & + [p_t (\sum_{j=1}^J q_{jt}^{nuc} + q_t^{th}) - \sum_{j=1}^J C_j^{nuc}(q_{jt}^{nuc}) - C^{th}(q_t^{th})] \end{aligned}$$

where  $D_t(\cdot)$  is the demand function at time  $t$  (supposed here constant). We recall that  $p_t$  is given by the supply-demand equilibrium. However, the formula defining consumer surplus does not make sense in the presence of inelastic demand (infinite value of surplus). In view of this remark, we focus on the variation of consumer surplus. Nevertheless, the infinite value of consumers surplus leads to an indeterminate form of the variation of consumers surplus. For this reason, we introduce here a definition of the variation of consumer surplus when the price evolves from  $\bar{p}_t$  which is a level of reference to  $p_t$  given by the following formula ( $\Delta$ ) which is true in the classical case (case of an affine demand). More precisely, we have

$$\Delta = - \int_{\bar{p}_t}^{p_t} D_t(p_t^*) dp_t^* = -D_t(p_t - \bar{p}_t) = D_t \bar{p}_t - D_t p_t = \left( \sum_{j=1}^J q_{jt}^{nuc} + q_t^{th} \right) \bar{p}_t - \left( \sum_{j=1}^J q_{jt}^{nuc} + q_t^{th} \right) p_t$$

In view of these remarks, we will maximize the function

$$\sum_{t=1}^T \left( [Constant - p_t (\sum_{j=1}^J q_{jt}^{nuc} + q_t^{th})] + [p_t (\sum_{j=1}^J q_{jt}^{nuc} + q_t^{th}) - \sum_{j=1}^J C_j^{nuc}(q_{jt}^{nuc}) - C^{th}(q_t^{th})] \right)$$

Thus, the social welfare maximization problem can be written as

$$\max_{((q_{jt}^{nuc})_{j=1}^J, q_t^{th})_{t=1}^T \in C} \sum_{t=1}^T \left( Constant - \sum_{j=1}^J C_j^{nuc}(q_{jt}^{nuc}) - C^{th}(q_t^{th}) \right)$$

or equivalently

$$\min_{((q_{jt}^{nuc})_{j=1}^J, q_t^{th})_{t=1}^T \in C} \sum_{t=1}^T \left( \sum_{j=1}^J C_j^{nuc}(q_{jt}^{nuc}) + C^{th}(q_t^{th}) \right) \quad (21)$$

Thus, we deduce that the social welfare maximization problem is equivalent to the total cost minimization problem (21) (same set of solutions). If the solution of the social welfare maximization problem belongs to  $F$ , we obtain a property given by the following proposition. Let us notice that the proof of this proposition is similar to the one of Proposition 4.3.

**Proposition 6.1** *If there exists a solution  $((\hat{q}_{jt}^{nuc})_{j=1}^J, \hat{q}_t^{th})_{t=1}^T \in F$  such that the social welfare is maximum on  $C$  then  $\hat{q}_1^{th} = \hat{q}_2^{th} = \dots = \hat{q}_T^{th}$ .*

**Proof**

The production vector  $\hat{q} = ((\hat{q}_{jt}^{nuc})_{j=1}^J, \hat{q}_t^{th})_{t=1}^T$  solves the equivalent with the social welfare maximization problem, total cost minimization problem (21)

$$\min_{((q_{jt}^{nuc})_{j=1}^J, q_t^{th})_{t=1}^T \in C} \sum_{t=1}^T \left( \sum_{j=1}^J C_j^{nuc}(q_{jt}^{nuc}) + C^{th}(q_t^{th}) \right)$$

We choose to apply the Karush - Kuhn - Tucker (KKT) conditions in order to determine an optimal solution of the problem (21). Let us recall that  $M$  is an affine set, hence Slater's condition is satisfied. Thus, there exists  $(\bar{\mu}_t)_{t=1}^T \in \mathbb{R}^T$  and  $(\bar{\lambda}_j^k)_{j=1}^J \in \mathbb{R}^{(J-2)4+6}$  such that the KKT conditions are satisfied, where  $\bar{\mu}_t$  denotes the Lagrange multiplier for the supply-demand equilibrium constraint at each month  $t$  and  $\bar{\lambda}_j^k$  is the Lagrange multiplier for the nuclear fuel constraint of the unit  $j$  during the campaign  $k$ . According to the KKT conditions of complementarity and since  $\hat{q} \in F$ , all the Lagrange multipliers associated with the min/max production constraints are equal to zero and they will be omitted in the Lagrangien function of this problem.

We call  $\bar{L}$  the reduced Lagrangien of the cost minimization problem (21)

$$\bar{L}(q) = \sum_{t=1}^T \left( \sum_{j=1}^J (C_j^{nuc}(q_{jt}^{nuc}) + C^{th}(q_t^{th})) - \bar{\sigma}(Dq - E) \right)$$

where  $q = ((q_{jt}^{nuc})_{j=1}^J, q_t^{th})_{t=1}^T$  is a production vector belonging to  $C$  and  $\bar{\sigma} = (\bar{\lambda}_j^k, \bar{\mu}_t)$  is the vector of the Lagrange multipliers. We recall from the proof of Proposition 4.3 that the vector  $E$  and the matrix  $D$  are defined so that  $M = \{q' \text{ s.t. } Dq' - E = 0\}$ .

Following some calculations and with the help of the supply-demand equilibrium constraints, we deduce the objective function of the cost minimization problem (21) is a quadratic function of the non-nuclear thermal production  $q_t^{th}$

$$c_{th} \sum_{t=1}^T (q_t^{th})^2 + d \quad (22)$$

where  $d = (\sum_{t=1}^T a_{th} + \sum_{t=1}^T \sum_{j=1}^J a_{nuc}^j + b_{th}((D_t - Q_{Tot,t}^{hyd}) - S_{reload}) + b_{nuc}S_{reload})$  is the constant part of the function.

According to the KKT conditions, we have

$$\frac{\partial \bar{L}}{\partial q_{jt}^{nuc}}(\hat{q}) = 0, \text{ for all } j, t$$

and

$$\frac{\partial \bar{L}}{\partial q_t^{th}}(\hat{q}) = 0, \text{ for all } t.$$

The derivative of Lagrangien with respect to the thermal production  $q_1^{th}$  at month 1 is

$$\frac{\partial \bar{L}}{\partial q_1^{th}}(\hat{q}) = 2c_{th}\hat{q}_1^{th} - \bar{\mu}_1 = 0 \Leftrightarrow \hat{q}_1^{th} = \frac{\bar{\mu}_1}{2c_{th}} \quad (23)$$

Symmetrically, the derivative of Lagrangien with respect to the thermal production  $q_2^{th}$  at month 2 is

$$\frac{\partial \bar{L}}{\partial q_2^{th}}(\hat{q}) = 2c_{th}\hat{q}_2^{th} - \bar{\mu}_2 = 0 \Leftrightarrow \hat{q}_2^{th} = \frac{\bar{\mu}_2}{2c_{th}} \quad (24)$$

