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Exit Polls and Voter Turnout

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Abstract

We set up a model of elections or referendums with two alternatives to study how voter turnout and election outcomes are affected by the publication of exit polls on election day. We find that the introduction of an exit poll influences the incentive to vote both before and after the poll is published, but the signs of the effects are generally ambiguous. The fact that exit polls influence the incentive to vote before they are even published is sometimes overlooked in the debate on their desirability. We show that this can lead to premature conclusions about the impact of exit polls on election outcomes.

Keywords: Elections, Exit Polls, Voter Turnout
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1 Introduction

On June 7 2009, Danish voters went to the polls to decide on a proposed change to the Danish Law of Succession, the law that governs the succession to the Danish throne. The proposed change would imply that sons would no longer have precedence over daughters in the line of succession, establishing so-called *equal primogeniture:* The right of the first-born child, whether male or female, to be first in line to inherit the throne.

The referendum was subject to the procedure that governs changes to the Danish constitution. In order to pass, the proposal therefore had to overcome two obstacles: One, a majority of the votes cast in the referendum must be in favor of the proposal. And two, at least 40% of all eligible voters must vote in favor of the proposal. In the weeks preceding the June 7 election, there was no doubt that only the latter of these requirements had the potential to become binding. In a Gallup poll released a week before the election, 84% of respondents indicated that they approved of the proposal to change the law. However, only 40.2% responded that they would show up at polls and vote in favor of the proposal.

On the afternoon of the election day, TV2, a major Danish TV channel, published the results of an exit poll, which predicted that 37.9% of all eligible voters would cast a vote in favor of the proposal to change the law. However, during the evening the situation turned around with pollsters reporting a considerable increase in turnout. In the end, the official result was that 45.1% of all eligible voters had voted in favor of the proposal, which corresponded to 85.4% of all votes cast. Thus, the proposal passed with a comfortable margin.

The discrepancy between the early exit poll and the final result sparked a lively public debate in the days following the referendum. The fact that voter turnout rose in the final hours before the polling stations closed led some observers to conclude that it was the publication of the exit poll itself, and the prospect of the proposal failing, that got potential yes-voters to the polling stations. Without the exit poll, the proposal would not have passed, the argument went.[1] The Social Democrats, the major opposition party, took the opportunity to propose that

[1] On June 9, the Danish newspaper *Politiken* ran a front page story under the headline "Equal rights in the Royal family were saved by exit poll". Professor Jørgen Elklit of Aarhus University was quoted for saying that "the prospect of a no made more people vote". A related article from the same day brought the following quote from Professor Johannes Andersen of Aalborg University: "It appears that 5 percent of voters changed their behavior in the last hours. They would not have voted if they had not been made aware that votes were lacking". ("Sofavælgere piskede til valgstederne", *Politiken*, June 9 2009).

Denmark follow a number of other countries in prohibiting the publication of exit polls before polling stations are closed.[2]

In this paper we present a theoretical model to analyze whether and how exit polls influence voter turnout and outcomes in referendums or elections with two alternatives. We assume that voting takes place over two stages. Voters can choose to vote in either stage, or to abstain. Before stage two, an exit poll reveals how voters voted in the first stage. Any remaining potential voters then use this information to refine their decision rule for stage two.

The basic set-up in our model follows the seminal contributions of Palfrey and Rosenthal (1983), and especially Palfrey and Rosenthal (1985). Voters face heterogeneous costs of participating in the referendum. Costs are net of any direct benefits of participating (e.g., from fulfillment of "civic duty") and may thus be positive or negative. Each voter knows her own cost, but has only probabilistic knowledge about others' costs. Thus, when the exit poll reveals all early votes it provides a remaining potential voter with information about, loosely speaking, how close the race is. But it also allows her to update her beliefs about the realized costs of the other remaining voters, i.e., those that did not vote early.

We restrict attention to referendums in which the outcome is determined not only by simple majority; one of the alternatives, typically a proposal to change a status quo, must also receive a certain number of votes in order to beat the other.[3] Moreover, we simplify our model by assuming that all voters prefer the same alternative. This may appear overly restrictive, but it closely mimicks the situation in the Danish referendum in 2009: Since more than 80% of voters were in favor of changing the law, it was clear that the proposal would pass if and only if at least 40% of the electorate voted for it. In that case, the opponents of the proposal would have no chance of getting a majority against it. This effectively made their participation decision irrelevant for the outcome of the referendum. Our simple model can also be seen as a building block that can be used in models of exit polls in more general electoral settings. For example, suppose that there are two alternatives and the electoral rule is simple majority. Then the supporters of each alternative are facing a problem that is similar to the one in our model: Taking the number of votes for the other alternative as given, they have to coordinate on beating this number of votes. Of course, since the required turnout for each group

[2] Countries that prohibit publication of exit polls on election day include Germany, United Kingdom and Norway.

[3] Such requirements, known as approval quorum requirements, are common in European referendums. See Aguiar-Conraria and Magalhães (2010) for a recent overview of quorum rules in national referendums in EU countries.

is endogenous, the general model becomes much more difficult to analyze. Still, the analysis in this paper can be seen as one step in this direction.

We restrict our analysis of the model to cases where the number of voters is either two or three. When there are two voters, we consider both the case where implementation of their commonly preferred alternative requires only one of them to vote, and the case where it requires both of them to vote. These two-voter cases are interesting because each of them contains in isolation one of two collective action issues that are relevant in real world elections. When only one vote is required there is a free riding problem: Each voter prefers that the other voter bears the cost of going to the polls. When two votes are needed free riding is not an issue, but instead there is an obvious coordination problem: Each voter wants to vote only if the other voter also votes. In our final case, two out of three voters need to vote in order for their preferred alternative to be implemented. Thus, both the free riding issue and the coordination issue are present and therefore the analysis of this case provides us with important insights that are also likely to be relevant for real world referendums and elections.

By focusing on a small number of players, we of course avoid the famous "paradox of not voting" formalized by Palfrey and Rosenthal (1985). Our goal here is solely to provide an analysis of how the publication of an exit poll influences the incentive to vote, relative to a situation with no exit poll, not to analyze why people vote in large elections.[4]

The introduction of an exit poll influences potential voters' incentive to vote both before and after the release of the poll. For individuals who have not yet voted when the exit poll is released, the poll's effect on the incentive to cast a late vote depends crucially on what it reveals. An exit poll that reveals a close race increases the probability of being pivotal, which stimulates late voting, while an exit poll revealing that the race is far from close does the opposite. While this is a rather basic insight, the exit poll's effect on the incentive to cast an early vote is more subtle: Intuitively, voters who face a positive participation cost may find it worthwhile to await the result of the poll before they decide whether to abstain or vote. This "wait-and-see" effect discourages early voting. But the exit poll may also give rise to a "first-mover effect" that stimulates early voting: By voting early, a voter may use the exit poll actively to convince other supporters of the same alternative that victory is within reach, and this could induce them to participate in the referendum. As a result, the exit poll's effect on the incentive to vote early is generally ambiguous.

[4] See Feddersen (2004) for a review of the literature on the paradox of not voting.

In relation to the debate on the effects of exit polls, the single most important insight from our analysis is the following: It may well happen that the incentive to vote increases after the revelation of an exit poll. It may also happen that this higher incentive is decisive for the outcome of the election, in the sense that the extra turnout it generates after the release of the poll is sufficient to ensure that voters' preferred alternative is implemented, whereas it would not have been implemented if voters had continued to behave as they did before the release of the exit poll. This is essentially the situation that led some commentators, politicians, and academics to conclude that exit polls changed the outcome of the 2009 Danish referendum. However, our model reveals that this conclusion could well be wrong. The problem is that it is not based on the correct counterfactual, which is what would have happened if there had been no exit poll, not what would have happened if people continued to behave as they did before the exit poll. Since the exit poll influences voting behavior *before as well as after* its release, these two counterfactuals are not identical. In the context of the Danish referendum, some of the late voters who showed up at the polls only after the release of the exit poll may just have postponed voting because of the "wait-and-see" effect. And thus, they may well have voted anyway if there had been no exit poll. If so, the outcome would have been no different in this case.

