working papers series
in Economics and Social Sciences
2010/21

Performance evaluation in competitive REE models

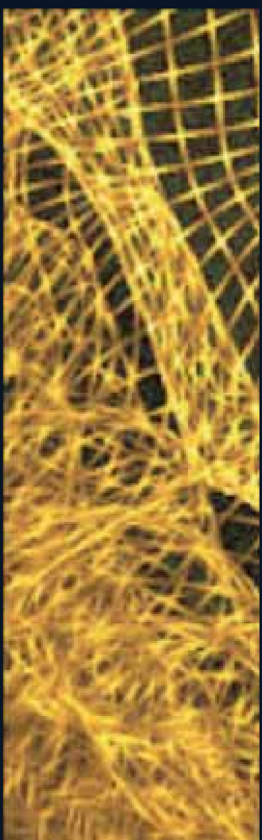
by Paolo Colla and José M. Marín

October 2010

Ctra. Colmenar Viejo km. 14, 28049 Madrid, Spain

http://www.imdea.org/socialsciences
http://repec.imdea.org
contacto.cienciassociales@imdea.org

www.imdea.org



Performance Evaluation in Competitive REE Models*

PAOLO COLLA† and JOSE M. MARIN‡

Abstract

Our basic premise is that fund managers performance is related to superior information about an asset payoff. We investigate the relationship between managerial skills and trading behavior within a two-period rational expectation equilibrium (REE) model where agents trade on private information in the first round, while a public signal arrives at the second date that makes traders revise their beliefs and retrade. The public signal can be related to the asset payoff, or to variables not related to fundamentals (noise), or both. We characterize the unique partially revealing REE and explore the drivers of price dynamics and trading behavior. Our main prediction is that good managers are contrarian traders, while bad managers are momentum traders when public news arrive to the market. Furthermore, the change in holdings of each type of trader is monotonic on the traders' skills. Based on these predictions, we propose new performance evaluation measures that rely on the manager's change in holdings around the arrival of public news rather than his past performance. A byproduct of our analysis is the proposal of a new protocol for performance evaluation and Due Diligence (DD) procedures.

Journal of Economic Literature Classification Numbers: G11, G12, G14.

Keywords: REE, Performance Evaluation, Mutual Funds, Hedge Funds, Talent, Informed Traders, Due Diligence.

*The authors are grateful to Peter Hammond, Jose Penalva, Motty Perry, Roberto Serrano, and participants at the M-SWET 2010 Conference (Madrid), the EUI Workshop on 'The acquisition, use and transmission of private information in financial markets' (Florence), and IGIER Seminar (Università Bocconi) for insightful comments. Paolo Colla gratefully acknowledges financial support from the "Paolo Baffi" Centre (Università Bocconi). Jose Marin acknowledges financial support from the Spanish Ministry of Science and Innovation (MICINN Grant ECO2008-05140.)

†Università Bocconi; paolo.colla@unibocconi.it.

‡IMDEA Social Sciences Institute; jose.marin@imdea.org.

1 Introduction

Delegated portfolio management is the norm rather than the exception in the asset management industry. Most individuals do not invest in securities markets directly but rather via mutual, pension and hedge funds[1]. These investment vehicles are managed by professionals who differ in their abilities to deliver good performance. Assessing these abilities is important for the well-being of individuals, for the efficient allocation of capital, and for market efficiency. The demand for assessing these abilities is probably maximal in the hedge fund industry where managers charge large management and incentive fees for implementing complex dynamic strategies. The title "Talent Required"[2] of a recent article on hedge funds by Sanford Grossman in the Wall Street Journal probably reflects the essence of this industry: talent is assumed but probably not always there. Being able to sort the talented managers is therefore crucial for the optimal investment in hedge funds. But, even if all managers were talented, it makes sense identifying the most talented ones, as these could be the only ones who merit the high fees charged to investors. In this paper we explicitly model the trading activities of talented versus non-talented traders in equilibrium and derive new performance evaluation measures which can be empirically tested. A by-product of our analysis is the proposal of new Due Diligence protocols for the proper assessment of managers' trading skills.

Although some traders' characteristics, such as age or academic background, could be predictors of management quality, in general both practitioners and academia have mostly relied on the manager's past portfolio performance to identify talent. In this paper

[1] For instance, as of April 2008, the assets under management reported by the 11,030 SEC-registered investment advisers totalled to $42.3 trillion (Investment Adviser Association (2008)). Further, the share of common equity held by mutual funds, hedge funds, pensions, bank trust departments, and other institutions increased from 32% to 68% of total market value during the period 1980 to 2007 (Lewellen (2009)). Finally, according to the 2007 Survey of Consumer Finance, US households direct stock holdings amount to 17.9% of their total financial assets (Bucks et al. (2009)). Indirect stock holdings (defined as the sum of retirement accounts and other managed assets) are twice as much important, with a 41.1% weight. Adding pooled investment funds (which include open-end and closed-end mutual funds, real estate investment trusts, and hedge funds) brings the US families' percentage of "delegated" investment to 60% of their total financial assets.

[2] The Wall Street Journal, September 29, 2005, Page A18.

we look at traders' actions (choice of portfolio), rather than portfolio returns, in order to assess managerial abilities. We do this by analyzing the general properties of trading in a competitive rational expectation equilibrium (REE) that directly map into testable hypothesis for performance evaluation. In typical REE settings, the inference of talent from traders' portfolios is severely limited as these are monotonic functions of both talent (private signals' precision) and private information (signals), which are not observable to a third party. This probably explains why the REE paradigm has been almost completely ignored in the performance evaluation literature. We circumvent this problem by looking at traders' *changes in portfolio holdings* when public news arrive to the market. We show that public news arrival induces a change in portfolio holdings which is monotonic in each trader's information precision and independent of each trader's private signal. In this setting, hence, the inference problem is trivial.

Should better informed traders behave differently from worse informed traders when public news arrive to the market? In general, indeed, we should expect different trading patterns both in terms of the direction of the trade and the trading volume. Let's illustrate this with a simple example. Suppose there are two types of traders, informed and uninformed, and in equal proportions in the market. Informed traders privately learn that the profits of a company will double next year (a good private signal). As long as the price today does not fully reflect this information (partially revealing equilibrium price), informed traders will buy shares today at a bargain. When public news reach the market, both the informed and the uninformed traders update their forecasts of the asset payoff. But, uninformed traders will always react *more* to the public information than informed traders. In particular, if good public news arrives to the market, then uninformed traders will increase their forecast proportionally more than the informed traders, and thus would end up buying, which implies that informed traders sell shares in the presence of good public news. In this case the price will increase as the average opinion on the asset payoff has improved. On the other hand, if the public news is negative (bad public news), then uninformed traders revise downwardly their asset payoffs forecast disproportionately and will end up selling. Since the average market forecast deteriorates, the price will fall. Therefore in the case of the arrival of bad public news, informed traders buy and uninformed traders sell.[3] Hence, good public news are associated with price increases, sales by informed, and purchases by uninformed traders. The arrival of bad public news generates the opposite change in these three variables. In this economy, informed traders are contrarian and uninformed traders are momentum traders when public news arrive to the market. Finally, notice that this portfolio rebalancing must be increasing in the quality of the information of the informed traders, as their initial purchases will be larger the more they trust their private information. So, the previous reasoning also maps into different volumes traded as a function of the quality of the private information, what we generically refer to as the trader's talent. This is great news for the success of our methodology for performance evaluation as differential trading patterns are a necessary condition for its implementability.

The previous example resembles a situation of *one-sided* private information, where only a group of traders is privately informed and prices transmit part of this private information to the remaining uninformed traders. In this paper we depart from this setting and model an economy with disperse information, where all traders are privately informed to different degrees. We show, however, that the results outlined in the previous paragraph hold true in our setting: better informed traders are contrarian and worse informed traders are momentum when public information arrives to the market. Further, the volume of trade of each trader is increasing in the distance between the quality of each trader information and the average quality in the market. These results are also good news as they suggest that the main predictions of our model are robust to the different type of informational asymmetries, one-sided versus disperse, we can encounter in financial markets.

[3] To simplify the exposition, in this example we ignore the impact of changes in the risk premium on trading when the public information arrives to the market.

The previous example is also specific to the type of public news that arrives to the market, i.e. public news related to the asset payoff. It is important however to notice that both in practice and in the context of REE models, asset prices are not only driven by information on assets payoffs but also by “noise”. Noise is just a shorthand expression for other underlying economic variables such as trading induced by changes in investors’ risk tolerance, labor income shocks, etc.[4] The arrival of public news on noise may also generate well defined differential trading patterns for all types of traders, as this information allows traders to update their beliefs about the asset’s fundamental value. Hence, we should also look at trade reactions to the arrival of public news on noise. For instance, to the extent that gold prices are a good proxy for changes in risk aversion, we may wish to look at managers’ trading activities around periods of large swings in gold prices.[5] In practice, however, we often find public information for which this separation between payoff versus noise relatedness is not straightforward. Analysts’ recommendations are probably the leading example. A recommendation to buy (sell) an asset could arise because the analyst predicts larger (smaller) profits for the company in the future (payoff relevant information), but also because the analyst believes the stock is currently underpriced (overpriced) due to, say, too much liquidity selling (buying). This means that a comprehensive model of trading around public information arrivals should also consider the case of public signals that simultaneously relate to payoffs and noise in the market. For this reason, in the present paper not only we explore the trading implications of the arrival of payoff relevant public information, but also public information related to noise, and public information related to both payoffs and noise. To the best of our knowledge, this is the first time this issue has been addressed in the literature.

[4] For instance, see Grossman (1995) or Black (1986) for a more comprehensive analysis of the economics of “noise”.

[5] The price of gold is just one of the many proxies for changes in risk aversion. For instance, both Bliss and Panigirtzoglou (2004) and Jackwerth (2000) use option data to estimate time-varying risk aversion by exploiting the difference between the risk neutral and the physical distributions of returns.

More specifically, in this paper we analyze a dynamic model where agents have different information and trade in a rational expectations equilibrium. The model is simple but general enough to accommodate the arrival of both public news on fundamentals, on noise, and on arbitrary combinations of these two. The basic setting is taken from Kim and Verrecchia (1991). The most important results of our analysis with direct bearing on performance evaluation can be summarized as follows:

- The arrival of public news generates the same qualitative trading patterns irrespective of whether the information is related to the asset payoffs, the noise in the market, or both.

- The change in each trader's holdings generated by the arrival of public news has the opposite sign for traders whose private information is more precise than the average precision of all traders (informed traders) and those traders whose information is less precise than the average precision of all traders (uninformed traders).

- The volume traded by each trader is increasing on the distance of the trader's information precision to the average precision of all traders. This means that, for instance, the largest trades in the market come from the most and the least talented traders in the market. But, following the previous point, these two type of traders will be trading in opposite directions always.

Setting up the performance evaluation problem in the context of competitive REE models offers many advantages. First, our approach to performance evaluation does not necessarily rely on security prices nor portfolio returns. The only requirement is prices not being fully efficient, as under full efficiency neither private nor public information would generate differential trading across agents. Second, in a REE equilibrium we analyze the *optimal* behavior of traders with different information. This means that we pose the performance evaluation problem in the context of optimizing agents whose trades satisfy market clearing. This contrasts with analyses where no optimizing behavior is assumed (or modeled at all), or where the portfolio problem is set in a partial equilibrium setting. Third, we explicitly model a potentially very important source of outstanding performance: the possession of superior information.[6] Further, private information can be interpreted as a proxy for other variables related to performance, such as talent. A trader with some 'special' (not modeled) talent or knowledge to select securities would end up trading in a similar fashion as our informed traders. Since our performance evaluation is based on actions, we would end up making the right conjectures about the trader's abilities.[7]

Often, the distinction between talent/knowledge and possession of private information is more formal than material. Consider, for instance, the case of a trader with great knowledge of the tax system who specializes on trading based on tax arbitrage and finds profits from buying shares of company A and selling shares of company B. This is no different to a trader who receives an accurate signal that tells him that company A is underpriced and company B overpriced. Again, to the extent that they trade similarly, our methodology will classify both as skillful traders.

These advantages are in contrast with serious weaknesses in the traditional approach to performance evaluation. Originally, performance evaluation was conducted in the context of equilibrium models, such as the CAPM, that assume homogeneous beliefs. The implicit assumption is that traders who might not share these beliefs do not affect the equilibrium. In this context, performance evaluation is indeed a joint test of the

[6] Informational asymmetries exist throughout the market. Further, there is explicit evidence that performance is related to superior information. For instance, studies such as Lakonishok and Lee (2001) and Jeng et al. (2003) show that the portfolios tracking the purchases done by corporate insiders exhibit outstanding performance. Unfortunately, as Admati and Ross (1985) point out, standard performance evaluation measures, such as the Sharpe ratio or Jensen's alpha, will often fail to indicate superior performance for the portfolios of traders with superior information. This issue is further analyzed in the main text.

