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# The Effects of Global Warming on Fisheries* 

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#### Abstract

This paper develops two fisheries models in order to estimate the effect of global warming (GW) on firm value. GW is defined as an increase in the average temperature of the earth's surface because of $\mathrm{CO}_{2}$ emissions. It is assumed that (i) GW exists, and (ii) higher temperatures negatively affect biomass. The literature on biology and GW supporting these two crucial assumptions is reviewed. The main argument presented is that temperature increase has two effects on biomass, both of which have an impact on firm value. First, higher temperatures cause biomass to oscillate. To measure the effect of biomass oscillation on firm value Pindyck's (1984) model is modified to include water temperature as a variable. The results indicate that a 1 to $20 \%$ variation in biomass causes firm value to fall from 6 to $44 \%$, respectively. Second, higher temperatures reduce biomass, and a modification of the Smith's (1968) model reveals that an increase in temperature anomaly between +1 and $+8^{\circ} \mathrm{C}$ causes fishery's value to decrease by 8 to $10 \%$.


Keywords: fisheries, fisheries economics, global warming, climate change.
JEL Classification: Q22, Q54.

[^0]
## 1 Introduction

The aim of this paper is to estimate the impact of global warming ( $\mathrm{GW}^{1}$ ) on fisheries. For the purpose of this paper, fishing is understood to be industrial deep-sea extraction and subsequent sale of marine resources that takes place in the Exclusive Economic Zone where there is international competition for these resources. Artisan fishing and fish farming are not included in this study since the variables that affect productivity in both can be closely controlled, whereas water temperature -the focus of this study- cannot be controlled.

Although GW is an issue of growing interest in many fields, this study only includes those aspects applicable to the field of marine biology, where efforts to understand the relationship between biomass and temperature changes have been on the rise, especially after the 2008 El Niño phenomenon (El-Niño-Southern-Oscillation, or ENSO).

This study considers two of the effects, oscillation and reduction, that GW causes in biomass ${ }^{2}$. When GW causes biomass to oscillate, a firm that uses technology designed for a non-oscillating biomass will be put at risk, and will have to increase efforts to remain competitive. Random oscillations are used when modeling this effect to reflect that resource availability is not always completely known. It is also assumed that fisheries participate in a competitive market.

Biomass reduction, the second effect, is assumed to be caused by increased mortality rates and/or the migration of species. Accordingly, firms must increase fishing efforts or extraction levels so that, depending on the amount of capital invested, they can reach a level of extraction that is both profitable and biologically sustainable.

The principal objective of this paper is to estimate the economic impact of biomass oscillation and reduction due to GW on fisheries. To do this, two models from the existing literature on fisheries economics are modified. First, for the case of stochastic biomass, the Pindyck (1984) model is adapted to include temperature as an explicit variable in biomass and an implicit variable in the profit function, in order to measure the economic cost faced by firms trying to reach an optimum extraction level. Then, to understand how biomass reduction affects firm's value, the Smith (1968) model is modified to include temperature as a variable in biomass and in the firm's profit function.

Both models are developed under two non-economic assumptions: the average temperature of the Earth's (marine) surface is rising and global warming affects biomass. The data and literature concerning these two assumptions is reviewed.

The paper is laid out as follows: the following section (Section 2) presents the arguments supporting the aforementioned assumptions and reviews the pertinent literature on biology and GW. Based on the existing literature it can be concluded that although temperature time series are still too short to indicate structural change on ecosystem, the Earth's temperature has been on the rise. Methodologically different studies concerning, for example, the consequences of ENSO in the Pacific Ocean and the warming of the sea floor, are also cited as indicators that the Earth's surface temperature is rising. Then, several specific cases studies that demonstrate the effect of elevated water temperature on biomass are analyzed, and the literature on ENSO, its impact on oceans in the southern hemisphere and other similar phenomena occurring in the northern hemisphere is presented.

This section also includes a literature review on the three issues that intersect in this paper: fisheries economics, GW and marine biology. The first fisheries economics models and the changes that have been made to these over time are explained in detail, as is the current literature on GW, much of which is still in the early stages of development and lacks precision.

Section 3 presents the model for stochastic biomass based on Pindyck (1984). Stochastic differ-

[^1]ential equations are used to model biomass and classic firm theory is used to represent the fishery. The model includes an equation that illustrates how firms react to biomass shocks (that is, how much is spent understanding and mitigating the problem). This equation is intended to create a more profound understanding of how firms react to stochastic biomass, whether they face it by increasing spending or simply enduring a higher number of shocks, both costly options.

In Section 4, the Smith (1968) model is adjusted to fit the purposes of this study. This model is used because the comparative static analysis that it provides simplifies situations where temperatures continue to rise as firms attempt to maximize profit. This model is also used to thoroughly analyze the effects of temperature on biomass.

In Section 5, both models are calibrated and the relevant numerical results indicate that stochastic biomass variations of 1 to $20 \%$ cause firm value to drop by 6 to $44 \%$. On the other hand, if a firm extracts resources from a biomass where temperatures have risen between +1 and $+9^{\circ} \mathrm{C}$, its annual value decreases by between 8 and $10 \%$.

The deterministic model also provides the optimal investment dynamic, showing that capital invested increases until the temperature anomaly has increased by $+4.3^{\circ} \mathrm{C}$, after which it falls and stabilizes at a negative value. In other words, it is economically advisable to withdraw capital from a firm if the temperature anomaly of the biomass has increased by $+4.3^{\circ} \mathrm{C}$. This corresponds to the "many boats and few fish" problem that makes investing in the fishing industry a less attractive option.

Section 6 describes the theoretical difficulties in fusing the two models into one and discusses using stochastic components in static models. The analysis presented in this section also justifies separating oscillation and reduction in biomass, since isolating them allows for a more direct estimation of their impact on firm value, although the literature suggests they occur together. Finally, Section 7 presents the principal conclusions gathered from these models.

## 2 Literature review: fisheries economics, global warming and marine biology

Initially, biology and economics were developed as separate sciences. Starting in the 1960's, research began to acknowledge the connection between the economic problems of fisheries (for example, fleet investment and optimal harvest levels) and biological issues, such as biomass sustainability and diseases in fish populations.

Fisheries economics begins with the work of Christy and Scott (1965), which tackles a number of topics relevant to fisheries, for example, how continual international competition, technological advances and the growing global demand for marine resources create a divergence between economic objectives and resources sustainability, and how fisheries can be regulated to assure resource renewability. Resource renewability is also the cornerstone of the work of Scott (1955), which argues that the sole ownership of a resource will exploit that resource in a sustainable way, based on monopolistic theory, as opposed to the theory of maximum extraction that assumed in a competitive market. Although the focus adopted by Christy and Scott (1965), is slightly more complicated because it assumes that firms are in a competitive market and are subject to international regulations. This paper is developed under the same assumption. In other words, for the purposes of this paper, the fishing industry is understood as the collection of firms that produce goods using common ${ }^{3}$ marine resources or transforms these goods into another product (a process known as "reduction"). Only deep-sea fishing, known for being highly technological and industrialized, is considered; rudimentary, artisan fishing operations are not included.

[^2]Here it is worth noting that the level of harvest proposed by Christy and Scott's (1965) static theoretical model, which by definition does not capture the contingent problems of biomass, actually threatened biodiversity. In fact, the regulations based on static theoretical models exacerbate the ecological damage of exceeding sustainable harvest quotas. Despite this problem, similar models such as those of Beverton and Holt (1957) and Schaefer and Beverton (1963), are used as a basis for more advanced models.

Since the 1970's, models have been incorporating mathematical elements that significantly improve both fishing efficiency and regulations. However, these improved models were not always taken into account by firms and governments facing the pressures of competition and demand, leading to major losses of biodiversity. The work of Pauly et al. (1998) for example, revealed that excessive fishing in tropical regions had reduced predator populations and caused permanent damage to biodiversity.

Most recently Bjørndal et al. (2007) updated a survey on fisheries economics carried out by Bjørndal and Munro (1998).

### 2.1 Fisheries economics: static models

Since GW is a relatively new line of research, before the 1970's fisheries economics research only considered the relationship between biological models and the classical theory of firm. Some of the most important works from this period are Beverton and Holt (1957), Schaefer and Beverton (1963), Gordon (1964), and later, Smith (1968, 1969), who picked up the earlier research, added aspects of the theory of the firm and expanded the analysis to other natural resources. The approach of these models is presented below (based on Bjørndal and Munro, 1998). The analysis focuses on the fishable biomass, that is to say the biomass that supports fish populations that can be industrially extracted. It is assumed that biomasses do not interact with each other, and their movement is affected by (i.) recruitment (new species entering the biomass), (ii.) individual growth, (iii.) natural mortality, and (iv.) fishing mortality (extraction). According to Schaefer and Beverton (1963), if $x$ is fishable biomass:

$$
\begin{equation*}
\dot{x}=z(x)+g(x)-m(x)-f(x, E), \tag{2.1}
\end{equation*}
$$

where $z(x), g(x), m(x)$ and $f(x, E)$ represent recruitment, individual growth, natural mortality and fishing mortality, with $x \equiv d x / d t$. Fishing mortality is dependent on $E$, the 'fishing effort', commonly measured in terms of boat-days per unit of time. These kinds of models are typically simplified due to the fact that it is impossible to know the exact functional form of the right side of equation 2.1. As such, it is assumed that:

$$
\begin{equation*}
\dot{x}=b(x, \bar{A})=b(x), \tag{2.2}
\end{equation*}
$$

where $\bar{A}$ is a constant that represents aquatic environment. In general, when talking about fish as opposed to other resources it is crucial to specify $b(x)$ in an inverted U-shaped curve in the plane $(x, \dot{x})$. The logistic model has been widely used for fisheries, because of the insight it provides. In effect:

$$
\begin{equation*}
b(x)=r x\left(1-\frac{x}{W}\right), \tag{2.3}
\end{equation*}
$$

where $r$ is the intrinsic population growth rate (constant), which incorporates recruitment and mortality, and $W$ denotes the biomass's maximum support capacity. The connection between the firm and the biomass is expressed by harvest. Therefore, according to Schaefer and Beverton (1963), the extraction ${ }^{4}$ function $f(x, E)$ in (2.1) can be expressed as:

$$
\begin{equation*}
h(E, x)=q E^{\alpha} x^{\beta}, \tag{2.4}
\end{equation*}
$$

[^3]where $q, \alpha$ and $\beta$ are constants. In general, it is assumed that $\alpha=\beta=1$ and $q \in[0,1]$. Under this assumption, biomass takes the form:
\[

$$
\begin{equation*}
\dot{x}=b(x)-h(E, x) . \tag{2.5}
\end{equation*}
$$

\]

The steady-state solution $(\dot{x}=0)$ occurs when extraction is positive, or $b\left(x^{*}\right)=h\left(E, x^{*}\right)>0$ with $0<x^{*}<W$. Given the solution $x^{*}$, effort and extraction can be written as a function of $x$, which in the function $b(x)$ gives sustainable yield $\left(Y_{s}\right)$, since $\dot{x}=0$. This is shown graphically in Illustration 1, where $Y_{s}^{*}$ corresponds to the maximum sustainable extraction.

Provided that $h(\ldots)$ is a function of $E$, sustainable yield can therefore also be written as a function of $E$, and is a decision variable of the firm. Therefore, sustainable yield is given by:

$$
\begin{equation*}
Y_{s}=\eta E-\theta E^{2}, \tag{2.6}
\end{equation*}
$$

with $\eta=q W$ and $\theta=q^{2} W / r$, conserving the inverted-U form. Equation 2.6 is the core of the static theory and can be used to find the optimal effort level which allows for the maximum degree of sustainable extraction.

The introduction of the cost function to this scenario is direct, $C(E)=\gamma E$ with $\gamma>0$. Thus, the firm's maximization occurs when:

$$
\begin{equation*}
\max _{\{E\}}[T I(E 0)-C(E)] \geq 0, \tag{2.7}
\end{equation*}
$$

where $T I(\ldots)$ corresponds to total income and $E=E_{0}$ represents the optimal level of effort obtained from (2.6). The firm's static problem is shown graphically in Illustration 2. The solution $E=E_{\infty}$ corresponds to a situation of perfect competition, where profits have been completely dissipated and there is a biological and economical equilibrium.

The first chapters of Anderson (1977) are dedicated to the derivation of this result. An additional microeconomic analysis is included to better compare a competitive situation and a monopoly. However, throughout this paper a competitive market is assumed.

