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Is private leverage excessive?

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Abstract

I examine whether a benevolent government can improve on the free market allocation by setting capital requirements for private borrowers in a stochastic model with collateral constraints. Previous theoretical studies have found that when asset prices enter into borrowing constraints, pecuniary externalities between atomistic agents can make the laissez faire equilibrium constrained inefficient. For reasonable parameter values, I find that, quantitatively, the answer is ‘no’ – private and government leverage choices coincide. Limiting private leverage by imposing capital requirements has the beneficial effect of dampening the effects of the ‘collateral amplification mechanism’. This reduces ‘fire sales’ in recessions and limits the negative externality that individual asset sales have on other credit constrained borrowers.

However, we find that capital requirements are a blunt tool. They tax the activities of highly productive entrepreneurs and reduce the amount they produce in equilibrium. This reduces total factor productivity and steady state consumption. In the end, society faces a choice between high but unstable consumption in the free borrowing world and low but stable consumption in the regulated world. The government chooses the former.

JEL Classification: E21.

Key Words: Collateral constraints, Capital Requirements.

1 Introduction

The 2007-09 financial crisis brought the world financial system to the brink of collapse, leading to calls for tighter regulation in order to prevent a repeat of the crisis. ‘Excessive leverage’ is thought to be one of the main culprits for the fragility of the economy in the face of shocks. This has re-opened the debate of whether private banks, corporates and households tend to take socially optimal borrowing decisions. In this paper we examine the optimality of firms’ leverage decisions using a standard macroeconomic model with credit frictions. We examine whether a benevolent government can improve ex ante welfare by imposing capital requirements which are different from those chosen by the market.

A growing academic literature has shown that the prevalence of uncontingent debt has the potential of interacting with binding collateral constraints in order to magnify the effects of shocks to the economy. The mechanism is based on different versions of the collateral amplification argument popularised by Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and Bernanke, Gertler and Gilchrist (1999). More recently, Lorenzoni (2008), Gromb and Vayanos (2002) and Korinek (2009) have shown that, in an environment of binding credit constraints, private leverage tends to be excessive from a social point of view due to the presence of a market price externality. This externality arises because private borrowers do not internalise the effects of their own financial distress on other borrowers. When collateral constraints tighten due to an adverse aggregate shock, leveraged debtors’ net worth declines and they need to sell assets in order to satisfy the collateral constraint. This ‘financial distress’ scenario leads to private losses which are fully taken into account by firms when they decide ex ante how much debt to take on.

What private borrowers ignore, however, is the market price externality of financial distress. The larger the volume of asset sales following an adverse shock to collateral values, the bigger the eventual decline in capital prices and the wider the spectre of financial distress. Individual borrowers, however, do not take such ‘general equilibrium’ effects into account. They take the state contingent evolution of market prices as exogenous, treating their own leverage decisions as irrelevant for aggregate outcomes. In contrast, the government takes the market price externalities in question into account when designing the optimal state

contingent capital adequacy rules.

This paper focuses on the quantitative question of whether taking the market price externality into account leads the government to choose very different capital requirements from those already required by the market. We use a business cycle model with credit constraints, which is similar to Kiyotaki (1998). In our environment borrowing and lending is motivated by a heterogeneity in the productivity of different firms. But because debt is assumed to be uncontingent and secured against collateral, aggregate shocks can damage the net worth of borrowers and reduce their access to finance. I assume that borrowing entrepreneurs in the model know that aggregate productivity shocks may hit and this gives them an incentive to hedge their net worth by borrowing less than the market determined debt limit. We nevertheless find that high productivity firms choose to take the maximum permitted leverage despite the risks to net worth this involves. The intuition for this is simple. High productivity entrepreneurs earn such a good return on their productive assets that insuring their net worth by leaving themselves with spare debt capacity is too costly. Because the owners of these fast growing firms have very good future consumption opportunities, saving at prevailing market prices is a very bad proposition for them. So they rationally choose to leverage up to the debt limit, accepting the ex post volatility in the rate of return on their portfolios.

The main result of the paper is the following. When we allow a benevolent government to choose state contingent capital requirements to maximise ex ante social welfare, we find that the government makes identical choices to the market for reasonable parameter values. In other words, the government chooses capital requirements which are equal to the incentive compatible debt limits. We find that this surprising result arises from the balance of the costs and benefits of regulation around the private optimum. Tightening capital requirements relative to the market-imposed borrowing limits has the benefit of dampening the collateral amplification mechanism and reducing the volatility of asset prices and consumption over the economic cycle. This cyclical volatility is ‘excessive’ from a social point of view because leveraged borrowers do not take into account the effect of their own forced asset sales on other leveraged borrowers. But the government considers the costs of regulation too. In our model, the flow of finance from low to high productivity entrepreneurs increases the economy’s TFP

by putting more of the economy's productive resources into the hands of those best able to make use of them. When the government regulates leverage, more production has to be undertaken by inefficient firms and this depresses average TFP and consumption over time.

How the government locates itself on this trade off between increasing the economy's average productivity and consumption and increasing its consumption volatility is a function of the costs of business cycles in the model. We find that, quantitatively, these costs are small. Because the government acts in the social interest, it allows private agents to borrow as much as can be credibly repaid without imposing tighter capital requirements than the market.

Interestingly, we find that the 'no overborrowing' result does not arise because amplification in the model is small. Contrary to the results of Cordoba and Ripoll (2004) we find that it is large, increasing the standard deviation of output by 40% higher than the first best without making any non-standard assumptions about preferences or the productive technology. The difference between our results and those of Cordoba and Ripoll (2004) arise out of our assumption of constant returns to scale to all factors, which helps to maintain productivity differences between firms even in the face of large shocks to their relative outputs. This result shows that the Kiyotaki and Moore (1997) framework is capable of generating quantitatively large amplification for reasonable calibrations. Nevertheless, despite generating a lot of amplification, the framework does not generate strong incentives to regulate financial transactions. This is because consumers care more about having a high rate of return on wealth and this dominates the welfare costs due to business cycle fluctuations.

Finally, we need to stress that the pecuniary externality our paper discusses is only one of the many reasons for capital regulation. Our framework misses out one very important reason for capital regulation - the risk shifting behaviour caused by the possibility of bankruptcy or a government bail-out. There is a large literature which has studied the incentives for banks and other private borrowers to take excessive risks when they know that losses in the worst case scenarios will be borne by lenders or the government. While such factors are undoubtedly an important cause of financial crises, we abstract from them in this paper in order to keep our framework tractable¹.

¹We study borrowing contracts which feature no bankruptcy in equilibrium. Also we assume that the

The rest of the paper is organised as follows. Section 2 discusses the related literature in a little more detail. Section 3 outlines the model environment. Section 4 outlines the competitive equilibrium for our model economy. Section 6 outlines the government's objective function and policy instrument. Section 5 compares private and government leverage choices and uses numerical simulation of the economy to illustrate the costs and benefits of tighter collateral requirements. Finally, Section 8 concludes.