[7] Often, the distinction between talent/knowledge and possession of private information is more formal than material. Consider, for instance, the case of a trader with great knowledge of the tax system who specializes on trading based on tax arbitrage and finds profits from buying shares of company A and selling shares of company B. This is no different to a trader who receives an accurate signal that tells him that company A is underpriced and company B overpriced. Again, to the extent that they trade similarly, our methodology will classify both as skillful traders.

pricing model *and* the manager's abilities. Hence the inability of the asset pricing model to explain asset prices directly maps into faulty performance evaluation. In particular, to the extent that asymmetric information matters for equilibrium, this approach to performance evaluation is bound to fail. Indeed, Admati and Ross (1985) show that the two most important performance evaluation measures in this framework, the Sharpe ratio and Jensen's alpha, will tend to understate the performance of the informed traders' portfolios. In particular, the portfolios of the most talented traders will exhibit low Sharpe ratios and lie below the securities market line. More recently, the literature has sidestepped this theoretical conundrum and drifted from theoretical to merely empirical considerations. The most typical setting nowadays builds on the APT model and assumes a multifactor specification in which asset returns are explained by the three Fama and French factors (Fama and French (1993)) plus a fourth momentum factor (Carhart (1997)). These factors are just the ones that work better in sample. They may capture the implications of asymmetric information for asset pricing. But we do not know it with certainty. Performance is evaluated by running a time series regression of the portfolio excess returns on these four factors. The regression intercept ("alpha") is the measure of unusual performance. But still it maybe the case that the intercept is biased because of some additional missing factor not accounted for.[8] In a nutshell, the standard performance evaluation methodology is handy, but lacks a solid microfoundation and can easily result in the wrong assessment of managers' abilities.

We do not propose, however, to replace these methods with ours, but rather to complement them. One of the weaknesses of our approach is that it is not 100% immunized to reverse engineering. More formally, our equilibrium is not manipulation-proof. If traders know that they will be evaluated as skillful when buying with bad public news and selling with good public news, untalented traders may end up doing so, mimicking

[8] For completeness, we should also refer to the performance evaluation in the context of Sharpe's (1964) style regressions and in the context of the Henriksson and Merton (1981) and Treynor and Mazuy (1966) models.

the talented ones. This type of strategic behavior that may result in pooling equilibria not considered in “competitive” REE models. But we must notice that, while the actions of traders at the arrival of public news can be mimicked, the actions at the arrival of the private information cannot (as only privately informed traders can implement a measurable trade). This means that the joint observation of traders holdings and change in holdings can be all we need to restore the separating equilibrium. More important for the purposes of the present paper, also notice that the optimality condition embedded in the REE implies that mimicking trades necessarily result in non-optimal portfolios. This means that mimicking traders would end up with portfolios with very poor returns. Besides their deficiencies, the traditional performance evaluation methods can therefore be helpful at assessing the poor performance of these mimicking traders. In summary, we believe that performance evaluation must be implemented combining the traditional return-based and the present portfolio-based methods. In this context, outstanding performance must be associated to funds that deliver both “alphas" and trading patterns consistent with the trading of informed traders according to our analysis. As previously stated, our measures of performance have a sound microfoundation. On the other hand “alpha” may or may not reflect outstanding managerial skills. The joint criteria for performance evaluation that we propose in this paper is bound to succeed as it filters alphas with the requirement of having a sound microfoundation.

We emphasize that our methodology is empirically testable, as the two variables involved, trading data and public news, are observable, and that some new institutional arrangements could be greatly beneficial for its implementation.

Data on quarterly mutual funds portfolio holdings is available in most developed countries. Looking at changes in holdings around public information events is a good starting point. On the other hand, we believe more of this data could be disclosed willingly. Hedge funds are probably the most reluctant to disclose portfolio holdings in order to keep secrecy of their trading strategies. However, to attract investors, they willingly offer Due Diligence (DD) questionnaires where their investment philosophy, risk management technology, etc., are disclosed. It would be interesting to include in the DD process the possibility for a prospective client to inspect the trading history for periods that include arrivals of public information.[9] The disclosing period should be short enough to avoid unravelling the trading strategy, but long enough to assess the manager's skills using the tools we develop in this paper.

Regarding the second variable of interest, public information, we have plenty of sources in the financial industry. First of all, security prices are public. As we will see, this is all we need to implement some of the performance evaluation measures suggested by the model. Second, we also have plenty of instances of public information related to asset payoffs. The typical examples are earnings announcements and analysts' (earnings) forecasts. Also, as we mentioned before, we have several pieces of information related to "noise" in the market. Finally, earnings recommendations are the leading example of public information related to both noise and payoffs. In the last part of the paper we explicitly address the empirical implementation of our performance evaluation measures.

The implications of REE models for performance evaluation have been analyzed in Admati and Ross (1985) and Kacperczyk and Seru (2007). Unlike these two papers, we consider a dynamic model. Furthermore, while Admati and Ross (1985) assess performance conditional on the realized fundamental value of the asset, we propose a more implementable procedure that relies on the arrival of public information. Like us, Kacperczyk and Seru (2007) also rely on public information arrivals but cannot derive all the trading implications due to the static nature of their model.[10] Furthermore, as previously stated, our model is the first one analyzing the trading implications of the

[9] Obviously, it should be the client the one choosing the period of analysis!

[10] We also highlight that the model prediction tested with mutual fund data in this paper is faulty. First, it is not always true, even in the context of their specific model (as it depends on the fraction of informed traders in the market). Second, it does not generalize to the case of disperse (as opposed to asymmetric) information. Indeed, as our model shows, many uninformed traders can be even more unresponsive to the arrival of public news than some informed traders. The sign of the trade, which is ignored in the analysis of Kacperczyk and Seru (2007), is critical to sort traders out, as they will always trade in opposite directions.

arrival of public news on fundamental, on noise, or on both, a critical issue when bringing the theory to the data.

We close acknowledging some limitations of our analysis. We only consider trade reactions to the arrival of public news in the context of competitive equilibria and with fully rational expectations agents. This leaves outside our analysis the class of models with strategic behavior led by the initial papers by Kyle (1985) and Foster and Viswanathan (1996). We also leave outside our analysis models where high order beliefs matter, such as Kondor (2009). Whether our trade predictions survive in these settings is an open question. We feel however comfortable with the chosen setting, as we believe it is the closest to the actual functioning of financial markets.

The rest of the paper is organized as follows. In Section 2 we set the primitives of a general model of public information arrivals and prove the existence of a unique linear partially revealing REE. In Section 3 we decompose equilibrium asset prices into an expectations and a risk premium components. This decomposition allows as to analyze the main drivers behind the change in prices when public news arrives to the market. In Section 4 we analyze the trading patterns generated by the arrival of public news. Finally, Section 5 is dedicated to the analysis of the empirical implication of the model in terms of performance evaluation, and Section 6 to conclusions.

2 A Model of Trading Around Public Information Events

In this section we describe our model of asset trading around public announcements. The basic setting is mainly taken from Kim and Verrecchia (1991). There are many advantages in using this setting. First, it allows for closed form solutions for all variables of interest. Further, we verify in this paper that the model still retains this property in several extensions we cover, such as the analysis of the arrival of public information on noise. Finally, although the model explicitly deals with the case of disperse information and prices as aggregators of information, the setting can easily be redefined to address the case of one-sided private information where prices transmit, rather than aggregate, information à la Grossman-Stiglitz (Grossman and Stiglitz (1980)). For all these reasons we believe the model delivers results that must be shared by most of the models in the competitive REE tradition. We now describe the model in detail.

2.1 The Primitives

We consider a pure exchange economy with a continuum of traders and three dates $t = 1, 2, 3$. Agents trade in the first two dates and consume the single good of the economy in the third date. There are two assets in the economy, a risk-free and a risky asset. The risk-free asset has a perfectly elastic supply and, without loss of generality, offers a rate of return equal to zero. The payoff of the risky asset, denoted by v, is realized at $t = 3$. It is assumed that v is normally distributed with mean $\overline{v}$ and precision (inverse of variance) τ_v. Each agent i, $i \in [0, 1]$, is endowed with $\overline{x}_i$ units of the risky asset and B_i units of the risk-free asset, whose price is normalized to 1 (cash). The aggregate supply of the risky asset, denoted by

$$x \equiv \int_0^1 \overline{x}_i di$$

is not known to traders and is normally distributed with mean $\overline{x}$ and precision τ_x. In this setting, x is the standard noisy supply that is introduced in REE models to prevent prices from fully revealing agents' private information on fundamentals. Further, each agent receives a signal

$$s_i = v + \epsilon_i,$$

where ϵ_i is independently and normally distributed with mean 0 and precision τ_i. The average of agents' precision is given by

$$\overline{\tau} \equiv \int_0^1 \tau_i di.$$

It is assumed that the set $\{\tau_i\}$ is uniformly bounded so that $\overline{\tau}$ is well-defined. This in turn will allow us to write $\int_0^1 \tau_i s_i di = \overline{\tau} v$ since the noise term ϵ_i in each trader's private signal vanishes in the aggregate.

Endowed with securities and private information, at date $t = 1$ the market opens and agents trade at competitive prices. Right before the market opens for trading at date $t = 2$, there is a public announcement. In principle this public signal, s_p, can be related to the risky asset fundamental value, v, or to noise, x. So, in general the public signal takes the form:

$$s_p \equiv a_f s_f + a_n s_n$$

$$s_f = v + \eta_f$$

$$s_n = x + \eta_n,$$

where η_f and η_n are two independent normally distributed random variables with mean 0 and precision τ_f and τ_n, respectively, and a_f and a_n are two positive and bounded constants. For the time being we do not impose any additional restrictions on these two constants[11]. The cases $(a_f = 1, a_n = 0)$ and $(a_f = 0, a_n = 1)$ have interest on their own as they correspond to the cases in which the public signal is only related to asset payoffs and to noise, respectively. We analyze these two cases separately as they allow for a very intuitive equilibrium characterization.

Agents are risk averse and have preferences represented by a CARA utility function over date $t = 3$ final wealth (or consumption), W_i, that takes the form $U_i(W_i) = -\exp(-rW_i)$. For simplicity, we assume all agents have the same risk aversion, r. Each trader i's final wealth is given by $W_i = B_i + P_1\overline{x}_i + (P_2 - P_1)H_{1,i} + (v - P_2)H_{2,i}$, where P_1 and P_2 are the prices of the risky asset at dates $t = 1$ and $t = 2$, and $H_{1,i}$ and $H_{2,i}$ are trader i's holdings (expressed in number of shares) of the risky asset at dates $t = 1$

[11] As we will see later on, the main results of this paper hold for arbitrary a_f and a_n. This means that they also hold for typical normalizations such as restricting the constants so that the variance of the public signal is constant.

and $t = 2$, respectively.

In this economy agents only differ in the quality of their private information. This is the only reason for trading to occur at date 1.[12] Large τ_i traders are better informed than small τ_i traders, regardless of the specific s_i they receive.[13] All of them, however, learn from prices, which in equilibrium are a noisy signal of the true value of the asset, v. The arrival of public information at the second round of trade generates a reassessment of agents' beliefs about the asset payoff and gives rise to new trading. Hence differences in information quality not only play a crucial role in trading before the arrival of the public signal, but also afterwards. This is the key aspect of the model that makes it relevant for our purposes, as we need different patterns of trading for informed versus uninformed traders so that we can infer traders' abilities from actions (trading). Finally, it is worthwhile noticing that the amount of public information (prices and the public signal) in the market is increasing over time, so the risk premium of the asset falls at date $t = 2$.

2.2 Equilibrium Analysis

Endowed with their private information and aware that a public signal will arrive at date $t = 2$, agents trade in a *Rational Expectations Equilibrium (REE)*.[14] In this equilibrium agents use their private information, the public information and the information contained in the equilibrium prices, P_1 and P_2, in the determination of their optimal asset holdings. In equilibrium agents make self-fulfilling conjectures about the prices and information in the market. In the remaining of this section we solve for the equilibrium in the general case of a signal that can be related to both the asset payoff and noise. Then, in the next sections we address the two extreme cases separately. We focus on

[12] Strictly speaking, agents also differ in their initial endowments. This would generate trade at $t = 1$ toward the optimal risk sharing allocation. The arrival of the public signal at date $t = 2$ would generate price changes, but no trade at all.

[13] In this paper, we indistinguishably refer to trader with $\tau_i > \overline{\tau}$ ($\tau_i < \overline{\tau}$) as better (worse) informed or just informed (uninformed) traders.

[14] See Radner (1979).

linear equilibria and conjecture that, in equilibrium, prices take the form:

$$P_1 = \alpha_1 + \beta_1 v - \gamma_1 x \tag{1}$$

$$P_2 = \alpha_2 + \beta_2 v - \gamma_2 x + \delta_2 s_p, \tag{2}$$

where the intercept terms α_1 and α_2 depend on $\bar{v}$ and $\bar{x}$. We denote with $\Phi_{1,i}$ the information set of trader i when trading at the first date, which includes his private signal, s_i, and the price, P_1. The price conjecture in (1) can be equivalently rewritten in normalized form as

$$\theta_1 = \frac{P_1 - \alpha_1}{\beta_1} = v - \frac{\gamma_1}{\beta_1} x$$

and we set $\Phi_{1,i} = \{s_i, \theta_1\}$. At the second trading round, each trader information consists of the private signal, the public signal s_p, the date 1 price, P_1, (or equivalently the normalized price θ_1) and the date 2 price, P_2. Since s_p is known, the date 2 equilibrium price can be normalized as follows

$$\theta_2 = \frac{P_2 - \alpha_2 - \delta_2 s_p}{\beta_2} = v - \frac{\gamma_2}{\beta_2} x,$$

and the information set is $\Phi_{2,i} = \{s_i, s_p, \theta_1, \theta_2\}$.