To compare the Lagrange multipliers  $\bar{\mu}_1$  and  $\bar{\mu}_2$ , we focus on a unit that is active during month 1 and 2. The derivative of Lagrangien with respect to the nuclear production  $q_{31}^{nuc}$  of unit 3 at month 1 is

$$\frac{\partial \bar{L}}{\partial q_{31}^{nuc}}(\hat{q}) = -\bar{\mu}_1 - \bar{\lambda}_3^1 = 0 \Leftrightarrow \bar{\mu}_1 = -\bar{\lambda}_3^1 \quad (25)$$

The derivative of Lagrangien with respect to the nuclear production  $q_{32}^{nuc}$  of unit 3 at month 2 is

$$\frac{\partial \bar{L}}{\partial q_{32}^{nuc}}(\hat{q}) = -\bar{\mu}_2 - \bar{\lambda}_3^1 = 0 \Leftrightarrow \bar{\mu}_2 = -\bar{\lambda}_3^1 \quad (26)$$

From the last two equations, we deduce that  $\bar{\mu}_1 = \bar{\mu}_2$ . This means that

$$\hat{q}_1^{th} = \frac{\bar{\mu}_1}{2c_{th}} = \frac{\bar{\mu}_2}{2c_{th}} = \hat{q}_2^{th}$$

By using a unit available at month  $t$  and  $t+1$ , we obtain  $\bar{\mu}_t = \bar{\mu}_{t+1}$ , which means that  $\hat{q}_t^{th} = \hat{q}_{t+1}^{th}$ . Thus,

$$\hat{q}_1^{th} = \hat{q}_2^{th} = \dots = \hat{q}_T^{th}$$

Consequently, we conclude that a solution of the total cost minimization problem (21) is determined by a non-nuclear thermal production that is constant during the entire time horizon of our model and a variable nuclear production which entirely follows the seasonal variations of the demand. □

Since  $\hat{q} \in F$ , the equation (23) (respectively (24)) implies that the sign of the multiplier  $\bar{\mu}_1$  (respectively  $\bar{\mu}_2$ ) is strictly positive. By a symmetric argument, the Lagrange multiplier  $\bar{\mu}_t$  is strictly positive ( $\bar{\mu}_t > 0$ ) for all  $t$ . Hence, in view of equations (25) and (26), the multiplier  $\bar{\lambda}_3^1$  (respectively  $\bar{\lambda}_j^k$ ) is strictly negative. Indeed, if an additional unit of nuclear fuel became available for the unit  $j$  during the campaign  $k$ , the non-nuclear thermal production would decrease which would lead to the augmentation of the nuclear production cost and the diminution of the non-nuclear thermal production cost. However, the second effect, thus the decrease of the non-nuclear thermal production cost is the most important. Consequently, the “additional” cost resulting from an additional nuclear fuel unit and thus the value of the multiplier  $\bar{\lambda}_j^k$  should be negative. The multiplier  $\bar{\lambda}_j^k$  indicates the “marginal value of nuclear fuel stock”, i.e. the additional cost  $|\bar{\lambda}_j^k|$  unit  $j$  would incur if the nuclear fuel stock decreased by one unit during the campaign  $k$ .

We remind that  $C$  is a compact set, thus the total cost minimization problem has solutions on  $C$ . Nevertheless, it may not have solutions on the set  $F$  since it is not compact. Hence, the existence of a solution of the problem (21) on  $F$  has the form of assumption in Proposition 6.1.

Let us now proceed with a proposition which shows that a constant non-nuclear thermal production is a sufficient condition for optimality on  $C$ . The proof of this proposition resembles the proof of Proposition 4.4.

**Proposition 6.2** *If  $((\hat{q}_{jt}^{nuc})_{j=1}^J, \hat{q}_t^{th})_{t=1}^T$  is a production vector belonging to  $C$  such that  $\hat{q}_1^{th} = \hat{q}_2^{th} = \dots = \hat{q}_T^{th}$  then  $((\hat{q}_{jt}^{nuc})_{j=1}^J, \hat{q}_t^{th})_{t=1}^T$  is a solution of the social welfare maximization problem (equivalently total cost minimization problem) on  $C$ .*

### Proof

In order to prove Proposition 6.2, we need to show that the production vector  $\hat{q} = ((\hat{q}_{jt}^{nuc})_{j=1}^J, \hat{q}_t^{th})_{t=1}^T$  is a solution of the total cost minimization problem on  $C$ . Thus, it suffices to show that there exist Lagrange multipliers such that the associated with the problem (21) KKT conditions are satisfied at  $\hat{q}$  given that the objective function of this problem is convex. We show the convexity of the total production cost by stating the following Lemma.

**Lemma 6.1** *The total production cost function of the total cost minimization problem on  $C$  is convex.*

### Proof of Lemma 6.1

The total production cost

$$\sum_{t=1}^T \left( \sum_{j=1}^J C_j^{nuc}(q_{jt}^{nuc}) + C^{th}(q_t^{th}) \right)$$

is a quadratic function of the non-nuclear thermal production  $q_t^{th}$  provided by the function (22)

$$c_{th} \sum_{t=1}^T (q_t^{th})^2 + d$$

(see proof of Proposition 6.1). We remark that (22) is a quadratic function of the form  $g(u) = \alpha(u^T \cdot u) + d$ , where  $\alpha = c_{th}$ . Since  $\alpha > 0$ , the function  $g(u)$  is convex. Moreover, it is strictly convex. Consequently, taking into consideration the other variables, we conclude that the total production cost function (22) is convex.  $\square$

By setting the Lagrange multipliers associated with the min/max production constraints equal to zero and since the nuclear fuel constraints as well as the supply-demand equilibrium constraints are pure equalities, we deduce that the KKT complementary conditions are satisfied at  $((\hat{q}_{jt}^{nuc})_{j=1}^J, \hat{q}_t^{th})_{t=1}^T$ . Then, we proceed with the Lagrange multipliers associated with the supply-demand equilibrium constraint in month  $t$  ( $\tilde{\mu}_t$ ) and the nuclear fuel constraints of the unit  $j$  during the campaign  $k$  ( $\tilde{\lambda}_j^k$ ). We set

$$\left\{ \begin{array}{l} \tilde{\mu}_t = 2c_{th}\hat{q}_t^{th}, \quad \text{for all } t \end{array} \right.$$

In view of the constant non-nuclear thermal production ( $\hat{q}_1^{th} = \hat{q}_2^{th} = \dots = \hat{q}_T^{th}$ ), we deduce that  $\tilde{\mu}_1 = \tilde{\mu}_2 = \dots = \tilde{\mu}_T$ . Hence, we set

$$\left\{ \begin{array}{l} \tilde{\lambda}_j^k = -\tilde{\mu}, \quad \text{for all } j, k \end{array} \right.$$

where  $\tilde{\mu}$  is the common value of the multipliers  $(\tilde{\mu}_t)_{t=1}^T$ .

