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# Capacity Constraint, Price Discrimination, and Oligopoly

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# Capacity Constaint, Price Discrimination, and Oligopoly

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**Abstract**

In the presence of market power in oligopolistic environment, price discrimination is a natural phenomenon. Surprisingly this setting has not been analyzed in depth in the literature. In contrast with existing literature, e.g., Hazledine (2006) and Kutlu (2009), we consider quantity setting games where firms compete in two stages. In the first stage firms decide on the choice of capacity and in the second stage they decide on the structure of price discrimination where the level of price discrimination is exogenous. In contrast to Hazledine (2006) we find that in the Cournot framework the quantity-weighted average price depends on the level of price discrimination. We also find that in the Stackelberg framework both the leader and the follower price discriminate as opposed to Kutlu (2009) which concludes that the leader doesn't price discriminate. Moreover, it is discovered that both the players (even the follower) prefer to be in the Stackelberg framework rather than the Cournot framework when price discrimination exists. Comparing welfare under various settings, it is found that competition is not always good for the total welfare if price discrimination exists.

## 1 Introduction

The strategic interactions of the firms in industries have been analyzed in many settings. The literature essentially has many strands originating from Cournot, Bertrand and Stackelberg. On the one hand, the outcome of the Cournot is more realistic, but on the other hand, the setup of price competition in Bertrand is more close to reality. The extremes of Cournot and Bertrand has been put together in the seminal paper by Kreps & Scheinkman [16] where capacity competition followed by price competition justifies the Cournot outcome. In many industries the existence of leaders and followers is a natural phenomenon. This is the source of another strand originating from Stackelberg [23]. Yet another dimension of firms' behavior when they have market power is that of price discrimination. Various kinds of price discrimination in monopoly and their effect on social welfare have interested economists from as early as Robinson [20] who considers third degree price discrimination. This question has been reexamined by Schmalensee [22] and Varian [27] where they find that increase in output is necessary for price discrimination to be welfare increasing. Formby and Millner [9] consider the relationship between price discrimination and competition.[1] More precisely, they compare the social welfare of price discrimination (in Varian's framework) and Cournot competition; and they find out that when the demand curve[2] is concave (convex, linear), price discrimination with $n$

[1] Formby and Millner [9] call it Stackelberg price discrimination.
[2] Whenever we mention "demand curve" we mean "inverse demand curve."

prices produces greater (lesser, equal) output and welfare than a Cournot oligopoly with $n$ competitors.

Next natural setup to analyze is the coexistence of price discrimination and oligopolistic competition. This happens invariably in all industries with airline industry being a good example. There is a class of recent literature focussing on this aspect [15, 13, 12, 14, 17].[3] Hazledine [13] analyzes the Cournot competition with Varian's framework of price discrimination. He finds out that the contrast from the single-price standard Cournot model is in the quantity produced in the market. He also finds out that the average price in the market is independent of the degree of price discrimination and thus the standard models' prediction is not misleading in terms of the average price. Hazledine [12] considers his earlier model of Cournot competition in the limiting case of infinitely many prices. He demonstrates that the total surplus is maximized. However, there is a difference from the monopoly case as the consumer surplus is non-zero.[4] Finally, Kutlu [17] incorporates price discrimination in the Stackelberg model and finds a counterintutive result that leader does not price-discriminate.

Our paper differs from the earlier works in that we analyze this situation as a two stage game. In the first stage, the firms compete on quantities that they put in the market and in the second stage they decide what fraction of the quantity they sell to different group of buyers. In other words, in the second stage of competition for price discrimination there is a capacity constraint. The mode of price discrimination based on the valuation of consumers is standard in the literature. Firms may have many instruments at their disposal for discrimination between buyers. As Varian [27] considers the valuation of consumers to be a function of age, firms may have discounts for senior citizens and students. We consider the example of airline industry where the valuation of the buyers is a function of the time when they are buying the tickets. The business travellers whose plans are generally last moment have less elastic demand whereas the tourists whose plans are almost always flexible have relatively more elastic demand. Thus different bins (groups) of buyers can be grouped based on the day they want to buy a particular airline seat. For example, higher bins consists of the likes of business travellers. The airline example also a good motivation for our two stage setting. In the first stage, when the firms enter the market, they buy certain number of planes; thus the total number of seats are decided for the second stage of the game. They cannot buy planes everyday but they can decide how to allocate the total number of tickets during a time frame. This critical assumption of the stages is missing in the literature that we just reviewed and we hope that this will explain the missing results.

In contrast to most of the other works, e.g., [15, 13, 12, 17], we find results for a general demand function rather than linear demand. We consider two firms in most of the paper (except in section 3.1) for simplicity. One of the main findings of our paper is that in the second stage both firms are active in the higher bins. If there are $K$ bins, the firm with higher capacity is active in all the $K$ bins. The smaller firm is active in the top $t$ bins. Moreover, in the bins $1, 2, ..., t-1$ it matches the quantity sold by the bigger firm. We characterize the behavior of the firms up to finding $\hat{\imath} = t + 1$, i.e., the first bin where the smaller firm is not active. Although the value of $\hat{\imath}$ is not explicitly provided for a given demand function, we describe a recursive algorithm for finding the unique $\hat{\imath}$. In section 3, we consider linear demand for expositional simplicity. We show firms' behavior in the benchmark Cournot case with $n$ firms and Stackelberg case with 2 firms - the leader and the follower. The total quantities sold by the Cournot oligopolist increase with the

[3] See Stole [24] and Armstrong [1] for two comprehensive surveys about price discrimination.

[4] Note that in the Varian's price discrimination model when the number of prices goes to infinity, it is equivalent to first degree price discrimination. Thus, the total surplus is maximized and goes to the monopolist.

level of price discrimination and decrease with the number of firms as expected. While in the Cournot competition all the firms are active in all the bins (due to the symmetry of firms), in the Stackelberg competition this is not the case. For the Stackelberg case, we calculate the bin number $\hat{\imath}$ for various levels of price discrimination. In section 4, as a function of the level of price discrimination, we compare various aspects of the competition in our two stage model with that of the single stage models of Kutlu [17] and Hazledine [13]. In our framework, among other results, we present two interesting results. First, the Stackelberg leader's profit is greater than the profit of the Stackelberg follower which in turn is greater than the profit of a Cournot duopolist for high levels of price discrimination. This contrasts with the well-known result without price discrimination where follower's profit is lower than that of a Cournot duopolist. Second, for higher levels of price discrimination the monopoly welfare beats the welfare of Cournot duopoly (with price discrimination). Hence, although both price discrimination and competition have positive effects on the total welfare, it is possible that their combination has a negative effect on total welfare. This negative effect disappears as the number of firms grows, since for high number of firms the full efficiency is reached. This negative effect adds one more possible reason for why mergers of duopolies might be beneficial (especially for the airlines) for social welfare. Of course, in practice the merger analysis is much more complicated than it is mentioned here. A better model for analyzing the effects of mergers would also incorporate many other factors such as the dynamic factors and efficiency.[5] Section 5 (the appendix) gathers all the major proofs.

## 2 The Model and Results

Assume for simplicity that there are only two firms, $A$ and $B$, in the market. We normalize the marginal costs of these firms to zero. Each consumer buys at most one unit of the good. The firms know valuations of the consumers and can prevent resale of the good. They divide the consumers into *bins* according to their reservation prices. The total capacities of the firms are exogenously given by $Q_A$ and $Q_B$ where $Q_A \leq Q_B$. Given these capacities firms are competing on the shares that they assign to each bin. Hence, firms choose $s_A = (s_A^1, s_A^2, ..., s_A^{K-1}, s_A^K)$ and $s_B = (s_B^1, s_B^2, ..., s_B^{K-1}, s_B^K)$ with $\sum_{i=1}^K s_A^i = 1$ and $\sum_{i=1}^K s_B^i = 1$ where $q_A^i = Q_A s_A^i$ and $q_B^i = Q_B s_B^i$. Going back to our example of airline seats offered for a specific route, from now on we can think of the product 'an airline seat' and a seller 'an airline'. Total number of seats of the airlines are exogenously given. The airlines simultaneously decide how many of these seats they sell to which customers.

The price of the good for the $k^{th}$ bin is given by:

$$P^k = P(Q^k) \tag{1}$$

where $q_A^i$ and $q_B^i$ are the quantities sold in bin $i$ by $A$ and $B$; $Q^k \equiv \sum_{i=1}^k (q_A^i + q_B^i)$ is the total quantity sold in all bins from 1 to $k$; and $P$ is a twice continuously differentiable, strictly decreasing demand function that represents consumers' valuations. Moreover, for a given combination of $Q_A$ and $Q_B$, we assume that the revenue function (which also is the profit function in our case given zero costs) of firm $A$ and $B$ are strictly concave in $s_A$ and $s_B$, respectively.[6]

[5] See Kutlu and Sickles [18] for a dynamic model considering the efficiencies of the firms when measuring market powers of firms.

[6] The demand curve not being too convex is one of the requirements. Notice for example, in the monopoly case with no price discrimination we require the inverse demand function to be 'less convex' than $\frac{1}{x}$. See ([19, 21, 25, 26, 28]) for conditions on existence in setups without price discrimination.

The optimization problem of the firm $A$ is given by:[7]

$$\begin{aligned} \max \pi_A &= Q_A \sum_{i=1}^{K} P^i s_A^i \qquad (2) \\ \text{st } s_A^i &\geq 0 \text{ and } \sum_{i=1}^{K} s_A^i = 1 \end{aligned}$$

The Lagrangian for the optimization problem (2) is given by:

$$\mathcal{L}_A = \pi_A + \mu_A\left(\sum_{j=1}^{K} s_A^j - 1\right) \qquad (3)$$

Let $\tilde{\mu}_A = \frac{\mu_A}{Q_A}$.[8] For any $i = 1, 2, ..., K$ the Kuhn-Tucker conditions are given by:[9]

$$\begin{aligned} P^i + A_i + \tilde{\mu}_A &\leq 0 \qquad (4) \\ (P^i + A_i + \tilde{\mu}_A)s_A^k &= 0 \qquad (5) \\ \sum_{k=1}^{K} s_A^k &= 1 \qquad (6) \\ s_A^i &\geq 0 \qquad (7) \end{aligned}$$

where $A_i = \sum_{k=i}^{K} \frac{\partial P^k}{\partial Q} \frac{\partial Q^k}{\partial s_A^k} s_A^k$.

In what follows we assume that $A_i = \sum_{k=i}^{K} \frac{\partial P^k}{\partial Q} \frac{\partial Q^k}{\partial s_A^k} s_A^k$ and $B_i = \sum_{k=i}^{K} \frac{\partial P^k}{\partial Q} \frac{\partial Q^k}{\partial s_B^k} s_B^k$ for the sake of notational simplicity.

In the following proposition we show that both of the firms are active in the top bin. Moreover, the bins where a firm sells are consecutive.

**Proposition 1***Assume that for some bin $i \in \{1, 2, ..., K\}$ we have $s_A^i = 0$, then $s_A^{i+1} = 0$.*

**Proof.** Assume to get a contradiction that $s_A^i = 0$ and $s_A^{i+1} > 0$ for some $i \in \{1, 2, ..., K-1\}$. Then we have:

$$P^i \leq -A_i - \tilde{\mu}_A = -A_{i+1} - \tilde{\mu}_A = P^{i+1}$$

Here the inequality comes from the Kuhn-Tucker conditions; the first equality follows from our assumption that $s_A^i = 0$; and the second equality follows from the Kuhn-Tucker conditions given that $s_A^{i+1} > 0$. Hence, $P^i \leq P^{i+1}$. But by the monotonicity of the

[7]Note that the optimization problem for firm $B$ is exactly the same.

[8]Note that we are solving the problem of an active firm. Therefore it is assumed that $Q_A > 0$.

[9]For notational simplicity we represent $\frac{\partial P(Q^j)}{\partial Q}$ by $\frac{\partial P^j}{\partial Q}$.

demand $P^i \geq P^{i+1}$ implying that $P^i = P^{i+1}$. This in turn implies that there are $K-1$ bins which is a contradiction.

■

The following lemma states that the bigger firm will be active in all bins.