Our paper is related to recent theoretical contributions by Goeree and Großer (2007) and Taylor and Yildirim (2010). Both of these papers focus on the effects of pre-election public opinion polls in two-candidate majoritarian elections and reach similar conclusions: Polls stimulate supporters of the alternative favored by a minority of the population to participate in the election, while individuals who support the alternative favored by a majority participate less frequently when they receive information about the distribution of preferences in the population because they free ride on the participation of other voters who support the same alternative. The result is that polls lower the probability that the alternative favored by a majority in the population is implemented. When it comes to expected turnout, the increase in participation for minority voters more than compensates the lower participation frequency for majority voters, so total expected turnout goes up, resulting in higher aggregate participation costs. The combination of higher costs and a lower probability that the majority's favored alternative is implemented leads to the conclusion that polls lower social welfare.[5]

[5] The effect of public opinion polls on turnout in the participation games of Palfrey and Rosenthal (1983, 1985) has also been studied experimentally by Klor and Winter (2007) and Großer and Schram (2010).

Although related, our paper addresses a different question than the papers mentioned above: First of all, we focus on exit polls, rather than pre-election polls. This is reflected in the fact that voters in our model have the opportunity to vote *before* the result of the poll is revealed. This makes the game studied here dynamic, and it opens up for strategic interactions among potential voters that are absent in the models of pre-election polls. Second, we do not study purely majoritarian elections. Instead, we focus on elections in which the majority requirement is supplemented with a requirement on the absolute number of votes for one of the candidates. Third and finally, uncertainty in our model comes from imperfect information about other players' voting costs, while we have assumed that preferences over alternatives are commonly known (e.g., from pre-election polls). In the pre-election poll papers mentioned above, it is exactly opposite: Uncertainty comes from imperfect information about other voters' preferences over alternatives, while all potential voters are assumed to share a commonly known participation cost. For the type of elections we wish to study, we believe that our approach is the appropriate one, as explained above.

Our model is also somewhat related to the theoretical literature on sequential voting (see, e.g., Battaglini 2004 and the references therein). This literature analyzes the aggregation of information in exogenously given sequential voting procedures and compares it with simultaneous voting. Our model is distinct from this literature on several dimensions. First, the order of voting is endogenous in our model, since all voters can vote in either period. Furthermore, all voters know their preferred alternative with certainty, so information aggregation is not an issue.

Throughout this paper, we present our model in the context of elections and exit polls, but our framework also lends itself to alternative interpretations. The fact that all agents prefer the same outcome means that we are effectively studying a game of voluntary private contributions to a threshold public good.[6] Such games have many applications. For example, we could translate "votes" with "charitable contributions" in fundraising campaigns where a certain number of contributions is required for the charity project to be carried out. The question we then ask is how the expected number of contributions, and the likelihood that the project is carried out, would change if campaign organizers during the campaign publicized

[6] A mechanism that provides a binary public if and only if the amount of private contributions is above a certain threshold is known in the literature as a *provision point mechanism* (Bagnoli and Lipman (1989)). In the terminology of this literature, the game studied in this paper is one of binary contributions, with no refund in case of insufficient contributions, and no rebate in case of excessive contributions. See Rapoport (1999) for a review of the theoretical and experimental literatures on these and related types of public good games.

how many people had contributed so far. Another example is general meetings, in which attendance is time-consuming and therefore individually costly, but where general attendance must be above a certain level for resolutions passed in the meeting to be valid. An exit poll would in this case correspond to a notice from the organizer revealing how many delegates had committed to showing up so far. These alternative interpretations of the model are perhaps more suitable in that the assumption of a small number of players is less restrictive. For consistency, we shall stick to the terminology of voting and exit polls, however.

2 The Model

We consider an election with two alternatives, A and B. The electoral rules are such that alternative A is implemented if the number of votes for A is at least M and higher than the number of votes for B. Otherwise alternative B is implemented. Thus, letting V_z denote the number of votes for alternative z, the outcome is A if and only if $V_A \geq \max\{M, V_B + 1\}$. We will only consider the special case where the number of B supporters in the population is commonly known (e.g., from pre-election polls) to be strictly smaller than M. So the behavior of the B supporters is irrelevant and alternative A is implemented if and only if it receives at least M votes.

The number of voters who prefer alternative A over alternative B is denoted N. They all receive a utility of 1 if A is implemented and a utility of 0 if B is implemented. Each voter i faces a cost c^i of participating in the election. The voting costs are independent and drawn randomly from a common distribution on $\mathbb{R}$, characterized by a cumulative distribution function F. The realized cost is private information, so that each voter knows only his own cost. The distribution function F is known to everyone (common knowledge).

Each voter can either vote early, vote late, or abstain. If there is no exit poll then no information is revealed during the election and the situation is obviously equivalent to simultaneous voting. If there is an exit poll then early votes are revealed before the second stage of voting and this is common knowledge at the beginning of the game. So we model an exit poll as complete revelation of all early votes. This is clearly a stylized way of modelling exit polls, but it is a reasonable starting point. Loosely speaking, by taking this extreme view of the quality of information revealed by an exit poll, we get results on the "upper limit" of its implications.

Each of the set-ups (with or without exit poll) represents a private information game and we use standard solution concepts. When there is no exit poll the game is static, so we use Bayesian Nash Equilibrium. With an exit poll the game is dynamic and we therefore use Perfect Bayesian Equilibrium. We will restrict attention to symmetric equilibria, i.e., we assume that all agents use the same strategy in equilibrium. Furthermore, we will assume that all agents use cut-off strategies. That is, we assume that there is a critical costs level, such that an agent i abstains if c^i exceeds the critical level, and votes if it is below it.[7] So in the simultaneous voting game an equilibrium can simply be described by a single cut-off cost $\bar{c}$. In the exit poll game an equilibrium can be described by a vector

$$c^* = (c_1^*, c_2^*(0), c_2^*(1), \ldots, c_2^*(M-1)),$$

where c_1^* is the cut-off cost in stage one and $c_2^*(n)$ is the cut-off cost in stage two if exactly $n \leq M-1$ agents voted in stage one.

If there is no exit poll then it is easily seen that the cut-off cost $\bar{c}$ is an equilibrium if and only if it is equal to the probability of some voter being pivotal given that all other voters use the $\bar{c}$ strategy.[8] Thus $\bar{c}$ is an equilibrium precisely if

$$\bar{c} = \binom{N-1}{M-1} F(\bar{c})^{M-1}(1-F(\bar{c}))^{N-M}. \tag{1}$$

Then consider the exit poll game. We want to find the conditions for c^* to be an equilibrium. First, suppose we are in stage two and that exactly $n \leq M-1$ voters voted in stage one. Agent i can then infer that the remaining $N-n-1$ potential voters must have costs in excess of the stage one cut-off, c_1^*. Then, by analogy with the no exit poll game, $c_2^*(n)$ must satisfy

$$c_2^*(n) = \binom{N-n-1}{M-n-1} \Pr(c^i \leq c_2^*(n) | c^i > c_1^*)^{M-n-1} \cdot$$
$$(1 - \Pr(c^i \leq c_2^*(n) | c^i > c_1^*))^{N-M}.$$

[7] In each of the specific cases we analyse below, it is straightforward to show that there is in fact no loss of generality in restricting attention to decision rules of this form.

[8] To see this, let V_A^{-i} denote the number of votes for A, excluding agent i's vote. The expected payoff from voting is then $\Pr(V_A^{-i} \geq M-1) - c^i$, while abstaining gives expected payoff $\Pr(V_A^{-i} \geq M)$. Voting therefore gives weakly higher expected payoff than abstaining if and only if $c^i \leq \bar{c} \equiv \Pr(V_A^{-i} = M-1)$. In a symmetric equilibrium all other agents also use the same cut-off strategy, and the probability that $V_A^{-i} = M-1$ is then equal to the binomial expression on the right-hand side of (1).

