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Exporting strategies of heterogeneous firms faced to export shocks and financial restraints

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This paper develops an open economy firm-heterogeneous model where the combination of market rigidities and exchange rate uncertainty acts like a barrier to trade and modifies a firm's optimal choice in terms of production and pricing. The existence of price and labor rigidities, coupled with imperfect financial development and exchange rate uncertainty, separates incumbent firms into (1) domestic producers, (2) exporters setting the price in national currency and (30 more productive exporters pricing in foreign currency. The model predicts that only where financial development is limited a reduction in exchange rate uncertainty raises a firm's profit, lowers prices, and induces new firms to export. Fully financially integrated countries are insulated from exchange rate risk.

Keywords: exchange rate uncertainty; firm heterogeneity; market rigidity; financial restraints.
JEL Classification: F1; F12; F16; F15

1 Introduction

The economic debate around the consequences of exchange rate volatility on international trade is ample and not yet settled. In 2004 the IMF completed a literature review about the topic concluding that neither the economic theory nor the empirical works have reached *"the undoubted conclusion that volatility necessarily lowers trade"*.

The reasons are many. Firms may decide to hedge against exchange rate fluctuations, although at a cost which increases with exchange rate volatility. Expected profits tend to rise with volatility according to the standard profit function. For multinationals, that engage in trade with many countries, the tendency of some exchange rates to move in opposite directions is a natural hedging for the overall exposure to currency risk. Finally, the impact of volatility may vary with the stage of development of a country ,its industrial structure, firm size and on market rigidities. Often, to justify that expected profits are reduced by uncertainty over the exchange rate evolution, convex objective functions were assumed that imply that either firms risk aversion or the existence of a risk averse manager.[1]

A different wave of empirical investigations on the topic, motivated by the birth of European Monetary union, has focused attention on examining ex-post the effects of a complete elimination of exchange rate uncertainty. At the same time, the possibility that other countries -South America, Africa or Middle East–currently in the process of considering the creation of a single currency area thus abandoning a national devise, keeps an intense interest on the subject and renews the need of a theoretical framework to provide precise predictions.

Following the path-breaking paper by Rose(1999), many studied price and trade evolution in the aftermath of the European Monetary Union. A niche of this literature focused on additional trade generated by new exporters entering a less uncertain international environment. The work by Baldwin and Di Nino(2006) tries to identify to what extent the boost in Eurozone trade is attributable to old exporters expanding

[1]Axel and Müller (1997) in a short note point out that firms face numerous risks and only financial ones are generally hedgeable. "Uncertainty over revenue such as uncertain demand, price risk, production risk, potential insolvency of customers, transportation risk cannot be generally covered". Asplund(2002), details other circumstances leading firms to behave as if they were risk-averse, "non-diversified owners, liquidity constraints, costly financial distress, and non-linear tax systems... delegation to risk averse manager".

their business rather than new players joining the international scenario. The results hint at the existence of a modest boost in trade via both the margins. Their analysis relies on the theoretical bases posed by Melitz(2003) which predicts that liberalization increases the number of exporters. Implicitly, this is equivalent to assimilating exchange rate uncertainty to some kind of trade barrier or fixed/sunk cost to export. What has not been previously modeled, within the firm-heterogeneous literature, is the mechanism that makes exchange rate volatility equivalent to a trade barrier .[2]

Finally, in more recent times, there is a special focus on financial development linked to a firm's growth, the ability to successful enter a market, the productivity level and the impact of financing possibilities on exporting decisions. Within a firm-heterogeneous model à la Melitz, Chaney (2005) introduces the existence of borrowing constraint as additional barrier to export. Bacchetta, Aghion and Ranciere suggest that exchange rate volatility affects productivity growth on less financially integrated countries, therefore fixing the exchange rate makes a good policy. Aghion, Fally, Scarpetta,(2007) when investigating the effect that financial development exercises on the entry possibilities of each firm, find that only small firms are affected. Financial integration, as shown by Alfaro and Charlton(2006) raises the number of small firms on a market and low financial development prevents companies from growing after their entry on the market.

The primary scope of this work is to set-up a model able to offer micro-foundations to the common currency effect, identify the channels through which currency risk depresses trade and identify the exact economic conditions for this to happen. It explains when the empirical evidences tend to favor the existence of a link between exchange rate volatility and trade. It also overcomes some limits of the previous literature, such as the assumptions of risk aversion and undiversified firms.

In this model the existence of price and labor rigidity combined with borrowing constraints and exchange rate uncertainty shapes a firm's optimization problem and determines which actions are viable to each firm. Considered in the model is a subset of the imperfections subsisting in the real world with showing that each of them

[2]A first attempt to justify the Rose's effect with a model of diversified firms is due to Baldwin and Taglioni (2004), their model suffer from the criticism of previous literature to the extent that producers are assumed to be risk averse and the effect is not micro-founded. The authors envisaged a mechanism where zeroing exchange rate volatility boosts the number of exporters and the effect is magnified when exchange rate volatility is low due to firm size distribution (many small firm with low productivity, very few firm with high productivity).

restrains producer's maximization possibilities. The same logic can be applied to alternative forms of rigidities. In our framework, whether exchange rate uncertainty depresses trade depends also on the level of financial development. Higher development lessens the exchange rate uncertainty. As in a standard firm-heterogeneous model, there exists one threshold domestic producer. Different from the previous literature, the model identifies two threshold exporters: (1) the marginal exporter who chose to set the export price in national currency (*the pnc strategy*) and (2) the marginal efficiency level which consents to set the price in foreign currency (*the ptm strategy*). The existence of borrowing constraints, which is more binding for smaller firms, implies that the *ptm* strategy is viable only to big firms. Finally, financial integration and exchange rate stabilization affect exports similarly.

An overview of the context and main assumptions of the model is given in Section (2), a description of the demand side in Section (3). The main body of this work is contained in Section (4) along with the analysis of the supply. The latter is further separated into domestic and export markets, respectively in Sections (4.1) and (4.2). Section (4.3), discusses the consequences of limited availability of financial resources on pricing and production alternatives. Section (4.4) and (4.6) investigate the relationship between domestic and export cut-offs. The number of active producers and exporters are computed in Section (4.5). The work measures the impact on the volume of exports of variation in exchange rate uncertainty and financial integration in Section (4.7). Section (5) describes the mechanism that guarantees the match of demand and supply, Concluding notes are collected in Section (6).

2 Framing the model

This model originates within the field of firm heterogeneity à la Melitz(2003) and permits a direct analysis of the effects that exchange rate volatility has on exports, on exporter profits, on the decision to enter the export market and on how exporters behave on the foreign market compared to the domestic market. The existence of market imperfections such as price stickiness in the goods market, labor rigidities and incomplete financial development determine the model predictions.

Households have constant elasticity of substitution (C.E.S) preferences over different varieties. Firms draw a heterogeneous but constant marginal cost of production from an exogenous distribution and compete in a monopolistic market. We assume full symmetry between home and abroad in terms of country size, sunk and fixed costs, distribution of marginal costs, exchange rate volatility and values of the exogenous parameters. In different terms, domestic and export markets are-in total-identical but segmented by the existence of exchange rate volatility. As they optimize, firms take the price index as given (as in standard Dixit-Stiglitz model) because they unable to influence it with their choices nor are they able to foresee what pricing strategy their competitors will choose.