2 Related Literature

2.1 The collateral amplification mechanism

This model is related to a large and rapidly growing literature on the credit amplification mechanism and on the pecuniary externalities this generates. The collateral amplification transmission channel was first popularised by the work of Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Kiyotaki (1998) Carlstrom and Fuerst (1997) and Bernanke, Gertler and Gilchrist (1999). All these models examine the effect of financing frictions on aggregate allocations. In them, the net worth of agents who have productive opportunities is key in determining the cost and availability of external finance. Adrian and Shin (2009) have explored this mechanism in the context of multiple leveraged traders in financial markets.

2.2 Pecuniary externalities and the efficiency of private leverage

The central question of this paper is related to an older literature which has examined the constrained efficiency of the competitive equilibrium in an economy with moral hazard and adverse selection. Arnott and Stiglitz (1986) showed using a simple insurance moral hazard example that the competitive equilibrium is constrained inefficient when prices affect insurees' incentives to take care. Kehoe and Levine (1993) show that the competitive equilibrium in their 'debt constrained' economy is only efficient in a single good world. Multi-good economies are not necessarily constrained efficient because relative prices affect the value of

government cannot make transfers. This rules out two of the most widely studied mechanism which generate overborrowing by private agents.

default and this introduces a market price externality which is not taken into account by atomistic private agents. What these papers show is that when relative prices determine the tightness of incentive compatibility constraints, this drives a wedge between the decisions of private agents and the decisions of the social planner. Private individuals take prices as given while the social planner recognises that manipulating prices can relax some of the constraints it is facing.²

Even more closely related to the topic of this paper, work by Lorenzoni (2008), Korinek (2009) and Gromb and Vayanos (2002) have shown rigorously that the presence of asset prices in the collateral constraint can generate a pecuniary asset price externality between leveraged borrowers. Distressed sales by one set of borrowers can push down asset prices, damaging the net worth and credit access of other borrowers. Private agents ignore this externality, generating incentives for government intervention in order to bring the social costs and benefits of leverage into line with one another. These papers provide the theoretical motivation in a simple three period framework for the quantitative investigation we undertake here in an infinite horizon macro model.

Korinek (2008) and Bianchi (2009) have also examined the possibility of excessive external debt in the an emerging market context. In Korinek (2008), borrowing in foreign currency is cheaper for individual firms because of the risk premium on domestic currency debt. However, foreign currency debt leaves domestic entrepreneurs vulnerable to a sharp appreciation of the domestic real exchange rate. In Bianchi (2009), fluctuations in the price of non-traded goods work in the same way to introduce sudden sharp changes in real debt values. In both of these models, just like in the model of this paper, the externality works through pecuniary externalities that affect the tightness of borrowing constraints.

2.3 The welfare costs of business cycles

How the government trades off average consumption against the volatility of consumption is an important reason behind the results of this paper. This issue connects with the literature

²Prescott and Townsend (1984) showed that introducing man-made lotteries into the economy can remove the externality in question and restore the constrained efficiency of the competitive equilibrium.

on the welfare costs of business cycles, which was started by Lucas (1987)'s seminal contribution. Lucas (1987) found that the cost of aggregate consumption volatility was of the order of 0.08% of annual consumption, implying that business cycle volatility is not an important determinant of social welfare. Lucas (1987), of course, recognised that imperfections in risk sharing had the potential of increasing the cost of business cycles at least for some groups in society.

This finding spurred a lot of research on the effect of risk sharing and consumer δ heterogeneity on the welfare costs of business cycles. Krusell and Smith (1998) examine this question in an infinitely lived economy with aggregate uncertainty in which individuals are subject to uninsurable idiosyncratic shocks. Storsletten et al. (2001) extended Krusell and Smith's analysis to an economy with finitely lived overlapping generations. They found that the welfare costs of the business cycle vary substantially across different groups in society and are larger than Lucas' original numbers but still far from enormous. We find that the small costs of business cycles play a substantial role in determining the costs and benefits of regulation in our framework too.

3 The Model

3.1 The Economic Environment

3.1.1 Population and Production Technology

The economy is populated with a continuum of infinitely lived entrepreneurs and a continuum of infinitely lived workers - both of measure 1. Each entrepreneur is endowed with a constant returns to scale production function which uses capital k , labour h and intermediate inputs x to produce gross output y .

$$y_t = a_t A_t \left(\frac{k_{t-1}}{\alpha} \right)^\alpha \left(\frac{x_{t-1}}{\eta} \right)^\eta \left(\frac{h_{t-1}}{1 - \alpha - \eta} \right)^{1 - \alpha - \eta}$$

where a is the idiosyncratic component of productivity which is revealed to the entrepreneur one period in advance and can be high a^H or low a^L . The idiosyncratic state evolves according to a Markov process. Following Kiyotaki (1998) let $n\delta$ be the probability that a

currently unproductive firm becomes productive and let β be the probability that a currently productive firm becomes unproductive. This implies that the steady state ratio of productive to unproductive firms is n . The aggregate state also evolves according to a persistent Markov process.

A_t is the aggregate component of productivity which also evolves according to a Markov process and alternates between high and low values. The realisation of the aggregate state A_t occurs at the beginning of time t .

Intermediate inputs x are produced one for one from consumption goods and fully depreciate between periods. Capital is in fixed aggregate supply and does not depreciate. The only financial asset is simple debt.

3.1.2 Commitment technology and private information

Agents suffer from limited commitment. They cannot make binding promises unless it is in their interests to do so. In addition, idiosyncratic productivity realisations and individual asset holdings are private information.

3.2 Entrepreneurs

3.2.1 Preferences

Entrepreneurs are ex-ante identical and have logarithmic utility over consumption streams

$$U^E = E_0 \sum_{t=0}^{\infty} \beta^t \ln c_t$$

3.2.2 Flow of Funds

Entrepreneurs purchase consumption (c), working intermediate inputs (x), capital (k) at price q and labour (h) at wage w . All inputs are chosen a period in advance. Entrepreneurs borrow using debt securities b_t at price $1/R_t$.

$$c_t + w_t h_t + x_t + q_t k_t - \frac{b_t}{R_t} = y_t + q_t k_{t-1} - b_{t-1}$$

Because we assume that idiosyncratic shocks and individual asset holdings are private information, securities contingent on the realisation of the idiosyncratic state will not trade in equilibrium.

3.2.3 Collateral constraints

Due to moral hazard in the credit market, agents will only honour their promises if it is in their interests to do so. We assume that only a fraction θ of capital holdings can be seized by creditors. We also assume that entrepreneurs only have the opportunity to default before the aggregate shock has been realised. Hence the collateral constraint limits the entrepreneur's debt to the expected value of collateralisable capital³:

$$b_t \leq \theta E_t q_{t+1} k_t \tag{1}$$

Note that θ here is assumed to be exogenously given by the underlying limited commitment problem in this economy. It therefore cannot be affected by the government. When we come to analyse the government's choice of capital requirements, we will allow it to choose the capital requirement $\tilde{\theta}_t \leq \theta$. This will then place a limit on private leverage over and above the limit imposed by the incentive compatibility constraint (1).