Under this conjecture we have two potential type of equilibria in the model as a function of its price informativeness:

- Partially revealing equilibrium prices. This corresponds to $\frac{\gamma_1}{\beta_1} = \frac{\gamma_2}{\beta_2} \neq 0$. In this case, $\theta_1 = \theta_2 = \theta$ which means that, conditional on s_p, P_2 and P_1 are equally informative about v and x so that agents only update their beliefs using the information contained in the public signal. We believe this is the natural candidate for equilibrium in our event-study like economy where the only innovation between $t = 1$ and $t = 2$ is the arrival of the public signal.

- Dynamically fully revealing equilibrium prices. This corresponds to $\frac{\gamma_1}{\beta_1} \neq \frac{\gamma_2}{\beta_2} \neq 0$. In this case, as before, the equilibrium price at date $t = 1$ is partially revealing; but at date 2 (when agents observe both P_1 and P_2), prices become fully revealing. For this reason, equilibrium holdings at date 2 are indeterminate. This can be corrected by introducing a common error in agents' private signals as in Grundy and McNichols (1989). This equilibrium tends to be not unique and it is computationally demanding. For these reasons we do not analyze it in this paper.[15] We conjecture, however, that the main results in this paper will also hold in this equilibrium. In both our equilibrium and in this one, agents update their beliefs at date $t = 2$ with the arrival of the public signal. The only difference is that in the dynamically fully revealing equilibrium agents learn "more" (to the point of learning the fundamental value of the asset, v). To the extent that our main results hold because there is learning at $t = 2$, we expect the dynamically fully revealing equilibrium to reinforce, rather than reverse, our conclusions.[16]

We focus on the partially revealing equilibrium prices and from now on we assume (and later corroborate) that:

$$\kappa \equiv \frac{\gamma_1}{\beta_1} = \frac{\gamma_2}{\beta_2}$$

so that $\Phi_{1,i} = \{s_i, \theta\}$ and $\Phi_{2,i} = \{s_i, s_p, \theta\}$. The distributional assumptions in the

[15] An additional reason for not analyzing this equilibrium in this paper is that, as shown in Grundy and McNichols (1989), this equilibrium may arise even in the absence of a public signal arriving at date $t = 2$. This means that this equilibrium may be driven by pure sunspot considerations.

[16] Just for completeness, we must notice that, as in all RRE models, there exists a third type of equilibrium in which, in some "magic" way, the price at date 1 already reveals the asset's fundamental value, v. At this equilibrium, $P_1 = P_2 = v$ and asset holdings are indeterminate in the two rounds of trade. Although in our economy these prices meet the standard Borel measurability condition imposed on equilibrium prices according to the REE definition (see Kreps (1977)), we believe this equilibrium does not constitute a good description of the functioning of financial markets.

present model can be summarized as follows:

$$\begin{pmatrix} v \\ s_i \\ s_p \\ \theta \end{pmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \bar{v} \\ \bar{v} \\ a_f\bar{v}+a_n\bar{x} \\ \bar{v}-\kappa\bar{x} \end{bmatrix}, \begin{bmatrix} \tau_v^{-1} & \tau_v^{-1} & a_f\tau_v^{-1} & \tau_v^{-1} \\ & \tau_v^{-1}+\tau_i^{-1} & a_f\tau_v^{-1} & \tau_v^{-1} \\ & & \tau_p^{-1} & a_f\tau_v^{-1}-a_n\kappa\tau_x^{-1} \\ & & & \tau_v^{-1}+\kappa^2\tau_x^{-1} \end{bmatrix}\right), \tag{3}$$

where

$$\tau_p = \frac{\tau_f\tau_n\tau_v\tau_x}{a_f^2\tau_n\tau_x\left(\tau_f+\tau_v\right)+a_n^2\tau_f\tau_v\left(\tau_n+\tau_x\right)} \tag{4}$$

is the precision of the public signal. We now solve our model by backward induction. The presentation in the text is heuristic and focuses on the main results. In Appendices A and B we include all the formal derivations.

2.2.1 Equilibrium at date $t = 2$

Due to CARA utility maximization and the normality of the distribution of final wealth, agent i's date 2 holdings are

$$H_{2,i} = \frac{1}{r}K_{2,i}\left(\mathbb{E}\left(v|\Phi_{2,i}\right)-P_2\right) \tag{5}$$

where, $K_{2,i} \equiv \mathbb{V}^{-1}(v|\Phi_{2,i})$, is trader i posterior precision of v at date 2. Hence, to derive the optimal holdings we need to compute the first two conditional moments of v:

Lemma 1. *The date 2 posterior expectation of v is*

$$\mathbb{E}\left(v|\Phi_{2,i}\right) = \frac{1}{K_{2,i}}\left[\tau_v\bar{v}+\frac{\tau_x}{\kappa}\bar{x}+\tau_i s_i+\tau_p' s_p+\left(\frac{\tau_p' a_n}{\kappa}+\frac{\tau_x}{\kappa^2}\right)\theta\right], \tag{6}$$

where τ_p' is defined as

$$\tau_p' \equiv \frac{\tau_f\tau_n\left(a_n+\kappa a_f\right)}{\kappa\left(a_f^2\tau_n+a_n^2\tau_f\right)}. \tag{7}$$

The date 2 posterior precision of v is

$$K_{2,i} = \omega_f K_{2,i}^{(f)} + (1-\omega_f) K_{2,i}^{(n)} + 2\frac{a_f a_n \tau_f \tau_n}{\kappa\left(a_f^2 \tau_n + a_n^2 \tau_f\right)}, \tag{8}$$

where $\omega_f \equiv a_f^2 \tau_n / \left(a_f^2 \tau_n + a_n^2 \tau_f\right)$, and $K_{2,i}^{(f)}$ and $K_{2,i}^{(n)}$ denote, respectively, trader i's posterior precision of v at date 2 when the public signal only contains information about the asset payoff, $s_p = s_f$, or about the noise, $s_p = s_n$,

$$K_{2,i}^{(f)} = \tau_f + \tau_i + \tau_v + \frac{\tau_x}{\kappa^2} \quad \textit{and} \quad K_{2,i}^{(n)} = \tau_i + \tau_v + \frac{\tau_n + \tau_x}{\kappa^2}. \tag{9}$$

Proof. See Appendix A ■

In order to gain some intuition on the updating problem of trader i described in Lemma 1, suppose the public signal only contains information about the asset payoff, $s_p = s_f$,[17] so that $a_f = 1$ and $a_n = 0$. In this case $\omega_f = 1$ and the last term on the right hand side of eq. (8) vanishes, which means that the conditional precision is $K_{2,i}^{(f)}$, as each trader only learns s_f. Moreover τ_p' in (7) is just τ_f, and eq. (6) becomes

$$\mathbb{E}(v|\Phi_{2,i}^{(f)}) = \frac{1}{K_{2,i}^{(f)}}\left(\tau_v \bar{v} + \frac{\tau_x}{\kappa}\bar{x} + \tau_i s_i + \tau_f s_f + \frac{\tau_x}{\kappa^2}\theta\right)$$

where $K_{2,i}^{(f)}$ is given in (9). The public signal s_f positively affects the conditional expectation, as s_f contains information about the asset payoff. Similarly, when the public signal is only related to noise, i.e. $a_f = 0$ and $a_n = 1$, then $\omega_f = 0$ and $K_{2,i} = K_{2,i}^{(n)}$ from (8) as each trader learns s_n only. In this case $\tau_p' = \tau_n/\kappa$ and eq. (6) yield

$$\mathbb{E}(v|\Phi_{2,i}^{(n)}) = \frac{1}{K_{2,i}^{(n)}}\left(\tau_v \bar{v} + \frac{\tau_x}{\kappa}\bar{x} + \tau_i s_i + \frac{\tau_n}{\kappa}s_n + \frac{\tau_n + \tau_x}{\kappa^2}\theta\right).$$

[17] To avoid confusion, in the rest of the paper we use upperscripts "(f)" and "(n)" in some variables to emphasize that the variable is computed in the "f" economy ($a_f = 1$, $a_n = 0$) and the "n" economy ($a_f = 0$, $a_n = 1$), respectively. For easiness in notation, we omit these upperscripts on endogenous variables such as prices and holdings.

In all intermediate cases, in which the public signal provides information about both v and x, traders learn from both components albeit in somehow different ways. On the one hand, since s_f is a measurement error around the asset payoff, traders *directly* extract information about v from s_f. On the other hand, the signal on noise trading does not contain payoff-relevant information *per se*. However s_n enables traders to improve their estimate of x and then to use this knowledge to extract more information about v from the equilibrium price. According to equation (8), the posterior precision can be decomposed into a linear combination of $K_{2,i}^{(f)}$ and $K_{2,i}^{(n)}$, plus an adjustment term.

Making use of the posterior expectation in (6) yields date 2 holdings in (5) as

$$H_{2,i} = \frac{1}{r}\left(\tau_v \bar{v} + \frac{\tau_x}{\kappa}\bar{x} + \tau_i s_i + \tau_p' s_p + \frac{a_n \kappa \tau_p' + \tau_x}{\kappa^2}\theta - K_{2,i} P_2\right). \qquad (10)$$

Market clearing requires the aggregate supply of the risky asset, x, to equal aggregate holdings

$$x = \frac{1}{r}\left(\tau_v \bar{v} + \frac{\tau_x}{\kappa}\bar{x} + \tau_i s_i + \tau_p' s_p + \frac{a_n \kappa \tau_p' + \tau_x}{\kappa^2}\theta - K_2 P_2\right), \qquad (11)$$

where K_2 is the average of the asset payoff posterior precisions

$$K_2 \equiv \int_0^1 K_{2,i} di.$$

How does trader i posterior precision $K_{2,i}$ relate to its average counterpart K_2? Aggregating all traders' posterior precisions in (8) reveals that

$$K_2 - K_{2,i} = \bar{\tau} - \tau_i, \qquad (12)$$

which says that a trader with better private information than the average, i.e. $\tau_i > \bar{\tau}$, will maintain his informational advantage (relative to the average trader) until the end of the trading period.

Rewriting eq. (11) using the definition of $\theta = v - \kappa x$ yields the date 2 equilibrium price as

$$P_2 = \frac{1}{K_2}\left[\tau_v \bar{v} + \frac{\tau_x}{\kappa}\bar{x} + \left(\bar{\tau} + \frac{a_n \kappa \tau_p' + \tau_x}{\kappa^2}\right) v + \tau_p' s_p - \left(r + \tau_p' a_n + \frac{\tau_x}{\kappa}\right) x\right], \tag{13}$$

which has the linear form conjectured in (2). Thus

$$\alpha_2 = \frac{1}{K_2}\left(\tau_v \bar{v} + \frac{\tau_x}{\kappa}\bar{x}\right), \beta_2 = \frac{1}{K_2}\left(\bar{\tau} + \frac{a_n \kappa \tau_p' + \tau_x}{\kappa^2}\right), \gamma_2 = \frac{1}{K_2}\left(r + \tau_p' a_n + \frac{\tau_x}{\kappa}\right), \delta_2 = \frac{\tau_p'}{K_2},$$

and the ratio $\kappa = \gamma_2/\beta_2$ becomes $\kappa = \left(r + \tau_p' a_n + \frac{\tau_x}{\kappa}\right) / \left(\bar{\tau} + \frac{a_n \kappa \tau_p' + \tau_x}{\kappa^2}\right)$ so that

$$\kappa = \frac{r}{\bar{\tau}}. \tag{14}$$

2.2.2 Equilibrium at date $t = 1$

By means of the holdings in (10), trader i's final wealth becomes

$$W_i = B_i + P_1 \overline{x}_i + (P_2 - P_1) H_{1,i} + (v - P_2)\frac{1}{r} K_{2,i}\left(\mathbb{E}\left(v|\Phi_{2,i}\right) - P_2\right).$$

At date 1 each trader chooses holdings $H_{1,i}$ given his private signal s_i and the price signal P_1. Thus trader i solves the following optimization

$$\max_{H_{1,i}} \mathbb{E}\left[-\exp\left(-rW_i\right)|\Phi_{1,i}\right] =$$
$$= \max_{H_{1,i}} \mathbb{E}\left[-\exp\left\{-r\left(P_2 - P_1\right) H_{1,i} - K_{2,i}\left(v - P_2\right)\left(\mathbb{E}\left(v|\Phi_{2,i}\right) - P_2\right)\right\}|\Phi_{1,i}\right],$$

where on the RHS we have omitted the value of initial endowment, $B_i + P_1 \overline{x}_i$, which is non-random as P_1 belongs to trader i's information set. The solution to this problem can be found in the Appendix B, and writes as

$$\begin{aligned} H_{1,i} &= \frac{1}{r}\left[-\left(\frac{K_2 K_1}{K_{2,i}-K_{1,i}}+(\tau_i-\bar{\tau})\right)P_1+\frac{K_2}{K_{2,i}-K_{1,i}}\left(\tau_v\bar{v}+\frac{\tau_x}{\kappa}\bar{x}\right)\right. \\ &\quad \left.+\tau_i s_i+\frac{1}{K_{2,i}-K_{1,i}}\left(K_1\bar{\tau}+\frac{\tau_x K_2}{\kappa^2}\right)\theta\right], \end{aligned} \tag{15}$$

where

$$K_{1,i} \equiv \mathbb{V}^{-1}\left(v|\Phi_{1,i}\right) = \tau_v + \tau_i + \frac{\tau_x}{\kappa^2} \tag{16}$$

is the asset payoff posterior precision at date 1, and

$$K_1 \equiv \int_0^1 K_{1,i}di.$$

Observe that $K_{1,i} - K_1 = \tau_i - \bar{\tau}$, so that a trader whose private signal is more precise than the average to start with, also more precisely estimates the asset payoff after date 1 –similarly to what equation (12) prescribes at date 2. Thus, the second round of trade brings in an increase in the precision of each trader's forecast of v which is identical to the one experienced by the market, on average, i.e.

