Graduate Institute of
International Studies | Geneva
Economics

HEI Working Paper No: 09/2007

A "Simple" Currency Union Model with Labor Market Frictions, Real Wage Rigidities and Equilibrium Unemployment

Mirko Abbritti
Graduate Institute of International Studies

Abstract

This paper derives a DSGE currency union model with labor market frictions, real wage rigidities and price staggering. The model combines many realistic features, but it is still tractable: like standard open-economy models, it can be closed in six equations. We derive and discuss the constrained efficient allocation and the decentralised equilibrium, under both flexible and sticky prices. We use the model to analyse how different labor market institutions or degrees of real wage rigidities influence the functioning of the currency union and the size and persistence of inflation and output differentials. We show that the presence of non trivial real imperfections affects substantially the transmission mechanism of shocks in general and, in particular, of monetary policy. Interestingly, we find that the implications of real wage rigidities and labor market frictions for business cycle fluctuations are likely to operate in opposite directions: a high degree of real wage rigidities tends to amplify the response of the real economy to shocks; when labor market are more sclerotic, instead, unemployment volatility tends to decrease while inflation volatility increases.

A "Simple" Currency Union Model with Labor Market Frictions, Real Wage Rigidities and Equilibrium Unemployment

Mirko Abbritti[1]
Graduate Institute of International Studies

This version: January 2007

[1]HEI Graduate Institute of International Studies, 132 Rue de Lausanne, CH-1211 Geneva, E-mail: abbritt3@hei.unige.ch. I am grateful to Charles Wyplosz, Stefano Manzocchi, Andrea Boitani, Mirella Damiani, Sebastian Weber, Mario Piacentini, Cinzia Alcidi, Alberto Amurgo Pacheco and Andreas Muller for very helpful comments and ideas.

Abstract

This paper derives a DSGE currency union model with labor market frictions, real wage rigidities and price staggering. The model combines many realistic features, but it is still tractable: like standard open-economy models, it can be closed in six equations. We derive and discuss the constrained efficient allocation and the decentralised equilibrium, under both flexible and sticky prices. We use the model to analyse how different labor market institutions or degrees of real wage rigidities influence the functioning of the currency union and the size and persistence of inflation and output differentials. We show that the presence of non trivial real imperfections affects substantially the transmission mechanism of shocks in general and, in particular, of monetary policy. Interestingly, we find that the implications of real wage rigidities and labor market frictions for business cycle fluctuations are likely to operate in opposite directions: a high degree of real wage rigidities tends to amplify the response of the real economy to shocks; when labor market are more sclerotic, instead, unemployment volatility tends to decrease while inflation volatility increases.

JEL Classification: E32, E52, F41

Keywords: Currency Union, labor market frictions, real wage rigidities, unemployment, sticky prices, inflation differentials

1 Introduction

Seven years after the launch of the Euro, participating countries still face important economic challenges. Economic growth has been quite disappointing. Structural unemployment is still high and labor force participation low. Inflation and output differentials are quite large and, more importantly, very persistent over time.

Following the seminal contribution by Mundell (1963), the structure of labor markets has often been seen as a crucial element in determining the costs and benefits of participating in a currency area. The old Mundell wisdom states that it may be very costly to abandon the devaluation tool when labor does not move easily and prices and wages are sticky. Adverse country-specific shocks, in this case, may trigger a long and painful adjustment process of low growth and rising unemployment, until the equilibrium is restored. Indeed, looking at Europe today, the general impression is that many States and politicians may have understated the economic costs of entering in a monetary union.

Three main elements seem to characterize European labor markets. First, unemployment is high and tends to be prolonged over time. As Blanchard notes in a recent paper, "being unemployed in Europe has always been a different experience from being unemployed in the United States (...) and has become increasingly so over time"[1]. Second, real wages seem to be rather inflexible. "Unemployment does eventually put some downward pressure on real wages in Europe, but a large share of the adjustment is borne by employment" (Mongelli 2002, p. 18). Third, labor market institutions are widely heterogeneous across countries, and this is likely to influence the functioning and performances of European economies. Indeed, the notion that labor market rigidities are at the core of European unemployment problem has now become widely accepted among policy makers.

Recent research, by integrating labor market frictions "à la Mortensen-Pissarides (1994)" in otherwise standard closed-economy New Keynesian (NK) models, has shown that the structure of labor markets influences substantially the transmission mechanism of monetary policy and, more generally, the overall adjustment of economic activity to shocks[2]. Labor market institutions are, in fact, an important determinant of the dynamics of real wages and of marginal costs of firms, which are in turn the main drivers of inflation. Specifically, it has been shown that the introduction of search and matching frictions "modifies the nature of real marginal costs faced by firms in a way that lowers the elasticity of marginal costs to output and thus help to account for the observed inertia in inflation and persistence in output" (Trigari 2005, p. 2). These results seem to suggest that the introduction of a more realistic labor market structure is needed in order to overcome some of the well-known weaknesses of the basic NK framework[3].

[1] Blanchard, "European Unemployment: The Evolution of Facts and Ideas" (2005, p. 6). The author notices that the proportion of long-term unemployment and unemployment's average duration are much larger in Europe than in the US. For instance, average duration, which in the US is around three months, in France stands over a year.

[2] Several authors have introduced a search model for the labor market in otherwise standard NK models. The main message is that labor market frictions and institutions matter for monetary policy. See, among others, Walsh(2005), Trigari(2005), Christoffel and Linzert(2005), Krause and Lubik (2005).

[3] See, e.g., Abbritti et al. (2006) for a discussion. The introduction of labor market imperfections allows one to overcome three main shortcomings of the basic NK model: 1) the absence of involuntary unemployment; 2) the lack of a meaningfull trade-off between inflation stabilization and output gap stabilization;

A few currency union models have been proposed in recent years[4]. Building on the standard NK framework, these models typically combine two "Keynesian" features - monopolistic competition in the goods markets and price stickyness - and Walrasian labor markets. The latter assumption has crucial implications. In fact, it implies that these models are unable to explain movements in involuntary unemployment: as workers are always on their supply curves, changes in hours of work or employment are only voluntary. Moreover, this assumption is at the heart of one key feature of the standard NK model, that is the lack of a trade-off between inflation stabilization and output gap stabilization. This in turn implies that, at the union level, some form of inflation targeting is optimal or nearly optimal[5]. Finally, it must be noticed that, by assuming perfectly competitive labor markets, we are implicitely ignoring a fundamental source of asymmetry among member countries, that is the wide heterogeneity in European labor market institutions.

In a very recent paper, Campolmi and Faia (2006) are the first to integrate labor markets frictions "à la Mortensen-Pissarides" into a DSGE currency union model. The currency union consists of two regions sharing the same currency and is characterized by a variety of frictions: matching frictions and wage rigidity in the labor market, monopolistic competition and price rigidity in the goods market. The result is a rich and quite complex model, which needs to be studied through calibration and simulations. The paper aims at studying the quantitative importance of labor market differences in generating differential inflation dynamics across euro area countries. Its complexity, however, makes it difficult to characterize analitically the solution and does not permit any normative analysis.

The aim of the present paper is to develop a *tractable* currency union model that fits the European empirical evidence and derive some policy implications. To this purpose, the model combines three key ingredients: (i) monopolistic competition and nominal rigidities "à la Calvo" in the goods market, which serve to give a role to monetary policy; (ii) labor market rigidities "à la Howitt (1988)", which generate involuntary unemployment; (iii) real wage rigidities, which hinder wage adjustments and shift the labor market adjustment from prices to quantities.

Following Blanchard and Galì (2006) and Abbritti et al. (2006), we model labor market imperfections in a very simple way. Drawing from Howitt (1988), we assume that firms face hiring costs which increase with the degree of labor market tightness[6]. Blanchard and Galì (2006) show how introducing hiring costs in a standard NK model results in a model simple enough "that its solution can be characterized analytically, the dynamic effects of shocks can be related to the underlying parameters, and optimal monetary policy can be derived"[7]. Abbritti et al. (2006) argue, in an independent work, that a NK model with hiring costs, while being considerably simpler, is able to generate dynamics that are very similar to the one of much more complex model which integrate nominal rigidities and

3) the great difficulty of the basic NK model in replicating the large and persistent response of output together with the sticky dynamics of inflation after nominal shocks.

[4] See, among others, Benigno (2004), Monacelli and Galì (2005), Benigno and Lopez-Salido (2002).

[5] See, e.g., Blanchard and Galì (2005) for a discussion.

[6] As in Blanchard and Galì (2006), labor market tightness is defined as the ratio of hires to the unemployment pool.

[7] Blanchard-Galì (2006), p. 34.

search models for the labor market.

Using this very simple modelling of the labor market, we develop a currency union model that incorporates many realistic features, but is still tractable. Interestingly, the model can be reduced to a six-equation model - like standard open-economy models. We regard the tractability of our framework as the key advantage of our approach.

The model provides a rigorous framework for the study of the potential causes of inflation and output differentials and, more generally, for the analysis of the funtioning of a currency union characterized by pervasive nominal and real rigidities. Moreover, it permits to study how different labor market institutions influence the dynamic behaviour of the member countries. Specifically, we study how labor market rigidities influence the dynamic behaviour of the member countries in response to three types of shocks: two asymmetric supply shocks (wage and productivity shocks) and one common demand shock (the monetary policy shock). We first explain the behaviour of the economy under the simple and highly special case of a currency union composed by symmetric countries. We then introduce some elements of asymmetry, considering first the impact of different degrees of real wage rigidities and then analysing what happens when the member countries are characterised by different labor market institutions.

Several interesting results emerge. First, when monetary policy is exclusively focused on inflation stabilisation, asymmetric wage or productivity shocks may have very long-lasting effects on unemployment, both at the country level and at the union level. The introduction of non trivial real imperfections creates, as in Blanchard and Galì (2005), a meaningfull trade-off between inflation stabilization and output gap stabilization. Second, even when the countries have similar structures, transitory asymmetric shocks can lead to large and long-lasting inflation and unemployment differentials. The size of these differentials is influenced by the degree of real wage rigidities and by labor market institutional parameters. Third, when member countries have different labor market fundamentals, symmetric policy shocks can have substantial asymmetric effects and lead to large differentials. Fourth and most importantly, not all sources of rigidity have the same effects: it does make a difference whether the rigidity lies in the wage determination mechanism or in the labor market structure. When the rigidity lies in the wage determination mechanism, real wages cannot fully adjust and shocks tend to be absorbed through changes in quantities - unemployment in our case. A higher degree of real wage rigidities thus amplifies the response of the real economy to shocks. When the rigidity lies in the labor market, it is more costly for firms to hire new workers and therefore unemployment does not vary as much as it would in a more flexible economy[8]. A country characterised by more sclerotic labor markets is thus likely to experience a smaller unemployment volatility. In other words, real wage rigidities and labor market imperfections are likely to have opposite effects on business cycle fluctuations. This is a very intuitive result, since (loosely speaking) in the first case the rigidity is in "prices", while in the second are "quantities" that cannot adjust.

The remainder of the paper is organised as follows. Section 2 briefly reviews and

[8] We define a labor market as "sclerotic" when it is characterized by low job-finding and separations rates.

discuss some stylised facts on inflation and output differentials in the Euro area. Section 3 is devoted to the building blocks of the model. Three different equilibrium allocations are derived: the constrained efficient, the flexible prices and the sticky prices allocations. Section 4 derives the constrained-efficient allocation, and shows that, under a standard parametrization, productivity shocks have no effect on unemployment. The fact that the welfare-relevant employment level is invariant to shocks considerably simplify the analysis, since it permits to say that all employment fluctuations are inefficient. Section 5 describes the decentralised equilibrium under flexible prices while Section 6 introduces nominal rigidities. The model is not solvable in closed-form, and to make progress requires log-linearizing the system. The dynamics of the model around the steady state are presented and discussed in Section 7. Section 8 describes the baseline calibration. Section 9 analyses the functioning of the currency union under different degrees of labor market rigidities or real wage rigidities. Finally, Section 10 compares the outcomes of different monetary policy rules and Section 11 concludes.

2 The Implications of a Suboptimal Currency Area

The European Monetary Union is not yet an optimal currency area. The empirical evidence suggests that labor mobility is low, prices and wages are sticky, and the labor and product markets are characterized by pervasive structural rigidities[9]. According to the Optimal Currency Area Theory, the presence of such rigidities hinder the adjustment process of member countries to changing economic conditions, as country-specific shocks can only be absorbed through a long and painful process of low growth and high unemployment. Indeed, the first seven years of the Euro seem to confirm this old intuition.

Inflation and output differentials, which reflect the efficiency of the adjustment process inside a currency union, are a big concern for policy-makers[10]. Fig. 1 shows the evolutions of inflation, output growth and of the current account in the "big four" countries of the euro area. Inflation and output growth differentials among Euro area countries are not unusually large, but they are very persistent[11]. As a result of lasting inflation differentials, a number of EMU economies are experiencing a sizeable loss of external price competitiveness vis-à-vis their peers[12]. Inevitably, overvaluation leads to low output growth, rising unemployment and large current account deficits (almost ten percent in Spain and Portugal). Large current account deficits - as well as large debt levels - pose the problem of

[9] See, e.g., Mongelli (2002) and ECB (2005) for some evidence.

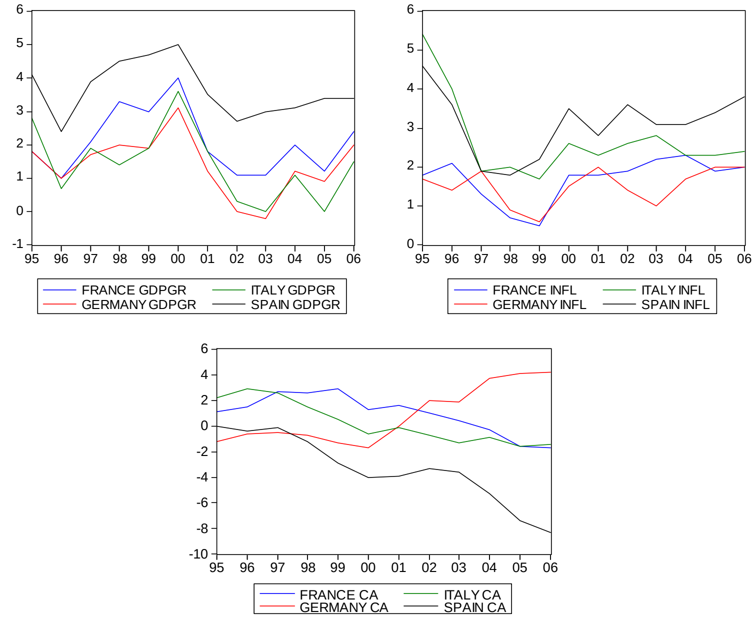
[10] See, e.g., ECB (2003 and 2005), Angeloni and Ehrmann (2004), Benalal, Diaz del Hoyo, Pierluigi and Vidalis (2006) for some evidence on inflation and output differentials and for analyses of the potential causes and policy implications.

[11] It must be noticed that inflation and output differentials are not undesirable per se, as they are a part of the equilibrating adjustment process inside the monetary union. Whether they are desirable or not depends on the causes of such differentials. In fact, inflation differentials may also be the product of "misaligned fiscal policies, diverging wage developments and deep-seated structural inefficiencies such as nominal and real rigidities in product and factor markets (ECB 2005, p.61)". The persistence of such differentials together with the empirical evidence on wage/price rigidities and on market imperfections, suggest that a large part of these divergences may indeed be inefficient and avoidable.

[12] This problem concerns, to different degrees, Greece, Italy, Portugal and Spain. See, e.g., Wyplosz (2006) and Ahearne and Pisani-Ferry (2006) for a discussion.

the vulnerability of certain countries to sudden stops of capital flows and/or to financial crises.

Figure 1: *The Implications of a Suboptimal Currency Area: Inflation, Output Growth and Current Accounts*



The traditional way to cope with a competitiveness problem is the devaluation of the exchange rate, but this is made impossible by Euro area membership. As Blanchard (2006) notes, there is no easy way out. A country has basically two options to improve its competitiveness. The first, is "to achieve a sustained increase in productivity growth"[13]; unfortunately, this is difficult to engineer, since it requires reforms in the goods and financial markets and is unlikely to work overnight. The second is lower nominal wage growth, which is equally difficult, especially from a political point of view. In the absence of any policy change, the most likely scenario is one of "competitive disinflation: a period of sustained high unemployment, leading to lower nominal wage growth until unit labor costs have decreased, competitiveness have improved, the current account deficit has decreased, and demand and output have recovered"[14]. This is typically a long and painfull process, and it does not come as a surprise the fact that some authors - and some politicians too - have started discussing about the risk of a EMU break-up.

Several interesting questions arise. For instance: why inflation and output differentials are so persistent? Which is the role of price/wage rigidities, and which is the one of different economic structures in the member states? How should the authorities deal with these differentials?

[13] Blanchard (2006) p.2.
[14] Ibid., p. 8.

In this paper we build a currency union model that may help to explain and characterize some of these facts, and to answer some of these questions. The attention is focused on the importance of different labor market structures and different degrees of real wage rigidities in explaining business cycle fluctuations and differentials. In particular, by determining the size of fluctations as deviations from the efficient level, we are able to focus on the *cyclical, inefficient* component of these differentials.