There exist two sunk costs (Fd) to produce and supply the domestic market and (Fx) to export. These permit partitioning of firms in three well known types: non producers, domestic producers and exporters (N, D and X type). As producers finance their exporting strategy in a liquidity constrained world and choose to set the export price in foreign or domestic currency, there will be, depending on the degree of financial development and volatility of the exchange rate, two kinds of exporters-setting the price in national currency (pnc type) and setting the price in foreign currency (ptm type). In other words, some exporters will adopt a full pass through strategy while others a pricing to market strategy. The latter produces higher expected profits but is more costly and due to borrowing constraints is not viable for smaller exporters

We assume that firms can only set the price at the beginning of the period. This hypothesis is in line with the idea that while the exchange rate evolves in continuous time, there exists a wide empirical literature proving that prices are sticky. Moreover, labor can only be hired at the beginning of each period and cannot be dismissed before next period comes. The period of time when rigidities are not resolved is in the short run and the time when it is possible to re-optimize is in the long run.

By assuming that export price and production capacity are set before the value of exchange rate is disclosed, we render firms vulnerable to exchange rate fluctuations and, eventually, household subject to rationing. Consumers and firms consider a different price index because of the temporal mismatch between consumption and production choices. Consumers observe the price index after the realization of the exchange rate shock, but producers base their optimization on the expected value of the price index. For this reason, the expected function of the price index will be simply P while the realized value will be labeled P_r; with the latter being a function of the exchange rate, the former not.

To simplify tractability, we eliminate the unnecessary complications of trade barriers and assume that the shocks in export sales due to exchange rate evolution are uniformly distributed around the equilibrium value of one.[3]

3 Households

The typical household maximizes the following C.E.S. utility function with constant elasticity of substitution equal to σ.

$$U\left(C\right)=C=\left(\int\limits_{v}c_{v}^{\frac{\sigma-1}{\sigma}}dv\right)^{\frac{\sigma}{\sigma-1}} \tag{3.1}$$

subject to the budget constraint: $\int\limits_{v}p_{v}c_{v}dv\leq E$ and to the condition that demand (c_{v}) must always be smaller or equal to supply (s_{v}).[4] The v subscript has been chosen as mnemonic for the generic variety,p stays for price and c for consumption, E is the overall expenditure. Similarly we will indicate the variables intended for the domestic market with the subscript d and for the export market with the subscript x. For instance the quantity demanded of a domestic variety and an imported one are c_{d} and c_{x} respectively. Each variety enters the utility function symmetrically no matter its origin.

Utility maximization results in the following iso-elastic demand function (c_{v}):

$$c_{v}=E\frac{p_{v}^{-\sigma}}{\left(P\right)^{1-\sigma}} \tag{3.2}$$

where the price index (P) is given by the equation just below 3.3

$$P=\left(\int p_{v}^{1-\sigma}dv\right)^{\frac{1}{1-\sigma}} \tag{3.3}$$

The demand schedule depends on the price relative to the price index which reflects the standard C.E.S properties.

[3]The last assumption is necessary in order to ensure the existence of a closed form solution.

[4]This condition becomes particularly relevant in our set-up as the supply is decided in advance based on expectations and may turn out insufficient once the exchange rate shock is realized.

4 FIRMS

This is a model of firm heterogeneity where each firm produces one variety, pays upon entry a common overhead sunk cost F_I, which is associated to the development of a new variety, before the constant marginal cost of production a is disclosed. The "a" is drawn from a cumulative distribution $G(a)$ common across countries and defined over a support $(0, a_{max})$. Firms in this model maximize profits (domestic and foreign) subject to a continuous risk of death with probability δ.

Similarly to Melitz(2003) in this model there are sunk costs for producing in the domestic market F_d and for exporting F_x that are paid only after the productivity level is revealed.

4.1 The domestic market

Production technology requires only one factor -labor- employed either for domestic or foreign production at a constant marginal cost a. l_d, is thus a linear function of domestic output c_d and the labor demand which represents at the same time the cost function equals $l_d = a \times c_d$.

Labor rigidity does not alter domestic market structure, since the latter affected by exchange rate shocks only indirectly via the price index. One period expected operating profits in the domestic market are given by $\mathbf{E}(\pi_d) = \mathbf{E}(p_d c_d - a c_d)$. Profit maximization implies a price like:

$$p_d = \frac{\sigma a}{(\sigma - 1)} \tag{4.1}$$

The price index can be expressed as a function of N, indicating the mass of domestic producers and Δ, the expected values of a weighted average sum of productivities on the market.[5]

[5]More precisely following Baldwin (2005) we define $\Delta = \int\limits_{a=0}^{a=a_d} a^{1-\sigma} dG[a_d] + \int\limits_{a=0}^{a=a_{ptm}} a^{1-\sigma} dG[a_d] + \tau^{1-\sigma} \int\limits_{a=a_{ptm}}^{a=a_{nc}} a^{1-\sigma} dG[a_d]$, this largely simplifies notation and conveys better the intuitions. The marginal costs distribution function is conditioned to a_d, the marginal domestic producer; notice that the second part of Δ is still unknown and we anticipate in this formula the solution that will be detailed in section 4.2. We call a_{ptm} and a_{pnc} the marginal cost associated to the least efficient of the two types of exporters, e is the shock of export sales while τ is a function of exchange rate shocks maximum magnitude.

$$P = \frac{\sigma}{\sigma - 1} N^{\frac{1}{1-\sigma}} \Delta^{\frac{1}{1-\sigma}}$$

Replacing the optimal price in the operating profit function we obtain the expected profits at the optimum:

$$\mathbf{E}\left(\pi_d\right) = \frac{E}{\sigma} \frac{a^{1-\sigma}}{N\Delta}$$

Having showed that on the domestic market there are no differences with respect to the baseline Melitz model, the novelty of this work concentrates on describing the export market.

4.2 The export market

In order to operate on the export market each firm needs to determine at the beginning of the period what pricing strategy is viable and, on this basis, the number of workers to hire and the export price. As workers cannot be dismissed at any point in time, labor cost can be assimilated to a fixed cost that must be paid irrespective of market conditions. Nevertheless its amount is optimally determined by each firm according to productivity, exchange rate volatility, elasticity of substitution among varieties and their pricing strategy. Firms can choose to set the price in national currency or in foreign currency. In the first case exchange rate fluctuations affect profits (expressed in national currency) via quantities and in the second case through prices. In both the cases we assume that the foreign sales are subject to continuous shocks, e, uniformly distributed between $-e_{max}$ and e_{max}. The density function of the probability distribution ($f(e)$) is then simply given by: $f(e) = \frac{1}{2e_{max}}$.