$$K_{2,i} - K_{1,i} = K_2 - K_1. \tag{17}$$

Equating the date 1 aggregate supply x to aggregate holdings from (15) then yields the date 1 equilibrium price as

$$P_1 = \frac{1}{K_1}\left[\tau_v\bar{v}+\frac{\tau_x}{\kappa}\bar{x}+\left(\bar{\tau}+\frac{\tau_x}{\kappa^2}\right)v-\left(r+\frac{\tau_x}{\kappa}\right)x\right]. \tag{18}$$

Finally, comparing the latter with the conjecture (1) reveals that the coefficients in the price function are

$$\alpha_1 = \frac{1}{K_1}\left(\tau_v\bar{v}+\frac{\tau_x}{\kappa}\bar{x}\right), \beta_1 = \frac{1}{K_1}\left(\bar{\tau}+\frac{\tau_x}{\kappa^2}\right), \gamma_1 = \frac{1}{K_1}\left(r+\frac{\tau_x}{\kappa}\right),$$

and the ratio γ_1/β_1 is $\left(r + \frac{\tau_x}{\kappa}\right) / \left(\bar{\tau} + \frac{\tau_x}{\kappa^2}\right)$, or equivalently

$$\frac{\gamma_1}{\beta_1} = \frac{r}{\bar{\tau}}$$

so that $\gamma_1/\beta_1 = \gamma_2/\beta_2 \neq 0$ as initially conjectured. We conclude that there exists a unique partially revealing rational expectations equilibrium where prices are linear as in (1) and (2).

3 Characterization of Equilibrium: Learning and Price Changes

Right after the first round of trade, agents hold an allocation which is not Pareto efficient. This means that the arrival of public news, the only innovation at date $t = 2$, generates changes in both the equilibrium price and the asset holdings. As a matter of fact, in Section 4 we show that each trader's change in holdings is an affine function of the price change. Hence, understanding the price change drivers is key to understand the changes in asset holdings, which is the variable that allows us to sort traders as a function of the precision of their private information. We first consider how the public signal affects trader i's learning over time about the final payoff:

Proposition 1 (individual filtering). *Traders update their posterior expectation and precision about the asset payoff according to*

$$\mathbb{E}\left(v|\Phi_{2,i}\right) - \mathbb{E}\left(v|\Phi_{1,i}\right) = \frac{\left(a_n + \kappa a_f\right)\tau_p'}{\kappa K_{2,i}}\left(\frac{\kappa s_p + a_n\theta}{a_n + \kappa a_f} - \mathbb{E}\left(v|\Phi_{1,i}\right)\right), \qquad (19)$$

and

$$K_{2,i} - K_{1,i} = \omega_f\tau_f + \left(1 - \omega_f\right)\frac{\tau_n}{\kappa^2} + 2\frac{a_f a_n \tau_f \tau_n}{\kappa\left(a_f^2\tau_n + a_n^2\tau_f\right)}. \qquad (20)$$

Proof. See Appendix C ■

Proposition 1 reveals that learning always takes place unless the public signal does not contain information on the asset payoff ($\tau_f = 0$) nor the noise ($\tau_n = 0$), in which case $\tau'_p = 0$ so that $\mathbb{E}(v|\Phi_{2,i}) = \mathbb{E}(v|\Phi_{1,i})$ from (19), and $K_{2,i} = K_{1,i}$ from (20). When the public signal provides some information, Proposition 1 yields $K_{2,i} > K_{1,i}$ so that equation (17) implies that the average precision increases over time as well, i.e. $K_2 > K_1$. The arrival of public news further causes traders to revise their expectations. Proposition 1 establishes that traders revise their expectation about the asset payoff in the same direction of the public signal, s_p. Moreover, the impact of this revision is inversely related to the quality of each trader's private information: better informed traders place a lower weight on the public signal, s_p, relative to worse informed traders.

To gain more insights about individual learning, we proceed to the analysis of the "f" and "n" economies. In the former case, since $\tau'_p = \tau_f$, it is easy to see from Proposition 1 that

$$\mathbb{E}(v|\Phi^{(f)}_{2,i}) - \mathbb{E}(v|\Phi^{(f)}_{1,i}) = \frac{\tau_f}{K^{(f)}_{2,i}} \left(s_f - \mathbb{E}(v|\Phi^{(f)}_{1,i}) \right). \quad (21)$$

Equation (21) reveals very intuitive results. First, a necessary condition for a trader to update his beliefs is that the public signal is related to v ($\tau_f \neq 0$). Second, the change in a trader's forecast is increasing in "his surprise", which is defined as the distance between the signal s_f and the trader's payoff forecast prior to the arrival of the public signal. Hence, the trader revises his forecast upwardly (downwardly) when good (bad) public news relative to his prior forecast arrive. Finally, notice that for a given surprise and public signal precision, the change in the forecast is decreasing in the precision of the trader's information. Hence, the change in the forecast is smaller for better informed traders than for worse informed traders. This means that, in a pure informational sense, worse informed traders "overreact", relative to better informed traders, to the arrival of public information on fundamentals. This explains why, as we document in the next section, when the market opens at $t = 2$ better and worse informed traders trade in opposite directions.

Consider now the "n" economy. In this case $\tau_p' = \tau_n/\kappa$ so that Proposition 1 yields

$$\mathbb{E}(v|\Phi_{2,i}^{(n)}) - \mathbb{E}(v|\Phi_{1,i}^{(n)}) = \frac{\tau_n}{\kappa^2 K_{2,i}^{(n)}} \left(\kappa s_n + \theta - \mathbb{E}(v|\Phi_{1,i}^{(n)}) \right). \tag{22}$$

At a first sight it may look awkward that trader i revises his expectations *on the asset payoff* upwardly when he learns that *noise* is high –which would be what a high signal s_n means. As a matter of fact, v and x are independent, so why does learning depend positively on s_n? The intuition for this result is as follows. Suppose that at date 2 the public signal s_n is large and positive. Since s_n is positively correlated with the noise, traders will infer that the realization of x at date 1 was large as well. Since the date 1 equilibrium price is negatively related to the amount of noise in the market, the public signal s_n therefore reveals to traders that v has to be large as well. As a result, trader i will revise upward his expectation of the asset payoff. In summary, when a trader learns that noise is higher than previously expected (bad news for the asset price), he concludes that the price can only clear the market if the expected cash flow is larger than previously expected (good news for the asset price). This explains why large realizations of s_n are associated with upwards revisions in the forecast of the asset payoff. According to (22), the signal on noise s_n needs to be larger than $\kappa^{-1}\left(\mathbb{E}(v|\Phi_{1,i}^{(n)}) - \theta\right)$ to sufficiently "surprise" trader i so that he revises his expectations upwardly. Finally, we emphasize that the rest of the conclusions of the previous case also hold here. In particular, s_n must be related to noise, i.e. $\tau_n > 0$, for traders to update their beliefs. Also, the change in a trader's forecast is increasing in "his surprise" and smaller for better informed traders than for worse informed traders. Again, worse informed traders "overreact" (in a pure informational sense), relative to better informed traders, to the arrival of public information on noise. As in the previous case, this explains why informed and uninformed traders trade in opposite directions at $t = 2$.

In Section 2 we proved the existence of a unique linear partially revealing equilibrium price. We are now ready to decompose the equilibrium prices in an expected cash flow and a risk premium component. This decomposition allows us to study separately how the arrival of public information generates changes in agents' beliefs and in the risk premium. To understand how traders' forecasts get impounded into prices we define the *market expectation* at date t as:

$$E_{t,M} \equiv \int_0^1 \frac{K_{t,i}}{K_t} \mathbb{E}\left(v|\Phi_{t,i}\right) di, \quad t = 1, 2, \tag{23}$$

i.e. $E_{t,M}$ is the date t precision-weighted average of traders' expectations of v. The next proposition establishes the relationship between equilibrium prices and market expectations.

Proposition 2 (price decomposition). *In equilibrium, prices and market expectations are related by:*

$$P_t = E_{t,M} - \frac{rx}{K_t}. \tag{24}$$

Proof. See Appendix D ■

According to Proposition 2, prices at date t can be decomposed into the market expectations of the asset payoff, $E_{t,M}$, and the risk premium, rx/K_t, components. This means that the change in the price generated by the public signal is equal to:

$$P_2 - P_1 = (E_{2,M} - E_{1,M}) - \frac{K_1 - K_2}{K_1 K_2} rx.$$

Hence, the price changes because the arrival of public news generates either a change in the market expectations, $\triangle ME$, or a change in the risk premium, $\triangle RP$, or both. For instance, in the case of $x > 0$, the arrival of public information lowers the risk premium as equation (20) in Proposition 1 certifies that learning reduces the uncertainty about the asset payoff, which implies that prices are less sensitive to the noisy asset supply. On the other hand, the public signal causes traders to revise their individual expectations (see equation (19) in Proposition 1), which in turn induces a change in *market expectations* as well. More formally, in Appendix D we show that

$$\triangle ME = \frac{\tau_p' (a_n + \kappa a_f)}{\kappa K_2} \left(\frac{\kappa s_p + a_n \theta}{a_n + \kappa a_f} - E_{1,M} \right), \tag{25}$$

which has the same form as the change in each trader posterior expectation of the asset payoff (see equation (19)).

Again, it is insightful to consider the two extreme cases of public information only related to the asset fundamentals ($a_f = 1$, $a_n = 0$) and to noise ($a_f = 0$, $a_n = 1$). In the former case, $\tau_p' = \tau_f$ and equation (25) readily gives

$$\triangle ME = \frac{\tau_f}{K_2^{(f)}} (s_f - E_{1,M}^{(f)}), \tag{26}$$

while $K_2^{(f)} - K_1 = \tau_f$ yields[18] the change in risk premium as

$$\triangle RP = -\frac{\tau_f}{K_1 K_2^{(f)}} rx. \tag{27}$$

First notice that a necessary condition for the price to change is that $\tau_f > 0$. This means that our equilibrium is sunspots free. When $\tau_f > 0$, the arrival of public information generates changes on *both* the market expectations and the asset's risk premium. In particular, the arrival of the public signal always lowers the risk premium, as long as there is a strictly positive supply of stock in the market $x > 0$. Market expectations revise upwardly (downwardly) provided the public signal, s_f, is larger (smaller) than $E_{1,M}^{(f)}$ –similarly to what equation (21) predicts for the learning of each individual trader. Further, notice that both the change in market expectations and in the risk premium are affine functions of τ_f. Hence, the more informative the public signal is, the higher its impact on expectations and the risk premium. Finally, notice that the price change

[18] Note that the average of the asset payoff posterior precisions after the first trading round is the same in both the "f" and "n" economies. Thus we omit upperscripts for K_1.

does not depend directly on traders' private signals: it just depends on the public signal, the precision of the public signal, and some parameters of the economy. As we will see in the next section, this is a key property of our model.

When the public signal is only related to noise ($a_f = 0$, $a_n = 1$) we have that $\tau_p' = \tau_n/\kappa$ and equation (25) becomes

$$\triangle ME = \frac{\tau_n}{\kappa^2 K_2^{(n)}}(\kappa s_n + \theta - E_{1,M}^{(n)}),$$

while the change in risk premium reads

$$\triangle RP = -\frac{\tau_n}{\kappa^2 K_1 K_2^{(n)}} rx.$$

The results in this case are qualitatively identical to those obtained in the previous one. In particular, a necessary condition for changes in market expectations and risk premium is $\tau_n > 0$ and the risk premium falls as long as this condition is met and $x > 0$. The only substantial difference comes from the condition on the signal to generate an upward versus downward revision in market expectations. For instance, in order for the price to increase we need a signal s_n large enough so that $\kappa s_n + \theta > E_{1,M}^{(n)}$. But, again, the left hand side of this inequality is related to what each agent learns about v when observing the public signal, as equation (22) reveals.

4 Trading Patterns Around Public Events

Having analyzed the economics behind price changes in our model, we now turn to characterize traders' responsiveness to the arrival of public news. In this paper we analyze the following measures of price responsiveness to the arrival of public news:

- Trader i's *change in holdings*, or trade: $\triangle H_i \equiv H_{2,i} - H_{1,i}$
- Trader i's *volume* of trade: $VOL_i \equiv |H_{2,i} - H_{1,i}|$

In the next proposition we state the main result of the present paper. The result applies to the general case of a public signal that can be arbitrarily related to noise and fundamentals, and corroborates that REE models can deliver sharp testable predictions on trading with bearing on performance evaluation.