For those multipliers the Lagrangien function of the total cost minimization problem on  $C$  has the following form

$$\bar{H}(q) = \sum_{t=1}^T \left( \sum_{j=1}^J (C_j^{nuc}(q_{jt}^{nuc}) + C^{th}(q_t^{th})) - \tilde{\sigma}(Dq - E) \right)$$

where  $\tilde{\sigma} = (\tilde{\mu}_t, \tilde{\lambda}_j^k)$ . We can see then that the Lagrangien  $\bar{H}$  coincides with the reduced Lagrangien  $\bar{L}$  of the total cost minimization problem (21).

In view of the analysis of (23) respectively (25) and by a symmetric argument, we realize that

$$\frac{\partial \bar{L}}{\partial q_t^{th}}(\hat{q}) = 0, \text{ for all } t$$

respectively

$$\frac{\partial \bar{L}}{\partial q_{jt}^{nuc}}(\hat{q}) = 0, \text{ for all } j, t.$$

Consequently, the production vector  $((\hat{q}_{jt}^{nuc})_{j=1}^J, \hat{q}_t^{th})_{t=1}^T$  is a solution of the KKT conditions associated with the total cost minimization problem determined on  $C$ . In addition, KKT conditions provide us with solutions that minimize the total production cost on  $C$ , since the objective function (total production cost function) is convex. Thus, a production vector  $((\hat{q}_{jt}^{nuc})_{j=1}^J, \hat{q}_t^{th})_{t=1}^T$  of the set  $C$  that is characterized by a constant non-nuclear thermal production is a solution of the total cost minimization problem on  $C$ .  $\square$

In view of propositions 6.1 and 6.2, we conclude that in the absence of binding production constraints, the optimal production vectors that maximize the social welfare in a competitive electricity market are such that the non-nuclear thermal generation units run all the time in a constant way to meet the demand, and thus the nuclear is entirely used to follow-up load. This means that the social planner chooses to produce the same quantity of non-nuclear thermal each month to cover the demand even during months of low demand, so it uses nuclear to follow the seasonal variations of the demand. Consequently, prices are determined permanently by the marginal cost of fossil fuel technologies and hence, they stay constant during the entire time horizon of the model. In addition, in view of the non-binding production constraints the nuclear units produce less than its maximum capacity during all months.

## 6.2 Numerical Illustration

In this section, we present the nuclear and non-nuclear thermal production levels as well as the storage levels resulting from the social welfare maximization problem, via a simple numerical model by using Scilab. Let us notice that the data used in this numerical model has been already used in section 5.

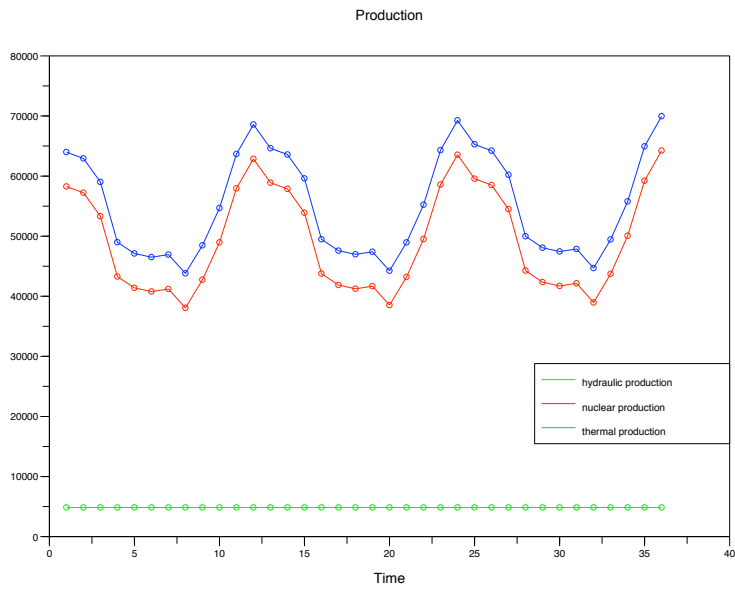


Figure 14: Simulated hydro/nuclear/non-nuclear thermal production (in MW)

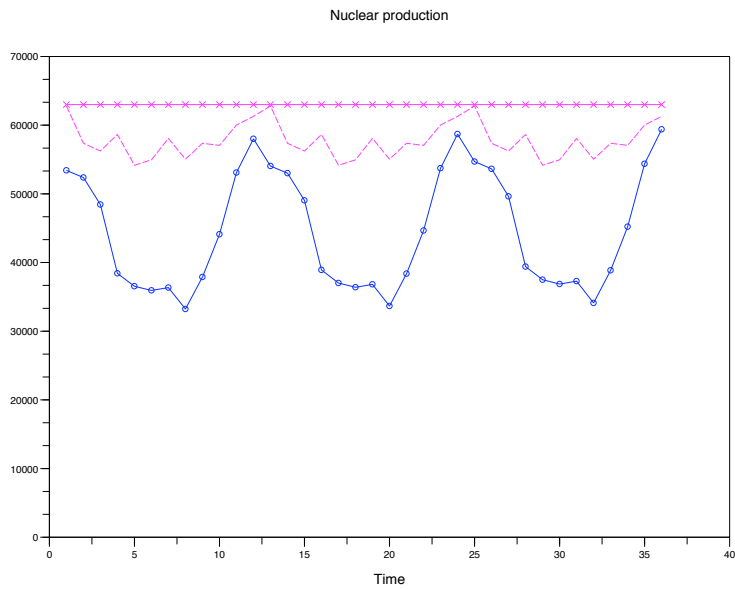


Figure 15: Simulated nuclear production (in MW)



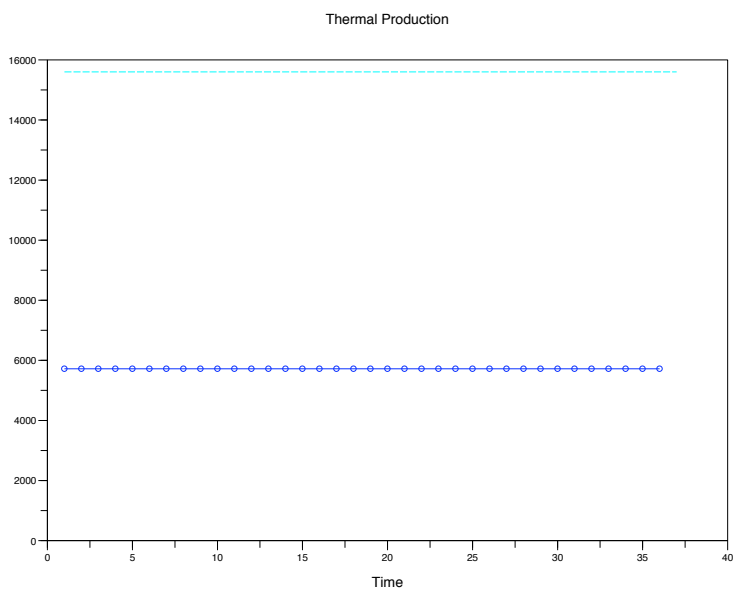


Figure 16: Simulated non-nuclear thermal production (in MW)

