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Arnaud Costinot
Guido Lorenzoni
Iván Werning

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**ABSTRACT**

This paper develops a simple theory of capital controls as dynamic terms-of-trade manipulation. We study an infinite horizon endowment economy with two countries. One country chooses taxes on international capital flows in order to maximize the welfare of its representative agent, while the other country is passive. We show that capital controls are not guided by the absolute desire to alter the intertemporal price of the goods produced in any given period, but rather by the relative strength of this desire between two consecutive periods. Specifically, it is optimal for the strategic country to tax capital inflows (or subsidize capital outflows) if it grows faster than the rest of the world and to tax capital outflows (or subsidize capital inflows) if it grows more slowly. In the long-run, if relative endowments converge to a steady state, taxes on international capital flows converge to zero. Although our theory emphasizes interest rate manipulation, the country's net financial position per se is irrelevant.

Arnaud Costinot
Department of Economics
MIT
50 Memorial Drive, E52-243B
Cambridge MA 02142-1347
and NBER
costinot@mit.edu

Guido Lorenzoni
Department of Economics
MIT
50 Memorial Drive, E52-251C
Cambridge, MA 02142-1347
and NBER
glorenzo@mit.edu

Iván Werning
Department of Economics
MIT
50 Memorial Drive, E51-251a
Cambridge, MA 02142-1347
and NBER
iwerning@mit.edu

# 1 Introduction

Since the end of World War II, bilateral and multilateral trade agreements have led to dramatic tariff reductions around the world, contributing to a spectacular increase in world trade; see Subramanian and Wei (2007) and Baier and Bergstrand (2007). Starting in the mid-1980*s*, the world has also experienced a dramatic increase in capital markets integration, with increased cross-border flows both across industrial countries and between industrial and developing countries; see Kose, Prasad, Rogoff, and Wei (2009). In sum, the world has experienced a dramatic increase in intratemporal and intertemporal trade, as Figure 1 illustrates.

The multilateral institutions that promote both types of trade, however, have followed two very different approaches. The primary goal of the World Trade Organization (WTO), and its predecessor the General Agreements on Tariffs and Trade, has been to reduce relative price distortions in intratemporal trade. The focus on relative price distortions and their associated terms-of-trade implications in static environments has a long and distinguished history in the international trade literature, going back to Torrens (1844) and Mill (1844). This rich history is echoed by recent theoretical and empirical work emphasizing the role of terms-of-trade manipulation in the analysis of optimal tariffs and its implication for the WTO; see Bagwell and Staiger (1999, 2011) and Broda, Limao, and Weinstein (2008).

By contrast, international efforts toward increased capital openness have emphasized the effects of capital controls on macroeconomic and financial stability. Consequently, the multilateral institutions that promote capital market integration, like the International Monetary Fund (IMF), have taken a different, more nuanced, approach to intertemporal trade, as exemplified in the recent IMF recommendations on the appropriate use of capital controls; see Ostry, Ghosh, Habermeier, Chamon, Qureshi, and Reinhardt (2010). Although the terms-of-trade effects emphasized in the international trade literature have natural implications for the analysis of optimal capital controls, these effects play little role in the existing international macro literature.

The objective of this paper is to bridge the gap between the trade approach to tariffs and the macroeconomic approach to capital controls. The basic idea is to use standard optimal tariff arguments to study taxes on international capital flows. Our objective is not to argue that the only motive for observed capital controls is the distortion of relative prices or that the removal of such distortions should be the only goal of international policy coordination. Rather, we want to develop some basic tools to think about capital controls as a form of intertemporal trade policy. For pedagogical purposes, we will develop these

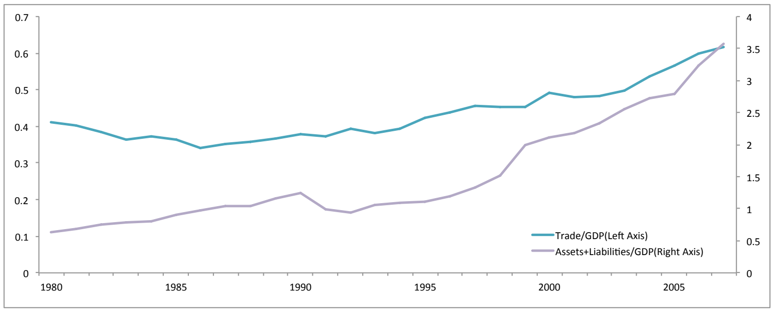


Figure 1: International Trade and Financial Integration

*Note*: The blue line represents the sum of world export and imports over world GDP (source: IMF World Economic Outlook). The purple line represents the sum of world assets and world liabilities over world GDP (source: updated and extended version of dataset constructed by Lane and Milesi-Ferretti (2007))

tools in a simple economic environment where other effects of capital controls are muted: namely, there are no effects on output levels, nominal prices and nominal exchange rates, and no effects on financial stability. Our hope is that the tools developed here can be incorporated in richer models where these other channels are active and that our analysis will help shed light on their interactions.

The starting point of our paper is that in an Arrow-Debreu economy there is no difference between intertemporal trade and intratemporal trade. In such an environment, one only needs to relabel goods by time period and the same approach used to study static terms-of-trade manipulation can be used to analyze dynamic terms-of-trade manipulation. Our results will build on this simple observation together with the time-separable structure of preferences typically used in macro applications. The only precedent that we know of our approach to intertemporal trade policy is the textbook treatment of a two-period, two-country, one-good endowment economy in Obstfeld and Rogoff (1996). By considering a richer economic environment, our analysis will not only provide more general results, but also allow us to uncover one key principle: unilaterally optimal capital controls are not guided by the absolute desire to alter the intertemporal price of goods produced in a given period, but rather by the relative strength of this desire between two consecutive periods. If a country is a net seller of goods dated $t$ and $t+1$ in equal amounts, and faces equal elasticities in both periods, there is no incentive for the country to distort the saving decisions of its consumers at date $t$. It is the time variation in the incentive to distort intertemporal prices that leads to non-zero capital controls.

Specifically, we consider an infinite horizon, two-country, one-good endowment economy. In this model the only relative prices are real interest rates. We solve for the unilaterally optimal taxes on international capital flows in one country, Home, under the assumption that the other country, Foreign, is passive.[1] In this environment, the principle described above has sharp implications for the direction of optimal capital flow taxes. In particular, it is optimal for Home to tax capital inflows (or subsidize capital outflows) in periods in which Home is growing faster than the rest of the world and to tax capital outflows (or subsidize capital inflows) in periods in which it is growing more slowly. Accordingly, if relative endowments converge to a steady state, then taxes on international capital flows converge to zero. Although our theory of capital controls emphasizes interest rate manipulation, the net financial position of Home per se, which might appear to be central in a two-period model, is shown to be irrelevant.

The intuition for our results is as follows. Consider Home's incentives to distort domestic consumption in each period. In periods of larger trade deficits, it has a stronger incentive, as a buyer, to distort prices downward by lowering domestic consumption. Similarly, in periods of larger trade surpluses, it has a stronger incentive, as a seller, to distort prices upward by raising domestic consumption. Since periods of faster growth at home tend to be associated with either lower future trade deficits or larger future trade surpluses, Home always has an incentive to raise future consumption relative to current consumption in such periods. This is exactly what taxes on capital inflows or subsidies on capital outflows accomplish through their effects on relative distortions across periods.

The next step in our paper is to extend the analysis to the case of multiple goods, thereby allowing for both intertemporal and intratemporal trade. In order to maintain the focus of our analysis on capital controls, we assume that Home can still choose its taxes on capital flows unilaterally, but that it is constrained by a free-trade agreement that prohibits good-specific taxes/subsidies in all periods. In this environment, we show that the incentive to distort trade over time does not depend only on the overall growth of the country's output relative to the world, but also on its composition. We do so in two ways. First, we establish a general formula that relates intertemporal distortions to the covariance between the price elasticities of different goods and the change in the value of home endowments. Ceteris paribus, we show that Home is more likely to raise aggregate consumption if a change in the value of home endowments is tilted towards goods whose prices are more manipulable. Second, we illustrate through a simple analytical example how such compositional issues relate to cross-country differences in demand. In a multi good world in which countries have different preferences, a change in the time profile of

[1] Throughout our analysis, we assume that the home government can freely commit at date 0 to a sequence of taxes. In the economic environment considered in this paper, this is a fairly mild assumption. As we formally establish in Section 3.4, if the home government can enter debt commitments at all maturities, as in Lucas and Stokey (1983), the optimal sequence of taxes under commitment is time-consistent.

consumption not only affects the interest rate but also the relative prices of consumption goods in each given period. This is an effect familiar from the literature on the transfer problem, which goes back to the debate between Keynes (1929) and Ohlin (1929). In our context this means that by distorting its consumers' decision to allocate spending between different periods a country also affects its static terms of trade. Even if all static trade distortions are banned by a free-trade agreement, our analysis demonstrates that intratemporal prices may not be at their undistorted levels if capital controls are allowed.

We conclude by considering the case of capital control wars in which the two countries simultaneously set taxes on capital flows optimally at date 0, taking as given the sequence of taxes chosen by the other country. In this situation, we show that results derived in the one-good case are affected in two ways. First, unlike in the case in which capital controls are set unilaterally, domestic consumption may decrease with domestic endowments, even though both countries would have liked domestic consumption to increase in the absence of policy response from the other country. Second, even if domestic consumption is increasing in domestic endowments, as in our earlier analysis, one can only relate periods of growth in one country to its taxes on capital flows relative to the other country, not their absolute level. In the simple quantitative example that we consider, we find that the net distortion on capital flows in the Nash equilibrium is larger than in the unilateral case: far from canceling each other out, capital flow taxes imposed by both countries make both countries worse off.

Our paper attacks an international macroeconomic question following a classical approach from the international trade literature and using tools from the dynamic public finance literature. In international macro, the closest literature is a growing theoretical literature demonstrating, among other things, how restrictions on international capital flows may be welfare-enhancing in the presence of various credit market imperfections; see e.g. Calvo and Mendoza (2000), Caballero and Lorenzoni (2007), Aoki, Benigno, and Kiyotaki (2010), Jeanne and Korinek (2010), and Martin and Taddei (2010). In addition to these second-best arguments, there also exists an older literature emphasizing the so-called "trilemma": one cannot have a fixed exchange rate, an independent monetary policy and free capital mobility; see e.g. McKinnon and Oates (1966), or more recently, Obstfeld, Shambaugh, and Taylor (2010). To the extent that having fixed exchange and an independent monetary policy may be welfare-enhancing, such papers offer a distinct rationale for capital controls.

On the international trade side, the literature on optimal taxes in open economies is large and varied; see Dixit (1985) for an overview. The common starting point of most trade policy papers, however, is that international trade is balanced. They therefore abstract from intertemporal considerations.[2] While one could, in principle, go from intratemporal to intertemporal trade policy by relabeling goods in an abstract Arrow-Debreu economy, existing trade policy papers typically focus on low-dimensional general equilibrium models, i.e., with only two goods. Exceptions featuring more than two goods only offer: $(i)$ partial equilibrium results under the assumption of quasi-linear preferences; $(ii)$ sufficient conditions under which seemingly paradoxical results may arise, see e.g. Feenstra (1986) and Itoh and Kiyono (1987); or $(iii)$ fairly weak restriction on the structure of optimal trade policy, see e.g. Bond (1990). Thus there are no 'off-the-shelf' results from the existing trade literature that directly apply to the dynamic environment considered in our paper.

In terms of methodology, we follow the dynamic public finance literature and use the primal approach to characterize first optimal wedges rather than explicit policy instruments; see e.g. Lucas and Stokey (1983). Since there are typically many ways to implement the optimal allocation in an intertemporal context, this approach will help us clarify the equivalence between capital controls and other policy instruments. Finally, since our theory of capital controls models one of the two governments as a dynamic monopolist optimally choosing the pattern of consumption over time, our analysis bears some resemblance to the problem of a dynamic monopolist optimally choosing the rate of extraction of some exhaustible resources; see Stiglitz (1976).

The rest of our paper is organized as follows. Section 2 describes a simple one-good economy. Section 3 characterizes the structure of optimal capital controls in this environment. Section 4 extends our results to the case of many goods. Section 5 considers the case of capital control wars. Section 6 offers some concluding remarks.