Illustration 1: Sustainable extraction as a function of fishing effort.


Illustration 2: Sustainable income and total cost of fishing effort.
${ }_{\mathrm{Y}}{ }^{\circ}$


### 2.2 Fisheries economics: dynamic models

The referential work for advanced models is Clark (1976), which proposes a complete dynamic theory of the fishing process and includes a comprehensive review of the existing models at that time, and then introduces the theory of optimal control to attain the appropriate level of extraction. The improvements to the static approach are presented below. The firm maintains its goal, now in terms of present value. In effect:

$$
\begin{equation*}
\max P V=\int e^{-\delta t} \pi\left(x_{t}, h_{t}\right) d t \tag{2.8}
\end{equation*}
$$

where $\delta$ is the social discount rate. The profit function corresponds to:

$$
\begin{equation*}
\pi\left(x_{t}, h_{t}\right)=\{p-c(x)\} h_{t} \tag{2.9}
\end{equation*}
$$

where $p$ is the unit price and $c(x)$ is the unit cost. The biomass is still represented by the equation 2.5. The Hamiltonian correspondent is:

$$
\begin{equation*}
H=-e^{\delta t}\{p-c(x)\} h_{t}+\lambda_{t}\left\{b(x)-h_{t}\right\}, \tag{2.10}
\end{equation*}
$$

where $\lambda_{t}$ is the dynamic Lagrange multiplier, which is interpreted as the resource's shadow price. This formulation emphasizes the temporary trade-off firms face between the level of investment to be made per period and the profits obtained in that period.

The solution is the fundamental equation of the utilization of natural resources, set out (for example) in Pearce and Turner (1990) and presented below,

$$
\begin{equation*}
b_{x}+{\frac{\partial \pi / \partial x^{*}}{\partial \pi / \partial h}}_{h=b\left(x^{*}\right)}=\delta . \tag{2.11}
\end{equation*}
$$

Equation 2.11 is interpreted as an investment decision rule: the marginal return on an investment in a resource should be equal to social discount rate. The first term on the left side is the impact of one additional unit of stock on the resource's return, while the second term reflects the fact that the level of stock has a different impact on extraction cost. Clark and Munro (1982), Bjørndal (1987) and Clark (1990) present different methods of deriving this result; which are brought together in this paper. Any differences are due only to the fact that the formulation of the prior equations focused on particular situations.

The extensions of this result are diverse. It is used in Clark et al. (1979) to better understand the effect of irreversible investment on the optimal extraction level, finding that at least in the
short-term, irreversibility is a relevant assumption forcing firms to increase fishing effort. Bjørndal (1987) analyzes the herring in Canada in 1977, where a ban on herring fishing narrowly avoided the extinction of this species in the area. Other applications include the bio-economic modeling of Atlantic Ocean harp seals (Conrad and Bjørndal, 1991), of sharks in the waters south of Australia (Pascoe et al., 1992) and of tiger prawns (a crustacean similar to the lobster) in Australia's Exmouth Gulf (Ye et al., 2005).

In addition to temporary decisions, the dynamic models also tend to be associated with the inclusion of random variables. In Pindyck (1984), a stochastic component dependent on biomass level is included. In that study, the biomass formulation is:

$$
\begin{equation*}
d x=\left\{b(x)-h_{t}\right\} d t+\sigma(x) d z, \tag{2.12}
\end{equation*}
$$

where $z=\varepsilon_{t} \sqrt{d t}$ is a Wiener process, or alternatively, $\varepsilon_{t}$ is a Brownian process. The variable $\sigma(x)$ indicates biomass variability and is specified in such a way that the resource is always nonnegative. The representative biomass described by equation 2.12 has been applied to various problems. De Leo y Gatto (2001) propose a model for the capture of eels on the coasts of Italy. In Levy et al. (2006), the result of (2.12) is extended to capture contingencies that can affect biomass growth. The specified function for biomass in that work takes the following form:

$$
\begin{equation*}
d x=\left\{b(x) s(x)-h_{t}\right\} d t+\sigma(x) d z \tag{2.13}
\end{equation*}
$$

where the function $s(x)$ captures the effect of disaster that reduces biomass.
Chong et al. (2006) take different approach using an advanced and complex mathematical analysis. The work's perspective better captures the time variable, allowing the model to be used to determine the optimum moment for extraction. Chong et al. (2005) use a similar methodology to develop a model for fishing in rivers.

The dynamic approach has also been refined by including rational expectations (Clark, 2007), game theory and incomplete information (Hannesson, 2007; Kobayashi, 2007; Lindroos et al., 2007; McKelvey et al., 2007).

This work adds to the already sophisticated models by including a recent and unprecedented problem, about which little is known and which could affect the performance of fisheries: the warming of the Earth's marine and land surfaces. This problem is considered recent because the trend of rising temperatures is present as recently as 2007, as can be seen in Illustration 3, and is unprecedented because this trend was not observated before 1990, as is depicted in Illustration 13 in Annex A.

### 2.3 On the existence of global warming

This sections review the interpretation and scope of Assumption 1: the average temperature of the Earth's (marine) surface is rising. GW is the increase in the Earth's average temperature due to $\mathrm{CO}_{2}$ emissions that prevent solar radiation absorbed by the Earth from completely returning to the atmosphere. The effect of GW is exacerbated by the emission of greenhouse gases like methane, ozone, nitrogen oxide and others into the atmosphere ${ }^{5}$. Annex A provides a graphic representation of $\mathrm{CO}_{2}$ emissions per continent from the year 1800 to 2000 and the relationship between temperature and $\mathrm{CO}_{2}$ emissions for the years 1000 through 2000, affirming that this relationship is not a cyclical phenomenon, and that GW is indeed a novel phenomenon. This definition does not explicitly differentiate between the causes of GW, since the increase in $\mathrm{CO}_{2}$ emissions can be the result of anthropogenic factors, natural factors (like forest fires) or a combination of both.

Methodologically speaking, the time series confirm that temperature is rising. However, biologically speaking this data should be interpreted with caution since longer time series than those currently available are needed to confirm structural change in ecosystems. Time series are reviewed

[^4]here only for the purpose of illustration. Studies from specific geographic zones better validate Assumption 1. Illustration 3 displays the Global Land-Ocean Temperature Anomaly Index from January 1979 through April 2008 provided by Goddard Institute for Space Studies (GISS) of the US National Aeronautics and Space Administration (NASA). The anomaly appears to have been on the rise since 1993, having increased at the peak of each cycle (3-5 years) between +0.02 and $+0.08^{\circ} \mathrm{C}$ per cycle.

Hansen et al. (2006) thoroughly analyze the GISS series from 1880 through 2005 and finds that from the beginning of last century through 1975, the temperature anomaly was around $+0.2^{\circ} \mathrm{C}$ per decade. However, between 1975 through the turn of the century the anomaly increased to $+0.7^{\circ} \mathrm{C}$. After reaching this point, the authors estimate that it returned to a level of $+0.2^{\circ} \mathrm{C}$ per decade. Illustration 14 in Annex B displays one of the series of temperature anomalies analyzed in Hansen et al. (2006) and the aforementioned results.

As far as rising water temperatures go, there is a great deal of data and specific studies that confirm this trend. Trathan et al. (2007) indicate that GW more severely impacts ecosystems located in low-temperature areas, or in other words the polar circles.

Illustration 15 of Annex B provides a graphic representation of the global oceanic anomalies from 1880 through 2005 from the GISS database. Although in comparison with land anomaly series the increase in temperature is less, ocean temperatures have also been on the rise since 1993.

Illustration 3: Global Land-Ocean Temperature Anomaly Index, 1979-2008.1Q.


Source: Goddard Institute for Space Studies, http://data.giss.nasa.gov.
Quayle et al. (2002) found that the temperature of Signey Island, located among the South Orkney Islands in the Antarctic Ocean (see Annex C), has increased by $+0.8^{\circ} \mathrm{C}$ in the last 50 years. 1998's ENSO phenomenon is also a relevant case study for GW research, since as Thompson and Ollason (2001) indicated, long and short-term changes affect ecosystems and ENSO was a sudden, short-term change with long term consequences. Forcada et al. (2006) found that ENSO increased the temperature of the South Orkney archipelago by $+2.0^{\circ} \mathrm{C}$. Chan and Liu (2004) also documented some of the consequences of this phenomenon, finding that the frequency of typhoons in the Asian-Pacific Ocean increased due to ENSO.

Trathan et al. (2007) also argued that since the Antarctic, Pacific, Atlantic and Indian Oceans are connected, the effects of higher temperatures will be felt throughout the entire southern hemisphere, from the arctic poles to the tropical zones, and will permanently affect the ecosystems of all these oceans.

In an important study, Johnson et al. (2007) collected temperature data from the Pacific Ocean floor and found that seafloor temperatures, like surface temperatures, are also on the rise.

The situation in the northern hemisphere appears to be quite similar. Illustration 16 from Annex B compares the temperature anomaly series from both hemispheres and reveals that as of 1987 the
average anomalies in the northern hemisphere are increasingly higher than those in the southern hemisphere. In 2005 the temperature anomaly in the northern hemisphere was approximately $+0.75^{\circ} \mathrm{C}$, while in the south it was around half this $\left(+0.36^{\circ} \mathrm{C}\right)$. Illustration 17 from Annex B verifies this behavior, presenting the anomaly series from between 90 and $23.6^{\circ} \mathrm{N}$ (the most arctic two thirds of the northern hemisphere) finding that the anomaly in this zone increased by around $+1.0^{\circ} \mathrm{C}$ in 2005.

On a global scale, a study by Goreau et al. (2005) takes a look at temperature change by dividing the earth into 21 oceanic zones and finds that since 1980 temperatures have been on the rise in all zones, including interior oceans.

The Intergovernmental Panel on Climate Change (IPCC, 2001) forecast future GW in order to assess its impact and work on political policies concerning elevated temperatures. Their projections through the year 2100 are presented in Annex D. The IPCC forecast an increase of between +1.0 and $+6.0^{\circ} \mathrm{C}$, which is used as the basis for the estimates found in the numerical findings section (Section 5).

### 2.4 Literature review: global warming

The research on GW that has been applied to economic phenomena is still being developed. One of the most significant problems researchers face is that the inter-sectoral consequences of GW are relatively unknown. This is known as the aggregation problem (Fankhauser et al., 1997). The fact that GW has only recently been recognized as a problem also contributes to the uncertainty surrounding its consequences.

A good introduction to the literature is a survey by Peterson (2006) which discusses recent discoveries concerning the economic consequences of GW. However, the studies reviewed by Peterson are multidisciplinary and there are no concrete principles used across models, leading to diverging estimates. Bosello et al. (2007), for example, predicts the economic consequences of rising sea level in coastal zones due to the melting of ice masses on land ${ }^{6}$. The estimates show that rising sea levels will create an economic loss, but establishing policies and protective technology to prevent these losses would create even more losses. The losses are asymmetric, and although the agricultural and livestock sectors in an economy could benefit from higher temperatures, the fishing sector could be seriously damaged. In other words, one sectors gain is less than the other's loss.

Estimates can be made at an aggregate level to avoid this difficulty. For example, Fankhauser and Tol (2002) adapts the Ramsey-Caas-Koopmans growth model to learn more about the macroeconomic effects of GW and conclude that it reduces savings and lowers the capital accumulation. Dumas and Ha-Duong (2008) assume growth with a GW adaption strategy that consists of protecting capital. They show that its early implementation would have negligible effects on annual consumption, with losses of $0.44 \%$ per year in the worst case and $0.00005 \%$ in the best case. Hübler et al. (2007) develop a deterministic model, calibrated for Germany, which finds that productivity falls and generates yearly gross domestic product (GDP) losses between $0.1 \%$ and $0.5 \%$. However, not all the results are categorical. Tol (2002) calculates how an average temperature increase of $+1^{\circ} \mathrm{C}$ affects GDP, resulting in $+2,-3$ and $0 \%$, depending on the aggregation method. A comprehensive study, the Stern Review (Stern, 2006), attempts to provide a base for a standard analysis of GW, but for the purpose of this paper it represents a generalization and does not provide the necessary depth.

At time, there is only one study directly related to the effect of GW on the fishing industry that uses elements of fisheries economics. Arnason (2007) assumes that temperature - which is considered an input in the production function-, is a Brownian process that directly impacts the firm. This paper, on the other hand, takes an additional step in-between, considering first the effect of temperatures on biomass and only then, how changes in biomass affect firm value. Also, this paper only considers the increase in temperature.