³We also consider an alternative collateral constraint which limits borrowing by the realisation of the land price in the worst case scenario. In our case there are only two aggregate productivity states so lenders look at the value of collateral in the low aggregate state.

$$b_{t+1} \leq \theta q_{t+1}^L k_{t+1}$$

Such a collateral constraint would obtain if borrowers were allowed to default after the realisation of the aggregate productivity shock. Lenders would then want to insure themselves against losses by only lending up to the value at which entrepreneurs would never default.

We found that using such a form of the collateral constraint did not significantly affect the results we get.

3.3 Workers

3.3.1 Preferences

Workers have the following preferences:

$$U^W = E_0 \sum_{t=0}^{\infty} \beta^t \ln \left(c_t - \varkappa \frac{h_t^{1+\omega}}{1+\omega} \right)$$

3.3.2 Flow of Funds

Workers do not have the opportunity to produce. They purchase consumption (c) and save using debt securities b_t at price $1/R_t$. Their net worth consists of labour income ($w_t h_t$) and bonds b_{t-1} .

$$c_t + \frac{b_t}{R_t} = w_t h_t + b_{t-1}$$

3.3.3 Collateral constraints

Due to moral hazard in the credit market, workers cannot borrow:

$$b_t \geq 0 \tag{2}$$

4 Competitive Equilibrium

4.1 Entrepreneurial behaviour

Entrepreneurs make decisions based on three key margins. First of all they decide how much to consume today and how much to save for future consumption. Secondly, they need to decide how to divide their savings between safe bonds and risky production - the portfolio problem. Thirdly, within the amount they invest in production, they need to decide on the input mix between capital, intermediate inputs and labour - the production problem.

Let $V(z_t, a_t, X_t)$ denote the value of an entrepreneur with wealth z_t , idiosyncratic productivity level a_t (determined and revealed to the entrepreneur at time $t - 1$) when the aggregate state is $X_t \equiv [A_t, Z_t, d_t]$. For now we simply assume that the aggregate state consists of the aggregate technology realisation A_t , total wealth in the economy Z_t as well as

the share of wealth held by high productivity entrepreneurs d_t . We will prove subsequently that this is the case.

The value function is defined recursively as follows:

$$V(z_t, a_t, X_t) = \max_{x_t, k_t, b_t, h_t, c_t} \{\ln c_t + \beta E_t V(z_{t+1}, a_{t+1}, X_{t+1})\} \quad (3)$$

where the maximisation is performed subject to the current resource constraint,

$$c_t + w_t h_t + x_t + q_t k_t - \frac{b_t}{R_t} \leq z_t$$

the transition law for individual wealth,

$$z_{t+1} = a_{t+1} A_{t+1} \left(\frac{k_t}{\alpha}\right)^\alpha \left(\frac{x_t}{\eta}\right)^\eta \left(\frac{h_t}{1 - \alpha - \eta}\right)^{1 - \alpha - \eta} + q_{t+1} k_t - b_t$$

the collateral constraint

$$b_t \leq \theta E_t q_{t+1} k_t$$

the Markov process for the idiosyncratic productivity shock and the transition law for the aggregate state. The aggregate technology shock evolves according to a Markov process. The share of wealth held by high productivity entrepreneurs is an endogenous variable and we will describe its evolution as part of our characterisation of the competitive equilibrium of our model economy.

4.1.1 Optimal consumption

In Appendix A we prove that the log utility assumption ensures that consumption is always a fixed fraction of wealth that depends upon the discount factor.

$$c_t = (1 - \beta) z_t$$

4.1.2 Optimal production

When borrowing constraints bind, high and low productivity entrepreneurs will make different production decisions. This is why we examine the optimal production decisions of the two groups separately.

High productivity entrepreneurs In equilibrium, the high productivity entrepreneurs will turn out to be the borrowers in this economy. Optimal production implies that the input mix between capital, labour and intermediate inputs is given by the following expressions:

$$x_t = \eta u_t^H k_t / \alpha \quad (4)$$

and

$$h_t = \frac{1 - \alpha - \eta u_t^H}{\alpha} \frac{k_t}{w_t} \quad (5)$$

where u_t^H is the user cost of capital faced by high productivity entrepreneurs.

When the borrowing constraint is binding, this means that the entrepreneur derives additional value from purchasing capital because this relaxes the collateral constraint. This value (in terms of goods) can be easily derived from the first order condition with respect to borrowing:

$$\begin{aligned} \frac{\mu_t}{\lambda_t} &= \frac{1}{R_t} - \beta E_t \left(\frac{c_t}{c_{t+1}} \right) \\ &= \frac{1}{R_t} - E_t \left(\frac{1}{R_{t+1}^H} \right) \end{aligned}$$

where R_{t+1}^H is the rate of return on wealth for high productivity entrepreneurs (to be pinned down later in the paper) and μ_t and λ_t are the Lagrange multipliers on the borrowing and resource constraints. The value of relaxing the borrowing constraint by a unit is equal to the difference between the market price of future consumption (the price of debt) and the private valuation of future consumption. Credit constrained borrowers are those who value future consumption less than the market because their wealth and consumption are growing fast. They would like to borrow unlimited amounts at prevailing market prices but are prevented from doing so by binding collateral constraints.

In general the user cost expression is given by:

$$u_t^H = q_t - E_t \left(\frac{q_{t+1}}{R_{t+1}^H} \right) - \theta E_t q_{t+1} \frac{\mu_t}{\lambda_t}$$

When credit constraints bind, the user cost expression is give by:

$$u_t^H = q_t - E_t \left(\frac{q_{t+1}}{R_{t+1}^H} \right) - \theta E_t q_{t+1} \left(\frac{1}{R_t} - E_t \left(\frac{1}{R_{t+1}^H} \right) \right)$$

while when they do not bind, the shadow price on the borrowing constraint $\mu_t = 0$ and the user cost is given by:

$$u_t^H = q_t - E_t \left(\frac{q_{t+1}}{R_{t+1}^H} \right)$$

Low productivity entrepreneurs In equilibrium, low productivity entrepreneurs are always unconstrained savers. When borrowing constraints bind sufficiently tightly, they also end up producing using their inefficient technology. Suppose that we are in such an environment where efficient and inefficient technologies are both used due to the borrowing constraint. Then the first order condition for optimal capital input by the low productivity producers is as follows:

$$u_t^L = q_t - E_t \left(\frac{q_{t+1}}{R_{t+1}^L} \right)$$

where $R_{t+1}^L \equiv \frac{z_{t+1}}{\beta z_t}$ is the rate of return on wealth for a low productivity entrepreneur (to be specified later on in the paper). This is a standard user cost expression. Because our economy has two aggregate states and two assets (debt and productive projects), markets for aggregate risk are complete and $\pi(s) / R_{t+1}^L(s)$ is the price of an Arrow security that pays a unit of consumption if state s is realised in the next period. The $E_t \left(\frac{q_{t+1}}{R_{t+1}^L} \right)$ term is the present value of the capital unit tomorrow evaluated at Arrow security prices.