**Lemma 1** *$s_B^j > 0$ for all $j \in \{1, 2, ..., K\}$.*

The following lemma states that in top bin(s), except the last bin where the smaller firm is active, firms match their quantities. By top bins we mean the bins with lower indices which represent the higher valuation customers.

**Lemma 2** *Let $\hat{\imath} \in \{3, ..., K, K+1\}$ be such that $s_A^{\hat{\imath}} = s_A^{\hat{\imath}+1} ... = s_A^{K+1} = 0$ and $s_A^j > 0$ for all $j < \hat{\imath}$. Then for $j < \hat{\imath} - 1$ we have:*

$$q_A^j = q_B^j \tag{8}$$

Now we provide a proposition which describes behavior of the firms in all the bins for a general demand function. Even though we don't have a closed form solution, this proposition gives a recursive way to get an explicit solution for a specific demand function up to finding $\hat{\imath}$. After the proposition we describe an algorithm to find such a solution. Later in this paper, in corollaries 1 and 2, we give an explicit solution for the linear demand case as an example.

**Proposition 2** *Assume that $Q_A \leq Q_B$. Let $\hat{\imath} \in \{2, 3, ..., K, K+1\}$ be such that $s_A^{\hat{\imath}} = s_A^{\hat{\imath}+1} ... = s_A^K = s_A^{K+1} = 0$ and $s_A^j > 0$ for all $j < \hat{\imath}$.[10] The optimal shares for $A$ and $B$ are described as follows:*

*Case I ($j < \hat{\imath} - 2$):*

$$P(2Q_A \sum_{k=1}^{j} s_A^k) - P(2Q_A \sum_{k=1}^{j+1} s_A^k) = -\frac{\partial P^j}{\partial Q} Q_A s_A^j \tag{9}$$

$$P(2Q_B \sum_{k=1}^{j} s_B^k) - P(2Q_B \sum_{k=1}^{j+1} s_B^k) = -\frac{\partial P^j}{\partial Q} Q_B s_B^j \tag{10}$$

*Case II ($j = \hat{\imath} - 2$):*

$$P(2Q_A \sum_{k=1}^{\hat{\imath}-2} s_A^k) - P(Q_A(1 + \sum_{k=1}^{\hat{\imath}-2} s_A^k) + Q_B s_B^{\hat{\imath}-1}) = -\frac{\partial P^{\hat{\imath}-2}}{\partial Q} Q_A s_A^{\hat{\imath}-2} \tag{11}$$

$$P(2Q_B \sum_{k=1}^{\hat{\imath}-2} s_B^k) - P(Q_A + Q_B \sum_{k=1}^{\hat{\imath}-1} s_B^k) = -\frac{\partial P^{\hat{\imath}-2}}{\partial Q} Q_B s_B^{\hat{\imath}-2} \tag{12}$$

[10] Even though there are $K$ bins, we are using the index up to $K+1$ in order to include the case where $Q_A = Q_B$ or they are so close that $A$ is active in all the bins. Hence, $\hat{\imath} = K+1$ means that $s_A^i > 0$ in all bins $i = 1, 2, ..., K$.

*Case III* ($j > \hat{\imath} - 2$):

$$
\begin{aligned}
s_A^j &= 0 \quad \textit{for } j > \hat{\imath} - 1 && (13) \\
s_A^{\hat{\imath}-1} &= 1 - \sum_{k=1}^{\hat{\imath}-2} s_A^k && (14) \\
P(Q_A + Q_B \sum_{k=1}^{j} s_B^k) - P(Q_A + Q_B \sum_{k=1}^{j+1} s_B^k) &= -\frac{\partial P^j}{\partial Q} Q_B s_B^j \quad \textit{for } j < K && (15) \\
s_B^K &= 1 - \sum_{k=1}^{K-1} s_B^k && (16)
\end{aligned}
$$

The solution algorithm is as follows. From the cases above, we can recursively solve $s_B^j$ in terms of $s_B^1$ for $j \leq K$. Moreover, since we have $\sum_{k=1}^{K} s_B^k = 1$, we can solve for $s_B^1$. Once we have the solution for $s_B$'s we can solve for $s_A$'s as follows. From Case I, we can recursively solve $s_A^j$ in terms of $s_A^1$ for $j < \hat{\imath} - 1$. Since we have $s_A^{\hat{\imath}-1} = 1 - \sum_{k=1}^{\hat{\imath}-2} s_A^k$, we can solve for $s_A^{\hat{\imath}-1}$ in terms of $s_A^1$ as well. In order to solve for $s_A^1$, we use Lemma 2. That is, given $s_B^1$ the solution for $s_A^1$ is given by:[11]

$$
s_A^1 = \frac{Q_B}{Q_A} s_B^1 \qquad (17)
$$

Note that depending on the value of $\hat{\imath}$ some of the cases disappear. Hence, the sequence of shares might start from Case II or Case III rather than Case I. Whenever $\hat{\imath} > 3$ the solution algorithm starts from Case I; if $\hat{\imath} = 3$, the the solution algorithm starts from Case II; and if $\hat{\imath} = 2$, the the solution algorithm starts from Case III.

In Proposition 2 we described the conditions for equilibrium shares for a general demand function. In the following proposition, we give more conditions which will help identifying $\hat{\imath}$. The first statement of the proposition along with Lemma 2 shows that there is no bin where the smaller firm puts more quantity than the bigger firm.

**Proposition 3** *The shares of the firms in the last bin where A is active i.e., the bin* $\hat{\imath}-1$, *must satisfy:*

$$
\begin{aligned}
Q_A s_A^{\hat{\imath}-1} &\leq Q_B s_B^{\hat{\imath}-1} \quad \textit{for any } \hat{\imath} = 2, 3, ..., K+1 && (18) \\
P(2Q_A \sum_{k=1}^{\hat{\imath}-2} s_A^k) - P(2Q_A \sum_{k=1}^{\hat{\imath}-1} s_A^k) &\leq -\frac{\partial P^{\hat{\imath}-2}}{\partial Q} Q_A s_A^{\hat{\imath}-2} \quad \textit{for any } \hat{\imath} = 3, 4, ..., K+1 && (19)
\end{aligned}
$$

While the following proposition is not a part of the solution algorithm of the equilibrium, it is useful in describing the behavior of the equilibrium for a general demand function. The proposition states that whenever the demand function is concave, the size of a bin is directly related to the valuation of the customer. In other words, the firms assign the larger bins to the higher valuation customers. In the bins up to $\hat{\imath} - 2$ the quantity decreases by less than half with the increase in the index of the bin. Notice

[11] Note that this statement holds for $\hat{\imath} > 2$ case. The $\hat{\imath} = 2$ case is trivial as $s_A^1 = 1$.

that this is a contrast to the monopoly case where the quantity will decrease but not as much as half. Due to this reason we see in the second statement of the proposition, where we discuss the convex demand, that even when the demand function is convex the quantity in the bins may be decreasing with the increase of the index of the bin.[12] This is because the condition only requires the bin size to be less than twice of the next bin's size. In the bins $\hat{\imath}-1, \hat{\imath}, ..., K$ where the firm $B$'s behavior mimics that of a monopoly, we have the shares decreasing (increasing) for concave (convex) demand function. Hence, for the linear case those bins will have equal shares. This result illustrates that competition between firms carries the prisioners-dilemma-symptom of Cournot. In Cournot (without price discrimination) there is overproduction compared to collusion (monopoly) whereas in this share game there is overproduction in higher bins as compared to that in monopoly with price discrimination. As it turns out, in the linear demand case there is in fact under production of total quantity in the share game as compared to the monopoly.

**Proposition 4** *If the demand function, $P$, is concave, then the shares of the firms are monotonically decreasing sequences. More precisely:*

$$s_A^j \geq 2s_A^{j+1} \text{ for } j < \hat{\imath} - 2 \quad (20)$$
$$Q_A s_A^{\hat{\imath}-2} \geq Q_A s_A^{\hat{\imath}-1} + Q_B s_B^{\hat{\imath}-1} \quad (21)$$

$$s_B^j \geq 2s_B^{j+1} \text{ for } j < \hat{\imath} - 2 \quad (22)$$
$$Q_B s_B^{\hat{\imath}-2} \geq Q_A s_A^{\hat{\imath}-1} + Q_B s_B^{\hat{\imath}-1} \quad (23)$$
$$s_B^j \geq s_B^{j+1} \text{ for } j > \hat{\imath} - 2 \quad (24)$$

*If the demand function is convex, then we have:*

$$s_A^j \leq 2s_A^{j+1} \text{ for } j < \hat{\imath} - 2 \quad (25)$$
$$Q_A s_A^{\hat{\imath}-2} \leq Q_A s_A^{\hat{\imath}-1} + Q_B s_B^{\hat{\imath}-1} \quad (26)$$

$$s_B^j \leq 2s_B^{j+1} \text{ for } j < \hat{\imath} - 2 \quad (27)$$
$$Q_B s_B^{\hat{\imath}-2} \leq Q_A s_A^{\hat{\imath}-1} + Q_B s_B^{\hat{\imath}-1} \quad (28)$$
$$s_B^j \leq s_B^{j+1} \text{ for } j > \hat{\imath} - 2 \quad (29)$$

At this point we would like to mention that the shares decided by the above results are invariant to any affine transformation of the demand function. In other words two demand functions $P$ and $\tilde{P}$ where $\tilde{P} = \alpha + \beta P$ would lead to the same solution for the shares. In what follows we consider the linear demand case for expositional simplicity. For a general demand function the equilibrium can be calculated in a similar fashion. Once we finish the full characterization of the linear demand case in this section, we provide Cournot and Stackelberg version of the our model.

[12] Note that we should be careful when we talk about convex demand. Recall that if the inverse demand function is too convex, then the profit function may not be concave.

## 3 The Linear Demand Case

We consider the linear demand given by $P^j = a - Q^j$ and using the above propositions solve completely for the equilibrium which turns out to be unique.[13] As expected, in the equilibrium both firms are active in the top bin(s). The bigger firm is active in all the $K$ bins. The firms match the quantities in the top bins till the bin $\hat{\imath} - 2$. Also, as it can be concluded from Proposition 4, in each bin till the bin $\hat{\imath} - 2$, the firms put exactly half of the quantity that they put in the previous bin. Starting from the bin $\hat{\imath} - 1$ the bigger firm splits the quantity equally in all the bins. Recall that this behavior of the bigger firm is like a monopolist in those bins.

**Corollary 1** *Let $\hat{\imath} \in \{2, ..., K, K+1\}$ be such that $s_A^{\hat{\imath}} = s_A^{\hat{\imath}+1} ... = s_A^K = 0$ and $s_A^j > 0$ for all $j < \hat{\imath}$. Moreover, assume that the demand is linear given by:*

$$P^j = a - Q^j \tag{30}$$

*The optimal shares for $A$ and $B$ are described as follows:*
*Case 1 ($\hat{\imath} = 2$):*

$$\begin{aligned} s_A^1 &= 1 && (31) \\ s_A^j &= 0 \textit{ if } j > 1 && (32) \\ s_B^j &= \frac{1}{K} \textit{ for } j = 1, 2, ..., K && (33) \end{aligned}$$

*Case 2 ($\hat{\imath} \geq 3$):*
*Case I ($j < \hat{\imath} - 1$):*

$$\begin{aligned} s_A^j &= \frac{1}{2^{j-1}} s_A^1 && (34) \\ s_B^j &= \frac{1}{2^{j-1}} s_B^1 && (35) \end{aligned}$$

*Case II ($j = \hat{\imath} - 1$):*

$$\begin{aligned} s_A^j &= 1 - (2 - \frac{1}{2^{\hat{\imath}-3}}) s_A^1 && (36) \\ s_B^j &= 2 s_B^1 - \frac{Q_A}{Q_B} && (37) \end{aligned}$$

*Case III ($j > \hat{\imath} - 1$):*

$$\begin{aligned} s_A^j &= 0 && (38) \\ s_B^j &= 2 s_B^1 - \frac{Q_A}{Q_B} && (39) \end{aligned}$$

The following corollary states the behavior of the firm $A$ in the last bins. In the bin $\hat{\imath} - 1$ it just puts the remainder which is no more than half of what he puts in the bin $\hat{\imath} - 2$.

[13] Note that any linear inverse demand function will lead to exactly the same solution as we have mentioned in the last paragraph.