[^5]Arnason's model makes its empirical estimate using the Solow decomposition method. According to this method, any change that cannot be attributed to another factor, is said to be caused by temperature. This could include changes in technology and temporary changes in the fishing efficiency among other factors. The model is calibrated for Greenland and Iceland and the results are similar to what is found in this paper, although they cannot be directly compared since this involve data from different geographical zones.

This paper and Arnason's work also differ in that this paper provides a more detailed model of how higher temperature are transferred onto firm value, and separates the two effects of temperature on biomass (oscillation and reduction). That said, Arnason's work is the best benchmark from current literature.

### 2.5 Literature review: the effects of global warming on biomass

This subsection reviews the literature on Assumption 2: global warming affects biomass. This is not intended to be an exhaustive review; rather it is a way of orientating and refining how this assumption is interpreted. Hannesson (2004) develops an economic model under a similar assumption in an effort to learn more about how species migration due to rising water temperatures affects firms, and finds that it is possible to quickly reach a level of extraction that is not economically viable. In finding that species migrate faster than firms can withdraw capital, which creates a very risky situation for the industry, this work is relevant to this paper.

While the fact that biomass is always changing due to natural causes is an important consideration, Pauly et al. (1998) shows that in the tropics the biggest biomass fluctuations are a result of anthropogenic factors, that is to say, human activity. Likewise, Christensen et al. (2002) estimates that since 1960 the biomass of pelagic fish species on the African coast has fallen by as much as 13 times due to a number of factors, including temperature.

It is important to reiterate that this paper only refers to those changes in biomass caused by increasing temperature, and only reviews the pertinent literature. For example, Suárez et al. (2004) analyzes the movements of ENSO toward the southern Pacific Ocean, in particular focusing on the biomass of a commercially very important species: tuna. One of the important conclusions from this work is that the reduction in tuna biomass exceeds recovery 3 to 1 . In other words, the biomass lost in one period is recovered over the following three periods. In a study of the northern Pacific, Hernández et al. (2004) find that ENSO was associated with the loss of 200 million tons of pelagic species.

Another way of proving the effect of temperature on biomass is by studying the behavior of predators in a set geographic area (Thompson and Ollason, 2001). This is the technique used by Trathan et al. (2003) in a study of krill, the main food source of predators in the Antarctic Ocean, in which the close relationship between temperature and the abundance of Antarctic krill is shown. This is consistent with the research of Trathan et al. (2006), which documents how variations in the krill stock due to ENSO caused species that depend on krill to survive to migrate, thus lowering biomass. Brierly et al. (1999) documents how the inter and intra-annual variations in krill affected the biomass in sectors near the South Georgia Islands in the Antarctic Ocean (see Annex C). Murphy et al. (2007) estimates that an increase of $+1.0^{\circ} \mathrm{C}$ in the Scotia Sea (also in the Antarctic Ocean) over 100 years would reduce the biomass and abundance of krill by $95 \%$.

In a study focusing on the coastal areas surrounding Tampa Bay in the US, Lipp et al. (2001) finds that higher temperatures incubate sicknesses and negatively affect biomass in a phenomenon known as acidification.

In the polar zones in the Northern hemisphere, ice thaws have also been studied as one of ways that GW affects biomass. Ice thaws influence water density and effect thermohaline circulation ${ }^{7}$.

[^6]Link and Tol (2005) show that changes in this circulation cause a significant reduction in the stock of cod and capulin ${ }^{8}$ in the Barents Sea, to the north of the Scandinavian Peninsula.

Stein (2007) finds that the marine temperature on the coasts of Greenland has increased by $+2.0^{\circ} \mathrm{C}$, damaging the stock of cod and pollock, two species with very high commercial value.

The literature on fisheries economics also includes studies on how anthropogenic factors affect nature. For example, Levy et al. (2006) develops a model that incorporates possible disasters caused by excessively high quotas, higher fishing efficiency and government subsidies. Industrial contamination is also a factor. Other works that include relevant biomass issues are cited in subsection 2.2.

Other fisheries economics studies, like Christensen et al. (2002), include temperature as part of their models, but only as a proxy for water salinity, an indicator of biomass quality.

This review is meant to contextualize the two assumptions and also to serve as an introduction to fisheries economics. Based on the literature, it can be concluded that higher temperature (i) causes biomass oscillation, and (ii) reduces biomass. This work models and quantifies the economic impacts that both oscillation and reduction in biomass have on fishing firms. In the case of oscillation, the Pindyck (1984) model is modified to better isolate biomass shocks. The second effect, reduction of biomass, can be measured by contrasting high temperature situations. The Smith (1968) model is updated for this purpose.

## 3 Stochastic biomass model: global warming shocks

This section develops a model for a fishery that extracts resources from a stochastic biomass. It is modeled in such a way that temperature, an exogenous factor, is the cause of biomass oscillation. The model is inspired by Pindyck (1984), but differs in the sense that temperature is relevant to firm value. Although this work complements that of Arnason (2007), the approach developed here is different. Arnason's work assumes a Brownian motion of temperature of the form:

$$
\begin{equation*}
d T_{t}=\mu_{t} d t+\sigma_{t} d z, \tag{3.1}
\end{equation*}
$$

and includes the variable $T$, temperature, as a fishing firm input to carry out a Solow decomposition. In this way, the impact of temperature change on the firm's value is understood. As mentioned, this work is different in that it assumes that temperature is always increasing $(d T / d t>0)$, and therefore analyzes the effect of stochastic biomass, not stochastic temperature, on the fishery's value.

### 3.1 Assumptions

It is assumed that there is perfect competition on the fishing market for final product, which means one firm cannot influence the market price. The fishing industry is understood as the collection of firms that produce goods using marine resources or transform them into another product. Based on the two previously mentioned assumptions, temperature increases that affect biomass are assumed to exist. Resources are extracted directly from the ocean, and species are treated as public goods, not common goods ${ }^{9}$ (which they really are). Hence, the firm's cost function is $c(x, j)=c(x)$, with $j \in \Upsilon=\{1, \ldots, J\}$, where $\Upsilon$ is the collection of firms that participate in the industry.

The $T$ variable, temperature, is the first difference between this study and Pindyck's work, and has only been included in Arnason's work, as mentioned above. Although temperature is included in other models as an explanatory variable ${ }^{10}$, in these cases it is only used as a proxy for water salinity, since this determines a biomass's maximum capacity. Temperature movements in this context do not necessarily involve biomass oscillations.

[^7]In this work, the $T$ variable causes biomass variations. It can be measured in traditional units $\left({ }^{\circ} \mathrm{C},{ }^{\circ} \mathrm{F}\right.$ or $\left.{ }^{\circ} \mathrm{K}\right)$, and can be defined as a continuous, increasing function of effective temperature $(E T)$, i.e. $T=v(E T)$, with $v_{E T}>0$. An alternative specification, which is useful for model calibration, is to define the variable in terms of categories or groups according to the GW projections specified by IPCC (2001). $T: E T \rightarrow \mathbb{R}^{+}$is assumed only because of its simplicity.

The social interest rate is $\delta$, which reflects the alternative cost of any investment in the economy. Capital is assumed to be homogenous, which is consistent with the assumption that there are no entry or exit barriers in a perfectly competitive market. Despite the fact that there are no barriers, there is incomplete information concerning when a shock occurs. In other words, the firm cannot know when a negative shock will occur, although it can know the variance of the biomass. This lack of information can potentially lead to short-term losses. However, the firm's reaction does not perpetuate negative results since, as proposed by Dumas and Ha-Duong (2008), firms have strategies for accommodating biomass shocks. This also goes along with the conclusions of Clark et al. (1979) regarding the high cost of adjusting capital investment, which show that in the short-term a firm will face financial stress, but in the long-term it will return to its competitive position.

Two elements are considered in the response to GW: spending per period on mitigating the problem and the direct economic impact of biomass variability. The reaction can be interpreted as a costly adjustment to new technology, since a fleet designed to extract from a biomass with a given oscillation must increase effort to compensate for lower production due to increasing oscillation. A firm can adjust either by facing a higher number of unfavorable events until completing the learning process or by spending more on adjusting to the problem. Both solutions are expensive.

### 3.1.1 Model

The model can be divided into two parts: biological (biomass) and economic (firm).

### 3.1.2 Biomass

According to Pindyck (1984) and Levy et al. (2006), the stochastic biomass responds to unanticipated movements in the components of equation 2.1 and its synthesized version (3.2). The model assumes that variability is a function of temperature, $\sigma=\sigma(T)$, which has two precautions with respect to the traditional formulation $\sigma=\sigma(x)$. First, that $\sigma(T)$ can be found by specifying an equation for how temperature affects fish metabolism and second, that $T$ is variable that cannot be controlled. Even with these considerations, it is biologically complex to establish the exact form of the mentioned functions. For this reason, biomass is given by equation 3.2:

$$
\begin{equation*}
d x=\left\{b(x)-h_{t}\right\} d t+\sigma(T) x d z, \tag{3.2}
\end{equation*}
$$

conserving the notation from the previous sections.

### 3.1.3 Firm

When temperature increases biomass variability, firm harvest is lower due to the fact that firm technology is not designed for the more difficult extraction that greater biomass variability entails. In this context, $i$ can be defined as the cost incurred by the firm to carry out an extraction plan that allows it to maintain its competitive position. That is to say that $i$ is defined as the firm's expenditure exclusively due to greater stock volatility. For simplicity, it is assumed that the transfer is direct, of the form ${ }^{11}$ :

$$
\begin{equation*}
d T=i d t . \tag{3.3}
\end{equation*}
$$

[^8]On the other hand, a fishery's reaction to GW occurs in the context of profit maximization and the consequent knowledge attained about how to deal with a biomass with greater oscillation. This gives:

$$
\begin{equation*}
G(i)=\frac{\sqrt{i}}{g} \tag{3.4}
\end{equation*}
$$

called the function for the firm's total expenditure for GW. Generically, it is required that $G_{i}>0$ and $G_{i i}<0$. Firms could respond in different ways, but because of the depth of the effect on the ocean, stand-alone solutions are not considered ${ }^{12}$.

From equation 3.4, it is possible to affirm that knowledge is attained ( $G(i)$ falls) as $i$ increases because $G(i)$ is concave in $i$ and also directly as a result of increases in $g$. Note that if $g=0$ then $G(i)=\infty^{+}$and the firm leaves the market.

The profit function is given by:

$$
\begin{equation*}
\pi(x, h, i)=\int_{0}^{h}\{p(h)-c(x)\} d h-G(i), \tag{3.5}
\end{equation*}
$$

where $p(h)$ is the demand function. The function $c(x)$ represents the marginal cost per unit and is decreasing in $x$. Note that:

$$
\begin{equation*}
\lim _{i \rightarrow \infty} \frac{\partial G(i)}{\partial i}=\frac{\partial G(i)}{\partial i}_{g=\infty}=0 \tag{3.6}
\end{equation*}
$$

Increases in $i$ as well as $g$ signify a proactive adaptation strategy where short-term losses are expected in order to gain biomass risk reduction know how. This also allows for the possibility of acquiring and/or maintaining a competitive position, at least in the short-term.

This function incorporates the effect of GW on profit, abstracting it from the effect on harvest ${ }^{13}$. In effect:

$$
\begin{equation*}
h(E, x)=q E^{\alpha} x^{\beta} . \tag{3.7}
\end{equation*}
$$

For simplicity, it is assumed that $E, \alpha, \beta=1$ and $q=0.10$.