Proposition 3. *Change in holdings and volume are*

$$\triangle H_i = -\frac{1}{r}\left(\tau_i - \bar{\tau}\right)\left(P_2 - P_1\right) \tag{28}$$

and

$$VOL_i = \frac{1}{r}\left|\tau_i - \bar{\tau}\right|\left|P_2 - P_1\right| \tag{29}$$

Proof. See Appendix E ■

The results stated in Proposition 3 are remarkable for several reasons. First, notice that by (28), each trader i's change in holdings, $H_{2,i} - H_{1,i}$, is proportional to the price change which, as we saw before, is driven by the change in market expectations, $\triangle ME$, and the change in the risk premium, $\triangle RP$. In particular, agent i's private information, s_i, does not enter directly in the trade. Traders private information only affects trade indirectly through the change in the average market belief, $\triangle ME$. In the cross section of traders, the change in holdings is therefore mainly driven by the trader's precision, τ_i. Second, by (28), we have that the sign of the trade is different for better informed traders, $\tau_i > \bar{\tau}$, and for less informed traders, $\tau_i < \bar{\tau}$. More formally, let's sort traders as a function of τ_i in the the set of informed traders, T_I, and uninformed traders, T_U,[19] where:

$$T_I \equiv \{\tau_i | \tau_i > \bar{\tau}\} \quad \text{and} \quad T_U \equiv \{\tau_i | \tau_i \leq \bar{\tau}\} \tag{30}$$

[19] We reiterate here that, strictly speaking, all traders are informed in our economy. These two sets are defined in the context of the abuse of language throughout the paper referring to better informed traders as "informed traders", and to worse informed traders as "uninformed traders".

Equation (28) states that informed agents trade against the market, while uninformed agents trade with the market. That is, informed and uninformed traders follow opposite strategies, as the former are contrarian traders, while the latter are momentum traders. Moreover, equation (28) establishes that, for a given price change $P_2 - P_1$, the trading intensity of trader i is proportional to $\tau_i - \bar{\tau}$, that is, to the "distance" between trader i's and the average trader's precisions. Consequently:

- When the arrival of public news generates a price increase:
  - All informed traders sell, but those relatively better informed sell more.
  - All uninformed traders buy, but those relatively worse informed buy more.
- When the arrival of public news generates a price decrease:
  - All informed traders buy, but those relatively better informed buy more.
  - All uninformed traders sell, but those relatively worse informed sell more.

In summary, equation (28) allows to globally rank all traders in the economy as it provides a one to one mapping between $\triangle H_i$ and τ_i for any given price change. On the other hand, note that looking at volume (see equation (29)) does not allow a one-to-one mapping into traders' talent. In essence for each well informed trader that trades a lot, there is an uninformed trader trading the same amount (but in the opposite direction). A one-to-one mapping, however, can be established conditional on the sets T_I or T_U. In other words, once we know that traders are informed or uninformed, trading volume allows for a full ranking of traders as a function of the talent.

In the previous paragraph we discussed the talent inference problem as a function of the price change generated by the arrival of a general public signal. We now turn to analyze the inference problem in our two particular economies separately to gain further intuition. In particular, we provide a characterization of the inference problem as a function of the size of public signal, instead of the price change.

4.1 Trading Patterns with Public News on Fundamentals

In this subsection we characterize trading in the equilibrium of an economy with a public signal which is only related to the asset payoff. Using expressions (26) and (27) we have the following two alternative characterizations of the price change:

$$P_2 - P_1 = \frac{\tau_f}{K_2^{(f)}} \left(s_f - E_{1,M}^{(f)} + \frac{rx}{K_1^{(f)}} \right) \tag{31}$$

$$P_2 - P_1 = \frac{\tau_f}{K_2^{(f)}} \left(s_f - P_1 \right). \tag{32}$$

Given that the risk premium falls (for $x > 0$), equation (31) implies that the price would increase even if there was no change in market expectations, $s_f = E_{1,M}^{(f)}$. Hence the condition for a price increase is weaker than requiring an increase in market expectations. Equation (32) states that for the price increase it is sufficient that $s_f > P_1$ (provided $\tau_f \neq 0$).

We now compute agent i's trade or change in holdings when public news arrive, which Proposition 3 delivers as

$$\triangle H_i = -\frac{\tau_f}{rK_2^{(f)}} (\tau_i - \bar{\tau})(s_f - P_1). \tag{33}$$

According to (33), the change in holdings is driven by the trader's precision, τ_i and the level of the public signal relative to the pre-announcement price, P_1. These two variables are the main drivers behind the trade direction as well as the number of shares bought or sold. As expected, given our discussion after Proposition 3, the sign of the trade is different for informed versus uninformed traders. In particular, informed traders sell when "good" public news arrive to the market, and buy when "bad" public news arrive to the market. They are in the right side of the trade as they sell at higher prices, and buy at lower prices relative to those prevailing in the previous period. Uninformed traders do exactly the opposite. Now, it is easy to see that the change in holdings is always decreasing for the informed traders and increasing for the uninformed traders in the public signal, s_f, or the public signal surprise, $s_f - P_1$. In particular:

$$\frac{\partial \triangle H_i}{\partial s_f} = \frac{\partial \triangle H_i}{\partial (s_f - P_1)} = -\frac{\tau_f}{rK_2^{(f)}}(\tau_i - \bar{\tau}), \tag{34}$$

which is always negative for the informed traders and positive for the uninformed traders. On the other hand, for a fixed s_f, the change in holdings is decreasing in τ_i when $s_f > P_1$, and increasing in τ_i otherwise. Figure 1 shows clearly that there is a one-to-one mapping from agents' τ_i and $\triangle H_i$ for any given public signal. This means that we can use standard econometric techniques to infer talent, τ_i, from $\triangle H_i$ around public announcements. In section 5 we address in more detail the empirical implications of the model.

The reaction to public news in terms of trading volume is slightly different. From (33) we have:

$$VOL_i = \frac{\tau_f}{rK_2^{(f)}} \left|\tau_i - \bar{\tau}\right| \left|s_f - P_1\right|.$$

Hence, for a fixed s_f, volume is increasing in the distance to average talent $|\tau_i - \bar{\tau}|$; that is, volume is increasing in τ_i for the informed traders ($\tau_i > \bar{\tau}$), and decreasing in τ_i for the uninformed traders ($\tau_i < \bar{\tau}$). On the other hand, for a fixed trader (τ_i), volume is increasing on the public news surprise $|s_f - P_1|$; that is, volume is increasing on s_f when $s_f > P_1$, and decreasing in s_f when $s_f < P_1$. Figure 2 shows how a global ranking is not possible in this case. As previously mentioned, It is possible, however, to sort traders conditional on τ_i in the set of informed traders, T_I, or uninformed traders, T_U.

4.2 Trading Patterns with Public News on Noise

Using the results in section 3, when $a_f = 0$ and $a_n = 1$ we have

$$P_2 - P_1 = \frac{\tau_n}{\kappa^2 K_2^{(n)}} \left(\kappa s_n + \theta - P_1\right). \tag{35}$$

As discussed in that section, when s_n is such that $\kappa s_n + \theta > P_1$, the increase in market expectations and the reduction in the risk premium result in a price increase. We now turn to the trade analysis. Proposition 3 yields

$$\triangle H_i = -\frac{\tau_n}{r\kappa^2 K_2^{(n)}}(\tau_i - \bar{\tau})(\kappa s_n + \theta - P_1).$$

Note that the main difference with respect to the "f" economy is that the explicit condition for price changes is different in this economy with public news on noise (equation (35)) and the economy with public news on fundamentals (equation (32)). Given that the change in holdings is qualitatively the same as before, so it is the volume of trade.

Hence, all the analysis of the previous subsection holds here. For instance, we have that

$$\frac{\partial \triangle H_i}{\partial s_n} = -\frac{\tau_n}{r\kappa^2 K_2^{(n)}}(\tau_i - \bar{\tau}), \tag{36}$$

which is always negative for the informed traders and positive for the uninformed traders. The only notorious exception with respect to the previous subsection is that the comparative exercises are not done splitting the public signal space between those s_f larger versus smaller than P_1, but rather for all s_n such that $\kappa s_n + \theta$ is larger versus smaller than P_1. This characterization gives rise to some empirical concerns, which we address in the next section.

5 Performance Evaluation: Empirical Issues

Our model maps agents' precisions into trading patterns. Based on these results, in this section we briefly discuss the empirical methods that can be used to invert the mapping and infer agents' precisions from trading data. In particular, we focus on the use of the basic event studies and time series regressions methodologies. The expert reader will certainly be able to come up with more sophisticated approaches.

5.1 Event Studies Methodology

The application of the event studies methodology is straightforward as our model fits the typical setting in these studies. In the case of a single event, the first task is to determine if it is associated to good versus bad fundamental or noise public news. In our model this corresponds to the cases:

$$s_f = \begin{cases} s_f > P_1 = E^{(f)}_{1,M} + \frac{rx}{K_1}, & \text{GOOD public news on fundamentals} \\ s_f < P_1 = E^{(f)}_{1,M} + \frac{rx}{K_1}, & \text{BAD public news on fundamentals} \end{cases}$$

$$s_n = \begin{cases} s_n > \frac{P_1-\theta}{\kappa} = \frac{E^{(n)}_{1,M}-\theta}{\kappa} + \frac{x}{K_1}, & \text{GOOD public news on noise} \\ s_n < \frac{P_1-\theta}{\kappa} = \frac{E^{(n)}_{1,M}-\theta}{\kappa} + \frac{x}{K_1}, & \text{BAD public news on noise} \end{cases}$$

Conditional on the particular type of event we have at hand, our model predicts that the *change in holdings* is either decreasing (case of good news) or increasing (case of bad public news) in talent. In the case of multiple event studies we just run separate regression of the change in holdings on the news that belong to the set of good news and the news in the set of bad news. Strictly speaking, to sort events we have to compare the realization of the public signal with past prices or market expectations and risk premia. Alternatively, we could use the price change, $P_2 - P_1$, as the sorting variables. There are good proxies for all these cases in actual financial markets. For instance, in the case of public news on fundamentals, we can define a GOOD event as an earnings announcement that generates a price increase. Alternatively, we could define it as an earnings announcement above the analysts' consensus on earnings forecasts, or simply an increase in earnings. In the last two cases we ignore the change in risk premium in the classification of the events. Given that the risk premium is positive in the data, in terms of our model, this means that we are classifying some of the GOOD events as BAD events. Consequently, the test in the set of GOOD events will be correct, while there could be problems with some "small" bad news. The econometrician can impose a suitable criteria to exclude these so that the ranking is accurate in the whole restricted set of bad news.

Unfortunately, *volume* is not mapped uniquely into talent and, consequently cannot be used as a sorting variable in this type of studies. In order to use volume we should be able to split the data of traders into informed versus uninformed *ex ante*. For instance, one might conjecture that all hedge fund managers are informed (i.e., belong to the set T_I defined in (30)) and that the task is to sort from most to least talented within this group. As we saw in section 4, once we are able to *ex ante* assign traders to the sets T_I or T_U, given in (30), we have a unique ranking of managers' talent as a function of their trading volume. Of course, for these conditional subsets we could also use the change in holdings as a predicting variable.

5.2 Times Series Methodology

We can also run unconditional regressions of the change in holdings on the *change in the public signal*, using our model implications outlined in (34) and (36).[20] The sign and size of the public signal coefficients (beta) will tell us all we need to know about the abilities of the traders in the sample. For instance, and for the sake of the argument, in the case of public information on fundamentals we could consider the change in quarterly earnings to measure the change in s_f, say $\Delta s_f = s_{f,t} - s_{f,t-1}$, and data on quarterly holdings of mutual funds to measure $\Delta H_i = H_{i,t} - H_{i,t-1}$, and then run the following regression:

$$\Delta H_i = \alpha_i + \beta_i \Delta s_f + \varepsilon_i. \tag{37}$$

Then, according to (34) we expect skilled managers to have negative betas whose absolute value is increasing in talent and unskilled traders will have positive betas which are decreasing in talent. In the case of public information on noise, (36) lends itself to an empirical counterpart very similar to regression (37) with Δs_n in-lieu of Δs_f. For

[20] See our previous discussion on how to proxy for this *change in the public signal*.

instance, one could proxy for changes in risk aversion by means of the change in gold price or the difference between the physical and the risk neutral distributions of returns.

As before, the *volume* of trade would not work on this type of regressions as volume is arbitrarily high for both very talented and not talented traders. This time series regressions would work if, as before, we are able to split the data of traders between informed and uninformed *ex ante*.

As we mentioned in the introduction, the availability of data both on portfolio holdings and public events makes these empirical exercises perfectly implementable in practice.

6 Conclusion

The main goal of our paper is to shed light on the relationship between fund managers' private information and their performance. As private information is very difficult –if not impossible– to measure, we bypass this problem and concentrate on the trading behavior of fund managers around public information arrivals. To this end, we develop a two-period rational expectation equilibrium (REE) model where agents possess private information about an asset payoff at the first trading round, and then public news flows to the market at the second trading round. The arrival of public information changes traders' beliefs about the asset payoff, and generates trading. The specification of the public signal is kept as general as possible, in order to encompass the different types of public information affecting prices, allowing for public signals related to the asset fundamentals, the noise in the market, or both. Real world examples of these signals include earnings announcements (payoff related), measures of risk aversion (noise related) and analyst recommendations (related to both). We focus on linear equilibria that partially reveal the asset payoff, and characterize the unique equilibrium within this class. We show that the equilibrium price can be decomposed in two parts: the market expectation of the asset payoff and a risk premium component. In order to gain an intuition on the drivers behind the changes in prices and asset holdings, we specialize our framework to a public signal that conveys information on either payoffs or noise. Our two main findings are: 1) good managers are contrarian traders and bad managers are momentum traders; and 2) the size of the trade is increasing in the distance of the traders' talent to the average talent in the market. Based on these predictions, we propose new performance evaluation measures that rely on the manager's change in holdings around the arrival of public news rather than his past performance. In general, we argue that disclosure of holdings may be included in hedge funds due DD procedures in order to better assess the manager's talent.