**Corollary 2** *For any* $\hat{\imath} = 3, ..., K+1$ *we have:*

$$2s_A^{\hat{\imath}-1} \leq s_A^{\hat{\imath}-2} \tag{40}$$

The following corollary together with Corollary 1 characterizes the solution for general $\hat{\imath} \geq 2$ . For a given $Q_A/Q_B$ ratio, the $\hat{\imath}$ is unique and therefore so is the equilibrium.

**Corollary 3** *The shares for the first bins are given as follows:*

$$s_A^1 = \begin{cases} \frac{Q_B}{Q_A} \frac{1+\frac{Q_A}{Q_B}K_{\hat{\imath}}}{\frac{H_{\hat{\imath}}}{2^{\hat{\imath}-3}}+2K_{\hat{\imath}}} & \text{if } \hat{\imath} > 2 \\ 1 & \text{if } \hat{\imath} = 2 \end{cases} \tag{41}$$

$$s_B^1 = \begin{cases} \frac{1+\frac{Q_A}{Q_B}K_{\hat{\imath}}}{\frac{H_{\hat{\imath}}}{2^{\hat{\imath}-3}}+2K_{\hat{\imath}}} & \text{if } \hat{\imath} > 2 \\ 1/K & \text{if } \hat{\imath} = 2 \end{cases} \tag{42}$$

*where the unique* $\hat{\imath}$ *is characterized by*

$$\frac{1+2H_{\hat{\imath}}}{K_{\hat{\imath}}+2H_{\hat{\imath}}} \geq \frac{Q_A}{Q_B} > \frac{H_{\hat{\imath}}}{K_{\hat{\imath}}+H_{\hat{\imath}}} \tag{43}$$

$$H_{\hat{\imath}} = 2^{\hat{\imath}-2} - 1 \tag{44}$$

$$K_{\hat{\imath}} = K - \hat{\imath} + 2 \tag{45}$$

Now that we have completely identified the unique equilibrium of the share allocation game with exogenously given capacities, we explore the equilibria in the games where the capacities themselves are endogenous. Two such natural settings are Cournot and Stackelberg where we assume that the firms involve in multi-stage games. In the beginning stages they compete in the quantities that they plan to put in the market. These quantities will serve as the capacity for the final stage of the game where they simultaneously decide the distribution of shares for the bins. In the following sections we show the equilibrium behavior of firms in such setups where the equilibrium concept is that of subgame perfection. We use backward induction where the final stage shares are borrowed from the previous results.

## 3.1 Cournot Competition with Price Discrimination

In this section we provide a generalization of the benchmark Cournot competition model. In this model, we assume that there are $n$ firms in the market. We normalize the costs of the firms to zero. The firms divide the consumers into $K$ *bins* according to their reservation prices. The demand is assumed to be linear and given by equation (30). We assume that firms are playing a two-stage game where in the first stage they choose the capacities and in the second stage they simultaneously choose the shares that they assign to each bin.

First, we solve the second stage of the game. The symmetric solution implies that $\hat{\imath} = K+1$. The shares are given by:

$$s_f^j = \frac{1}{n^{j-1}} s_f^1 \tag{46}$$

Now, we solve for the first stage of the game to get the equilibrium capacities. Let $Q = \sum_{f=1}^n Q_f$. The following proposition provides the equilibrium quantities and equilibrium profits of the firms.

**Proposition 5** *The equilibrium profits of firm $f$ is given by*

$$\pi_f \quad = \quad (a-Q)Q_f + g_{n,K}Q^2 \tag{47}$$

*the equilibrium quantities and output weighted price are given by:*

$$Q_f \quad = \quad \frac{1}{n+1-2ng_{n,K}} a \tag{48}$$

$$\bar{P} \quad = \quad \frac{\sum_{i=1}^n \pi_i}{\sum_{i=1}^n Q_i} = \frac{1+(n^2-2n)g_{n,K}}{n+1-2ng_{n,K}} \tag{49}$$

*where* $g_{n,K} = \frac{1}{n(n+1)} \frac{n^K - n}{n^K - 1}$ *for* $n > 1$.

For a given $n > 1$ ($K$), $g_{n,K}$ is an increasing (decreasing) function of $K$ ($n$) which implies that $Q_f$ is increasing (decreasing) in $K$ ($n$) as well. The only exception to this monotonicity is observed when we compare the quantities for monopoly and oligopoly. This result has important welfare implications that we examine later. For a given $K$, switching from monopoly to oligopoly leads to a decrease in quantity. From the profit function we can see that, for a fixed $n$, $g_{n,K}Q^2$ term reflects the upwards pressure of price discrimination on the reaction functions of the firms. This leads to higher total capacity choice in equilibrium. Thus, the equilibrium profits are also growing in $K$. For a given $n$, while the average price[14] is invariant to $K$ in Hazledine [13], it is an increasing function of $K$ in our setup.

### 3.2 Stackelberg Competition with Price Discrimination

In this section we provide a generalization of the benchmark Stackelberg competition model. In this model, as in the former section, we assume that there are two firms, the leader ($B$) and the follower ($A$), in the market. We normalize the costs of the firms to zero. The firms divide the consumers into $K$ *bins* according to their reservation prices. The demand is assumed to be linear and given by equation (30). We assume that firms are playing a three-stage game where in the first stage $B$ chooses its capacity; in the second stage $A$ chooses its capacity; and in the final stage they simultaneously choose the shares that they assign to each bin. Unlike the Cournot case where symmetric quantities in the symmetric solution implies $\hat{\imath} = K+1$, here the $\hat{\imath}$ must be endogenously determined in the equilibrium. In the following proposition we provide the profits of the firms as functions of $\hat{\imath}$, $K$ and the quantities.

[14] By average price we mean output weighted average price.

**Proposition 6** *The profit function of A and B is given by:*

$$\pi_A = \begin{cases} (a - f_{\hat{\imath}} Q_{\hat{\imath}})Q_A + g_{\hat{\imath}} Q_{\hat{\imath}}^2 & \text{if } \hat{\imath} > 2 \\ (a - Q_A - \frac{1}{K} Q_B) Q_A & \text{if } \hat{\imath} = 2 \end{cases}$$

$$\pi_B = \begin{cases} aQ_B - h_{\hat{\imath}} Q_{\hat{\imath}}^2 + K_{\hat{\imath}} Q_{\hat{\imath}} Q_A - \frac{(K_{\hat{\imath}}-1)K_{\hat{\imath}}}{2} Q_A^2 & \text{if } \hat{\imath} > 2 \\ (a - Q_A)Q_B - \frac{K+1}{2K} Q_B^2 & \text{if } \hat{\imath} = 2 \end{cases}$$

*where*

$$\begin{aligned} f_{\hat{\imath}} &= \frac{2(1-2x_{\hat{\imath}})}{1-4x_{\hat{\imath}}+K_{\hat{\imath}}} \\ g_{\hat{\imath}} &= \frac{2}{3}\frac{8x_{\hat{\imath}}^2 - 6x_{\hat{\imath}} + 1}{(1-4x_{\hat{\imath}}+K_{\hat{\imath}})^2} \\ h_{\hat{\imath}} &= \frac{8(8x_{\hat{\imath}}^2 - 6x_{\hat{\imath}} + 1) + 12K_{\hat{\imath}}(1-2x_{\hat{\imath}}) + 3K_{\hat{\imath}}(K_{\hat{\imath}}-1)}{6(1-4x_{\hat{\imath}}+K_{\hat{\imath}})^2} \\ Q_{\hat{\imath}} &= Q_B + K_{\hat{\imath}} Q_A \\ x_{\hat{\imath}} &= \frac{1}{2^{\hat{\imath}}} \end{aligned}$$

The following proposition provides a simultaneous system which characterizes the equilibrium quantities and the equilibrium $\hat{\imath}$.

**Proposition 7** *The equilibrium quantities are given by:*

$$Q_A = \frac{a - (f_{\hat{\imath}} - 2g_{\hat{\imath}} K_{\hat{\imath}}) Q_B}{2K_{\hat{\imath}}(f_{\hat{\imath}} - g_{\hat{\imath}} K_{\hat{\imath}})} \tag{50}$$

$$Q_B = \frac{K_{\hat{\imath}}(4(f_{\hat{\imath}} - g_{\hat{\imath}} K_{\hat{\imath}})^2 - f_{\hat{\imath}}(2h_{\hat{\imath}} - 1) + 2g_{\hat{\imath}}) - f_{\hat{\imath}}}{K_{\hat{\imath}}(4(f_{\hat{\imath}} - g_{\hat{\imath}} K_{\hat{\imath}})^2 + f_{\hat{\imath}}^2(2h_{\hat{\imath}} - 1)) - (f_{\hat{\imath}} - 2g_{\hat{\imath}} K_{\hat{\imath}})^2} a \tag{51}$$

*where $\hat{\imath}$ is characterized by:*

$$\frac{1 + 2H_{\hat{\imath}}}{K_{\hat{\imath}} + 2H_{\hat{\imath}}} \geq \frac{f_{\hat{\imath}}(2h_{\hat{\imath}} + 1) - 2(f_{\hat{\imath}} - g_{\hat{\imath}} K_{\hat{\imath}})(f_{\hat{\imath}} - 2g_{\hat{\imath}} K_{\hat{\imath}}) - 2g_{\hat{\imath}} K_{\hat{\imath}}}{K_{\hat{\imath}}(4(f_{\hat{\imath}} - g_{\hat{\imath}} K_{\hat{\imath}})^2 - f_{\hat{\imath}}(2h_{\hat{\imath}} - 1) + 2g_{\hat{\imath}}) - f_{\hat{\imath}}} > \frac{H_{\hat{\imath}}}{K_{\hat{\imath}} + H_{\hat{\imath}}} \tag{52}$$

By utilizing the Propositions 6 and 7 we solve the equilibrium values up to $K = 100$. We proceed as follows: For all $\hat{\imath} = 2, 3, ..., K + 1$ we calculate the corresponding optimal quantities and check the inequality (52). Except the $K = 3$ case we get a unique consistent $(Q_A, Q_B)$ pair. For $K = 3$ case, we eliminate the equilibrium candidate $\hat{\imath} = 3$ by utilizing the fact that $\hat{\imath} = 4$ case gives more profit to the firm $B$. Hence, in the first stage $B$ picks its capacity in such a way as to maximize his equilibrium profit i.e. $\hat{\imath}$ must be 4 for $K = 3$.

In Table 1 below, we provide the equilibrium values of individual quantities, individual profits, and total joint profits for selected values of $K$. Apart from the results for $K = 2, 3, ..., 10$, we provide the values for critical $K$'s, i.e., the $K$'s where the value of $\hat{\imath}$ changes.[15]. Even though the total quantity, average price, and profits are monotonically increasing in $K$ for both the leader and the follower, the individual quantities do not follow the pattern.

[15] Note that $\hat{\imath}$ is non-decreasing in $K$.

| $K$ | $\hat{\imath}$ | $Q_A$ | $Q_B$ | $\pi_A$ | $\pi_B$ | $\pi_T$ |
|---|---|---|---|---|---|---|
| 2 | 3 | 0.3519 | 0.4815 | 0.1358 | 0.1574 | 0.2932 |
| 3 | 3,4 | 0.3702 | 0.5227 | 0.1535 | 0.1721 | 0.3256 |
| 4 | 4 | 0.3784 | 0.5231 | 0.1650 | 0.1842 | 0.3492 |
| 5 | 4 | 0.3782 | 0.5455 | 0.1695 | 0.1910 | 0.3605 |
| 6 | 4 | 0.3785 | 0.5593 | 0.1727 | 0.1953 | 0.3680 |
| 7 | 5 | 0.3794 | 0.5586 | 0.1756 | 0.1988 | 0.3744 |
| 8 | 5 | 0.3787 | 0.5685 | 0.1772 | 0.2015 | 0.3787 |
| 9 | 5 | 0.3783 | 0.5757 | 0.1784 | 0.2036 | 0.3820 |
| 10 | 5 | 0.3780 | 0.5813 | 0.1794 | 0.2052 | 0.3846 |
| 13 | 6 | 0.3776 | 0.5888 | 0.1815 | 0.2084 | 0.3900 |
| 24 | 7 | 0.3764 | 0.6069 | 0.1845 | 0.2135 | 0.3979 |
| 47 | 8 | 0.3757 | 0.6156 | 0.1860 | 0.2162 | 0.4022 |
| 89 | 9 | 0.3754 | 0.6203 | 0.1868 | 0.2174 | 0.4042 |

Table 1

# 4 Model Comparisons

In this section we compare the equilibrium properties of a variety of models: Kutlu's Stackelberg competition model (Stackelberg K); Hazledine's Cournot competition model (Cournot H); Stackelberg competition with share allocation model (Stackelberg KK); and Cournot competition with share allocation model (Cournot KK). Since the framework of Stackelberg K and Cournot H are essentially the same, we call it the HK framework and analogously we call our framework as the KK framework.