### 3.1.4 Equilibrium and model dynamics

This subsection closely follows the derivation of Jin and Herrera (2005), who present the problem of extracting from a stochastic biomass where variability is reduced by research. The problem of maximization on an infinite horizon means repeating the maximization infinite times. Since the formulation is similar each time, one period can be optimized to find the solution for the infinite horizon. This is the Bellman equation, which for the firm corresponds to the maximization of equation 3.8 subject to equation 3.2 :

$$
\begin{equation*}
\delta V(x, T)=\max _{\{h, i\}}\left\{\pi(x, h, i)+\frac{d}{d t} E_{t} V(x, T)\right\} . \tag{3.8}
\end{equation*}
$$

This expression is equivalent to equation 9, p. 293, in Pindyck's work, except that there the univariate case is presented. The first term on the right side corresponds to the current profit, while the second term on the right represents the expected appreciation. $h$ and $i$ are the decision variables and $x$ and $T$ are the state variables. To find the solution, the first-order conditions (FOC) are derived, noting that the second term on the right side is a diffusion process whose stochastic differential can be found using Itô's Lemma:

$$
\begin{equation*}
d V=\frac{d V}{d t}+V_{x} d x+V_{T} d T+\frac{1}{2}\left\{V_{x x}\left(d x^{2}\right)+V_{T T}\left(d T^{2}\right)\right\}+V_{x} T \tag{3.9}
\end{equation*}
$$

[^9]Since the problem is time-independent, $d V / d t=0$. Substituting in the equations for $d x$ and $d T$, and applying Itô's Lemma:

$$
\begin{equation*}
d V=\left[V_{x}\{b(x)-h\}+i V_{T}\right] d t+\frac{1}{2} V_{x x} \sigma^{2} x^{2} d t+\sigma V_{x} x d z \tag{3.10}
\end{equation*}
$$

Considering that $E_{t} d z=0$, since $z_{t}$ is a Wiener process with mean zero, and substituting equations 3.5 and 3.10 into equation 3.8 gives:

$$
\begin{equation*}
\delta V(x, T)=\max _{\{h, i\}}\left[\int_{0}^{h}\{p(h)-c(x)\} d h-G(i)+V_{x}\{b(x)-h\}+i V_{T}+\frac{1}{2} V_{x x} \sigma^{2} x^{2}\right] . \tag{3.11}
\end{equation*}
$$

The FOC are:

$$
\begin{gather*}
\delta \frac{\partial V(x, T)}{\partial h}=\{p(h)-c(x)\}-V_{x}=0 \Longrightarrow\{p(h)-c(x)\}=V_{x}  \tag{3.12}\\
\delta \frac{\partial V(x, T)}{\partial i}=-G_{i}(i)+V_{T}=0 \Longrightarrow G_{i}(i)=V_{T} . \tag{3.13}
\end{gather*}
$$

Both FOC represent partial results of the economic effects of GW. Later, the direct effects of GW on the fishing industry in terms of social welfare will be derived.

The first FOC represents the standard condition of optimality for $h$. The marginal extraction value is equal to the shadow price of one additional extracted unit. The second FOC represents firm transfer when faced with a shock, and may be different for each firm. It is interpreted as the marginal expense incurred by the firm, which is equivalent to the change in firm value because of an increase in temperature. In summary, this FOC can be used to find the optimal firm response when faced with temperature shocks.

The value of $V_{T}$ will be less as $G_{i}$ falls. This occurs with increases of $i$ and/or $g$, that is to say, when the impact is perceived as high, and spending on mitigating GW increasing. The firm is completely isolated from temperature when $g \rightarrow \infty$ and/or $i \rightarrow \infty$.

To determine how the industry is affected, the optimal values for $h^{*}$ and $i^{*}$ are substituted into equation 3.8 and the first derivative with respect to $x$ is found. In effect:
$\delta V_{x}=\left\{p\left(h^{*}\right)-c(x)-V_{x}\right\} \frac{d h^{*}}{d x}-c_{x} h^{*}+b_{x} V_{x}+\sigma^{2}(T) x V_{x x}+\left\{b(x)-h^{*}\right\} V_{x x}+i V_{T x}+\frac{1}{2} \sigma^{2}(T) x^{2} V_{x x x}$.
Deriving the result of $d V$ with respect to $x$ (equation 3.10) gives an expression that contains the last three terms on the right side of equation 3.14:

$$
\begin{equation*}
\frac{d}{d t} E_{t} V_{x}=\left\{b(x)-h^{*}\right\} V_{x x}+i V_{T x}+\frac{1}{2} \sigma^{2}(T) x^{2} V_{x x x} \tag{3.15}
\end{equation*}
$$

and from the FOC 3.12 it follows that this expression is equivalent to:

$$
\begin{equation*}
\frac{d}{d t} E_{t} V_{x}=\frac{d}{d t}\left\{p\left(h^{*}\right)-c(x)\right\} . \tag{3.16}
\end{equation*}
$$

Substituting this result into equation 3.14 and considering that the first term is zero (due to the CPO 3.12) gives:

$$
\begin{equation*}
\delta V_{x}=-c_{x} h^{*}+b_{x} V_{x}+\sigma^{2}(T) x V_{x x}+\frac{d}{d t}\left\{p\left(h^{*}\right)-c(x)\right\} . \tag{3.17}
\end{equation*}
$$

Combining similar terms, simplifying and solving gives a modified version of the fundamental equation of the utilization of natural resources, similar to equation 18, p. 294, in Pindyck's work:

$$
\begin{equation*}
\delta+\sigma^{2}(T) x A R A(x, x)=b_{x}+\left[\frac{d\left\{p\left(h^{*}\right)-c(x)\right\} / d t}{p\left(h^{*}\right)-c(x)}-\frac{c_{x} h^{*}}{p\left(h^{*}\right)-c(x)}\right], \tag{3.18}
\end{equation*}
$$

where $A R A(x, x)=-V_{x x} / V_{x}$ is the coefficient of absolute risk aversion (Pratt, 1964). The left side of the equation shows that the opportunity cost increases when biomass oscillates. The right side shows that the profit in-situ fish unit breaks down into the profit conferred by greater biomass availability $\left(b_{x}\right)$, plus the economic change divided into (i) earnings due to higher margins, and (ii.) reduction of the marginal cost.

This equation can be used to assess the economic effect of biomass oscillation on the fishing industry. The opportunity cost increases because $\sigma(T) \neq 0, \sigma_{T}>0, x>0$ and $A R A(x, x)>0$. This result is an algebraic representation of GW's harmful effect on the industry. The traditional proposal for the extraction of natural resources is returned to in the event that the resource is completely controlled.

The same methodology is used to determine the increase in opportunity cost caused by a rise in temperature. Deriving equation 3.14 with respect to $T$ gives:

$$
\begin{equation*}
\delta V_{T}=\left\{p\left(h^{*}\right)-c(x)-V_{x}\right\} \frac{d h^{*}}{d T}+\sigma_{T} \sigma(T) x^{2} V_{x x}+\left\{b(x)-h^{*}\right\} V_{x T}+i V_{T T}+\frac{1}{2} \sigma^{2}(T) x^{2} V_{x x T} . \tag{3.19}
\end{equation*}
$$

On the other hand, the derivative of $d V$ (equation 3.10) with respect to $T$ equals:

$$
\begin{equation*}
\frac{d}{d t} E_{t} V_{T}=\left\{b(x)-h^{*}\right\} V_{x T}+i V_{T T}+\frac{1}{2} \sigma^{2}(T) x^{2} V_{x x T} \tag{3.20}
\end{equation*}
$$

Substituting into equation 3.19, simplifying, dividing by $V_{T}$ and noting that the first term is zero (because of FOC 3.12), gives:

$$
\begin{equation*}
\delta+\sigma_{T} \sigma(T) x^{2} A R A(x, T)=\frac{\frac{d}{d t} E_{t} V_{T}}{V_{T}} \tag{3.21}
\end{equation*}
$$

where $\operatorname{ARA}(x, T)=-V_{x x} / V_{T}$ is an indicator of the coefficient of absolute risk aversion. On the left side an increase in firm opportunity cost due to biomass variability and the fact that $\sigma(T)>0$, $\sigma_{T}>0$ and $A R A(x, T)>0$ can be observed. Optimally, higher opportunity cost is equal to the expected change in firm value due to temperature increase (in percentile units).

The result gives the coefficient for the transfer of higher temperature onto the fishery's opportunity cost. From FOC 3.13 it is inferred that:

$$
\begin{equation*}
\frac{\frac{d}{d t} E_{t} V_{T}}{V_{T}}=\frac{\frac{d}{d t} G_{i}}{G_{i}} . \tag{3.22}
\end{equation*}
$$

That is to say, once knowledge has been attained and/or adaptation is complete, spending on this costs the same as any other investment in the economy. Thus, equation 3.22 summarizes the economic disincentive caused by biomass oscilation.

## 4 Deterministic biomass model: the direct effect of global warming

This section studies the other effect of GW from a different perspective than the one used in the previous section. Keep in mind that the effects are complementary and occur simultaneously. Later in the paper the theoretical difficulties in joining these two models will be explained.

A modification to the fundamental equation of the utilization of natural resources is proposed in this model due to the introduction of $T$.

The harvesting path that maximizes firm value is derived from the dynamic models. This section models how GW-induced temperature increase damages harvesting path, and thus, firm value. The introduction of $T$ is associated to a decrease in biomass and the consecuent rise in funding cost and risk.

Assuming the biomass and profit function are dependent on $T$, the discount rate therefore includes an element of risk when it is influenced by an exogenous circumstance, as described in equation 4.1:

$$
\begin{equation*}
b_{x}(x, T)+{\frac{\partial \pi(T, \ldots) / \partial x^{*}}{\partial \pi(T, \ldots) / \partial h}}_{h=b\left(x^{*}, T\right)}=\delta(T), \tag{4.1}
\end{equation*}
$$

with $\delta_{T}>0$. Comparative statics are used because of the simplified view of the impact of less biomass on the firm that they provide. If the model is dependent on $x$ and $T$, then the long-term movements (when $\dot{x}=0$ ) are exclusively due to increases in $T$, through a function of mortality and/or species migration.

Unlike the previous model, all of the equations are deterministic. For simplicity, the work is mainly framed around Smith (1968) model. However, this study is an advance on this model not only because it incorporates GW but also because it calibrates the model specifically for fisheries.

### 4.1 Assumptions

The model is developed for a fishery that participates in a competitive environment. The social interest rate is $\delta(T)$ with $\delta_{T}>0$, and can be asymmetrical depending on the sign of $\pi(\ldots)$. In effect:

$$
\delta(T)= \begin{cases}\delta_{1}(T) & \text { if } \pi \geq 0  \tag{4.2}\\ \delta_{2}(T) & \text { if } \pi<0 .\end{cases}
$$

This rate is assumed to be exogenous to the firm but endogenous to the industry. Investment adjustments are assumed to be instantaneous, as are capital increases and reductions.

### 4.1. 1 Model

The model is divided into two parts: biological (biomass) and economic (firm).

### 4.1.2 Biomass

As Smith's work shows, the biomass is the natural resource's "technological restriction": a population that exceeds the biomass's capacity cannot survive. As such, within that environment, the climatic variable is introduced in a manner similar to the methodology of Levy et al. (2006) for the effect of sicknesses, in the sense that a disturbance is added that modifies the deep parameters of recruitment, growth, and mortality. In effect, if $M(T)$ is a function of mortality and/or migration caused exclusively by higher temperature, then the biomass corresponds to:

$$
\begin{equation*}
\dot{x}=b(x) M(T), \tag{4.3}
\end{equation*}
$$

where $b_{x}>0$ if $x \in\left\{x_{m}, x_{e}\right\}, b_{x}<0$ if $x \in\left\{x_{e}, x_{M}\right\}$ and $M_{T}<0^{14}$. All of the variables are time-dependent and each one is represented by a differential equation.

The previous assumptions about $T$ 's measurement are maintained despite the fact that in this model it makes even more sense to define the effect of $T$ by categories, and thus analyze the static comparative of moving from one category to another. This specification also allows nonlinearities of the effect of warming the water on the population to be captured. Based on this argument, $M(T)$ can represent mortality levels (severe $\rightarrow$ mild), depending on the temperature range being measured. Formally it corresponds to a function $M(T)$ that collapses the effective temperature into some category, which numerically defines the effect on the population:

$$
\begin{equation*}
M(T): \mathbb{R}_{T}^{+} \rightarrow M\left(\omega_{i}\right) \rightarrow \mathbb{R}_{\tau}^{+} \tag{4.4}
\end{equation*}
$$

[^10]As such, each $T$ has a correspondent in the $\tau$ set:

$$
\begin{equation*}
T: \mathbb{R}_{T}^{+} \rightarrow \tau \tag{4.5}
\end{equation*}
$$

The set $\tau$ is a finite union that excludes subsets $\omega_{i}$ :

$$
\begin{gather*}
\tau=\bigcup_{i=1}^{\infty} \omega_{i} ; \bigcap_{i=1}^{\infty} \omega_{i}=\varnothing  \tag{4.6}\\
\omega_{i}=\left[T_{i-j}, T_{i}\right] ; i, j \in \mathbb{R}^{++} ; i>j \tag{4.7}
\end{gather*}
$$

In this way, the temperature interval $\omega_{i}$ produces a lower rate of biomass mortality than $\omega_{i-j}$, with $i>j$. There exist inambiguity on the effect caused by temperature $T=T_{0}$ on the mortality $M\left(T_{0}\right)=M_{0}$, but not in that mortality $M_{0}$ is due uniquely to temperature $T_{0}$.