This is equivalent to $c_2^*(n) = 0$ or

$$\begin{aligned} c_2^*(n) &> c_1^* \text{ and} \\ c_2^*(n) &= \binom{N-n-1}{M-n-1}\left(\frac{F(c_2^*(n)) - F(c_1^*)}{1-F(c_1^*)}\right)^{M-n-1}\left(\frac{1-F(c_2^*(n))}{1-F(c_1^*)}\right)^{N-M}. \end{aligned} \quad (2)$$

The condition that must be satisfied by the stage one cut-off c_1^* is more complicated. Consider a voter i and assume that all other voters use the strategy c^* and that i himself will use the stage two cut-offs from c^* if he does not vote in stage one. If i votes in stage one then his expected utility is

$$\Pr(V_A^{-i} \geq M-1) - c_i,$$

where V_A^{-i} denotes the realized final number of votes for A from all other voters than i. His expected utility if he does not vote in stage one is much more complex because it depends on whether he will vote in stage two which again depends on the number of stage one votes from all other agents (and of course the expected utility also depends on the number of stage two votes from other potential voters not voting in stage one). Therefore, we will not write it out in detail, we simply note that the two expected utilities must be equal at the cut-off $c^i = c_1^*$. In the following section we will solve the model in several special cases and in each case we write down explicitly the equation that determines c_1^*.

3 Analysis

In this section we will solve the model described above when $(N, M) = (2, 1)$, $(2, 2)$, and $(3, 2)$. In each (N, M)-case we will compare the no exit poll solution to the exit poll solution. Clearly, the $N = 2$ cases are extremely simplistic. However, they are interesting because each of them contains in isolation one of two collective action issues that are relevant in real world elections. When $M = 1$ there is a free riding problem: Each voter (with a positive voting cost) wants alternative A to be implemented but prefers to stay home while the other person votes. Obviously, the free riding problem is not present when $M = 2$. In that case we instead have a coordination problem: Each voter (with a voting cost below one) wants to vote if and only if the other person votes as well. Therefore, the $N = 2$ cases provide us with valuable insights on the effects of exit polls when only the free riding problem or only the coordination problem is present. The case of $N = 3$, $M = 2$ is the simplest case in which both problems are present. So, even though this case is obviously still very simplistic as a model of real world elections, it takes us a substantial step in the right direction.

As described earlier, the voting cost of each voter is drawn from a common distribution on the real axis represented by a distribution function F. We will make some assumptions about the properties of F. First of all we will assume that F is twice differentiable on $[0,1]$ with $F' > 0$ and either $F'' \leq 0$ or $F'' \geq 0$ on this interval. The most restrictive of these assumptions is that the second derivative of F is either non-positive or non-negative. This means that, for example, F cannot be given by a density function f with mode in $(0,1)$. Furthermore, we will assume that $F(0) > 0$ and $F(1) < 1$. This means that there is a positive probability that a given voter will always vote (i.e., that his cost is negative) and also a positive probability that he wil never vote (i.e., that his cost is above one). All of these assumptions are made for existence and uniqueness purposes. In some cases they are substantially stronger than we need, but for simplicity we keep the assumptions throughout the section.

As an example of a voting cost distribution we will use the uniform distribution on some interval $[-\varepsilon_L, 1+\varepsilon_H]$, where $\varepsilon_L, \varepsilon_H > 0$. Such a distribution clearly satisfies all the assumptions above. Note that we can write F as $F(c) = F(0) + (F(1) - F(0))c$ for all c's in the support of the distribution. In fact, under the assumption of a uniform distribution we will treat the values $F(0)$ and $F(1)$ (with $0 < F(0) < F(1) < 1$) as parameters. When we do that it is of course important to remember that by changing $F(0)$ or $F(1)$ we change the support of the uniform distribution.[9] Finally, in the $N = 3$, $M = 2$ case we need to assume that F is given by a uniform distribution to get existence and uniqueness of equilibrium when there is no exit poll. This is necesssary to get meaningful comparisons between this situation and the exit poll situation.

3.1 $N = 2$, $M = 1$

Equation (1), the equation that must be satisfied for $\bar{c}$ to be an equilibrium in the no exit poll game, is in this case

$$\bar{c} = 1 - F(\bar{c}). \tag{3}$$

[9] ε_L and ε_H can of course be expressed in terms of $F(0)$ and $F(1)$ (and vice versa). By simple calculations we get

$$\varepsilon_L = \frac{F(0)}{F(1) - F(0)} \text{ and } \varepsilon_H = \frac{1 - F(1)}{F(1) - F(0)}.$$

The function $1 - F(c)$ is positive at $c = 0$, below one at $c = 1$, and decreasing. From these observatons it is easily seen that there exists precisely one equilibrium $\bar{c}$ which must be in the open interval between zero and one. Note that we can get this result with much weaker assumptions on F than the ones made above. Table 1 summarizes voter turnout (i.e., the total number of votes given) in the game with no exit poll for allcombinations of participation costs.

Table 1: Turnout without exit poll when $N = 2$, $M = 1$

$\downarrow$Voter 1, $\rightarrow$Voter 2	$c^2 \leq \bar{c}$	$\bar{c} < c^2$
$c^1 \leq \bar{c}$	2	1
$\bar{c} < c^1$	1	0

Then consider the exit poll game. First consider stage two when neither voter voted before the exit poll (otherwise stage two is obviously trivial). For $c^* = (c_1^*, c_2^*(0))$ to be an equilibrium, $c_2^*(0)$ must satisfy

$$c_2^*(0) = 1 - \Pr(c^j \leq c_2^*(0) | c^j > c_1^*).$$

If $c_2^*(0) \leq c_1^*$ then the conditional probability on the right-hand side of this equation is equal to zero, which means that the equation cannot be satisfied (under the assumption that $c_1^* < 1$, which is easily seen to be true in any equilibrium, since $F(1) < 1$). Thus, we must have $c_2^*(0) > c_1^*$ and the equation above can be written

$$c_2^*(0) = 1 - \frac{F(c_2^*(0)) - F(c_1^*)}{1 - F(c_1^*)},$$

or

$$c_2^*(0) = \frac{1 - F(c_2^*(0))}{1 - F(c_1^*)}.$$

Now consider stage one. We first claim that in equilibrium we must have $c_1^* \leq 0$. Suppose that we had an equilibrium with $c_1^* > 0$ and consider a voter i with $c^i \in (0, c_1^*)$. By definition, he would then vote in stage one. However, abstaining in stage one would actually ensure him a higher expected payoff. To see this, note that if no voters vote in stage one, then voter i would vote in stage two (since $c^i < c_1^* < c_2^*(0)$) and his payoff would be the same as if he had voted in stage one. But if the other voter does in fact vote in stage one (i.e., if his cost is below c_1^*) then alternative A would be implemented without voter i having to vote, so his payoff would be strictly higher than if he had voted in stage one. From this observation we immediately conclude that we cannot have $c_1^* > 0$ in equilibrium, so $c^* = (c_1^*, c_2^*(0))$ must satisfy

$$c_1^* \leq 0 \text{ and } c_2^*(0) = \frac{1 - F(c_2^*(0))}{1 - F(c_1^*)}.$$

A voter with $c^i \leq 0$ will clearly always vote, but is indifferent between voting early and voting late, since it only takes one vote for A to be implemented. However, by voting early he increases the expected utility of the other voter - he may save him a costly vote. So if we assume that a voter who is indifferent with respect to his own payoff will take the action that maximizes the expected payoff of the other voter then we must have $c_1^* = 0$. And then the conditions for c^* to be an equilibrium become

$$c_1^* = 0 \text{ and } c_2^*(0) = \frac{1 - F(c_2^*(0))}{1 - F(0)}. \tag{4}$$

It is easy to see that there is a unique $c_2^*(0)$ satisfying (4), and that it must be in the open interval between zero and one (mimick the argument for existence and uniqueness of an equilibrium in the no exit poll situation). Furthermore, $c_2^*(0)$ must be strictly higher than the no exit poll equilibrium cut-off, $\bar{c}$. To see this, simply note that, since $F(0) > 0$, we have $\frac{1-F(c)}{1-F(0)} > 1 - F(c)$ for all $c \in [0, 1]$, and thus the function $(1-F(c))/(1-F(0))$ must intersect the diagonal at a higher cost than the function $1 - F(c)$.[10] The intuition behind this result is straightforward: Consider the problem facing a voter i in stage two. He can infer that the other voter does not have a negative voting cost, since he did not vote in stage one. Thus, for a given cut-off strategy of the other voter, the probability that he votes is lower than in the no exit poll game. This makes free-riding less attractive and voter i will therefore vote for higher realizations of c^i.

Table 2 summarizes the total number of votes for all combinations of realized voting costs in the exit poll game.