In Figure 1 we compare equilibrium quantities for our models as a function of the degree of price discrimination, $K$. A remarkable observation is that while in the HK framework the quantity choices of the leader, the follower, and a Cournot firm are converging to a unified quantity of 0.5, this is not the case for the KK framework. In the earlier levels of price discrimination, the price discrimination helps the follower to gain a considerable amount of market share. For both of the Cournot and Stackelberg cases the total quantity is higher relative to the uniform price case. While for the Cournot case the overproduction is higher for the HK framework, it is lower for the Stackelberg case.

In Figure 3 we compare equilibrium profits for our models as a function of the degree of price discrimination. The profit of the leader of KK is lower than that of the leader of HK; and the profit of the follower of KK is higher than that of the follower of HK. This is because of the third stage of the game which decreased the competitive advantage of the leader. Typically, in quantity competitions – traditional Stackelberg and Cournot – the leader position is preferred to the follower position; and simultaneous move results in an intermediate payoff. Whereas price competition contrasts with quantity competition in that the follower position is preferred to the leader position which is preferred to the simultaneous move competition. For example, Gal-Or (1985) shows that for identical firms if the follower's reaction function is upwards (downwards) sloping, then the firms would like to be the follower (the leader). In the case of upwards sloping reaction functions, the follower copies or undercuts the leader. Hence, this makes moving first less attractive.[16] Gal-Or gives a simple sequential quantity (price) choice game where the leader (the follower) position is preferred.[17] Gal-Or's example is in line with the intuition that quantity (price) reaction functions are expected to slope downward (upward).[18] The Stackelberg competition in HK framework contrasts with the standard Stackelberg dynamics where the follower's output is inversely related to the leader's output. In order to see the contrast, consider the two bin case. If the leader expands output in bin 2, the follower reduces its output in bin 2 and expands it in bin 1. Hence, this framework has an essence of both downwards and upwards sloped reaction functions. The net outcome for the preference of timing is still in line with the standard Stackelberg competition, i.e., the firms' preference ordering is given by: The leader $\succ$ simultaneous move $\succ$ the follower. On the contrary, while the KK framework accords with the typical quantity results (in terms of firms timing preferences) for lower degrees of price discrimination, as the degree of price discrimination becomes large firms prefer being a leader to being a follower; and prefer being a follower to fighting simultaneously.[19] In this game as the leader expands its capacity, the follower reduces it capacity. This reduction rate is a decreasing function of the degree of price discrimination.[20] Hence, for high degrees of price discrimination the follower is less sensitive to the changes in the leader's capacity.

As pointed out by Corts (1998), note that while in the monopoly case price discrimination certainly increases the profit, it might not be the case for an oligopoly environment. The reason is that firms might use their increased power for fighting each other rather than extracting more profits from the customers. In such cases firms would like to commit themselves to uniform pricing. Hence, in order to examine whether the firms are using this price discrimination power against each other or not, we use a simple market power index as a function of $K$. Our measure of market power is defined as the ratio of profits of firms to the monopoly profits as a function of $K$. Hence, if this index is increasing as $K$ increases, this would be a sign of increase in coordination among firms. In Figures 4 and 5 we plot both firm specific and market specific market power measures. From Figure 5 we deduce that the third stage of the KK framework enables a higher degree of coordination among firms. Finally, we consider the relationship between market power and price discrimination. From Figure 5 we observe that for the Cournot of HK setting, the competition increases as the price discrimination increases. This is in line with Borenstein (1985), Holmes (1989), Stole (1995), Valetti (2000), and Valetti (2002). In contrast to this, the market power of Stackelberg of HK is invariant to the degree of price discrimination meaning that price discrimination does not effect the aggregate market power. For the KK setting, initially the aggregate market power indices for both the Cournot and Stackelberg show some tendency to increase; but for higher levels of price discrimination, as the degree of price discrimination increases the aggregate market power decreases.

In Figure 6 we compare the equilibrium welfares. It turns out that the welfare always converges to full efficiency except for the Cournot duopoly case where the firms move simultaneously in the first stage. Therefore, in contrast to the conventional wisdom where competition is almost always supposed to increase the social welfare, this is not the case when there is price discrimination. However, as we see in Figure 7, the full welfare is again achieved when the number of firms grows. For the monopoly case the total welfare increases as the number of bins increases; and for the single price case the total welfare increases as the number of firms increases. Although the outcomes, in terms

[16] The preferences of the firms are affected by asymmetries in the cost function or capacity. For example, a limited capacity would reduce the incentive of the follower to undercut the price of the leader. For more details see Deneckere and Kovenock (1992), Furth and Kovenock (1993), and Canoy (1996).

[17] See also Boyer and Moreaux (1987) and Dowrick (1986).

[18] See Dowrick (1986) for more details.

[19] See Von Stengel [29] for sufficient conditions for non-existence of such examples.

[20] This is verified by numerically evaluating the symbolic solution of the reaction functions.

of total welfare, are similar for price discrimination levels and number of firms[21], the effects of these factors are in opposite direction in terms of the distribution of the total welfare. That is, while an increase in the price discrimination level favors the producer, an increase in the number of firms favors the consumers. Hence, increase in the number of firms decreases the producers' ability to benefit the price discrimination fully. Finally, in Figure 8 we provide the consumer surplus as a function of $K$ and $n$. Except the relative

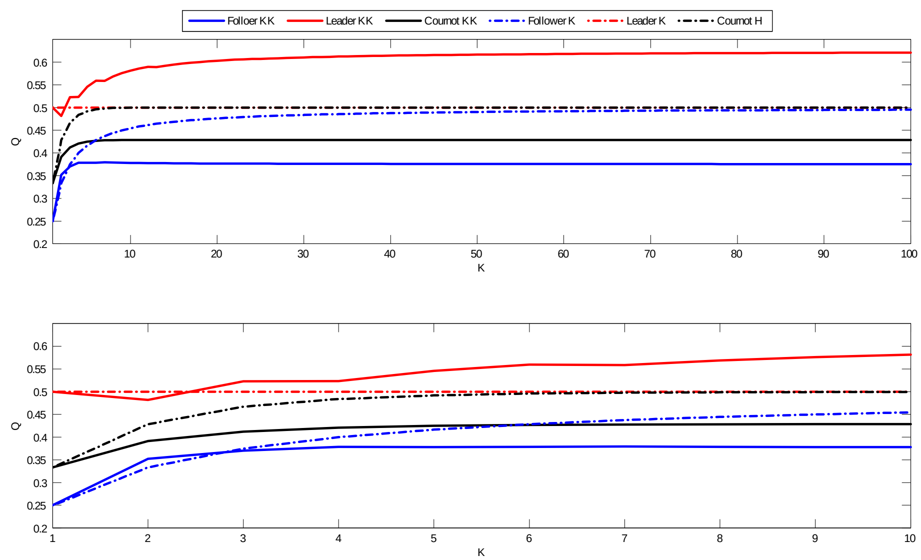


Figure 1: Quantity

[21]Remember that for $n = K$ and linear demand, Formby and Millner (1989) showed that the outcomes for $n$ firm single price and $K$ bin monopoly are the same in terms of total welfare.