3 The Model

A currency union is a group of regions or countries sharing the same currency, with a single central bank entitled to conduct the monetary policy. To keep things as simple as possible, we consider a currency union that consists of two regions, Home and Foreign, taken of the same size (normalised to 1). Each economy, which is populated by identical, infinitively lived households, is specialised in the production of a bundle of differentiated goods. There is no migration across regions. Capital markets are complete. Countries are symmetric for everything apart from labor market institutions. The labor market is characterised by hiring costs, leading to involuntary unemployment in equilibrium[15].

3.1 Households

The representative household within a country is thought of as a continuum of members with names on the unit interval. Each household purchases consumption goods, holds money and supplies labor. Wages are fixed by bargaining, and, given the presence of involuntary unemployment, the labor supply is not binding. Household members can be employed or unemployed. To avoid distributional issues, we assume that households pool their income and consumption.

The representative household in country i ($i = H$ or F) seeks to maximize lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t - \chi_0 \frac{N_t^{1+\phi}}{1+\phi} \right\}, \quad E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t^* - \chi_1 \frac{(N_t^*)^{1+\phi}}{1+\phi} \right\} \tag{1}$$

where variables with star are referred to the foreign country. N_t^i denotes the number of employed individuals in the representative household of country i while C_t and C_t^* are the composite consumption indexes for the Home and Foreign country respectively, defined as:

$$C_t = \frac{(C_{Ht})^{1-\alpha} (C_{Ft})^{\alpha}}{(1-\alpha)^{1-\alpha} \alpha^{\alpha}}, \quad C_t^* = \frac{(C_{Ft}^*)^{1-\alpha} (C_{Ht}^*)^{\alpha}}{(1-\alpha)^{1-\alpha} \alpha^{\alpha}} \tag{2}$$

C_{jt}^i is the quantity of the good produced in country j and consumed by residents of country i. $\alpha \in [0,1]$ is the weight on the imported goods in the utility of private consumption; a value for α strictly less than $\frac{1}{2}$ reflects the presence of home bias in consumption.

[15] The basic framework of the currency union is inspired by the work of Benigno (2004) and Galì-Monacelli (2006). The structure of the labor market builds on Blanchard-Galì (2006) and Abbritti et al. (2006).

The production sectors are characterised by monopolistic competition. The index of country i's consumption of the good produced in country j, C^i_{jt}, is given by the usual CES aggregator:

$$C^i_{jt} = \left(\int_0^1 \left(C^i_{jt}(z) \right)^{\frac{\epsilon-1}{\epsilon}} dz \right)^{\frac{\epsilon}{\epsilon-1}}, \ i = H \ or \ F; \ j = H \ or \ F$$

The parameter ϵ $(\epsilon^*) > 1$ is the elasticity of substitution between varieties produced within Home (Foreign) country.

Utility maximization for the Home household is subject to a sequence of budget constraints of the form[16]:

$$\int_o^1 P_{Ht}(j) C_{Ht}(j) dj + \int_0^1 P_{Ft}(j) C_{Ft}(j) dj + E_t \left\{ Q_{t,t+1} D_{t+1} \right\} \leq D_t + W_t N_t - T_t$$

for $t = 0, 1, 2, ...,$ where $P_{it}(j)$ is the price of good j produced in country i (expressed in the units of the single currency). D_t is the nominal payoff in period t of the portfolio held at the end of period $t-1$; W_t is the nominal wage and T_t denotes lump-sum taxes.

We assume complete securities markets; $Q_{t,t+1}$ is the stochastic discount factor for one-period ahead nominal payoffs, which is common across countries. Implicit in the budget constraint is the assumption that the law of one price holds across the union.

The demands for the generic goods produced at Home and Foreign are respectively:

$$C^i_{Ht}(z) = \left(\frac{P_{Ht}(z)}{P_{Ht}} \right)^{-\epsilon} C^i_{Ht}\,; \ C^i_{Ft}(z) = \left(\frac{P_{Ft}(z)}{P_{Ft}} \right)^{-\epsilon^*} C^i_{Ft} \tag{3}$$

for $i = H, F(^*), z \in [0, 1]$. The appropriate domestic (producer) price indexes of the Home and Foreign countries are:

$$P_{Ht} = \left(\int_0^1 \left(P_{Ht}(z) \right)^{1-\epsilon} dz \right)^{\frac{1}{1-\epsilon}}; \ P_{Ft} = \left(\int_0^1 \left(P_{Ft}(z) \right)^{1-\epsilon^*} dz \right)^{\frac{1}{1-\epsilon^*}}$$

Since the law of one price holds, P_{Ht} represents both the price index for the bundle of goods imported from country H as well as H's domestic price index. From the demand functions (3), we get (for the Home country): $\int_o^1 P_{Ht}(j) C_{Ht}(j) dj = P_{Ht} C_{Ht}$ and $\int_o^1 P_{Ft}(j) C_{Ft}(j) dj = P_{Ft} C_{Ft}$.

Furthermore, the optimal allocation of expenditures by country of origin implies, for the Home country:

$$P_{Ht} C_{Ht} = (1 - \alpha) P_t C_t, \quad P_{Ft} C_{Ft} = \alpha P_t C_t \tag{4}$$

[16] The utility maximization problem for the foreign household is completely analogous.

while for the foreign country:

$$P_{Ht}C^*_{Ht} = \alpha P^*_t C^*_t, \quad P_{Ft}C^*_{Ft} = (1-\alpha) P^*_t C^*_t \tag{5}$$

where $P_t = (P_{Ht})^{1-\alpha} (P_{Ft})^{\alpha}$ and $P^*_t = (P^*_{Ht})^{\alpha} (P^*_{Ft})^{1-\alpha}$ are respectively the Home and the Foreign CPI indexes. As usual with Cobb-Douglas preferences, households allocate a fixed proportion of income to each consumption bundle.

Under the assumption of "home bias" in consumption (i.e. $\alpha < \frac{1}{2}$) different regions consume goods in different proportions; therefore, even if the Law of One Price holds for all goods, the Purchasing Power Parity (PPP) may not hold at the aggregate level ($P_t \neq P^*_t$).

Combining all previous results, we can write total consumption expenditures by Home's households as $P_{Ht}C_{Ht} + P_{Ft}C_{Ft} = P_tC_t$. Thus, conditional on optimal allocation of expenditures, the period budget constraint is given by:

$$P_tC_t + E_t\left\{Q_{t,t+1}D_{t+1}\right\} \leq D_t + W_tN_t - T_t \tag{6}$$

The remaining optimality conditions for country i are given by:

$$\beta R_t E_t \left(\frac{C^i_t}{C^i_{t+1}} \frac{P^i_t}{P^i_{t+1}} \right) = 1 \tag{7}$$

$$\chi_i C^i_t \left(N^i_t\right)^{\phi} \leq \frac{W^i_t}{P^i_t} \tag{8}$$

where $R_t = \frac{1}{E_tQ_{t,t+1}}$ is the (gross) nominal interest rate. The first condition is a conventional Euler condition. The second, which is similar to a standard labor supply equation, determines how many individuals within the representative household join the labor force. The introduction of hiring costs implies that this condition is not binding in equilibrium. In the following we assume that, in equilibrium, the wage is set at a level that guarantees full participation ($N^i_t = 1$). The assumption of full participation allows us to say that all unemployment is "involuntary".

3.2 The Terms of Trade and the Real Exchange Rate

In this section we introduce some definitions and identities that are used extensively below. First, we define the bilateral term of trade between the Home and Foreign countries as the ratio of the price of goods produced in country F to that produced in country H:

$$S_t = \frac{P_{Ft}}{P_{Ht}} \tag{9}$$

The terms of trade, which represent an index of competitiveness, plays a central role in our model. Movements in the terms of trade are crucial for understanding the response of the economy to asymmetric shocks and the transmission mechanism of monetary policy.

As the Law of One Price holds for all goods - which implies $P_{Ft} = P^*_{Ft}$ and $P_{Ht} = P^*_{Ht}$ - the CPI and the domestic price indexes in the two regions are related according to:

$$P_t = P_{Ht}\left(S_t\right)^{\alpha}, \quad P^*_t = P_{Ft}\left(S_t\right)^{-\alpha} \tag{10}$$

Let domestic (i.e. producer prices') inflation be defined as the rate of change of domestically produced goods, i.e. as $\pi_t^i \equiv \log \frac{P_t^i}{P_{t-1}^i} = p_t^i - p_{t-1}^i$, where $p_t^i = \log P_t^i$. Taking logs of the above identities, we obtain a relation between Domestic and CPI inflation:

$$\pi_t = \pi_t^H + \alpha \Delta s_t, \quad \pi_t^* = \pi_t^F - \alpha \Delta s_t \tag{11}$$

for the Home and the Foreign country respectively[17].

Finally, the real exchange rate V_t is defined as the ratio between foreign and home CPIs and is related to the terms of trade according to:

$$V_t = \frac{P_t^*}{P_t} = (S_t)^{1-2\alpha} \tag{12}$$

3.3 International Risk Sharing

Capital markets are complete: each household has access to a complete set of contingent claims, traded internationally. Combining the first order conditions relative to state contingent securities in the two countries, we obtain the usual result:

$$V_t = \psi \frac{u'(C_t^*)}{u'(C_t)} = \psi \frac{C_t}{C_t^*} \tag{13}$$

where $\psi = V_0 \frac{u'(C_0^*)}{u'(C_0)} = \frac{P_0^*}{P_0} \frac{u'(C_0^*)}{u'(C_0)}$ is a constant, reflecting initial conditions regarding relative net asset positions. If PPP holds (and this will occur in this model for $\alpha = 1/2$), the real exchange rate $V_t = 1$ and the marginal utilities of consumption are equated up to a constant ψ. In general, movements in the real exchange rate will be reflected in different consumption rates:

$$C_t = \frac{1}{\psi} V_t C_t^* \tag{14}$$

Therefore, even with complete financial markets, it is not efficient to equalize consumption across countries when there is a Home Bias in consumption ($\alpha < \frac{1}{2}$).

Henceforth, to keep the analysis as simple as possible, we assume initial conditions are such that $\psi = 1$.

3.4 Firms and the labor market

The setup of the supply side of the economy follows Blanchard-Galì (2006).

The production in each country is composed by a continuum of firms, indexed by $j \in [0, 1]$. Each firm in a country produces a differentiated good with an identical technology:

$$Y_t^i(j) = A_t^i N_t^i(j), \ for \ i = H, F(^*) \tag{15}$$

where the variables A_t^i represent the state of technology in country i.

[17] Notice that the distinction between CPI inflation and domestic inflation, while important at the country level, vanishes for the monetary union as a whole. In fact, summing up the equation for the logs in prices, yields the equality $p_{CPI,t}^U = p_t^U$.

In each period a fraction δ_i of the employed loses their jobs and joins the unemployment pool. Employment in firm j evolves according to:

$$N_t^i(j) = (1-\delta_i)N_{t-1}^i(j) + H_t^i(j),\ \ for\ i = H, F(^*) \tag{16}$$

where $H_t^i(j)$ is the the number of new hires for firm j in country i. We assume that the job destruction rate δ_i is exogenously given[18].

We denote by U_t^i the number of searching workers who are available for hire in country i. Since we make assumptions below that guarantee full participation, U_t^i is defined as[19]

$$U_t^i = 1 - (1-\delta_i)N_{t-1}^i,\ \ for\ i = H, F(^*) \tag{17}$$

"After hiring" unemployment, instead, is defined as the fraction of the population who are left without a job after hiring takes place, $u_t^i = 1 - N_t^i$.

Aggregate hiring $H_t^i \equiv \int_0^1 H_t^i(j)dj$ evolves according to

$$H_t^i = N_t^i - (1-\delta_i)N_{t-1}^i \tag{18}$$

Where $N_t^i \equiv \int_0^1 N_t^i(j)dj$ denotes aggregate employment[20].

Firms face a cost of searching and recruiting new workers "à la Howitt"[21]. Hiring costs for firm j in country i are:

$$G_t^i H_t^i(j),\ \ for\ i = H, F(^*) \tag{21}$$

where G_t^i is the cost per hire in country i (expressed in terms of the domestic CES bundle of goods), which is taken as given by the individual firm. Following Blanchard–Galì (2006), we assume

$$G_t^i = A_t^i B^i \left(\frac{H_t^i}{U_t^i}\right)^{\varphi},\ \ for\ i = H, F(^*) \tag{22}$$

where $\varphi > 0$ and B^i is a positive scaling parameter that may be influenced by the authorities. The marginal cost of hiring is increasing in the aggregate hiring rate H_t^i: this captures the idea that a high rate of hiring may force firms to increase their search

[18] There is some empirical evidence showing that the job destruction rate remains rather constant over time. See Christoffel and Linzert (2005), p. 12.

[19] We assume that the wage is set at a level such that at all times all individuals are either employed or willing to work. Notice that the labor force is normalized to unity.

[20] We define the aggregate output for Home as $Y_t = \left(\int_0^1 (Y_t(j))^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}$. The amount of labor employed is thus given by:

$$N_t = \int_0^1 N_t(j)dj = \frac{Y_t Z_t}{A_t} \tag{19}$$

where $Z_t = \int_0^1 \frac{Y_t(j)}{Y_t} dj$. It can be shown that equilibrium variations in $z_t = \log Z_t$ around the perfect foresight steady state are of second order. Thus, up to a first order approximation,

$$y_t = a_t + n_t \tag{20}$$

[21] Cfr. Howitt (1988).

intensity. The marginal cost is decreasing in U_t^i: a high aggregate unemployment makes it easier and cheaper for firms to find willing and competent workers. Notice that there are two externalities at work in the model. When a firm hires new workers, she does not internalize the effect that her action has on the cost of hiring for other firms through H_t^i and U_t^i.

In this framework, the presence of hiring costs creates a friction in the labor market similar to that of standard, but much more complex, search models.

If we define the labor market tightness index as

$$x_t^i \equiv \frac{H_t^i}{U_t^i},\ for\ i = H, F(^*) \tag{23}$$

i.e. as the ratio of aggregate hires to the employment rate, we can rewrite the cost per hire for H and F as

$$G_t = A_t B\,(x_t)^{\varphi}\,,\ G_t^* = A_t^* B^*\,(x_t^*)^{\varphi} \tag{24}$$

Recruitment costs are increasing in the labor market tightness index. Since by assumption firms can hire workers only from the pool of unemployed, $x_t \in [0, 1]$.

Note that, from the viewpoint of the unemployed, x_t can be interpreted as the probability of finding a new job in period t, i.e. as the job-finding rate.

3.5 Market Clearing Conditions

Consider the Home country. The clearing of the market for good j requires:

$$\begin{aligned} Y_t(j) &= C_{Ht}(j) + C_{Ht}^*(j) + G_t H_t(j) \\ &= \left(\frac{P_{Ht}(j)}{P_{Ht}}\right)^{-\epsilon} [C_{Ht} + C_{Ht}^* + G_t H_t] \\ &= \left(\frac{P_{Ht}(j)}{P_{Ht}}\right)^{-\epsilon} [C_t (S_t)^{\alpha} + G_t H_t] \\ &= \left(\frac{P_{Ht}(j)}{P_{Ht}}\right)^{-\epsilon} Y_t \end{aligned} \tag{25}$$

Plugging (25) in the definition of aggregate output, $Y_t = \left(\int_0^1 (Y_t(j))^{\frac{\epsilon-1}{\epsilon}} dz\right)^{\frac{\epsilon}{\epsilon-1}}$, we obtain the aggregate goods market clearing condition for Home:

$$Y_t = C_t (S_t)^{\alpha} + G_t H_t \tag{26}$$

Similar conditions hold for the Foreign country:

$$Y_t^*(j) = \left(\frac{P_{Ft}(j)}{P_{Ft}}\right)^{-\epsilon} Y_t^* \tag{27}$$

$$Y_t^* = C_t^* (S_t)^{-\alpha} + G_t^* H_t^* = C_t (S_t)^{-(1-\alpha)} + G_t^* H_t^* \tag{28}$$

The assumption of Cobb-Douglas preferences over the Home and Foreign goods allows us to derive a simple relation between the terms of trade and relative output[22]:

$$S_t = \frac{P_{Ft}}{P_{Ht}} = \frac{Y_t - G_t H_t}{Y_t^* - G_t^* H_t^*} \tag{29}$$

Equation (29) simply states the relative price of domestic (foreign) goods is inversely related to the quantity produced in the two regions (net of aggregate hiring costs)[23].

This expression allows us to highlight one simple, but interesting, point. First, note that in this model inflation differentials are simply represented by variations in the terms of trade, i.e. $\Delta s_t = \pi_t^F - \pi_t^H$. Suppose that in a long run equilibrium the Home and the Foreign country are characterised by different productivity growth rate, which we call γ_a and γ_a^*. Assume employment is constant in the long run equilibrium; it is easy to check that inflation differentials evolves according to:

$$\Delta s_t = \pi_t^F - \pi_t^H = \gamma_a - \gamma_a^*$$

This shows one possible explanation for the persistent inflation differentials we see in the European Union: persistent differences in the productivity growth rates[24]. Inflation differentials, in this case, are an *equilibrium phenomenon*, performing a positive role: they allow the clearings of all markets. In other words, inflation differentials are not undesirable per se. As Ahearne and Pisani-Ferry puts it, "whether the observed differences are desirable or undesirable, depends in large part on the nature of shocks that are causing that divergences"[25].

In this paper, by determining inflation and output fluctuations in terms of deviations from the corresponding efficient levels, we are able to focus on the cyclical, undesired, components of inflation and output differentials. To this purpose, we need to analyse three different equilibrium allocations: the constrained efficient, the flexible prices and the sticky prices allocations. This is the task to which we turn.