Table 2: Turnout with exit poll when $N = 2$, $M = 1$

↓Voter 1, →Voter 2	$c^2 \leq 0$	$0 < c^2 \leq \bar{c}$	$\bar{c} < c^2 \leq c_2^*(0)$	$c^2 > c_2^*(0)$
$c^1 \leq 0$	2	1	1	1
$0 < c^1 \leq \bar{c}$	1	2	2	1
$\bar{c} < c^1 \leq c_2^*(0)$	1	2	2	1
$c^1 > c_2^*(0)$	1	1	1	0

[10] Even if $c_1^* < 0$ we get the same conclusion as long as $F(c_1^*) > 0$. However, note that the closer $F(c_1^*)$ is to zero, the closer is $c_2^*(0)$ to $\bar{c}$.

Comparing Table 1 and Table 2, we see that there is no general conclusion as to whether turnout is higher with or without the exit poll. For example, turnout is always higher with an exit poll when $\bar{c} < c^1 \leq c_2^*(0)$ and $c^2 > 0$, while it is the other way around when $0 < c^1 \leq \bar{c}$ and $c^2 \leq 0$. However, note that if the realized voting cost of each candidate is positive then the exit poll always leads to a level of turnout that is at least as high as without an exit poll and sometimes strictly higher. Also note that the exit poll situation "dominates" the no exit poll situation with respect to implementation of alternative A: If A is implemented without an exit poll then it is also implemented with an exit poll and sometimes it is implemented with an exit poll but not without it. This does not, however, mean that we have the same dominance with respect to efficiency (measured by simply aggregating the utility of the two voters). For some combinations of strictly positive voting costs, the introduction of an exit poll leads to two votes for A instead of one, which is obviously inefficient.

So, to sum up our analysis of this case, we have seen that when there is an exit poll, only voters with non-positive voting costs will vote in stage one. Intuitively, the anticipation of the exit poll induces potential voters to "wait and see": While there is nothing to lose from voting late rather than early, awaiting the result of the poll can potentially allow voters to free-ride on other supporters of alternative A who choose to vote early. Thus, the number of early votes is likely to be lower when an exit poll is introduced. However, if nobody votes in stage one, then voters find free-riding less attractive in stage two, and the number of late votes is then likely to be higher. These observations lead to the result that the introduction of an exit poll implies that alternative A is implemented for a strictly larger set of cost realizations for the two voters. In this sense, an exit poll mitigates the free riding problem that is the central aspect of this case.

3.2 $N = M = 2$

This case highlights another collective action problem, namely a coordination problem: For voters with participation cost between zero and one, voting is preferable to abstaining if and only if the other voter also participates. A voter's probability of being pivotal in the no exit poll game is now equal to the probability that the other voter votes. So the condition for $\bar{c}$ to be an equilibrium is

$$\bar{c} = F(\bar{c}). \tag{5}$$

Since $F(0) > 0$, $F(1) < 1$, and F' is either non-increasing or non-decreasing in the interval $[0, 1]$ (we have assumed that F'' is either non-negative or non-positive), it easily follows that F intersects the diagonal precisely once and that this intersection happens in $(0,1)$. Thus we have a unique equilibrium $\bar{c} \in (0,1)$ of the simultaneous (no exit poll) voting game. Voter turnout for different combinations of participation costs are as in Table 1, except that $\bar{c}$ is now defined as the solution to equation (5), rather than (3).

Then consider the exit poll game. In stage two, if one of the voters contributed in stage one then the other voter knows he is pivotal. Thus he will vote if his cost is not above one, i.e., we have $c_2^*(1) = 1$. If nobody voted before the exit poll, then voters in stage two play a game that is very similar the no exit poll game. The only difference is that each voter's belief about the cost of the other voter is different, because he can infer from the stage one actions that it must be above c_1^*. So $c_2^*(0)$ must satisfy

$$c_2^*(0) = \Pr(c^j \leq c_2^*(0) | c^j > c_1^*),$$

which is equivalent to

$$c_2^*(0) = \frac{\Pr(c_1^* < c^j \leq c_2^*(0))}{\Pr(c_1^* > c^j)},$$

or

$$c_2^*(0) = \max\{0, \frac{F(c_2^*(0)) - F(c_1^*)}{1 - F(c_1^*)}\}. \tag{6}$$

Now consider stage one. Voter i's payoff from voting in this stage is (let j denote the other voter)

$$1 - \Pr(c^j > 1) - c^i.$$

His payoff from not voting early is (assuming that j follows the strategy c^* and that i follows c^* in stage two)

$$\begin{array}{rl} 1 - \Pr(c^j > \max\{c_1^*, c_2^*(0)\}) - c^i & \text{if } c^i \leq c_2^*(0) \\ \Pr(c^j \leq c_1^*)(1 - c^i) & \text{if } c_2^*(0) < c^i \leq 1 \\ 0 & \text{if } c^i > 1 \end{array}$$

It is easily seen that in any equilibrium we must have $c_1^*, c_2^*(0) < 1$ (since $F(1) < 1$). And then it immediately follows from the expressions above that voting early dominates not voting early for voter i if $c^i \leq c_2^*(0)$. That is, if $c^i \leq c_2^*(0)$, then it must also be true that $c^i \leq c_1^*$. This means that we must have $c_1^* \geq c_2^*(0)$, and it then follows from equation (6) that $c_2^*(0) = 0$. So we have that, in equilibrium, if neither voter votes in stage one then they will also not vote in stage two. We also see that the equilibrium condition for c_1^* is

$$1 - c_1^* - \Pr(c^j > 1) = \Pr(c^j \leq c_1^*)(1 - c_1^*),$$

which is equivalent to

$$F(1) - c_1^* = F(c_1^*)(1 - c_1^*).$$

Solving this equation we get

$$c_1^* = \frac{F(1) - F(c_1^*)}{1 - F(c_1^*)}.$$

So, summing up, $c^* = (c_1^*, c_2^*(0), c_2^*(1))$ is an equilibrium if and only if

$$c_1^* = \frac{F(1) - F(c_1^*)}{1 - F(c_1^*)}, \; c_2^*(0) = 0, \text{ and } c_2^*(1) = 1.$$

By standard arguments we see that there is a unique solution $c_1^* \in (0, 1)$ to the first equation, so there is a unique equilibrium in the exit poll game.

Our next step is to compare the outcome of the exit poll situation with the outcome when there is no exit poll. We split the analysis into two cases: $c_1^* \geq \bar{c}$ and $c_1^* < \bar{c}$. We shall later present a condition on F that determines which of the two cases that is relevant.

First, consider the case $c_1^* \geq \bar{c}$. Voter turnout as a function of the realized costs of the voters (c^1 and c^2) is summarized in Table 3. To ease comparisons with the game without an exit poll, we illustrate the corresponding voter turnouts in this game in Table 4.

Table 3: Turnout with exit poll when $N = M = 2$ and $c_1^* \geq \bar{c}$

↓Agent 1, →Agent 2	$c^2 \leq \bar{c}$	$\bar{c} < c^2 \leq c_1^*$	$c_1^* < c^2 \leq 1$	$c^2 > 1$
$c^1 \leq \bar{c}$	2	2	2	1
$\bar{c} < c^1 \leq c_1^*$	2	2	2	1
$c_1^* < c^1 \leq 1$	2	2	0	0
$c^1 > 1$	1	1	0	0

Table 4: Turnout without exit poll when $N = M = 2$ and $c_1^* \geq \bar{c}$

↓Agent 1, →Agent 2	$c^2 \leq \bar{c}$	$\bar{c} < c^2 \leq c_1^*$	$c_1^* < c^2 \leq 1$	$c^2 > 1$
$c^1 \leq \bar{c}$	2	1	1	1
$\bar{c} < c^1 \leq c_1^*$	1	0	0	0
$c_1^* < c^1 \leq 1$	1	0	0	0
$c^1 > 1$	1	0	0	0

The tables illustrate that with an exit poll the turnout is always at least as high as when there is no exit poll and often higher. It is also easy to see that we have an analogous domination result with respect to the implementation of A: If A is implemented without an exit poll, then it is also implemented with an exit poll and there are many combinations of costs such that it is implemented with an exit poll but not without it. Also note that we have domination in the opposite direction, no exit poll dominates exit poll, with respect to the outcome that no votes are cast.