4.2.1 Pricing to market strategy

When the exporters set the price in foreign currency *(no pass-through)*, foreign demand does not fluctuate but export sales denominated in domestic currency do via the price channel (a currency depreciation increases sales), as shown by the first part of the expression in square brackets in equation 4.2. Knowing the demand schedule; the expected operating profits are given by expected sales minus labor costs:

$$\mathbf{E}(\pi_{ptm}) = \int_{-e_{max}}^{e_{max}} \frac{E * p_{ptm}^{-\sigma}}{P^{(1-\sigma)}} [p_{ptm}(1-e) - a] f(e) de \qquad (4.2)$$

In the above equation we indicated with p_{ptm} the price the producer charges for exports denominated in consumer currency and with $\mathbf{E}(\pi_{ptm})$ the one period expected operating profit function. From the maximization of 4.2 we obtain that the optimal price coincides with the one that would have prevailed in the absence of exchange rate uncertainty:

$$p_{ptm} = \frac{\sigma a}{(\sigma - 1)} \tag{4.3}$$

As standard in monopolistic competition models, the optimal export price is, , a constant mark-up over the marginal cost exactly as the one the producer charges in the domestic market. Therefore choosing the pricing to market strategy permits to pin-down the foreign demand and hire labor accordingly. Expected ex ante foreign sales are not a function of volatility and similarly expected profits at the optimum.

$$\mathbf{E}(\pi_{ptm}) = \frac{E}{\sigma} \frac{a^{1-\sigma}}{N\Delta}$$

4.2.2 Full pass through strategy

When the export price is set in national currency (full pass-through), firms needs to decide independently over price and labor because the export demand fluctuates with the exchange rate. Since the marginal cost is constant, depending on the amount of labor hired at the beginning of the period, each firm has a determined production capacity (K), that can be changed only through hiring or layoffs when the next period comes. Therefore K represents a capacity constraint which expressed in terms of units of production is given by the following expression:

$$K = \frac{E * p_{pnc}^{-\sigma}\,(1 - e_k)}{P^{(1-\sigma)}} = c_{pnc}\,(1 - e_k) \tag{4.4}$$

c_{pnc} represents the export demand when the shock is zero and e_k determine the maximum shock such that ex-ante production capacity is still sufficient to satisfy demand ex-post. Notice that when a firm chooses over p_{pnc} and K, it is implicitly determining e_k to the extent that equation 4.4 can be rewritten as $e_k = (c_{pnc} - K)/c_{pnc}$. In this set-up then the expected profits are represented by the following function:

$$\mathbf{E}\ (\pi_{pnc}) = \int_{ek}^{e_{max}} \frac{E * p_{pnc}^{1-\sigma}(1 - e)}{P^{(1-\sigma)}} f(e)de + \int_{-e_{max}}^{ek} p_{pnc} K f(e)de - aK \tag{4.5}$$

The expected profits are a function of the export price p_{pnc} expressed in producer's national devise, the price index P, the marginal cost of production a but also of K, the maximum supply given the labor hired that period. Any time the shock is smaller than the threshold value e_k, production is constrained to K, when it is above e_k, the producer is in duty bound of paying workers (aK), even tough the demand turns out to be smaller than the planned capacity. The first addendum of equation (4.5) represents cases of overcapacity (the capacity constraint is not binding), while in the second addendum foreign demand exceed K. In other words for any shock smaller than e_k, a firm will be able to meet the foreign demand at a cost which is fixed ex-ante $(a \times K)$ but proportional to firm productivity.
The maximization problem of a firm that follows this strategy is then :

$$\max_{p_{pnc},K} \mathbf{E}\left(\pi_{pnc}\right) = \int_{ek}^{e_{max}} \frac{E * p_{pnc}^{1-\sigma}\left(1-e\right)}{P^{(1-\sigma)}} f(e)de + \int_{-e_{max}}^{ek} p_{pnc} K f(e)de - aK \tag{4.6}$$

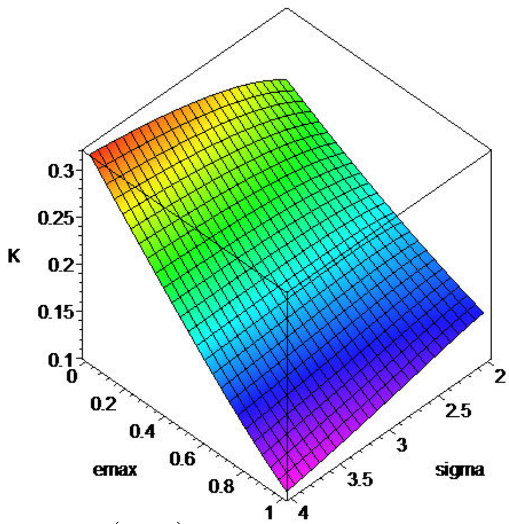
$$s.t. : K = \frac{E * p_{pnc}^{-\sigma}\left(1-e_k\right)}{P^{(1-\sigma)}}$$

The solution to (4.6) is given by the pair of optimal price and optimal capacity.

$$K = \left[1 - e_{max}\frac{2a - p_{pnc}}{p_{pnc}}\right] c_{pnc} \tag{4.7}$$

Figure 4.1: The optimal production capacity

The production capacity gets smaller for higher e_{max}. Firms set a larger K as σ rises when e_{max} is low and reduces it if σ rises ife_{max} is high. Clearly it is more and more risky for a firm to hire workers ex-ante when the volatility of export sales augments. The picture 4.1 plots the value of K as function of parameters treating $\left(\frac{E}{P^{1-\sigma}}\right)$ as a constant and set equal to 1



$\left(\frac{E}{P^{1-\sigma}}\right)$ *is treated as a constant*

Two different price values satisfy the first order condition one is positively related to the marginal production cost a, the second one is negatively related to it.

$$p_{pnc} = a\frac{\sigma\left(1+e_{\max}\right) \pm \sqrt{\sigma^2\left(1-e_{\max}\right)^2 + 4e_{\max}}}{2\left(\sigma-1\right)} = a\frac{\sigma\tau}{\left(\sigma-1\right)} = p_d\tau \tag{4.8}$$

where $\tau = \frac{1+e_{\max}}{2} + \frac{\sqrt{\sigma^2(1-e_{\max})^2+4e_{\max}}}{2\sigma}$.
Investigations of the definitiveness of the Hessian matrix prove the first to be the unique optimal price (see A.1 for details).
The mark up is an increasing function of sales volatility. The capacity constraint is negatively related to volatility and coincides with the expected demand when volatility is zero.
The expected profit evaluated at the optimal K and p_{pnc} are:

$$\pi_{pnc} = \frac{E}{\sigma}\frac{\tau^{1-\sigma}a^{1-\sigma}}{N\Delta}\Omega \tag{4.9}$$

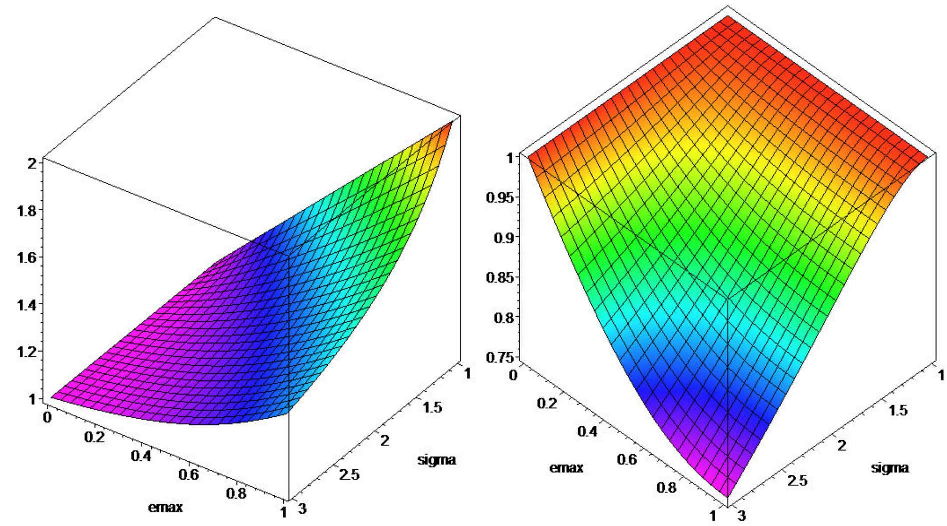
where $\Omega = \frac{\left[\sigma\tau e_{\max}(1-\sigma)+e_{\max}(1-\sigma)^2+\sigma\tau(\sigma\tau+1-\sigma)\right]}{\sigma\tau^2}$[6].