Conditional upon the user cost of capital, low productivity entrepreneurs have the same input mix as high productivity types. However, high productivity entrepreneurs will use less capital intensive production strategies because they face a higher cost of capital compared to low productivity ones. We will return to the link between downpayment requirements and the user cost of capital later because it is key to the policy conclusions of the paper.

4.1.3 The portfolio problem

In the previous two subsections we characterised the solution of two of the consumer's three decision margins: the consumption function and the optimal input mix into production. Now what remains is to solve for the optimal mix between productive projects and loans to other entrepreneurs. For the high productivity entrepreneurs who are the borrowers in our economy this problem boils down to choosing optimal leverage. For the low productivity savers, it will be a choice of whether to produce or lend at the margin.

High productivity entrepreneurs In equilibrium, high productivity entrepreneurs have investment opportunities in excess of the rates of return available on market securities (in this model, simple debt). Consequently they will want to leverage up in order to take advantage of this (temporary) investment opportunity. Let $l_t \equiv b_t/E_t q_{t+1} k_t$ denote the fraction of the entrepreneur's capital purchase which is financed by debt. This fraction is bounded from above by the collateral constraint, which states that, in the laissez faire economy, at most θ fraction can be borrowed. In the regulated economy l_t will be bounded by the capital requirement chosen by the government, $\tilde{\theta}_t$.

In Appendix B we show that a high productivity entrepreneur who borrows a fraction $l_t \leq \theta$ to fund his capital purchases will earn the following rate of return:

$$R_{t+1}^H = \frac{(A_{t+1} a^H / \alpha) w_t^{\alpha+\eta-1} (u_t^H)^{1-\alpha} + q_{t+1} - l_t E_t q_{t+1}}{q_t + (1 - \alpha) u_t^H / \alpha - (l_t / R_t) E_t q_{t+1}} \quad (6)$$

The numerator of the above expression denotes project revenues consisting of output per unit of capital $((A_{t+1} a^H / \alpha) w_t^{\alpha+\eta-1} (u_t^H)^{1-\alpha})$ and the value of capital (q_{t+1}) net of debt repayments $l_t E_t q_{t+1}$. The denominator denotes the total cost of undertaking the project. It consists of the total cost of capital (q_t) and other inputs $((1 - \alpha) u_t^H / \alpha)$ minus the amount of financing the entrepreneur chose to undertake via debt markets $(l_t / R_t) E_t q_{t+1}$. So in other words, R_{t+1}^H is the leveraged rate of return on production.

In Appendix C we show that the entrepreneur's value function depends on the net present value of future expected rates of return on wealth. The entrepreneur, therefore, chooses l_t in order to maximise the expected log rate of return on wealth.

$$\ln R^{H*} = \max_{l_t} E_t \ln \left[\frac{(A_{t+1} a^H / \alpha) w_t^{\alpha+\eta-1} (u_t^H)^{1-\alpha} + q_{t+1} - l_t E_t q_{t+1}}{q_t + (1 - \alpha) u_t^H / \alpha - (l_t / R_t) E_t q_{t+1}} \right] \quad (7)$$

subject to the constraint:

$$l_t \leq \tilde{\theta}_t \quad (8)$$

To get a more intuitive understanding of the leverage decision, we can think of the entrepreneur's leverage decision as a standard portfolio problem in which the entrepreneur chooses how much of his savings to put into a risky and a safe asset. We define the return on the risky asset as the return on a productive project together with the returns from the

capital holding that goes with it:

$$R_{t+1}^k = \frac{(A_{t+1}a^H/\alpha) w_t^{\alpha+\eta-1} (u_t^H)^{1-\alpha} + q_{t+1}}{q_t + (1-\alpha) u_t^H/\alpha}$$

Then we can write the rate of return on the entrepreneur's total portfolio as the weighted average between the risky and the safe rate of return:

$$R_{t+1}^H = \varpi_t^H R_{t+1}^k + (1 - \varpi_t^H) R_t$$

where

$$\varpi_t^H \equiv \frac{q_t + (1-\alpha) u_t^H/\alpha}{q_t + (1-\alpha) u_t^H/\alpha - (l_t/R_t) E_t q_{t+1}} > 1 \quad (9)$$

is the share of the risky asset in the high productivity entrepreneur's portfolio. Entrepreneurs are free to choose a value of l_t below θ if they are unconstrained. However, the maximum share of the risky asset is determined by the borrowing constraint and is given by:⁴

$$\varpi_{\max}^H \equiv \frac{q_t + (1-\alpha) u_t^H/\alpha}{q_t + (1-\alpha) u_t^H/\alpha - (\theta/R_t) E_t q_{t+1}} > 1 \quad (10)$$

In Appendix E we show that we can take a second order approximation to the portfolio problem as follows:

$$\ln R^{H*} \approx \max_{\varpi_t^H} \left[\ln R_t + \varpi_t^H (E_t \rho_{t+1}^H - 1) - \frac{(\varpi_t^H)^2}{2} \sigma_{R_{t+1}}^2 \right]$$

where the expected excess return on production for high productivity agents is defined as follows:

$$E_t \rho_{t+1}^H = \frac{E_t R_{t+1}^k}{R_t} = E_t \left(\frac{(A_{t+1}a^H/\alpha) w_t^{\alpha+\eta-1} (u_t^H)^{1-\alpha} + q_{t+1}}{q_t + (1-\alpha) u_t^H/\alpha} \right) / R_t \quad (11)$$

⁴The larger l_t the higher the share of risky assets in the entrepreneur's portfolio. As (9) shows, when $l_t > 0$, the share of the risky asset ϖ_t^H is greater than unity. But even when the entrepreneur borrows the full value of her capital purchases, this does not mean that she is unconstrained in her borrowing. As long as the expected return on the risky asset R_{t+1}^k is sufficiently greater than the interest rate on safe debt R_t to compensate for risk, the entrepreneur will remain credit constrained and would like to borrow against the value of her future output as well.

Reducing the value of l_t below the market determined θ is tantamount to the entrepreneur choosing to reduce his holdings of the risky asset. As the entrepreneur borrows less and less, l_t falls and with it ϖ_t^H falls too. If the entrepreneur decides to become a net saver, l_t falls below zero. In the limit, as l_t becomes large and negative, ϖ_t^H tends to zero and the portfolio of the entrepreneur consists of only the safe asset.