We believe our methodology should complement, rather than substitute, the more traditional return-based performance evaluation methodology. More specifically, we propose a new protocol for performance evaluation that requires outstanding performance both in terms of "alpha" and in terms of trading patterns consistent with informed trading. Given the sound microfoundation of our measures of performance we would then identify the true alphas arising from superior management skills.

References

Admati, Anat R., and Stephen A. Ross, 1985, Measuring Investment Performance in a Rational Expectations Equilibrium Model, *Journal of Business*, 58, 1-26.

Black, Fisher, 1986, Noise, *Journal of Finance*, 41, 529-543.

Bliss, Robert R., and Nikolaos Panigirtzoglou, 2004, Option-Implied Risk Aversion Estimates, *Journal of Finance*, 59, 407-446.

Bucks, Brian K., Arthur B. Kennickell, Traci L. Mach, and Kevin B. Moore, 2009, Changes in U.S. Family Finances from 2004 to 2007: Evidence from the Survey of Consumer Finances, Federal Reserve Bulletin.

Carhart, Mark M., 1997, On Persistence in Mutual Fund Performance, *Journal of Finance*, 52, 57-82.

Fama, Eugene F., and Kenneth R. French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics*, 33, 3-56.

Foster, F. Douglas, and S. Viswanathan, 1996, Strategic Trading When Agents Forecast the Forecast of Others, *Journal of Finance*, 51, 1437-1478.

Grossman, Sanford J., 1995, Dynamic Asset Allocation and the Informational Efficiency of Markets, *Journal of Finance*, 50, 773-787.

Grossman, Sanford J., and Joseph E. Stiglitz, 1980, On the Impossibility of Informationally Efficient Markets, *American Economic Review*, 70, 393-408.

Grundy, Bruce D., and Maureen McNichols, 1989, Trade and the Revelation of Information through Prices and Direct Disclosure, *Review of Financial Studies*, 2, 495-526.

Henriksson, Roy D., and Robert C. Merton, 1981, On Market Timing and Investment Performance. II. Statistical Procedures for Evaluating Forecasting Skills, *Journal of Business*, 54, 513-533.

Investment Adviser Association, 2008, Evolution on Revolution: A Profile of the Investment Adviser Profession, available at www.investmentadviser.org.

Jackwerth, Jens Carsten, 2000, Recovering Risk Aversion from Option Prices and Realized Returns, *Review of Financial Studies*, 13, 433-451.

Jeng, Leslie, Andrew Metrick, and Richard Zeckhauser, 2003, Estimating the Returns to Insider Trading: A Performance Evaluation Perspective, *Review of Economics and Statistics*, 85, 453-471.

Kacperczyk, Marcin, and Amit Seru, 2007, Fund Manager Use of Public Information: New Evidence on Managerial Skills, *Journal of Finance*, 62, 485-528.

Kim, Oliver, and Robert E. Verrecchia, 1991, Trading Volume and Price Reactions to Public Announcements, *Journal of Accounting Research*, 29, 302-321.

Kondor, Peter, 2009, The More We Know, the Less We Agree: Higher-order Expectations, Public Announcement and Rational Inattention, mimeo, Central European University.

Kreps, David M., 1977, A Note on "Fulfilled Expectations" Equilibria, *Journal of Economic Theory*, 14, 32-43.

Kyle, Albert S., 1985, Continuous Auctions and Insider Trading, *Econometrica*, 53, No. 6, 1315-1335.

Lakonishok, Joseph, and Inmoo Lee, 2001, Are Insider Trades Informative?, *Review of Financial Studies*, 14, 79-111.

Lewellen, Jonathan, 2009, Institutional Investors and the Limits of Arbitrage, *Journal of Financial Economics* (forthcoming).

Marin, Jose M., and Rohit Rahi, 2000, Information Revelation and Market Incompleteness, *Review of Economic Studies*, 67, 563-579.

Radner, Roy, 1979, Rational Expectations Equilibrium: Generic Existence and the Information Revealed by Prices, *Econometrica*, 47, 655-678.

Sharpe, William, 1964, Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risks, *Journal of Finance*, 19, 425-442.

Treynor, Jack L., and Kay K. Mazuy, 1966, Can Mutual Funds Outguess the Market?, *Harvard Business Review*, 44, 131-136.

Appendix

A. Proof of Lemma 1

Consider the first two moments of the distribution of the vector (v, s_i, s_p, θ) in (3) and define as M_i the variance-covariance matrix of $\Phi_{2,i}$. The determinant of M_i is

$$|M_i| = \frac{m_i}{\tau_i \tau_p \tau_v^2 \tau_x^2}$$

with

$$m_i = \tau_v \tau_x \left(\tau_x + \kappa^2 \tau_i + \kappa^2 \tau_v\right) - \tau_p \left[\left(a_f \tau_x - a_n \kappa \tau_v\right)^2 + \kappa^2 \tau_i \left(a_f^2 \tau_x + a_n^2 \tau_v\right)\right]. \quad \text{(A1)}$$

We will throughout assume that the parameter restrictions guaranteeing the matrix M_i to be a valid variance-covariance matrix hold true.

By properties of the normal distribution we have that

$$\mathbb{E}\left(v|\Phi_{2,i}\right) = m_i^{-1}\left(a_2 \bar{v} + b_2 \bar{x} + c_{2,i} s_i + d_2 s_p + e_2 \theta\right), \quad \text{(A2)}$$

where

$$a_2 = m_i - c_{2,i} - a_f d_2 - e_2 \quad \text{(A3.1)}$$

$$b_2 = \kappa e_2 - a_n d_2 \quad \text{(A3.2)}$$

$$c_{2,i} = \kappa^2 \tau_i \left(\tau_v \tau_x - \tau_p \left(a_f^2 \tau_x + a_n^2 \tau_v\right)\right) \quad \text{(A3.3)}$$

$$d_2 = \kappa \tau_p \tau_v \tau_x \left(a_n + a_f \kappa\right) \quad \text{(A3.4)}$$

$$e_2 = \tau_x \left(\tau_v \tau_x - a_f \tau_p \left(a_f \tau_x - a_n \kappa \tau_v\right)\right) \quad \text{(A3.5)}$$

Since the coefficients d_2 and e_2 are identical for all traders, so is b_2 in (A3.2). To see that also a_2 is identical for all traders, one substitutes m_i, $c_{2,i}$, d_2 and e_2 (defined, respectively, in eqs. (A1),(A3.3),(A3.4) and (A3.5)) in (A3.1) and gets

$$a_2 = \kappa^2 \tau_v \left(\tau_v \tau_x - \tau_p \left(a_f^2 \tau_x + a_n^2 \tau_v\right)\right), \quad \text{(A4)}$$

which allows to rewrite (A3.2) and (A3.3) as

$$b_2 = \frac{\tau_x a_2}{\kappa \tau_v} \quad \text{and} \quad c_{2,i} = \frac{\tau_i a_2}{\tau_v}. \quad \text{(A5)}$$

Making use of τ_p from (4) in the expression (A4) allows to rewrite a_2 as

$$a_2 = \frac{\left(a_f^2 \tau_n + a_n^2 \tau_f\right) \kappa^2 \tau_v^3 \tau_x^2}{a_f^2 \tau_n \tau_x \left(\tau_f + \tau_v\right) + a_n^2 \tau_f \tau_v \left(\tau_n + \tau_x\right)}, \quad \text{(A6)}$$

which is clearly positive. It follows from (A5) that b_2 and $c_{2,i}$ are positive as well. Finally, substituting τ_p from (4) in eqs. (A3.4) and (A3.5) yields

$$d_2 = \frac{\tau'_p a_2}{\tau_v} \quad \text{and} \quad e_2 = \left(\frac{\tau'_p a_n}{\kappa} + \frac{\tau_x}{\kappa^2} \right) \frac{a_2}{\tau_v} \tag{A7}$$

where τ'_p is defined in (7). Since $a_2 > 0$, from eqs. (7) and (A7) we have that $\kappa > 0$ is a sufficient condition for d_2 and e_2 to be positive -in the main text we indeed show this condition is satisfied in equilibrium, see eq. (14).

The posterior variance of the asset payoff at date 2 is

$$\begin{aligned} \mathbb{V}(v|\Phi_{2,i}) &= \frac{1}{\tau_v} - \frac{1}{\tau_v m_i}(c_{2,i} + a_f d_2 + e_2) \\ &= \frac{a_2}{\tau_v m_i}, \end{aligned}$$

where the last line follows from (A3.1).

Equivalently, the date 2 posterior precision of v is

$$K_{2,i} = \frac{m_i \tau_v}{a_2}. \tag{A8}$$

Making use of (A8) in the date 2 posterior expectation (A2) yields

$$\mathbb{E}(v|\Phi_{2,i}) = \frac{1}{K_{2,i}} \left(\tau_v \bar{v} + \frac{\tau_v b_2}{a_2} \bar{x} + \frac{\tau_v c_{2,i}}{a_2} s_i + \frac{\tau_v d_2}{a_2} s_p + \frac{\tau_v e_2}{a_2} \theta \right),$$

and the expectation in (6) follows from the latter and the expressions for the projection coefficients in (A5) and (A7).

Making use of eqs. (4),(A1) and (A6), the posterior precision $K_{2,i}$ in (A8) becomes

$$K_{2,i} = \frac{a_f^2 \tau_n \left(\tau_f + \tau_i + \tau_v + \frac{\tau_x}{\kappa^2}\right) + a_n^2 \tau_f \left(\tau_i + \tau_v + \frac{\tau_n + \tau_x}{\kappa^2}\right) + 2\frac{a_f a_n \tau_f \tau_n}{\kappa}}{a_f^2 \tau_n + a_n^2 \tau_f},$$

which is (8) with $\omega_f = a_f^2 \tau_n / \left(a_f^2 \tau_n + a_n^2 \tau_f\right)$. $\square$

B. Derivation of date $t = 1$ holdings

First define:

$$y_i \equiv r(P_1 - P_2) H_{1,i} - K_{2,i}(v - P_2)(\mathbb{E}(v|\Phi_{2,i}) - P_2),$$

and using the LIE, trader i objective function at date 1 rewrites as $E[E(-e^{y_i}|\Phi_{2,i})|\Phi_{1,i}]$. Note that, conditional on $\Phi_{2,i}$, the asset payoff v is the only source of randomness in y_i.

Therefore e^{y_i} is lognormally distributed as

$$y_i|\Phi_{2,i} \sim \mathcal{N}\left(r\left(P_1 - P_2\right) H_{1,i} - K_{2,i}\left(\mathbb{E}\left(v|\Phi_{2,i}\right) - P_2\right)^2, \underbrace{K_{2,i}^2\left(\mathbb{E}\left(v|\Phi_{2,i}\right) - P_2\right)^2 \frac{1}{K_{2,i}}}_{=K_{2,i}\left(\mathbb{E}\left(v|\Phi_{2,i}\right)-P_2\right)^2} \right).$$

It then follows that

$$\mathbb{E}\left(-e^{y_i}|\Phi_{2,i}\right) = -\exp\left\{ r\left(P_1 - P_2\right) H_{1,i} - \frac{1}{2} K_{2,i}\left(\mathbb{E}\left(v|\Phi_{2,i}\right) - P_2\right)^2 \right\},$$

and trader i objective function becomes $\mathbb{E}\left(-e^{z_i}|\Phi_{1,i}\right)$, where

$$z_i \equiv r\left(P_1 - P_2\right) H_{1,i} - \frac{1}{2} K_{2,i}\left(\mathbb{E}\left(v|\Phi_{2,i}\right) - P_2\right)^2. \tag{B1}$$

Before proceeding, we have rewrite the term $\mathbb{E}\left(v|\Phi_{2,i}\right) - P_2$. The date 2 posterior expectation in (A2) can be rewritten as

$$\begin{aligned}
\mathbb{E}\left(v|\Phi_{2,i}\right) &= \frac{\tau_i s_i}{K_{2,i}} + \frac{1}{K_{2,i}}\left(\tau_v \bar{v} + \frac{\tau_x}{\kappa}\bar{x} + \bar{\tau} v + \frac{d_2 \tau_v}{a_2} s_p + \frac{e_2 \tau_v}{a_2}\left(v - \kappa x\right) - rx \right) - \frac{1}{K_{2,i}}\left(\bar{\tau} v - rx\right) \\
&= \frac{\tau_i s_i}{K_{2,i}} + \frac{K_2}{K_{2,i}} P_2 + \frac{1}{K_{2,i}}\left(rx - \bar{\tau} v\right) \\
&= \frac{1}{K_{2,i}}\left(\tau_i s_i + K_2 P_2 - \bar{\tau}\theta\right),
\end{aligned}$$

where the definition of θ together with (A5-A8) give the first line, the equilibrium price in (13) yields the second line, and the last line obtains as $v = \theta + \kappa x$ and from (14). Using the latter and (12) yields

$$\mathbb{E}\left(v|\Phi_{2,i}\right) - P_2 = \frac{1}{K_{2,i}}\left(-\left(\tau_i - \bar{\tau}\right) P_2 + \tau_i s_i - \bar{\tau}\theta\right),$$

so that z_i in (B1) rewrites as

$$z_i = r\left(P_1 - P_2\right) H_{1,i} - \frac{1}{2K_{2,i}}\left(-\left(\tau_i - \bar{\tau}\right) P_2 + \tau_i s_i - \bar{\tau}\theta\right)^2.$$