Profits are proportional to productivity and inversely related to the number of operating firms, as varieties are more easily substitutable (bigger σ) the optimal mark-up shrinks and profits follows. While higher σ values tend to depress the value of τ and Ω, volatility unambiguously depresses profits on an overall basis as it raises the value of τ and lowers that of Ω. In the two subfigures 3(a) and 3(b) τ and Ω are plotted as function of the parameters σ and e_{max}.

Comparing the two strategies in terms of profits and prices, we conclude that the *"no pass through"* pricing dominates the *"full pass through"* pricing. It yields higher expected profits, lower prices and a larger market share. At the same time it is more expensive as it entails a larger number of workers to be hired. The presence of liquidity constraints and a low degree of financial sector development makes this alternative viable only to the most efficient producers that can employ their pure profit-earned on the domestic market- to finance exporting plans.

[6]Sales from selling abroad are given by: $S_{pnc} = E\frac{(\tau * a)^{1-\sigma}}{N\Delta} * [\frac{\sigma}{\sigma-1} - \frac{e_{max}(\sigma-1)}{\sigma * \tau^2}]$

Figure 4.2: τ and Ω: the effect of volatility on prices and profits



(a) (b)

4.3 Liquidity constraint

Upon payment of sunk and labor costs, firms will be able to produce and sell. Consequently financing possibilities become very important to determine the viability of attractive investment strategies. Firms should use export revenues as collateral in financing operations, but export revenues are subject to exchange rate volatility. The domestic financial sector-that is not model explicitly,-cannot fully hedge against exchange rate volatility because it hits every firm symmetrically while an internationally integrated financial system is able to pool the risk across firms of different countries.[7] Therefore a firm's financing possibilities strictly depends on the degree of financial development of a nation. Firms operating in integrated countries face weaker liquidity constraints and are able to obtain greater profits on average.

In this model, an exogenous parameters λ, ranging between 0 and 1 proxies the level of financial development (development falls when λ rises. The financial system will accept as collateral the entire value of domestic sales (since these are not subject to exchange rate volatility shocks, but only a fraction proportional to λ and e_{max} of volatile foreign sales. Therefore in the absence exchange rate uncertainty, exports will be treated as domestic production, the same is true when financial integration is

[7]We are implicitly assuming that the level of development is strictly related to the degree of financial integration of a country with its trading partners. Developed systems are able to share the risk also with foreign financial operators.

complete. Firms need to cover labor and sunk costs with these resources.

While the choice of entering the domestic market is not affected by liquidity constraints, exporting will be a viable option only when a fraction of expected foreign sales and pure domestic profit cover both labor and exporting sunk costs. This conditions will be exploited to find the two cut-off exporters, one for each pricing strategy by imposing that financial resources at disposal of a firm are just sufficient to cover costs.

The model shares with the recent works by Chaney and Aghion, Fally and Scarpetta, the assumption of the existence of liquidity constraint in a heterogeneous world but formalizes and justifies it on different grounds. Chaney(2005) assumes that firms are endowed with a random liquidity shock, additional to domestic profits, which may or may not be correlated with firm productivity and justifies it as coming from an inheritance. In addition the financial system applies different criteria to finance domestic and export production to the extent that *" a firm may find investors for any investment regarding domestic activities, but none whatsoever for exporting activities".* In his model, similarly to this one, liquidity constraints reveal to be important barrier to export; otherwise from Chaney's model in this model liquidity constraints are correlated to firm's efficiency and determine also the pricing policy of exporting firms.". Again similar, liquidity constraints reveal to be an important barrier to export;, Dissimilar from Chaney's model, liquidity constraints are correlated to a firm's efficiency and it also determines the pricing policy of exporting firms. Aghion, Fally and Scarpetta(2007) instead use the credit constraint in order to study a firm's entry dynamics and post-entry growth. According to their empirical evidences small firms are affected by credit restrictions and benefit the most from financial development. Also, the level of financial development fosters the growth of successful firms after entry. Finally in their model small does not mean inefficient. There are small firms which are more productive than large ones.

4.4 The zero profit condition and the borrowing constraints

When the resources borrowed from the financial system at zero cost are sufficient to cover the cost of operating on the domestic or on the export market, production decisions will be implementable. The three conditions to determine the domestic cut-off and the two exporting cut-offs, one for each pricing strategy become:

$$a_d : \pi_d = \delta * F_d \tag{4.10}$$

The above equation says that the stream of discounted operating domestic profits (given by the one period profits divided the constant firm death-rate δ) must cover the sunk cost of producing domestically.

$$a_{pnc} : \pi_d + (1 - \lambda * e_{max}) * E \frac{(\tau * a_{pnc})^{1-\sigma}}{N\Delta} * [\frac{\sigma}{\sigma - 1} - \frac{e_{max}(\sigma - 1)}{\sigma * \tau^2}] - K * a_{pnc} = \delta * (F_x + F_d) \tag{4.11}$$

In equation 4.11 the stream of operating domestic profits plus a fraction proportional to volatility and financial integration $(1 - \lambda * e_{max})$ of the stream of expected export sales cover the two sunk costs for producing domestically and exporting (F_x and F_d) plus the labor cost identified by $K * a_{pnc}$.
The cut-off condition for *"pricing to market"* to be feasible (equation 4.12) is similar to the previous expression with the only difference that export sales and labor costs to be sustained are larger.

$$a_{ptm} : \pi_d + E \frac{p_{ptm}^{-\sigma} \sigma}{N\Delta(\sigma - 1)} [(1 - \lambda * e_{max}) * p_{ptm} - a_{ptm}] = \delta * (F_x + F_d) \tag{4.12}$$

Notice that only the domestic cut-off condition is still a zero-profit condition. There are firms which are prevented from operating abroad because of borrowing constraints, whose efficiency would ensure an expected positive profit from exporting.

In the absence of sunk costs the two exporting strategies will respectively be feasible provided that:

$$\lambda \leq \frac{2}{\sigma * e_{max}} = \lambda_{ptm}$$

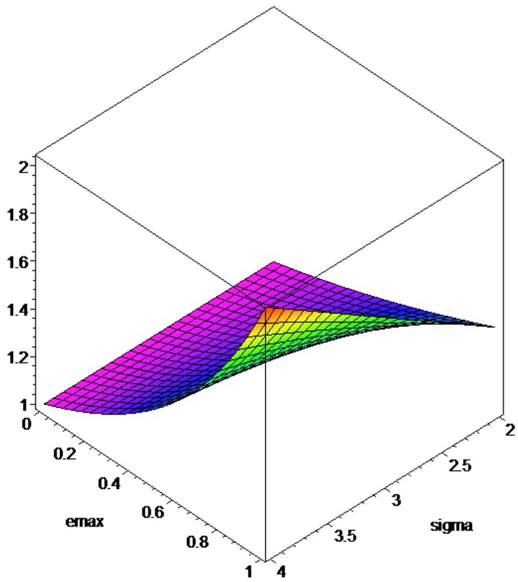
for the pricing in consumer's currency strategy and

$$\lambda \leq \frac{\sigma\tau[2 + \sigma(\tau - 1) - e_{\max}(\sigma - 1)] + e_{\max}\left(\sigma - 1\right)^2}{e_{\max}\left(\sigma^2\tau^2 - e_{\max}(\sigma - 1)^2\right)} = \lambda_{pnc}$$

for pricing in exporter's currency. The existence of a positive threshold also when sunk costs are zero is clearly due to the existence of labor costs that firms pay regardless of consumer demand realization ex-post.