The conditional variance of the log rate of return of the risky asset σ_{Rt+1}^2 is dominated by the variance of the capital price as well as the covariance of the capital price with the technology shock (for more details see Appendix E). Both of these terms increase strongly as the collateral amplification mechanism becomes stronger. The first order condition is:

$$\frac{\partial \ln R^{H*}}{\partial \varpi_t^H} \approx E_t \rho_{t+1}^H - 1 - \varpi_t^H \sigma_{Rt+1}^2 \geq 0 \quad (12)$$

It holds with equality if the collateral constraint does not bind. Re-arranging we get:

$$\varpi_t^H \approx \frac{S_{t+1}^H}{\sigma_{Rt+1}}$$

where $S_{t+1}^H \equiv \frac{E_t \rho_{t+1}^H - 1}{\sigma_{Rt+1}}$ is the conditional Sharpe ratio on the risky asset for the high productivity entrepreneur. σ_{Rt+1} is determined by the volatility of the technology shock σ_A^2 as well as the volatility of the capital price σ_{qt+1}^2 . The higher these are, the smaller the share of the risky asset chosen by the entrepreneur. Equally a higher premium $E_t \rho_{t+1}^H - 1$ leads to a larger share invested in the risky asset.

This means that, in general, the share of the risky asset in the high productivity entrepreneur's portfolio is given by:

$$\varpi_t^H = \min \left[\frac{E_t \rho_{t+1}^H - 1}{\sigma_{Rt+1}^2}, \frac{q_t + (1 - \alpha) u_t^H / \alpha}{q_t + (1 - \alpha) u_t^H / \alpha - (\theta / R_t) E_t q_{t+1}} \right]$$

where $\frac{q_t + (1 - \alpha) u_t^H / \alpha}{q_t + (1 - \alpha) u_t^H / \alpha - (\theta / R_t) E_t q_{t+1}}$ is the share of the risky asset when the constraint is binding.

Low productivity entrepreneurs Low productivity entrepreneurs may or may not produce in equilibrium, depending on the tightness of the collateral constraint. When the constraint binds very tightly, high productivity firms will be constrained in their ability to purchase the productive assets in the economy and some of them will have to be bought by low productivity firms. Consistent with the large variance of plant level productivity, we focus on a level of θ such that low productivity firms do end up producing in equilibrium, financing themselves using their own net worth. In Appendix D we show that the rate of return on their net worth is given by:

$$R_{t+1}^L = \frac{\left[(A_{t+1} / \alpha) w_t^{\alpha + \eta - 1} (u_t^L)^{1 - \alpha} + q_{t+1} \right] k_t + b_t}{[q_t + (1 - \alpha) u_t^L / \alpha] k_t + b_t / R_t}$$

where the numerator consists of the revenues from production as well as debt repayments received from other entrepreneurs, while the denominator is the cost of purchasing the portfolio. Unlike, high productivity entrepreneurs who leverage up in order to invest in production, low productivity entrepreneurs have more balanced portfolios, consisting of loans to other entrepreneurs as well as own productive projects.

The portfolios of high and low productivity entrepreneurs are linked by the market clearing conditions in the capital and debt markets. This means that once we have solved for the optimal portfolio of the high productivity entrepreneurs, this also gives us the investment choices of low productivity ones. In Appendix D we show that the equilibrium rate of return on wealth for the low types is given below:

$$R_{t+1}^L = \varpi_t^L \left[\frac{(A_{t+1}/\alpha) w_t^{\alpha+\eta-1} (u_t^L)^{1-\alpha} + q_{t+1}}{q_t + (1-\alpha) u_t^L/\alpha} \right] + (1 - \varpi_t^L) R_t$$

where

$$\varpi_t^L \equiv \frac{[q_t + (1-\alpha) u_t^L/\alpha] (1 - K_t)}{[q_t + (1-\alpha) u_t^L/\alpha] (1 - K_t) + l_t E_t q_{t+1}/R_t} < 1$$

is the share of the risky asset in the low productivity entrepreneur's portfolio. Note that this is always less than one because this entrepreneur invests part of his savings into risk free loans to other entrepreneurs. The risky asset available to the low productivity entrepreneur earns a lower rate of return compared to the one held by high productivity ones. The excess return for the 'low' type is given by:

$$E_t \rho_{t+1}^L = E_t \left(\frac{(A_{t+1}/\alpha) w_t^{\alpha+\eta-1} (u_t^L)^{1-\alpha} + q_{t+1}}{q_t + (1-\alpha) u_t^L/\alpha} \right) / R_t \quad (13)$$

The conditions for the optimal portfolio composition of the low productivity type are similar to those in the previous subsection:

$$\varpi_t^L \approx \frac{S_{t+1}^L}{\sigma_{rt+1}}$$

where $S_{t+1}^L \equiv \frac{E_t \rho_{t+1}^L - 1}{\sigma_{rt+1}}$ is the conditional Sharpe ratio on the risky asset for the low productivity entrepreneur and σ_{rt+1} is the standard deviation of the log return on the risky asset. Analogously with σ_{Rt+1} , σ_{rt+1} is determined by the volatility of the technology shock σ_A^2 as well as the volatility of the capital price σ_{qt+1}^2 .

4.2 Behaviour of Workers

Let $V^W(b_{t-1}, X_t)$ denote the value function of a worker with individual financial wealth b_t when the aggregate state is X_t . The value function is given by:

$$V^W(b_{t-1}, X_t) = \max_{c_t, h_t, b_{t+1}} \left\{ \ln \left(c_t - \varkappa \frac{h_t^{1+\omega}}{1+\omega} \right) + \beta E_t V^W(b_t, X_{t+1}) \right\}$$

subject to the flow of funds constraint and the borrowing constraint. The first order conditions are given by:

$$w_t = \varkappa h_t^\omega \tag{14}$$

$$\frac{1}{c_t - \varkappa \frac{h_t^{1+\omega}}{1+\omega}} = \beta R_t E_t \left(\frac{1}{c_{t+1} - \varkappa \frac{h_{t+1}^{1+\omega}}{1+\omega}} \right)$$

In equilibrium, workers will not save as long as the volatility of the aggregate wage is not too great. This is because the risk free interest rate is below the workers' rate of time preference. This means that workers will consume their entire wage income in equilibrium and their welfare will be dominated by the stochastic process for the aggregate wage rate⁵.

The result that workers consume their entire labour income allows us to drop the financial wealth state variable and simplify their value function considerably. Using the optimal labour supply condition (14) we get to the following simple expression:

$$V^W(X_t) = \Theta + \frac{\omega}{1+\omega} \ln w_t + \beta E_t V^W(X_{t+1})$$

where Θ is a constant that depends on parameter values.

4.3 Aggregation and Market Clearing

We complete the characterisation of the competitive equilibrium of our model economy by specifying the evolution equations for the endogenous state variables well as the market clearing conditions.