# 2 Basic Environment

## 2.1 A Dynamic Endowment Economy

There are two countries, Home and Foreign. Time is discrete and infinite, $t = 0, 1, ...$ and there is no uncertainty. The preferences of the representative consumer at home are represented by the additively separable utility function:

$$\sum_{t=0}^{\infty} \beta^t u(c_t),$$

[2] A notable exception is Bagwell and Staiger (1990), though their focus is on self-enforcing trade agreements. See Staiger (1995) for an overview of that literature.

where $c_t$ denotes consumption, $u$ is an increasing, concave function, and $\beta \in (0,1)$ is the discount factor. The preferences of the representative consumer abroad have a similar form, with asterisks denoting foreign variables.

Both domestic and foreign consumers receive an endowment sequence denoted by $\{y_t\}$ and $\{y_t^*\}$, respectively. We make two simplifying assumptions: world endowments are fixed across periods, $y_t + y_t^* = Y$, and the home and foreign consumer have the same discount factor, $\beta = \beta^*$. Accordingly, in the absence of distortions, there should be perfect consumption smoothing across time in both countries.

We assume that both countries begin with zero assets at date 0. Let $p_t$ be the price of a unit of consumption in period $t$ on the world capital markets. In the absence of taxes, the intertemporal budget constraint of the home consumer is

$$\sum_{t=0}^{\infty} p_t(c_t - y_t) \leq 0.$$

The budget constraint of of the foreign consumer is the same expression with asterisks on $c_t$ and $y_t$.

## 2.2 A Dynamic Monopolist

For most of the paper, we will focus on the case in which the home government sets taxes on capital flows in order to maximize domestic welfare, assuming the foreign government is passive: it does not have any tax policy in place and does not respond to variations in the home policy. We will look at the case where both governments set taxes strategically in Section 5.

In order to characterize the optimal policy of the home government, we follow the dynamic public finance literature and use the primal approach. That is, we approach the optimal policy problem of the home government by studying a planning problem in which equilibrium quantities are chosen directly and address implementation issues later.

Formally, we assume that the objective of the home government is to maximize the lifetime utility of the representative domestic consumer subject to $(i)$ utility maximization by the foreign consumer at (undistorted) world prices $p_t$, and $(ii)$ market clearing in each period. The foreign consumer first-order conditions are given by

$$\begin{aligned} \beta^t u^{*\prime}(c_t^*) &= \lambda^* p_t, & (1) \\ \sum_{t=0}^{\infty} p_t(c_t^* - y_t^*) &= 0, & (2) \end{aligned}$$

where $\lambda^*$ is the Lagrange multiplier on the foreign consumer's budget constraint. Moreover, goods market clearing requires

$$c_t + c_t^* = Y. \tag{3}$$

Combining equations (1)-(3), we can express the planning problem of the home government as

$$\max_{\{c_t\}} \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{$P$}$$

subject to

$$\sum_{t=0}^{\infty} \beta^t u^{*\prime}(Y - c_t)(c_t - y_t) = 0. \tag{4}$$

Equation (4) is an implementability constraint, familiar from the optimal taxation literature. Note that given a sequence of domestic consumption, condition (4) is also sufficient to ensure the existence of a feasible, utility-maximizing consumption sequence for Foreign. The argument is constructive: given $\{c_t\}$, the proposed sequence $\{c_t^*\}$ is obtained from market clearing (3) and the sequence of prices is computed from (1), so that (2) is implied by (4), ensuring that the foreign consumer's sufficient conditions for optimality are met.

# 3 Optimal Capital Controls

## 3.1 Optimal Allocation

We first describe how home consumption $\{c_t\}$ fluctuates with home endowments $\{y_t\}$ along the optimal path. Next we will show how the optimal allocation can be implemented using taxes on international capital flows.

The first-order condition associated with Home's planning problem is given by

$$u'(c_t) = \mu \left[ u^{*\prime}(Y - c_t) - u^{*\prime\prime}(Y - c_t)(c_t - y_t) \right], \tag{5}$$

where $\mu$ is the Lagrange multiplier on the implementability constraint. This condition immediately leads to our first observation. Although the entire sequence $\{y_t\}$ affects the level of current consumption through their effects on the Lagrange multiplier $\mu$, we see that variations in current consumption $c_t$ along the optimal path only depend on variations in the current value of $y_t$.

The next proposition shows that, whatever the shape of the utility functions $u$ and $u^*$,

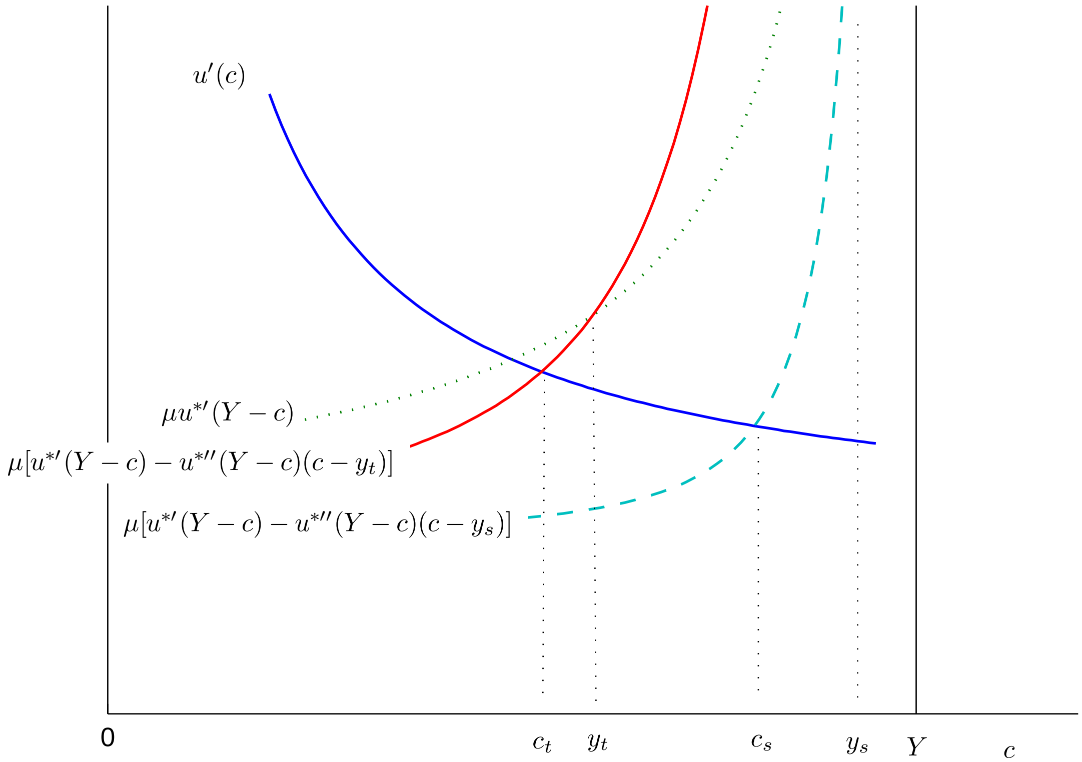


Figure 2: Consumption is Procyclical Along the Optimal Path

there is a monotonic relationship between domestic consumption and domestic endowments along the optimal path.

**Proposition 1** *(Procyclical consumption) For any two periods $t$ and $s$, if the home endowment is larger in $s$, $y_s > y_t$, then the home consumption is also higher, $c_s > c_t$.*

Figure 2 provides a graphical representation of Home's planning problem. On the x-axis we have domestic consumption $c$, which determines foreign consumption, $Y - c$, by market clearing. The decreasing blue curve labeled $u'(c)$ represents the marginal utility of the home consumer. The increasing red curve represents the marginal revenue associated with reducing consumption at home by a small amount and thus $(i)$ increasing net sales, and $(ii)$ decreasing the intertemporal price of the $t$-dated good. The optimal consumption choice at $t$ corresponds to the point where the two curves meet, that is, where marginal revenue is equalized with the marginal cost of reducing home consumption, which is just $u'(c)$. The same reasoning applies in period $s$, but with the marginal revenue represented by the light blue dashed curve.

Figure 2 gives an intuition for Proposition 1. As the endowment increases from $y_t$ to $y_s$, the curve $u'(c)$ does not move. At the same time, the marginal revenue curve shifts down, as the price decrease associated to a reduction in $c$ applies to a larger amount of inframarginal units sold. This induces Home to consume more, explaining why consumption is procyclical along the optimal path.

As a preliminary step in the analysis of optimal capital flow taxes, we conclude this section by describing how the "wedge" between the marginal utility of domestic and foreign consumption varies along the optimal path. Formally, define

$$\tau_t \equiv \frac{u'(c_t)}{\mu u^{*\prime}(c_t^*)} - 1. \tag{6}$$

By market clearing, we know that $c_t^* = Y - c_t$. Thus combining the definition of $\tau_t$ with the strict concavity of $u$ and $u^*$, we obtain the following corollary to Proposition 1.

**Corollary 1** *(Countercyclical wedges) For any two periods $t$ and $s$, if the home endowment is larger in $s$, $y_s > y_t$, then the wedge is lower, $\tau_s < \tau_t$.*

The existing trade policy literature provides a useful way to interpret Corollary 1. By equations (5) and (6), we have

$$\tau_t = -\frac{u^{*\prime\prime}(Y - c_t)}{u^{*\prime}(Y - c_t)}(c_t - y_t). \tag{7}$$

Condition (7) is closely related to the well-known optimal tariff formula involving the elasticity of the foreign export supply curve in static trade models with two goods and/or quasi-linear preferences. This should not be too surprising since $\tau_t$ measures the difference between the marginal utility of domestic and foreign consumption. According to equation (7), the wedge $\tau_t$ is positive in periods of trade deficit and negative in periods of trade surplus. This captures the idea that if (time-varying) trade taxes were available, Home would like to tax imports if $c_t - y_t > 0$ and tax exports if $c_t - y_t < 0$. Corollary 1, however, goes beyond this simple observation by establishing a monotonic relationship between $\tau_t$ and $y_t$. This insight will play a key role in our analysis of optimal capital controls, to which we now turn.[3]

[3] A natural question is whether the same logic implies a monotonic relationship between $\tau_t$ and net imports, $c_t - y_t$. Perhaps surprisingly, this is not necessarily the case. While the sign of the slopes of $u'(c)$ and $\mu u^{*\prime}(Y - c)$ is unambiguous (by concavity) the sign of the slope of the marginal return curve could in general be positive or negative. This opens up the possibility that a positive output shock may lead to an increase in import volumes, and so by Proposition 1, that an increase in import volumes may be accompanied by a decrease in $\tau_t$ along the optimal path.

## 3.2 Optimal Taxes on International Capital Flows

It is well-known from the Ramsey taxation literature that there are typically many combinations of taxes that can implement the optimal allocation; see e.g. Chari and Kehoe (1999). Here, we focus on the tax instrument most directly related to world interest rate manipulation: taxes on international capital flows.[4]

For expositional purposes, we assume that consumers can only trade one-period bonds on international capital markets, with the home government imposing a proportional tax $\theta_t$ on the gross return on net asset position in international bond markets. Standard arguments show that any competitive equilibrium supported by intertemporal trading of consumption claims at date 0 can be supported by trading of one-period bonds. As we discuss later in Section 3.4, none of the results presented here depend on the assumption that one-period bonds are the only assets available.

With only one-period bonds, the per-period budget constraint of the home consumer takes the form

$$q_t a_{t+1} + c_t = y_t + (1-\theta_t)\, a_t - l_t, \tag{8}$$

where $a_t$ denotes the current bond holdings, $l_t$ is a lump sum tax, and $q_t \equiv p_{t+1}/p_t$ is the price of one-period bonds at date $t$. In addition, consumers are subject to a standard no-Ponzi condition, $\lim_{t\to\infty} p_t a_t \geq 0$. In this environment the home consumer's Euler equation takes the form

$$u'(c_t) = \beta(1-\theta_t)(1+r_t)u'(c_{t+1}). \tag{9}$$

where $r_t \equiv 1/q_t - 1$ is the world interest rate. Given a solution $\{c_t\}$ to Home's planning problem ($P$), the world interest rate is uniquely determined as

$$r_t = \frac{u^{*\prime}(Y-c_t)}{\beta u^{*\prime}(Y-c_{t+1})} - 1,$$

by equations (1) and (3). Thus, given $\{c_t\}$, we can use (9) to construct a unique sequence of taxes $\{\theta_t\}$. We can then set the sequence of assets positions and lump-sum transfers

$$\begin{aligned} a_t &= \sum_{s=t}^{\infty} (p_s/p_t)\,(c_s - y_s)\,, \\ l_t &= -\theta_t a_t, \end{aligned}$$

[4]Other tax instruments that could be used to implement the optimal allocation include time-varying trade and consumption taxes (possibly accompanied by production taxes in more general environments). See Jeanne (2011) for a detailed discussion of the equivalence between capital controls and trade taxes.

which ensures that the per-period budget constraint (8) and the no-Ponzi condition are satisfied. Since (8), (9), and the no-Ponzi condition are sufficient for optimality it follows that given prices and taxes, $\{c_t\}$ is optimal for the home consumer. This establishes that any solution $\{c_t\}$ of ($P$) can be decentralized as a competitive equilibrium with taxes.