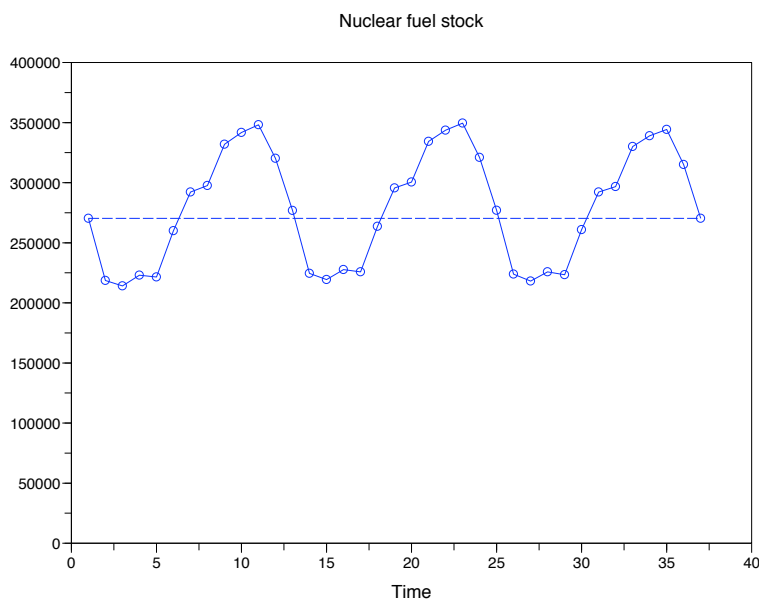


Figure 17: Simulated nuclear fuel stock (in MW)

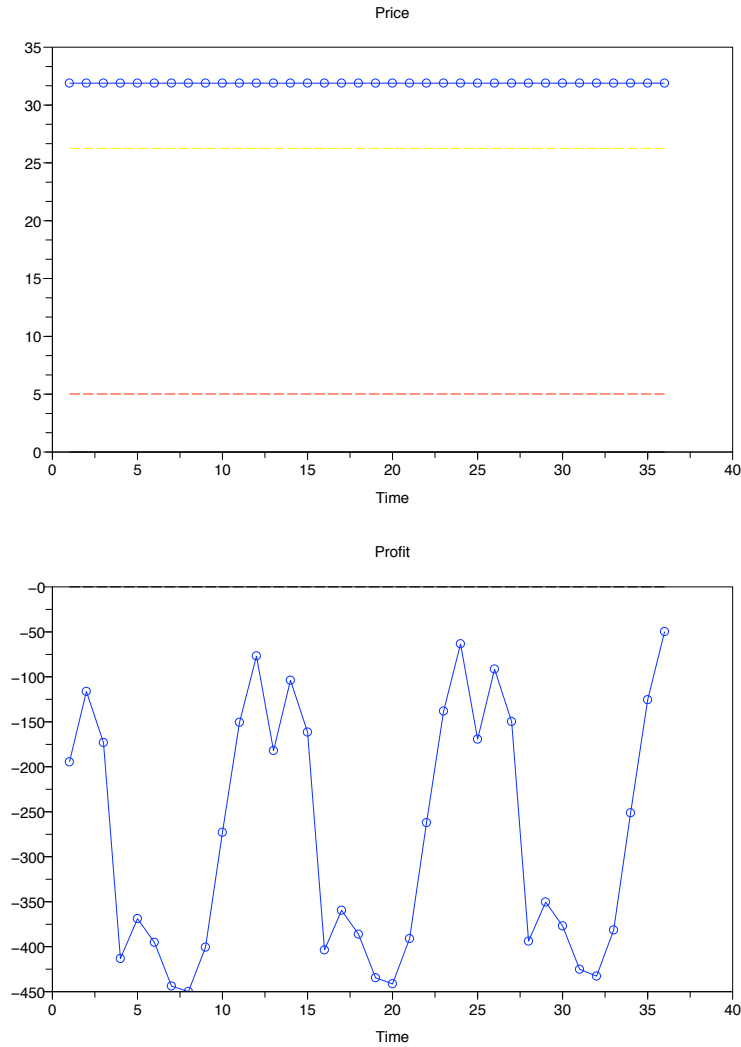


Figure 18: Simulated price (in Euro/MWh)/Aggregated total profit (in Euro (million))

### General simulation results

In our numerical model, we maximize social welfare (equivalently we minimize total production cost) on the entire set of feasible solutions  $C$ .

Simulation results obtained by our numerical model show that nuclear entirely follows the seasonal variations of the demand by decreasing during summer and increasing during winter while non-nuclear thermal is constant during the entire time horizon of our model without saturating the minimum/maximum non-nuclear thermal production constraints. Thus, the non-nuclear thermal generation remains marginal during the entire time horizon of the model while the nuclear technology is never marginal, even during the months of low demand (see Figure 14, Figure 15, Figure 16). Furthermore, we verify (through a numerical test) that the numerical solution described in this section belongs to  $F$  which shows that Proposition 6.1 is applicable since the non-nuclear thermal component of this solution is constant. We also deduce that Proposition 6.2 is applied within our numerical model since we find that a production vector belonging to  $C$  such that the non-nuclear thermal production is constant is a solution of the social welfare maximization problem. It should be noticed that the strict convexity of the total cost function with respect to the non-nuclear thermal production (see

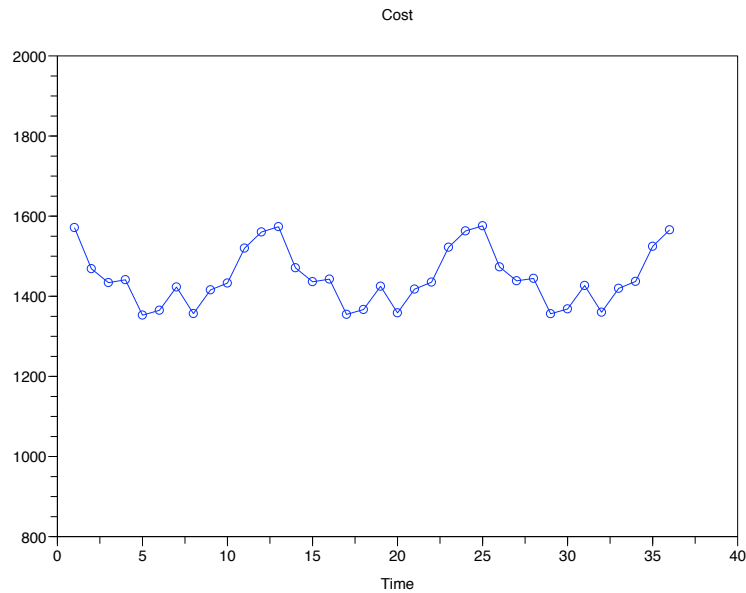


Figure 19: Aggregated total cost (in Euro (million))

proof of Lemma 6.1) implies the unicity of solutions with respect to the non-nuclear thermal component which in view of Proposition 6.1 is constant. However, taking into account the other variables the total cost function is convex, thus the entire solution is not unique (see Lemma 6.1).