⁵In solving the model we verify at each point in time that the condition for no saving holds

$$\frac{1}{c_t - \varkappa \frac{h_t^{1+\omega}}{1+\omega}} > \beta R_t E_t \left(\frac{1}{c_{t+1} - \varkappa \frac{h_{t+1}^{1+\omega}}{1+\omega}} \right)$$

There are three market clearing conditions. The bond,

$$\int b_{t+1}(i) di = 0 \quad (15)$$

capital

$$\int k_{t+1}(i) di = 1 \quad (16)$$

and goods markets

$$C_t^H + C_t^L + C_t^W + X_{t+1}^H + X_{t+1}^L = Y_t^H + Y_t^L \quad (17)$$

all clear.

Finally the economy's endogenous state variables evolve according to the following transition law.

$$\begin{aligned} Z_{t+1} &= R_{t+1}^H \beta Z_t^H + R_{t+1}^L \beta Z_t^L \\ &= [d_t R_{t+1}^H + (1 - d_t) R_{t+1}^L] \beta Z_t \end{aligned} \quad (18)$$

$$\begin{aligned} d_{t+1} &= \frac{Z_{t+1}^H}{Z_{t+1}} \\ &= \frac{(1 - \delta) d_t R_{t+1}^H + n \delta (1 - d_t) R_{t+1}^L}{d_t R_{t+1}^H + (1 - d_t) R_{t+1}^L} \end{aligned} \quad (19)$$

4.4 Equilibrium Definition

Recursive competitive equilibrium of our model economy is a price system $w_t, u_t^H, u_t^L, q_t, R_t$, value functions V_t^E and V_t^W , entrepreneur decision rules k_t, x_t, b_t^e, h_t^e and c_t^e , worker decision rules b_{t+1}^w, h_{t+1}^w and c_t^w , and equilibrium laws of motion for the endogenous state variables (18) and (19) such that

(i) The value function V_t^E and the decision rules k_t, x_t, h_t^e, b_t^e and c_t^e solve the entrepreneur's decision problem conditional upon the price system $w_t, u_t^H, u_t^L, q_t, R_t$, the value function V_t^W and the decision rules b_t^w, h_t^w and c_t^w solve the worker's decision problem conditional upon the price system $w_t, u_t^H, u_t^L, q_t, R_t$.

(ii) The process governing the transition of the aggregate productivity and the household decision rules $k_t, x_t, b_t^e, h_t^e, c_t^e, b_t^w, h_t^w$ and c_t^w induce a transition process for the aggregate state given by (18) and (19).

(iii) All markets clear

5 The Economic Impact of Capital Requirements

Capital requirements are the main policy instrument for the government in our framework. In this section we examine using numerical solutions of our model economy what their effect is on economic outcomes. We focus on the ways in which tighter borrowing limits affects the different distortions in the credit constrained economy in order to see how the government trades them off against one another. Section 6 will derive the optimal capital requirement.

5.1 Baseline Calibration

In this section we outline the basic features of the baseline calibration. More details can be found in Appendix G.

We calibrate η , the share of intermediate inputs in gross output to 0.45 using data from the 2007 BEA Industrial Accounts. Using the Cooley and Prescott (1995) methodology we calibrate α (the share of capital in gross output) to 0.2 which gives a share of 0.36 in value added. We set θ (the share of capital which can be collateralised for loans) to 1.0 in line with the value used in Kiyotaki (1998) and Aoki et al (2009). However, since there is very little information on the collateralisability of capital goods we conduct extensive sensitivity analysis due to the highly uncertain value of this parameter.

The technology process at the firm level consists of an aggregate and an idiosyncratic component. Because TFP is endogenous in the Kiyotaki-Moore framework we pick the process for the aggregate exogenous technology shock to match the standard deviation of HP-filtered real GDP. The high (low) realisations of the aggregate TFP shock are 0.6% above (below) the steady state TFP level. The probability that the economy remains in the same aggregate state it is today is equal to 0.8.

Calibrating the cross-sectional dispersion of TFP is important because the quantitative importance of the pecuniary externality studied in our paper is related to the productivity gap between high and low productivity firms. Bernard et al. (2003) report an enormous cross-sectional variance of plant level value added per worker using data from the 1992 US Census of Manufactures. The standard deviation of the log of value added per worker is 0.75 in the data while their model is able to account for only around half this number. The

authors argue that imperfect competition and data measurement issues can account for much of this discrepancy between model and data. In addition, the study assumes fixed labour share across plants so any departures from this assumption would lead to more variations in the measured dispersion of labour productivity.

In a comprehensive review article on the literature on cross-sectional productivity differences, Syverson (2009) documents that the top decile of firms has a level of TFP which is almost twice as high as the bottom decile. He finds that unobserved inputs such as the human capital of the labour force, the quality of management and plant level ‘learning by doing’ can account for much of the observed cross-sectional variation in TFP.

This model does not have intangible assets of the sort discussed in Syverson (2009) and consequently calibrating the model using the enormous productivity differentials identified in the productivity literature would overestimate the true degree of TFP differences. In addition, the Kiyotaki-Moore model would need very tight borrowing constraints or a very small number of high productivity entrepreneurs in order for credit constraints to be binding if some firms are so much more productive than others. And within the framework we have, binding credit constraints are the only mechanism for generating cross-sectional differences in productivity. Aoki et al. (2009) also consider these issues in their calibration of a small open economy version of Kiyotaki and Moore (1997). They argue that a ratio of the productivities of the two groups of 1.15 is broadly consistent with the empirical evidence and I choose this number for the baseline case. However I conduct extensive sensitivity analysis on this hard to pin down parameter because there is very little strong evidence for how to calibrate the productivity dispersion across firms.

Moving on to the parameters governing labour supply we set ω^{-1} (the Frisch elasticity of labour supply) to 3. This is higher than micro-data estimates (references) but is consistent with choices made in the macro literature. We then pick \varkappa , a parameter governing the disutility of labour to get a value of labour supply as a fraction of workers’ time endowment which is equal to 0.33.

The discount factor β , the probability that a highly productive entrepreneur switches to low productivity δ , and the ratio of high to low productivity entrepreneurs n are parameters I pick in order to match three calibration targets - the ratio of tangible assets to GDP,

aggregate leverage and the leverage of the most indebted decile of firms.

I use data on tangible assets and GDP from the BEA National Accounts in the 1952-2008 period. The concept of tangible assets includes Business and Household Equipment and Software, Inventories, Business and Household Structures and Consumer Durables. GDP excludes government value added so it is a private sector output measure.