A positive $\theta_t$ can be interpreted as imposing simultaneously a tax $\theta_t$ on capital outflows and a subsidy $\theta_t$ to capital inflows. Obviously, since there is a representative consumer, only one of the two is active in equilibrium: the outflow tax if the country is a net lender, $a_t > 0$, and the inflow subsidy if it is a net borrower, $a_t < 0$. Similarly, a negative $\theta_t$ can be interpreted as a subsidy on capital outflows plus a tax on capital inflows. The bottom line is that $\theta_t > 0$ discourages domestic savings while $\theta_t < 0$ encourages them.

In Foreign, since there are no capital flow taxes, the Euler equation takes the form

$$u^{*\prime}(c_t^*) = \beta(1 + r_t)u'(c_{t+1}^*). \tag{10}$$

Combining the definition of the wedge (6) with the Euler equations at home and abroad (9) and (10), we obtain the following relationship between wedges and taxes on capital flows:

$$\theta_t = 1 - \frac{1 + \tau_t}{1 + \tau_{t+1}}. \tag{11}$$

The previous subsection has already established that variations in domestic consumption $c_t$ along the optimal path are only a function of the current endowment $y_t$. Since $\tau_t$ is only a function of $c_t$, equation (11) implies that variations in $\theta_t$ are only a function of $y_t$ and $y_{t+1}$. Combining equation (11) with Corollary 1, we then obtain the following result about the structure of optimal capital controls.

**Proposition 2** *(Optimal capital flow taxes) Suppose that the optimal policy is implemented with capital flows taxes. Then it is optimal:*

1. *to tax capital inflows/subsidize capital outflows ($\theta_t < 0$) if $y_{t+1} > y_t$;*

2. *to tax capital outflows/subsidize capital inflows ($\theta_t > 0$) if $y_{t+1} < y_t$;*

3. *not to distort capital flows ($\theta_t = 0$) if $y_{t+1} = y_t$.*

Proposition 2 builds on the same logic as Proposition 1. Suppose, for instance, that Home is running a trade deficit in periods $t$ and $t + 1$. In this case, the home government wants to exercise its monopsony power by lowering domestic consumption in both periods. But if Home grows between these two periods, $y_{t+1} > y_t$, the number of units imported from abroad is lower in period $t+1$. Thus the home government has less incentive to lower consumption in that period. This explains why a tax on capital inflows is optimal in period $t$: it reduces borrowing in period $t$, thereby shifting consumption from period $t$ to period $t+1$. The other results follow a similar logic.

It is worth emphasizing that, although the only motive for capital controls in our model is interest rate manipulation, the net financial position of Home in any particular period, debtor or creditor, is not the relevant variable to look at to sign the optimal direction of the tax. This is because the effect of a capital flow tax is to affect the relative distortion in consumption decisions between two consecutive periods. Therefore, what matters is whether the monopolistic/monopsonistic incentives to restrict domestic consumption are stronger in period $t$ or $t+1$. In our simple endowment economy, these incentives are purely captured by the growth rate of the endowment, but the same broad principle would extend to more general environments.

Proposition 2 has a number of interesting implications. Consider first an economy that is catching up with the rest of the world in the sense that $y_{t+1} > y_t$ for all $t$. According to our analysis, it is optimal for this country to tax capital inflows and to subsidize capital outflows. The basic intuition is that a growing country will export more tomorrow than today. Thus it has more incentive to increase export prices in the future, which it can achieve by raising future consumption through a subsidy on capital outflows. For an economy catching up with the rest of the world, larger benefits from future terms-of-trade manipulation are associated with taxes and subsidies that encourage domestic savings.

Consider instead a country that at time $t$ borrows from abroad in anticipation of a temporary boom. In particular, suppose that $y_{t+1} > y_t$ and $y_s = y_t$ for all $s > t+1$. In this situation, the logic of Proposition 2 implies that, at time $t$, at the onset of the boom, it is optimal to impose restrictions on short-term capital inflows, i.e., to tax bonds with one-period maturity and leave long-term capital inflows unrestricted.[5] This example provides a different perspective on why governments may try to alter the composition of capital flows in favor of longer maturity flows in practice; see Magud, Reinhart, and Rogoff (2011). In our model, incentives to alter the composition of capital flows do not come from the fear of "hot money" but from larger benefits of terms-of-trade manipulation in the short run.

Finally, Proposition 2 has sharp implications for the structure of optimal capital controls in the long-run.

[5]The tax on two period bonds is easily shown to be $(1-\theta_t)(1-\theta_{t+1})-1$ and Proposition 1 implies that it is zero in our example.

**Corollary 2** *(No tax in steady state) In the long run, if endowments converge to a steady state, $y_t \to y$, then taxes on international capital flows converge to zero, $\theta_t \to 0$.*

Corollary 2 is reminiscent of the Chamley-Judd result (Judd, 1985; Chamley, 1986) of zero capital income tax in the long-run. Intuitively, the home government would like to use its monopoly power to influence intertemporal prices to favor the present value of its income. However, at a steady state all periods are symmetric, so it is not optimal to manipulate relative prices. Note that a steady state may be reached with trade balance, trade deficit or trade surplus. Which of these cases applies depends on the entire sequence $\{y_t\}$. Our analysis demonstrates that taxes on international capital flows are unaffected by these long-run relative wealth dynamics. For instance, even if Home, say, becomes heavily indebted, it is not optimal to lower long run interest rates. In our model, even away from a steady state, taxes on international capital flows are determined by the endowments at $t$ and $t+1$ only.

## 3.3 An Example with CRRA Utility and Aggregate Fluctuations

Up to now we have focused on the case of a fixed world endowment. Thus we have looked at how optimal capital controls respond to a reallocation of resources between countries, keeping the total pie fixed. This provides a useful benchmark in which all fluctuations in consumption reflect the incentives of the home government to manipulate the world interest rate. Here we show that if domestic and foreign consumers have identical CRRA utility functions, then our results extend to economies with aggregate fluctuations. We also take advantage of this example for a simple exploration of the magnitudes involved with optimal capital controls in terms of quantities and welfare.

Our characterization of the optimal policy of the home government extends immediately to the case of a time-varying world endowment: one just needs to replace $Y$ with $Y_t$ in equation (5). Under the assumption of identical CRRA utility functions, $u(c) = u^*(c) = c^{1-\gamma}/\,(1-\gamma)$ with $\gamma \geq 0$, this leads to a simple relationship between the home share of world endowments, $y_t/Y_t$, and the home share of world consumption, $c_t/Y_t$:

$$\left(\frac{c_t/Y_t}{1-c_t/Y_t}\right)^{-\gamma} = \mu\left[1+\gamma\left(\frac{c_t/Y_t - y_t/Y_t}{1-c_t/Y_t}\right)\right].$$

The left-hand side is decreasing in $c_t/Y_t$, whereas the right-hand side is increasing in $c_t/Y_t$ and decreasing in $y_t/Y_t$. Thus the implicit function theorem implies that, along the optimal path, the home share of world consumption, $c/Y$, is strictly increasing in

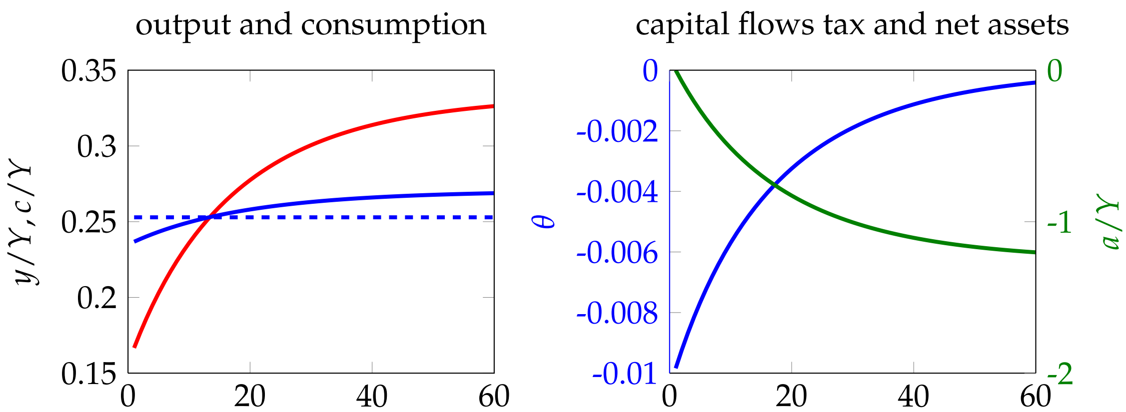


Figure 3: Optimal Allocation and Taxes for the Country Catching Up

*Note*: In the left panel, the red line is the exogenous path for the endowment, the blue line is consumption at the optimal policy of the home government, and the dashed line is the efficient no-tax benchmark. In the right panel, the blue line is the capital flow tax and the green line the home assets-to-world-GDP ratio.

the home share of world endowments, $y/Y$. Put simply, if utility functions are CRRA, Proposition 1 generalizes to environments with aggregate fluctuations.

Now consider the wedge $\tau_t$ between the marginal utility of domestic and foreign consumption in period $t$. Under the assumption of CRRA utility functions we have

$$\tau_t = \frac{1}{\mu}\left(\frac{c_t/Y_t}{1-c_t/Y_t}\right)^{-\gamma} - 1.$$

According to this expression, if $c/Y$ is strictly increasing in $y/Y$ along the optimal path, then $\tau$ is strictly decreasing. The same logic as in Section 3.2 therefore implies that optimal taxes on capital flows must be such that $\theta_t < 0$ if and only if $y_{t+1}/Y_{t+1} > y_t/Y_t$. In other words, if utility functions are CRRA, Proposition 2 also generalizes to environments with aggregate fluctuations.

As a quantitative illustration of our theory of capital controls as dynamic terms-of-trade manipulation, suppose that foreign endowments $\{y_t^*\}$ are growing at the constant rate $g = 3\%$ per year and that Home is catching up with the rest of the world. To be more specific, suppose that the home endowment is 20% of world endowments at date 0 and that it is converging towards being 50% in the long run, with the ratio $y_t/y_t^*$ converging to its long run value at a constant speed $\eta = 0.05$.[6]

[6]That is, we assume that

$$y_t/y_t^* - a = (y_0/y_0^* - a)\, e^{-\eta t},$$

with $a = 0.5/0.5 = 1 > y_0/y_0^* = 0.2/0.8 = 0.25$.

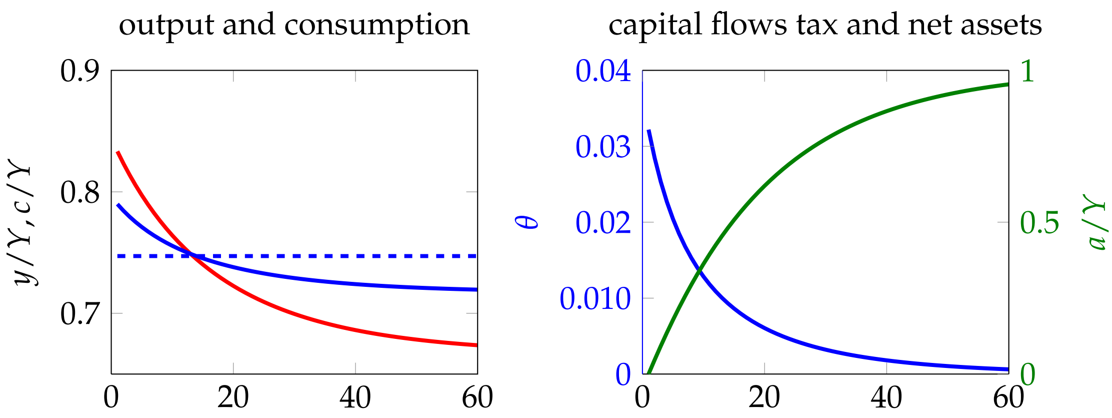


Figure 4: Optimal Allocation and Taxes for the Rest of the World

*Note*: In the left panel, the red line is the exogenous path for the endowment, the blue line is consumption at the foreign optimal policy, and the dashed line is the efficient no-tax benchmark. In the right panel, the blue line is the capital flow tax and the green line the foreign assets-to-world-GDP ratio.