4 The Social Planner 's Problem

In our framework there are three sources of inefficiency; two of them are standard in the New Keynesian literature, and serve to give a role to monetary policy. The first consists

[22] The Cobb-Douglas assumption in fact imply that the percentage variation in relative prices is equal, and opposite in sign, to the percentage variation in relative quantities.

[23] Notice that equation (29) allows us to pin down the steady state level of S_t:

$$S = \frac{AN(1 - g\delta_H)}{A^* N^* (1 - g^* \delta_F)}$$

where $g = B(x)^{\varphi}$ and $g^* = B^*(x^*)^{\varphi}$.

[24] This mechanism differs from the famous Balassa-Samuelson effect for two main reasons: (i) in the standard Balassa-Samuelson, each country produces two goods, one tradable and one nontradable; (ii) in our case PPP does not hold.

[25] See Ahearne and Pisani-Ferry (2006), p. 3. These authors compare the cases of Ireland, where the real exchange rate appreciation can be explained by "advances in productivity and movements up the value chain in the traded goods sector", with the cases of Portugal and Italy, where competitiveness loss has contributed to poor economic performances.

in market structures characterized by monopolistic competition. The second comes from the assumption of price rigidities. The third source of inefficiency, which is specific to this model, is the presence of frictional costs for hiring new workers.

In this section we derive the so-called "constrained efficient allocation". Following Blanchard and Gali' (2006), we assume that the social planner maximizes the welfare of the Union, taking as given the technological constraints and the labor market frictions that are present in the decentralised economy. In other words, the social planner cannot eliminate or reduce hiring costs, which are simply taken as a fact of life; he can, however, internalize the effects of variations in employment on labor market tightness and, hence, on hiring costs[26].

Given simmetry in preferences and technology, the social planner chooses an equilibrium in which the goods, in each countries, are produced and consumed in identical quantities $C^i_{jt}(z) = C^i_{jt}$. Moreover, since participation in the labor market has no individual costs but some social benefits (it lowers hiring costs), the social planner chooses an allocation with full participation.

Hence, the Union's optimal allocation can be described as the solution of the following social planner's problem:

$$Max \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t - \chi_0 \frac{N_t^{1+\phi}}{1+\phi} + \log C_t^* - \chi_1 \frac{(N_t^*)^{1+\phi}}{1+\phi} \right\}$$

$$s.t.$$

$$C^W_{Ht} \equiv C_{Ht} + C^*_{Ht} = A_t N_t - G_t H_t$$

$$C^W_{Ft} \equiv C_{Ft} + C^*_{Ft} = A^*_t N^*_t - G^*_t H^*_t$$

where C^i_t, C^i_{jt}, G^i_t and H^i_t are as defined before. Notice that the previous constraints already embed the optimal condition whereby the different good types in any given country should be produced and consumed in identical quantities.

Maximization with respect to consumption leads to the following optimality conditions:

$$\frac{1-\alpha}{\alpha} = \frac{C_{Ht}}{C^*_{Ht}} = \frac{C^*_{Ft}}{C_{Ft}}$$

$$\frac{\nu_t}{\xi_t} = \frac{\alpha}{1-\alpha}\frac{C_{Ht}}{C_{Ft}} = MRS^H = \frac{1-\alpha}{\alpha}\frac{C^*_{Ht}}{C^*_{Ft}} = MRS^F$$

where ξ_t is the shadow value of an additional unit of the good produced at Home and ν_t is the shadow value of an additional unit of the foreign good. As usual, the optimal allocation of consumption implies that the marginal rate of substitution between the two goods is equal across agents. Moreover, the Cobb-Douglas preferences imply that the ratio of domestic goods over imported goods consumption is constant and equal to $\frac{1-\alpha}{\alpha}$.

Much more interesting is the optimality condition with respect to employment. For ease of exposition we will focus on the Home country. Solving the social planner's problem

[26] Blanchard-Galì (2006), p.9

with respect to Home employment, we obtain:

$$\chi_0 \frac{N_t^{\phi} C_{Ht}^W}{A_t} \leq 1 - B(1+\varphi)x_t^{\varphi} \qquad (30)$$
$$+B(1-\delta_H)E_t \left\{ \frac{C_{Ht}^W}{C_{Ht+1}^W} \frac{A_{t+1}}{A_t} B \left[\begin{array}{c} x_{t+1}^{\varphi} + \\ \varphi x_{t+1}^{\varphi}(1 - x_{t+1}) \end{array} \right] \right\}$$

which must hold with strict equality if $N_t < 1$. The above condition states that the marginal rate of substitution between labor and consumption (the left hand side) has to be less or equal to the corresponding marginal rate of transformation (the right hand side) - both normalised by productivity. Hiring an additional worker at time t has three effects. First, it generates additional output. Second, it increases the recruitment costs at time t. This effect is represented by the term $-B(1+\varphi)x_t^{\varphi}$. Third, it reduces the costs of hiring new workers in period t+1. This effect is captured by the last term at the right hand side.

The important point to note is that the above expression implies a *constant level of employment*. Note in fact that, world consumption of the home good is proportional to productivity

$$C_{Ht}^W \equiv C_{Ht} + C_{Ht}^* = A_t(N_t - Bx_t^{\varphi} H_t) \qquad (31)$$

It follows that the optimality condition does not depend on the productivity levels prevailing at Home (or Foreign). This invariance is the result of two main assumption:

1. The utility function is log in consumption: this implies that income and substitution effects offset each other on the labor supply.

2. Unit hiring costs varies one-for-one with productivity shocks.

The fact that employment is constant is a very useful result, since it allows us to say that any fluctuation in employment is inefficient[27].

To determine the efficient level of employment we can proceed in two steps. First, the efficient level for the labor market tightness indicator, x_E, is implicitely determined as the solution to

$$\chi_0 \left(\frac{x}{\delta_H + (1-\delta_H)x} \right)^{1+\phi} (1 - \delta_H B(x)^{\varphi}) \leq 1 - (1 - \beta(1-\delta_H))(1+\varphi)B(x)^{\varphi} \qquad (32)$$
$$-\beta(1-\delta_H)\varphi B x^{1+\varphi}$$

Second, the optimal level of employment at Home is given by

$$N_E = \frac{x_E}{\delta_H + (1-\delta_H)x_E} \qquad (33)$$

The optimal employment level depends therefore on the separation rate δ_H, on the hiring costs' scaling parameter B, on the sensitivity of hiring costs to labor market conditions φ and on parameters influencing the disutility of working (ϕ and χ_0).

A constant employment level implies that output is proportional to Home productivity ($Y_t = A_t N_E$) while consumption depends on both Home and Foreign productivity ($C_t = A_t^{1-\alpha} (A_t^*)^{\alpha} N_E(1 - \delta_H B(x_E)^{\varphi})$.

Similar conditions and the same conclusions hold for the Foreign country[28].

[27] Blanchard and Galì (2006, p. 9-11) get the same result in the context of a one-country model.

[28] The appendix contains the key expressions for the foreign country.

5 Equilibrium under Flexible Prices

In this section we derive the equilibrium under the assumption that prices are flexible. We first describe the optimal price setting of a firm, given the wage. We then characterize the equilibrium that emerges with Nash bargained wages. Finally, we introduce real wage rigidities in form of a Hall (2005) type wage norm.

We focus on the Home country; the solution for the Foreign country is completely symmetric.

5.1 Optimal Price Setting

Suppose that all firms adjust prices optimally each period to maximize the present discounted value of expected profits:

$$E_t \sum_{s=0}^{\infty} Q_{t,t+s} \left\{ P_{Ht+s}(i) Y_{t+s}(i) - P_{Ht+s} G_{t+s} H_{t+s}(i) - W_{t+s} N_{t+s}(i) \right\} \tag{35}$$

subject to the sequence of demand constraints $Y_t(i) = \left(\frac{P_{Ht}(j)}{P_{Ht}}\right)^{-\epsilon} Y_t$, the production function and the employment evolution equation. $Q_{t,t+s} = \beta^s \frac{C_t}{C_{t+s}} \frac{P_t}{P_{t+s}}$ is the relevant stochastic discount factor for nominal payoffs. Notice that the unit recruitment costs are expressed in units of *domestic* goods.

The optimal price setting rule takes the form of a markup $\mu = \frac{\epsilon}{\epsilon - 1}$ over the real marginal cost:

$$\frac{P_{Ht}(i)}{P_{Ht}} = \frac{\epsilon}{\epsilon - 1} MC_t = \mu MC_t \tag{36}$$

where the firm's real marginal cost is (expressed in terms of domestic goods):

$$MC_t = \frac{W_t^R}{A_t} (S_t)^{\alpha} + B (x_t)^{\varphi} - \beta(1 - \delta_H) E_t \left\{ \frac{C_t (S_t)^{\alpha}}{C_{t+1} (S_{t+1})^{\alpha}} \frac{A_{t+1}}{A_t} B (x_{t+1})^{\varphi} \right\} \tag{37}$$

and $W_t^R = \frac{W_t}{P_t}$ is the real wage expressed in terms of the consumption good.

The key difference between the supply side in our model and in a standard New Keynesian model with a neoclassical labor market is the behaviour of the real marginal cost[29]. In a model with a competitive labor market the real marginal cost is strictly related to the evolution of the real wage:

$$MC_t = \frac{W_t^R}{A_t} (S_t)^{\alpha} \tag{38}$$

As for Home, it can be shown that the optimal level of employment does not depend on productivity, and is given by:

$$N_E^* = \frac{x_E^*}{\delta_F + (1 - \delta_F) x_E^*} \tag{34}$$

where x_E^* is the efficient level for the tightness indicator for the foreign country, implicitely given as the solution to the optimality condition for N_t^*.

The levels of consumption and output are given by $C_t^* = A_t^{\alpha} (A_t^*)^{1-\alpha} N_E^* (1 - \delta_F B (x_E^*)^{\varphi})$ and $Y_t^* = A_t^* N_E^*$.

[29] Krause and Lubik (2005) make a similar argument comparing a standard NK model with a model with search and matching frictions in the labor market. See p.10-11.

In our model, which embeds the NK model as a special case, the presence of hiring costs creates a wedge between the real wage and the marginal costs relevant for the firm, which in turn are essential to explain inflation dynamics. This wedge consist of two terms. The first, $B(x_t)^{\varphi}$, represents the additional cost the firm faces to hire a new worker; the second - the last term in (37) - reflects the savings in future hiring costs resulting from increasing the number of employees today. The cyclical behaviour of marginal costs in a model with labor market frictions can thus depart substantially from that of real wages. As Krause and Lubik (2005) notice, "Hiring frictions generate a surplus for existing matches which give rise to long-term employment relationships. These, in turn, reduce the allocative role of current real wages. As a consequence, the effective real marginal cost can change even if the wage does not change"[30].

In a symmetric equilibrium, $P_{Ht}(i) = P_{Ht}$ for all $i \in [0, 1]$, and hence the optimal price setting implies:

$$MC_t = \frac{1}{\mu} \tag{39}$$

for all t. When shocks occur, each firm varies its prices and hiring decisions to keep the marginal cost constant. It follows that in equilibrium[31]:

$$\frac{W_t^R}{A_t}(S_t)^{\alpha} = \frac{1}{\mu} - B(x_t)^{\varphi} + \beta(1-\delta_H)E_t\left\{\frac{C_t (S_t)^{\alpha}}{C_{t+1}(S_{t+1})^{\alpha}}\frac{A_{t+1}}{A_t}B(x_{t+1})^{\varphi}\right\} \tag{40}$$

Similar conditions hold for the foreign country (See Appendix).

To get a full characterization of the equilibrium, we now need to specify a mechanism of wage determination.

5.2 Equilibrium with Nash Bargained Wages

In this model, the presence of hiring costs creates a positive rent for existing employment relationships. Following much of the literature, we assume wages are bargained to split this rent between the firm and the employee, according to their respective bargaining power.

Consider the generic firm j in the Home country.

The value of a job for firm j "is simply given by the hiring costs G_t, as a firm can always replace a worker at that cost"[32]. Notice however that hiring costs are expressed in terms of the domestic goods, while wages are set in terms of the consumption goods. The relevant firm's surplus - expressed in terms of consumption goods - is therefore

$$\frac{P_{Ht}G_t}{P_t} \tag{41}$$

Turning to the problem of the worker, let W_t^E and W_t^U denote the value of being employed or unemployed, expressed in consumption units.

[30] Krause and Lubik (2005), p. 11.

[31] Note that real wages for Home (and the same is true for Foreign) are determined in terms of the corresponding consumption good - i.e. are the ratio of the nominal wage rate and the corresponding CPI index. Unit recruitment costs, instead, are defined in terms of the home good for Home (i.e. using as deflator P_{Ht}) and in terms of the foreign good for Foreign (i.e. divided for P_{Ft}).

[32] Blanchard-Galì (2006), p. 14. This section follows closely their paper.

The marginal value of an employment relationship is given by:

$$W_t^E = W_t^R - \chi_0 C_t (N_t)^{\phi} \\ +\beta E_t \left\{ \frac{C_t}{C_{t+1}} \left[(1-\delta_H(1-x_{t+1})) W_{t+1}^E + \delta_H (1-x_{t+1}) W_{t+1}^U \right] \right\} \tag{42}$$

The first term represents the worker's wage income; the second the disutility of work and the last the discounted expected continuation value. $\delta_H(1-x_{t+1})$ is the probability of being unemployed at time t conditional on being employed at time t.

The corresponding value for a member who remains unemployed after hiring take place is:

$$W_t^U = \beta E_t \left\{ \frac{C_t}{C_{t+1}} \left[x_{t+1} W_{t+1}^E + (1-x_{t+1}) W_{t+1}^U \right] \right\} \tag{43}$$

Combining both conditions we obtain the household's surplus from an established relationship:

$$W_t^E - W_t^U = W_t^R - \chi_0 C_t (N_t)^{\phi} + \beta(1-\delta_H) E_t \left\{ \frac{C_t}{C_{t+1}} \left[(1-x_{t+1})(W_{t+1}^E - W_{t+1}^U) \right] \right\} \tag{44}$$

Let ζ denote the share of the surplus going to the worker. The bargaining solution is given by:

$$W_t^N - W_t^U = \frac{\zeta}{1-\zeta} \frac{P_{Ht} G_t}{P_t} = \eta G_t (S_t)^{-\alpha} \tag{45}$$

where we make use of the fact that $\frac{P_{Ht}}{P_t} = (S_t)^{-\alpha}$ and we define $\eta = \frac{\zeta}{1-\zeta}$ as the relative weight of workers in the Nash bargaining, which reflects workers' bargaining power.

Imposing this condition to (44) and rearranging, we get the *Nash wage schedule*:

$$\frac{W_t^R}{A_t} (S_t^H)^{\alpha} = \frac{\chi_0 C_{Ht}^W (N_t)^{\phi}}{A_t} + \eta B (x_t)^{\varphi} \\ -\beta(1-\delta_H) E_t \left\{ \frac{C_{Ht}^W}{C_{Ht+1}^W} \frac{A_{t+1}}{A_t} \left[(1-x_{t+1})(\eta B (x_{t+1})^{\varphi}) \right] \right\} \tag{46}$$

where we use the fact that $C_t (S_t)^{\alpha} = C_{Ht} + C_{Ht}^* = C_{Ht}^W$.

Finally, substituting this wage rule in the (40), we obtain the *equilibrium under Nash bargaining*:

$$\frac{\chi_0 C_{Ht}^W (N_t)^{\phi}}{A_t} = \frac{1}{\mu} - (1+\eta) B (x_t)^{\varphi} \\ +\beta(1-\delta_H) E_t \left\{ \frac{C_{Ht}^W}{C_{Ht+1}^W} \frac{A_{t+1}}{A_t} \left[(1+\eta(1-x_{t+1})) B (x_{t+1})^{\varphi} \right] \right\} \tag{47}$$

This condition determines the evolution of (un)employment under Nash bargaining.

It is easy to verify that the decentralised equilibrium with Nash bargained wages involves a constant job-finding rate and, hence, a constant level of unemployment[33].

Again, this crucial result derives from two assumptions: a utility function that is log in consumption and recruitment costs that vary one for one with producivity shocks.

Combining the equilibrium under Nash bargaining and the Nash wage rule, we can determine the actual behaviour of real wages:

$$\frac{W_t^R}{A_t}(S_t)^\alpha = \frac{1}{\mu} - [1-\beta(1-\delta)]\, B\,(x_M)^\varphi \tag{48}$$

where x_M is the (constant) equilibrium job-finding rate, which is solution of (47).

The equilibrium wage moves one for one with $A_t(S_t)^{-\alpha}$. Notice that, since the employment level is constant at Home and abroad, the terms of trade S_t varies proportionally to $\frac{A_t}{A_t^*}$. Changes in productivity at Home or at Foreign are neutralized by changes in the wage rate and thus do not affect firms' incentives to hire people; as a result, unemployment is unchanged. In other words, movements in the term of trade, reflected in movements in the real wage, imply that employment does not vary with productivity shocks.

Compare the equilibrium under the efficient allocation (30) and under the decentralised equilibrium (47). While the (un)employment level is constant in both cases, these levels generally differs. Mainly due to the monopolistic distortions, the unemployment level under the efficient allocation is higher than the one prevailing in the decentralised solution. It is easy to verify that the conditions under which the two equilibria correspond, are the following[34]:

1. Perfect competition in the goods market, i.e. $\mu = 1$ (or "alternatively a production subsidy which exactly offset the market power distortions"[35]).