λ_{ptm} is always smaller than λ_{pnc} regardless of other exogenous parameters, therefore as λ or e_{max} shrinks the first concrete possibility of exporting entails pricing in national currency; only when integration deepens or exchange rate stabilize, the more profitable option of pricing in consumer's currency becomes valid. The plot below,confirms this premise by showing that the ratio of λ_{pnc} and λ_{ptm}, stays always above one.

Figure 4.3: $\frac{\lambda_{pnc}}{\lambda_{ptm}}$



This implies that if $\lambda > \lambda_{pnc}$ no firm will be able to export and for $\lambda_{ptm} < \lambda < \lambda_{pnc}$ only the *"full pass through strategy"* will be implementable.

Under perfect symmetry, the domestic and export cut-offs, the number of firms, as well as the Δ coincide in the two countries D and F (for domestic and foreign country respectively); the three cut-off conditions offer then a short-cut to obtain the relation between marginal producer and the two marginal exporter types:

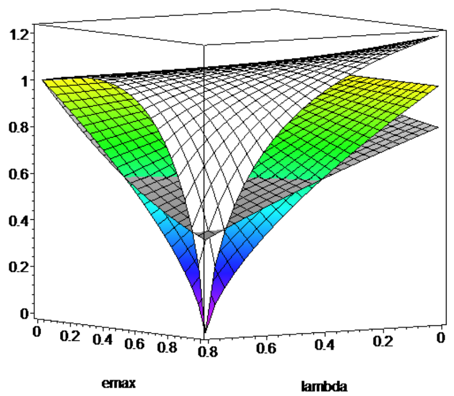
$$a_{ptm} = a_d * \{(2 - \lambda * \sigma * e_{max}) \frac{F_d}{F_d + F_x}\}^{\frac{1}{\sigma - 1}} \tag{4.13}$$

$$a_{pnc} = a_d \left\{ 1 + \sigma\tau^{-\sigma} [\lambda e_{\max}^2 \left(\frac{\sigma - 1}{\sigma}\right)^2 + (1 - \lambda e_{\max})\,\tau - \frac{\sigma - 1}{\sigma}\,(1 + e_{\max})] \frac{F_d}{F_{d+Fx}} \right\}^{\frac{1}{\sigma - 1}} \tag{4.14}$$

$$a_{ptm} = a_{pnc} \left\{ \frac{1 + \sigma\tau^{-\sigma}[\lambda e_{\max}^2 \left(\frac{\sigma-1}{\sigma}\right)^2 + (1-\lambda e_{\max})\,\tau - \frac{\sigma-1}{\sigma}(1+e_{\max})]}{2-\lambda\sigma e_{\max}} \right\}^{\frac{1}{1-\sigma}} \quad (4.15)$$

Being that *ptm* is a dominating strategy as soon as $a_{ptm} > a_{pnc}$ every exporter will implement the *"no pass through"* strategy. The condition is verified when $\lambda \leq \frac{1}{\sigma * e_{\max}} = \lambda_{aptm}$. Between λ_{aptm} and λ_{ptm} the two types of exporters coexist.
Similarly to Melitz(2003) there are conditions to be imposed on the exogenous parameters for exporting firms to be more efficient than domestic ones. Assuming F_x is equal to zero , this condition is verified for $\lambda > \lambda_{aptm}$. Intuitively if exporting does not entail additional cost then at the point when *pricing to market* strategy becomes feasible to every firm, they will start exporting by choosing not to pass through exchange rate fluctuation onto consumers. Finally, as soon as $F_x \geq F_d$, exporters are a subset of the most efficient producers. This is graphically proved in figure 4.4 which plots equations 4.13, 4.14, 4.15 and illustrate the relationship existing among the three cut-offs when $F_x = F_d$.

Figure 4.4: Cut-offs ratios



The only plot to go above one is $\frac{a_{ptm}}{a_{pnc}}$ when financial integration is strong and exchange rate volatility volatility is low. The ratio of $\frac{a_{pnc}}{a_d}$ is less than one for $e_{max} \neq 0$, $\frac{a_{ptm}}{a_d}$ for $e_{max} \neq 0$ and for $\lambda \neq 0$, confirming that exporters are always more productive than domestic producers.

4.5 The equilibrium number of producers and exporters

To obtain an explicit solution for the number of firms and the cut-offs, the cumulative density function so far named simply $G[a]$ has to be modeled explicitly. It is usual in this literature to assume a pareto cumulative distribution of marginal costs: $G[a] = (\frac{a}{a_{max}})^{b}$[8].

[8] As in Melitz(2003), for this model to be solvable, we need to guarantee through a restriction on the parameter of the pareto distribution and the elasticity of substitution that the integral in Δ converges by assuming that $b + 1 \geq \sigma$

Replacing the marginal cost distribution in the definition of Δ and solving the domestic cut-off condition, equation (4.10), for N we obtain:

$$N = \frac{E\,(b-\sigma+1)}{F_d\sigma b\left[1+(1-\tau^{1-\sigma})\left[\frac{a_{ptm}}{a_d}\right]^{\frac{b-\sigma+1}{\sigma-1}}+\tau^{1-\sigma}\left[\frac{a_{nc}}{a_d}\right]^{\frac{b-\sigma+1}{\sigma-1}}\right]} \tag{4.16}$$

We know that only a fraction of domestic firms will export and this fraction is defined by the ratio of $\frac{G[a_{pnc}]}{G[a_d]}$. The number of exporters by type is hence equivalent to:

$$N_{ptm} = N\left(\frac{a_{ptm}}{a_d}\right)^b$$

$$N_{pnc} = N\left(\frac{a_{pnc}-a_{ptm}}{a_d}\right)^b$$

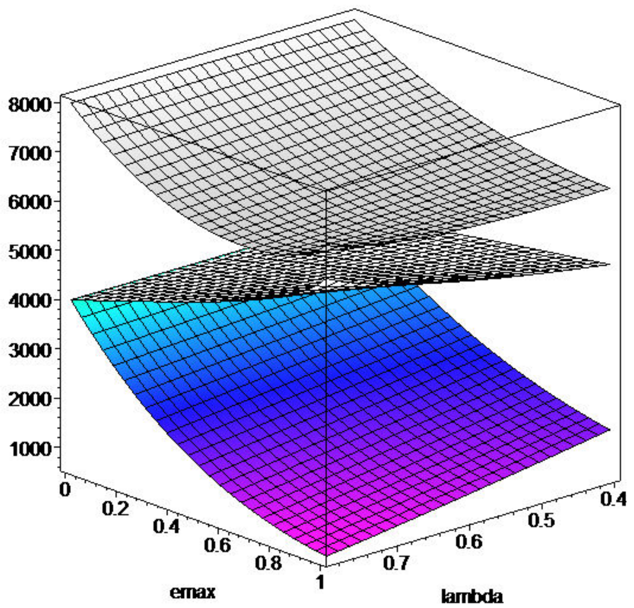
Volatility together with a low level of financial integration impedes some firms to export but allows relatively inefficient ones (those around the a_d) to stay on the domestic stage since it lowers the degree of competition within a country. Reducing uncertainty or increasing financial integration raises the number of exporters but reduces the domestic producers. In the range of λ values where both pricing strategies are present, it also allows some exporters to replace the *pnc* with the *ptm* strategy. Doing this lessens the price index and via the competition channel boosts the average productivity. This happens in addition to the standard selection effect typical of firm heterogeneous models where new producers start exporting because of trade liberalization and the least efficient domestic producers exit the market.