Figure 3 shows the path of the home share of world endowments and consumption, assuming a unit elasticity of intertemporal substitution, $\gamma = 1$. For comparison, we also plot the path for consumption in the benchmark case with no capital controls. In this case, consistently with consumption smoothing, Home consumes a fixed fraction of the world endowment in all periods. Although optimal capital controls reduce consumption smoothing, intertemporal trade flows are several times larger than domestic output. The optimal tax on capital inflows is less than 1% at date 0 and vanishing in the long run, following the same logic as for Corollary 2. Perhaps surprisingly, the optimal tax on capital inflows decreases as the value of the home debt increases. Compared to the benchmark with no capital controls, optimal taxes are associated with an increase in domestic consumption of 0.12% and a decrease in foreign consumption of 0.07%. Though the welfare impact of optimal capital controls is admittedly not large in this particular example, it is not much smaller than either the estimated gains of international trade or financial integration.[7]

Figure 4 considers the same exogenous path for endowments, but assumes that it is the foreign country, instead of the home country, that is imposing capital flow taxes optimally. Formally, the home share of world endowments is now 80% at date 0 and is

[7]According to a fairly large class of trade models, the welfare gains from international trade in the United States are between 0.7% and 1.4% of real GDP; see Arkolakis, Costinot, and Rodríguez-Clare (2009). Similarly, the welfare gains from switching from financial autarky to perfect capital mobility is roughly equivalent to a 1% permanent increase in consumption for the typical non-OECD country; see Gourinchas and Jeanne (2006).

converging towards being 50% in the long-run. In this situation, we see that optimal taxes on international capital flows are about three times as large as in the previous case, reflecting the greater size of the rest of the world, and hence its greater ability to manipulate world interest rates. The impact on welfare is larger as well. In this second example, domestic consumption increases by 0.27% compared to the benchmark with no capital controls, whereas foreign consumption decreases by 1.27%. With higher values of $\gamma$, i.e. lower intertemporal elasticity of substitution, we obtain similar though slightly higher numbers. For instance, for $\gamma = 2$, the increase in domestic consumption is equal to 0.48% and the decrease in foreign consumption to 2.5%.[8]

## 3.4 Initial Assets, Debt Maturity, and Time-Consistency

So far, we have focused on environments in which: $(i)$ there are no initial assets at date 0 and $(ii)$ one-period bonds are the only assets available. We now briefly discuss how relaxing both assumptions affects our results. We also show that if more debt instruments are available, the optimal allocation is time-consistent: a future government free to choose future consumption, but forced to fulfill previous debt obligations would not want to deviate from the consumption path chosen by its predecessors.

Let $a_{t,s}$ represents holdings at time $t$ of bonds maturing at time $s$. Suppose the home consumer enters date 0 with initial asset holdings $\{a_{0,t}\}_{t=0}^{\infty}$. The asset holdings now enter the intertemporal budget constraints of the home and foreign consumers. In particular, the budget constraint of the foreign consumer generalizes to

$$\sum_{t=0}^{\infty} p_t (c_t^* - y_t^* - a_{0,t}^*) = 0,$$

where $a_{0,t}^* = -a_{0,t}$ denotes initial asset holdings abroad. The other equilibrium conditions are unchanged, so Home's planning problem becomes

$$\max_{\{c_t\}} \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{$P_0$}$$

subject to

$$\sum_{t=0}^{\infty} \beta^t u^{*\prime}(Y - c_t)(c_t - y_t + a_{0,t}^*) = 0. \tag{12}$$

[8]In general, changing $\gamma$ has non-monotonic effects on the welfare impact of optimal capital controls. Intuitively, since Home and Foreign have the same intertemporal elasticity of substitution, changing $\gamma$ affects both the marginal revenue and the marginal cost of a dynamic monopolist, as described in Figure 2.

Compared to the case without initial assets, the only difference is the new implementability constraint (12) that depends on $\{y_t - a^*_{0,t}\}$ rather than $\{y_t\}$. Accordingly, Proposition 1 and Corollary 1 simply generalize to environments with initial assets $\{a^*_{0,t}\}_{t=0}^{\infty}$ provided that they are restated in terms of changes in $y_t - a^*_{0,t}$ rather than changes in $y_t$.

Throughout our analysis, we have assumed that the home government can freely commit at date 0 to a consumption path $\{c_t\}$. Now that we have recognized the role of the initial asset positions, this assumption may seem uncomfortably restrictive. After all, along the optimal path, the debt obligations $\{a^*_{t,s}\}_{s=t}^{\infty}$ held at date $t$ will typically be different from the obligations $\{a^*_{0,s}\}_{s=t}^{\infty}$ held at date 0. Accordingly, a government at later dates may benefit from deviating from the consumption chosen at date 0.

We now demonstrate that this is not the case if the government has access to bonds of arbitrary maturity. The basic idea builds on the original insight of Lucas and Stokey (1983). At any date $t$, the foreign consumer is indifferent between many future asset holdings $\{a^*_{t+1,s}\}_{s=t+1}^{\infty}$. Given a consumption sequence $\{c^*_t\}$ that maximizes her utility subject to her budget constraint, she is indifferent between any bond holdings satisfying

$$\sum_{s=t+1}^{\infty} p_s(c^*_s - y^*_s - a^*_{t+1,s}) = 0. \tag{13}$$

As we show in the appendix, this degree of freedom is sufficient for the home government at date 0 to construct sequences of debt obligations $\{a^*_{t,s}\}_{s=t}^{\infty}$ for all $t \geq 1$ such that the solution of

$$\max_{\{c_s\}} \sum_{s=t}^{\infty} \beta^s u(c_s)$$

subject to

$$\sum_{s=t}^{\infty} \beta^s u^{*\prime}(Y - c_s)(c_s - y_s + a^*_{t,s}) = 0 \tag{14}$$

coincides with the solution of ($P_0$) at all dates $t \geq 0$. In short, if the home government can enter debt commitments at all maturities, the optimal allocation derived in Section 3.1 is time-consistent.

# 4 Intertemporal and Intratemporal Trade

How do the incentives to tax capital flows change in a world with many goods? In a one-good economy, the only form of terms-of-trade manipulation achieved by taxing capital flows is to manipulate the world interest rate. In a world with many goods, distorting the borrowing and lending decisions of domestic consumers also affects the relative prices of the different goods traded in each period. In this section, we explore how these new intratemporal considerations change optimal capital flow distortions.

In order to maintain the focus of our analysis on optimal capital controls, we proceed under the assumption that Home is constrained by an international free-trade agreement that prohibits good specific taxes/subsidies in all periods. As in the previous section, Home is still allowed to impose taxes on capital flows that distort intertemporal decisions. This means that while Home cannot control the path of consumption of each specific good $i$, it can still control the path of aggregate consumption. As we shall see, in general, the path of aggregate consumption can affect relative prices at any point in time, thus creating additional room for terms-of-trade manipulation.

## 4.1 The Monopolist Problem Revisited

The basic environment is the same as in Section 2.1, except that there are $n > 1$ goods. Thus domestic consumption and output, $c_t$ and $y_t$, are now vectors in $R^n_+$. We assume that the domestic consumer has additively separable preferences represented by

$$\sum_{t=0}^{\infty} \beta^t U(C_t),$$

where $U$ is increasing and strictly concave, $C_t \equiv g(c_t)$ is aggregate domestic consumption at date $t$, and $g$ is increasing, concave, and homogeneous of degree one. Analogous definitions apply to $U^*$ and $C_t^* \equiv g^*(c_t^*)$.

In the absence of taxes, the intertemporal budget constraint of the home consumer is now given by

$$\sum_{t=0}^{\infty} p_t \cdot (c_t - y_t) \leq 0,$$

where $p_t \in R^n_+$ denotes the intertemporal price vector for period-$t$ goods and $\cdot$ is the inner product. A similar budget constraint applies in Foreign.

As in Section 2.2, we use the primal approach to characterize the optimal policy of the home government. In this new environment, the home government's objective is to set consumption $\{c_t\}$ in order to maximize the lifetime utility of its representative consumer subject to $(i)$ utility maximization by the foreign representative consumer at (undistorted) world prices $p_t$; $(ii)$ market clearing in each period; and $(iii)$ a free trade agreement that rules out good specific taxes or subsidies.

Constraint $(i)$ can be dealt with as we did in the one-good case. In vector notation, the first-order conditions associated with utility maximization by the foreign consumer generalize to

$$\beta^t U^{*\prime}(C_t^*)\, g_c^*(c_t^*) = \lambda^* p_t, \tag{15}$$

$$\sum_{t=0}^{\infty} p_t \cdot (c_t^* - y_t^*) = 0. \tag{16}$$

Next, note that if Home cannot impose good specific taxes or subsidies, the relative price of any two goods $i$ and $j$ in period $t$, $p_{it}/p_{jt}$, must be equal in the two countries and equal to the marginal rates of substitution $g_i(c_t)/g_j(c_t)$ and $g_i^*(c_t^*)/g_j^*(c_t^*)$. Accordingly, the consumption allocation $(c_t, c_t^*)$ in any period $t$ is Pareto efficient and solves

$$C^*(C_t) = \max_{c,c^*}\{g^*(c^*) \qquad \text{subject to} \quad c + c^* = Y \text{ and } g(c) \geq C_t\} \tag{17}$$

for some $C_t$. Therefore, constraints $(ii)$ and $(iii)$, can be captured by letting Home choose an aggregate consumption level $C_t$, which identifies a point on the static Pareto frontier. The consumption vectors at time $t$ are then given by the corresponding solutions to problem (17), which we denote by $c(C_t)$ and $c^*(C_t)$.

We can then state Home's planning problem in the case of many goods as

$$\max_{\{C_t\}} \sum_{t=0}^{\infty} \beta^t U(C_t) \tag{P'}$$

subject to

$$\sum_{t=0}^{\infty} \beta^t \rho(C_t) \cdot (c(C_t) - y_t) = 0, \tag{18}$$

where $\rho(C_t) \equiv U^{*\prime}(C^*(C_t))\, g_c^*(c^*(C_t))$ and equation (18) is the counterpart of the implementability constraint in Section 2.2.

## 4.2 Optimal Allocation

With many goods, the first-order condition associated with Home's planning problem generalizes to

$$U'(C_t) = \mu \left[\rho(C_t) \cdot \frac{\partial c(C_t)}{\partial C_t} + \frac{\partial \rho(C_t)}{\partial C_t} \cdot (c(C_t) - y_t)\right], \tag{19}$$

where $\mu$ still denotes the Lagrange multiplier on the implementability constraint. Armed with condition (19), we can now follow the same strategy as in the one-good case. First we will characterize how $\{C_t\}$ covaries with $\{y_t\}$ along the optimal path. Second we will derive the associated implications for the structure of optimal capital controls.

The next proposition describes the relationship between domestic consumption and domestic endowments along the optimal path.

**Proposition 3** *(Procyclical aggregate consumption) Suppose that between periods $t$ and $t+1$ there is a small change in the home endowment $dy_{t+1} = y_{t+1} - y_t$. Then the home consumption is higher in period $t+1$, $C_{t+1} > C_t$, if and only if $\partial\rho(C_t)/\partial C_t \cdot dy_{t+1} > 0$.*

In the one good case, $\partial\rho(C_t)/\partial C_t$ simplifies to $-u^{*\prime\prime}(Y - c_t)$, which is positive by the concavity of $u^*$. Therefore, whether domestic consumption grows or not only depends on whether the level of domestic endowments is increasing or decreasing. In the multi-good case, by contrast, this also depends on the composition of domestic endowments and on how relative prices respond to changes in $C_t$.

In order to highlight the importance of these compositional effects, in an economy with many goods, consider the effect of a small change in domestic endowment that leaves its market value unchanged at period $t$ prices. That is, suppose $\rho(C_t) \cdot dy_{t+1} = 0$. In the one good case this can only happen if the endowment level does not change, thereby leading to a zero capital flow tax. In the multi-good case this is no longer true. According to Proposition 3, consumption would grow if and only if

$$\mathrm{Cov}\left(\frac{\rho_i'(C_t)}{\rho_i(C_t)}, \rho_i(C_t) dy_{it+1}\right) > 0.$$

Here, what matters is whether the composition of endowments tilts towards goods that are more or less price sensitive to changes in $C_t$. We will come back to the role of this compositional effects in more detail in Section 4.4.