Then letting

$$\hat{P}_2 \equiv P_2 - \mathbb{E}\left(P_2|\Phi_{1,i}\right) \tag{B2}$$

be the deviation of the second period price from its conditional mean (note in general this depends on trader i, but we suppress this dependence), one has

$$z_i = a\hat{P}_2^2 + b\hat{P}_2 + c$$

where

$$a = -\frac{(\tau_i - \bar{\tau})^2}{2K_{2,i}} \tag{B3.1}$$

$$b = -\left(rH_{1,i} - \frac{\tau_i - \bar{\tau}}{K_{2,i}} \left(\tau_i s_i - (\tau_i - \bar{\tau}) \mathbb{E}\left(P_2|\Phi_{1,i}\right) - \bar{\tau}\theta \right) \right) \tag{B3.2}$$

$$c = r\left(P_1 - \mathbb{E}\left(P_2|\Phi_{1,i}\right)\right) H_{1,i} - \frac{1}{2K_{2,i}} \left(\tau_i s_i - (\tau_i - \bar{\tau}) \mathbb{E}\left(P_2|\Phi_{1,i}\right) - \bar{\tau}\theta \right)^2. \tag{B3.3}$$

Following Marin and Rahi (2000) we then have that the expected utility takes the form

$$\mathbb{E}\left(-e^{z_i}|\Phi_{1,i}\right) = -\left(1 - 2a\mathbb{V}\left(\hat{P}_2|\Phi_{1,i}\right)\right)^{-1/2} \exp\left(\frac{b^2}{2}\psi_i + c\right), \tag{B4}$$

where we set

$$\psi_i \equiv \frac{\mathbb{V}\left(\hat{P}_2|\Phi_{1,i}\right)}{1 - 2a\mathbb{V}\left(\hat{P}_2|\Phi_{1,i}\right)}. \tag{B5}$$

From eq. (B2) we immediately get $\mathbb{V}\left(\hat{P}_2|\Phi_{1,i}\right) = \mathbb{V}\left(P_2|\Phi_{1,i}\right)$. Thus to evaluate the date 1 expected utility we need closed forms for $\mathbb{E}\left(P_2|\Phi_{1,i}\right)$ –which enters a and b– and $\mathbb{V}\left(P_2|\Phi_{1,i}\right)$, according to:

Lemma 2. *The first two moments of P_2 conditional on date 1 information are*

$$\mathbb{E}\left(P_2|\Phi_{1,i}\right) = \frac{1}{K_{1,i}K_2}\left[\left(\tau_v\bar{v} + \frac{\tau_x}{\kappa}\bar{x}\right)K_{2,i} + \left(K_{2,i} - K_{1,i}\right)\tau_i s_i + \left(K_{1,i}\bar{\tau} + \frac{\tau_x K_{2,i}}{\kappa^2}\right)\theta\right] \tag{B6}$$

$$\mathbb{V}\left(P_2|\Phi_{1,i}\right) = \frac{K_{2,i}}{K_{1,i}K_2^2}\left(K_{2,i} - K_{1,i}\right) \tag{B7}$$

Proof. Rewrite P_2 in eq. (2) as $P_2 = \alpha_2 + \beta_2\theta + \delta_2 s_p$ and note that, conditional on $\Phi_{1,i} = \{s_i, \theta\}$, we have

$$\mathbb{E}\left(P_2|\Phi_{1,i}\right) = \alpha_2 + \beta_2\theta + \delta_2\mathbb{E}\left(s_p|\Phi_{1,i}\right) \tag{B8}$$

$$\mathbb{V}\left(P_2|\Phi_{1,i}\right) = \delta_2^2\mathbb{V}\left(s_p|\Phi_{1,i}\right) \tag{B9}$$

so that the problem boils down to projecting s_p onto $\Phi_{1,i}$. Letting $K_{1,i}$ be as in (16), the distributional assumptions in (3) yield

$$\mathbb{E}\left(s_p|\Phi_{1,i}\right) = \left(K_{1,i}\kappa^2\right)^{-1}\left(a_1\bar{v} + b_1\bar{x} + c_{1,i}s_i + d_{1,i}\theta\right),$$

where

$$
\begin{aligned}
a_1 &= a_f K_{1,i}\kappa^2 - c_{1,i} - d_1 && \text{(B10.1)}\\
b_1 &= a_n K_{1,i}\kappa^2 + d_{1,i}\kappa && \text{(B10.2)}\\
c_{1,i} &= \kappa\tau_i\left(a_n + a_f\kappa\right) && \text{(B10.3)}\\
d_{1,i} &= a_f\tau_x - a_n\kappa\left(\tau_i + \tau_v\right) && \text{(B10.4)}
\end{aligned}
$$

Substituting $K_{1,i}$ from eq. (16) into the expression for a_1 in (B10.1) gives

$$
a_1 = \kappa\tau_v\left(a_n + a_f\kappa\right) \tag{B11}
$$

which does not depend on i. Moreover, from (B10.2-B10.3) and (B11) we have that

$$
b_1 = \frac{a_1\tau_x}{\kappa\tau_v} \quad \text{and} \quad c_{1,i} = \frac{a_1\tau_i}{\tau_v}.
$$

Thus the conditional expectation of the public signal is

$$
\mathbb{E}\left(s_p|\Phi_{1,i}\right) = \left(K_{1,i}\kappa^2\right)^{-1} a_1\left(\bar{v} + \frac{\tau_x}{\kappa\tau_v}\bar{x} + \frac{\tau_i}{\tau_v}s_i + \frac{d_{1,i}}{a_1}\theta\right). \tag{B12}
$$

Now consider the conditional variance

$$
\begin{aligned}
\mathbb{V}\left(s_p|\Phi_{1,i}\right) &= a_f^2\tau_f^{-1} + a_n^2\tau_n^{-1} + \frac{a_f^2\tau_x + a_n^2\tau_v}{\tau_v\tau_x} - \frac{\left(a_f\tau_x - a_n\kappa\tau_v\right)^2 + \kappa^2\tau_i\left(a_f^2\tau_x + a_n^2\tau_v\right)}{K_{1,i}\kappa^2\tau_v\tau_x}\\
&= a_f^2\tau_f^{-1} + a_n^2\tau_n^{-1} + \frac{\left(\kappa^2\tau_v + \tau_x\right)\left(a_f^2\tau_x + a_n^2\tau_v\right) - \left(a_f\tau_x - a_n\kappa\tau_v\right)^2}{K_{1,i}\kappa^2\tau_v\tau_x}, \quad \text{(B13)}
\end{aligned}
$$

where we have used (4) in the first line. Before moving to the second date price posterior expectation and variance, we obtain a number of results. First, substituting $K_{1,i}$ from (16) into the expression for $K_{2,i}$ in (8) yields the following recursion on the asset payoff posterior precision

$$
K_{2,i} = K_{1,i} + \frac{\tau_f\tau_n\left(a_n + a_f\kappa\right)^2}{\kappa^2\left(a_n^2\tau_f + a_f^2\tau_n\right)}. \tag{B14}
$$

Making use of d_2 in (A7) and the definition of τ_p' in (7) and a_1 in (B11) allows to rewrite the recursive formula (B14) rewrites as

$$
K_{2,i} - K_{1,i} = \frac{a_1 d_2}{a_2\kappa^2}. \tag{B15}
$$

Second, making use of (A7) together with (A6) and $d_{1,i}$ in (B10.4) gives

$$
\frac{e_2}{a_2}K_{1,i}\kappa^2 + \frac{d_{1,i}d_2}{a_2} = \frac{\tau_x}{\tau_v}K_{2,i}. \tag{B16}
$$

After having established these results, we go back to the conditional moments of P_2. The conditional expectation of s_p in (B12) allows to rewrite $\mathbb{E}\left(P_2|\Phi_{1,i}\right)$ in (B8) as

$$\begin{aligned}\mathbb{E}\left(P_2|\Phi_{1,i}\right) &= \left(\tau_v\bar{v}+\frac{\tau_x}{\kappa}\bar{x}\right)\left(\frac{1}{K_2}+\frac{\delta_2 a_1}{K_{1,i}\kappa^2\tau_v}\right)+\frac{a_1 d_2}{a_2 K_{1,i}K_2\kappa^2}\tau_i s_i \\ &+\frac{1}{K_2}\left(\bar{\tau}+\frac{\tau_v e_2}{a_2}+\frac{\tau_v d_{1,i}d_2}{a_2 K_{1,i}\kappa^2}\right)\theta,\end{aligned}$$

which, together with (B15) and (B16), yields (B6). As for the conditional variance, $V\left(s_p|\Phi_{1,i}\right)$ in (B13) provides

$$\mathbb{V}\left(P_2|\Phi_{1,i}\right)=\left(\frac{1}{K_2}\right)^2\frac{\tau_f\tau_n\left(a_n+a_f\kappa\right)^2}{\kappa^2\left(a_n^2\tau_f+a_f^2\tau_n\right)}\frac{K_{2,i}}{K_{1,i}},$$

and (B9) follows using the recursive formula (B15). After having established these results, we go back to conditional moments of P_2. The conditional expectation of s_p in (B12) allows to write

$$\begin{aligned}\mathbb{E}\left(P_2|\Phi_{1,i}\right) &= \left(\tau_v\bar{v}+\frac{\tau_x}{\kappa}\bar{x}\right)\left(\frac{1}{K_2}+\frac{\delta_2 a_1}{K_{1,i}\kappa^2\tau_v}\right)+\frac{a_1 d_2}{a_2 K_{1,i}K_2\kappa^2}\tau_i s_i \\ &+\frac{1}{K_2}\left(\bar{\tau}+\frac{\tau_v e_2}{a_2}+\frac{\tau_v d_{1,i}d_2}{a_2 K_{1,i}\kappa^2}\right)\theta,\end{aligned}$$

which, together with (B15) and (B16), yields (B6). As for the conditional variance. eq. (B13) yields

$$\mathbb{V}\left(P_2|\Phi_{1,i}\right)=\left(\frac{1}{K_2}\right)^2\frac{\tau_f\tau_n\left(a_n+a_f\kappa\right)^2}{\kappa^2\left(a_n^2\tau_f+a_f^2\tau_n\right)}\frac{K_{2,i}}{K_{1,i}}$$

and (B9) follows using the recursive formula (B15) ■

We now proceed in determining date 1 holdings. From eqs. (B3.1) and (B9) neither a nor $\mathbb{V}\left(P_2|\Phi_{1,i}\right)$ –and a fortiori, ψ_i defined in (B5)– depend on $H_{1,i}$ so that the the first order condition for maximizing (B4) is

$$\exp\left(\frac{b^2}{2}\psi_i+c\right)\left(b\psi_i\frac{\partial b}{\partial H_{1,i}}+\frac{\partial c}{\partial H_{1,i}}\right)=0.$$

As the first term in the latter is strictly positive, optimality then boils down to

$$b\psi_i\frac{\partial b}{\partial H_{1,i}}+\frac{\partial c}{\partial H_{1,i}}=0. \tag{B17}$$

Eqs. (B3.2)-(B3.3) readily give

$$\frac{\partial b}{\partial H_{1,i}} = -r \quad \text{and} \quad \frac{\partial c}{\partial H_{1,i}} = r\left(P_1 - \mathbb{E}\left(P_2|\Phi_{1,i}\right)\right),$$

so that date 1 holdings from condition (B17) are

$$rH_{1,i} = \frac{\mathbb{E}\left(P_2|\Phi_{1,i}\right) - P_1}{\psi_i} + \frac{\tau_i - \bar{\tau}}{K_{2,i}}\left(\tau_i s_i - (\tau_i - \bar{\tau})\,\mathbb{E}\left(P_2|\Phi_{1,i}\right) - \bar{\tau}\theta\right), \tag{B18}$$

which depend on the posterior expectation and variance of P_2. Eqs. (B3.1), (B7) and (B5) give

$$\begin{aligned} \psi_i^{-1} &= \frac{K_{1,i}K_2^2}{K_{2,i}\left(K_{2,i} - K_{1,i}\right)} + \frac{\left(\tau_i - \bar{\tau}\right)^2}{K_{2,i}} \\ &= \frac{K_2 K_1}{K_{2,i} - K_{1,i}} + \left(\tau_i - \bar{\tau}\right) \end{aligned}$$

where the last line follows from eq. (12) and some rearranging. The latter allows to rewrite (B18) as

$$rH_{1,i} = -\left(\frac{K_2 K_1}{K_{2,i} - K_{1,i}} + \left(\tau_i - \bar{\tau}\right)\right) P_1 + \frac{\tau_i - \bar{\tau}}{K_{2,i}}\left(\tau_i s_i - \bar{\tau}\theta\right) + \frac{K_{1,i}K_2^2}{K_{2,i}\left(K_{2,i} - K_{1,i}\right)}\mathbb{E}\left(P_2|\Phi_{1,i}\right),$$

and using $\mathbb{E}\left(P_2|\Phi_{1,i}\right)$ from eq. (B6) in the latter yields holdings as in (15). □

C. Proof of Proposition 1

We first consider the change in posterior precision. From the date 1 posterior precision in (16) we have that $K_{2,i}^{(f)}$ and $K_{2,i}^{(n)}$ in (9) can be equivalently rewritten as

$$K_{2,i}^{(f)} = K_{1,i} + \tau_f \quad \text{and} \quad K_{2,i}^{(n)} = K_{1,i} + \frac{\tau_n}{\kappa^2}$$

so that the date 2 posterior precision in (8) becomes

$$K_{2,i} = K_{1,i} + \omega_f \tau_f + \left(1 - \omega_f\right)\frac{\tau_n}{\kappa^2} + 2\frac{a_f a_n \tau_f \tau_n}{\kappa\left(a_f^2 \tau_n + a_n^2 \tau_f\right)},$$

which is eq. (20).