Below we provide a graphical representation of what was just affirmed by plotting the number of producers, exporters and the total number of firms active on a single market as function of volatility and financial integration for given values of the other exogenous variables.

The effect of exchange rate uncertainty on the overall number of producers operating on the domestic market is slightly non monotonic in (e_{max}): it tends to decrease starting from a very high value of e_{max} and to increase if the reduction occurs when volatility is low. Financial integration has the unambiguous effect of raising the number of firms. Despite the non linear behavior it is possible to assess that the number is unquestionably higher when $e_{max} = 0$ than when $e_{max} = 1$.

Melitz(2003) reaches the same result when he states that in countries that are marginally

Figure 4.5: Number of active producers (white scale), of exporters (colored scale), of active firm on each market(grey scale)



(E=1000000, FI=Fd=Fx=100, $\delta = 0.2$, $b = 2.5$ $\sigma = 2.5$,$a_{max} = 1$*)*

liberalized, liberalization may lead to fewer active firms on the market.[9]

4.6 Free entry

While the number of active firms is related to the size of the economy and sunk costs F_x and F_d,the existence of an innovation cost F_I guarantees the existence of an equilibrium in this economy, despite the continuous process of entering and exiting due to the exogenous probability of dying in each period (δ).F_I is such that potential entrants will earn in equilibrium an expected profit equal to zero from entering the market. This extra condition of equilibrium pins down the domestic cut-off a_d[10]:

[9]Melitz adds that the productivity selection mechanism outweighs the variety effect in terms of welfare and liberalization is always welfare improving.

[10]Replacing the solution for N,the definition of Δ, using the distribution of marginal cost and the relationship between the exporting and producing cut-offs, we simplify the above expression as follows

$$\frac{f_I}{F_d}\left(\frac{a_{\max}}{a_d}\right)^b = \frac{1}{\delta}\frac{b}{b+1-\sigma}\left[1+\left(1-\tau^{1-\sigma}\Omega\right)\left(\frac{a_{ptm}}{a_d}\right)^{b+1-\sigma}+\tau^{1-\sigma}\Omega\left(\frac{a_{pnc}}{a_d}\right)^{b+1-\sigma}\right]-\left[1+\frac{F_x}{F_d}\left(\frac{a_{pnc}}{a_d}\right)^b\right]$$

$$f_I = \int_{a=0}^{a=a_d} \left(\frac{E}{\sigma\delta} \frac{a^{1-\sigma}}{N\Delta} - F_d \right) dG[a] + \int_{a=0}^{a=a_{ptm}} \left(\frac{E}{\sigma\delta} \frac{a^{1-\sigma}}{N\Delta} - F_x \right) dG[a] + \int_{a=a_{ptm}}^{a=a_{pnc}} \left(\frac{E}{\sigma\delta} \frac{\tau^{1-\sigma} a^{1-\sigma}}{N\Delta} \Omega - F_x \right) dG[a]$$

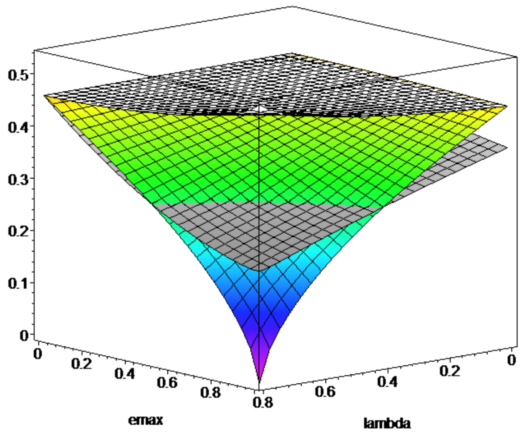
Solving for a_d we are able to determine the domestic marginal producer:

$$a_d = \left[\frac{a_{\max}^b \frac{F_I}{F_d}}{\frac{1}{\delta} \frac{b}{b+1-\sigma} \left[1 + (1 - \tau^{1-\sigma}\Omega) \left(\frac{a_{ptm}}{a_d} \right)^{b+1-\sigma} + \tau^{1-\sigma}\Omega \left(\frac{a_{pnc}}{a_d} \right)^{b+1-\sigma} \right] - \left[1 + \frac{F_x}{F_d} \left(\frac{a_{pnc}}{a_d} \right)^b \right]} \right]^{\frac{1}{b}} \tag{4.17}$$

To obtain the cut-off exporters a_{ptm} and a_{pnc} ,equation 4.17 is replaced in 4.13 and 4.14.

The three cut-offs obtained are plotted in figure 4.6

Figure 4.6: Marginal producer (white plot) and exporter type *ptm* (colored plot) and exporter type *pnc* (grey plot)



(E=1000000 FI=Fd=Fx=100, $\delta = 0.2$,$b = 2.5$, $a_{max} = 1$*)*

In line with the findings about the number of firms, uncertainty and lack of financial development, acting on competition, tend to increase the threshold for domestic producers and to make exporting harder for both types of exporters. Nevertheless the effect on the two types of exporters differs in magnitude as the a_{ptm} is more responsive.

In figure 4.6 the *ptm* are represented by the segment on the z axis between the origin and the colored line, the *pnc* exporters by the distance between the colored plot and the grey one and the domestic producers (which only serve the domestic market) by the distance between the highest of the colored and grey and the white plot. Finally *pnc* exporters disappear from the market when λ and e_{max} are low (below a_{ptm}), confirming what said in section 4.4.

4.7 The volume of trade

Similarly to trade restrictions the lack of financial integration and the existence of exchange rate uncertainty affects cut-offs and in some cases export prices. At the same time it is not obvious that this similarity carries to the impact they have on the volume of exports. So far we have studied policies of firms in terms of pricing and capacity, computed the expected value of profits, obtained the number of producers per market and the three type of thresholds. Addressed next is what happens to the total volume of trade when these two exogenous variables are modified by some external event. For instance the consequences of the common currency adoption on the overall volume of trade are very important to the policy makers, more so than on any single firm. Policy maker evaluate the effectiveness of a plan based mainly on its macroeconomic effects.

In a standard firm heterogeneous model, trade liberalization entails an increase of trade volumes. The same conclusion is not granted in this model where the impact differs depending on a firm's productivity. Therefore this section is dedicated to evaluate the impact of exchange rate stabilization and financial integration on overall export volumes.

At the optimum, the volume of total exports represented by *ptm* and *pnc* type of exporters is given by the following expression:

$$V_x = \frac{E}{\Delta a_d^b}\frac{\sigma-1}{\sigma}\frac{b}{b-\sigma}\left\{a_{ptm}^{b-\sigma} + (\tau)^{-\sigma}\left[1-\frac{e_{\max}}{\tau^2}\left(\frac{\sigma-1}{\sigma}\right)^2\right]\left(a_{pnc}^{b-\sigma} - a_{ptm}^{b-\sigma}\right)\right\}$$

The first addendum in curly brackets is the volume of trade associated to *ptm* exporters, the remaining part of the expression within brackets is the volume of exports generated by *pnc* exporters. In first place the effect depends on presence of *pnc* type of exporters: as long as both the kind of exporters are on the market, which means as far as integration is low enough and export shocks are sufficiently volatile, then these two variables have the power to influence trade. When the *pnc* type disappears, then they stop affecting volume of trade.