Moreover, one observes that the monthly nuclear production as well as the monthly non-nuclear thermal production never reach their maximum value (see Figure 15, Figure 16). In addition, we obtain “high” levels of nuclear fuel stock during summer and “low” levels of nuclear fuel stock during winter because of the seasonality that characterizes the variations of the nuclear production (high production during winter – low production during summer). Consequently, we observe a periodical evolution for the nuclear fuel stock due to the periodical evolution of the nuclear production as well as an oscillation around the “stock of reference” (see Figure 15, Figure 17).

The price<sup>18</sup> is constant since it is determined by the non-nuclear thermal marginal cost during the entire period  $T$  (see Figure 18). Furthermore, the value of the aggregated total cost evolves periodically by increasing during winter and decreasing during summer (see Figure 19). It should be noticed that within this numerical model the aggregated total cost resulting from the optimal inter-temporal production problem is relatively higher than the aggregated total cost coming from the optimal per month production problem. As expected the value of the aggregate total cost resulting from both optimization problems is higher than the optimal value of the aggregated total cost determined in this section (see Table 5, section 9).

Let us also remark that despite the variations in the temporal horizon of the model the periodical evolution of the nuclear and non-nuclear thermal production during the entire time horizon does not change (e.g. for  $T = 84$ , see Figure 20).

<sup>18</sup>The red (respectively yellow) dotted line indicates the price level when nuclear (respectively non-nuclear thermal) is the marginal technology.

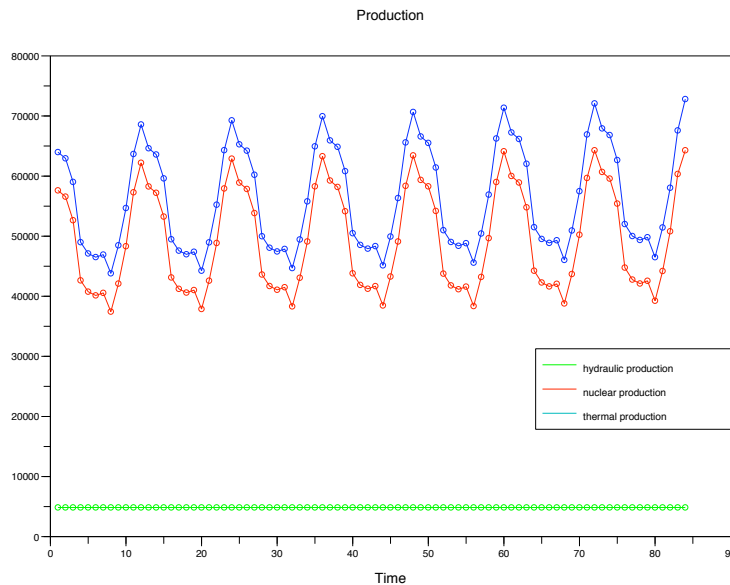


Figure 20: Aggregated total cost (in Euro)

## 7 Conclusion

In this paper, we have examined the optimal inter-temporal management of a flexible nuclear generation set in a perfectly competitive regime. Once again, we focused on a “medium-term approach” which takes into account the seasonal variation of the demand between winter (high demand) and summer (low demand). As we mentioned in our previous paper, the nuclear fuel functions like a “reservoir”, which allows different allocations of the nuclear production during the different demand seasons of the year. In this paper, the modelling of the optimal management of a flexible nuclear reservoir consists in maximizing the value of generation during the whole time horizon  $T$  (36 months) and it leads to the determination of the global optimum of the optimal production problem. We proposed a deterministic multi-period model to study the perfect competitive case in a market where producers use both nuclear and non-nuclear generations units. Under the assumption of symmetry of capacity among producers, we demonstrated that the equilibrium of the optimal inter-temporal production problem is “almost” symmetric. More precisely, we proved that its non-nuclear thermal component is symmetric while its nuclear component is symmetrisable and we focused on the symmetric solution (Proposition 3.1). We also showed that under some assumptions producer’s inter-temporal profit decreases when nuclear is the marginal technology which leads us to the exclusion of zero non-nuclear thermal production levels from the domain of definition of our optimization problem (Proposition 4.2). Then, we looked at the “interior” optimal solutions and we proved that if such a solution exists then it is characterized by a constant nuclear production (Proposition 4.3). In addition, we showed that a production vector with strictly positive non-nuclear thermal production levels (thus, belonging to  $F^{th}$ ) characterized by this property is a solution of the inter-temporal profit maximization problem (Proposition 4.4).

In the following section, we presented a numerical simulation by taking into account the actual size of a given nuclear set (the French) vis-à-vis the non-nuclear generation set. We treated the price discontinuity (more precisely the price decrease), which under some assumptions is responsible for the decrease of the inter-temporal profit when the non-nuclear thermal

units are not active, by “regularizing” the merit order price. This price “regularization” led to an alternative optimal inter-temporal production problem (the “regularized” problem) different from the economical problem (presented in subsection 2.6) by its objective function. However, as we proved, the value of both optimization problems is identical which allowed us to obtain a “good” approximation of our economical problem (see Annex, Proposition 8.1). In addition, we determined the set of solutions of the economical problem as the intersection of the set of solutions of the “regularized” problem and the set  $F^{th}$  (see Annex, Proposition 8.6). We deduced for our numerical example that the set of solutions of the economical problem is empty since the solution obtained by the “regularized” problem does not belong to the set  $F^{th}$  (see Annex, Proposition 8.7). Nevertheless, this numerical solution is only an “approximate” solution of our economical problem. It is characterized by high levels of nuclear generation during the months of high demand (winter) and low levels during the months of low demand (summer) which confirms the “theoretical” ability of the nuclear generation set to follow-up load variations. As expected, this leads to low levels of nuclear fuel stock during winter and high levels of nuclear fuel stock during summer. In addition, we noticed that the values of the nuclear fuel stock oscillate around the “reference” value of this fuel stock. Moreover, to ensure the equilibrium between supply and demand during winter, the producer has to take into account the thermal non-nuclear generation capacity. Non-nuclear thermal generation is marginal only during some months of the year (months of high demand) since the nuclear production is no more penalized by a low price within our numerical model. Consequently, the price is determined by the non-nuclear thermal marginal cost only during high demand’s seasons. In particular, we observed that market price peaks during winter and reaches its lowest during summer. Accordingly, producers obtain higher profits during winter and lower profits during summer. It should be noticed that solutions fully characterized by a constant nuclear production do not exist within our numerical model because of non-nuclear thermal production constraints. Furthermore, we drew two interesting conclusions: (i) the average nuclear cost estimated here is close to the range of nuclear electricity prices (37.5 and 38.8 euros per MWh) given by the Commission for Energy Regulation (CRE) in 2010, (ii) given the approximations that we did do, a non-nuclear thermal producer does not cover its total monthly production cost.