### 4.1.3 Firm

The firm's decisions are synthesized in the dynamics of the invested capital. The reason for this is that the firm always extracts the maximum amount permitted by the biomass subject to its capital restriction, then, the optimal extraction decision is subordinate to the investment decision.

Investment is $K$ (i.e. boats). There is an immediate capital adjustment, and thus if $K$ corresponds to an acquired boat, all are assumed as equal and active secondary market is also assumed. The cost function of the representative firm is:

$$
\begin{equation*}
C(h, x, K, T)=\varphi(h, x, K)+G(h, T, M(T)) \tag{4.8}
\end{equation*}
$$

resembling equation 3.2 in Smith's work, p. 413. Harvest (extraction) corresponds to $h \in$ $\left[x_{m}, x_{M}\right]$, where $x$ is still fishable biomass.

Based on an argument similar to that presented in the previous section, the function $G(h, T, M(T))$ represents the firm's response to temperature increases. It is assumed that $G_{h}>0, G_{T}>0$ and $G_{M(T)}<0$. On the other hand, and following $\operatorname{Smith}$, the function $\varphi$ is characterized by $\varphi_{h}>0$, $\varphi_{x} \leq 0$ and $\varphi_{K} \geq 0$.

The term $\varphi_{x}<0$, called stock externality, implies that improvements in biomass quality are interpreted as a less costly harvest. The term $\varphi_{K}>0$, called crowding externality, appears when the amount of boats is increased above the optimal level, causing congestion in resource extraction. Under this condition, fish cease to be public goods and become rival goods. Consequently, this serves as the capital adjustment mechanism: above average profits create incentive for new competitors to enter, which in turns generates crowding externalities that increase the cost of extraction until profits return to their normal level. The industry's competitive environment is constructed by applying the same logic to stock externalities.

Each boat allows for a maximum extraction level $h$, where the firm's total extraction is $K h$. With this intervention, biomass takes the form:

$$
\begin{equation*}
\dot{x}=b(x) M(T)-K h . \tag{4.9}
\end{equation*}
$$

The firm's total income depends on the level of extraction and the level of capital invested $\rho(K h)$, therefore the profit is:

$$
\begin{equation*}
\pi(h, K, T)=\frac{\rho(K h)}{K}-C(h, x, K, T) \tag{4.10}
\end{equation*}
$$

where $\rho(K h) / K$ is the income obtained by extraction $h$. The industry's price level, then, is $\rho(K h) / K h$. In a perfect competition environment, it holds that:

$$
\begin{equation*}
\frac{\rho(K h)}{K h}=\varphi_{h}+G_{h} \tag{4.11}
\end{equation*}
$$

equivalent to Smith's equation 4.2 , p. 414. New firms enter the market when they observe $\pi>0$, and firms that are already participating in the market leave when $\pi<0$. This decision is in line with the amount of capital invested, and therefore the dynamic equation corresponds to:

$$
\begin{equation*}
\dot{K}=\delta(T)\left[\frac{\rho(K h)}{K}-C(h, x, K, T)\right], \tag{4.12}
\end{equation*}
$$

with:

$$
\begin{equation*}
\frac{\partial \dot{K}}{\partial T}=\delta_{T}\left[\frac{\rho(K h)}{K}-C(h, x, K, T)\right]-\delta(T) C_{T} \tag{4.13}
\end{equation*}
$$

When equation 4.13 is positive the firm remains in the industry, although it requires more capital in order to compensate for losses due to GW. Nevertheless, this capital is invested with a lower rate of return, since the opportunity cost is greater $\left(\delta_{T}>0\right)$. This partial result provides a picture of the mechanism through which higher temperature affects investment dynamics, making the industry less attractive.

From a dynamic perspective, for there to be investment, profits must continue to increase in order to compensate for the cost $\delta(T) C_{T}$ (increasing in $T$ ), even though returns are still smaller.

### 4.1.4 Development and model equilibrium

The model is summarized by the following system of equations:

$$
\begin{gather*}
\dot{x}=b(x) M(T)-K h,  \tag{4.14}\\
p=\varphi_{h}+G_{h},  \tag{4.15}\\
\dot{K}=\delta(T)[p h-C(h, x, K, T)], \tag{4.16}
\end{gather*}
$$

because price is equal to marginal cost all the time, equation 4.15 is solved instantaneously, and $h$ is exogenously determined. Then, the dynamic system to solve is:

$$
\begin{align*}
& \dot{x}=F(x, K, T),  \tag{4.17}\\
& \dot{K}=I(x, K, T), \tag{4.18}
\end{align*}
$$

with initial conditions $x(0)=x_{0}$ and $K(0)=K_{0}>0$. Should price movements cause the margin per unit to fluctuate, the form of $I(x, K, T)$ is nonlinear. On the other hand, if the price is constant, then the form is a horizontal line in the plane $(x, K)$.

Illustration 4 presents the model's solution in a phase diagram, that is to say, when $\dot{x}=K=0$. $F\left(x^{*}, K^{*}, T\right)=0$ corresponds to a point of biological and economic equilibrium $\left(x^{*}, K^{*}\right)$, which represents equilibrium between resource biomass and its environment. $I\left(x^{*}, K^{*}, T\right)=0$ represents equilibrium between the resource exploiting firm and any investment made in the economy. The phase diagram indicates how quickly equilibrium can be reached from any point in the plane, starting from initial conditions. Superimposing both equations divides the first quadrant into five regions. Each region contains the direction from a point towards the steady-state equilibrium.

Without the firm's intervention the equilibrium is $x=x_{M}$. With the firm's introduction, there are two equilibriums, $P^{I}$ and $P^{I I}$, both of which are unstable. As indicated in Illustration 4 the firm rests on point $P^{I I}$, which corresponds to the equilibrium reached once $x_{M}$ has been abandoned.

However, the phase diagram shows equilibrium in a steady-state. Assuming that temperature is non-stationary, its increase moves the curves in the direction shown in Illustration 5 . In such a situation, and following the previous logic, the new equilibrium occurs at point $P^{I I I}$, with
lower capital levels extracting fewer resources. The move from $P^{I I}$ to $P^{I I I}$ implies firms leaving the industry and lowered biomass capacity.

If the firms continue to operate with the same technology, the result of the exercise is predictable: the temperature increase moves the equilibrium to a point $\left(0, K_{t}>0\right)$, similar to $P^{I V}$ in Illustration 6. In this scenario $h^{*}=0$ since $h<x_{m}$, coinciding with a capital investment level with a return rate of $\infty^{-}$if $\lim _{h \rightarrow 0} C(h, x, K, T)=\infty$, with $K_{0}>0$. In other words, for a positive level of initial capital, the firm that does not extract resources gets a return rate of $\infty^{-}$for that capital.

The definition of the firm's value considered in this model is expressed as ${ }^{15}$ :

$$
\begin{equation*}
V=[p h-C(h, x, K, T)]-\dot{K}[\delta(T)] . \tag{4.19}
\end{equation*}
$$

From this equation it can be concluded that while $p h$ increases monotonically by increments of $h$ ( $p$ is constant), the cost function $C(h, x, K, T)$ increases through $h$ and $T$, reducing the firm's value.

In the same way, the effect of greater investment is added through $\dot{K}$, since it is expected to increase up to an economically sustainable level and then decline as a result of the "many boats and few fish" effect, accompanied by a return that makes investment less and less attractive. This argument proves that the non-stationary nature of temperature has harmful effects on the industry, even in the long-run.

Illustration 4: Initial phase diagram.


Source: Own elaboration.

[^11]Illustration 5: Intermediate phase diagram.


Source: Own elaboration.

Illustration 6: Final phase diagram.


## 5 Numerical findings

This section presents some of the numeric results from both models, calibrated according to the studies presented in Section 2.

### 5.1 Stochastic biomass model

The analytical form of the model is similar to that presented in Charles (2007), Doyle et al. (2007), Levy et al. (2006), McDonald et al. (2002), Munro (1992) and Pindyck (1984). These works use equations similar to the equations $5.2,5.3,5.4$ and 5.6 shown in Table 1. The parameters used are taken from Clark (2007), McDonald et al. (2002), and De Leo and Gatto (2001), despite being responses to different situations than those presented in this paper. Nonetheless, these parameters are used because they provide a convenient description of a mid to large-sized fishery that does not affect industry price levels.

Since it is difficult to know the exact analytical form of a biomass variation function, the function $\sigma(T)$ takes on different values in the biomass equation (3.2):

$$
\begin{equation*}
d x=\left\{b(x)-h_{t}\right\} d t+\sigma(T) x d z . \tag{3.2}
\end{equation*}
$$

The values are considered reasonable in light of a review of Murphy et al. (2007), Stein (2007), Trathan et al. (2007), Hernández et al. (2004), Suárez et al. (2004) and Christensen et al. (2002). In practice, seven values are considered for $\sigma(T)$ :

$$
\begin{gathered}
\sigma(T) \in \Sigma, \\
\Sigma=\{1 \%, 2 \%, 3 \%, 4 \%, 5 \%, 10 \%, 20 \%\} .
\end{gathered}
$$

Illustration 20 of Annex E shows equation 3.2 with the $\Sigma$ values. Unlikely cases (10 and 20\%) are included to see how robust the results are. The estimation corresponds to the annual value of a firm that extracts resources from a biomass with different volatilities. The definition of value is the result of annual profit maximization, subject to the availability provided by the biomass. That is to say, the estimation is given by equation 5.1:

$$
\begin{equation*}
\delta V(x, T)=\int_{0}^{h}\{p(h)-c(x)\} h_{t}^{*}-G_{i}(i)+V_{x}\left\{b(x)-h_{t}^{*}\right\}+i^{*} V_{T}+\frac{1}{2} V_{x x} \sigma^{2} x^{2} . \tag{5.1}
\end{equation*}
$$

To normalize the units of account, the results are a benchmark for the case $\sigma(T)=0$. Other partial results are not included in order to focus the analysis exclusively on the impact on value.

### 5.1.1 Calibration

The calibration of the equations is presented in Table 1. The values of equation 5.2 are measured in thousands of metric tons and the values of equations 5.4 and 5.5 are measured in monetary units (i.e. millions of dollars). The $a_{2}$ parameter of equation 5.2 is used as an adjustment parameter for units of measurement. Firm spending on GW is assumed to increase as biomass variability rises, according to what is presented in Table 2. An extraction of $h \in[0,100]$ (thousands of tons) is assumed, divided across 1,000 observations. The firm's response series is presented in Illustration 7, this is obtained assuming a temperature transfer equation of $197,894.63 \cdot T^{0.9}$, with $T \in[0,10]$ distributed across 1,000 observations.

Illustration 7: Firm response to GW across harvesting.


Source: Own elaboration.

Table 1: Calibration of stochastic biomass model.

| Equation | Function | Analytical form | Parameters |
| :---: | :---: | :---: | :---: |
| $(5.2)$ | Biomass | $b(x)=a_{1} x+a_{2} x\left(1-\frac{x}{a_{3}}\right)$ | $a_{1}=0.40, a_{2}=0.14, a_{3}=320$ |
| $(5.3)$ | Demand | $p(h)=b_{1}-b_{2} h$ | $b_{1}=20,000, b_{2}=-0.09$ |
| $(5.4)$ | Marginal cost | $c(x)=c_{1} x^{-c_{2}}$ | $c_{1}=15,000, c_{2}=-0.05$ |
| $(5.5)$ | Spending due to GW | $G(i)=\frac{\sqrt{i}}{g_{k}}$ | $g_{k}=\{100 ; 500 ; 7,000 ; 10,000\}$ |
| $(5.6)$ | Extraction | $h=q E^{\alpha} x^{\beta}$ | $q=0.10, E=\alpha=\beta=1$ |

Source: Own elaboration based on Charles (2007), Doyle et al. (2007), Levy et al. (2006),
McDonald et al. (2002), Munro (1992) and Pindyck (1984).