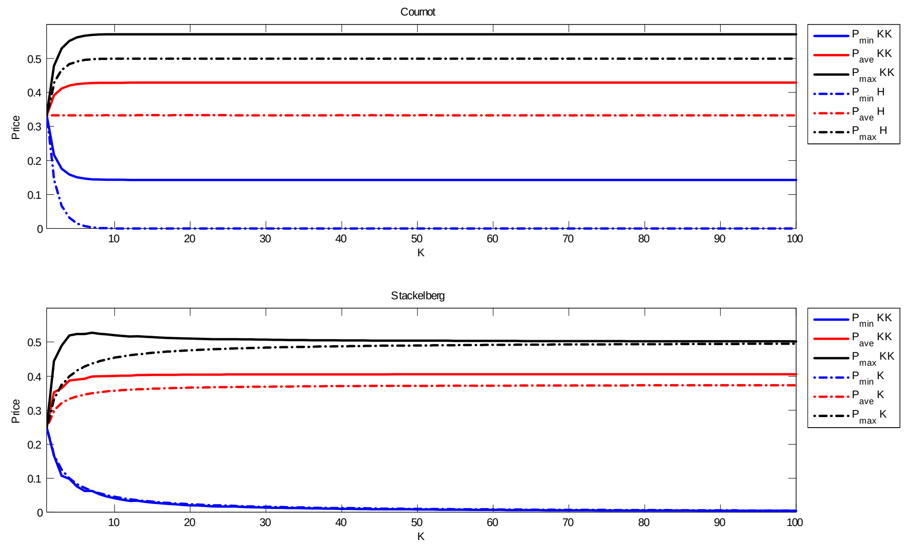


Figure 2: Price

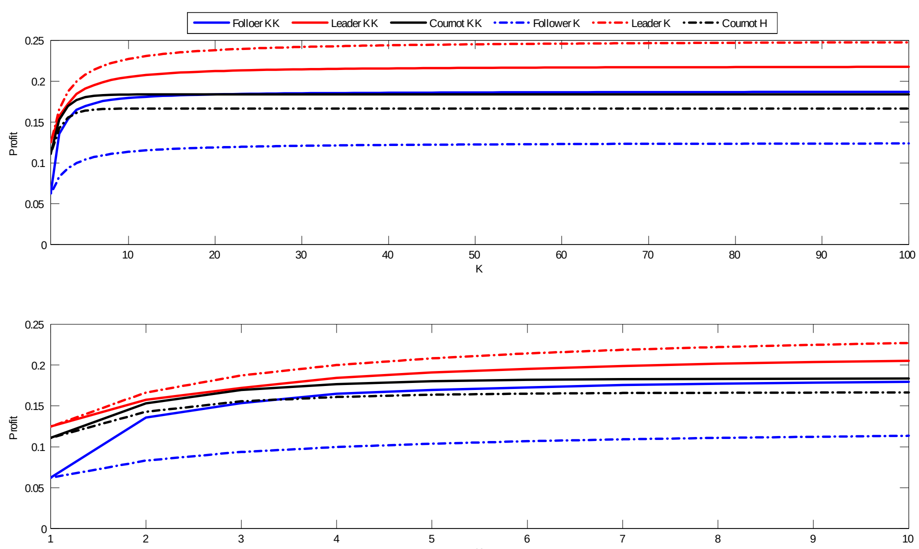


Figure 3: Profit

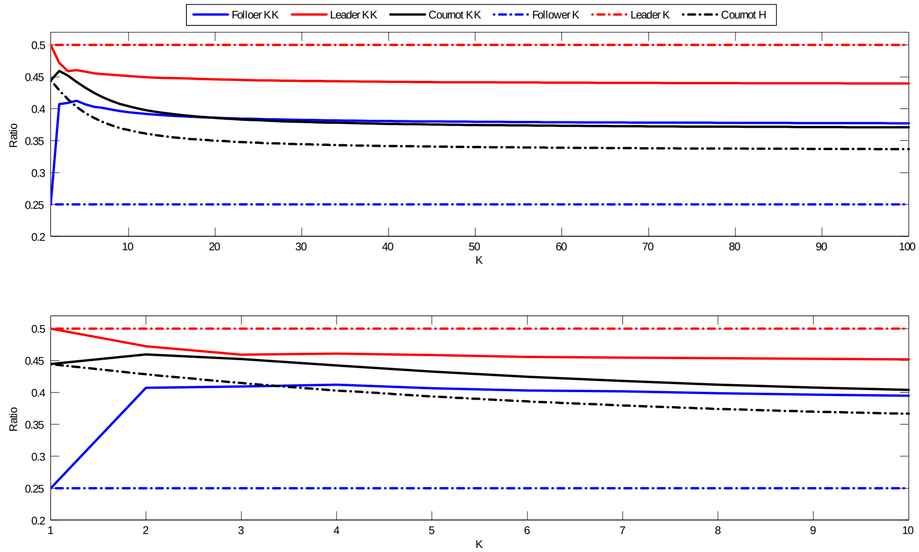


Figure 4: Ratio of firm profit to the monopoly profit

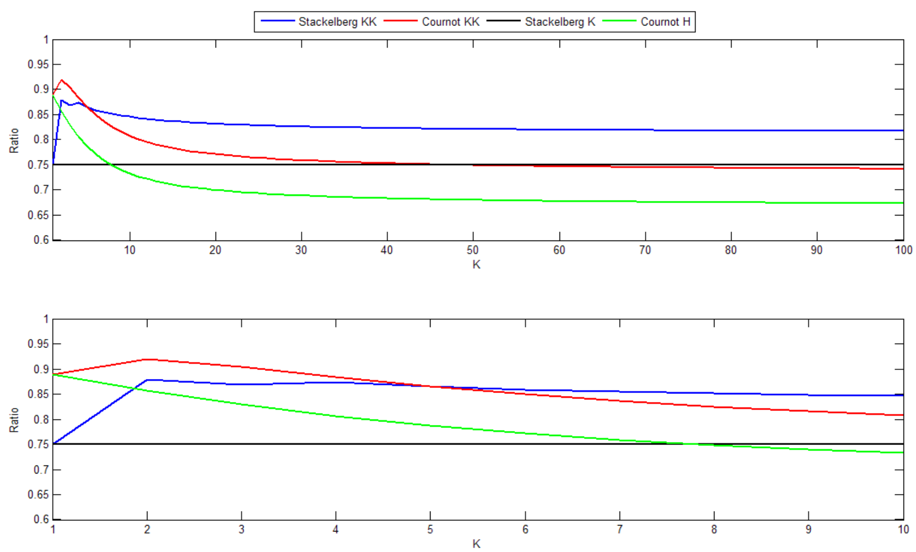


Figure 5: Ratio of industry profit to monopoly profit

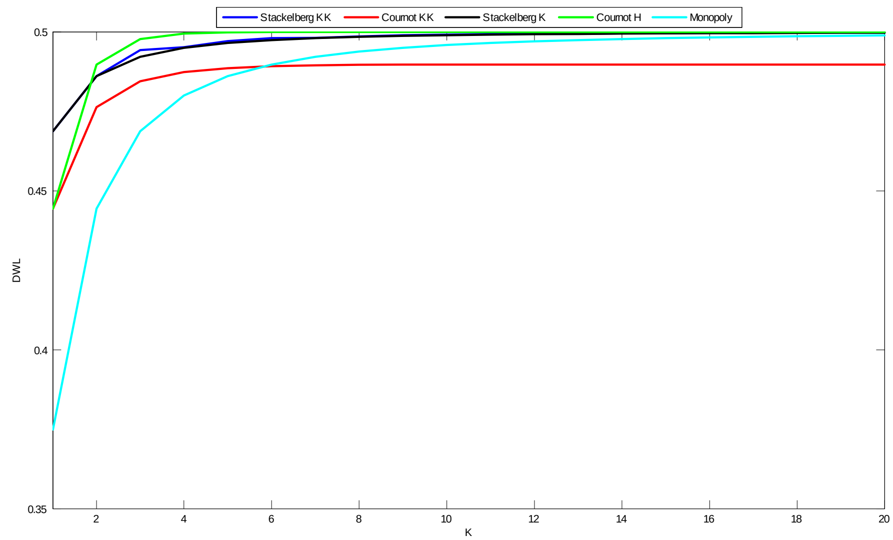


Figure 6: Welfare

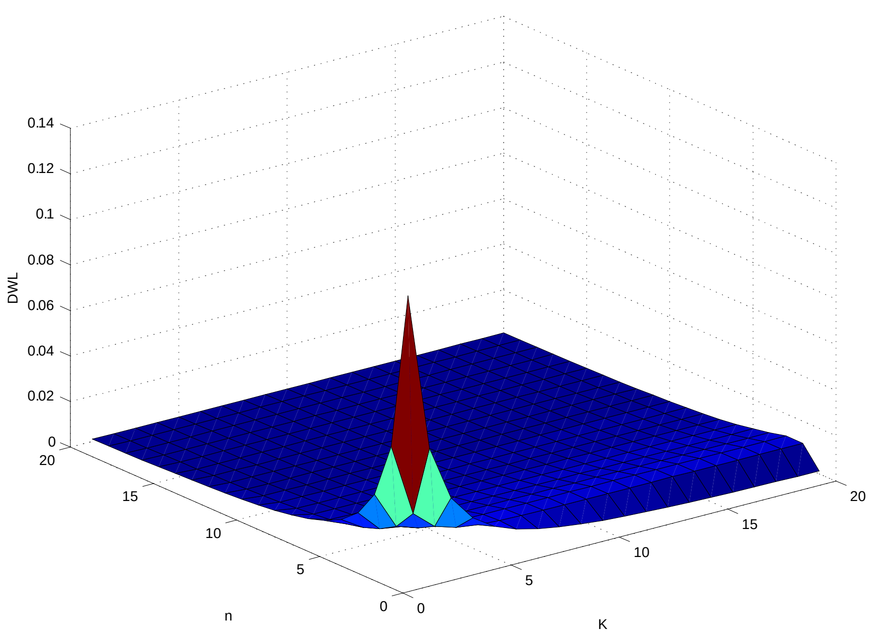


Figure 7: Deadweight loss for Cournot (KK)

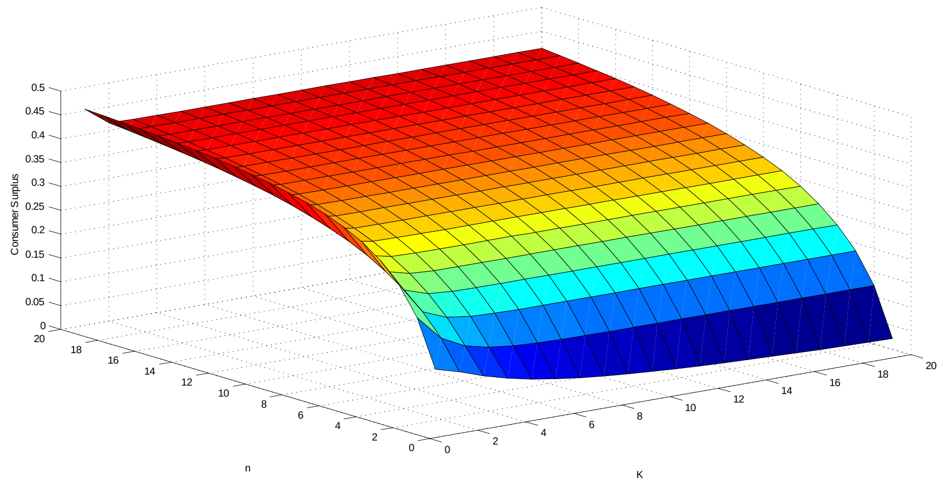


Figure 8: Consumer surplus for Cournot (KK)

# 5 Appendix: Proofs

## 5.1 Proof of Lemma 1

Let $\hat{\imath} \in \{2, ..., K, K+1\}$ be such that $s_A^{\hat{\imath}} = s_A^{\hat{\imath}+1} ... = s_A^{K+1} = 0$ and $s_A^j > 0$ for all $j < \hat{\imath}$. In order to prove the lemma, we consider two cases.

Case 1 ($\hat{\imath} \leq K$): If $\hat{\imath} \leq K$, then $s_B^K > 0$. Otherwise, there will not be $K$ bins which is a contradiction.

Case 2 ($\hat{\imath} = K+1$): From Proposition 1 it follows that $s_B^1 > 0$. Let's assume that $s_B^j > 0$ for all $j < t$. We will show that $s_B^t > 0$. Assume not, i.e. $s_B^t = s_B^{t+1} = ... = s_B^K = 0$. From the Kuhn-Tucker conditions we know that:

$$P^j + A_j + \tilde{\mu}_A = 0 \quad (53)$$

$$P^j + B_j + \tilde{\mu}_B = 0 \quad (54)$$

$$P^t + A_t + \tilde{\mu}_A = 0 \quad (55)$$

$$P^t + B_t + \tilde{\mu}_B \leq 0 \quad (56)$$

Subtracting the equality (55) from the inequality (56) gives:

$$B_t - A_t + \tilde{\mu}_B - \tilde{\mu}_A \leq 0 \quad (57)$$

From (53) and (54), we know that:

$$\tilde{\mu}_B - \tilde{\mu}_A = A_j - B_j \tag{58}$$

Therefore, we have:

$$\begin{aligned} B_t - A_t + A_j - B_j &\leq 0 && (59) \\ B_j - A_j + A_{j-1} - B_{j-1} &= 0 && (60) \end{aligned}$$

From equations (59) and (60), we have:

$$\begin{aligned} -Q_B \frac{\partial P^{t-1}}{\partial Q} s_B^{t-1} + Q_A \frac{\partial P^{t-1}}{\partial Q} s_A^{t-1} &\leq 0 && (61) \\ -Q_B \frac{\partial P^{t-2}}{\partial Q} s_B^{t-2} + Q_A \frac{\partial P^{t-2}}{\partial Q} s_A^{t-2} &= 0 \\ &\vdots \\ -Q_B \frac{\partial P^{1}}{\partial Q} s_B^{1} + Q_A \frac{\partial P^{1}}{\partial Q} s_A^{1} &= 0 && (62) \end{aligned}$$

From monotonicity of demand, we have $\frac{\partial P^j}{\partial Q} < 0$. Therefore:

$$Q_B s_B^{t-1} \leq Q_A s_A^{t-1} \tag{63}$$

Summing over bins $1, 2, ..., t-1$ we get:

$$Q_B \sum_{k=1}^{t-1} s_B^k \leq Q_A \sum_{k=1}^{t-1} s_A^k \tag{64}$$

or

$$Q_B \leq Q_A \sum_{k=1}^{t-1} s_A^k < Q_A \tag{65}$$

The strict inequality follows from the fact that $A$ is active in all bins until bin $K$. This is a contradiction.
■

## 5.2 Proof of Lemma 2

Note that by Lemma 1 we have $s_B^j > 0$ for $j \in \{1, 2, ..., K\}$. Hence, for all $j < \hat{\imath}$ we have $P^j = -A_j - \tilde{\mu}_A = -B_j - \tilde{\mu}_B$. Hence, $P^j - P^{j+1} = q_A^j = q_B^j$.
■

## 5.3 Proof of Proposition 2

Note that for $j < \hat{\imath} - 1$ we have :

$$
\begin{aligned}
P(Q^j) - P(Q^{j+1}) &= (-A_j - \tilde{\mu}_A) - (-A_{j+1} - \tilde{\mu}_A) && (66)\\
&= -\frac{\partial P^j}{\partial Q} Q_A s_A^j && (67)
\end{aligned}
$$

Also, by Lemma 1 using the similar steps as above we get:

$$
P(Q^j) - P(Q^{j+1}) = -\frac{\partial P^j}{\partial Q} Q_B s_B^j \qquad (68)
$$

For case I, we have $j < \hat{\imath} - 2$. Therefore $Q^j = Q_A \sum_{k=1}^{j} s_A^k + Q_B \sum_{k=1}^{j} s_B^k$. Since, $j < \hat{\imath} - 2$ by Lemma 2 we have $Q_A s_A^j = Q_B s_B^j$. Hence, $Q^j = 2Q_A \sum_{k=1}^{j} s_A^k = 2Q_B \sum_{k=1}^{j} s_B^k$ and $Q^{j+1} = 2Q_A \sum_{k=1}^{j+1} s_A^k = 2Q_B \sum_{k=1}^{j+1} s_B^k$.