Then, the numerical results of the optimal inter-temporal production problem (the “regularized” problem) were contrasted with the numerical results of the optimal per month production problem by taking into consideration that nuclear is paid at price  $b_{th}$  in both problems when it is the marginal technology. We did find that producer’s profit is greater in the inter-temporal optimization case than in the per month optimization case given that in the first case the producer has “discovered” how competitive wholesale markets work, hence it is interested in determining the global optimum of the optimal production problem. In addition, a comparison between the simulated production levels resulting from both optimization problems showed that the period during which the non-nuclear thermal technology is marginal is longer for the optimal per month production problem than for the optimal inter-temporal production problem since price’s “regularization” makes nuclear production profitable for the producer.

Finally, in the last section, we studied the social welfare maximization problem by taking into account the seasonal variations of the demand (medium-term horizon) in order to determine social planner’s optimal decisions in a competitive electricity market. After showing the equivalence between the social welfare maximization problem and the total cost minimization problem, we demonstrated that the “interior” solutions of the social welfare maximization problem are characterized by a constant non-nuclear thermal production (Proposition 6.1). Then, we proved that this property is a sufficient condition for optimality on the entire set of feasible solutions  $C$  meaning that a production vector of  $C$  that has this property constitutes a solution of the social welfare maximization on  $C$ . As one might see the production scheduling resulting

from the social welfare maximization problem is diametrically opposite from the one coming from the optimal inter-temporal production problem. Simulation results showed that the non-nuclear thermal production is constant through the entire time horizon of the model forcing nuclear to follow demand's seasonal variations (high production levels during winter – low production levels during summer). Moreover, we observed that this numerical solution belongs to  $F$ , hence the Proposition 6.1 is applicable within our numerical example and we also deduced the applicability of Proposition 6.2. As expected, the periodical evolution of the nuclear production cause the opposite periodical evolution of the nuclear fuel stock. More precisely, we observed an increase of the nuclear fuel stock during summer and a decrease during winter. Furthermore, the price is determined only by the non-nuclear thermal generation since it is the only marginal technology during the entire period  $T$ . Hence, the price remains constant over the whole time horizon of the model. We also noticed the periodical evolution of the aggregated total cost which increases during winter and decreases during summer.

## 8 Annex

Let us introduce some notations used in the annex. We call  $P_1$  the economical problem (presented in subsection 2.6)

$$\max_{q \in C} \pi(q)$$

and  $P_2$  the “regularized” problem (presented in subsection 5.2)

$$\max_{q \in C} \psi(q)$$

where  $\pi$  and  $\psi$  represent the inter-temporal profit functions of problem  $P_1$  and  $P_2$  respectively.

We will recall the properties proved in the paper in order to allow a self-content annex to the maximum extent. Functions  $\pi$  and  $\psi$  are such that  $\pi \leq \psi$  on  $C$ . This is because  $\pi = \psi$  on  $F^{th}$  which is a subset of  $C$  since the price is determined by the non-nuclear thermal production in both problems and  $\pi < \psi$  on  $C \setminus F^{th}$  because when nuclear is marginal, it is paid at price  $b_{th}$  within the “regularized” problem while it is paid  $b_{nuc}$  inside the economical problem and  $b_{nuc} < b_{th}$  (see Proposition 4.2). Let us also mention that  $\psi$  is a continuous function which, in addition, is strictly concave with respect to the non-nuclear thermal production  $q_t^{th}$  (see proof of Lemma 4.1). Moreover, the set  $F^{th}$  is dense in  $C$  (see Proposition 4.1).

We are now ready to state the following propositions.

**Proposition 8.1** *The value of the economical problem ( $val(P_1)$ ) and the value of the “regularized” problem ( $val(P_2)$ ) are the same ( i.e.  $val(P_1) = val(P_2)$ ).*

### Proof

First, since  $\pi \leq \psi$ , we deduce that  $val(P_1) \leq val(P_2)$ .

Secondly, we prove that  $val(P_1) \geq val(P_2)$ . The set  $C$  is compact, hence the set of solutions of the “regularized” problem is non empty. This means that there exists  $q \in C$  such that  $\psi(q) = val(P_2)$ . In view of the density of  $F^{th}$  in  $C$ , the production vector  $q \in \overline{F^{th}}$ . It follows that there exists a sequence  $(q_h)_{h \in \mathbb{N}}$  such that  $q_h \in F^{th}$  and  $\lim_{h \rightarrow \infty} q_h = q$ . Since  $\pi = \psi$  on  $F^{th}$  and  $\pi(q_h) \leq val(P_1)$ , at the limit we obtain

$$val(P_2) = \psi(q) = \lim_{h \rightarrow \infty} \psi(q_h) = \lim_{h \rightarrow \infty} \pi(q_h) \leq val(P_1)$$

Consequently, the inequality  $val(P_1) \geq val(P_2)$  is proved.

From the first and the second part of the proof, we conclude that  $val(P_1) = val(P_2)$ . □

**Proposition 8.2** *The set of solutions of the economical problem ( $Sol(P_1)$ ) is contained on the set  $F^{th}$  ( i.e.  $Sol(P_1) \subset F^{th}$ ).*

### Proof

If  $Sol(P_1) = \emptyset$  then there is nothing to prove. Let  $\hat{q} \in Sol(P_1)$ . This means that for all  $q \in C$ ,  $\pi(q) \leq \pi(\hat{q})$ . Especially, for all  $q \in F^{th}$ ,  $\psi(q) = \pi(q) \leq \pi(\hat{q}) \Rightarrow \psi(q) \leq \pi(\hat{q})$ . In particular, for  $q = \hat{q}$ , we obtain  $\psi(\hat{q}) \leq \pi(\hat{q})$  which means that  $\hat{q} \notin C \setminus F^{th}$  and thus,  $\hat{q} \in F^{th}$ . Hence, we deduce that  $Sol(P_1) \subset F^{th}$ . □

**Proposition 8.3** *The set of solutions of the economical problem ( $Sol(P_1)$ ) is contained on the set of solutions of the “regularized” problem ( $Sol(P_2)$ ) ( i.e.  $Sol(P_1) \subset Sol(P_2)$ ).*

**Proof**

In the case that  $Sol(P_1) = \emptyset$ , there is nothing to prove. If  $Sol(P_1)$  is a non-empty set then there exists  $\tilde{q} \in Sol(P_1)$ , which means that: (i)  $val(P_1) = \pi(\tilde{q})$ , (ii) in view of Proposition 8.2,  $\tilde{q} \in F^{th}$ , hence  $\psi(\tilde{q}) = \pi(\tilde{q})$ . Consequently, we obtain

$$val(P_1) = \pi(\tilde{q}) = \psi(\tilde{q}) \leq val(P_2) = val(P_1) \Rightarrow \psi(\tilde{q}) = val(P_2)$$

Thus,  $\tilde{q} \in Sol(P_2)$  and the proposition is proved.  $\square$

We will reinforce the last two propositions.