Then, we derive the date 1 posterior expectation of the asset payoff. First, observe that the LIE implies $\mathbb{E}\left(v|\Phi_{1,i}\right) = \mathbb{E}\left[\mathbb{E}\left(v|\Phi_{2,i}\right)|\Phi_{1,i}\right]$ since $\Phi_{2,i} \supset \Phi_{1,i}$. Thus, from eq. (D1)

$$\mathbb{E}\left(v|\Phi_{1,i}\right) = \frac{1}{K_{2,i}}\left(\tau_v \bar{v} + \frac{\tau_x}{\kappa}\bar{x} + \tau_i s_i + \frac{\tau_v}{a_2} d_2 \mathbb{E}\left(s_p|\Phi_{1,i}\right) + \frac{\tau_v}{a_2} e_2 \theta\right),$$

which, substituting the conditional expectation in (B12), becomes

$$\begin{aligned}\mathbb{E}(v|\Phi_{1,i}) &= \frac{1}{K_{2,i}}\left(\tau_v\bar{v}+\frac{\tau_x}{\kappa}\bar{x}+\tau_i s_i+\frac{a_1 d_2}{a_2 K_{1,i}\kappa^2}\left(\tau_v\bar{v}+\frac{\tau_x}{\kappa}\bar{x}+\tau_i s_i+\frac{\tau_v d_{1,i}}{a_1}\theta\right)+\frac{\tau_v}{a_2}e_2\theta\right)\\ &= \frac{1}{K_{2,i}}\left(\tau_v\bar{v}+\frac{\tau_x}{\kappa}\bar{x}+\tau_i s_i+\left(\frac{K_{2,i}}{K_{1,i}}-1\right)\left(\tau_v\bar{v}+\frac{\tau_x}{\kappa}\bar{x}+\tau_i s_i\right)+\tau_x\frac{K_{2,i}}{K_{1,i}\kappa^2}\theta\right)\\ &= \frac{1}{K_{1,i}}\left(\tau_v\bar{v}+\frac{\tau_x}{\kappa}\bar{x}+\tau_i s_i+\frac{\tau_x}{\kappa^2}\theta\right), \qquad \text{(C1)}\end{aligned}$$

where we have used the recursion (B15) and eq. (B16) to get the second line. consider changes in posterior precision first. From eqs. (6) and (C1) the date 2 posterior expectation of v reads

$$\mathbb{E}(v|\Phi_{2,i})=\frac{1}{K_{2,i}}\left(K_{1,i}\mathbb{E}(v|\Phi_{1,i})+\tau_p' s_p+\frac{\tau_p' a_n}{\kappa}\theta\right), \qquad \text{(C2)}$$

so that the change in posterior expectation is

$$\mathbb{E}(v|\Phi_{2,i})-\mathbb{E}(v|\Phi_{1,i})=\left(\frac{K_{1,i}}{K_{2,i}}-1\right)\mathbb{E}(v|\Phi_{1,i})+\frac{\tau_p'}{\kappa K_{2,i}}(\kappa s_p+a_n\theta). \qquad \text{(C3)}$$

The dynamics of the posterior precision in (B15) can be equivalently rewritten as

$$K_{2,i}-K_{1,i}=\frac{\tau_p'(a_n+\kappa a_f)}{\kappa} \qquad \text{(C4)}$$

where τ_p' is defined in (7). Making use of (C4) in eq. (C3) then yields the expression for changes in expectations in (19). □

D. Proof of Proposition 2

The product between the asset payoff posterior expectation and precision at date 2 is given by eqs. (6) and (A8) as

$$K_{2,i}\mathbb{E}(v|\Phi_{2,i})=\tau_v\bar{v}+\frac{\tau_x}{\kappa}\bar{x}+\tau_i s_i+\tau_p' s_p+\left(\frac{\tau_p' a_n}{\kappa}+\frac{\tau_x}{\kappa^2}\right)\theta, \qquad \text{(D1)}$$

which, together with the definition of θ and the date 2 price in (13), gives

$$\int_0^1 K_{2,i}\mathbb{E}(v|\Phi_{2,i})\,di=K_2P_2+rx$$

which is eq. (24) for $t = 2$. We now establish (24) for date 1. Multiplying both sides of eq. (C1) by $K_{1,i}/K_1$, and aggregating gives

$$E_{1,M} = \frac{1}{K_1}\left(\tau_v \bar{v} + \frac{\tau_x}{\kappa}\bar{x} + \tau_i s_i + \frac{\tau_x}{\kappa^2}\theta\right), \tag{D2}$$

and eq. (24) obtains by comparing the latter with the date 1 equilibrium price (18). Finally, we show that market expectations evolve according to . Multiplying both sides of eq. (C2) by $K_{2,i}/K_2$ yields

$$\frac{K_{2,i}}{K_2}\mathbb{E}\left(v|\Phi_{2,i}\right) = \frac{1}{K_2}\left(K_{1,i}\mathbb{E}\left(v|\Phi_{1,i}\right) + \tau_p' s_p + \frac{\tau_p' a_n}{\kappa}\theta\right),$$

and aggregating across traders gives the date 2 market expectation as

$$E_{2,M} = \frac{1}{K_2}\left(K_1 E_{1,M} + \tau_p' s_p + \frac{\tau_p' a_n}{\kappa}\theta\right).$$

Then the change in market expectations is

$$\begin{aligned} E_{2,M} - E_{1,M} &= \left(\frac{K_1}{K_2} - 1\right) E_{1,M} + \frac{\tau_p'}{\kappa K_2}\left(\kappa s_p + a_n \theta\right) \\ &= \frac{\tau_p'}{\kappa K_2}\left(\kappa s_p + a_n\theta - \left(a_n + \kappa a_f\right) E_{1,M}\right), \end{aligned}$$

where the last line follows from eq. (C4) and the fact that $K_{2,i} - K_{1,i} = K_2 - K_1$. □

E. Proof of Proposition 3

We first rewrite date 2 holdings in (10) as

$$\begin{aligned} rH_{2,i} &= \tau_v \bar{v} + \frac{\tau_x}{\kappa}\bar{x} + \bar{\tau} v + \tau_p' s_p + \frac{a_n \kappa \tau_p' + \tau_x}{\kappa^2}\theta - rx + \tau_i s_i - \bar{\tau} v + rx - K_{2,i} P_2 \\ &= \left(K_2 - K_{2,i}\right) P_2 + \tau_i s_i - \bar{\tau} v + rx \\ &= -\left(\tau_i - \bar{\tau}\right) P_2 + \tau_i s_i - \bar{\tau} v + rx \end{aligned} \tag{E1}$$

where we use the date 2 equilibrium price in (13) and $\theta = v - rx$ to get the second line, and the last line follows from (12). Next, we rearrange date 1 holdings in (15) as follows

$$\begin{aligned} rH_{1,i} = {} & -\left(\frac{K_2 K_1}{K_{2,i} - K_{1,i}} + \left(\tau_i - \bar{\tau}\right)\right) P_1 + \frac{K_2}{K_{2,i} - K_{1,i}}\left(\tau_v \bar{v} + \frac{\tau_x}{\kappa}\bar{x} + \bar{\tau} v + \frac{\tau_x}{\kappa^2}\theta - rx\right) \\ & + \tau_i s_i + \frac{K_1}{K_{2,i} - K_{1,i}}\bar{\tau}\theta - \frac{K_2}{K_{2,i} - K_{1,i}}\left(\bar{\tau} v - rx\right). \end{aligned}$$

Comparing the latter expression with the date 1 equilibrium price in (18) yields

$$
\begin{aligned}
rH_{1,i} &= -(\tau_i - \bar{\tau}) P_1 + \tau_i s_i + \frac{K_1}{K_{2,i} - K_{1,i}} \bar{\tau}\theta - \frac{K_2}{K_{2,i} - K_{1,i}} (\bar{\tau} v - rx) \\
&= -(\tau_i - \bar{\tau}) P_1 + \tau_i s_i + \frac{K_1 - K_2}{K_{2,i} - K_{1,i}} (\bar{\tau} v - \kappa x) \\
&= -(\tau_i - \bar{\tau}) P_1 + \tau_i s_i - (\bar{\tau} v - \kappa x), \qquad \text{(E2)}
\end{aligned}
$$

where the second line follows from $\theta = v - rx$, and eq. (17) yields the last line. Then Proposition 3 follows from eqs. (E1) and (E2). □

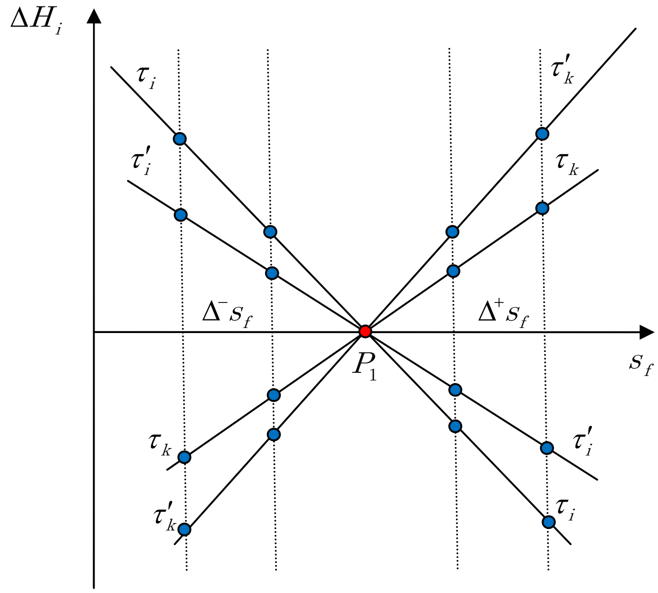


Figure 1. Public information and change in holdings. In the case of payoff-relevant information, the arrival of a public signal (horizontal axis, s_f) different from the date 1 price generates trading. Changes in holdings (ΔH_i) are represented on the vertical axis. The figure portrays the trading reaction of four different traders (k', k, i', i) to the arrival of both good and bad news on fundamentals and, as prescribed by the equation $\Delta H_i = -\frac{\tau_f}{rK_2^{(f)}}(\tau_i - \bar{\tau})(s_f - P_1)$. Each trader has a different precision of private information, with $\tau_i > \tau_i' > \bar{\tau} > \tau_k > \tau_k'$.

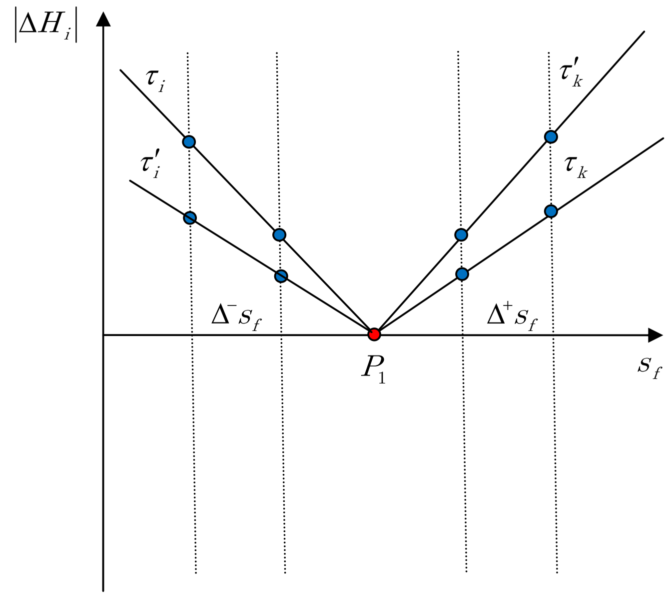


Figure 2. Public information and volume. In the case of payoff-relevant information, the arrival of a public signal (horizontal axis, s_f) different from the date 1 price generates trading. Absolute changes in holdings (volume, $|\Delta H_i|$) are represented on the vertical axis. The figure portrays the volume reaction of four different traders (k', k, i', i) to the arrival of both good and news on fundamentals, as prescribed by the equation $|\Delta H_i| = \frac{\tau_f}{rK_2^{(f)}} |\tau_i - \bar{\tau}| |(s_f - P_1)|$. Each trader has a different precision of private information, with $\tau_i > \tau'_i > \bar{\tau} > \tau_k > \tau'_k$.