## 4.3 Optimal Taxes on International Capital Flows

In line with Section 3.2, let us again assume that consumers can only trade one-period bonds on international capital markets. But compared to Section 3.2, suppose now that there is one bond for each good. Since the home government cannot impose good specific taxes/subsidies, it must impose the same proportional tax $\theta_t$ on the gross return on net lending in all bond markets. So the per period budget constraint of the domestic consumer takes the form

$$p_{t+1} \cdot a_{t+1} + p_t \cdot c_t = p_t \cdot y_t + (1 - \theta_t)\,(p_t \cdot a_t) - l_t,$$

where $a_t \in R^n_+$ now denotes the vector of current asset positions and $l_t$ is a lump sum tax. As before, the domestic consumer is subject to the no-Ponzi condition, $\lim_{t\to\infty} p_t \cdot a_t \geq 0$. The first-order conditions associated with utility maximization at home are given by

$$U'(C_t)\, g_i(c_t) = \beta(1-\theta_t)(1+r_{it})U'(C_{t+1})g_i(c_{t+1}), \text{ for all } i = 1, ..., n. \tag{20}$$

where $r_{it} \equiv p_{it}/p_{it+1} - 1$ is a good-specific interest rate. Let $P_t \equiv \min_c \{p_t \cdot c : g(c) \geq 1\}$ denote the home consumer price index at date $t$. Using this notation, the previous conditions can be rearranged in a more compact form as

$$U'(C_t) = \beta(1-\theta_t)(1+R_t)U'(C_{t+1}), \tag{21}$$

where $R_t \equiv P_t/P_{t+1} - 1$ is the home real interest rate at date $t$.[9] Since there are no taxes abroad, the same logic implies

$$U^{*\prime}(C^*_t) = \beta(1+R^*_t)U^{*\prime}(C^*_{t+1}), \tag{22}$$

where $R^*_t \equiv P^*_t/P^*_{t+1} - 1$ is the foreign real interest rate at date $t$. Equations (21) and (22) are the counterparts of the Euler equations (9) and (10) in the one-good case. Combining these two expressions we obtain

$$\theta_t = 1 - \frac{U'(C_t)}{U^{*\prime}(C^*_t)} \frac{U^{*\prime}(C^*_{t+1})}{U'(C_{t+1})} \frac{(1+R^*_t)}{(1+R_t)}.$$

If we follow the same approach as in the one-good case and let $\tau_t \equiv U'(C_t)/\mu U^{*\prime}(C^*_t) - 1$ denote the wedge between the marginal utility of domestic and foreign consumption, we can rearrange Home's tax on international capital flows as

$$\theta_t = 1 - \left(\frac{1+\tau_t}{1+\tau_{t+1}}\right)\left(\frac{P_{t+1}/P^*_{t+1}}{P_t/P^*_t}\right).$$

With many goods, the sign of $\theta_t$ depends on $(i)$ whether the wedge $\tau_t$ between the marginal utility of domestic and foreign consumption is increasing or decreasing and $(ii)$ whether Home's real exchange rate, $P_t/P^*_t$, appreciates or depreciates between $t$ and $t+1$. Like in the one-good case, one can check that the wedge is a decreasing function of home aggregate consumption $C_t$. In the next proposition we further demonstrate that an increase in $C_t$ is always associated with an appreciation of Home's real exchange rate. Combining

[9] In the proof of Proposition 4 in the Appendix, we formally establish that $P_t = p_{it}/g_i(c_t)$ for all $i = 1, ..., n$. Equation (21) directly derives from this observation and equation (20).

these two observations with Proposition 3, we obtain the following result.

**Proposition 4** *(Optimal capital flow taxes revisited) Suppose that the optimal policy is implemented with capital flow taxes and that between periods $t$ and $t+1$ there is a small change in the home endowment $dy_{t+1} = y_{t+1} - y_t$. Then it is optimal:*

1. *to tax capital inflows/subsidize capital outflows ($\theta_t < 0$) if $\partial\rho(C_t)/\partial C_t \cdot dy_{t+1} > 0$;*
2. *to tax capital outflows/subsidize capital inflows ($\theta_t > 0$) if $\partial\rho(C_t)/\partial C_t \cdot dy_{t+1} < 0$;*
3. *not to distort capital flows ($\theta_t = 0$) if $\partial\rho(C_t)/\partial C_t \cdot dy_{t+1} = 0$.*

In order to understand better how intertemporal and intratemporal considerations affect the structure of the optimal tax schedule, let us decompose the price vector in period $t$ into an intertemporal price and an intratemporal vector of relative prices: $p_t = P_t^* \pi_t$, where $\pi_{it} \equiv p_{it}/P_t^*$ denotes the price of good $i$ in terms of foreign consumption at date $t$.

Using the previous decomposition, we see that the sign of the expression $\partial\rho(C_t)/\partial C_t \cdot dy_{t+1}$ in Proposition 4 is the same as the sign of the following expression[10]

$$\frac{P^{*\prime}(C_t)}{P^*(C_t)} \sum_i \pi_i(C_t)\, dy_{it+1} + \sum_i \frac{\pi_i'(C_t)}{\pi_i(C_t)} \pi_i(C_t)\, dy_{it+1}. \tag{23}$$

The first term captures the intertemporal price channel and is proportional to the change in the value of output. It is possible to show that $P^{*\prime}(C_t) > 0$. Thus an increase in the value of home output—all else equal—pushes in the direction of a tax on capital inflows/subsidy to capital outflows. This follows the same logic as in the one-good case.

The new element is the second term in (23), which captures intratemporal terms-of-trade effects. The sign of this term depends on the elasticity of relative prices to changes in domestic consumption. To sign this term we need to know more about preferences. The simplest case is the case of symmetric preferences in which $g$ and $g^*$ are the same. In that case, the Pareto set in the Edgeworth box is a straight line and relative prices are independent of the point we choose (i.e., of $C_t$). Not surprisingly, in this case the analysis boils down to the one-good case. Therefore, the interesting case is the case of asymmetric preferences, which we now turn to.

[10] Just notice that

$$\rho_t = \lambda^* \beta^{-t} p_t = \lambda^* \beta^{-t} P_t^* \pi_t,$$

from the optimality condition of the foreign consumer and so

$$\frac{\rho_i'(C_t)}{\rho_i(C_t)} = \frac{P^{*\prime}(C_t)}{P^*(C_t)} + \frac{\pi_i'(C_t)}{\pi_i(C_t)}.$$

## 4.4 An Example with CRRA and Asymmetric Cobb-Douglas Utility

In this subsection we focus on a simple example in which the effects of intratemporal considerations can be captured analytically. There are two goods. The upper-level utility function at home is CRRA and the lower-level utility is Cobb-Douglas:

$$U(C) = \frac{1}{1-\gamma} C^{1-\gamma}, C = c_1^{\alpha} c_2^{1-\alpha}, \tag{24}$$

where $\gamma \geq 0$ and $\alpha > 1/2$. Foreign utility functions take the same form, but the roles of goods 1 and 2 are reversed

$$U^*(C^*) = \frac{1}{1-\gamma} (C^*)^{1-\gamma}, C^* = (c_2^*)^{\alpha} (c_1^*)^{1-\alpha}. \tag{25}$$

Since $\alpha > 1/2$, Home has a higher relative demand for good 1 in all periods. Without risk of confusion, we now refer to good 1 and good 2 as Home's "import-oriented" and "export-oriented" sectors, respectively. The next proposition highlights how this distinction plays a key role in linking intertemporal and intratemporal terms-of-trade motives.[11]

**Proposition 5** *(Import- versus export-oriented growth) Suppose that equations (24)-(25) hold with $\gamma \geq 0$ and $\alpha > 1/2$ and that between periods $t$ and $t+1$ there is a small change in the home endowment $dy_{t+1} = y_{t+1} - y_t$. If growth is import-oriented, $dy_{1t+1} > 0$ and $dy_{2t+1} = 0$, it is optimal to tax capital inflows/subsidize capital outflows ($\theta_t < 0$). Conversely, if growth is export-oriented, $dy_{1t} = 0$ and $dy_{2t+1} > 0$, it is optimal to tax capital inflows/subsidize capital outflows ($\theta_t < 0$) if and only if $\gamma > \left(\frac{2\alpha-1}{\alpha}\right)\left(\frac{P_t^* C_t^*}{P_t^* C_t^* + P_t C_t}\right)$.*

The idea behind the first part of Proposition 5 is closely related to Proposition 2. In periods in which Home controls a larger fraction of the world endowment of good 1, the incentive to subsidize consumption $C$ increases. Here, however, the reason is twofold. First, a larger endowment of good 1 means that Home is running a smaller (net) trade deficit, which reduces the incentive to depress the intertemporal price $P^*$. Second, it means that within the period the country is selling more of good 1. Since home preferences are biased towards good 1, an increase in $C$ drives up the intratemporal price of good 1, which further increases the incentives to subsidize aggregate consumption.

[11] Another simple example that can be solved analytically is the case of tradable and non-tradable goods. If there only is one tradable good, then Proposition 2 applies unchanged to changes in the endowment of the tradable good. The only difference between this case and the one-good case studied in Section 3 is that taxes on capital inflows/subsidies on capital outflows ($\theta_t < 0$) now are always accompanied by a real exchange rate appreciation, whereas taxes on capital outflows/subsidies on capital inflows ($\theta_t > 0$) now are always accompanied by a real exchange rate depreciation

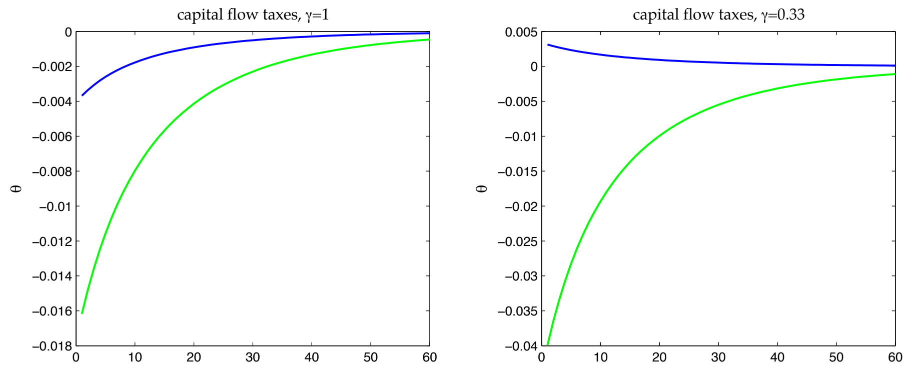


Figure 5: Optimal Taxes under Import- and Export-Oriented Scenarios

*Note*: In both panels, the green line is the optimal tax on capital flows under the import-oriented scenario (growth in sector 1) and the blue line is the optimal tax on capital flows under the export-oriented scenario (growth in sector 2).

By contrast, when endowment growth is export-oriented, intertemporal and intratemporal considerations are not aligned anymore. If the elasticity of intertemporal substitution, $1/\gamma$, is low enough, the intertemporal motive for terms-of-trade manipulation dominates and we get the same result as in the one good economy. If instead that elasticity is high enough, the result goes in the opposite direction. Namely, it is possible that when Home receives a larger endowment of good 2, it decides to subsidize aggregate consumption less, even though the increase in $y_2$ is reducing its (net) trade deficit. Intuitively, Home now benefits from reducing its own consumption since this increases the intratemporal price of good 2 due again to the fact that, relative to foreign preferences, home preferences are biased towards good 1. Proposition 5 formally demonstrates that the intratemporal terms-of-trade motive is more likely to dominate the intertemporal one if demand differences between countries are large and/or Foreign accounts for a large share of world consumption.