For case II, we have $j = \hat{\imath} - 2$. Therefore $Q^j = 2Q_A \sum_{k=1}^{\hat{\imath}-2} s_A^k = 2Q_B \sum_{k=1}^{\hat{\imath}-2} s_B^k$ and $Q^{j+1} = Q_A(1 + \sum_{k=1}^{\hat{\imath}-2} s_A^k) + Q_B s_B^{\hat{\imath}-1} = Q_A + Q_B \sum_{k=1}^{\hat{\imath}-1} s_B^k$.

For case III, notice that $Q_A$ is exhausted after bin $\hat{\imath} - 1$. For bin $\hat{\imath} - 1$, $s_A^{\hat{\imath}-1}$ is the residual share for $A$. By Lemma 1, $B$ is active in bins $\hat{\imath}, \hat{\imath}+1, ..., K$, i.e. $s_B^{\hat{\imath}}, s_B^{\hat{\imath}+1}, ..., s_B^K > 0$. Therefore from the Kuhn-Tucker conditions for all $j = \hat{\imath}, \hat{\imath}+1, ..., K-1$ we have:

$$
\begin{aligned}
P^j + B_j + \tilde{\mu}_B &= 0 && (69)\\
P^{j+1} + B_{j+1} + \tilde{\mu}_B &= 0 && (70)
\end{aligned}
$$

Hence, we have:

$$
P^j - P^{j+1} = -\frac{\partial P^j}{\partial Q} Q_B s_B^j \qquad (71)
$$

or

$$
P(Q_B \sum_{k=1}^{j} s_B^k + Q_A) - P(Q_B \sum_{k=1}^{j+1} s_B^k + Q_A) = -\frac{\partial P^j}{\partial Q} Q_B s_B^j \qquad (72)
$$

■

## 5.4 Proof of Proposition 3

First, we prove the inequality (18). From the Kuhn-Tucker conditions we know that:

$$\begin{aligned} P^{\hat{\imath}} + A_{\hat{\imath}} + \tilde{\mu}_A &\leq 0 && (73)\\ P^{\hat{\imath}} + B_{\hat{\imath}} + \tilde{\mu}_B &= 0 && (74)\\ P^{\hat{\imath}-1} + A_{\hat{\imath}-1} + \tilde{\mu}_A &= 0 && (75)\\ P^{\hat{\imath}-1} + B_{\hat{\imath}-1} + \tilde{\mu}_B &= 0 && (76) \end{aligned}$$

Then we have:

$$\begin{aligned} P^{\hat{\imath}} - P^{\hat{\imath}-1} + A_{\hat{\imath}} - A_{\hat{\imath}-1} &\leq 0 && (77)\\ P^{\hat{\imath}} - P^{\hat{\imath}-1} + B_{\hat{\imath}} - B_{\hat{\imath}-1} &= 0 && (78) \end{aligned}$$

Hence:

$$A_{\hat{\imath}} - A_{\hat{\imath}-1} \leq B_{\hat{\imath}} - B_{\hat{\imath}-1} \tag{79}$$

or

$$-\frac{\partial P^{\hat{\imath}-1}}{\partial Q} Q_A s_A^{\hat{\imath}-1} \leq -\frac{\partial P^{\hat{\imath}-1}}{\partial Q} Q_B s_B^{\hat{\imath}-1} \tag{80}$$

By monotonicity of the demand we know that $\frac{\partial P^{\hat{\imath}-1}}{\partial Q} < 0$. Therefore we have:

$$Q_A s_A^{\hat{\imath}-1} \leq Q_B s_B^{\hat{\imath}-1} \tag{81}$$

Now, we prove the inequality (19). From Proposition 2 we know that:

$$P(2Q_A \sum_{k=1}^{\hat{\imath}-2} s_A^k) - P(Q_A(1 + \sum_{k=1}^{\hat{\imath}-2} s_A^k) + Q_B s_B^{\hat{\imath}-1}) = -\frac{\partial P^{\hat{\imath}-2}}{\partial Q} Q_A s_A^{\hat{\imath}-2} \tag{82}$$

or

$$P(2Q_A \sum_{k=1}^{\hat{\imath}-2} s_A^k) - P(2Q_A \sum_{k=1}^{\hat{\imath}-2} s_A^k + Q_A s_A^{\hat{\imath}-1} + Q_B s_B^{\hat{\imath}-1}) = -\frac{\partial P^{\hat{\imath}-2}}{\partial Q} Q_A s_A^{\hat{\imath}-2} \tag{83}$$

Since $Q_A s_A^{\hat{\imath}-1} \leq Q_B s_B^{\hat{\imath}-1}$ by monotonicity of the demand we have:

$$P(2Q_A\sum_{k=1}^{\hat{\imath}-2}s_A^k+Q_As_A^{\hat{\imath}-1}+Q_Bs_B^{\hat{\imath}-1})\leq P(2Q_A\sum_{k=1}^{\hat{\imath}-1}s_A^k) \tag{84}$$

Therefore:

$$P(2Q_A\sum_{k=1}^{\hat{\imath}-2}s_A^k)-P(2Q_A\sum_{k=1}^{\hat{\imath}-1}s_A^k)\leq -\frac{\partial P^{\hat{\imath}-2}}{\partial Q}Q_As_A^{\hat{\imath}-2} \tag{85}$$

■

## 5.5 Proof of Proposition 4

First assume that the demand function is concave. Note that since the demand function is concave we have:

$$P(x)-P(x+y)\geq -\frac{\partial P(x)}{\partial x}y \text{ for any } x,y \tag{86}$$

Let $j<\hat{\imath}-2$. By case I of Proposition 2 and inequality (86) we have:

$$P(2Q_A\sum_{k=1}^{j}s_A^k)-P(2Q_A\sum_{k=1}^{j+1}s_A^k) = -\frac{\partial P^j}{\partial Q}Q_As_A^j \tag{87}$$

$$P(2Q_A\sum_{k=1}^{j}s_A^k)-P(2Q_A\sum_{k=1}^{j+1}s_A^k) \geq -\frac{\partial P^j}{\partial Q}2Q_As_A^{j+1} \tag{88}$$

Thus:

$$-\frac{\partial P^j}{\partial Q}Q_As_A^j \geq -\frac{\partial P^j}{\partial Q}2Q_As_A^{j+1} \tag{89}$$

$$s_A^j \geq 2s_A^{j+1} \tag{90}$$

Proof of $s_B^j\geq 2s_B^{j+1}$ is the same.

Now, let $j=\hat{\imath}-2$. By case II of Proposition 2 and inequality (86) we have:

$$P(2Q_A\sum_{k=1}^{\hat{\imath}-2}s_A^k)-P(2Q_A\sum_{k=1}^{\hat{\imath}-2}s_A^k+Q_As_A^{\hat{\imath}-1}+Q_Bs_B^{\hat{\imath}-1}) = -\frac{\partial P^{\hat{\imath}-2}}{\partial Q}Q_As_A^{\hat{\imath}-2} \tag{91}$$

$$P(2Q_A\sum_{k=1}^{\hat{\imath}-2}s_A^k)-P(2Q_A\sum_{k=1}^{\hat{\imath}-2}s_A^k+Q_As_A^{\hat{\imath}-1}+Q_Bs_B^{\hat{\imath}-1}) \geq -\frac{\partial P^{\hat{\imath}-2}}{\partial Q}(Q_As_A^{\hat{\imath}-1}+Q_Bs_B^{\hat{\imath}-1}) \tag{92}$$

Thus:

$$-\frac{\partial P^j}{\partial Q} Q_A s_A^{\hat{\imath}-2} \geq -\frac{\partial P^j}{\partial Q}(Q_A s_A^{\hat{\imath}-1} + Q_B s_B^{\hat{\imath}-1}) \tag{93}$$

$$Q_A s_A^{\hat{\imath}-2} \geq Q_A s_A^{\hat{\imath}-1} + Q_B s_B^{\hat{\imath}-1} \tag{94}$$

Similarly, we have $Q_B s_B^j \geq Q_A s_A^{\hat{\imath}-1} + Q_B s_B^{\hat{\imath}-1}$.

Finally, assume that $j > \hat{\imath} - 2$. By case III of Proposition 2 and inequality (86) we have:

$$P(Q_A + Q_B \sum_{k=1}^{j} s_B^k) - P(Q_A + Q_B \sum_{k=1}^{j+1} s_B^k) = -\frac{\partial P^j}{\partial Q} Q_B s_B^j \tag{95}$$

$$P(Q_A + Q_B \sum_{k=1}^{j} s_B^k) - P(Q_A + Q_B \sum_{k=1}^{j+1} s_B^k) \geq -\frac{\partial P^j}{\partial Q} Q_B s_B^{j+1} \tag{96}$$

Hence:

$$s_B^j \geq s_B^{j+1} \tag{97}$$

The proof of convex demand function is similar.
■

## 5.6 Proof of Corollary 1

For Case 1, note that by definition of $\hat{\imath}$ and Lemma 1 we have $s_A^1 = 1$. From equations (15) and (16) we have:

$$(a - Q_A - Q_B \sum_{k=1}^{j} s_B^k) - (a - Q_A - Q_B \sum_{k=1}^{j+1} s_B^k) = Q_B s_B^j \text{ for } j < K \tag{98}$$

$$s_B^K = 1 - \sum_{k=1}^{K-1} s_B^k \tag{99}$$

Hence:

$$s_B^{j+1} = s_B^j \text{ for } j < K \tag{100}$$

$$s_B^K = 1 - \sum_{k=1}^{K-1} s_B^k \tag{101}$$

This implies that:

$$s_B^j = \frac{1}{K} \tag{102}$$

For Case 2, we only prove the $\hat{\imath} > 3$ case. The $\hat{\imath} = 3$ is case is similar. For Case I, by equations (9) and (10) for any $j < \hat{\imath} - 2$ we have:

$$(a - 2Q_A \sum_{k=1}^{j} s_A^k) - (a - 2Q_A \sum_{k=1}^{j+1} s_A^k) = Q_A s_A^j \tag{103}$$

$$(a - 2Q_B \sum_{k=1}^{j} s_B^k) - (a - 2Q_B \sum_{k=1}^{j+1} s_B^k) = Q_B s_B^j \tag{104}$$

Hence:

$$2s_A^{j+1} = s_A^j \tag{105}$$

$$2s_B^{j+1} = s_B^j \tag{106}$$

Hence, for any $j < \hat{\imath} - 1$ we have:

$$s_A^j = \frac{1}{2^{j-1}} s_A^1 \tag{107}$$

$$s_B^j = \frac{1}{2^{j-1}} s_B^1 \tag{108}$$

Equation (36) follows from equation (34) and the fact that $s_A^{\hat{\imath}-1} = 1 - \sum_{k=1}^{\hat{\imath}-2} s_A^k$. Now, we derive equation (37). From equation (12) we know that:

$$(a - 2Q_B \sum_{k=1}^{\hat{\imath}-2} s_B^k) - (a - Q_A - Q_B \sum_{k=1}^{\hat{\imath}-1} s_B^k) = Q_B s_B^{\hat{\imath}-2} \tag{109}$$

$$s_B^K = 1 - \sum_{k=1}^{K-1} s_B^k \tag{110}$$

Hence:

$$(-2Q_B \sum_{k=1}^{\hat{\imath}-2} s_B^k) - (a - Q_A - Q_B \sum_{k=1}^{\hat{\imath}-1} s_B^k) = Q_B s_B^{\hat{\imath}-2} \tag{111}$$

Hence, we have:

$$\begin{aligned}
Q_A + Q_B s_B^{\hat{\imath}-1} &= Q_B s_B^{\hat{\imath}-2} + Q_B \sum_{k=1}^{\hat{\imath}-2} s_B^k \quad (112)\\
Q_A + Q_B s_B^{\hat{\imath}-1} &= Q_B s_B^{\hat{\imath}-3} + Q_B \sum_{k=1}^{\hat{\imath}-3} s_B^k \\
&\vdots \\
Q_A + Q_B s_B^{\hat{\imath}-1} &= 2Q_B s_B^1
\end{aligned}$$

This implies that:

$$s_B^{\hat{\imath}-1} = 2s_B^1 - \frac{Q_A}{Q_B} \tag{113}$$

Case III directly follows from equations (13), (15), and (113).
■

## 5.7 Proof of Corollary 2

By Proposition 3 we know that:

$$(a - 2Q_A \sum_{k=1}^{\hat{\imath}-2} s_A^k) - (a - 2Q_A \sum_{k=1}^{\hat{\imath}-1} s_A^k) \leq Q_A s_A^{\hat{\imath}-2} \tag{114}$$