**Proposition 8.4** *The set of solutions of the economical problem ( $Sol(P_1)$ ) is determined as following:*

$$Sol(P_1) = \begin{cases} Sol(P_2), & \text{if } Sol(P_2) \cap F^{th} \neq \emptyset \\ \emptyset, & \text{otherwise} \end{cases}$$

**Proof**

If  $Sol(P_2) \cap F^{th} \neq \emptyset$  then there exists a production vector  $\bar{q}$  such that  $\bar{q} \in F^{th}$  and  $\bar{q} \in Sol(P_2)$ . Since  $\bar{q} \in Sol(P_2)$  one has, for all  $q \in C$ ,  $\psi(q) \leq \psi(\bar{q})$ . If  $q \in F^{th}$  then  $\pi(q) = \psi(q) \leq \psi(\bar{q}) = \pi(\bar{q})$ . If  $q \in C \setminus F^{th}$  then  $\pi(q) < \psi(q) \leq \psi(\bar{q}) = \pi(\bar{q})$ . Hence, we conclude that  $\bar{q}$  corresponds to a solution of the problem  $P_1$ , which means that  $Sol(P_2) \subset Sol(P_1)$ . Together with Proposition 8.3, we obtain  $Sol(P_1) = Sol(P_2)$ .

In view of Proposition 8.2, one has  $Sol(P_1) \subset F^{th}$ . In addition,  $Sol(P_1) \subset Sol(P_2)$  (see Proposition 8.3). Consequently, we deduce that  $Sol(P_1) \subset Sol(P_2) \cap F^{th}$ . If  $Sol(P_2) \cap F^{th} = \emptyset$  then  $Sol(P_1) \subset \emptyset \Leftrightarrow Sol(P_1) = \emptyset$ , thus our economical problem  $P_1$  has no solutions, which concludes our proof.  $\square$

**Proposition 8.5** *The set of solutions of the “regularized” problem ( $Sol(P_2)$ ) is such that either  $Sol(P_2) \subset F^{th}$  or  $Sol(P_2) \subset C \setminus F^{th}$ .*

**Proof**

In view of the strict concavity of the inter-temporal profit function  $\psi$  with respect to  $q_t^{th}$ , the non-nuclear thermal component of the solutions of the problem  $P_2$  is unique. Consequently, either  $Sol(P_2) \subset F^{th}$  or  $Sol(P_2) \subset C \setminus F^{th}$ .  $\square$

**Proposition 8.6** *The set of solutions of the economical problem ( $Sol(P_1)$ ) and the set of solutions of the “regularized” problem ( $Sol(P_2)$ ) are such that  $Sol(P_1) = Sol(P_2) \cap F^{th}$ .*

**Proof**

If  $Sol(P_2) \cap F^{th} = \emptyset$  then, according to Proposition 8.4,  $Sol(P_1) = \emptyset$ . Consequently,  $Sol(P_1) = Sol(P_2) \cap F^{th}$ .

If  $Sol(P_2) \cap F^{th} \neq \emptyset$  then, in view of Proposition 8.4, one has  $Sol(P_1) = Sol(P_2)$ . However, from Proposition 8.5, we obtain  $Sol(P_2) \subset F^{th}$ . Thus,  $Sol(P_1) = Sol(P_2) \cap F^{th}$ .  $\square$

**Proposition 8.7** *If  $q$  is a production vector that does not belong to  $F^{th}$  and  $q \in Sol(P_2)$  (it constitutes a solution of the “regularized” problem), then  $Sol(P_1) = \emptyset$ .*



**Proof**

In view of the hypothesis,  $q \in \text{Sol}(P_2)$  but it is not a production vector of the set  $F^{th}$ . Thus, according to Proposition 8.5,  $\text{Sol}(P_2) \subset C \setminus F^{th}$ . This means that  $\text{Sol}(P_2) \cap F^{th} = \emptyset$ . Hence, in view of Proposition 8.4 (or equivalently Proposition 8.6), the set of solutions of the economical problem ( $\text{Sol}(P_1)$ ) is empty, which proves our proposition.  $\square$

## 9 Tables

In this section, we present the values of the exogenous variables of our numerical model. We also provide the value of the aggregated total “regularized” profit obtained by the optimal inter-temporal production problem as well as by the optimal per month production problem. In addition, we present the value of the aggregated total “regularized” nuclear and non-nuclear thermal profit. Finally, we give the aggregated total cost and aggregated total variable cost resulting from all three optimization problems (optimal inter-temporal production problem, optimal per month production problem, social welfare maximization problem).

Nuclear capacity of unit $j$ (in MW)	$j = 1$	$j = 2$	$j = 3$	$j = 4$	$j = 5$	$j = 6$	
$Capacity^{j,nuc,Tot}$	201	5641	6758	4359	8838	8045	
	$j = 7$	$j = 8$	$j = 9$	$j = 10$	$j = 11$	$j = 12$	Total
	4923	7956	5634	5950	2972	1723	63000

Table 1

The level of nuclear capacity that the unit  $j$  disposes is denoted by  $Capacity^{j,nuc,Tot}$ .

Capacity (in MW)	Nuclear	Non-nuclear thermal	Hydro
$Q_{min}^{j,nuc,Tot}$	$0.25 \times Capacity^{j,nuc,Tot}$	—	—
$Q_{max}^{j,nuc,Tot}$	$1 \times Capacity^{j,nuc,Tot}$	—	—
$Q_{min}^{th}$	—	0	—
$Q_{max}^{th,Tot}$	—	15600	—
$Q_{Tot,t}^{hyd}$	—	—	4851.6
Stock of reloading of unit $j$ (in MWh)			
$S_{reload}^{j,Tot}$	$1 \times Capacity^{j,nuc,Tot} \times 24 \times 258$	—	—

Table 2

where 258 corresponds to the number of days during which a nuclear unit can operate at full capacity.

Parameters	Nuclear	Non-nuclear thermal
$a_{nuc}^{j,Tot}$ (in Euro)	$22.79 \times Capacity^{j,nuc,Tot} \times (8760/11)$	—
$b_{nuc}$ (in Euro/MWh)	5.01	—
$a_{th}^{Tot}$ (in Euro)	—	$11.5 \times 10^7$
$b_{th}$ (in Euro/MWh)	—	26.24
$c_{th}^{Tot}$ (in Euro/MWh <sup>2</sup> )	—	$6.76 \times 10^{-7}$

Table 3

where 22.79 represents the total fixed cost of nuclear in Euro/MWh.