In order to illustrate the quantitative importance of this effect, we return to the exercise presented in Section 3.3 in which Home is catching up with the rest of the world. For simplicity, the world endowments of both goods are assumed to be constant over time. In the first panel of Figure 5, the intertemporal elasticity of substitution is set to unity, $\gamma = 1$, and demand differences are set such that $\alpha = 3/4$. The green curve represents the optimal tax on capital flows in the import-oriented scenario: the home endowment of good 2 is fixed, but the home endowment of good 1 is 20% of world endowments at date 0 and is converging towards being 50% in the long run, with the ratio $y_{1t}/y^*_{1t}$ converging to its long run value at a constant speed $\eta = 0.05$. The blue curve instead represents the optimal tax on capital flows in the export-oriented scenario: the home endowment of good 1 is fixed, but the home endowment of good 2 is growing. In order to make the two scenarios comparable, the growth rate of good 2's endowments is chosen such that the home share of world income in all periods is the same as in the import-oriented scenario. In all periods we see that the optimal tax on capital inflows is lower in the export-oriented scenario. While taxes converge to zero under both scenarios, the tax on capital inflows at date 0 is four times larger in the import-oriented scenario than in the export-oriented one: 1.6% versus 0.4%. In the second panel of Figure 5, we repeat the same experiments under the assumption that $\gamma = 0.33$. In this situation, the intratemporal terms-of-trade motives now dominate the intertemporal ones under the export-oriented scenario. When there is growth at home relative to the rest of the world, but growth is concentrated in sector 2, Home finds it optimal to subsidize rather than tax capital inflows. At date 0, the optimal *subsidy* on capital inflows is now around 0.4%.

# 5 Capital Control Wars

In this section we go back to the one-good case, but consider the case in which both countries set capital controls optimally, taking as given the capital controls chosen by the other country. As before, we assume that consumers can only trade one-period bonds on international capital markets, but we now let both the home and foreign government impose proportional taxes $\theta_t$ and $\theta_t^*$, respectively, on the gross return on net asset position in international bond markets. At date 0, we assume that the two governments simultaneously choose the sequences $\{\theta_t\}$ and $\{\theta_t^*\}$, and commit to them. While this assumption is admittedly strong in this context, it has one key benefit: it will allow us to use the same primal approach developed in previous sections to offer a first look at the outcome of capital control wars.

## 5.1 Nash Equilibrium

We look for a Nash equilibrium, so we look at each government's optimization problem taking the other government's tax sequence as given. Focusing on the problem of the home government, the optimal taxes can be characterized in terms of a planner problem involving directly the quantities consumed, as in the unilateral case. Given the sequence $\{\theta_t^*\}$ the foreign consumer's Euler equation can be written as

$$u^{*\prime}(c_t^*) = \beta(1 - \theta_t^*)(1 + r_t)u^{*\prime}(c_{t+1}^*).$$

Since $1 + r_t = p_t / p_{t+1}$, a standard iterative argument then implies

$$p_t = \beta^t \left[ \prod_{s=0}^{t-1} (1 - \theta_s^*) \right] \left[ p_0 u^{*\prime}(c_t^*) / u^{*\prime}(c_0^*) \right]. \tag{26}$$

Accordingly, Home's planning problem is now given by

$$\max_{\{c_t\}} \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{$P_N$}$$

subject to

$$\sum_{t=0}^{\infty} \beta^t u^{*\prime}(Y - c_t) \left[ \prod_{s=0}^{t-1} (1 - \theta_s^*) \right] (c_t - y_t) = 0,$$

where the new implementability constraint captures the fact that the home government now takes foreign capital flow taxes as given. This yields the optimality condition

$$u'(c_t) = \mu \left[ \prod_{s=0}^{t-1} (1 - \theta_s^*) \right] \left[ u^{*\prime}(c_t^*) - u^{*\prime\prime}(c_t^*)(c_t - y_t) \right], \tag{27}$$

which further implies

$$\frac{u'(c_t)}{u'(c_{t+1})} = \frac{1}{(1 - \theta_t^*)} \frac{u^{*\prime}(c_t^*) - u^{*\prime\prime}(c_t^*)(c_t - y_t)}{u^{*\prime}(c_{t+1}^*) - u^{*\prime\prime}(c_{t+1}^*)(c_{t+1} - y_{t+1})}. \tag{28}$$

From the domestic consumer's Euler equation, we also know that

$$u'(c_t) = \beta(1 - \theta_t)(1 + r_t) u'(c_{t+1}). \tag{29}$$

Combining equations (28) and (29) with equation (26), we obtain after simplifications

$$1 - \theta_t = \frac{1 - \frac{u^{*\prime\prime}(c_t^*)}{u^{*\prime}(c_t^*)}(c_t - y_t)}{1 - \frac{u^{*\prime\prime}(c_{t+1}^*)}{u^{*\prime}(c_{t+1}^*)}(c_{t+1} - y_{t+1})}.$$

The planning problem of the foreign government is symmetric. So the same logic implies

$$1 - \theta_t^* = \frac{1 - \frac{u''(c_t)}{u'(c_t)}(c_t^* - y_t^*)}{1 - \frac{u''(c_{t+1})}{u'(c_{t+1})}(c_{t+1}^* - y_{t+1}^*)}.$$

Substituting for the foreign tax on international capital flows in equation (28) and using the good market clearing condition (3), we obtain

$$\frac{u'(c_t)+u''(c_t)(c_t-y_t)}{u^{*\prime}(Y-c_t)-u^{*\prime\prime}(Y-c_t)(c_t-y_t)}=\frac{u'(c_{t+1})+u''(c_{t+1})(c_{t+1}-y_{t+1})}{u^{*\prime}(Y-c_{t+1})-u^{*\prime\prime}(Y-c_{t+1})(c_{t+1}-y_{t+1})},$$

which can be rearranged as

$$\frac{u'(c_t)+u''(c_t)(c_t-y_t)}{u^{*\prime}(Y-c_t)-u^{*\prime\prime}(Y-c_t)(c_t-y_t)}=\alpha,\text{ for all } t\geq 0, \tag{30}$$

where $\alpha \equiv [u'(c_0)+u''(c_0)(c_0-y_0)]/[u^{*\prime}(Y-c_0)-u^{*\prime\prime}(Y-c_0)(c_0-y_0)] > 0$. This is the counterpart of equation (5) in Section 2. In particular, using equations (27) and (29) and their counterparts in Foreign, one can check that $\alpha = \lambda\mu^*/\lambda^*\mu$, where $\lambda$ and $\lambda^*$ are the Lagrange multipliers associated with the intertemporal budget constraints in both countries.

## 5.2 Main Results Revisited

With the analysis of Section 3 in mind, one might expect that an increase in $y$ would necessarily lead to an increase in $c$. Indeed, we have established that if Home were to impose taxes unilaterally, it would like to increase $c$ in response to a positive shock in $y$. The same logic implies that if Foreign were to impose taxes unilaterally, it would like to decrease $c^*$, i.e., to increase $c$ as well, in response to a positive shock in $y$. Thus both unilateral responses point towards an increasing relationship between $c$ and $y$. Yet, as the next lemma demonstrates, the relationship between endowments and consumption along the Nash equilibrium is more subtle.

**Lemma 1** *Along the Nash equilibrium, consumption $c$ is increasing in $y$ if and only if the following condition holds:*

$$\frac{(y_t-c_t)}{2}\frac{\partial\left[u''(c_t)+\alpha u^{*\prime\prime}(Y-c_t)\right]/\partial c_t}{\left[u''(c_t)+\alpha u^{*\prime\prime}(Y-c_t)\right]}<1. \tag{31}$$

In addition to the effects that were present in Section 3, the change in foreign taxes may now lead Home to operate in a region in which prices are more manipulable. Broadly speaking, Lemma 1 states that if there is a large number of inframarginal units and if changes in consumption have large effects on changes in the price of these inframarginal units, then $c$ may be decreasing in $y$ along the Nash equilibrium. A sufficient condition to rule out this scenario is that net exports, $y-c$, is not too large in equilibrium. Another sufficient condition is that utility functions are quadratic.[12] In this case, changes in consumption have no effects on changes in prices.

Under the assumption that inequality (31) holds, we can use Lemma 1 together with the domestic and foreign consumer's Euler equations to characterize capital control wars the same way we characterized optimal capital controls in Section 3. Our main result about capital control wars can be stated as follows.

**Proposition 6** *(Capital control wars) Suppose that inequality (31) holds. Then along the Nash equilibrium, the home and foreign capital flow taxes are such that:*

1. *Home interest rates are higher than foreign interest rates ($\theta_t < \theta_t^*$) if $y_{t+1} > y_t$;*

2. *Home interest rates are lower than foreign interest rates ($\theta_t > \theta_t^*$) if $y_{t+1} < y_t$;*

3. *Home and foreign interest rates are equal ($\theta_t = \theta_t^*$) if $y_{t+1} = y_t$.*

If there are no intertemporal distortions abroad, $\theta_t^* = 0$, then like in Section 3, an increase in domestic endowments, $y_{t+1} > y_t$, leads to a tax on capital inflows or a subsidy to capital outflows, $\theta_t < 0$, which is associated with higher domestic interest rates, $(1 - \theta_t)(1 + r_t) > 1 + r_t$. In general, however, we cannot sign $\theta_t$ and $\theta_t^*$. The intuition for this result is a combination of the intuition for the unilateral policy of Home and Foreign. Suppose, for instance, that Home is running a trade deficit in period $t$. An increase in the home endowment reduces the trade deficit and reduces the incentives of the home government to repress domestic consumption. Foreign incentives are symmetric, meaning that the foreign government has less incentives to stimulate foreign consumption. The increase in domestic consumption and the reduction in foreign consumption between periods $t$ and $t + 1$ can be achieved in two ways: by a tax on capital inflows at home, $\theta_t < 0$, or by a tax on capital outflows abroad, $\theta_t^* > 0$. Because of the general equilibrium response of world prices, we do not necessarily need both. All we need is that $\theta_t < \theta_t^*$, i.e. that domestic interest rates are higher than foreign interest rates.

To conclude, we compare the Nash equilibrium capital flow taxes to the unilaterally optimal taxes for both countries, i.e. the best response to a zero tax, using again the parameterized example presented in Section 3.2. In Figure 6, we see that a capital control war leads to a larger interest rate differential between the two countries (as a percentage of the world return to net lending) than either one of the two unilateral outcomes considered in Section 3.2. Far from canceling each other out, the net distortion on capital flows is therefore larger when both countries set capital controls optimally. Compared to

[12] This immediately implies $\partial \left[ u''(c_t) + \alpha u^{*\prime\prime}(Y - c_t) \right] / \partial c_t = 0$.

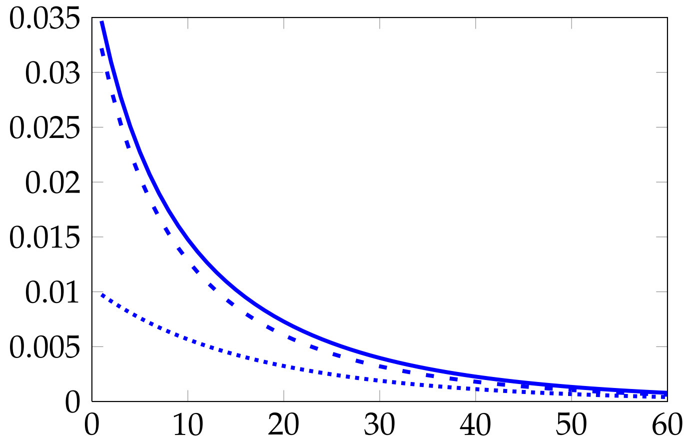


Figure 6: Interest rate differentials with capital control wars and unilaterally optimal taxes

*Note*: The solid line represents the Nash equilibrium interest rate differential (as a percentage of the world return to net lending), while the dotted and dashed lines show the interest rate differentials under the unilaterally optimal capital flow taxes considered in Figures 3 and 4, respectively.

the benchmark with no capital controls, a capital control war here decreases consumption by 0.49% in the country catching-up and by 0.05% in the rest of the world. Interestingly, even though the interest rate differential is close to its value when the rest of the world sets capital controls unilaterally, both countries are worse off in the Nash equilibrium. In this particular example, neither country wins the capital control war.

# 6 Concluding Remarks

In this paper we have developed a theory of capital controls as dynamic terms-of-trade manipulation. We have studied an infinite horizon endowment economy with two countries where one country chooses taxes on international capital flows in order to maximize the welfare of its representative agent, while the other country is passive. We have shown that capital controls are not guided by the absolute desire to alter the intertemporal price of the goods produced in any given period, but rather by the relative strength of this desire between two consecutive periods. Specifically, it is optimal for the strategic country to tax capital inflows (or subsidize capital outflows) if it grows faster than the rest of the world and to tax capital outflows (or subsidize capital inflows) if it grows more slowly. In the long-run, if relative endowments converge to a steady state, taxes on international capital flows converge to zero. Although our theory emphasizes interest rate manipulation, the country's net financial position per se is irrelevant.