2. $\varphi = \eta$, i.e. the share of the surplus that goes to workers has to coincide with the elasticity of hiring costs with respect to the job-finding rate.

Similar conditions and exactly the same conclusions hold for the Foreign country (See Appendix).

We restrict ourselves to equilibria in which, for all t:

$$\frac{W_t^R}{A_t} > \chi_0(1-\delta_H B) \tag{49}$$

$$\frac{W_t^{R*}}{A_t^*} > \chi_1(1-\delta_F B^*) \tag{50}$$

These conditions require that the real wage in each country remains above the corresponding marginal rate of substitution, when the latter is evaluated at full employment. They guarantee "full participation (as all the employed would rather work than not) and that those without a job in any given period are involuntary unemployed"[36].

[33] To see this, notice that $C_{Ht}^W = C_{Ht} + C_{Ht}^* = A_t(N_t - Bx_t^\varphi H_t)$. It follows that $\frac{C_{Ht}^W}{A_t}$ does not depend on A_t.

[34] Blanchard-Gali (2006) obtain the same conditions in the context of a one country model.

[35] Blanchard-Gali (2006), p. 16.

[36] See Blanchard-Gali' (2006), p. 17. These assumptions are needed in order to define the (before hiring)

5.3 Introducing Real Wage Rigidities

As Christoffel and Linzert note, especially in Europe, "sudden and significant shifts in the aggregate wage level are not observed. Due to collective wage bargaining agreements, wage changes only take place on a quite infrequent basis. Therefore, a wage that can be freely adjusted each period assumes a degree of wage flexibility that is hardly consistent with actual practises".[37]

Hall (2005) shows that the introduction of sticky wages improves the behaviour of labor market models, as it increases the sensitivity of labor-market conditions - and hence unemployment - to productivity shocks. Blanchard and Gali (2005) demonstrate that the introduction of real wage rigidities is a natural way to overcome one of the shortcomings of the standard New Keynesian model, namely the lack of a meaningful trade-off between output stabilization and inflation stabilization.

Accordingly, and following much of the recent literature, we introduce real wage rigidity by employing a version of Hall's (2005) notion of wage norm. A wage norm may arise as a result of social conventions that constrain wage adjustment for existing and newly hired workers. One way to model this is to assume that the real wage is a weighted average of the Nash bargained wage W_t^n and a wage norm $\bar{W}$, which it is simply assumed to be the wage prevailing in steady state.

Specifically, we assume the real wage is determined as follows:

$$W_t^R = (W_t^n)^{1-\gamma} \left(\bar{W}\right)^{\gamma} e^{\varepsilon_t^w} \tag{52}$$

$$W_t^{R*} = (W_t^{*n})^{1-\gamma^*} \left(\bar{W}^*\right)^{\gamma^*} e^{\varepsilon_t^{*w}} \tag{53}$$

For Home and Foreign respectively. The wage norm $\bar{W}$ ($\bar{W}^*$ for Foreign) is simply the wage prevailing in steady state while γ (γ^*) is an index of the real wage rigidities present in the economy, with $0 \leq \gamma^i \leq 1$. Notice that we introduce an exogenous shock to the wage determination process, $e^{\varepsilon_t^w}$ ($e^{\varepsilon_t^{*w}}$). This "wage shock", which has the nature of a standard cost-push shock, can be interpreted as an unexpected deviation from the wage rule determined by some - not specified - political reason.

The introduction of such a wage rule modifies the decentralised equilibrium solution. Consider for instance the Home country. In equilibrium:

$$\frac{(W_t^n)^{1-\gamma} \left(\bar{W}\right)^{\gamma} e^{\varepsilon_t^w}}{A_t} (S_t)^{\alpha} = \frac{1}{\mu} - B\,(x_t)^{\varphi} + \beta(1-\delta_H) E_t \left\{ \frac{C_{Ht}^W}{C_{Ht+1}^W} \frac{A_{t+1}}{A_t} B\,(x_{t+1})^{\varphi} \right\} \tag{54}$$

As shown before, the Nash bargained wage varies proportionally to $A_t(S_t)^{-\alpha}$ and thus neutralizes the effect of productivity changes on employment. When real wage rigidities are present, instead, the wages do not move enough to absorb the impact of technology shocks. As a result, in a decentralised equilibrium with sticky wages, employment will *not*

unemployment rate as

$$U_t^i = 1 - (1-\delta_i) N_{t-1}^i, \;\; for\ i = H, F(^*) \tag{51}$$

[37] Christoffel and Linzert (2005), p. 17-18.

be constant. As in Blanchard-Galì (2005), the presence of real wage rigidities introduces a substantial difference between the efficient solution (where employment is constant) and the decentralised solution (where employment varies with productivity shocks). For this reason, to the extent that γ or γ^* are different from zero, it is not possible for the monetary authority to stabilize simultaneously inflation and unemployment. There is no "Divine Coincidence".

6 Introducing Sticky Prices

We introduce nominal price rigidity using a model à la Calvo (1983). Each period, a firm faces a fixed probability $(1-\theta)$ of adjusting its price, irrespective of the time elapsed since it last reset its price. The firm resets the price in order to maximize its present discounted value, while taking into consideration that the price it chooses will remain effective for a (random) number of periods. It can be shown that the optimal price setting rule for a Home firm resetting prices in period t is given by:[38]

$$E_t \left\{ \sum_{s=0}^{\infty} \theta^s Q_{t,t+s} Y_{t+s/t} \left(\tilde{P}_{Ht} - \frac{\epsilon}{\epsilon - 1} P_{Ht+s} MC_{t+s} \right) \right\} = 0 \tag{55}$$

where $\tilde{P}_{Ht}$ denotes the price newly set at time t, $Y_{t+s/t}$ is the level of output in period t+s for a firm resetting its price in period t and $\mu = \frac{\epsilon}{\epsilon-1}$ is the gross desired markup. The real marginal cost takes the usual form:

$$MC_t = \frac{W_t^R}{A_t} (S_t)^{\alpha} + B (x_t)^{\varphi} - \beta(1-\delta_H) E_t \left\{ \frac{C_{Ht}^W}{C_{Ht+1}^W} \frac{A_{t+1}}{A_t} B (x_{t+1})^{\varphi} \right\} \tag{56}$$

As Blanchard and Galì (2006) note, the two previous equations embody the essence of the integration of hiring costs in a standard NK model. In fact:

1. Taking as given the path of marginal costs, the optimal price setting rule takes the same form as in the standard Calvo model.

2. The dynamics of the marginal costs are however deeply influenced by the introduction of hiring costs and real wage rigidities (which enters through $W_t^R = (W_t^n)^{1-\gamma} \left(\bar{W}\right)^{\gamma} e^{\varepsilon_t^w}$).

Log-linearizing around a zero inflation steady state the optimal price setting rule and the price index equation $P_{Ht} = \left[(1-\theta)(\tilde{P}_{Ht})^{1-\epsilon} + \theta (P_{Ht-1})^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}$, we get the New Keynesian Phillips curve:

$$\hat{\pi}_t^H = \beta E_t \left\{ \hat{\pi}_{t+1}^H \right\} + \lambda \widehat{mc}_t \tag{57}$$

where $\hat{\pi}_t^H$ is domestic (i.e. producer prices') inflation, $\widehat{mc}_t$ represent the log deviation of real marginal cost from its steady state value and $\lambda = (1-\beta\theta)(1-\theta)/\theta$. Note that, while (57) looks like a standard New Keynesian Phillips curve, the dynamics of the real marginal costs are now substantially different from the ones of a standard NK model. We defer a full discussion of this important point to later.

[38] See Blanchard-Galì (2006) for a detailed description of the derivation of the New Keynesian Phillips Curve with Hiring Costs.

Similarly, it can be shown that domestic inflation in the Foreign country evolves according to

$$\hat{\pi}_t^F = \beta E_t \left\{ \hat{\pi}_{t+1}^F \right\} + \lambda \widehat{mc}_t^* \tag{58}$$

where the real marginal costs are given by:

$$MC_t^* = \frac{W_t^{R*}}{A_t^*} \left(S_t^H\right)^{-\alpha} + B^* \left(x_t^*\right)^{\varphi} - \beta(1-\delta_F) E_t \left\{ \frac{C_{Ft}^W}{C_{Ft+1}^W} \frac{A_{t+1}}{A_t} B^* \left(x_{t+1}^*\right)^{\varphi} \right\} \tag{59}$$

6.1 Monetary Policy

In order to close the model, a characterization of monetary policy is needed.

We assume the Central Bank sets the short term nominal interest rate by reacting to the average inflation and output gap levels in the currency area. Consistently with empirical evidence, we also assume that monetary policy displays a certain degree ρ_m of interest rate smoothing[39]. Specifically, the monetary authority follows the Taylor-type rule:

$$(1+i_t) = \beta^{1-\rho_m} (1+i_{t-1})^{\rho_m} E_t (\pi_{t+1}^U)^{\phi_\pi (1-\rho_m)} (n_t^U)^{\phi_x (1-\rho_m)} e^{\varepsilon_t^m} \tag{60}$$

Where π_{t+1}^u and n_t^U are respectively the union-wide inflation and output gap, defined as:

$$\pi_{t+1}^U = \frac{\pi_{t+1}^F + \pi_{t+1}^H}{2}, \quad n_t^U = \frac{n_t^F + n_t^H}{2} \tag{61}$$

Log-linearizing (60) around the steady state, we get:

$$\hat{\imath}_t = \rho_m \hat{\imath}_{t-1} + \phi_\pi (1-\rho_m) E_t \hat{\pi}_{t+1}^U + \phi_x (1-\rho_m) \tilde{n}_t^U + \varepsilon_t^m \tag{62}$$

where ε_t^m is an i.i.d monetary policy shock.

6.2 Log-linearized Marginal Costs

The model is not solvable in a closed form solution. To proceed, we need an approximation around the steady state. In this section we show how to approximate marginal cost dynamics. The log-linearization of the other equations characterizing the economies does not pose particular problems.

Consider the Home country. Log-linearization of the marginal cost around the steady state gives[40]:

$$\begin{aligned} \widehat{mc}_t = & \mu W^R (S)^\alpha \left\{ \hat{w}_t - \hat{a}_t + \alpha \hat{s}_t^H \right\} + \varphi g \mu \hat{x}_t \\ & - \beta(1-\delta_H) g \mu E_t \left\{ (\hat{c}_{Ht}^w - \hat{a}_t) - \left(\hat{c}_{Ht+1}^w - \hat{a}_{t+1}\right) + \varphi \hat{x}_{t+1} \right\} \end{aligned} \tag{63}$$

Variables with "hat" denote log-deviations from steady state; W^R and $g = B(x)^\varphi$ are the steady state values for real wages and unit recruitment costs, and μ is the markup. Note that we have normalized the steady state value of productivity to unity ($A = 1$).

[39] See, e.g, Clarida, Gali and Gertler (1999).

[40] Notice that in steady state: $MC = \frac{W^R}{A} \left(S^H\right)^\alpha + g(1-\beta(1-\delta_H)) = \frac{1}{\mu}$

The loglinear approximations for the labor market tightness $x_t = \frac{H_t}{U_t}$ and for the world consumption of the home good $C_{Ht}^W = A_t(N_t - Bx_t^{\varphi} H_t)$ are given by:

$$\delta_H \hat{x}_t = \hat{n}_t - (1-\delta_H)(1-x)\hat{n}_{t-1} \tag{64}$$

$$\hat{c}_{Ht}^w = \hat{a}_t + \frac{1-g}{1-\delta_H g}\hat{n}_t + \frac{g(1-\delta_H)}{1-\delta_H g}\hat{n}_{t-1} - \frac{\varphi g}{1-\delta_H g}\delta_H \hat{x}_t \tag{65}$$

Following Blanchard-Galì (2006), we introduce two approximations that considerably simplify the characterization of the equilibrium:

1. Hiring costs are small relative to output, so that we can approximate $\hat{c}_{Ht}^w$ with $\hat{c}_{Ht}^w = \hat{a}_t + \hat{n}_t$. More precisely, we assume that δ and g are of the same order of magnitude as $\hat{n}_t$, implying that terms involving $g\hat{n}_t$ or $\delta\hat{n}_t$ are of second order.

2. Fluctuations in $\hat{x}_t$ are large relative to those in $\hat{n}_t$, an approximation that follows from the log-linearization of the labor tightness index (64) and the assumption of a low separation rate. This implies that terms involving $g\hat{x}_t$ or $\delta\hat{x}_t$ cannot be ignored.

We can therefore rewrite the expression for marginal cost as:

$$\widehat{mc}_t = \mu W^R (S)^{\alpha} \left\{\hat{w}_t - \hat{a}_t + \alpha \hat{s}_t^H\right\} + \varphi g \mu \left\{\hat{x}_t - \beta E_t \hat{x}_{t+1}\right\} \tag{66}$$

To fully determine the marginal costs, we need a characterisation of the processes for the real wages. Log-linearization of the Home wage rule, $W_t^R = (W_t^n)^{1-\gamma} \left(\bar{W}\right)^{\gamma} e^{\varepsilon_t^w}$, gives

$$\hat{w}_t = (1-\gamma)\hat{w}_t^{Nash} + \varepsilon_t^w \tag{67}$$

where $\hat{w}_t^{Nash}$ comes from the log-linearization of the Nash wage schedule and can be shown to be equal to

$$\hat{w}_t^{Nash} = \frac{1}{W^R (S)^{\alpha}} \left\{ \begin{array}{c} \chi_0 (N)^{1+\phi} (1+\phi)\hat{n}_t \\ +\eta g \varphi \hat{x}_t - \beta(1-x)\eta g \left(\varphi - \frac{x}{1-x}\right) E_t \hat{x}_{t+1} \end{array} \right\} \tag{68}$$
$$-\hat{a}_t + \alpha \hat{s}_t^H$$

Using these results, we finally obtain the dynamics of the real marginal costs for the Home country:

$$\begin{aligned} \widehat{mc}_t = {} & \Psi_2 \mu \hat{n}_t + g\mu\Psi_0 \hat{x}_t - g\mu\Psi_1 E_t\left\{\hat{x}_{t+1}\right\} \\ & -\gamma\mu W^R (S)^{\alpha} (\hat{a}_t - \alpha\hat{s}_t) + \mu W^R \left(S^H\right)^{\alpha} \varepsilon_t^w \end{aligned} \tag{69}$$

Where the structural parameters Ψ_0, Ψ_1, and Ψ_2 depend on the bargaining power of workers, on labor market conditions and on the degree of real wage stickyness[41]. The steady state level of the terms of trade is $S = \frac{P_{Ft}}{P_{Ht}} = \frac{AN(1-g\delta_H)}{A^* N^* (1-g^* \delta_F)}$.

Equation (69) highlights the determinants of marginal costs. Marginal costs increase with employment ($\hat{n}_t$) as the firm has to pay higher wages to persuade households to

[41] It can be shown that $\Psi_0 = \varphi(1+\eta(1-\gamma))$, $\Psi_1 = \beta\left[\varphi + \eta(1-x)(1-\gamma)\left(\varphi - \frac{x}{1-x}\right)\right]$, and $\Psi_2 = \chi_0 (N)^{1+\phi} (1+\phi)(1-\gamma)$.

provide more labor. This is the only channel at work in the standard NK model. Marginal costs increases with the worsening of labor market conditions at time t (i.e. with an increase of $\hat{x}_t$); an expected increase of $E_t\hat{x}_{t+1}$, instead, has the opposite effect, as it becomes convenient for the firm to hire at time t in order to be ready for a more difficult labor market in time $t+1$. Finally, marginal costs depend negatively on productivity shocks ($\hat{a}_t$) and positively on wage shocks (ε_t^w) and on the terms of trade ($\hat{s}_t$).

In order to express marginal costs in terms of unemployment, let $\hat{u}_t = u_t - u$ denote the deviations of (after-hiring) unemploment from its steady state value u. Taking a first order Taylor expansion of $u_t^i = 1 - N_t^i$, it can be shown that

$$\hat{u}_t^i = -\left(1 - u^i\right)\hat{n}_t^i$$

Using this approximation and (64), we can rewrite the evolution of marginal costs as a function of unemployment and shocks:

$$\begin{aligned} \widehat{mc}_t = & -\kappa_0\hat{u}_t + \kappa_1\hat{u}_{t-1} + \kappa_2 E_t\hat{u}_{t+1} \\ & -\gamma\kappa_3\hat{a}_t + \alpha\gamma\kappa_3\hat{s}_t + \kappa_3\varepsilon_t^w \end{aligned} \tag{70}$$

where the coefficients κ_i depends on the structural parameter of the model. Similarly for the foreign country[42]:

$$\widehat{mc}_t^* = -\kappa_0^* \hat{u}_t^* + \kappa_1^* \hat{u}_{t-1}^* + \kappa_2^* E_t \hat{u}_{t+1}^* \\ -\gamma^* \kappa_3^* \hat{a}_t^* - \alpha \gamma^* \kappa_3^* \hat{s}_t + \kappa_3^* \varepsilon_t^w \tag{71}$$

We are now ready to characterize - in terms of loglinear approximations - the constrained efficient, the natural and the sticky prices outcomes.