Finally, the tables show that the exit poll improves efficiency (aggregate utility) for many cost combinations. However, if $c^1 > 1$ and $\bar{c} < c^2 \leq c_1^*$, and in the symmetric situation where $c^2 > 1$ and $\bar{c} < c^1 \leq c_1^*$, the exit poll leads to lower effciency (one instead of zero votes). So there is no domination result with respect to efficiency.

Then consider the case $c_1^* < \bar{c}$. Voter turnout for all combinations of participation costs is summarized in Table 5, For each combination, the corresponding turnout in the game with no exit poll is shown in Table 6.

Table 5: Turnout with an exit poll when $N = M = 2$ and $c_1^* < \bar{c}$

$\downarrow$Agent 1, $\rightarrow$Agent 2	$c^2 \leq c_1^*$	$c_1^* < c^2 \leq \bar{c}$	$\bar{c} < c^2 \leq 1$	$c^2 > 1$
$c^1 \leq c_1^*$	2	2	2	1
$c_1^* < c^1 \leq \bar{c}$	2	0	0	0
$\bar{c} < c^1 \leq 1$	2	0	0	0
$c^1 > 1$	1	0	0	0

Table 6: Turnout without an exit poll when $N = M = 2$ and $c_1^* < \bar{c}$

$\downarrow$Agent 1, $\rightarrow$Agent 2	$c^2 \leq c_1^*$	$c_1^* < c^2 \leq \bar{c}$	$\bar{c} < c^2 \leq 1$	$c^2 > 1$
$c^1 \leq c_1^*$	2	2	1	1
$c_1^* < c^1 \leq \bar{c}$	2	2	1	1
$\bar{c} < c^1 \leq 1$	1	1	0	0
$c^1 > 1$	1	1	0	0

In this case we have that for some combinations of costs the exit polls leads to higher turnout, for other combinations it is the other way around. So there is no domination result with respect to turnout. The same is true with respect to implementation of A and efficiency. However, it is easy to see that, with the exit poll, the set of combinations of positive costs for which exactly one voter votes is smaller than if there was no exit poll. So in this sense the introduction of an exit poll leads to a decrease in "wasteful voting", which is at the heart of the coordination problem that this simple case highlights.

As is clear from the analysis above, the conclusions about the effects of an exit poll in this case is highly dependent on whether $c_1^* \geq \bar{c}$ or $c_1^* < \bar{c}$. If the first inequality holds then an exit poll has an unambiguous positive effect on the level of turnout and the implementation of A. If the second inequality holds then the effect of an exit poll on turnout and implementation of A are ambiguous, i.e., positive for some combinations of realized voting costs and negative for others. Therefore, it is important to find out when we have $c_1^* \geq \bar{c}$ and when it is the opposite inequality that holds.

We first show the following lemma.

Lemma 1

$$c_1^* \gtreqless \bar{c} \iff \bar{c} \lesseqgtr 1 - \sqrt{1 - F(1)}.$$

Proof. See the appendix.

Suppose that the distribution of voting costs is uniform on some interval containing $[0, 1]$ such that we can write $F(c) = F(0) + (F(1) - F(0))c$ for $c \in [0, 1]$. Then, using the lemma above, we immediately get a condition on $F(0)$ and $F(1)$ which determines whether $c_1^* \geq \bar{c}$ or $c_1^* < \bar{c}$. Because with the uniform distribution a straightforward calculation shows that

$$\bar{c} = \frac{F(0)}{1 - (F(1) - F(0))}.$$

And then, by a bit of algebra, we see that $\bar{c} \lesseqgtr 1 - \sqrt{1 - F(1)}$ is equivalent to

$$F(0) \lesseqgtr (\sqrt{1 - F(1)})(1 - \sqrt{1 - F(1)}).$$

Finally, note that if $F'' > 0$ (on $[0, 1]$) then F must intersect the diagonal at a lower cost than if we had a uniform distribution with the same $F(0)$ and $F(1)$ values (as illustrated in Figure 1 below), which means that $\bar{c}$ must be lower when $F'' > 0$ than when $F'' = 0$. It then follows from the arguments above and Lemma 1 that if

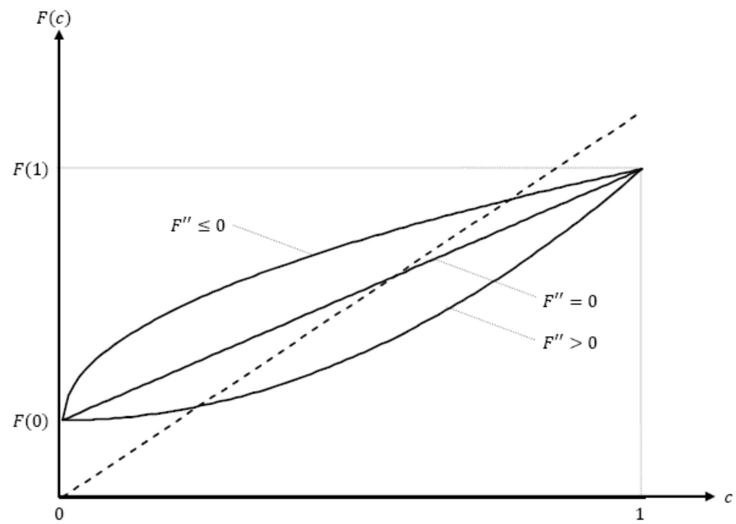
$$F'' \geq 0 \text{ and } F(0) \leq (\sqrt{1 - F(1)})(1 - \sqrt{1 - F(1)})$$

then we have $c_1^* \geq \bar{c}$. Analogously, if

$$F'' \leq 0 \text{ and } F(0) > (\sqrt{1-F(1)})(1-\sqrt{1-F(1)})$$

then we have $c_1^* < \bar{c}$.

Figure 1: $\bar{c}$ when $F'' < 0$, $F'' = 0$ and $F'' > 0$



We sum up our results in the following proposition.

Proposition 2 (The effects of an exit poll when $N = M = 2$)

1. *If the distribution function F satisfies*

$$F'' \geq 0 \text{ (on } [0,1]) \text{ and } F(0) \leq (\sqrt{1-F(1)})(1-\sqrt{1-F(1)})$$

then the equilibrium period one cut-off cost in the exit poll game will be equal to or greater than the equilibrium cut-off cost in the no exit poll game ($c_1^* \geq \bar{c}$). This implies that an exit poll has unambiguous positive effects on the level of turnout and the implementation of A.

2. *If the distribution function F satisfies*

$$F'' \leq 0 \text{ (on } [0,1]) \text{ and } F(0) > (\sqrt{1-F(1)})(1-\sqrt{1-F(1)})$$

then the equilibrium period one cut-off cost in the exit poll game will be lower than the equilibrium cut-off cost in the no exit poll game ($c_1^* < \bar{c}$). This implies that an exit poll does not have an unambiguous effect on the level of turnout or the implementation of A (for some combinations of realized voting costs the effect is positive, for others it is negative). However, an exit poll has an unambiguous negative effect on "wasteful voting", i.e., there are fewer combinations of positive voting costs such that precisely one voter votes.

Let us explore some of the intuition behind the results above. For simplicity, assume that $F'' = 0$, i.e., consider a uniform distribution of voting costs. We are especially interested in why $c_1^* \geq \bar{c}$ if $F(0)$ is sufficiently low, while we have the opposite inequality when it is not. Therefore, we hold $F(1)$ fixed. This of course means that a change in $F(0)$ corresponds to a change in both the upper and lower limit of the support of the uniform cost distribution.

An exit poll influences a voter's incentive to vote early in two ways. First, the exit poll gives rise to a "first-mover effect", which stimulates early voting: If agent i votes early then he might induce the other voter to vote as well. Intuitively, if the other player observes that player i voted in stage one, he will know with certainty that he is pivotal in stage two, which makes voting more attractive. Second, the exit poll produces a "wait-and-see effect" that discourages early voting. This effect parallels the wait-and-see effect from the $N = 2$, $M = 1$ case studied in the previous section, but now the incentive to wait and see comes from a desire to avoid casting a costly but useless vote, and not from a desire to free-ride on the other voter. If he abstains in stage one, voter i will choose to vote late if and only if his vote is pivotal, i.e., if and only if the other agent voted early.