Table 2: $g_{k}$ values for different $\sigma(T)$ values.

| Value of $\sigma(T)$ | Value of $g$ |
| :---: | :---: |
| $0 \%, 1 \%, 2 \%, 3 \%$ | $g_{1}=100$ |
| $4 \%, 5 \%$ | $g_{2}=500$ |
| $10 \%$ | $g_{3}=7,000$ |
| $20 \%$ | $g_{4}=10,000$ |
| Source: Own elaboration. |  |

### 5.1.2 Results

The results are displayed in Table 3. The "Average biomass" row provides the average firm value with respect to the benchmark for the firm's fishable biomass, calculated using equation 5.7:

$$
\begin{equation*}
\text { Average biomass }=\frac{1}{h^{M A X}} \sum_{h=0}^{h^{M A X}} \frac{\left.V_{h}\right|_{\sigma(T) \in \Sigma}}{\left.V_{h}\right|_{\sigma(T)=0}} \tag{5.7}
\end{equation*}
$$

This calculation is repeated for all elements of $\Sigma$. The value of $h^{M A X}$ corresponds to the maximum value of the firm's extraction, which is assumed to be proportional to the total biomass, and $V_{h}$ is the annual value of the firm that sells $h$ amount of tons. Illustration 8 graphs the percent change in value per biomass unit as the firm extracts larger and larger quantities of resources. As expected, value falls as biomass variability increases, from $-6.4 \%$ loss when $\sigma=1 \%$, to $-44.6 \%$ when $\sigma=20 \%$.

The harmful effects of biomass variation on firms can also be calculated by assuming that a firm decides to extract resources from a biomass with a known variation of $\Sigma$. Each marginal unit extracted causes exposure to temperature shocks. Exposure is then calculated by estimating the noise around a trend, which is understood to be the expected value of each extraction. In Table 3 the "Trend" row shows the estimation given by equation 5.8:

$$
\begin{equation*}
\frac{\left.V_{h}\right|_{\sigma(T) \in \Sigma}}{\left.V_{h}\right|_{\sigma(T)=0}}=\mu_{h} h+\varepsilon_{h} \tag{5.8}
\end{equation*}
$$

where $\mu_{h} h$ is the trend and $\varepsilon_{h}$ is white noise. The coefficient $\mu_{h}$ is the percent change in firm value (dependent variable) as harvesting increases (independent variable).

Table 3: Change in firm value due to stochastic biomass (base: $\sigma(T)=0 \%$ ).

| $\sigma(T)$ | $1 \%$ | $2 \%$ | $3 \%$ | $4 \%$ | $5 \%$ | $10 \%$ | $20 \%$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Average biomass | $-6.40 \%$ | $-10.85 \%$ | $-14.42 \%$ | $-17.57 \%$ | $-20.32 \%$ | $-31.10 \%$ | $-44.79 \%$ |
| Standard errors | 0.10 | 0.13 | 0.15 | 0.17 | 0.18 | 0.19 | 0.20 |
| Trend | $-0.01 \%$ | $-0.02 \%$ | $-0.03 \%$ | $-0.04 \%$ | $-0.04 \%$ | $-0.06 \%$ | $-0.08 \%$ |
| Residual std. err. | 0.09 | 0.10 | 0.11 | 0.11 | 0.11 | 0.10 | 0.10 |
| $\mathrm{R}^{2}$ | $30.66 \%$ | $43.17 \%$ | $51.03 \%$ | $57.54 \%$ | $62.66 \%$ | $76.02 \%$ | $73.50 \%$ |

Source: Own elaboration.

Illustration 8: Changes in firm value per different biomass volatility.


Source: Own elaboration.
Since the variance of the stochastic term stabilizes around $10 \%$ the $\mu_{h}$ values for all $\Sigma$ elements can be compared to judge, with a certain amount of confidence, the loss of value as more resources are extracted. The coefficient of this trend (value loss as extraction increases) increases (in absolute terms) as $\sigma$ grows. When $\sigma=1 \%$, firm value is reduced by $-0.014 \%$ for each marginal unit extracted, and when $\sigma=20 \%$, exposure causes a $-0.081 \%$ reduction per marginal unit extracted, confirming then the detrimental effects of stochastic biomass on fisheries.

### 5.2 Deterministic biomass model

This model is calibrated based on studies by McDonald et al. (2002), De Leo and Gatto (2001), Bjørndal and Munro (1998), Conrad and Bjørndal (1991), and Smith (1968, 1969). Although these studies have different focuses and use different processes, they are still useful for the bioeconomic purposes of this work.

IPCC (2001) forecast, reproduced in Illustration 19 of Annex D, are used for the temperature anomaly. This paper presents the results considering 12 points from a series described by equation 5.9:

$$
\begin{equation*}
T_{t}=0.3+0.8(t-1), \tag{5.9}
\end{equation*}
$$

with $t \in[1,12]$. Illustration 21 from Annex E provides a graphic illustration of the biomass equation (4.4):

$$
\begin{equation*}
\dot{x}=b(x) M(T), \tag{4.4}
\end{equation*}
$$

for different values of $T \in \Theta$ measured in degrees Celsius and presented in Illustration 9 .
Illustration 9: $T$ values used in estimations.


Source: Own elaboration.

In this model the estimate focuses on firm value and the dynamics of invested capital. Applied value is defined as:

$$
V=[p h-C(h, x, K, T)]-\dot{K}[\delta(T)] .
$$

As the previous case, results are a benchmark for the case of a temperature anomaly of $+0.3^{\circ} \mathrm{C}$, in order to standardize the unit of measurement.

### 5.2.1 Calibration

The calibration is presented in Table 4. The values of equations 5.10 and 5.15 are measured in thousands of metric tons, the values for equations 5.11 and 5.12 are measured as percentages, and finally the values of equations 5.13 and 5.14 are measured in monetary units (i.e. millions of dollars). The parameter $k_{2}$ from equation 5.10 is used as an adjustment parameter for units of measurements. An extraction of $h \in[0,100]$ (thousands of tons), divided across 1,000 observations, is assumed for the estimate.

Table 4: Deterministic biomass model calibration.

| Equation | Function | Analytical form | Parameters |
| :---: | :---: | :---: | :---: |
| $(5.10)$ | Partial biomass | $b(x)=k_{1} x+k_{2} x\left(1-\frac{x}{k_{3}}\right)$ | $k_{1}=0.40, k_{2}=0.50, k_{3}=166,33$ |
| $(5.11)$ | Mortality and/or migration | $M(T)=l_{1} T^{l_{2}}$ | $l_{1}=1.00, l_{2}=-0.30$ |
| $(5.12)$ | Interest rate | $\delta(T)=m_{1} T^{m_{2}}$ | $m_{1}=0.09, m_{2}=-0.20$ |
| $(5.13)$ | Firm reaction to GW | $G(h, T, M(T))=n_{1} h M(T)^{n_{2}}$ | $n_{1}=1.00, n_{2}=-050$ |
| $(5.14)$ | Partial cost | $\varphi(h, x, K)=r_{1} h+r_{2} x+r_{3} K$ | $r_{1}=2.00, r_{2}=-2.00, r_{3}=-0.01$ |
| $(5.15)$ | Extraction | $h=q E^{\alpha} x^{\beta}$ | $q=0.10, E=\alpha=\beta=1$ |

Source: Own elaboration based on McDonald et al. (2002), De Leo and Gatto (2001),
Bjørndal and Munro (1998), Conrad and Bjørndal (1991) and Smith (1968, 1969).

### 5.2.2 Results

The results for firm value are presented in Table 5. The "Average biomass" row provides changes (\%) in value with regard to the benchmark from the results of equation 5.16:

$$
\begin{equation*}
\text { Average biomass }=\frac{1}{h^{M A X}} \sum_{h=0}^{h^{M A X}} \frac{\left.V_{h}\right|_{T \in \Theta}}{\left.V_{h}\right|_{T=0.3^{\circ} C}} \tag{5.16}
\end{equation*}
$$

The calculation is repeated for all $\Theta$ elements. $h^{M A X}$ is the firm's maximum extraction level, which is assumed to be proportional to total biomass, and $V_{h}$ is the annual value of a firm that sells $h$ tons.

This confirms that when temperature increases, value falls due to less fishable biomass, from $-8.68 \%$ when the anomaly is $+1.1^{\circ} \mathrm{C}$ to $-9.98 \%$ when it is $+9.1^{\circ} \mathrm{C}$.

The methodology from the previous section provides another way of investigating how temperature influences value. It is assumed that a firm extracts resources from a biomass with a given temperature, expressed by $\Theta$. Marginally increasing the level of extraction over time exposes the firm to a reduction in biomass that could affect the economic yield of the harvest. This exposure is quantified in terms of a noise around a trend, which is understood to be each extraction's expected value. The results are presented in the "Trend" row of Table 5, and correspond to the estimate provided by equation 5.17 :

$$
\begin{equation*}
\frac{\left.V_{h}\right|_{T \in \Theta}}{\left.V_{h}\right|_{T=0.3^{\circ} \mathrm{C}}}=\mu_{h} h+\varepsilon_{h}, \tag{5.17}
\end{equation*}
$$

where $\mu_{h} h$ is the trend and $\varepsilon_{h}$ is a white noise. The coefficient $\mu_{h}$ indicates the percentage change in firm value (dependent variable) as harvest (independent variable) increases.

On average, the biomass trend and the cyclical component are stable as temperature increases. Consequently, the anomaly does not substantially disturb the firm's risk profile, although it does hurt its rate of return because of lower annual profit.

An analysis of the amount of capital invested, which is related to the movement of the discount rate and annual profits, contributes to the understanding of this phenomenon by indicating the direction of capital contributions or withdrawals both in transitory and steady-state. Once a steady-state has been reached, capital only moves because of increases in temperature. In effect, fisheries increase (lower) capital as $\delta(T)$ increases (lowers) and/or the profits are positive (negative), as equation 4.16 indicates. Measuring capital levels is understood to be a measurement of how attractive the industry is.

The results are presented in Table 6. The "Average investment" row provides the average capital per biomass unit for the different temperature anomalies, provided by equation 5.18:

$$
\begin{equation*}
\text { Average investment }=\frac{1}{h^{M A X}} \sum_{h=0}^{h^{M A X}} \frac{\left.K_{h}\right|_{T \in \Theta}}{\left.b(x) M(T)_{h}\right|_{T \in \Theta}} . \tag{5.18}
\end{equation*}
$$

Capital appears to grow as the anomaly grows until reaching around $+4.3^{\circ} \mathrm{C}$. Before reaching this temperature investment is around 0.015 monetary unites per biomass unit (i.e. millions of dollars per metric ton). After surpassing this temperature, however, capital falls and stabilizes in negative terms for higher temperature values. This trajectory can be interpreted similar to how fleet adaption to resource availability is interpreted. In fact, temperatures lower than $+4.3^{\circ} \mathrm{C}$ indicate that the investment flow should be positive, which suggests that -economically speakinghigher extraction capacity is required. After this point, temperature reduces biomass to levels where it is more convenient to extract from biomass at a less then maximum capacity. The result suggests capital withdrawal each time the anomaly surpasses $+4.3^{\circ} \mathrm{C}$, in which case a reduction of 0.2 monetary units per biomass unit is expected. This is the very circumstance that creates the "many boats and few fish" problem.

Table 5: Change in firm value due
to biomass reduction (base: $T=0.3^{\circ} C$ ).

| TEMPERATURE | 1.1 | 1.9 | 2.7 | 3.5 | 4.3 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Average biomass | -8.69\% | -9.00\% | -9.21\% | -9.37\% | -9.49\% |
| Standard errors | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 |
| Trend | -0.14\% | -0.15\% | -0.15\% | -0.15\% | -0.16\% |
| Residual std. err. | 0.03 | 0.03 | 0.03 | 0.03 | 0.04 |
| TEmperature | 5.9 | 6.7 | 7.5 | 8.3 | 9.1 |
| Average biomass | -9.24\% | -9.78\% | -9.86\% | -9.93\% | -9.99\% |
| Standard errors | 0.02 | 0.01 | 0.01 | 0.01 | 0.01 |
| Trend | -0.15\% | -0.16\% | -0.16\% | -0.16\% | -0.16\% |
| Residual std. err. | 0.03 | 0.04 | 0.04 | 0.04 | 0.04 |

Table 6: Investment per biomass unit.

| TEMPERATURE | 1.1 | 1.9 | 2.7 | 3.5 | 4.3 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Average investment | 1.38\% | 1.68\% | 1.90\% | 2.11\% | 2.27\% |
| Standard errors | 5.76 | 7.02 | 7.96 | 8.74 | 9.42 |
| Trend | -0.15\% | -0.19\% | -0.21\% | -0.26\% | -0.25\% |
| Residual std. err. | 5.76 | 7.01 | 7.96 | 9.27 | 9.41 |
| TEmperature | 5.9 | 6.7 | 7.5 | 8.3 | 9.1 |
| Average investment | -19.50\% | -20.51\% | -20.56\% | -20.60\% | -20.63\% |
| Standard errors | 30.83 | 32.59 | 32.84 | 33.07 | 33.28 |
| Trend | -1.16\% | -1.23\% | -1.24\% | -1.24\% | -1.25\% |
| Residual std. err. | 30.82 | 32.58 | 32.83 | 33.06 | 33.27 |

Source: Own elaboration.