7 Equilibrium Fluctuations

7.1 Constrained Efficient Allocation

The efficient constrained allocation is characterized by the following relationships (in loglinear form):

$$\begin{aligned} \bar{u}_t &= \bar{u}_t^* = 0 \\ \bar{y}_t^U &= \bar{c}_t^U = \frac{1}{2}\left(\hat{a}_t + \hat{a}_t^*\right) = \hat{a}_t^U \\ \bar{y}_t - \bar{y}_t^* &= \hat{a}_t - \hat{a}_t^* \\ \bar{c}_t - \bar{c}_t^* &= (1-2\alpha)\left(\hat{a}_t - \hat{a}_t^*\right) \end{aligned}$$

[42] It can be shown that:

$$\begin{aligned} \kappa_0 &= \frac{1}{1-u}\left(g\Psi_0 b_0 + g\Psi_1 b_1 + \Psi_2\right)\mu \\ \kappa_1 &= \frac{1}{1-u}\mu g \Psi_0 b_1 \\ \kappa_2 &= \frac{1}{1-u}\mu g \Psi_0 b_0 \\ \kappa_3 &= \mu W^R (S)^\alpha \\ b_0 &= \frac{1}{\delta_H} \\ b_1 &= \frac{1}{\delta_H}(1-\delta_H)(1-x) \end{aligned}$$

where Ψ_0, Ψ_1 and Ψ_2 have already been defined. Similarly for the foreign country:

$$\begin{aligned} \kappa_0^* &= \frac{1}{1-u^*}\left(g^*\Psi_0^* b_0^* + g^*\Psi_1^* b_1^* + \Psi_2^*\right)\mu^* \\ \kappa_1^* &= \frac{1}{1-u^*}\mu^* g^* \Psi_0^* b_1^* \\ \kappa_2^* &= \frac{1}{1-u^*}\mu^* g^* \Psi_0^* b_0^* \\ \kappa_3^* &= \mu^* W^{*R} (S)^\alpha \\ b_0^* &= \frac{1}{\delta_F} \\ b_1^* &= \frac{1}{\delta_F}(1-\delta_F)(1-x^*) \end{aligned}$$

while $\Psi_0^* = \varphi\left(1+\eta^*(1-\gamma^*)\right)$, $\Psi_1^* = \beta\left[\varphi + \eta^*(1-\gamma^*)(1-x^*)\left(\varphi - \frac{x^*}{1-x^*}\right)\right]$ and $\Psi_2^* = (1-\gamma)\chi_1 \left(N^*\right)^{1+\phi}(1+\phi)$.

Where variables with bars denote the constrained efficient outcome and a union-wide variable is a simple average of the corresponding country-specific variables: $X_t^U = \frac{X_t^H + X_t^F}{2}$. As shown before, in the efficient equilibrium (un)employment is invariant to shocks and thus constant across time. Union-wide output and consumption depend only on the union's supply shocks. Asymmetric shocks influence the relative output of Home and Foreign. Finally notice that as long as there is home bias in consumption (i.e. $\alpha < \frac{1}{2}$) the PPP does not hold and consumption is not equated in equilibrium.

7.2 The Flexible Price Equilibrium

Given a variable X, we denote with $\tilde{X}$ the deviation of a variable from its constrained efficient level, i.e. $\tilde{X} = \hat{X} - \bar{X}$. With flexible prices, monetary policy is neutral and real variables are affected only by real disturbances. We can characterize the flexible price equilibrium *in terms of deviations from the efficient solutio*n as follows:

$$
\begin{aligned}
\tilde{u}_t &= \frac{1}{\kappa_0}\begin{pmatrix} \kappa_1 \tilde{u}_{t-1} + \kappa_2 E_t \tilde{u}_{t+1} \\ +\alpha\gamma\kappa_3 \tilde{s}_t - \gamma\kappa_3\left[(1-\alpha)\hat{a}_t + \alpha\hat{a}_t^*\right] + \kappa_3 \varepsilon_t^w \end{pmatrix} \\
\tilde{u}_t^* &= \frac{1}{\kappa_0^*}\begin{pmatrix} \kappa_1^* \tilde{u}_{t-1}^* + \kappa_2^* E_t \tilde{u}_{t+1}^* \\ -\alpha\gamma^*\kappa_3^* \tilde{s}_t - \gamma^*\kappa_3^*\left[(1-\alpha)\hat{a}_t^* + \alpha\hat{a}_t\right] + \kappa_3^* \varepsilon_t^{*w} \end{pmatrix} \\
\tilde{y}_t^U &= \tilde{c}_t^U = \tilde{n}_t^U = -\tilde{u}_t^A \\
\tilde{y}_t &= -\tilde{u}_t; \qquad \tilde{y}_t^* = -\tilde{u}_t^* \\
\tilde{s}_t &= \tilde{y}_t - \tilde{y}_t^* \\
\tilde{c}_t &= \tilde{c}_t^* + (1-2\alpha)\tilde{s}_t
\end{aligned}
$$

where, to simplify the notation, we define aggregate unemployment as a weighted average of the unemployment rates prevailing in each country: $\tilde{u}_t^A = \frac{1}{2}\left(\frac{\tilde{u}_t^F}{1-u} + \frac{\tilde{u}_t^H}{1-u^*}\right)$.

The crucial conditions are the first two, which describe the evolution of unemployment gap. Note that, since employment is constant under the efficient allocation, any (un)employment fluctuation is inefficient. It is useful to highlight some facts. First, unemployment displays a substantial degree of inertia. In fact, on the one side, the expectations about future labor market conditions affect today's firms' decisions; on the other side, today's hiring costs depend on the employment level at time $t-1$, which is inherited from the past. Second, if real wages are sticky (i.e. γ or γ^* are different from zero), productivity shocks at Home or Foreign, as well as deviations of the terms of trade from its efficient level, influence the *unemployment gap*, i.e. affect the wedge between the "efficient" and the "natural" unemployment level. Third, even when real wage rigidities are absent, shocks to the wage determination mechanism - which have the nature of "cost-push shocks" - create inefficient fluctuations of the unemployment gap.

Blanchard and Galì (2005) show that one of the shortcomings of the standard NK models, namely the lack of a meaningful policy trade off between output and inflation stabilization (which they call "Divine Coincidence"), comes from the fact that the gap between the efficient and the natural levels of output is constant and invariant to shocks. In this model, the introduction of real wage rigidities and of exogenous wage shocks decouples the evolution of unemployment under the two regimes. Unemployment would be characterized by inefficient fluctuations *even if prices were flexible*. In terms of monetary policy, and contrary to what happens in standard NK models, stabilizing inflation does not stabilize output. A pure inflation targeting strategy is not optimal anymore.

Under flexible prices, the terms of trade, which are affected only by relative disturbances, exert their crucial role of balancing the burden of production across regions. A larger supply shock in the Home region depreciates the terms of trade and shifts, optimally, a part of the production burden from country F to country H.

Finally notice that the implied path of the nominal interest rate, in an equilibrium in which union's inflation is zero, is only a function of union wide variables:

$$\begin{aligned}\tilde{r}_t^U &= \frac{1}{2}(E_t \Delta \hat{a}_{t+1} + E_t \Delta \tilde{n}_{t+1} + E_t \Delta \hat{a}_{t+1}^* + E_t \Delta \tilde{n}_{t+1}^*) \\ &= E_t \Delta \hat{a}_{t+1}^U + E_t \Delta \tilde{n}_{t+1}^U\end{aligned}$$

7.3 Sticky Prices

If prices are sticky, monetary policy matters. In a closed economy model, the presence of staggered price setting typically leads to an inefficient dispersion of inflation and output across resources produced using the same technology. In an open economy model, price stickyness creates an additional source of distortion: as prices are not free to adjust, the terms of trade typically follow an inefficient path in response to asymmetric disturbances[43]. In this model, two new elements deeply affect the economy: the presence of labor market imperfections and real wage rigidities. In the following we show that these elements matter substantially for the dynamic behaviour of the economy and, in particular, for the transmission mechanism of monetary policy.

It can be shown that the union-wide IS equation takes the form:

$$\tilde{u}_t^A = E_t \tilde{u}_{t+1}^A + (r_t - E_t \tilde{\pi}_{t+1}^U - E_t \Delta \hat{a}_{t+1}^U) \tag{72}$$

where $\tilde{\pi}_t^U$ and $\tilde{u}_t^A$ are the union-wide inflation and unemployment gap and $E_t \Delta \hat{a}_{t+1}^U = \frac{1}{2}(E_t \Delta a_{t+1} + E_t \Delta a_{t+1}^*)$. The IS equation - in terms of union-wide variables - takes the same form as in a standard closed economy model. Solving (72) forward, we get

$$\tilde{c}_t^U = -\tilde{u}_t^A = -E_t \sum_{j=0}^{\infty} \left[\left(r_t - E_t \tilde{\pi}_{t+1}^U \right) - E_t \Delta \hat{a}_{t+1}^U \right]$$

Hence, the unemployment and consumption gaps are driven by the expected future path of real interest rates and productivity differentials (the latter mimic the natural interest rate that would prevail under the constrained efficient allocation). While the real interest rate affects aggregate (union) unemployment, terms of trade movements distribute production among the two countries and explain unemployment and consumption

[43] In an open economy model, this problem has typically a (at least partial) solution: the exchange rate. Movements in the exchange rate in fact may provide some additional flexibility to the terms of trade. This instrument, however, is absent in a monetary union. See, for instance, Benigno (2004) and Pappa (2002) for a discussion of the welfare properties of monetary unions.

differentials:

$$\begin{aligned}\tilde{s}_t &= \frac{1}{1-u^*}\tilde{u}_t^* - \frac{1}{1-u}\tilde{u}_t \\ \tilde{c}_t &= \tilde{c}_t^* + (1-2\alpha)\tilde{s}_t\end{aligned} \tag{73}$$

The supply block of the model contains the aggregate supply equations for Home:

$$\begin{aligned}\tilde{\pi}_t^H &= \beta E_t\left\{\tilde{\pi}_{t+1}^H\right\} + \lambda\widehat{mc}_t \\ &= \beta E_t\left\{\tilde{\pi}_{t+1}^H\right\} - \lambda\kappa_0\tilde{u}_t + \lambda\kappa_1\tilde{u}_{t-1} + \lambda\kappa_2 E_t\tilde{u}_{t+1} \\ &\quad -\gamma\lambda\kappa_3\hat{a}_t + \alpha\gamma\lambda\kappa_3\tilde{s}_t + \lambda\kappa_3\varepsilon_t^w\end{aligned} \tag{74}$$

and Foreign:

$$\begin{aligned}\tilde{\pi}_t^F &= \beta E_t\left\{\tilde{\pi}_{t+1}^F\right\} + \lambda^*\widehat{mc}_t^* \\ &= \beta E_t\left\{\tilde{\pi}_{t+1}^F\right\} - \lambda^*\kappa_0^*\tilde{u}_t^* + \lambda^*\kappa_1^*\tilde{u}_{t-1}^* + \lambda^*\kappa_2^* E_t\tilde{u}_{t+1}^* \\ &\quad -\gamma^*\lambda^*\kappa_3^*\hat{a}_t^* - \alpha\lambda^*\gamma^*\kappa_3^*\tilde{s}_t + \lambda^*\kappa_3^*\varepsilon_t^{*w}\end{aligned} \tag{75}$$

Note that the coefficients are functions of the structural parameters characterizing the two economies: workers' bargaining power, hiring costs, separation rates, markups, degree of nominal stickiness or of real wage rigidity, and so on. The introduction of hiring costs and real wage rigidities substantially change the dynamics of the marginal costs, which in turn influence the firms' optimal price setting and the inflation dynamics.

In open economy models, an implicit inertia in the inflation rate is inherent. In fact, from the definition of the terms of trade $S_t = \frac{P_{Ft}}{P_{Ht}}$ we get the following relationship between the terms of trade and the domestic inflation rates:

$$\tilde{s}_t - \tilde{s}_{t-1} = \tilde{\pi}_t^F - \tilde{\pi}_t^H \tag{76}$$

As Benigno (2004) notes, "If monetary policy is not able to eliminate the link between the inflation rate and the terms of trade, inflation itself will be a function of its past values".[44]

Equations (72), (73), (74), (75), (76), together with the interest rate rule

$$\hat{\imath}_t = \rho_m\hat{\imath}_{t-1} + \phi_\pi(1-\rho_m)E_t\tilde{\pi}_{t+1}^U - \phi_x(1-\rho_m)\tilde{u}_t^A + \varepsilon_t^m$$

completely characterize our equilibrium dynamics.

7.4 A Special Case: Complete Symmetry

In this section, we characterize the dynamics of a currency union composed by two countries that are perfectly symmetric. This is not a realistic scenario, but it allows us to highlight some interesting facts of the model and it can constitute a good benchmark for comparing more general frameworks.

[44] See Benigno (2004), p. 11.

The "complete symmetry" assumption implies that the two regions have the same parameter values:

$$\kappa_i^* = \kappa_i,\ \lambda^* = \lambda \text{ and } \gamma = \gamma^*$$

In this case, we can find a simple expression for the union-wide NK Phillips Curve:

$$\begin{aligned}\tilde{\pi}_t^U = &\beta E_t \left\{ \tilde{\pi}_{t+1}^U \right\} - \lambda\kappa_0 \tilde{u}_t^U + \lambda\kappa_1 \tilde{u}_{t-1}^U + \lambda\kappa_2 E_t \tilde{u}_{t+1}^U \\ &-\gamma\lambda\kappa_3 \hat{a}_t^U + \lambda\kappa_3 \varepsilon_t^{wU}\end{aligned} \tag{77}$$

where union wide variables are defined as before and we use the fact that, under complete symmetry, $\tilde{u}_t^U = \tilde{u}_t^A$. Equation (77) has the same interpretation as a closed-economy AS equation, in which all variables are substituted with their union correspondents. This is the equation the common central bank takes into consideration when it implements monetary policy. What it is important to note is that in our model - even in the case of complete symmetry - stabilizing inflation *does not* stabilize the output gap. To see this, consider first a "Pure Inflation Targeting" strategy, i.e. a strategy aimed at stabilizing inflation at all horizons ($\hat{\pi}_t^U = 0$ at all t). It is well known that, under this strategy, firms have no incentive to change their prices[45]. Accordingly, the dynamics of the unemployment replicate exactly the dynamics under flexible prices. From (77) it follows that the unemployment gap evolves according to:

$$\kappa_0 \tilde{u}_t^U = \kappa_1 \tilde{u}_{t-1}^U + \kappa_2 E_t \tilde{u}_{t+1}^U - \gamma\kappa_3 \hat{a}_t^U + \kappa_3 \varepsilon_t^{wU} \tag{78}$$

Thus, we see that a pure inflation targeting strategy is unable to stabilize the unemployment gap in the presence of wage and of productivity shocks (to the extent that real wage rigidities are present). Moreover, the unemployment deviations from the efficient level are large and display a high degree of inertia. As in Blanchard-Galì (2006), the extent of real wage rigidities γ tends to increase the size of unemployment fluctuations, while labor market imperfections increase the persistency of these fluctuations.

Secondly, consider a "Pure Unemployment Targeting" policy, a strategy aimed at stabilizing the unemployment gap in each period, i.e. $\tilde{u}_t^U = 0$ at all t.

Iterating forward (77), one gets:

$$\begin{aligned}\tilde{\pi}_t^U &= \lambda \sum_{s=0}^{\infty} \beta^s E_t \left\{ -\kappa_0 \tilde{u}_{t+s}^U + \kappa_1 \tilde{u}_{t+s-1}^U + \kappa_2 E_t \tilde{u}_{t+s+1}^U - \gamma\kappa_3 \hat{a}_{t+s}^U + \kappa_3 \varepsilon_{t+s}^{wU} \right\} \\ &= \lambda\mu \sum_{s=0}^{\infty} \beta^s \left\{ \kappa_3 E_t \varepsilon_{t+s}^{wU} - \gamma\kappa_3 E_t \hat{a}_{t+s}^U \right\}\end{aligned}$$

A "Pure Unemployment Targeting" strategy is thus unable to stabilize inflation in face of productivity or wage shocks.

Therefore, adverse realizations of wage or productivity shocks necessarily lead to a rise in inflation and/or a negative unemployment gap. The presence of real wage rigidities and of wage shocks, by affecting the distance between the first best and the natural level

[45] See Galì (2002) for a discussion of this point.

of output, creates a non trivial trade-off between output and inflation stabilization: the "divine coincidence" does not hold. A strategy that is exclusively focused on inflation stabilization it is not optimal anymore, as it leads to large and persistent unemployment fluctuations.

In the "complete symmetry" special case it is also possible to find a simple expression for inflation differentials in terms of unemployment differentials and shocks:

$$\begin{aligned}\tilde{\pi}_t^R = & \beta E_t\left\{\tilde{\pi}_t^R\right\} - \lambda\kappa_0 \tilde{u}_t^R + \lambda\kappa_1 \tilde{u}_{t-1}^R + \lambda\kappa_2 E_t\left\{\tilde{u}_{t+1}^R\right\} \\ & -\gamma\lambda\kappa_3\left(\hat{a}_t - \hat{a}_t^*\right) + \lambda\kappa_3\left(\varepsilon_t^w - \varepsilon_t^{*w}\right) + 2\alpha\gamma\lambda\kappa_3\tilde{s}_t\end{aligned} \tag{79}$$

where $\tilde{\pi}_t^R = \tilde{\pi}_t^H - \tilde{\pi}_t^F$ and $\tilde{u}_t^R = (\tilde{u}_t - \tilde{u}_t^*)$ denote respectively inflation and unemployment differentials.