Hence:

$$2s_A^{\hat{\imath}-1} \leq s_A^{\hat{\imath}-2} \tag{115}$$

■

### 5.8 Proof of Corollary 3

First, assume that $\hat{\imath} > 2$. Using Case II in Proposition 2 we have:

$$Q_A s_A^{\hat{\imath}-1} + Q_B s_B^{\hat{\imath}-1} = Q_A s_A^{\hat{\imath}-2} \tag{116}$$

Also from Corollary 2 we have:

$$s_A^{\hat{\imath}-1} \leq \frac{1}{2} s_A^{\hat{\imath}-2} \tag{117}$$

Assume that the quantities that firm $A$ puts in the bins $\hat{\imath}-1$ and $\hat{\imath}-2$ are $y$ and $x$ respectively. Then, from equation (116), Corollary 1, and Corollary 2 we get the following system which will characterize $\hat{\imath}$:

$$\begin{aligned} y + x + 2x + 4x + ... + 2^{\hat{\imath}-3}x &= Q_A && (118)\\ (K - \hat{\imath} + 2)(x - y) + x + 2x + 4x + ... + 2^{\hat{\imath}-3}x &= Q_B && (119)\\ 0 &\leq y \leq \frac{x}{2} && (120)\\ Q_A &\leq Q_B && (121) \end{aligned}$$

Letting $H_{\hat{\imath}} = 2^{\hat{\imath}-2} - 1$ and $K_{\hat{\imath}} = K - \hat{\imath} + 2$ we have:

$$\begin{aligned} y + H_{\hat{\imath}}x &= Q_A && (122)\\ -K_{\hat{\imath}}y + (K_{\hat{\imath}} + H_{\hat{\imath}})x &= Q_B && (123)\\ 0 &< y \leq \frac{x}{2} && (124)\\ Q_A &\leq Q_B && (125) \end{aligned}$$

Solving for $x$ and $y$ we have:

$$\begin{aligned} x &= \frac{K_{\hat{\imath}}Q_A + Q_B}{K_{\hat{\imath}}H_{\hat{\imath}} + K_{\hat{\imath}} + H_{\hat{\imath}}} && (126)\\ y &= \frac{K_{\hat{\imath}}Q_A + H_{\hat{\imath}}(Q_A - Q_B)}{K_{\hat{\imath}}H_{\hat{\imath}} + K_{\hat{\imath}} + H_{\hat{\imath}}} && (127) \end{aligned}$$

From the inequality (124) we have:

$$\frac{1 + 2H_{\hat{\imath}}}{K_{\hat{\imath}} + 2H_{\hat{\imath}}} \geq \frac{Q_A}{Q_B} > \frac{H_{\hat{\imath}}}{K_{\hat{\imath}} + H_{\hat{\imath}}} \tag{128}$$

Now, assume that $\hat{\imath} = 2$. Then, by Proposition 3 and Corollary 1 we have:

$$Q_A \leq \frac{1}{K} Q_B \tag{129}$$

Note that $H_2 = 0$ and $K_{\hat{\imath}} = K$. Hence, the system (128) holds for $\hat{\imath} = 2$ as well.

Let $\theta_{\hat{\imath}} = \frac{1+2H_{\hat{\imath}}}{K_{\hat{\imath}}+2H_{\hat{\imath}}}$ and $\lambda_{\hat{\imath}} = \frac{H_{\hat{\imath}}}{K_{\hat{\imath}}+H_{\hat{\imath}}}$. Now, we show the uniqueness of the equilibrium. Note that it is enough to show that $\{(\lambda_{\hat{\imath}}, \theta_{\hat{\imath}}]\}_{\hat{\imath}}$ partitions $(0, 1]$. This simply means that for any given $\frac{Q_A}{Q_B}$ value, there will be one and only one corresponding set $(\lambda_{\hat{\imath}}, \theta_{\hat{\imath}}]$. This set identifies the $\hat{\imath}$ that gives the equilibrium. First, note that $\frac{\partial \theta_{\hat{\imath}}}{\partial \hat{\imath}} \geq 0$ and $\frac{\partial \lambda_{\hat{\imath}}}{\partial \hat{\imath}} \geq 0$. Moreover, we know that $\theta_{K+1} = 1$ and $\lambda_2 = 0$. Hence, if $\lambda_{\hat{\imath}} = \theta_{\hat{\imath}-1}$ for any $\hat{\imath} = 3, ..., K$, then $\{(\lambda_{\hat{\imath}}, \theta_{\hat{\imath}}]\}_{\hat{\imath}}$ partitions $[0, 1]$. We want to show that:

$$\theta_{\hat{\imath}-1} = \frac{1 + 2H_{\hat{\imath}-1}}{K_{\hat{\imath}-1} + 2H_{\hat{\imath}-1}} = \frac{H_{\hat{\imath}}}{K_{\hat{\imath}} + H_{\hat{\imath}}} = \lambda_{\hat{\imath}} \tag{130}$$

or

$$\begin{aligned}
(1 + 2H_{\hat{\imath}-1})(K_{\hat{\imath}} + H_{\hat{\imath}}) - (K_{\hat{\imath}-1} + 2H_{\hat{\imath}-1})H_{\hat{\imath}} &= 0 \\
K_{\hat{\imath}} + 2K_{\hat{\imath}}H_{\hat{\imath}-1} + H_{\hat{\imath}} + 2H_{\hat{\imath}}H_{\hat{\imath}-1} - K_{\hat{\imath}-1}H_{\hat{\imath}} - 2H_{\hat{\imath}}H_{\hat{\imath}-1} &= 0 \\
K_{\hat{\imath}} + 2K_{\hat{\imath}}H_{\hat{\imath}-1} + H_{\hat{\imath}} - K_{\hat{\imath}-1}H_{\hat{\imath}} &= 0 \\
K_{\hat{\imath}} + (H_{\hat{\imath}} - 1)K_{\hat{\imath}} + H_{\hat{\imath}} - K_{\hat{\imath}-1}H_{\hat{\imath}} &= 0 \\
H_{\hat{\imath}}(K_{\hat{\imath}} + 1) - K_{\hat{\imath}-1}H_{\hat{\imath}} &= 0 \\
H_{\hat{\imath}}(K - \hat{\imath} + 2 + 1) - (K - (\hat{\imath} - 1) + 2)H_{\hat{\imath}} &= 0 \\
0 &= 0
\end{aligned} \tag{131}$$

We conclude that for any given $\frac{Q_A}{Q_B}$ there exists a unique equilibrium for the quantity choices of $A$ and $B$. The equilibrium is determined by the conditions from Corollary 1 and inequality system (128). Finally, the shares for the first bins are given as follows:

$$s_A^1 = \frac{Q_B}{Q_A} \frac{1 + \frac{Q_A}{Q_B} K_{\hat{\imath}}}{\frac{H_{\hat{\imath}}}{2^{\hat{\imath}-3}} + 2K_{\hat{\imath}}} \tag{132}$$

$$s_B^1 = \frac{1 + \frac{Q_A}{Q_B} K_{\hat{\imath}}}{\frac{H_{\hat{\imath}}}{2^{\hat{\imath}-3}} + 2K_{\hat{\imath}}} \tag{133}$$

■

## 5.9 Proof of Proposition 5

Let $Q_f$ and $s_f^k$ denote the total quantity and share for the $k^{th}$ bin for firm $f$, respectively. As in the proof of Proposition 2, the first order conditions for firm $f$ are given by:

$$P(nQ_f \sum_{k=1}^{j} s_f^k) - P(nQ_i \sum_{k=1}^{j+1} s_f^k) = -\frac{\partial P^j}{\partial Q} Q_i s_f^j \text{ for } j < K \tag{134}$$

Hence, we have:

$$(a - nQ_f \sum_{k=1}^{j} s_f^k) - (a - nQ_f \sum_{k=1}^{j+1} s_f^k) = Q_i s_f^j \text{ for any } j < K \tag{135}$$

or:

$$s_f^{j+1} = \frac{1}{n} s_f^j \text{ for any } j < K \tag{136}$$

Similar to the proof of Corollary 3, we have to solve the following system:

$$(1 + n + n^2 + ... + n^{K-1})x = Q_f \tag{137}$$

$$Q_f = Q_j \text{ for any } j \leq K \tag{138}$$

Then, $x = \frac{n-1}{n}\frac{1}{n^K-1}Q$ and $s_f^1 Q_f = n^{K-1}x = \frac{n-1}{n}\frac{n^{K-1}}{n^K-1}Q$ implying that $s_f^1 = \frac{n-1}{n^2}\frac{n^K}{n^K-1}\frac{Q}{Q_f}$. Therefore, the the profit function of firm $f$ in the first stage is given by:

$$
\begin{aligned}
\pi_f &= Q_f\left[\sum_{i=1}^{K-1}(a - nQ_f\sum_{k=1}^{i} s_f^k)s_f^i) + (a-Q)s_f^K\right] \qquad (139)\\
&= Q_f\left[a - nQ_f\sum_{i=1}^{K-1}(\sum_{k=1}^{i} s_f^k)s_f^i) - Q(1-\sum_{k=1}^{K-1} s_f^k)\right]\\
&= Q_f\left[a - nQ_f\sum_{i=1}^{K-1}(\sum_{k=1}^{i}\frac{1}{n^{k-1}}s_f^1)\frac{1}{n^{i-1}}s_f^1) - Q(1-s_f^1\sum_{k=1}^{K-1}\frac{1}{n^{k-1}})\right]\\
&= Q_f\left[a - \frac{n^3}{n-1}Q_f[s_f^1]^2\sum_{i=1}^{K-1}(\frac{1}{n^i}-\frac{1}{n^{2i}}) - Q(1-s_f^1\frac{n}{n-1}(1-n\frac{1}{n^K}))\right]\\
&= Q_f\left[a - \frac{n^3}{n-1}Q_f[s_f^1]^2\sum_{i=1}^{K-1}(\frac{1}{n^i}-\frac{1}{n^{2i}}) - Q(1-s_f^1\frac{n}{n-1}(1-n\frac{1}{n^K}))\right]\\
&= aQ_f - \frac{n^4}{(n+1)(n-1)^2}(1-n\frac{1}{n^K})(1-\frac{1}{n^K})Q_f^2[s_f^1]^2 - Q(Q_f - Q_f s_f^1\frac{n}{n-1}(1-n\frac{1}{n^K}))\\
&= (a-Q)Q_f - \frac{n^4}{(n+1)(n-1)^2}(1-n\frac{1}{n^K})(1-\frac{1}{n^K})Q_f^2[s_f^1]^2 + QQ_f s_f^1\frac{n}{n-1}(1-n\frac{1}{n^K})\\
&= (a-Q)Q_f + (\frac{1}{n}\frac{n^K-n}{n^K-1} - \frac{1}{(n+1)}\frac{n^K-n}{n^K-1})Q^2\\
&= (a-Q)Q_f + \frac{1}{n(n+1)}\frac{n^K-n}{n^K-1}Q^2\\
&= (a-Q)Q_f + g_{n,K}Q^2
\end{aligned}
$$

where $g_{n,K} = \frac{1}{n(n+1)}\frac{n^K-n}{n^K-1}$ for $n > 1$.