How does the size of $F(0)$ affect the relative strengths of these two opposite effects? As $F(0)$ gets larger it becomes less likely that the first-mover effect from an early vote will kick in. This is because, for any $c \in (0, 1)$, the probability that the other voter has a cost in the interval $(c, 1)$ becomes lower. Therefore, it becomes less likely that an early vote from agent i will make another agent switch from abstention to late participation. On the other hand, an increase in $F(0)$ makes it more attractive to wait and see in stage one, since the probability that the other voter votes early is now higher. In total, these observations imply that as $F(0)$ rises, not voting in stage one becomes relatively more attractive than voting. And that explains the role of $F(0)$ in the results in the proposition.

The role of $1 - F(1)$ in the results is more subtle. An increase in $1 - F(1)$ (we now hold $F(0)$ fixed) makes it less likely that an early vote will make the other voter vote in stage two, thus weakening the first-mover effect. But the effect on the attractiveness of waiting is also negative because it becomes less likely that the other voter will vote early. Thus, the relative effect of a change in $1 - F(1)$ cannot be determined from these intuitive arguments. And indeed, it follows from the proposition that the effect can go both ways, the expression $(\sqrt{1 - F(1)})(1 - \sqrt{1 - F(1)})$ is not monotone with respect to $1 - F(1)$.

We end the analysis of this case by considering the simple example where the cost distribution is given by the uniform distribution on $[-\varepsilon, 1+\varepsilon]$ for some $\varepsilon > 0$. Thus we assume that the probability that a voter will always vote is the same as the probability that he will never vote: $F(0) = 1 - F(1)$. In this case the condition

$$F(0) \leq (\sqrt{1 - F(1)})(1 - \sqrt{1 - F(1)})$$

is equivalent to

$$F(0) = 1 - F(1) \leq \frac{1}{4}.$$

So with this type of cost distribution we have that $c_1^* \geq \bar{c}$, and thus that an exit poll has an unambiguous positive effect on turnout and the implementation of A, if and only if there is at least a fifty percent chance that a given voter has a voting cost between zero and one.

3.3 $N = 3$, $M = 2$

Our final analysis combines the insights from the previous two analyses. With three potential voters and two votes required for implementation, the free riding problem and the coordination problem are both relevant. A voter is now pivotal if precisely one of the other two agents votes. Thus $\bar{c}$ is an equilibrium in the no exit poll game if and only if

$$\bar{c} = 2F(\bar{c})(1 - F(\bar{c})).$$

As mentioned earlier, we will in this case assume that the distribution of voting costs is given by a uniform distribution on some interval containing $[0, 1]$. So we have $F(c) = F(0) + (F(1) - F(0))c$ for all c's in the support of the distribution. It is easy to see that, with this assumption, the function on the right hand side of the equilibrium equation above becomes a second order polynomial with negative second derivative. Furthermore, note that the polynomial (we simply refer to the variable as c) is positive at $c = 0$ and that its maximum value is $\frac{1}{2}$. From these observations it follows easily that there is precisely one equilibrium $\bar{c}$ which must be in the open interval between zero and one. It is straightforward to solve explicitly for the equilibrium, we simply have to find the largest root of a second order polynomial. The explicit solution, which is of course a function of $F(0)$ and $F(1)$, can be found in the appendix.

Then consider the exit poll game. In this game an equilibrium is written $c^* = (c_1^*, c_2^*(0), c_2^*(1))$. The following proposition establishes that there always exists an equilibrium. Note that our assumption about a uniform cost distribution is not necessary for these results to hold, only our general assumptions about the cost distribution are used in the proof.

Proposition 3 (Existence of exit poll equilibrium when $N = 3$, $M = 2$) *Let $c_1^*, c_2^*(1) \in (0, 1)$. Then $c^* = (c_1^*, 0, c_2^*(1))$ is an equilibrium of the exit poll game if and only if the following two equations are satisfied:*

$$c_1^* = \frac{(1 - F(c_1^*))^2 - (1 - F(c_2^*(1)))^2}{F(c_1^*)^2 + (1 - F(c_1^*))^2}$$

and

$$c_2^*(1) = \frac{1 - F(c_2^*(1))}{1 - F(c_1^*)}.$$

There exists a solution $c_1^*, c_2^*(1)$ (with $c_1^*, c_2^*(1) \in (0, 1)$) to this pair of equations. All solutions satisfy $c_1^*, \bar{c} < c_2^*(1)$.

Proof. See the appendix.

If we have an equilibrium of the exit poll game with $c_1^* \geq \bar{c}$ then it is easy to check that the turnout will always be at least as high as in the no exit poll equilibrium and sometimes higher. Furthermore, it is also easy to check that we have the same domination result with respect to implementation of A. On the other hand, if we have an equilibrium with $c_1^* < \bar{c}$ and $c_2^*(0) = 0$ then there are cost realizations such that the turnout is zero in this equilibrium while it is at least two without an exit poll. For example, consider a combination of costs where $c^i \in (c_1^*, \bar{c})$ for all voters i. Then nobody will vote in the exit poll equilibrium while all voters would vote if there were no exit poll. In Proposition 4 we find a condition that guarantees the existence of an exit poll equilibrium that dominates the no exit poll equilibrium with respect to turnout and implementation of A. When this condition is not satisfied, then there exists an exit poll equilibrium with zero turnout for some cost realizations that would give a turnout of at least two if there were no exit poll. Finally, we also show that in any exit poll equilibrium there are cost realizations such that the turnout is at least two while it would be zero without an exit poll. So the no exit poll equilibrium cannot dominate an exit poll equilibrium with respect to turnout or implementation of A. Note that this holds for *all* equilibria of the exit poll game, not only equilibria with $c_2^*(0) = 0$.

For the proposition we need the following definition. Let $\underline{c}_2(\bar{c}) \in (0, 1)$ be the unique number given by the equation

$$\underline{c}_2(\bar{c}) = \frac{1 - F(\underline{c}_2(\bar{c}))}{1 - F(\bar{c})}.$$

This is the equation for the cut-off strategy $\underline{c}_2(\bar{c})$ to be optimal in stage two after one vote in stage one, given that all voters used the cut-off strategy $\bar{c}$ in stage one and that the other remaining voter also uses the $\underline{c}_2(\bar{c})$ strategy in stage two.

Proposition 4 (The effects of an exit poll when $N = 3$, $M = 2$)

1. *Suppose the following condition is satisfied:*

$$(1 - F(\bar{c}))^2 - (1 - F(\underline{c}_2(\bar{c})))^2 \geq \bar{c}(1 - \bar{c}).$$

   Then there exists an equilibrium of the exit poll game with $c_1^* \geq \bar{c}$ (and $c_2^*(0) = 0$), i.e., an equilibrium that dominates the no exit poll equilibrium with respect to both turnout and implementation of A.

2. *Suppose that the condition from part one is not satisfied, i.e., that*

$$(1 - F(\bar{c}))^2 - (1 - F(\underline{c}_2(\bar{c})))^2 < \bar{c}(1 - \bar{c}).$$

   Then there exists an equilibrium of the exit poll game with $c_1^* < \bar{c}$ (and $c_2^*(0) = 0$). For such an equilibrium there are cost realizations such that turnout is zero while it would be at least two if there were no exit poll.

3. *For any equilibrium of the exit poll game there exist cost realizations such that the turnout is at least two while it would be at most one if there were no exit poll.*

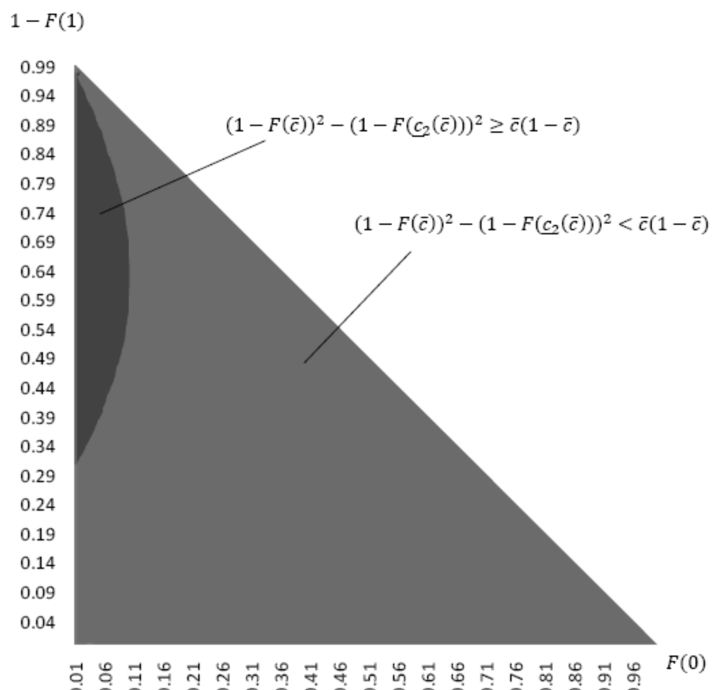
Proof. See the appendix.