Nonetheless, when the effect exists, we still recognize that borrowing restraints and export shocks alter the volume of trade through different channels. Restraints modify trade indirectly via the two exporting thresholds. Shocks via a twofold conduit by varying the two exporting threshold and more directly also the optimal price and capacity building.

Analyzing the expression we are able to ascertain that lower volatility reduces Δ, a_d, τ, increases the expression in square brackets, raises a_{ptm} and diminishes a_{pnc}, this we knew from figure 4.6; $(a_{pnc} - a_{ptm})$ shrinks as e_{max} tends to zero.

In conclusion, researchers should expect to find a significant relation of causality between exchange rate volatility and trade depending on the stage of development of a country and the level of volatility .Therefore, there are relevant threshold effects to be considered in empirical estimations.

5 Searching costs fill the gap between demand and supply

In this model consumers purchasing possibilities are constrained by supply, which is set ex ante due to market rigidities.

In the case of the imported varieties, whose price is set in the exporter currency, a depreciation of exporter's currency below $(1 - e_k)$, by making them cheaper, increases their demand, beyond maximum available supply (K). This is a consequence of the fact that producers decide both price and capacity in advance.

To re-establish the equilibrium between demand and supply we assume the existence of a costly research of the import variety. More precisely, for each variety the additional searching cost, expressed in units of consumer currency, is proportional to the wedge between demand and supply as follow:

$$c = \frac{E_v}{K} - \frac{p_{pnc}}{1 - e}$$

where c is the cost of searching and E_v is the share of income spent on variety v. The price paid by the consumer (p_c)is endogenously determined to clear the market:

$$p_c = c + \frac{p_{pnc}}{1-e}$$

where $\frac{p_{pnc}}{1-e}$ is the price set by the exporter in the consumer's currency and c is endogenously determined.

A similar situation happens for domestically produced varieties and imported varieties (whose price is set in consumer's currency). When the exporter's currency appreciates above the equilibrium level, the indirect channel of transmission is represented by the price index. This is taken as given by each producer when deciding over production strategy since it is unknown to each of them the proportion of firms that will opt for *ptm* rather than *pnc* strategies and what will be the shock. When the currency appreciates, these varieties become relatively cheaper and the demand rises above the maximum supply. Equilibrium is reestablished between demand and supply by a mechanism very much like the one just illustrated.

6 Final remarks

This model is unique in that it introduces- for the first time- real rigidities and market imperfections in a firm heterogeneous framework. It presents a set of results which enable the formulation of clear predictions about the characteristics a firm must have to be shielded from exchange rate uncertainty. In particular, a firm must belong to the subset of the most efficient and operate within a country whose financial system is sufficiently integrated. This happens because the financial system accepts as collateral the entire domestic sales to the extent that they are not subject to uncertainty, and a fraction of the operation of export sales, varying with the maturity of the system itself and the riskiness.

It is worth noting that even the least integrated financial system ($\lambda = 1$) will lend no less than the amount of resources equivalent to exporting sales under the worst case ($e = e_{max}$). As the country progresses, operation of international hedging against the exchange rate variations becomes available encouraging more lending. In this situation only large firms expect to gain enough extra profits from domestic sales to finance the exporting choice. As a consequence, two exporter types will exist in the same market. A first set operating via a pricing to market strategy and a second set composed by smaller, and less efficient producers which can only access resources for pricing the foreign market in national currency. Due to rigidities, the last strategy implies price and production capacity set before the currency value is disclosed. The price is still a mark-up over the marginal cost but it becomes a function of exchange rate volatility- integration and stabilization lead to a lower price index. In terms of production, the fact that exporters respond to uncertainty under building capacity depends on profitability of exporting and by the same token, firms may ask to be compensated for the extra risk they face on the export market charging higher mark-ups. Finally, being that production capacity and price is set in advance, the model turns out to be a disequilibrium model, when consumers see their demand unsatisfied.

Stability of exchange rates encourages new exporters to join the international trade system, increases international competition and forces out of the market less efficient domestic firms. In addition it extends the range of producers that implement the *no pass through strategy.* Similar changes are brought about by financial integration. There exist a threshold value of λ below which all exporters will adopt a *ptm* pricing and exchange rate volatility stops reducing export.

In the light of our findings, the modest size of the "euro effect" on both price and trade in Europe is the outcome of a financially developed system and very low exchange rate volatility pre-existing the monetary union. Next, empirical investigations must be directed at measuring the effect volatility exercises on the trade of developing countries and on the exports of small firms. In terms of pricing, it would be very interesting to explore whether small versus large firm size implies full versus no pass through of exchange rate on price.

A Appendix:

A.1 Derivation of optimality conditions for *pnc* exporters

Most of the derivations in this paper are straightforward. Probably the only passages which deserve some attention are those concerning the optimization problem of exporters who set the price in national currency. Solving the integral of equation 4.5, the optimization problem is represented by the following equations:

$$\max_{p_{pnc},K} E\left(\pi_{pnc}\right) = \frac{E * p_{pnc}^{1-\sigma}}{P^{(1-\sigma)} 2 * e_{\max}} \left[\left(\frac{K}{c_{pnc}}\right)^2 - \left(1 - e_{\max}\right)^2 \right] + \frac{p_{pnc} K}{2 * e_{\max}} \left(1 - \frac{K}{c_{pnc}} + e_{\max}\right) - aK$$

$$s.t. : K = \frac{E * p_{pnc}^{-\sigma} \left(1 - e_k\right)}{P^{(1-\sigma)}}$$

This is a simple maximization subject to a constraint. We replace the constraint in the objective function and derive the f.o.c.s with respect to the price and the capacity constraint. Below are reported the passages to obtain K.

$$\frac{\partial E\left(\pi_{pnc}\right)}{\partial K} = 0 = \frac{1}{2 * e_{\max}} \left[\frac{E * p_{pnc}^{1-\sigma}}{P^{(1-\sigma)}} \left(\frac{K}{c_{pnc}^2}\right) + p_{pnc} \left(1 - \frac{K}{c_{pnc}} + e_{\max}\right) - \frac{p_{pnc} K}{c_{pnc}} \right] - a$$

$$\frac{1}{2 * e_{\max}} \left[\frac{p_{pnc} K}{c_{pnc}} + p_{pnc} - \frac{p_{pnc} K}{c_{pnc}} + p_{pnc} e_{\max} - \frac{p_{pnc} K}{c_{pnc}} \right] - a = 0$$

$$\left[p_{pnc} - \frac{p_{pnc} K}{c_{pnc}} + p_{pnc} e_{\max} \right] = 2 * e_{\max} * a$$

$$\frac{K}{c_{pnc}} = \left[1 - \frac{2 * e_{\max} * a}{p_{pnc}} + e_{\max} \right]$$

$$K = \left[1 - e_{\max} \frac{2a - p_{pnc}}{p_{pnc}} \right] c_{pnc} \tag{A.1}$$

Similarly taking the first order derivative of the profit function with respect to p_{pnc} and replacing the optimal K in the f.o.c we obtain two prices which satisfy the f.o.c.s. Only one satisfies all the condition to be a maximum. See next section for the proof.