	Optimal inter-temporal production problem	Optimal per month production problem
Aggregated total “regularized” profit (in Euro)	$-9.147 \times 10^9$	$-15.130 \times 10^9$
Aggregated total “regularized” nuclear profit (in Euro)	$-5.957 \times 10^9$	$-11.726 \times 10^9$
Aggregated total “regularized” non-nuclear thermal profit (in Euro)	$-3.189 \times 10^9$	$-3.403 \times 10^9$

Table 4

	Social welfare maximization problem	Optimal inter-temporal production problem	Optimal per month production problem
Aggregated total cost (in Euro)	$5.209 \times 10^{10}$	$5.261 \times 10^{10}$	$5.240 \times 10^{10}$
Aggregated variable total cost (in Euro)	$1.023 \times 10^{10}$	$1.075 \times 10^{10}$	$1.054 \times 10^{10}$

Table 5

## References

- [1] J. Bushnell, (1998), "Water and Power: Hydroelectric Resources in the Era of Competition in the Western US". Power Working Paper PWP-056r. Revised version published in Operations Research, Jan/Feb 2003.
- [2] M. Soledad Arellano, (2004), "Market-power in mixed hydro-thermal electric systems", Universidad de Chile.
- [3] Yves Smeers, (2007), "How well can one measure market power in restructured electricity systems", CORE Discussion Papers No 2005050.
- [4] M. Ventosa, A. Baillo, A. Ramos, M. Rivier, (2005), "Electricity market modelling trends", Energy Policy, Vol. 33, pp. 897-913.
- [5] M. Ventosa, A. Ramos, M. Rivier, (1997), "Modeling profit maximisation in deregulated power markets by equilibrium constraints", 13th PSCC Conference, Norway. Vol 1. pp. 231-237.
- [6] R. Green, D. Newbery, (1992), "Competition in the British electricity spot market", Journal of Political Economy 100, 929-953.
- [7] Simon Taylor, (2008), "Nuclear power and deregulated electricity markets: lessons from british energy", EPRG Working Paper 0808, JEL Classification G32, L94, Q48, University of Cambridge.
- [8] Laurent Pouret and William J. Nuttall, (2007), "Can nuclear power be flexible?" EPRG Draft Working Paper, Judge Business School, University of Cambridge.
- [9] David Spector, (2007), "*Électricité : Faut-il désespérer du marché?*", Editeur: Rue d'Ulm, Collection: CEPREMAP.
- [10] Jean-Michel Glachant and Dominique Finon, (2007), "The inevitable rise of the prices of electricity in France: Is-it necessary to correct the effects of the European continental market"? Working Paper, LARSEN, Paris.
- [11] World Nuclear Association, (2006): "The nuclear fuel cycle".
- [12] Fabien Roques, (2007), "Nuclear power and the world energy outlook", International Energy Agency, 1st CeSSA Conference, "Natural Gas, Nuclear Energy and Security of Supply"-Berlin.
- [13] European Economic and Social Committee, (2004), "The issues involved in using nuclear power in electricity generation", Official Journal of the European Union, Vol. 47, 2004/C 110/14.
- [14] B. Guesdon, C. Martin, C. Miossec, (1985), "L'adaptation des réacteurs nucléaires au suivi du réseau", Energy Review, Vol. 36, No. 372, pp. 148-155.
- [15] Commission of Regulation of Energy, (2007), Report of monitoring: "The operation of the French wholesale markets of electricity and of natural gas in 2007".
- [16] DIDEME and DGEMP, "Costs of reference of the electricity production", Ministry of Economy, of Finance and of Industry, Paris, December 2003.
- [17] CEA (Centre d'Énergie Atomique), (2007), "*Elecnuc, Nuclear plants in the world*".
- [18] CEA (Centre d'Énergie Atomique), (2008), Energy Handbook, "*Memorandum on the energy*".
- [19] E. Bertel, G. Naudet, EDP Sciences (2004), Génie Atomique, "*L'économie de l'énergie nucléaire*".
- [20] Andreu Mas Colell, Michael D. Whinston, and Jerry R. Green, (1995), "*Microeconomic Theory*", Oxford University Press, New York.
- [21] M. Florenzano, C. Le Van, Springer - Verlag, (2001), "*Finite Dimensional Convexity and Optimization*".
- [22] Le Monde, (01/02/2011), Newspaper: "Le prix de l'électricité nucléaire serait proposé entre 37,5 et 38,8 euros le MWh".

- [23] Maria Lykidi, Jean-Michel Glachant, Pascal Gourdel, (2010), "Modelling the effects of nuclear fuel reservoir operation in a competitive electricity market", Working Papers 1009, Massachusetts Institute of Technology, Center for Energy and Environmental Policy Research.
- [24] Hal R. Varian, (2006), "*Intermediate microeconomics*", 7th Edition.
- [25] Charles Chapman Pugh, (2002), "*Real Mathematical Analysis*", New York: Springer.
- [26] Menahem E. Yaari, (Jan., 1987), "The dual theory of choice under risk", *Econometrica*, Volume 55, Issue 1, 95-115.
- [27] Geoff Renshaw, (2005), "*Maths for Economics*", Oxford, 2nd Edition.
- [28] M. S. Bazaraa, H. D. Sherali, and C. M. Shetty, (1993), "*Nonlinear Programming Theory and Algorithms*", John Wiley, New York, 2nd edition.
- [29] Daniel Léonard, Ngo Van Long (1992), "*Optimal Control Theory and Static Optimization in Economics*", Cambridge University Press.
- [30] Stephen Boyd, Lieven Vandenbergh, (2004) "*Convex Optimization*", Cambridge University Press.
- [31] Philippe Bich, Rida Laraki (2011), "Relaxed Nash equilibrium concepts for discontinuous games", Working Paper.
- [32] London Economics, Global Energy Decisions, (2007), "Structure and Performance of Six European Wholesale Electricity Markets in 2003, 2004 and 2005".
- [33] Jean-Marie Chevalier, (2004), "*Les grandes batailles de l'énergie*", Editions Gallimard.
- [34] Jean-Michel Glachant, Dominique Finon, (2003), "*Competition in European Electricity Markets*", Edward Elgar.
- [35] Philippe Vassilopoulos, (2007), "Les prix des marchés de gros de l'électricité donnent-ils les bons signaux et les bonnes incitations pour l'investissement en capacité de production électrique?", PhD Dissertation.
- [36] Les Echos, (20/04/2011), Newspaper: "Prix de l'électricité : l'Etat tranche en faveur d'EDF".
- [37] NEA/IEA, (2005), Projected costs of Generating Electricity: 2005 Update.
- [38] C. Crampes, M. Moreaux, (2001), "Water resource and power generation", *International journal of Industrial Organization*, Vol. 19, pp. 975-997.
- [39] C. Crampes, M. Moreaux, (2008), "Pumping water to compete in electricity markets", Institut d'Économie Industrielle (IDEI) Working Papers, Number 507, University of Toulouse.
- [40] An interdisciplinary study of MIT (2003), "The future of nuclear power".