With many goods, we have shown that optimal capital controls depend both on the level and composition of growth across goods. If countries have different preferences, a change in the time profile of consumption not only affects the interest rate but also the relative prices of consumption goods in each given period. Accordingly, even if all static trade distortions are banned by a free-trade agreement, intratemporal prices may not be at their undistorted levels if capital controls are allowed. Finally we have studied capital control wars in which the two countries simultaneously set taxes on capital flows. In the simple quantitative example that we consider, far from canceling each other out, the net distortion on capital flows is larger than in the unilateral case.

In order to maintain the focus of our analysis on optimal capital controls, we have abstracted from environments with intratemporal and intertemporal trade in which countries can set good-and-time specific trade taxes. An interesting question that arises once these more general policy instruments are allowed is whether restricting static trade distortions may contribute to reduce the incentives to use capital controls. Broadly speaking, could free-trade agreements act as a substitute for agreements on capital controls? We hope that our analysis will provide a useful starting point for thinking about this issue as well as other related questions at the frontier of international macro and international trade policy.

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# 7 Appendix

**Proof of Proposition 1.** The first- and second-order conditions associated with Home's planning problem imply:

$$u'(c_t) - \mu \left[ u^{*\prime}(Y - c_t) - u^{*\prime\prime}(Y - c_t)(c_t - y_t) \right] = 0, \tag{32}$$

$$\frac{\partial}{\partial c_t} \left\{ u'(c_t) - \mu \left[ u^{*\prime}(Y - c_t) - u^{*\prime\prime}(Y - c_t)(c_t - y_t) \right] \right\} < 0. \tag{33}$$

Differentiating equation (32), we get after simple rearrangements

$$\frac{\partial c_t}{\partial y_t} = \frac{\mu u^{*\prime\prime}(Y - c_t)}{\frac{\partial}{\partial c_t} \left\{ u'(c_t) - \mu \left[ u^{*\prime}(Y - c_t) - u^{*\prime\prime}(Y - c_t)(c_t - y_t) \right] \right\}} > 0, \tag{34}$$

where the inequality directly derives from inequality (33) and the strict concavity of $u^*$. Inequality (34) implies that for any pair of periods, $t$ and $s$, such that $y_s > y_t$, we must have $c_s > c_t$. ■

**Section 3.4.** Let us focus on date 0 and date 1. Let $\{c_t\}_{t=1}^{\infty}$ and $\{c'_t\}_{t=1}^{\infty}$ denote the optimal consumption paths for all dates $t \geq 1$ from the point of view of the home government at date 0 and date 1, respectively. We want show that one can construct $\{a^*_{1,s}\}_{s=1}^{\infty}$ satisfying equation (13) at $t = 0$ such that $c'_t = c_t$ for all $t \geq 1$. As in Lucas and Stokey (1983), we focus on the first-order conditions associated with Home's planning problem at dates $t = 0$ and $t = 1$. In the present environment, they imply

$$u'(c_t) = \mu_0 \left[u^{*\prime}(Y - c_t) - u^{*\prime\prime}(Y - c_t)(c_t - y_t + a^*_{0,t})\right], \tag{35}$$

$$u'(c'_t) = \mu_1 \left[u^{*\prime}(Y - c'_t) - u^{*\prime\prime}(Y - c'_t)(c'_t - y_t + a^*_{1,t})\right], \tag{36}$$

where $\mu_0$ and $\mu_1$ are the Lagrange multipliers associated with the implementability constraints of the home government at dates 0 and 1, respectively. For a given value of $\mu_1$, let us construct $a^*_{1,t}(\mu_1)$ such that

$$\mu_0 \left[u^{*\prime}(Y - c_t) - u^{*\prime\prime}(Y - c_t)(c_t - y_t + a^*_{0,t})\right] = \mu_1 \left[u^{*\prime}(Y - c_t) - u^{*\prime\prime}(Y - c_t)(c_t - y_t + a^*_{1,t}(\mu_1))\right],$$

which can be rearranged as

$$a^*_{1,t}(\mu_1) = \frac{u^{*\prime}(Y - c_t)}{u^{*\prime\prime}(Y - c_t)} - (c_t - y_t) + \frac{\mu_0}{\mu_1}\left[a^*_{0,t} - \frac{u^{*\prime}(Y - c_t)}{u^{*\prime\prime}(Y - c_t)} + (c_t - y_t)\right]. \tag{37}$$

By construction, if the previous condition holds, then, for any $\mu_1$, equation (36) holds as well if $c'_t = c_t$ for all $t \geq 1$. Now let us choose $\mu_1$ such that:

$$\mu_1 = \mu_0 \sum_{s=1}^{\infty} \nu_s \left[1 - \frac{u^{*\prime\prime}(Y - c_s)\left(c_s - y_s + a^*_{0,s}\right)}{u^{*\prime}(Y - c_s)}\right], \tag{38}$$

where

$$\nu_s \equiv \frac{\left[u^{*\prime}(Y - c_s)\right]^2 / u^{*\prime\prime}(Y - c_s)}{\sum_{r=1}^{\infty}\left[u^{*\prime}(Y - c_r)\right]^2 / u^{*\prime\prime}(Y - c_r)} \in [0, 1]. \tag{39}$$

By equation (35), we know that $1 - \frac{u^{*\prime\prime}(Y - c_s)(c_s - y_s + a^*_{0,s})}{u^{*\prime}(Y - c_s)} > 0$. Thus we have $\mu_1 > 0$. One can check that, by construction, equations (37)-(39) further imply

$$\sum_{s=1}^{\infty} u^{*\prime}(Y - c_s)\left[c_s - y_s + a_{1,s}^{*}(\mu_1)\right] = 0.$$

Thus, equation (13) is satisfied at $t = 0$. Since equations (13) and (14) evaluated at $t = 0$ and $t = 1$, respectively, are identical, we have constructed $\{a_{1,s}^{*}\}_{s=1}^{\infty}$ satisfying equation (13) at $t = 0$ such that $c_t' = c_t$ for all $t \geq 1$. The argument for other dates is similar. ■

**Proof of Proposition 3.** The basic strategy is the same as in the proof of Proposition 1. The first- and second-order conditions associated with Home's planning problem imply:

$$U'(C_t) - \mu\left(\sum_i \rho_i(C_t)\frac{\partial c_i(C_t)}{\partial C_t} + \sum_i \frac{\partial \rho_i(C_t)}{\partial C_t}(c_i(C_t) - y_{it})\right) = 0, \tag{40}$$

$$\frac{\partial}{\partial C_t}\left\{U'(C_t) - \mu\left(\sum_i \rho_i(C_t)\frac{\partial c_i(C_t)}{\partial C_t} - \sum_i \frac{\partial \rho_i(C_t)}{\partial C_t}(c_i(C_t) - y_{it})\right)\right\} < 0. \tag{41}$$

Differentiating equation (40), we get after simple rearrangements

$$dC_t = -\frac{\mu \sum_i \frac{\partial \rho_i(C_t)}{\partial C_t} dy_{it}}{\frac{\partial}{\partial C_t}\left\{U'(C_t) - \mu \sum_i \rho_i(C_t)\frac{\partial c_i(C_t)}{\partial C_t} - \sum_i \frac{\partial \rho_i(C_t)/\partial C_t}{\rho_i(C_t)}\rho_i(C_t)(c_i(C_t) - y_{it})\right\}}.$$

By inequality (41), we therefore have $dC_t > 0$ if and only if $\sum_i \frac{\partial \rho_i(C_t)}{\partial C_t} dy_{it} > 0$. ■

**Proof of Proposition 4.** In the main text, we have already established that

$$\theta_t = 1 - \left(\frac{1 + \tau_t}{1 + \tau_{t+1}}\right)\left(\frac{P_{t+1}/P_{t+1}^{*}}{P_t/P_t^{*}}\right), \tag{42}$$

with the wedge $\tau_t$ such that

$$\tau_t = \frac{U'(C_t)}{\mu U^{*\prime}(C^{*}(C_t))} - 1.$$

Since $U$ and $U^*$ are concave and $C^*$ is decreasing in $C_t$ along the Pareto frontier, we already know from Proposition 3 that

$$\tau_{t+1} < \tau_t \text{ if and only if } \sum_i \frac{\partial \rho_i(C_t)}{\partial C_t} dy_{it} > 0. \tag{43}$$

Now notice that by the envelope theorem, $C^{*\prime}(C_t)$ is equal to the opposite of the Lagrange multiplier associated with the constraint $g(c) \geq C_t$ in (17). Thus the first-order conditions associated with that program imply

$$g_i^*\left(c_t\right) = -C^{*\prime}\left(C_t\right) g_i\left(c_t\right). \tag{44}$$

Let us now show that $P_t = p_{it}/g_i\left(c_t\right)$. Let us denote $c_t\left(1\right) \equiv \arg\min_c \left\{p_t \cdot c : g\left(c\right) \geq 1\right\}$. The associated first-order conditions are given by $(i)$ $p_i = \lambda g_i\left[c_t\left(1\right)\right]$ and $(ii)$ $g\left[c_t\left(1\right)\right] = 1$. This implies

$$P_t = \sum_i p_{it} c_{it}\left(1\right) = \lambda \sum_i g_i\left[c_t\left(1\right)\right] c_{it}\left(1\right) = \lambda g\left[c_t\left(1\right)\right] = \lambda,$$

where the third equality uses the fact that $g$ is homogeneous of degree one. Combining this equality with condition $(i)$, we obtain $P_t = p_{it}/g_i\left[c_t\left(1\right)\right]$. Since $g_i$ is homogeneous of degree zero, this further implies $P_t = p_{it}/g_i\left(c_t\right)$ for all $i = 1, ..., n$. The same logic applied to Foreign implies $P_t^* = p_{it}/g_i^*\left(c_t\right)$. Combining the two previous observations with equation (44), we obtain

$$\frac{P_t}{P_t^*} = -C^{*\prime}\left(C_t\right).$$

Since $g$ and $g^*$ are concave and homogeneous of degree one, standard arguments imply that the solution $C^*$ of (17) is (weakly) concave in $C_t$. By Proposition 3, we therefore have

$$P_{t+1}/P_{t+1}^* > P_t/P_t^* \text{ if and only if } \sum_i \frac{\partial \rho_i(C_t)}{\partial C_t} dy_{it} > 0. \tag{45}$$

Combining equation (42) with conditions (43) and (45), we finally get $\theta_t < 0$ if and only if $\sum_i \frac{\partial \rho_i(C_t)}{\partial C_t} dy_{it} > 0$. ■

**Proof of Proposition 5.** Suppose that $dy_{it+1} > 0$ and $dy_{jt+1} = 0$. By Proposition 4, we know that $\theta_t < 0$ if and only if $\partial\rho(C_t)/\partial C_t \cdot dy_{t+1} > 0$, where $\rho(C_t) \equiv U^{*\prime}\left(C^*(C_t)\right) g_c^*(c^*\left(C_t\right))$. Thus if $dy_{it+1} > 0$ and $dy_{jt+1} = 0$, $\theta_t < 0$ if and only if

$$\frac{U^{*\prime\prime}\left(C^*(C_t)\right)}{U^{*\prime}\left(C^*(C_t)\right)} \frac{\partial C^*(C_t)}{\partial C_t} + \left[\frac{g_{ii}^*(c^*\left(C_t\right))}{g_i^*(c^*\left(C_t\right))} \frac{\partial c_i^*(C_t)}{\partial C_t} + \frac{g_{ij}^*(c^*\left(C_t\right))}{g_i^*(c^*\left(C_t\right))} \frac{\partial c_j^*(C_t)}{\partial C_t}\right] > 0. \tag{46}$$

Consider the first term on the left-hand side of inequality (46). In the proof of Proposition 4, we have already established that

$$\frac{\partial C^*(C_t)}{\partial C_t} = -\frac{P_t}{P_t^*}.$$

Since $U^*(C^*) = \frac{1}{1-\gamma}(C^*)^{1-\gamma}$, we therefore have

$$\frac{U^{*\prime\prime}(C^*(C_t))}{U^{*\prime}(C^*(C_t))}\frac{\partial C^*(C_t)}{\partial C_t} = \frac{\gamma P_t}{P_t^* C_t^*}. \tag{47}$$