## 6 Discussion

This section takes a look at the problems found when the two models are joined in order to discuss the shared effects of GW. The equation for biomass (6.1) indicates that the joint model does not allow both effects to coexist and thus, it is impossible to calculate numerical results:

$$
\begin{equation*}
d x=\left\{f(x, T)-h_{t}\right\} d t+\sigma(T) x d z . \tag{6.1}
\end{equation*}
$$

For the purposes of the arguments presented in this paper, it is worth mentioning some similarities between both models. $f(x, T)=b(x)$ and $d x \neq 0$ represent the stochastic model. When $d x=0$ and $\sigma(T)=0$, this represents the deterministic model. Equation 6.1, when $d x=0, f(x, T)=b(x)$ and $\sigma(T) \neq 0$, is graphed in the phase diagram in Illustration 10 , showing that the expected biomass is within the confidence interval from the stochastic term $\sigma(T) x d z$ of equation 6.1. In a steady-state, when $d x=0$, equilibrium is $P^{I I}$.

The problem with doing this is that it makes the initial equilibrium is unstable. At the equilibrium point, the functions of the derived probabilities are degenerate: with a probability equal to one the system lands on $\left(0, K_{t}\right)$, as is shown in Illustration $10^{16}$. In effect, around equilibrium there is an area that has been divided in four quadrants because of biomass variation that does not disappear in the long term. Supposing that $\sigma(T)>0$, an increase in temperature will move equilibrium to the upper quadrant, even when $\sigma(T)$ is small. The phase diagram indicates when equilibrium is located around this area, when $K>K_{1}$, the systems moves towards ( $0, K_{t}$ ), which makes it impossible to model the effect with biomass oscillations in a steady-state.

There is also a more direct way to verify this argument. Assuming that $\sigma(T) \neq 0, d x=0$ and $f(x, T)=b(x)$, equation 6.1 is graphed in Illustration 11. The initial equilibrium is $P^{I I}$. If a temperature shock reduces biomass ${ }^{17}$ this can either move the system to a point like $A$ or to point $B$. In both cases some firms abandon the market.

[^12]Illustration 10: Joint model.


Source: Own elaboration.

Illustration 11: Shocks in the deterministic model.


Source: Own elaboration.
If the impact moves equilibrium to quadrant $A$ then the system continues in initial equilibrium. If a stronger shock moves the equilibrium to a point such as $B$ not only do firms leave from the market, but harvest will also be close to the biomass's minimum capacity. This is the equivalent of some firms closing, at least temporarily, until the remaining stock generates enough population to be sustainable. The more biomass variability, the higher the possibility of equilibrium being located in $\left(0, K_{t}\right)$. As such, the conclusion here is that separating both models allows for a more direct estimate.

## 7 Concluding remarks

While GW is an issue of growing interest among diverse disciplines, this paper focuses on the biological side of GW in order to uncover its economic consequences on the fishing industry.

As a starting point, the paper argues that higher temperature anomalies cause oscillation and reduction in biomass and then moves on to discuss how these affect fisheries' value.

Two important assumptions are included in the models: the average temperature of the Earth's marine surface is increasing and $G W$ affects biomass. These assumptions are reviewed and time series and studies from specific geographic zones are highlighted to validate assumptions.

The literature on fisheries economics is also reviewed, and it is found that although the subject has made considerable advances over the past several decades, only on rare occasions has GW been
considered as one of the problems of managing a fishery. This study contributes to the literature by providing estimates of the economic consequences of a biomass that has been affected by GW.

The Pindyck (1984) model, which includes current elements from fisheries economics, is adapted to investigate the effect of biomass oscillation on fisheries. The Smith (1968) model, which is used because it simplifies comparative statics, is adapted to analyze the consequences of biomass reduction on firm value.

Then, the arguments for separating oscillation and reduction even though these occur together are discussed.

The results indicate that if there is a $1 \%$ variation in biomass, annual firm value drops by around $6 \%$, while a $20 \%$ variation means values could fall by as much as $44 \%$.

Reduced biomass, which is assumed to be the result of increased mortality and/or fishing, forces firms to increase extraction level, which also requires more capital. The results indicate that if the temperature anomaly increases between +1 and $+8^{\circ} \mathrm{C}$, annual value will fall between 8 and $10 \%$. This calculation also provides optimal capital investment trajectories: investment is positive until the increase in temperature hits $+4.3^{\circ} \mathrm{C}$, after which point it is advisable to withdraw capital, creating the problem of "many boats and few fish" problem.

The results of both models demonstrate the negative effect of GW on the fishing industry.