The first order condition for $f$ is given by:

$$
\begin{aligned}
\frac{\partial \pi_f}{\partial Q_f} &= a - Q - Q_f + 2g_{n,K}Q \qquad (140)\\
&= a - Q_f + (2g_{n,K}-1)(Q_f + \sum_{i\neq f} Q_i)\\
&= 0
\end{aligned}
$$

From symmetry we get:

$$
Q_f = \frac{1}{n+1-2ng_{n,K}}a \qquad (141)
$$

Now, we calculate the output weighted price $\bar{P}$.

$$
\begin{aligned}
\bar{P} &= \frac{\sum_{i=1}^{n}\pi_i}{\sum_{i=1}^{n}Q_i} = \frac{\pi_f}{Q_f} \\
&= \frac{(a-Q)Q_f + g_{n,K}Q^2}{Q_f} \\
&= (a-Q) + \frac{g_{n,K}Q^2}{Q_f} \\
&= (a-nQ_f) + n^2 g_{n,K}Q_f \\
&= a-(n-n^2 g_{n,K})Q_f \\
&= a-\frac{n-n^2 g_{n,K}}{n+1-2ng_{n,K}}a \\
&= a(1-\frac{n-n^2 g_{n,K}}{n+1-2ng_{n,K}}) \\
&= a\frac{1+(n^2-2n)g_{n,K}}{n+1-2ng_{n,K}}
\end{aligned} \tag{142}
$$

■

## 5.10 Proof of Proposition 6

Derivation of $\hat{\imath}=2$ case is transparent and we leave it to the reader. Assume that $\hat{\imath}>2$ and let $x_{\hat{\imath}}=\frac{1}{2^{\hat{\imath}}}$. Before deriving the profit functions, we present some expressions that will be useful for deriving the profit functions:

$$
Q_A s_A^1 = Q_B s_B^1 = \frac{Q_B + K_{\hat{\imath}}Q_A}{2(1-4x_{\hat{\imath}}+K_{\hat{\imath}})} \tag{143}
$$

$$
2Q_A^2\sum_{i=1}^{\hat{\imath}-2}(\sum_{k=1}^{i}s_A^k)s_A^i = \frac{16}{3}(8x_{\hat{\imath}}^2-6x_{\hat{\imath}}+1)[Q_A s_A^1]^2 \tag{144}
$$

$$
2Q_B^2\sum_{i=1}^{\hat{\imath}-2}(\sum_{k=1}^{i}s_B^k)s_B^i = \frac{16}{3}(8x_{\hat{\imath}}^2-6x_{\hat{\imath}}+1)[Q_B s_B^1]^2 \tag{145}
$$

$$
(2Q_A^2\sum_{k=1}^{\hat{\imath}-2}s_A^k + Q_A^2 s_A^{\hat{\imath}-2})s_A^{\hat{\imath}-1} = -8(8x_{\hat{\imath}}^2-6x_{\hat{\imath}}+1)[Q_A s_A^1]^2 + 4(1-2x_{\hat{\imath}})[Q_A s_A^1]Q_A \tag{146}
$$

$$
(2Q_B^2\sum_{k=1}^{\hat{\imath}-2}s_B^k + Q_B^2 s_B^{\hat{\imath}-2})s_B^{\hat{\imath}-1} = 8(1-2x_{\hat{\imath}})[Q_B s_B^1]^2 - 4(1-2x_{\hat{\imath}})[Q_B s_B^1]Q_A \tag{147}
$$

$$
\begin{aligned}
&\sum_{i=\hat{\imath}}^{K}(2Q_B^2\sum_{k=1}^{\hat{\imath}-2}s_B^k + Q_B^2 s_B^{\hat{\imath}-2} + Q_B^2\sum_{k=\hat{\imath}}^{i}s_B^k)s_B^i \\
&= (K_{\hat{\imath}}-1)\left[4(1-2x_{\hat{\imath}})[Q_B s_B^1]Q_B s_B^{\hat{\imath}-1} + \frac{K_{\hat{\imath}}}{2}[Q_B s_B^{\hat{\imath}-1}]^2\right] \\
&= (K_{\hat{\imath}}-1)\left[4(1-2x_{\hat{\imath}})[Q_B s_B^1](2[Q_B s_B^1]-Q_A) + \frac{K_{\hat{\imath}}}{2}[2[Q_B s_B^1]-Q_A]^2\right] \\
&= (K_{\hat{\imath}}-1)\left[(8(1-2x_{\hat{\imath}})+2K_{\hat{\imath}})[Q_B s_B^1]^2 - (4(1-2x_{\hat{\imath}})+2K_{\hat{\imath}})[Q_B s_B^1]Q_A + \frac{K_{\hat{\imath}}}{2}Q_A^2\right]
\end{aligned} \tag{148}
$$

Let $Q_{\hat{\imath}} = Q_B + K_{\hat{\imath}} Q_A$. Now, using the above equalities we derive the profit function for $A$. The profit function of $A$ is given by:

$$\begin{aligned}
\pi_A &= Q_A(\sum_{i=1}^{\hat{\imath}-2}(a - 2Q_A \sum_{k=1}^{i} s_A^k)s_A^i + (a - 2Q_A \sum_{k=1}^{\hat{\imath}-2} s_A^k - Q_A s_A^{\hat{\imath}-2})s_A^{\hat{\imath}-1}) \\
&= aQ_A - \left[-\frac{8}{3}(8x_{\hat{\imath}}^2 - 6x_{\hat{\imath}} + 1)[Q_A s_A^1]^2 + 4(1 - 2x_{\hat{\imath}})[Q_A s_A^1]Q_A\right] \\
&= aQ_A - \left[-\frac{2}{3}\frac{8x_{\hat{\imath}}^2 - 6x_{\hat{\imath}} + 1}{(1 - 4x_{\hat{\imath}} + K_{\hat{\imath}})^2}Q_{\hat{\imath}}^2 + \frac{2(1 - 2x_{\hat{\imath}})}{1 - 4x_{\hat{\imath}} + K_{\hat{\imath}}}Q_{\hat{\imath}}Q_A\right] \\
&= (a - f_{\hat{\imath}}Q_{\hat{\imath}})Q_A + g_{\hat{\imath}}Q_{\hat{\imath}}^2
\end{aligned} \tag{149}$$

where

$$\begin{aligned}
f_{\hat{\imath}} &= \frac{2(1 - 2x_{\hat{\imath}})}{1 - 4x_{\hat{\imath}} + K_{\hat{\imath}}} \\
g_{\hat{\imath}} &= \frac{2}{3}\frac{8x_{\hat{\imath}}^2 - 6x_{\hat{\imath}} + 1}{(1 - 4x_{\hat{\imath}} + K_{\hat{\imath}})^2}
\end{aligned}$$

The profit function of $B$ is given by:

$$\begin{aligned}
\pi_B &= Q_B(\sum_{i=1}^{\hat{\imath}-2}(a - 2Q_B \sum_{k=1}^{i} s_B^k)s_B^i + (a - 2Q_B \sum_{k=1}^{\hat{\imath}-2} s_B^k - Q_B s_B^{\hat{\imath}-2})s_B^{\hat{\imath}-1} \\
&\quad + \sum_{i=\hat{\imath}}^{K}(a - 2Q_B \sum_{k=1}^{\hat{\imath}-2} s_B^k - Q_B s_B^{\hat{\imath}-2} - Q_B \sum_{k=\hat{\imath}}^{i} s_B^k)s_B^i) \\
&= aQ_B - (\frac{16}{3}(8x_{\hat{\imath}}^2 - 6x_{\hat{\imath}} + 1)[Q_B s_B^1]^2 + 8(1 - 2x_{\hat{\imath}})[Q_B s_B^1]^2 - 4(1 - 2x_{\hat{\imath}})[Q_B s_B^1]Q_A \\
&\quad + (K_{\hat{\imath}} - 1)\left[(8(1 - 2x_{\hat{\imath}}) + 2K_{\hat{\imath}})[Q_B s_B^1]^2 - (4(1 - 2x_{\hat{\imath}}) + 2K_{\hat{\imath}})[Q_B s_B^1]Q_A + \frac{K_{\hat{\imath}}}{2}Q_A^2\right]) \\
&= aQ_B - (\frac{16}{3}(8x_{\hat{\imath}}^2 - 6x_{\hat{\imath}} + 1) + 8K_{\hat{\imath}}(1 - 2x_{\hat{\imath}}) + 2(K_{\hat{\imath}} - 1)K_{\hat{\imath}}))[Q_B s_B^1]^2 \\
&\quad + 2K_{\hat{\imath}}(1 - 4x_{\hat{\imath}} + K_{\hat{\imath}})[Q_B s_B^1]Q_A - \frac{(K_{\hat{\imath}} - 1)K_{\hat{\imath}}}{2}Q_A^2 \\
&= aQ_B - h_{\hat{\imath}}Q_{\hat{\imath}}^2 + K_{\hat{\imath}}Q_{\hat{\imath}}Q_A - \frac{(K_{\hat{\imath}} - 1)K_{\hat{\imath}}}{2}Q_A^2
\end{aligned} \tag{150}$$

where

$$h_{\hat{\imath}} = \frac{8(8x_{\hat{\imath}}^2 - 6x_{\hat{\imath}} + 1) + 12K_{\hat{\imath}}(1 - 2x_{\hat{\imath}}) + 3K_{\hat{\imath}}(K_{\hat{\imath}} - 1)}{6(1 - 4x_{\hat{\imath}} + K_{\hat{\imath}})^2}$$

■

### 5.11 Proof of Proposition 7

The best response capacity of $A$ and corresponding $Q_{\hat{\imath}}$ is given by:

$$Q_A = \frac{a - (f_{\hat{\imath}} - 2g_{\hat{\imath}}K_{\hat{\imath}})Q_B}{2K_{\hat{\imath}}(f_{\hat{\imath}} - g_{\hat{\imath}}K_{\hat{\imath}})} \tag{151}$$

$$Q_{\hat{\imath}} = \frac{a + f_{\hat{\imath}}Q_B}{2(f_{\hat{\imath}} - g_{\hat{\imath}}K_{\hat{\imath}})} \tag{152}$$

By plugging equations (151) and (152) into the profit function of $B$, we calculate the optimal capacity choices of $A$ and $B$ as follows:

$$Q_A = \frac{a - (f_{\hat{\imath}} - 2g_{\hat{\imath}}K_{\hat{\imath}})Q_B}{2K_{\hat{\imath}}(f_{\hat{\imath}} - g_{\hat{\imath}}K_{\hat{\imath}})} \tag{153}$$

$$Q_B = \frac{K_{\hat{\imath}}(4(f_{\hat{\imath}} - g_{\hat{\imath}}K_{\hat{\imath}})^2 - f_{\hat{\imath}}(2h_{\hat{\imath}} - 1) + 2g_{\hat{\imath}}) - f_{\hat{\imath}}}{K_{\hat{\imath}}(4(f_{\hat{\imath}} - g_{\hat{\imath}}K_{\hat{\imath}})^2 + f_{\hat{\imath}}^2(2h_{\hat{\imath}} - 1)) - (f_{\hat{\imath}} - 2g_{\hat{\imath}}K_{\hat{\imath}})^2} a \tag{154}$$

The ratio of $Q_A$ and $Q_B$ is given by:

$$\frac{Q_A}{Q_B} = \frac{f_{\hat{\imath}}(2h_{\hat{\imath}} + 1) - 2(f_{\hat{\imath}} - g_{\hat{\imath}}K_{\hat{\imath}})(f_{\hat{\imath}} - 2g_{\hat{\imath}}K_{\hat{\imath}}) - 2g_{\hat{\imath}}K_{\hat{\imath}}}{K_{\hat{\imath}}(4(f_{\hat{\imath}} - g_{\hat{\imath}}K_{\hat{\imath}})^2 - f_{\hat{\imath}}(2h_{\hat{\imath}} - 1) + 2g_{\hat{\imath}}) - f_{\hat{\imath}}} \tag{155}$$

Equations (128) and (155) help us to identify the equilibrium. If $Q_A$ and $Q_B$ are on the equilibrium, then their ratio should be consistent with the equilibrium value of $\hat{\imath}$ in the final stage. Hence, on the equilibrium $\hat{\imath}$ should satisfy the following condition:

$$\frac{1 + 2H_{\hat{\imath}}}{K_{\hat{\imath}} + 2H_{\hat{\imath}}} \geq \frac{f_{\hat{\imath}}(2h_{\hat{\imath}} + 1) - 2(f_{\hat{\imath}} - g_{\hat{\imath}}K_{\hat{\imath}})(f_{\hat{\imath}} - 2g_{\hat{\imath}}K_{\hat{\imath}}) - 2g_{\hat{\imath}}K_{\hat{\imath}}}{K_{\hat{\imath}}(4(f_{\hat{\imath}} - g_{\hat{\imath}}K_{\hat{\imath}})^2 - f_{\hat{\imath}}(2h_{\hat{\imath}} - 1) + 2g_{\hat{\imath}}) - f_{\hat{\imath}}} > \frac{H_{\hat{\imath}}}{K_{\hat{\imath}} + H_{\hat{\imath}}} \tag{156}$$

■

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