Aggregate leverage is defined as the average ratio of the value of the debt liabilities of the non-financial corporate sector to the total value of assets. Leverage measures can be obtained from a number of sources. In the US Flow of Funds, aggregate leverage is approximately equal to 0.5 for the 1948-2008 period. This is broadly consistent with the findings of den Haan and Covas (2007) who calculate an average leverage ratio of 0.587 in Compustat data from 1971 to 2004. Den Haan and Covas (2007a) also examine the leverage of large firms and find that it is slightly higher than the average in the Compustat data set. Firms in the top 5% in terms of size have leverage of around 0.6. Den Haan and Covas (2007b) have similar findings in a panel of Canadian firms. There the top 5% of firms have leverage of 0.7-0.75 compared to an average of 0.66 for the whole sample. High productivity entrepreneurs in our economy run larger firms so differences in productivity and therefore leverage could be one reason for the findings of Den Haan and Covas (2007a and 2007b). But the perfect correlation of firm size and leverage that holds in our model will not hold in the data. So if we are interested in the distribution of firm leverage, the numbers in Den Haan and Covas will be an underestimate. This is why we pick a target for the average leverage of the top 10% most indebted firms to be equal to 0.75. This number is broadly consistent with the findings in Den Haan and Covas.

Table 1 below summarises the calibration targets we match while Table 2 summarises the baseline parameter values used in the paper.

Table 1: Calibration targets

Target	Value	Source
Tangible Assets to GDP = $q / (Y^H + Y^L - X^H - X^L)$	3.49	BEA National Accounts
Aggregate Leverage = $L^A = B / (q + Y^H + Y^L)$	0.50	Flow of Funds
Leverage of indebted firms = $L^H = B / (qK + Y^H)$	0.75	Den Haan-Covas (2007a)
Share of intermediate inputs in gross output = η	0.45	BEA National Accounts
Share of capital in GDP = $\alpha / (1 - \eta)$	0.36	BEA National Accounts
Cross sectional productivity dispersion = a^H / a^L	1.15	Aoki et al. (2009)
Collateralisability of capital = θ	1.00	Aoki et al. (2009)
Standard deviation of annual real GDP	2.01	BEA National Accounts

Table 2: Summary of baseline model calibration

Parameter Name	Parameter Value
β	0.896
δ	0.145
n	0.084
α	0.20
η	0.45
ω	0.33
\varkappa	2.29
p_{gg}	0.80
p_{bb}	0.80
A^h	1.006
A^l	0.994
a^H / a^L	1.15
θ	1.00

5.2 Model evaluation

Having chosen parameter values to match the first moments of the model to those in the data and to match the volatility of real GDP, in this section we evaluate the model by analysing how key moments of the model compare to those in the data. All variables have been detrended using the HP filter (for more details see Appendix G) Table 3 below compares the second moments of the model relative to the data⁶. The numbers we focus on is the standard deviation of annual aggregate non-durable consumption, aggregate labour hours and the stock market

Table 3: Model second moments

	Data	Model
σ_c	1.55	2.01
σ_h	1.32	1.25
σ_v	6.06	2.55

Note: σ_c is the standard deviation of the logarithm of aggregate consumption, σ_h is the standard deviation of the logarithm of aggregate labour hours, σ_v is the standard deviation of the logarithm of stock prices

The standard deviation of aggregate labour hours in the model are broadly in line with those in the data. The model does less well in the other two key dimensions we use in our evaluation. Aggregate consumption is too volatile relative to the data. This is a feature of the model that can be improved upon in future work by adding a better means of aggregate saving. Capital is fixed and the only means of aggregate saving for agents in the model is to purchase intermediate inputs. In addition, due to the low risk free interest rate, workers do not save and their consumption is as volatile as labour income. In future work I intend to extend the model by adding capital which does not depreciate fully and which can, therefore, be accumulated in the aggregate, allowing households to smooth consumption better. The volatility of the real value of the S&P 500 in the data is also considerably higher than the volatility of asset prices in the model.

⁶More details on how the data moments were computed are in Appendix G.

5.3 Borrowing Constraints and Steady State Productive Efficiency

In this subsection we consider what would happen in the steady state (i.e. in the absence of aggregate shocks) if the government chooses to impose tighter capital requirements (a lower value of $\tilde{\theta}$). Perhaps the biggest welfare cost of tighter borrowing constraints arises because borrowing constraints reduce the efficiency of the economy. This happens for two reasons. Firstly, the downpayment requirements on capital acts as a tax on the capital purchases of high productivity entrepreneurs and distorts their production mix relative to the first best. Secondly, borrowing constraints increase the share of low productivity firms in economic activity, reducing aggregate TFP. Below we explain both of these sources of inefficiency.

5.3.1 Capital requirements and the ‘downpayment tax’ on high productivity entrepreneurs

In Appendix H we show that we can write the steady state user cost of capital for high productivity entrepreneurs in the tax wedge form popularised by Chari, Kehoe and McGrattan (2007):

$$\begin{aligned} u_t^H &= q_t - \left[\frac{\tilde{\theta}_t}{R_t} + \frac{1 - \tilde{\theta}_t}{R_{t+1}^H} \right] q_{t+1} \\ &= u_t^L \left(1 + \tau_t(\tilde{\theta}) \right) \end{aligned}$$

where the tax is given by the following expression

$$\tau_t(\tilde{\theta}_t) = (1 - \tilde{\theta}_t) \left(\frac{q_t}{u_t^L} - 1 \right) \left(1 - \frac{R_t}{R_{t+1}^H} \right) \quad (20)$$

The collateral requirement acts like a tax on the capital purchases of constrained producers. The size of the tax is determined by the following factors. First of all, the tax is increasing in the required downpayment on capital goods $1 - \tilde{\theta}_t$. This fraction determines how much of the capital purchase needs to be financed by expensive own savings as opposed to cheap external funds. The difference between the valuation of internal funds and the market price of loans is given by the $1 - \frac{R_t}{R_{t+1}^H}$ term in (20). It arises when the borrowing constraint leads to a deterioration in consumption smoothing. High productivity entrepreneurs experience faster consumption growth making them less willing to save. And because

the collateral constraint forces them to save, this acts to increase their user cost relative to unconstrained low productivity agents. Secondly, the tax is increasing in the price to rent ratio of capital. This is because a high price to rent ratio increases the internal funds required by a constrained borrower (who needs to have a fraction of the cost of capital as downpayment) relative to an unconstrained borrower (who effectively faces only the user cost). The first row in Table 4 below shows how the ‘downpayment tax’ varies with the value of downpayment requirement. As $\tilde{\theta}_t$ - the collateralisability of capital - declines from 1.0 and 0.8, the ‘tax’ increases from 0 to 20%.

Interestingly the impact of capital requirements on the real wage is very small due to two opposing effects. Lower $\tilde{\theta}_t$ allows high productivity entrepreneurs to expand production which boosts TFP and increases wages. But there is another effect. Lower $\tilde{\theta}_t$ increases the user cost of capital and skews the input mix by high productivity entrepreneurs towards intermediate inputs and labour. The higher labour demand increases the wage. At high levels of $\tilde{\theta}_t$, the share of production done by the efficient producers is high and the two effects offset each other leaving the real wage broadly unchanged..