Because of our assumption about uniform distribution of voting costs it is straightforward to explicitly write $\underline{c}_2(\bar{c})$ as a function of $\bar{c}$:

$$\underline{c}_2(\bar{c}) = \frac{1 - F(0)}{1 - F(0) + (F(1) - F(0))(1 - \bar{c})}.$$

And thus, given that we have already found an explicit expression for $\bar{c}$ in terms of $F(0)$ and $F(1)$ we can in principle write the conditions from part one and two of Proposition 4 as conditions on $F(0)$ and $F(1)$. However, the expressions on either side of the inequalities are complicated and it seems impossible to analytically derive insights from them. So instead we nummerically calculate each side of the inequalities. And then, treating $F(0)$ and $1-F(1)$ as parameters, we plot whether it is the condition from part one or the condition from part two that holds. Note that we use $1-F(1)$ instead of $F(1)$ as a parameter because then our parameters are, respectively, the probability that a voter will always vote and the probability that a voter will never vote. For any feasible combination of $F(0)$ and $1-F(1)$, figure 2 below illustrates which of the inequalities in Proposition 4 is satisfied.

Figure 2: The effects of an exit poll when $N = 3$, $M = 2$



We see that if the probability that a voter will always vote is above (roughly) ten percent then the condition from part two of the proposition is satisfied no matter what the probability that a voter will never vote is. Similarly, if the probability that a voter will never vote is below (roughly) thirty percent then it is also the case that the condition from part two is satisfied (no matter what the value of

$F(0)$ is). So in a large region of the $F(0), 1 - F(1)$ parameter space we can only conclude that the introduction of an exit poll *can* have both positive and negative effects on turnout and the implementation of A. The sign of the effect depends on the realizations of voting costs. Only in a much smaller subset of the parameter space - in which $F(0)$ is below ten percent and $1 - F(1)$ is above thirty percent - can we guarantee the existence of an equilibrium such that the introduction of an exit poll has an unambiguous positive effect on the level of turnout and the implementation of A.

The intuition behind these results is similar to the one explained in the case of $N = M = 2$. A larger value of $F(0)$ means that there is a higher probability that the other players will vote early (not because of strategic considerations, but for example because they enjoy fulfilling their "civic duty"). This makes it more attractive for a player with a positive participation cost to await the result of the exit poll before going to the polls, since this may allow her to free-ride on the other players, or at least avoid wasting a costly vote on a losing candidate. As a result, the incentive to vote early falls. When $F(0)$ becomes suffiently large, we can therefore guarantee the existence of an equilibrium such that the exit poll lowers the incentive to vote early ($c_1^* < \bar{c}$), and the number of voters participating may be lower than if there had been no poll.

We end our analysis with an important example. Suppose that the parameters $F(0)$ and $1 - F(1)$ are such that the inequality in part two of proposition 4 is satisfied (the light grey shaded area in Figure 2). We then know that there exists an equilibrium with $c_1^* < \bar{c} < c_2^*(1)$ and $c_2^*(0) = 0$. Assume that the three voters face cost realizations such that $c^1 \leq c_1^*$, $c^2 \in (c_1^*; \bar{c})$, and $c^3 > c_2^*(1)$. With this combination of costs, voter 1 will vote early, voter 2 will vote late, and voter 3 will abstain. Alternative A is therefore implemented with a minimal margin of victory. To the casual observer, this outcome may suggest the following interpretation: "The release of the exit poll raised the incentive to vote (since $c_2^*(1) > c_1^*$). This led player 2, whose vote was pivotal to the outcome of the election, to cast a late vote. Without the exit poll, alternative A would therefore not have been implemented." This interpretation is wrong. In the absence of an exit poll, voter 1 and voter 2 would both have voted (since $c^1, c^2 < \bar{c}$) and the outcome of the election would in fact have been the same. The problem with the erroneous interpretation is that it presupposes that voter 2's behavior before the release of the poll is indicative of how she would have behaved if there were no exit poll. In doing so, it ignores that it is in fact the exit poll itself that, through the wait-and-see effect, causes voter 2 to abstain in stage one.

4 Conclusion

This paper has studied the impact of exit polls in elections with two alternatives, where the alternative favored by a majority in the population must receive a certain number of votes in order to win. We have shown that the introduction of an exit poll influences the incentive to vote both before and after the results of the poll are released. Before the exit poll is released, potential voters may find it worthwhile to await its result before they decide on whether to stay home or go to the polls. That way, they may hope to free ride on other voters who support the same alternative, or they may avoid wasting time and effort on participating in the election if their preferred alternative is bound to lose anyway. This effect thus discourages early voting. On the other hand, supporters of the alternative favored by a majority may also use the exit poll actively to coordinate their efforts: By influencing the result of the exit poll, early voters can induce fellow supporters to vote after the result of the poll is revealed, and this may stimulate early voting. In sum, the total effect of an exit poll on the incentive to vote early is ambiguous. Once the results of the exit poll is released, the effect on remaining potential voters' incentive to participate depends on the information revealed by the poll. Voting becomes more attractive if the poll reveals a close race, but less attractive if it reveals the opposite. As a result of these opposite effects, we find that an exit poll's effect on voter turnout and election outcomes is ambiguous.

Much of the skepticism towards exit polls comes from the belief that such polls may change the outcome of the election, a possibility that some commentators consider undemocratic. While we certainly agree that exit polls have the potential to change election outcomes, we also believe that the empirical case for this hypothesis is sometimes overstated. This was for example the case, we believe, in the debate following the Danish referendum in June 2009. The problem with this debate was that it did not recognize the possibility that voter behavior was influenced by exit polls not only *after*, but also *before* the release of the first exit poll result. Hence, the low turnout before the release of the first poll was interpreted as an indicator for what would have happened later in the day, had the results of the exit poll not been published. In reality, however, the low early turnout itself could very well be a direct consequence of the exit poll if voters postponed voting until after the results of the poll were revealed.

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5 Appendix

Proof of Lemma 1.
First note that the functions $F(c)$ and $\frac{F(1)-F(c)}{1-F(c)}$ intersects at precisely one cost $\check{c} \in (0,1)$. By solving a second order equation we get that $F(\check{c}) = 1 - \sqrt{1 - F(1)}$.

Now suppose that $c_1^* > \bar{c}$. Since $\frac{F(1)-F(c)}{1-F(c)}$ is decreasing and $F(c)$ is increasing this is equivalent to these two functions intersecting below the 45° line, which is obviously equivalent to $F(\check{c}) < \check{c}$. Further, since F is increasing and intersects the diagonal precisely once, this is equivalent to

$$\bar{c} = F(\bar{c}) < F(\check{c}) = 1 - \sqrt{1 - F(1)}.$$

Thus we have shown that

$$c_1^* > \bar{c} \iff \bar{c} < 1 - \sqrt{1 - F(1)}.$$

The arguments showing that

$$c_1^* = \bar{c} \iff \bar{c} = 1 - \sqrt{1 - F(1)}$$

and

$$c_1^* < \bar{c} \iff \bar{c} > 1 - \sqrt{1 - F(1)}$$

are completely analogous. □

Equilibrium in the no exit poll game when $N = 3$, $M = 2$.
CLAIM: In the $N = 3$, $M = 2$ case (where we assume that the cost distribution is uniform), the unique equilibrium of the no exit poll game is

$$\bar{c} = \frac{2(F(1) - F(0))(1 - 2F(0)) - 1}{4(F(1) - F(0))^2} + \frac{\sqrt{(2(F(1) - F(0)) - 1)^2 + 8F(0)(F(1) - F(0))}}{4(F(1) - F(0))^2}.$$