$$\frac{\partial E\left(\pi_{pnc}\right)}{\partial p_{pnc}}=0=\frac{(1-\sigma)E*p_{pnc}^{-\sigma}}{P^{(1-\sigma)}4*e_{\max}}\left[\left(\frac{K}{c_{pnc}}\right)^{2}-\left(1-e_{\max}\right)^{2}\right]+\frac{K}{2*e_{\max}}\left(1-\frac{K}{c_{pnc}}+e_{\max}\right)$$

replacing the solution for K into the f.o.c for the price and rearranging the expression we obtain the optimal price.

$$E\left(\pi_{pnc}\right)=-\frac{p_{pnc}^{\sigma+1}E^{-1}P^{1-\sigma}K^{2}}{4*e_{\max}}-\frac{p_{pnc}^{1-\sigma}EP^{\sigma-1}}{4*e_{\max}}\left(1-e_{\max}\right)^{2}+\frac{p_{pnc}K}{2*e_{\max}}\left(1+e_{\max}\right)-aK \tag{A.2}$$

$$\frac{\partial E\left(\pi_{pnc}\right)}{\partial p_{pnc}}=0=-\frac{(\sigma+1)}{4*e_{\max}}\left(\frac{K^{2}}{c_{pnc}}\right)-(1-\sigma)\frac{c_{pnc}}{4*e_{\max}}\left(1-e_{\max}\right)^{2}+\frac{K}{2*e_{\max}}\left(1+e_{\max}\right) \tag{A.3}$$

$$-(\sigma+1)\left[1-e_{\max}\frac{2ae_{\max}}{p_{pnc}}+e_{\max}\right]^{2}+2*\left(1+e_{\max}\right)\left(1-\frac{2ae_{\max}}{p_{pnc}}+e_{\max}\right)+(\sigma-1)\left(1-e_{\max}\right)^{2}=0 \tag{A.4}$$

$$p_{pnc}=a\frac{\sigma\left(1+e_{\max}\right)\pm\sqrt{\sigma^{2}\left(1-e_{\max}\right)^{2}+4e_{\max}}}{2\left(\sigma-1\right)}=a\frac{\sigma\left(1+e_{\max}\right)\pm\Psi}{2\left(\sigma-1\right)} \tag{A.5}$$

A.2 Proving uniqueness of optimal export price.

The Hessian matrix is reported below.

$$H=\begin{bmatrix}\frac{\partial\pi_{pnc}}{\partial^{2}p_{pnc}} & \frac{\partial\pi_{pnc}}{\partial p_{x}\partial K}\\ \frac{\partial\pi_{pnc}}{\partial p_{pnc}\partial K} & \frac{\partial\pi_{pnc}}{\partial^{2}K}\end{bmatrix}$$

$$H=\begin{bmatrix}-\frac{\sigma}{2p_{pnc}(2*e_{max})}\left[c_{pnc}\left(e_{max}+1\right)^{2}+\frac{K^{2}(1+\sigma)}{c_{pnc}}-c_{pnc}\left(1+\sigma\left(2e_{max}-1\right)\right)\right] & \frac{1+e_{max}}{2*e_{max}}-\frac{K}{c_{pnc}}\frac{(1+\sigma)}{(2*e_{max})}\\ \frac{1+e_{max}}{2*e_{max}}-\frac{K}{c_{pnc}}\frac{(1+\sigma)}{(2*e_{max})} & \frac{-p_{pnc}}{c_{pnc}(2*e_{max})}\end{bmatrix}$$

In order to prove that there is an optimum and that is unique we investigate the sign of $\frac{\partial\pi_{pnc}}{\partial^{2}p_{x}}$ and of the determinant. If $\frac{\partial\pi_{pnc}}{\partial^{2}p_{v}}$ is negative definite and the determinant is positive then we have a point of maximum, If $\frac{\partial \pi_{pnc}}{\partial^2 p_{pnc}}$ is positive definite and the determinant is positive then we have a point of minimum and otherwise we have a saddle point.

The determinant of H evaluated at K and

$$p_{pnc} = a\frac{\sigma\left(1+e_{\max}\right)+\Psi}{2\left(\sigma-1\right)} \tag{A.6}$$

is given by:

$$\det(H) = (\sigma-1)\frac{\sigma(1+e_{max})\Psi+\Psi^2}{e_{max}(\sigma(1+e_{max})+\Psi)^2}$$

and is positive for any value. The determinant of H evaluated at K and

$$p_{pnc} = a\frac{\sigma\left(1+e_{\max}\right)-\Psi}{2\left(\sigma-1\right)} \tag{A.7}$$

is given by:

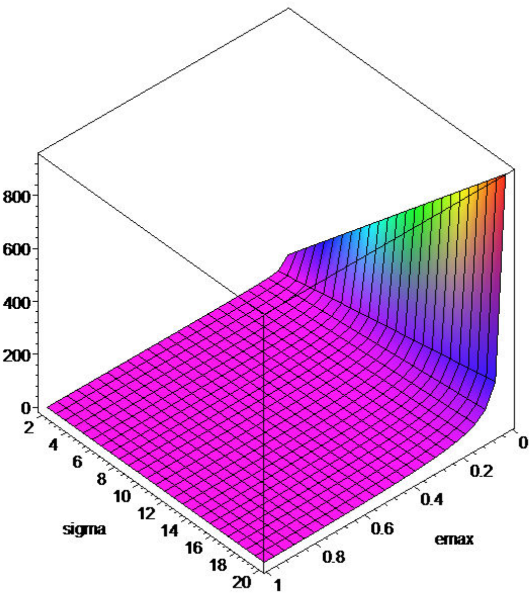
$$\det H = (1-\sigma)\frac{\sigma(e_{max}+1)\Psi-\Psi^2}{e_{max}(\sigma(e_{\max}+1)-\Psi)^2}$$

In this case the determinant is negative for $\sigma > \frac{2e_{max}}{1+e_{max}^2}$ which is always verified. Plotting the two determinants as function of the two variables σ and e_{max} we convey graphical intuition.
For the first value of p_{pnc} to be a maximum we still need to prove that $\frac{\partial \pi_{pnc}}{\partial^2 p_{pnc}}$ is negative.

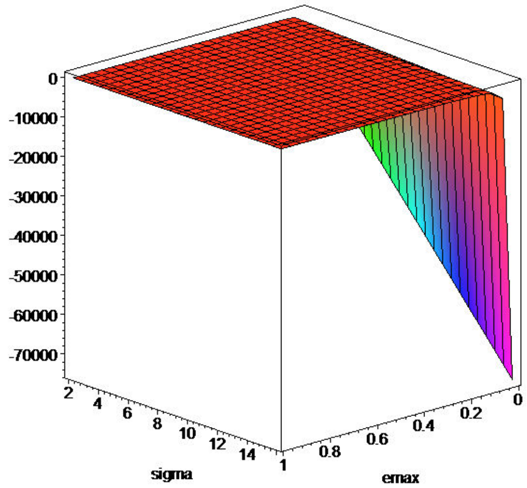
$$\frac{\partial \pi_{pnc}}{\partial^2 p_{pnc}} = -\frac{\sigma c_{pnc}}{4 p_{pnc} e_{max}}[(\sigma-1)(1-e_{max})^2+2(1-e_k)^2]$$

The sign depends on the expression in square brackets which is clearly always positive. Therefore the overall expression is proved to be negative. The expresssion $\frac{\partial \pi_{pnc}}{\partial^2 p_{pnc}}$ is negative for any value of p_{pnc} therefore we can conclude that A.6 is a point of maximum and A.7 is a saddle point.

Figure A.1: Hessian determinant



(a)



(b)