## 8 References

1. Anderson, L. G. (1977): "The Economics of Fisheries Management", The Johns Hopkins University Press, Baltimore and London, USA.
2. Arnason, R. (2007): "Climate Change and Fisheries: Assesing the Economic Impact in Iceland and Greenland", Natural Resource Modelling, Vol. 20, $\mathrm{N}^{\circ} 2$.
3. Bjørndal, T. (1987): " Production Economics and the Optimal Stock Size in a North Atlantic Fishery", Scandinavian Journal of Economics, Vol. 89, N², pp. 145-162.
4. Bjørndal, T. and G. Munro (1998): "The Economics of Fisheries Management: A Survey", in Tom Tietenberg and Henk Folmer (eds.), "The International Yearbook of Environmental and Resource Economics 1998/1999: A Survey of Current Issues", Cheltenham, Edward Elgar, pp. 153-188.
5. Bjørndal, T., D. Gordon, R. Arnason and R. Sumaila (eds.) (2007): "Advances in Economic Fisheries", Blackwell Publishing, USA.
6. Beverton, R. and S. Holt (1957): "On The Dynamics of Exploited Fish Populations", Ministry of Agriculture, Fisheries and Food, Fishery Investigations, Ser. II, Vol. XIX, London 1957.
7. Bosello, F., R. Roson and R. Tol (2007): "Economy-wide Estimates of the Implications of Climate Change: Sea Level Rise", Environmental and Resource Economics, European Association of Environmental and Resource Economists, Vol. 37, N ${ }^{\circ} 3$, pp. 549-571.
8. Brierly, A. S., J. L. Watkins, C. Goss, M. Wilkinson and I. Everson (1999): "Acoustic Estimates of Krill Density at South Georgia, 1981 to 1998", CCAMLR Sci., Vol. 6, pp. 47-57.
9. Christy, F. and A. Scott (1965): "The Common Wealth in Ocean Fisheries. Some Problems of Growth and Economic Allocation", The Johns Hopkins University Press, Baltimore and London, USA.
10. Chan, J. C. and K. S. Liu (2004): "Typhoon Intensity in a Warming World", Bulletin American Meteorological Society, Vol. 85, pp. 660-661.
11. Christensen, V., Amorim, P., Diallo, I. Diouf, T., Guénette, S. Heymans, J.J., Mendy, A.N., Mahfoudh Sidi, T., Palomares, M.L.D., Samb, B., Stobberup, K., Vakily, J.M., Vasconcellos, M., Watson, R., Pauly, D. (2002): "Trends in Fish Biomass Off Northwest Africa, 19602000", pp. 215-220, en Palomares, M.L.D., Pauly, D. (eds.) (2004): "West African Marine Ecosystems: Models and Fisheries Impacts", Fisheries Centre Research Reports, Vol. 12, No.7. Fisheries Centre, UBC, Vancouver.
12. Charles, A. (2007): "Linking Natural Capital and Physical Capital: A Review of Natural Resource Investment Models", en Bjørndal, T., D. Gordon, R. Arnason and R. Sumaila (eds.) (2007): "Advances in Economic Fisheries", Blackwell Publishing, USA.
13. Chong, O., D. Iglesias and G. Diniz (2006): "Modelo Cuadrático para Simular el Crecimiento de una Población de Peces", Biomatematica, No. 16, pp. 117-129.
14. Chong, O. A. G., G. L. Diniz, and F. R. Villatoro (2005): "Dispersal of Fish Populations in Dams: Modelling and Simulation", Ecological Modelling, 186, pp. 290-298.
15. Clark, C. W. (1976): "Mathematical Bioeconomics: The Optimal Management of Renewable Resources", First Edition, Wiley-Interscience, New York, USA.
16. Clark, C. W. (1990): "Mathematical Bioeconomics", Wiley and Sons, New York, USA.
17. Clark, C. W. (2007): "Rational Expectations and Fisheries Management", en Bjørndal, T., D. Gordon, R. Arnason and R. Sumaila (eds.) (2007): "Advances in Economic Fisheries", Blackwell Publishing, USA.
18. Clark, C. W., F. H. Clarke and G. Munro (1979): "The Optimal Exploitation of Renewable Resource Stocks: Problems of Irreversible Investment", Econometrica, Vol. 47, No. 1, pp. 25-47.
19. Clark, C.W. and G.R. Munro (1982): "The Economics of Fishing and Modern Capital Theory: A Simplified Approach", en L. J. Mirman and D. F. Spulber (eds.), "Essays in the Economics of Renewable Resources", North-Holland, Amsterdam, Netherlands, pp. 31-54.
20. Conrad, J. and T. Bjørndal (1991): "A Bioeconomic Model of the Harp Seal in the Northwest Atlantic", Land Economics 67, pp. 158-171.
21. De Leo, G. A. and M. Gatto (2001): "A Stochastic Bioeconomic Analysis of Silver Eel Fisheries", Ecological Applications, Vol. 11, No. 1, pp. 281-294.
22. Doyle, M., R. Singh and Q. Weninger (2007): " Fisheries Management with Stock Uncertainty and Costly Capital Adjustment", en Bjørndal, T., D. Gordon, R. Arnason and R. Sumaila (eds.) (2007): "Advances in Economic Fisheries", Blackwell Publishing, USA.
23. Dumas, P. and M. Ha-Duong (2008): "Optimal Growth with Adaptation to Climate Change", http://ideas.repec.org/p/hal/papers/halshs-00207621_v1.html.
24. Fankhauser, S. and R.S.J. Tol (2005), "On Climate Change and Economic Growth", Resource and Energy Economics, 27, pp. 1-17.
25. Fankhauser, S., R.S.J. Tol and D.W. Pearce (1997): " The Aggregation of Climate Change Damages: A Welfare Theoretic Approach", Environmental and Resource Economics 10, pp. 249-266.
26. Forcada J., P. N. Trathan, K. Reid, E. J. Murphy and J. P. Croxall (2006): "Contrasting Population Changes in Sympatric Penguin Species in Association with Climate Warming", Global Change Biology 12, pp. 411-423.
27. Goreau, T., R. Hayes and D. McAllister (2005): "Regional Patterns Of Sea Surface Temperature Rise: Implications For Global Ocean Circulation Change And The Future Of Coral Reefs And Fisheries", World Resource Review 17, pp. 350-374.
28. Gordon, H. (1954): "The Economic Theory of a Common Property: The Fishery", Journal of Political Economy, Vol. 62, No. 2, pp. 124-142.
29. Hannesson, R. (2004): "Global Warming and Fish Migrations", mimeo, Centre for Fisheries Economics, The Norwegian School of Economics and Business Administration.
30. Hannesson, R. (2007): "Incentive Compatibility of Fish-Sharing Agreements" en Bjørndal, T., D. Gordon, R. Arnason and R. Sumaila (eds.) (2007): "Advances in Economic Fisheries", Blackwell Publishing, USA.
31. Hansen, J., M. Sato, R. Ruedy, K. Lo, D.W. Lea and M. Medina-Elizade (2006): " Global Temperature Change", Proceedings of the National Academy of Sciences, 103, pp. 1428814293.
32. Hernández, B., G. Gaxiola-Castro and S. Nájera Martínez (2004): "Efectos del ENSO en la Producción Primaria frente a Baja California", Ciencias Marinas, Vol. 30, ${ }^{\circ} 3$, pp. 427-441.
33. Hübler, M., Klepper, G. and Peterson, S. (2007): "Costs of Climate Change - The Effects of Rising Temperatures on Health and Productivity in Germany", Kiel Working Paper 1321, Institut für Weltwirtschaft, Kiel, Alemania.
34. Intergovernmental Panel of Climate Change (IPCC) (2001): "Climate Change 2001: Third Assessment Report", Synthesis Report, Cambridge, UK.
35. Jin, D. and G. E. Herrera (2005): "A Stochastic Bioeconomic Model with Research", Marine Resource Economics, Vol. 20, pp. 249-261.
36. Johnson, G. C., S. Mecking, B. M. Sloyan and S, E. Wijffels (2007): "Recent Bottom Water Warming in the Pacific Ocean", Journal of Climate, Vol. 20, No. 21, pp. 5365-5375.
37. Kemfert, C. (2005): "The Economic Costs of Climate Change", German Institute for Economic Research, Vol. 1, No. 2, pp. 43-49.
38. Kobayashi, S. (2007): "A Dynamic Game of Renewable Natural Resource Explotation and Markov Perfect Equilibrium" en Bjørndal, T., D. Gordon, R. Arnason and R. Sumaila (eds.) (2007): "Advances in Economic Fisheries", Blackwell Publishing, USA.
39. Levy, J. K., D. M. Kilgour and K. W. Hipel (2006): "Reducing The Risk Of Fishery Resource Disasters: A Bioeconomic Approach To Sustainable Resource Management", Journal of the American Water Resources Association, Vol. 42, No. 6, pp. 1451-1463.
40. Lindroos, M., V. Kaitala and L. G. Kronbak (2007): "Coalition Games in Fisheries Econom$i c s "$, en Bjørndal, T., D. Gordon, R. Arnason and R. Sumaila (eds.) (2007): "Advances in Economic Fisheries", Blackwell Publishing, USA.
41. Link, P. M. and R. S. J. Tol (2005): "Economic Impacts on Key Barents Sea Fisheries Arising from Changes in the Strength of the Atlantic Thermohaline Circulation", Working Papers FNU-104, Research Unit Sustainability and Global Change, Hamburg University.
42. Lipp, E., N. Schmidt, M. Luther and J. Rose (2001): "Determining the Effects of El NiñoSouthern Oscillation Events on Coastal Water Quality", Estuaries, Vol. 24, No. 4, pp. 491-497.
43. Lorenzen, K. (2000): "Allometry of Natural Mortality as a Basis for Assessing Optimal Release Size in Fish-Stocking Programmes", Canadian Journal of Fisheries and Aquatic Sciences, Vol 57, pp. 2374-2381.
44. McDonald, A., L. K. Sandal and S. I. Steinshamn (2002): "Implications of a Nested Stochastic / Deterministic Bio-economic Model for a Pelagic Fishery", Ecological Modelling 149, pp. 193-201.
45. McKelvey, R., P. V. Golubtsov, G. Cripe and K. Miller (2007): "The Incomplete Information Stochastic Split-Stream Model: An Overview" en Bjørndal, T., D. Gordon, R. Arnason and R. Sumaila (eds.) (2007): "Advances in Economic Fisheries", Blackwell Publishing, USA.
46. Munro, G. (1992): "Mathematical Bioeconomics and the Evolution of Modern Fisheries Economics", Bulletin of Mathematical Biology 54, pp. 163-184.
47. Murphy, E., P. Trathan, J. Watkins, K. Reid, M. Meredith, J. Forcada, S. Thorpe, N. Johnston and P. Rothery (2007): "Climatically Driven Fluctuations in Southern Ocean Ecosystems", Philosophical Transactions of The Royal Society B, Vol. 274, pp. 3057-3067.
48. Pascoe, S., T. Battaglene and D. Campbell (1992): "A Bioeconomic Model of the Southern Shark Fishery", ABARE Research Report 92.1, www.abareconomics.com.
49. Pauly, D., V. Christensen, J. Dalsgaard, R. Froese and F. Torres (1998): "Fishing Down Marine Food Webs", Science 279, pp. 860-863.
50. Pearce, D. W., and R. K. Turner (1990): "The Economics of Natural Resources and the Environment", The Johns Hopkins University Press, Baltimore, USA.
51. Peterson, S. (2006): "Uncertainty and Economic Analysis of Climate Change: A Survey of Approaches and Findings", Kiel Institute for World Economics, http://www.ifw-kiel.de/.
52. Pindyck, R. (1984): "Uncertainty in the Theory of Renewable Resource Markets", The Review of Economics Studies, Vol. 51, No. 2, pp. 289-303.
53. Pratt, J. (1964): "Risk Aversion in the Small and the Large", Econometrica 32, pp.122-136.
54. Quayle, W. C., L. S. Peck, H. Peat, J. C. Ellis-Evans and P. R. Harrigan (2002): "Extreme Responses to Climate Change in Antarctic Lakes", Science 295, p. 645.
55. Schaefer, M.B. and Beverton, R.J.H. (1963): "Fishery dynamics-their analysis and interpretation", en Hill, M.N. (Ed.): "The Sea", Vol. 2, Interscience, New York, USA.
56. Scott, A. (1955): "The Fishery: The Objectives of Sole Ownership", Journal of Political Economy 63, pp. 116-124.
57. Smith, V. (1968): "Economics of Production from Natural Resources", The American Economic Review, Vol. 58, No. 3, Part I, pp. 409-431.
58. Smith, V., (1969): "On Models of Commercial Fishing", Journal of Political Economy 77, pp. 181-198.
59. Stein, M. (2007): "Warming Periods off Greenland during 1800-2005. Their Potential Influence on the Abundance of Cod (Gadus morhua) and Haddock (Melanogrammus aeglefinus) in Greenlandic Waters", Journal of Northwestern Atlantic Fishery Science 39, pp. 1-20.
60. Stern, N. (2007): "The Economics of Climate Change: The Stern Review", Cambridge University Press, Cambridge, UK.
61. Suárez, J., W. Ritter, C. Gay and J. Torres (2004): "ENSO-Tuna Relations in the Eastern Pacific Ocean and its Prediction as a Non-Linear Dynamic System", Atmosfera, pp. 245-258.
62. Tol, R.S.J. (2002): "New Estimates of the Damage Costs of Climate Change, Part I: Benchmark Estimates", Environmental and Resource Economics, Vol. 21, N ${ }^{\circ} 1$, pp. 47-73.
63. Trathan, P., J. Forcada and E. Murphy (2007): "Environmental Forcing and Southern Ocean Marine Predator Populations: Effects of Climate Change and Variability", Philosophical Transactions of The Royal Society B, Vol. 362, pp. 2351-2365.
64. Trathan, P. N., Brierley, A. S., Brandon, M. A., Bone, D. G., Goss, C., Grant, S. A., Murphy, E. J. and Watkins, J. L. (2003): "Oceanographic Variability and Changes in Antarctic Krill (Euphausia Superba) Abundance at South Georgia", Fisheries Oceanography 12, pp. 569-583.
65. Trathan, P.N., Murphy, E. J., Forcada, J., Croxall, J. P., Reid, K. and Thorpe, S.E. (2006): "Physical Forcing in the Southwest Atlantic: Ecosystem Control" en Boyd, I.L., Wanless, S., Camphuysen, C.J. (eds.) "Top Predators in Marine Ecosystems", Cambridge University Press.
66. Thompson, P. M. and J. C. Ollason (2001): "Lagged Effects of Ocean Climate Change on Fulmar Populations Dynamics", Nature 413, pp. 417-420.
67. Ye, Y., N. Loneragan, D. J. Die, R. Watson and B. Harch (2005): "Bioeconomic Modelling and Risk Assessment of Tiger Prawn (Penaeus Esculentus) Stock Enhancement in Exmouth Gulf, Australia", Fisheries Research 73, pp. 231-249.

## 9 Annexes

## A Annex A



Source: Carbon Dioxide Information Analysis Center, http://cdiac.ornl.gov/.

Illustration 13: Global temperature and $\mathrm{CO}_{2}$ concentration, 1000 - 2000.


Source: Etheridge, D.M., L.P. Steele, R.L. Langenfelds, R.J. Francey, J.-M. Barnola, V.I. Morgan (1998): "Historical CO2 Records from the Law Dome DE08, DE08-2, and DSS Ice Cores", http://cdiac.esd.ornl.gov/trends/co2/lawdome.html.

## B Annex B

Illustration 14: Temperature anomaly of the Earth's (land) surface, $1880-2005$.


Illustration 15: Temperature anomaly of the Earth's land and ocean surfaces, 1880 - 2005.


Source: Goddard Institute for Space Studies, http://data.giss.nasa.gov.

Illustration 16: Temperature anomaly per hemisphere, 1880 - 2005.


Source: Goddard Institute for Space Studies, http://data.giss.nasa.gov.

Illustration 17: Temperature anomaly for $90^{\circ} \mathrm{N}-23,6^{\circ} \mathrm{N}$ zone, $1900-2005$.


Source: Goddard Institute for Space Studies, http://data.giss.nasa.gov.

## C Annex C

Illustration 18: Antarctic Ocean, $60^{\circ} \mathrm{S}$.


Source: Trathan P.N, Forcada J. and Murphy E.J. (2007): "Environmental Forcing and Southern Ocean Marine Predator Populations: Effects of Climate Change and Variability", Philosophical Transactions of The Royal Society B, $\mathrm{N}^{\circ} 362$, pp. $2351-2365$.

## D Annex D

Illustration 19: Historic and forecasted temperature anomalies for the Earth's surface, 1000-2100.


[^13]
## E Annex E

Illustration 20: Stochastic biomass dependent on $T$.


Source: Own elaboration.

Illustration 21: Deterministic biomass dependent on $T$.


Source: Own elaboration.


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[^1]:    ${ }^{1}$ GW is understood as the increase in the Earth's average temperature due to $\mathrm{CO}_{2}$ emissions that prevent solar radiation absorbed by the Earth from completely returning to the atmosphere.
    ${ }^{2}$ Biomass is understood as the abbreviation for biological mass, the living material produced in a determined area of land or water.

[^2]:    ${ }^{3}$ When a good is common the use of this good by a consumer lowers the consumption of another good (rival) and it is impossible to stop other consumers from using this good (non-excludable). On the other hand, when a consumer uses a public good it does not reduce the consumption of another good (non-rival) and does not stop other consumers from using it (non-excludable).

[^3]:    ${ }^{4}$ Although they are not identical, the terms harvesting and extraction will not be differentiated in this paper. This does not affect the relevant results.

[^4]:    ${ }^{5}$ Kemfert (2005) provides the times series of greenhouse gas emissions on a global level.

[^5]:    ${ }^{6}$ The melting of ice already in water does not cause changes in sea level.

[^6]:    ${ }^{7}$ Thermohaline circulation is the name for the convective circulation that affects oceanic bodies of water on a global scale. Global circulation can be described as relatively superficial flow of water, which is heated in the tropical zones of the Pacific, Indian, and Atlantic Oceans, before dropping to the depths of the northern Atlantic Ocean.

[^7]:    ${ }^{8}$ Both with high commercial value for the zone.
    ${ }^{9}$ See footnote 3.
    ${ }^{10}$ Such as Christensen et al. (2002).

[^8]:    ${ }^{11}$ This follows the work of Jin and Herrera (2005).

[^9]:    ${ }^{12}$ As opposed to fish farming, where temperature can be controlled.
    ${ }^{13}$ However, the decision is affected in a tangential way by temperature, since movements of $x$ are caused only by this decision.

[^10]:    ${ }^{14}$ The fact that $M_{T}<0$ indicates that mortality is high when temperature levels are low, and later decreases as temperature increases. The reason for this is that when temperature increases the mortality of less adaptable species increases, later stabilizing for species that are more resistant to habitat changes. Following this reasoning, Lorenzen (2000) argues that mortality depends on the size-shape relation of species.

[^11]:    ${ }^{15}$ Similar to that used by Doyle et al. (2007).

[^12]:    ${ }^{16}$ Capital dynamic (equation 4.16) is also graphed, identical to in Section 4.
    ${ }^{17}$ The shock is necessarily a product of temperature since it is the only variable that does not behave in a stationary manner in the long term.

[^13]:    Source: Intergovernmental Panel on Climate Change (2001): "Climate Change 2001: Synthesis Report", http://www.ipcc.ch/.