Proof. Remember that the equilibrium condition is

$$\bar{c} = 2F(\bar{c})(1 - F(\bar{c})).$$

Since the cost distribution is uniform we have $F(c) = F(0) + (F(1) - F(0))c$ (for c's in the support of the distribution) and thus the equilibrium condition becomes

$$\bar{c} = 2(F(0) + (F(1) - F(0))\bar{c})(1 - F(0) - (F(1) - F(0))\bar{c}),$$

which, by straightforward calculations, becomes

$$\begin{aligned} 2(F(1) - F(0))^2\bar{c}^2 \\ + (4F(0)(F(1) - F(0)) - 2(F(1) - F(0)) + 1)\bar{c} \\ - 2F(0)(1 - F(0)) = 0. \end{aligned}$$

The equilibrium is the positive solution to this second order equation. By solving for this solution we get the expression from the claim. □

Proof of Proposition 3.
First, suppose $c^* = (c_1^*, 0, c_2^*(1))$ is an equilibrium of the exit poll game such that $c_1^*, c_2^*(1) \in (0, 1)$. It is easy to see that then we must have

$$c_2^*(1) = \Pr(c^j > c_2^*(1) | c^j > c_1^*).$$

If $c_2^*(1) \leq c_1^*$ the probability on the right-hand side of this expression is equal to 1. But since $c_2^*(1) < 1$ we must then have

$$c_2^*(1) > c_1^* \text{ and } c_2^*(1) = \frac{1 - F(c_2^*(1))}{1 - F(c_1^*)}.$$

To get the equation for c_1^*, first note that if some voter i votes in stage one then his expected payoff is

$$\begin{aligned} 1 - \Pr(c^j > c_2^*(1))^2 - c^i &= 1 - (1 - F(c_2^*(1)))^2 - c^i \\ &= F(c_2^*(1))(2 - F(c_2^*(1)) - c^i. \end{aligned}$$

If $0 < c^i \leq c_2^*(1)$ then his payoff from not voting in stage one is[11]

$$\begin{aligned} \Pr(c^j \leq c_1^*)^2 + 2\Pr(c^j \leq c_1^*)(1 - \Pr(c^j \leq c_1^*))(1 - c^i) \\ = F(c_1^*)^2 + 2F(c_1^*)(1 - F(c_1^*))(1 - c^i). \end{aligned}$$

[11] If $c^i > c_2^*(1)$ it is

$$\Pr(c^j \leq c_2^*(1))^2$$

and if $c^i \leq 0$ it is

$$\Pr(c^j \leq c_1^*)^2 + 2\Pr(c^j \leq c_1^*)(1 - \Pr(c^j \leq c_1^*)) - c^i.$$

But since we already know that $c_2^*(1) > c_1^* > 0$ we do not need to consider these cases.

Thus c_1^* must satisy

$$F(c_2^*(1))(2 - F(c_2^*(1)) - c_1^* = F(c_1^*)^2 + 2F(c_1^*)(1 - F(c_1^*))(1 - c_1^*).$$

This equation is easily seen to be equivalent to

$$c_1^* = \frac{F(c_2^*(1))(2 - F(c_2^*(1)) - F(c_1^*)(2 - F(c_1^*))}{1 - 2F(c_1^*)(1 - F(c_1^*))}.$$

From this equation we easily get the equation from the proposition.

Then suppose that we have $c_1^*, c_2^*(1) \in (0, 1)$ such that the equations are satisfied (note that the equation for c_1^* implies that $c_1^* < c_2^*(1)$) . Consider the strategy $c^* = (c_1^*, 0, c_2^*(1))$. From the arguments above it easily follows that c_1^* and $c_2^*(1)$ are optimal cut-off costs at the history in the game where they are used, given that the other voters use c^* (and, of course, given that beliefs are updated correctly in stage two). Also, since $c_1^* > 0$, it is easy to see that $c_2^*(0) = 0$ is optimal in stage two if there is no early votes. So when the equations from the proposition are satisfied, $c^* = (c_1^*, 0, c_2^*(1))$ is indeed an equilibrium.

Next, we will show that we always have existence of a solution $c_1^*, c_2^*(1) \in (0, 1)$ to the equations. First, for each $c_1 \in [0, 1]$ let $\underline{c}_2(c_1)$ be the unique solution to the equation

$$\underline{c}_2(c_1) = \frac{1 - F(\underline{c}_2(c_1))}{1 - F(c_1)}.$$

Then $\underline{c}_2$ is a continuous function of c_1 on $[0, 1]$ and we have $\underline{c}_2(c_1) > c_1$ for all $c_1 \in [0, 1)$ (note that $\underline{c}_2(1) = 1$).

Now consider the equation

$$c_1^* = \frac{(1 - F(c_1^*))^2 - (1 - F(\underline{c}_2(c_1^*)))^2}{F(c_1^*)^2 + (1 - F(c_1^*))^2}.$$

The right hand side of this equation (considered as a function of $c_1^* \in [0, 1]$) is positive at $c_1^* = 0$, zero at $c_1^* = 1$, and continuous. Thus it must intersect the diagonal at least once between zero and one, so we have at least one solution in $(0, 1)$ to this equation. Pick a solution c_1^* and let $c_2^*(1) = \underline{c}_2(c_1^*)$. Then we have a solution to the equations from the proposition and the solution obviously satisfies $c_1^* < c_2^*(1)$.

Finally, it remains to be shown that $c_2^*(1) > \bar{c}$. Since $\bar{c} = 2F(\bar{c})(1 - F(\bar{c})) \leq \frac{1}{2}$, it suffices to show that $c_2^*(1) > \frac{1}{2}$. We have that $c_2^*(1)$ satisfies

$$c_2^*(1) = \frac{1 - F(c_2^*(1))}{1 - F(c_1^*)}.$$

Thus $c_2^*(1)$ is given as the intersection between the function

$$\frac{1 - F(\cdot)}{1 - F(c_1^*)}$$

and the diagonal. This function is a straight line (because F is linear) with a negative slope, is above one at zero, and above zero at one. So it is easy to see that its intersection with the diagonal must happen at a cost above one half. $\square$

Proof of Proposition 4.
1. Since $\bar{c} = 2F(\bar{c})(1 - F(\bar{c})) = 1 - F(\bar{c})^2 + (1 - F(\bar{c}))^2$, the inequality can be rewritten as

$$\bar{c} \leq \frac{(1 - F(\bar{c}))^2 - (1 - F(\underline{c}_2(\bar{c})))^2}{F(\bar{c})^2 + (1 - F(\bar{c}))^2}.$$

Consider, for a moment, the right hand side as a function on $[0, 1]$. At one it is equal to zero and thus below the diagonal so (because of continuity) there must exist a $c_1^* \in [\bar{c}, 1)$ with

$$c_1^* = \frac{(1 - F(c_1^*))^2 - (1 - F(\underline{c}_2(c_1^*)))^2}{F(c_1^*)^2 + (1 - F(c_1^*))^2}.$$

And thus (see Proposition 3 and its proof) we have an exit poll equilibrium with $c_1^* \geq \bar{c}$ (and $c_2^*(0) = 0$).

2. Analogously to above, it follows from the inequality in the proposition that there exists a $c_1^* \in (0, \bar{c})$ such that

$$c_1^* = \frac{(1 - F(c_1^*))^2 - (1 - F(\underline{c}_2(c_1^*)))^2}{F(c_1^*)^2 + (1 - F(c_1^*))^2}.$$

And thus we have an exit poll equilibrium with $c_1^* < \bar{c}$ (and $c_2^*(0) = 0$).

3. Suppose $c^* = (c_1^*, c_2^*(0), c_2^*(1))$ is an exit poll equilibrium. Analogously to the proof of proposition 3 it can be shown that $c_2^*(1) > \bar{c}$. Any cost realization satisfying, for example, $c^1 < c_1^*$, $\bar{c} < c^2 \leq c_2^*(1)$, and $c^3 > 1$ would result in two votes if there is an exit poll and at most one vote if there is not (one if $c^1 \leq \bar{c}$, zero if $c^1 > \bar{c}$). $\square$