Intuitively, the presence of labor market imperfections amplify the persistence of inflation differentials following asymmetric shocks, while the degree of real wage rigidity γ mainly influences the size of these differentials. In particular, as long as $\gamma > 0$, a positive productivity shock in the Home country generates a competiveness gain for Home ($\tilde{\pi}_t^R$ decreases). An unexpected wage increase at Home (ε_t^w rises) translates instead into a positive inflation differential. Finally, notice that shocks also have an indirect impact through the terms of trade $\tilde{s}_t$. Thus, a productivity shock $\hat{a}_t$ not only influences (79) directly, it also generates a terms of trade depreciation, partially offsetting the direct impact of the shock. The strength of the impact of terms of trade shocks depends on the degree of trade between the two countries, as represented by the home bias parameter α.

8 Baseline Calibration

In our baseline calibration, we assume that Home and Foreign are perfectly symmetric. In this highly special case, Home and Foreign share the same structural parameters and the results of the previous section apply. In the following, we will then relax the "perfect symmetry" assumption and analyse what happens when the two countries have different productive structures.

The parameters of the baseline calibration are chosen to be largely consistent with those standard in the New Keynesian literature. The following table summarises the values for the key parameters of our model (for $i = H$ or F):

Preferences	β	ϕ^i	ϵ^i	μ^i	α	
	0.99	0.2	6	1.2	0.2	
Technology	A^i	φ^i				
	1	1				
Labour market	u^i	x^i	δ_i	η^i	B^i	
	0.08	0.45	0.07	1	0.47	
Price and Real Wage rigidities	θ^i	γ^i				
	0.75	0.5				
Interest Rate rule	ρ_m	ϕ_π	ϕ_x			
	0.9	1.5	0			
Shocks' Persistence and Volatility	ρ_w^i	ρ_a^i	ρ_ε	σ_w^i	σ_a^i	σ_ε
	0.9	0.9	0	0.01	0.01	0.002

Preferences: Time is taken as quarters. The discount factor β is set equal to 0.99, which implies a riskless annual return of about 4 percent. We assume the labor supply elasticity to be $\phi^i = 0.2$, which implies that disutilty of labor is almost linear. We see this as a reasonable assumption, given that in our model labor changes on the extensive margin and not on the intensive one. The elasticity of substitution between differentiated goods ϵ^i is set equal to 6, corresponding to a markup $\mu^i = 1.2$. Finally, the home bias parameter α, representing the share of imported goods on total consumption, is set to 0.2.

Technology: Following Blanchard and Galì (2006) we set the parameter φ^i in the hiring cost function, representing the sensitivity of hiring costs to labor market conditions, to be $\varphi^i = 1$[46]. The steady state level of productivity A^i is also set to 1.

The labor market: In the baseline calibration, we set unemployment in country i to be $u^i = 0.08$, which is roughly consistent with the average unemployment in Europe. The job-finding rate x^i is set to 0.45, which corresponds approximately to a monthly rate of 0.15. Given u^i and x^i, it is possible to determine the separation rate using the relation $\delta_i = u^i x^i / \left(\left(1 - u^i\right)\left(1 - x^i\right)\right)$. We obtain a value $\delta^i = 0.07$. The relative bargaining power η^i is set to 1, which implies that firms and workers have the same bargaining power. The scaling parameter B^i is chosen such that hiring costs represent a 1.5 percent fraction of steady state output[47]. The parameters χ_i can then be determined using steady state identities.

The degree of Price rigidity θ^i is set equal to 0.75, as in Galì (2002), implying an average duration of price contracts of one year. In the baseline calibration, following Campolmi and Faia (2006) and Blanchard and Galì (2006), we set the degree of real wage rigidity γ^i equal to 0.5.

Following Campolmi and Faia (2006) and Walsh (2005), we adopt an *interest rate rule* for monetary policy where the central bank responds to inflation but not to the output gap. Furthermore, we assume that the degree of inertia in the policy rule ρ_m equals 0.9, a value consistent with the empirical evidence on policy rules[48].

Shocks: we assume that wage shocks are as persistent as productivity shocks ($\rho_w^i = \rho_a^i = 0.9$). Following Walsh (2005), we set the standard deviation of the policy shock $\sigma_\varepsilon = 0.002$ and the standard deviation of productivity shocks to $\sigma_a^i = 0.01$. We have no reference value for the standard deviation of wage shocks; to facilitate comparisons, we assume the standard deviation of wage shocks is equal to the one of productivity shocks, i.e. $\sigma_w^i = 0.01$.

9 The Dynamics of the Currency Union

In this section we describe the dynamic behaviour of the model in response to three types of shocks: two asymmetric supply shocks (wage and productivity shocks) and one symmetric demand shock (monetary policy shock). We start explaining the behaviour of the economy under the baseline calibration, which represents the simple and highly special case of a

[46] In order to calibrate φ^i, Blanchard and Gali exploit a simple mapping between their model and the standard search and matching model. See Blanchard and Galì (2006), p. 28.

[47] To pin down B^i, we use the fact that in steady state hiring costs represent a fraction $\delta_i g^i = \delta_i B^i \left(x^i\right)^{\varphi^i}$ of GDP.

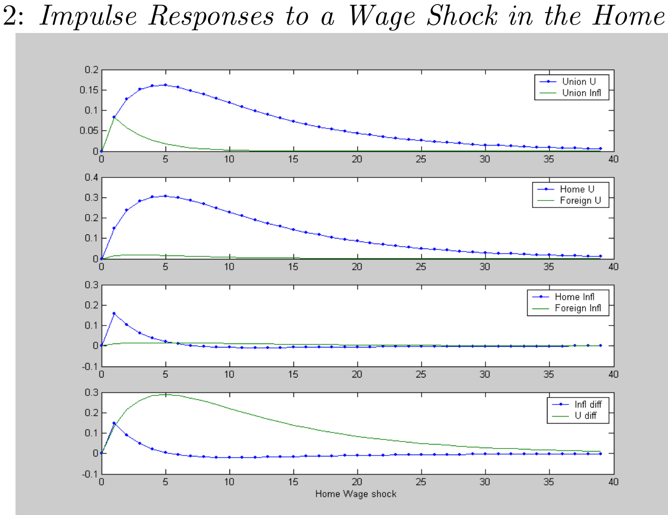
[48] See, e.g, Clarida et al. (2000).

currency union composed by *symmetric* countries. We then introduce some elements of asymmetry, looking first at the impact of different degrees of real wage rigidities and then analysing what happens when the two regions are characterized by different labor market institutions.

9.1 The "Perfect Symmetry" Case

Under the baseline calibration, the two regions are exactly symmetric and the transmission mechanism of shocks is identical. Fig. 2 shows the impact of an unexpected wage increase in the Home country. A wage increase forces the firms to rise prices and to reduce the quantities produced. On impact, union-wide inflation increases by 0.1 percent, and then decreases smoothly towards the equilibrium level. More interesting it is the behaviour of union-wide unemployment: the corresponding impulse response function displays a prominent hump-shaped dynamic and is highly persistent. The effect of the shock is not very strong, as unemployment reaches a maximum of 0.15 after 5 quarters, but it is very prolonged - after 40 quarters it has not disappeared. The persistency comes from the interaction of some realistic elements in our framework: labor market imperfections, nominal prices and real wages rigidities, the persistence in the monetary policy rule and an exclusive focus of the central bank on inflation. The interaction of these features amplify considerably the persistency of otherwise transitory shocks. As it can be observed in the second and the third quadrant, it is the Home country (i.e. the country hit by the shock) that absorb nearly all the effect of the shock. Unemployment and inflation movements in the Foreign country are very small. Accordingly, inflation and unemployment differentials tend to be quite large and (especially with respect to unemployment) very persistent[49].

Figure 2: *Impulse Responses to a Wage Shock in the Home Country*



[49] A different monetary policy rule, that puts a lower weight on inflation and a higher weight on unemployment, would partially modify these results, increasing the response of inflation while decreasing unemployment fluctuations.

Figure 3: *Impulse Responses to a Productivity Shock in the Home Country*

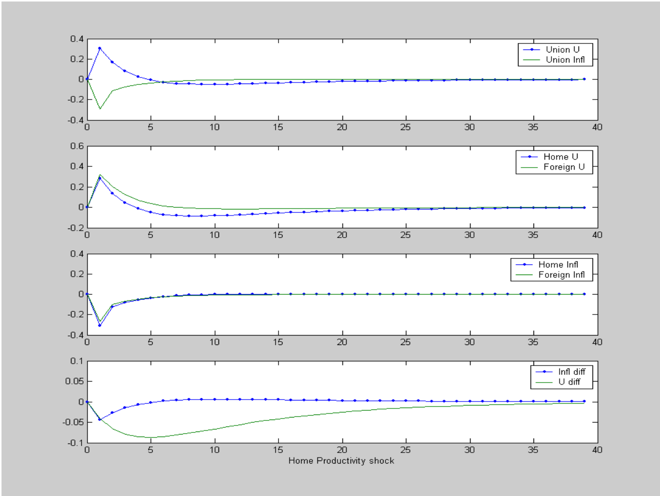


Figure 4: *Impulses Responses to a common Monetary Policy Shock*

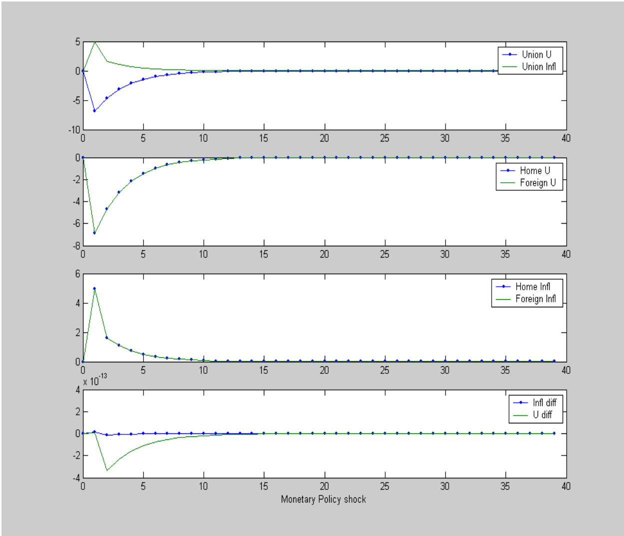


Fig. 3 displays the responses of unemployment and inflation to a positive technology shock in the Home country. On impact, union inflation decreases while unemployment increases. The latter is due to the presence of price rigidity. Because of the price stickyness assumption, not all firms are able to reduce prices as they would have done under flexible prices. The productivity increase allows firms that cannot reset prices to produce the same amount with less work; consequently, unemployment rises on impact. This unemployment increase is shortlived, as over time more firms can reset their prices and the effect of the productivity shock fades away.

Contrary to wage shocks, productivity changes in one country have a large spillover effect in the other country. As shown in the second and the third quadrant of Fig. 3, an asymmetric productivity shock in one country generates inflation and unemployment dynamics which are very similar among the two countries. The reason for these larger spillovers lies in the fact that productivity shocks influence the terms of trade much more than wage shocks[50]. Accordingly, inflation and unemployment differentials are now much smaller - even if equally persistent.

Fig. 4 shows the response of the economies following a one percent unexpected decrease of the nominal interest rate. On impact, union inflation increases while unemployment decreases. Since the two countries are symmetric, the transmission mechanism of monetary policy is identical and inflation and unemployment differentials do not arise.

9.2 Different Degrees of Real Wage Rigidities

Let now introduce a first element of asymmetry between the two countries. Specifically, we assume that in the Home country real wages are more sticky, i.e. respond less to market forces, than in the Foreign country:

Degree of real wage rigidity at Home	$\gamma = 0.9$
Degree of real wage rigidity at Foreign	$\gamma^* = 0.5$

What happens when the two countries have different degrees of real wage rigidities?

Fig. 5 shows the impulse response functions following wage shocks at Home (first column) and at Foreign (second column). Qualitatively, the dynamics are very similar to the "perfect simmetry" case. However, the effects of the shocks are now larger, since both union unemployment and union inflation tend to be more responsive to shocks.

When the member countries have different degrees of real rigidities, it does matter *where* the shock materialize. In the Home country, as wages cannot easily adjust, shocks tend to have larger real effects than in the Foreign country. As in Christoffel and Linzert (2005), wage rigidities, by limiting the adjustment capabilities of wages, increase the business cycle fluctuations of unemployment and output. This is true not only at the

[50] Notice in fact that in equilibrium the terms of trade satisfy

$$S_t = \frac{P_{Ft}}{P_{Ht}} = \frac{A_t N_t \left(1 - \delta_H B\left(x_t\right)^{\varphi}\right)}{A_t^* N_t^* \left(1 - \delta_F B^*\left(x_t^*\right)^{\varphi}\right)}$$

While a wage shock influences the terms of trade only through variation in (un)employment, a productivity shock has not only a indirect effect, through N_t and N_t^*, but also a direct effect, as it enters directly in the expression for S_t.

union level but also for what concerns differentials: a higher degree of real wage rigidities substantially increases the size and persistence of inflation and unemployment differentials.

Similar conclusions, as can be seen by analyzing Fig. 6, hold with respect to productivity shocks. A positive productivity shock in the Home country generates, in the medium term, a large and persistent unemployment decrease at Home; when a similar shock hits the Foreign country, instead, medium run unemployment in both country is only slightly affected. Therefore, unemployment fluctuations, as well as inflation and unemployment differentials, are much larger when the shock hit the "rigid wages" country. Notice, in particular, that unemployment differentials after a Home productivity shock are 4 time bigger than unemployment differentials after a Foreign productivity shock.

The presence of different degrees of real wage rigidities deeply affects the transmission mechanism of monetary policy (Fig. 7). After the common monetary policy shock, unemployment fluctuations are larger in the Home country while inflation reacts more in the Foreign country. As a consequence, large differentials arises: on impact, both inflation and output differentials are larger than 1 percentage point. This result confirms the old intuition that a common symmetric shock can have large asymmetric effects when the countries have different economic structures. Note however that while inflation differentials decrease rapidly, unemployment differentials are large and quite persistent[51].

9.3 The Role of Labor Market Imperfections

After having analyzed the role of different degrees of wage stickyness, in this section we study the role of labor market imperfections. Specifically, we assume that Home is the "rigid" country, i.e. the country that has more sclerotic labor markets. Foreign is instead the "flexible" country, where "flexibility" means higher job-finding and separation rates. The parameters are calibrated as shown in the following table:[52]

	Job-Finding rate x^i	Separation rate δ_i	Natural unemployment
Home	0.3	0.048	0.10
Foreign	0.6	0.096	0.06

Fig. 8 and 9 display, respectively, the responses of inflation and unemployment to wage and productivity shocks. The asymmetries generated by different labor market institutions are smaller than the ones generated by different degrees of real wage rigidities, but they are still not negligeable. Interestingly, unemployment fluctuations are larger in the "flexible"

[51] This result partly depends on the characteristics of our model, that displays persistence on the quantity side but not on the price side. Notice that it would be easy to generate inflation persistence (and a backward looking term for inflation in the Phillips Curve) by simply assuming that the wage norm in the wage rule is given by the past wage level (instead that by the steady state wage level): $W_t^R = (W_t^n)^{1-\gamma} (W_{t-1})^{\gamma} e^{\varepsilon_t^w}$. With this specification of the wage rule, which can be written in log-linear form as $\hat{w}_t = \gamma \hat{w}_{t-1} + (1-\gamma)\hat{w}_t^{Nash} + \varepsilon_t^w$, we would obtain a *hybrid* Phillips Curve. To avoid tractability problems, we have chosen not to introduce this element in our framework.

[52] As in Blanchard and Galì (2006), we calibrate the value of the job-finding rate and of the unemployment rate, while the separation rate is determined using the steady state relationship $\delta_i = u^i x^i / \left(\left(1-u^i\right)\left(1-x^i\right)\right)$. Notice also that in these simulations, to insulate the impact of different labor market institutions, we have set the real wage rigidities in Home and Foreign as in the baseline calibration, i.e. $\gamma = \gamma^* = 0.5$.