Let us now turn to the second term on the left-hand side of inequality (46). Our goal is to establish that

$$\begin{aligned}
\frac{g_{11}^*(c_t^*)}{g_1^*(c_t^*)}\frac{\partial c_1^*(C_t)}{\partial C_t} + \frac{g_{12}^*(c_t^*)}{g_1^*(c_t^*)}\frac{\partial c_2^*(C_t)}{\partial C_t} &= \frac{\alpha}{(1-\alpha)C_t}\frac{(2\alpha-1)\,c_2(C_t)}{(1-\alpha)\,c_2^*(C_t)+\alpha c_2(C_t)}, \quad (48)\\
\frac{g_{12}^*(c_t^*)}{g_2^*(c_t^*)}\frac{\partial c_1^*(C_t)}{\partial C_t} + \frac{g_{22}^*(c_t^*)}{g_2^*(c_t^*)}\frac{\partial c_2^*(C_t)}{\partial C_t} &= -\frac{1}{C_t}\frac{(2\alpha-1)\,c_2(C_t)}{(1-\alpha)\,c_2^*(C_t)+\alpha c_2(C_t)}. \quad (49)
\end{aligned}$$

Since $g^*(c^*) = (c_2^*)^{\alpha}(c_1^*)^{1-\alpha}$, simple algebra implies

$$\begin{aligned}
\frac{g_{11}^*(c_t^*)}{g_1^*(c_t^*)} &= -\frac{\alpha}{c_{1t}^*}, \frac{g_{12}^*(c_t^*)}{g_1^*(c_t^*)} = \frac{\alpha}{c_{2t}^*},\\
\frac{g_{12}^*(c_t^*)}{g_2^*(c_t^*)} &= \frac{(1-\alpha)}{c_{1t}^*}, \frac{g_{22}^*(c_t^*)}{g_2^*(c_t^*)} = -\frac{(1-\alpha)}{c_{2t}^*}.
\end{aligned}$$

Using the previous expressions, we obtain

$$\begin{aligned}
\frac{g_{11}^*(c_t^*)}{g_1^*(c_t^*)}\frac{\partial c_1^*(C_t)}{\partial C_t} + \frac{g_{12}^*(c_t^*)}{g_1^*(c_t^*)}\frac{\partial c_2^*(C_t)}{\partial C_t} &= \alpha\frac{\partial \ln\left[c_2^*(C_t)/c_1^*(C_t)\right]}{\partial C_t}, \quad (50)\\
\frac{g_{12}^*(c_t^*)}{g_2^*(c_t^*)}\frac{\partial c_1^*(C_t)}{\partial C_t} + \frac{g_{22}^*(c_t^*)}{g_2^*(c_t^*)}\frac{\partial c_2^*(C_t)}{\partial C_t} &= -(1-\alpha)\frac{\partial \ln\left[c_2^*(C_t)/c_1^*(C_t)\right]}{\partial C_t}. \quad (51)
\end{aligned}$$

Let us compute $\frac{\partial \ln\left[c_2^*(C_t)/c_1^*(C_t)\right]}{\partial C_t}$. By definition, $c(C_t)$ and $c^*(C_t)$ are the solution of

$$\max_{c,c^*} \; (c_2^*)^{\alpha}(c_1^*)^{1-\alpha}$$

subject to

$$\begin{aligned}
c_1 + c_1^* &\leq Y_1,\\
c_2 + c_2^* &\leq Y_2,\\
c_1^{\alpha}c_2^{1-\alpha} &\geq C_t.
\end{aligned}$$

The associated first-order conditions imply

$$\frac{c_2^*(C_t)}{c_1^*(C_t)} = \beta\left(\frac{Y_2 - c_2^*(C_t)}{Y_1 - c_1^*(C_t)}\right) \text{ with } \beta \equiv [\alpha/(1-\alpha)]^2, \tag{52}$$

and

$$\frac{Y_2 - c_2^*(C_t)}{Y_1 - c_1^*(C_t)} = \left(\frac{C_t}{Y_2 - c_2^*(C_t)}\right)^{-\frac{1}{\alpha}}. \tag{53}$$

Combining the two previous expressions, we obtain

$$\frac{\partial \ln[c_2^*(C_t)/c_1^*(C_t)]}{\partial C_t} = -\frac{1}{\alpha}\left(\frac{1}{C_t} - \frac{\partial \ln[Y_2 - c_2^*(C_t)]}{\partial C_t}\right) = -\frac{1}{\alpha}\left(\frac{1}{C_t} - \frac{\partial \ln[c_2(C_t)]}{\partial C_t}\right). \tag{54}$$

Let us compute $\frac{\partial \ln[c_2(C_t)]}{\partial C_t}$. Using the resource constraint, we can express equation (52) as

$$c_1(C_t) = \frac{\beta c_2(C_t) Y_1}{Y_2 - (1-\beta) c_2(C_t)}.$$

Together with equation (53), using again the resource constraint, this implies

$$c_2(C_t) = C_t\left[\frac{Y_2 - (1-\beta)c_2(C_t)}{\beta Y_1}\right]^{\alpha}.$$

Taking the log and differentiating, we obtain after rearrangements

$$\frac{\partial \ln[c_2(C_t)]}{\partial C_t} = \frac{Y_2 - (1-\beta)c_2(C_t)}{C_t[Y_2 - (1-\alpha)(1-\beta)c_2(C_t)]}. \tag{55}$$

Equations (54) and (55) imply

$$\frac{\partial \ln[c_2^*(C_t)/c_1^*(C_t)]}{\partial C_t} = \frac{1}{C_t}\frac{(\beta-1)c_2(C_t)}{Y_2 + (1-\alpha)(\beta-1)c_2(C_t)}.$$

Using the definition of $\beta \equiv [\alpha/(1-\alpha)]^2$, we can rearrange the previous expression as

$$\frac{\partial \ln[c_2^*(C_t)/c_1^*(C_t)]}{\partial C_t} = \frac{1}{(1-\alpha)C_t}\frac{(2\alpha-1)c_2(C_t)}{(1-\alpha)c_2^*(C_t) + \alpha c_2(C_t)}.$$

Equations (48) and (49) directly derive from the previous expression and equations (50) and (51), respectively.

To conclude the proof of Proposition 5, first note that equations (47) and (48) imply

$$\frac{U^{*\prime\prime}\left(C^{*}(C_{t})\right)}{U^{*\prime}\left(C^{*}(C_{t})\right)}\frac{\partial C^{*}(C_{t})}{\partial C_{t}}+\left[\frac{g_{11}^{*}(c_{t}^{*})}{g_{1}^{*}(c_{t}^{*})}\frac{\partial c_{1}^{*}(C_{t})}{\partial C_{t}}+\frac{g_{12}^{*}(c_{t}^{*})}{g_{1}^{*}(c_{t}^{*})}\frac{\partial c_{2}^{*}(C_{t})}{\partial C_{t}}\right]$$
$$=\frac{\gamma P_{t}}{P_{t}^{*}C_{t}^{*}}+\frac{\alpha}{(1-\alpha)C_{t}}\frac{(2\alpha-1)\,c_{2}\left(C_{t}\right)}{(1-\alpha)\,c_{2}^{*}\left(C_{t}\right)+\alpha c_{2}\left(C_{t}\right)}>0.$$

Thus if $dy_{1t+1}>0$ and $dy_{2t+1}=0$, then $\theta_{t}<0$. Second note that equations (47) and (49) imply

$$\frac{U^{*\prime\prime}\left(C^{*}(C_{t})\right)}{U^{*\prime}\left(C^{*}(C_{t})\right)}\frac{\partial C^{*}(C_{t})}{\partial C_{t}}+\left[\frac{g_{12}^{*}(c_{t}^{*})}{g_{2}^{*}(c_{t}^{*})}\frac{\partial c_{1}^{*}(C_{t})}{\partial C_{t}}+\frac{g_{22}^{*}(c_{t}^{*})}{g_{2}^{*}(c_{t}^{*})}\frac{\partial c_{2}^{*}(C_{t})}{\partial C_{t}}\right]$$
$$=\frac{\gamma P_{t}}{P_{t}^{*}C_{t}^{*}}-\frac{1}{C_{t}}\frac{(2\alpha-1)\,c_{2}\left(C_{t}\right)}{(1-\alpha)\,c_{2}^{*}\left(C_{t}\right)+\alpha c_{2}\left(C_{t}\right)}.$$

According to this expression, if $dy_{1t+1}=0$ and $dy_{2t+1}>0$, then $\theta_{t}<0$ if and only if

$$\gamma>\frac{P_{t}^{*}C_{t}^{*}}{P_{t}C_{t}}\frac{(2\alpha-1)\,c_{2}\left(C_{t}\right)}{(1-\alpha)\,c_{2}^{*}\left(C_{t}\right)+\alpha c_{2}\left(C_{t}\right)}. \tag{56}$$

Since utility functions are Cobb-Douglas, $g(c)=c_{1}^{\alpha}c_{2}^{1-\alpha}$ and $g^{*}(c^{*})=\left(c_{2}^{*}\right)^{\alpha}\left(c_{1}^{*}\right)^{1-\alpha}$, we know that

$$\begin{aligned}p_{2t}c_{2}\left(C_{t}\right)&=(1-\alpha)\,P_{t}C_{t}\\ p_{2t}c_{2}^{*}\left(C_{t}\right)&=\alpha P_{t}^{*}C_{t}^{*}\end{aligned}$$

Combining these two observations with inequality (56), we conclude that if $dy_{1t+1}=0$ and $dy_{2t+1}>0$, then $\theta_{t}<0$ if and only if $\gamma>\left(\frac{2\alpha-1}{\alpha}\right)\left(\frac{P_{t}^{*}C_{t}^{*}}{P_{t}^{*}C_{t}^{*}+P_{t}C_{t}}\right)$. ■

**Proof of Lemma 1.** Let us rearrange equation (30) as

$$F(c_{t},y_{t})\equiv\left[u'(c_{t})-u''(c_{t})(y_{t}-c_{t})\right]-\alpha\left[u^{*\prime}(Y-c_{t})-u^{*\prime\prime}(Y-c_{t})(c_{t}-y_{t})\right]=0. \tag{57}$$

Note that $\partial F(c_{t},y_{t})/\partial y_{t}>0$ by strict concavity of $u$ and $u^{*}$. Thus

$$dc_{t}/dy_{t}=-\,\partial F(c_{t},y_{t})/\partial y_{t}/\,\partial F(c_{t},y_{t})/\partial c_{t}>0$$

if and only if $\partial F(c_{t},y_{t})/\partial c_{t}<0$. Differentiating equation (57) we get

$$\frac{\partial F(c_{t},y_{t})}{\partial c_{t}}=\frac{\partial}{\partial c_{t}}\left[u'(c_{t})-u''(c_{t})(y_{t}-c_{t})\right]-\alpha\frac{\partial}{\partial c_{t}}\left[u^{*\prime}(Y-c_{t})-u^{*\prime\prime}(Y-c_{t})(c_{t}-y_{t})\right],$$

which can be rearranged as

$$\frac{\partial F(c_t,y_t)}{\partial c_t} = \left[u''(c_t) + \alpha u^{*\prime\prime}(Y - c_t)\right] \left\{2 - (y_t - c_t)\frac{\partial \left[u''(c_t) + \alpha u^{*\prime\prime}(Y - c_t)\right] / \partial c_t}{\left[u''(c_t) + \alpha u^{*\prime\prime}(Y - c_t)\right]}\right\}.$$

Since $u$ and $u^*$ are strictly concave, this implies that

$$\partial F(c_t,y_t)/\partial c_t < 0 \text{ if and only if } \frac{(y_t - c_t)}{2}\frac{\partial \left[u''(c_t) + \alpha u^{*\prime\prime}(Y - c_t)\right] / \partial c_t}{\left[u''(c_t) + \alpha u^{*\prime\prime}(Y - c_t)\right]} < 1.$$

Lemma 1 directly derives from the previous equivalence. ■

**Proof of Proposition 6.** The foreign and domestic consumers' Euler equations imply

$$\frac{1-\theta_t}{1-\theta_t^*} = \frac{u'(c_t)}{u^{*\prime}(c_t^*)}\frac{u^{*\prime}(c_{t+1}^*)}{u'(c_{t+1})}$$

Using the good market clearing condition (3), we can rearrange this expression as

$$\frac{1-\theta_t}{1-\theta_t^*} = \frac{u'(c_t)}{u^{*\prime}(Y-c_t)}\frac{u^{*\prime}(Y-c_{t+1})}{u'(c_{t+1})}.$$

By Lemma 1, we know that if inequality (31) holds, then $c_t$ is increasing in $y_t$. Since $u$ and $u^*$ are strictly concave, the previous expression therefore implies

$$\frac{1-\theta_t}{1-\theta_t^*} < 1 \text{ if and only if } y_t > y_{t+1}.$$

Proposition 6 directly derives from the previous equivalence. ■