Figure 5: *Impulse Responses to Asymmetric Wage Shocks*

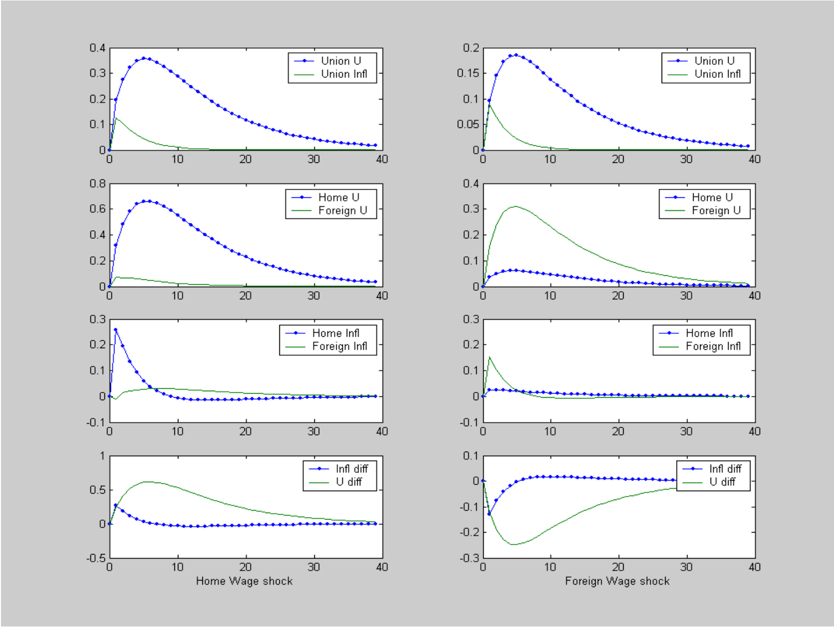


Figure 6: *Impulse Responses to Asymmetric Productivity Shocks*

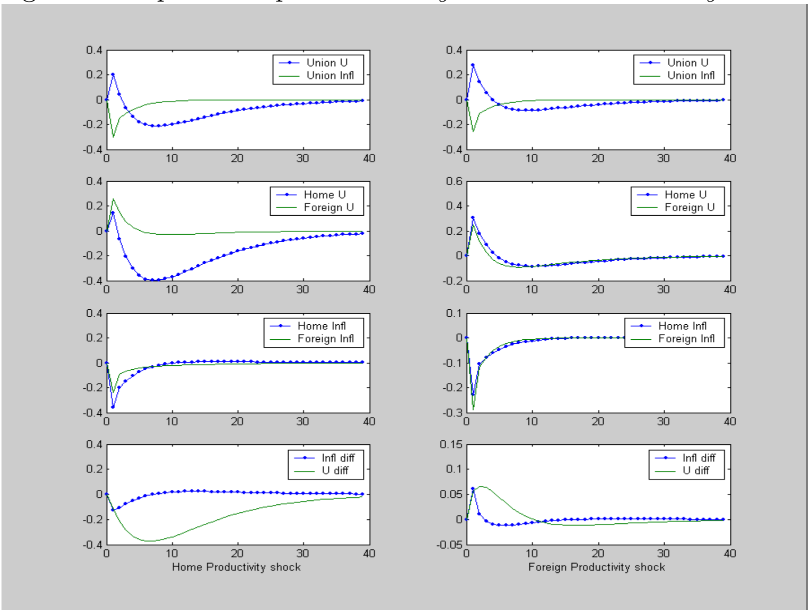
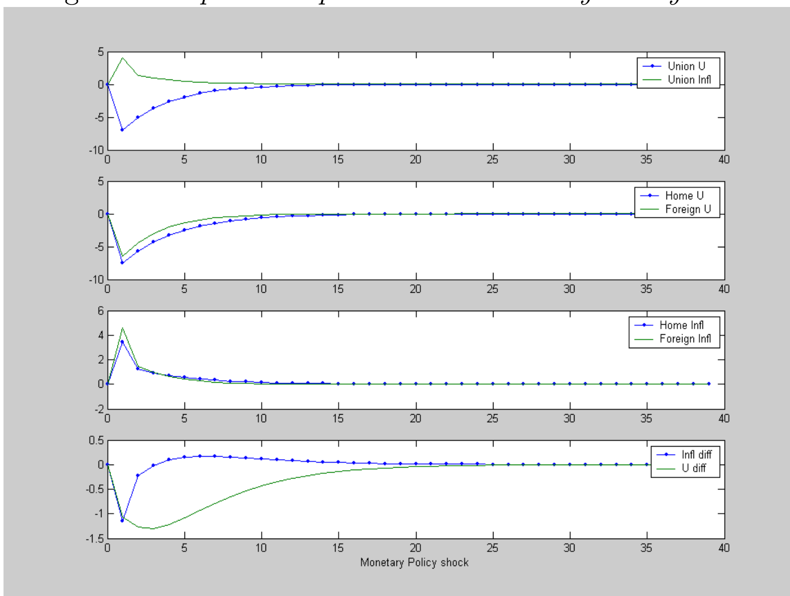


Figure 7: *Impulse Responses to a Monetary Policy Shock*



country - the Foreign country in our case. Intuitively, when hiring costs are high and job turnovers are low, a firm finds it relatively more costly to hire new workers. Therefore, when a wage or a productivity shock hit the economy, it is relatively more convenient for the firm to absorb the shock by changing prices than by changing the quantities produced. A "sclerotic" economy thus tends to be characterized by higher inflation volatility and lower unemployment volatility.

Fig. 10 shows the impulse responses to a one percent decrease of the nominal interest rate. Following a common interest rate shock, unemployment reacts more in the flexible country, while inflation on impact increases faster at Home - but then it also dies out more rapidly. The differentials generated by a monetary policy shock can be substantial: on impact, unemployment and inflation differentials increase by more than 2%.

9.4 Discussion

Several interesting results emerge from the previous analysis.

First, when monetary policy is exclusively focused on inflation stabilization, asymmetric wage or productivity shocks may have very long-lasting effects on unemployment, both at the union level and at the country level. Under the baseline calibration, an asymmetric unexpected wage increase has still some effect after 40 quarters. This persistency comes from some realistic assumptions of our framework: nominal prices and real wage rigidities, interest rate smoothing in monetary policy, the presence of labor market imperfections.

Second, even when the countries have similar structures, large and long-lasting differentials can arise as a consequence of asymmetric shocks. The size of these differentials is influenced by the degree of real wage rigidities and by labor market institutionals para-

Figure 8: *Impulse Responses to Asymmetric Wage Shocks*

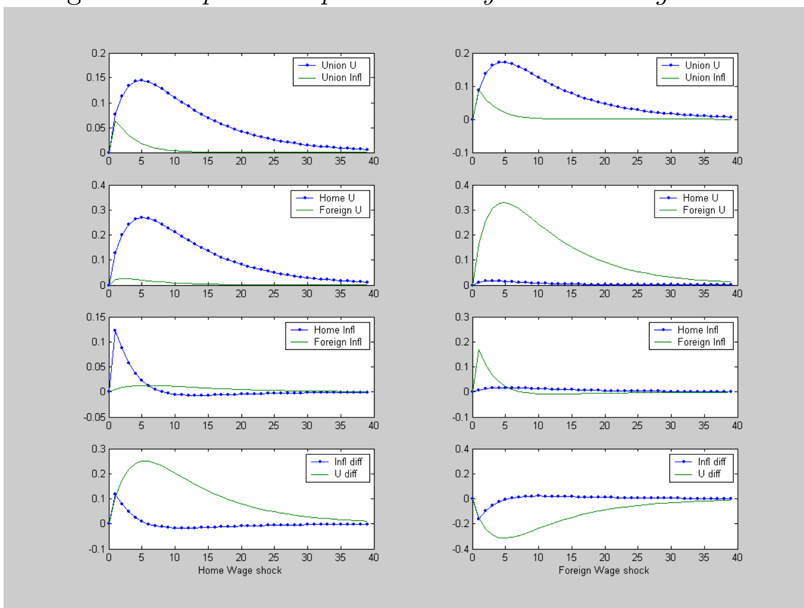


Figure 9: *Impulse Responses to Asymmetric Productivity Shocks*

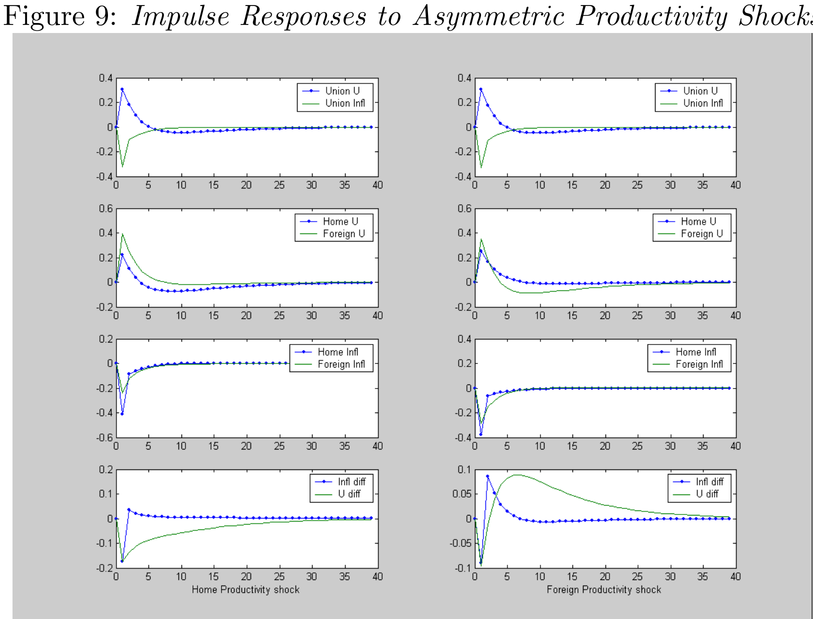
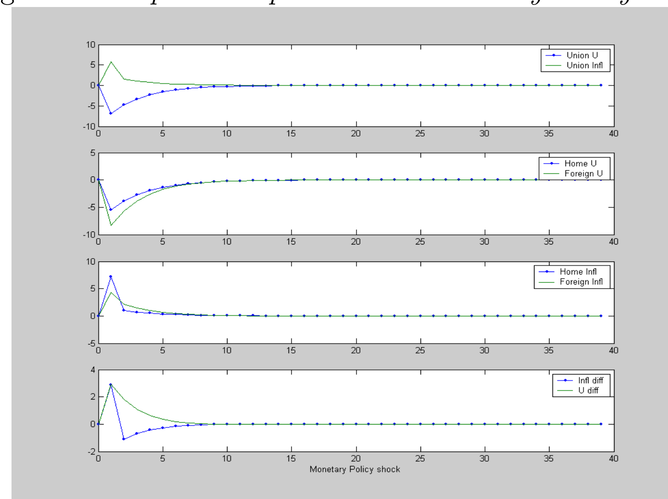


Figure 10: *Impulse Responses to a Monetary Policy Shock*



meters. Interestingly, productivity shocks in our model have larger spillover effects than wage shocks and, accordingly, tend to generate smaller differentials.

Third and most importantly, understanding the labor market structure and the sources of rigidities in the wage determination mechanism is crucial. These rigidities have an important impact on the overall behaviour of an economy and, in particular, on the trasmission mechanism of monetary policy. Interestingly, the simulations show that not all sources of rigidity have the same effects. It does make a difference whether the rigidity lies in the wage determination mechanism or in the labor market structure. To understand this point, consider Fig 11, which show the implied volatilities of the key variables under two alternative calibrations. In the first table Home and Foreign, as in section 9.2, differ in the degree of real wage rigidity; in the second, they differ in the degree of labor market imperfections - as in section 9.3.

A key fact emerges: the relative size of business cycle fluctuations depends substantially on the source of the rigidity:

1. A higher degree of real wage rigidities amplifies the response of the real economy to shocks. Unemployment volatility in the "rigid wages" country is much higher than in the "flexible wages" country. The real wage rigidities, in fact, limit wage adjustments and shift the labor market adjustment from prices to quantities. If the rigidity is in the wage determination mechanism, the "rigid" country is more volatile.

2. A country that has more sclerotic labor markets typically experiences a lower unemployment volatility, and a higher inflation volatility. Intuitively, when labor markets are rigid, it is more costly for the firm to hire new people and therefore unemployment does not vary as much as it would in a more flexible economy. If the rigidity lies in the labor market, the country is less volatile, since in this case are "quantities" that cannot adjust.

Figure 11: *Real Wage Rigidities VS Labor Market Imperfections*

Flexible VS Rigid Wages						
	Shocks					
	Wage (Home and Foreign)	Prod. (Home and Foreign)	Home (Wage and Prod.)	Foreign (Wage and Prod.)	Monetary Policy	All
Union Level						
$\sigma(\pi_u)$	0,23	0,47	0,40	0,32	0,87	1,02
$\sigma(U_u)$	1,23	0,85	1,25	0,70	2,14	2,57
$\sigma(ToT)$	2,04	1,23	2,15	0,74	0,64	2,48
$\sigma(i)$	0,47	0,69	0,67	0,44	0,90	1,20
Home (Rigid Wages)						
$\sigma(\pi_H)$	0,37	0,53	0,58	0,27	0,77	1,01
$\sigma(U_H)$	1,19	1,27	2,12	0,48	2,22	3,13
Foreign (Flex Wages)						
$\sigma(\pi_F)$	0,22	0,42	0,28	0,37	0,98	1,10
$\sigma(U_F)$	0,87	0,50	0,36	0,91	1,73	2,00

Flexible VS Rigid Labor Markets						
	Shocks					
	Wage (Home and Foreign)	Prod. (Home and Foreign)	Home (Wage and Prod.)	Foreign (Wage and Prod.)	Monetary Policy	All
Union Level						
$\sigma(\pi_u)$	0,15	0,50	0,36	0,37	1,22	1,32
$\sigma(U_u)$	0,67	0,61	0,61	0,65	2,01	2,16
$\sigma(ToT)$	1,25	0,43	0,86	0,94	0,70	1,44
$\sigma(i)$	0,27	0,51	0,39	0,41	0,82	0,98
Home (Rigid Labor Markets)						
$\sigma(\pi_H)$	0,17	0,58	0,45	0,38	1,47	1,58
$\sigma(U_H)$	0,76	0,48	0,82	0,33	1,50	1,71
Foreign (Flex Labor Markets)						
$\sigma(\pi_F)$	0,22	0,45	0,28	0,40	1,02	1,13
$\sigma(U_F)$	0,92	0,70	0,49	1,01	2,21	2,44

The implications of real wage rigidities and labor market imperfections for business cycle flucuations are thus likely to operate in opposite directions. This is a very intuitive result, as (loosely speaking) in the first case the rigidity is on the “labor prices” side, while in the second are “labor quantities” that cannot adjust.

Furthermore, we have shown that when member countries have different labor market structures, symmetric shocks can have substantial asymmetric effects and lead to large differentials. As shown in Fig. 11, the impact of common monetary policy shocks may differ substantially when the member countries have different degrees of wage stickyness or labor market institutions.

These results have strong policy implications. They suggest that the central bank may benefit from closely monitoring the development of labor markets in member countries, as this would permit a deeper understanding of the transmission mechanism of shocks in general, and of monetary policy in particular[53]. Moreover, they suggest that a monetary policy strategy that gives equal weights to different countries may not be optimal. As we have shown, when member countries have different economic structures, it does make a difference where the shock takes place. For instance, a shock that takes place in the “rigid wages” country generates much larger union-wide inflation and unemployment fluctuations than an equal shock that hit the “flexible wages” country. The central bank would presumably benefit from giving more weight, in its monetary policy rule, to the country whose functioning creates more distortions - the “rigid wages” country in our example.[54] A more rigorous discussion of these important implications is beyond the scope of this

[53] See, e.g., Christoffel and Linzert (2005) for a similar claim.

[54] See Benigno (2004) for a similar argument. Benigno shows that, when the degree of nominal rigidities in the regions forming a currency union is not equal, it is not optimal to give equal weights to the two regions in the monetary policy rule. Instead, an inflation targeting policy in which higher weight is given to the inflation in the region with higher degree of nominal rigidity is nearly optimal.

paper and we leave it to future research.

10 Comparing Monetary Policy Rules

Which is the impact of monetary policy on the functioning of the currency union? In this section we try to tackle this question by comparing three simple monetary policy rules.

The *Pure Inflation Targeting (IT)* strategy represents the extreme case of a central bank that seeks to stabilize inflation at all horizons, setting $\hat{\pi}_t^U = 0$ at all t. Under a *Pure Output (or Unemployment) Targeting (OT)* strategy, on the contrary, the central bank only cares about unemployment and, by setting $\tilde{u}_t^A = \tilde{n}_t^U = \tilde{y}_t^U = 0$ at all t, stabilizes the gap between unemployment and its efficient level. The third strategy represents the intermediate case of a central bank that cares about both inflation and unemployment. Specifically, we assume the central bank sets the nominal rate following the standard *Taylor rule*:

$$\hat{\imath}_t = 1.5\tilde{\pi}_t^U - 0.5\tilde{u}_t^A \tag{80}$$

Throughout this analysis, we assume the Home and Foreign regions differ in both the degree of wage rigidities and in the labor market structure, with the Home country being more "rigid" in both sense. Specifically, we assume:[55]

	Job-Finding rate x^i	Separation rate δ_i	Unemployment	Wage Rigidities γ^i
Home	0.3	0.048	0.10	0.7
Foreign	0.6	0.096	0.06	0.4

Fig. 12 shows the response of union-wide variables to an unexpected wage increase in the Home country[56]. If the Central Bank completely stabilizes unemployment, union-wide inflation rises by 0.40 points on impact, and then decreases very slowly towards equilibrium. Under a pure IT strategy, instead, the monetary authority completely stabilizes union-wide inflation, but at the cost of quite high and persistent unemployment fluctuations. Union unemployment reaches a peak increase of almost 0.35 after 3 quarters and then start to decrease.

Of course, there is no reason why the central bank would want to respond to a shock by allowing only inflation or unemployment fluctuations. As long as the central bank cares about both unemployment and inflation fluctuations, she would spread the costs of the adjustment among the two variables. The first panel in Fig. 12 shows the response of union-wide variables under the Taylor rule. As the Taylor rule divides the burden of adjustment between the two variables, the impulse responses of inflation and unemployment are very similar. Both variable display a peak increase of about 0.2 after two quarters, and then decrease smoothly towards equilibrium. Notice that in each of the three cases, the impulse responses of the adjusting variables remain above the initial values well after the shock has vanished.

[55] The main results of this section hold also in the perfect symmetry case. We have chosen to show the asymmetric case because both more interesting and more realistic.

[56] In this section, since all types of shocks provides similar insights, we only show the effects of Home wage shock.

Figure 12: *Comparing different policy rules: Home Wage shock - Union Level*

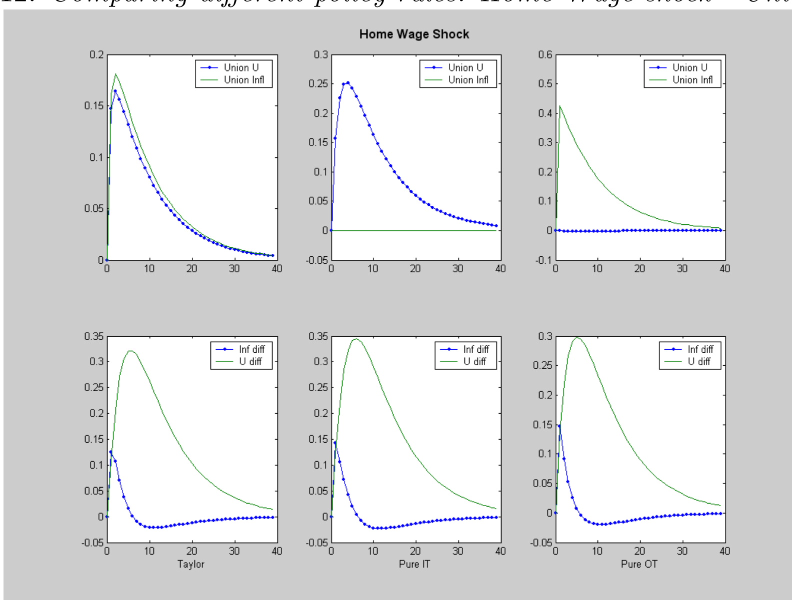


Figure 13: *Comparing different policy rules: Home Wage shock - Country Level*

Figure 14: *The Properties of Three Simple Policy Rules*

	Taylor rule					**Output Targeting**					**Inflation Targeting**				
	Shocks					Shocks					Shocks				
	Wage (Home and Foreign)	Prod. (Home and Foreign)	Home (Wage and Prod.)	Foreign (Wage and Prod.)	All	Wage (Home and Foreign)	Prod. (Home and Foreign)	Home (Wage and Prod.)	Foreign (Wage and Prod.)	All	Wage (Home and Foreign)	Prod. (Home and Foreign)	Home (Wage and Prod.)	Foreign (Wage and Prod.)	All
Union Level															
$\sigma(\pi_u)$	0,54	0,45	0,56	0,42	0,69	1,16	0,67	1,08	0,78	1,33	0,00	0,00	0,00	0,00	0,00
$\sigma(U_u)$	0,54	0,21	0,47	0,34	0,58	0,00	0,00	0,00	0,00	0,00	0,92	0,54	0,85	0,68	1,11
$\sigma(ToT)$	1,32	0,52	1,11	0,85	1,37	1,31	0,47	1,02	0,96	1,31	1,39	0,55	1,21	0,80	1,51
$\sigma(i)$	2,18	2,28	2,49	1,90	3,06	4,19	2,99	4,15	2,99	5,08	0,51	0,63	0,60	0,55	0,81
Home (Rigid Wages and Labor Markets)															
$\sigma(\pi_H)$	0,57	0,47	0,62	0,39	0,72	1,17	0,68	1,14	0,73	1,35	0,14	0,05	0,12	0,10	0,15
$\sigma(U_H)$	0,85	0,39	0,90	0,15	0,91	0,59	0,21	0,46	0,43	0,59	1,17	0,69	1,30	0,30	1,40
Foreign (Flex Wages and Labor Markets)															
$\sigma(\pi_F)$	0,55	0,44	0,52	0,47	0,69	1,16	0,65	1,03	0,84	1,32	0,14	0,05	0,12	0,10	0,15
$\sigma(U_F)$	0,72	0,18	0,21	0,71	0,72	0,61	0,22	0,48	0,45	0,62	0,92	0,38	0,29	1,00	1,02

Fig. 14, which displays the implied volatilities of the key variables under different shocks and policy rules, confirms this basic result: in the model, an important trade-off between unemployment and inflation stabilization arises. A pure inflation stabilization strategy leads to a large unemployment volatility; similarly, a pure output targeting strategy entails the cost of high inflation volatility. The Taylor rule makes a balance of the two, and trades some unemployment volatility with some inflation volatility. Notice that interest rate volatility is five time larger under a pure OT strategy than under a pure IT strategy.

Different monetary policy rules does not seem to have a big impact on differentials, as the dynamics of inflation and unemployment differentials under the three rules are very similar. Interestingly, unemployment differentials in our model are always much larger and more persistent than inflation differentials.

Turning at the country level (see Fig. 13 and Fig.14), a strict inflation targeting strategy stabilizes inflation at the union level, but not at the country level The same is true for unemployment under a strict unemployment stabilization policy. Notice however that while inflation fluctuations under pure IT are small, a pure output targeting strategy leads to non-negligeable unemployment fluctuations at the country level.

Different degrees of real rigidities in the two economies deeply influence the volatilities of the two regions. Asymmetric regions react differently to similar shocks. Interestingly, under our calibration, whether the Home country has larger business cycle fluctuations than Foreign depends on the policy rule implemented. Unemployment fluctuations at Home are larger than at Foreign under the Taylor rule and the pure IT strategy; the opposite is true under pure unemployment targeting. Intuitively, different labor market structures or wage determination mechanisms influence the slopes of the Phillips curves of the two countries. Different monetary policy rules may exploit this asymmetry and produce different results.[57]

[57] See Blanchard-Galì (2006), p. 31, for a similar point.

11 Conclusion

In this paper we have introduced labor market frictions and real wage rigidities in a standard DSGE currency union model. The model provides a rigorous, but still tractable, framework for the analysis of the functioning of a currency union characterised by pervasive nominal and real rigidities.

As in Blanchard and Galì (2006), labor market frictions are introduced by assuming the presence of hiring costs, which increase in the degree of labor market tightness. The introduction of hiring costs has two important consequences. First, it leads to involuntary unemployment. Second, it fundamentally changes the nature of marginal costs, which now depends not only on the evolution of real wages and productivity, but also on the evolution of marginal hiring costs. The presence of real wage rigidities, on the other side, hinders wage adjustments and hence increases the adjustments on the labor quantity side. The degree of labor market frictions and real wage rigidities, by influencing the incentives and constraints faced by firms, significantly affect the transmission mechanism of shocks in general, and of monetary policy in particular.

In the presence of real wage rigidities or exogenous wage shocks, strict inflation targeting is not optimal, as it entails large and persistent unemployment fluctuations, both at the union and at the country level.

The presence of real rigidities considerably amplifies the persistency of transitory shocks and helps to explain the existence of long-lasting inflation and unemployment differentials. Moreover, when member countries have different labor market institutions, symmetric shocks (in our case, monetary policy shocks) can have large asymmetric effects.

Finally, it does make a difference if the rigidities lies in the wage determination mechanism or in the labor market structure. The implications of real wage rigidities and labor market frictions for business cycle fluctuations are in fact likely to operate in opposite directions: a high degree of real wage rigidities tends to amplify the response of the real economy to shocks; when labor market are more sclerotic, instead, unemployment volatility tends to decrease while inflation volatility increases.

All these results suggest that understanding the labor market structures and the sources of rigidities in the wage determination mechanism is crucial for policy analysis.

12 References

Abbritti M., Boitani A. and M. Damiani, (2006), "Unemployment, Inflation and Monetary Policy in a Dynamic New Keynesian Model with Hiring Costs", *Quaderni del Dipartimento di Economia, Finanza e Statistica*, Perugia, n. 22.

Ahearne A. and Pisani-Ferry J., (2006), "The Euro: Only for the Agile", *Bruegel Policy brief*, 2006/01.

Alexopoulos M. (2004), "Unemployment and the Business Cycle", *Journal of Monetary Economics*, vol. 51, pp. 277-298.

Altissimo F., Benigno P., Rodriguez Palenzuela D.(2005), "Long Run Determinants of Inflation Differentials in a Monetary Union", *NBER Working Paper*, n. 11473.

Angeloni I. and Ehrmann M., (2004), "Euro Area Inflation Differentials", *European Central Bank Working Paper Series*, n. 388

Benalal N., Diaz del Hoyo J. L., Pierluigi B. and Vidalis, N., (2006), "Output Growth Differentials across the Euro Area Countries. Some Stylised Facts", *European Central Bank Occasional Paper Series*, n. 45.

Benigno P. and Lopez-Salido, J. D., (2002), "Inflation Persistence and Optimal Monetary in the Euro Area", *European Central Bank Working Paper Series*, n. 178

Benigno P.(2004), "Optimal Monetary in a Currency Area", *Journal of International Economics*, vol 63, issue 2, 293-320

Blanchard O., (2006), "Adjustment within the Euro. The Difficult Case of Portugal", *mimeo.*

Blanchard O., (2005), "European Unemployment: the Evolution of Facts and Ideas", *mimeo*, MIT.

Blanchard O., Galí J., (2006), "A New Keynesian model with Unemployment", *mimeo.*

Blanchard O., Galí J., (2005), "Real Wage Rigidities and the New Keynesian model", *NBER Working Paper*, n. 11806.

Blanchard O., Kahn C., (1980), "The solution of linear difference models under Rational Expectations", *Econometrica* 48 (5), 1305-1312.

Campolmi F. and Faia E., (2006), "Cyclical Inflation Divergence and Different Labor Market Institutions in the EMU", *European Central Bank Working Paper Series*, n. 619

Christoffel K, Linzert T (2005), "The Role of Real Wage Rigidity and Labor Market Frictions for Unemployment and Inflation Dynamics", *European Central Bank Working Paper Series*, n. 556

Christoffel K, Kuester K., Linzert T (2006), "Identifying the Role of Labor Markets for Monetary Policy in an Estimated DSGE Model", *European Central Bank Working Paper Series*, n. 635

Clarida R., Galí J., Gertler M. (1999), "The Science of Monetary Policy: A New Keynesian Perspective", *Journal of Economic Literature*, vol. 37, pp. 1661-1707.

ECB (2003), "Inflation Differentials in the Euro Area: Potential Causes and Policy Implications", *ECB Report*, September 2003.

ECB (2005), "Monetary Policy and Inflation Differentials in a Heterogeneous Currency Area", *ECB Monthly Bullettin*, May 2005, pp. 61-77.

Galì J., Monacelli T.(2005), "Monetary Policy and Exchange Rate Volatility in a Small Open Economy", *Review of Economic Studies*, vol. 72, issue 3, pp. 707-734.

Galì J., Monacelli T.(2005), “Optimal Monetary and Fiscal Policy in a Currency Union”, *NBER Working Paper*, n. 11815

Galì J., (2002), “New Perspectives on Monetary Policy, Inflation and the Business Cycle”, *CEPR Discussion Paper*, n. 3210

Hall R. E. (2005), “Employment Fluctuations with Equilibrium Wage Stickiness”, *American Economic Review*, vol. 95, pp. 50- 65.

Howitt P. (1988), “Business Cycles with Costly Search and Recruiting”, *Quarterly Journal of Economics*, vol. 103, n.1, pp.147-165.

Krause and Lubik, (2003), “The (Ir)relevance of Real Wage Rigidity in the New keynesian Model with Search Frictions”, *Economics Department Working Paper*, n. 504, John Hopkins University.

Mortensen D.T., Pissarides C.A.(1994), “Job Creation and Job Destruction in the Theory of Unemployment”, *Review of Economic Studie*s, vol. 61, pp. 397-416.

Mongelli F. P., (2002), “New Views on the Optimum Currency Area Theory: What is the EMU Telling us?", *European Central Bank Working Paper Series*, n. 138

Obsfeld M., Rogoff K.(2001), *Foundations of International Macroeconomics*, MIT press

Pappa E., (2004), “Should the Fed and the ECB Cooperate? Optimal Monetary Policy in a Two-Country World”, *Journal of Monetary Economics*, 51 (4) pp. 753-779.

Shimer R. (2005), “The Cyclical Behavior of Equilibrium Unemployment and Vacancies”, *American Economic Review*, vol. 95, pp. 25-49.

Trigari A. (2005), “Equilibrium Unemployment, Job Flows and Inflation Dynamics”, *mimeo*, IGIER, Bocconi University, Milan.

Trigari A. (2004), “Labor Market Search, Wage Bargaining and Inflation Dynamics”, *mimeo*, IGIER, Bocconi University, Milan.

Uhlig H., (1997), “A toolkit for analyzing non-linear dynamics stochastic models easily”, *mimeo*

Walsh C.E. (2005), “Labor Market Search, Sticky prices, and Interest Rate Policies”, *Review of Economic Dynamics*, vol. 8, pp. 829 - 849.

Weber S. (2006), “Labor Market Structures, Trade and their Effect on Unemployment: A Theoretical Analysis and Empirical Investigation ”, *mimeo*, HEI, Geneva.

Woodford M. (2003), *Interest and Prices*, Priceton and Oxford, Princeton University Press.

Wyplosz C. (2006), "Diverging Tendencies of Competitiveness", *Briefing Notes to the Committee for Economic and Monetary Affaires of the European Parliament*, Third Quarter 2006.

Appendix: Key Conditions for the Foreign Country

A. 1: The Social Planner Problem

The optimality condition with respect to employment in the foreign country, N_t^*, implies:

$$\chi_1 \frac{(N_t^*)^{\phi} C_{Ft}^W}{A_t^*} \leq 1 - B^*(1+\varphi)\,(x_t^*)^{\varphi} \tag{81}$$

$$+\beta(1-\delta_F)E_t \left\{ \frac{C_{Ft}^W}{C_{Ft+1}^W} \frac{A_{t+1}^*}{A_t^*} B^* \left[\begin{array}{c} \left(x_{t+1}^*\right)^{\varphi} + \\ \varphi \left(x_{t+1}^*\right)^{\varphi} (1 - x_{t+1}^*) \end{array} \right] \right\} \tag{82}$$

It can be shown that the optimal level of employment does not depend on productivity, and is given by:

$$N_E^* = \frac{x_E^*}{\delta_F + (1-\delta_F)x_E^*} \tag{83}$$

where x_E^* is the efficient level for the tightness indicator for the foreign country, implicitely given as the solution to the optimality condition for N_t^*.

The levels of consumption and output are given by $C_t^* = A_t^{\alpha} \left(A_t^*\right)^{1-\alpha} N_E^*(1 - \delta_F B \left(x_E^*\right)^{\varphi})$ and $Y_t^* = A_t^* N_E^*$.

A. 2: Equilibrium under Flexible Prices

The following conditions hold for Foreign under flexible prices:

- Optimal price setting rule

$$\frac{P_{Ft}(i)}{P_{Ft}} = \frac{\epsilon^*}{\epsilon^* - 1} MC_t^* = \mu^* MC_t^* \tag{84}$$

- Real Marginal costs

$$MC_t^* = \frac{W_t^{R*}}{A_t^*} \left(S_t\right)^{-\alpha} + B^* \left(x_t^*\right)^{\varphi} \tag{85}$$

$$-\beta(1-\delta_F)E_t \left\{ \frac{C_t^* \left(S_t\right)^{-\alpha}}{C_{t+1}^* \left(S_{t+1}\right)^{-\alpha}} \frac{A_{t+1}^*}{A_t^*} B^* \left(x_{t+1}^*\right)^{\varphi} \right\} \tag{86}$$

- Equilibrium marginal costs

$$MC_t^* = \frac{1}{\mu^*} = MC_t \tag{87}$$

- Equilibrium requires

$$\frac{W_t^{R*}}{A_t^*} \left(S_t\right)^{-\alpha} = \frac{1}{\mu^*} - B^* \left(x_t^*\right)^{\varphi} \tag{88}$$

$$+\beta(1-\delta_F)E_t \left\{ \frac{C_t^* \left(S_t\right)^{-\alpha}}{C_{t+1}^* \left(S_{t+1}\right)^{-\alpha}} \frac{A_{t+1}^*}{A_t^*} B^* \left(x_{t+1}^*\right)^{\varphi} \right\} \tag{89}$$

A. 3: Nash Bargained Wages

It can be shown that the Nash wage schedule for Foreign is

$$\frac{W_t^{R*}}{A_t^*}(S_t^H)^{-\alpha} = \frac{\chi_1 C_{Ft}^W (N_t^*)^{\phi}}{A_t^*} + \eta^* B^* (x_t^*)^{\varphi} \tag{90}$$

$$-\beta(1-\delta_F)E_t \left\{ \frac{C_{Ft}^W}{C_{Ft+1}^W} \frac{A_{t+1}^*}{A_t^*} \left[(1 - x_{t+1}^*)(\eta^* B^* \left(x_{t+1}^* \right)^{\varphi}) \right] \right\} \tag{91}$$

It follows that the equilibrium under Nash Bargaining in the Foreign region is characterised by

$$\frac{\chi_1 C_{Ft}^W (N_t^*)^{\phi}}{A_t^*} = \frac{1}{\mu^*} - (1+\eta^*) B^* (x_t^*)^{\varphi} \tag{92}$$

$$+\beta(1-\delta_F)E_t \left\{ \frac{C_{Ft}^W}{C_{Ft+1}^W} \frac{A_{t+1}^*}{A_t^*} \left[\left(1 + \eta^*(1 - x_{t+1}^*)\right) B^* \left(x_{t+1}^* \right)^{\varphi} \right] \right\} \tag{93}$$

As for Home - and for the same reasons - the employment level is constant in an equilibrium with Nash Bargaining.

In equilibrium, the real wage is determined by:

$$\frac{W_t^{R*}}{A_t^*}(S_t^H)^{-\alpha} = \frac{1}{\mu} - B^* (x^*)^{\varphi} (1 - \beta(1-\delta)) \tag{94}$$

The equilibrium wage moves one for one with $A_t^*(S_t^H)^{\alpha}$. Productivity shocks at Home or at Foreign are completely neutralized by changes in the wage rate and thus do not affect the unemployment level, which is unchanged.