As the last four rows of Table 4 show, the decline in the economy’s efficiency due to higher capital requirements leads to a fall in the steady state consumption of all groups in society. Most strongly affected are high productivity entrepreneurs; their consumption (C^H) declines by more than 30% as $\tilde{\theta}$ falls towards 0.8 from the baseline of 1.0. But other agents in the economy are negatively affected too. Low productivity entrepreneurs’ consumption (C^L) falls 10% largely as a result of the lower wealth of these consumers who accumulate less wealth during previous productive spells due to the effect of capital requirements. Workers’ consumption posts a more modest 1% decline largely as a result of the small impact on the real wage.

Table 4: Selected first moments under different capital requirements

Capital Requirements	$\tilde{\theta} = 0.8$	$\tilde{\theta} = 0.9$	$\tilde{\theta} = 1.0$	1st best
τ	0.20	0.12	0.00	0.00
w	1.582	1.586	1.586	1.744
K^H	0.29	0.41	0.69	1.00
TFP	1.048	1.064	1.104	1.15
C^E	0.464	0.489	0.549	0.643
C^H	0.942	1.122	1.540	0.643
C^L	0.427	0.439	0.471	0.643
C^W	0.518	0.523	0.523	0.767

Notes: τ is the 'downpayment tax' rate, w is the wage rate, K^H is the share of the capital stock held by high productivity entrepreneurs, TFP is aggregate total factor productivity, C^E is average entrepreneurs' consumption, C^H is average high productivity entrepreneurs consumption, C^L is average low productivity entrepreneurs consumption, C^W is average workers' consumption.

5.3.2 Capital requirements and the level of TFP

The aggregate level of TFP in this economy is given by the ratio of aggregate output in the economy to the inputs that are used in production.

$$TFP_t = A_t \frac{a^H (K)^\alpha (X^H)^\eta (H^H)^{1-\alpha-\eta} + (1-K)^\alpha (X^L)^\eta (H^L)^{1-\alpha-\eta}}{(X^H + X^L)^\eta (H^H + H^L)^{1-\alpha-\eta}}$$

In Appendix I we show that aggregate TFP in the economy is given by the following expression:

$$TFP_t = \frac{1 + K_t [a^H (1 + \tau(\theta))^{1-\alpha} - 1]}{1 + \tau(\theta) K_t}$$

The downpayment tax and the existence of inefficient production under binding borrowing constraints endogenously reduces the economy's level of TFP. This can be seen in the last row of Table 5 above. As $\tilde{\theta}$ declines from unity to 0.8, the share of capital held by high productivity entrepreneurs declines from 0.69 to 0.30, bringing about a decline in aggregate TFP of more than 5%. This is a crucial feature of the Kiyotaki (1998) framework. When

borrowing constraints bind tightly, not enough funds get into the hands of the high productivity firms. As a result, the economy operates within the production possibility frontier because some of the scarce capital input is held by low productivity firms.

5.4 Borrowing Constraints and Aggregate Volatility in the Stochastic Economy

In this subsection we consider how the imposition of capital requirements affect the equilibrium of the economy with aggregate uncertainty. Here we focus on the ways in which capital requirements affect the volatility of aggregate consumption as well as the consumption of different groups and link it to the endogenous fluctuations in TFP which arise due to the amplification mechanism.

Leverage leads to a reallocation of capital between high and low productivity entrepreneurs over the business cycle. This happens through the standard collateral amplification mechanism of Kiyotaki and Moore (1997), which can cause substantial endogenous fluctuations in TFP amplifying the normal shocks to technology over the business cycle. The mechanism which generates this amplification is the following. When the aggregate productivity state A_t changes (say, it falls), this reduces the capital price in both the borrowing constrained and in the ‘first best’ economy. But whereas in the ‘first best’ world, there is very little additional propagation, in the credit constrained (leverage financed) economy, the fall in asset prices impacts the wealth of high productivity and low productivity agents differently. Because they are leveraged, high productivity entrepreneurs are badly affected and have to scale down their capital investments because they can no longer afford the required downpayment as well as the cost of the capital input needed to operate productive projects with a large capital input. The purchasers of capital are the low productivity entrepreneurs and consequently the economy’s aggregate TFP declines as inefficient production expands. The additional fall in TFP puts further downward pressure on capital prices and on the wealth and borrowing capacity of high productivity entrepreneurs. This is the amplification channel of Kiyotaki and Moore (1997): small declines in the economy’s aggregate technology can set off a self-reinforcing spiral of falling TFP and asset prices, magnifying the effect of

the original technology shock. The amplification mechanism is very important because its quantitative strength will be a crucial determinant of whether capital requirements can be welfare improving or not.

Table 5: Selected second moments under different capital requirements

Capital Requirements	$\tilde{\theta} = 0.80$	$\tilde{\theta} = 0.90$	$\tilde{\theta} = 1.00$	1st best
σ_y	1.48	1.57	2.01	1.37
σ_q	1.52	1.68	2.55	1.37
σ_c	1.48	1.57	2.01	1.37
σ_w	0.57	0.59	0.65	0.34
σ_{TFP}	0.53	0.66	0.85	0.60

Note: σ_y is the standard deviation of the log of output, σ_q is the standard deviation of the log of the capital price, σ_c is the standard deviation of the log of aggregate consumption, σ_{TFP} is the standard deviation of the log of aggregate total factor productivity, σ_w is the standard deviation of the log of the real wage rate.

Cordoba and Ripoll (2004) have argued that the amount of amplification in the Kiyotaki and Moore (1997) framework is very small when one assumes concave utility and decreasing returns to scale in production. They show that large amplification needs a large productivity gap, a large share of constrained agents in production and substantial reallocation of collateral in response to shocks. Cordoba and Ripoll (2004) find that, in particular, there is a trade off between having a large productivity gap and having a lot of production in the hands of constrained entrepreneurs. This is because they assume decreasing returns to scale at the plant level. When constrained firms are very small and their output is low they are much more productive than the larger unconstrained firms. But the downside is that their share in total output is low. At the other extreme, when constrained firms are large, their productivity advantage relative to unconstrained ones is small. In both cases, at least one condition for large amplification is not satisfied and so the additional volatility from the model is negligible.

As Table 5 shows, the amplification we obtain from our calibrated version of the Kiyotaki (1998) model is very substantial. In the baseline case, the standard deviations of TFP and

output are, respectively, 38% and 45% higher compared to the first best while the standard deviation of the capital price is 84% higher. So contrary to the results in Cordoba and Ripoll (2004) we get quantitatively large amplification from the framework. Our differences from Cordoba and Ripoll (2004) arise from one main source - our assumption of constant returns to scale to all factors at the plant level. Even though we have decreasing returns to the collateral factor (capital), the production function is constant returns in all the three factors. This means that in our calibration we do not face the trade off between the size of the productivity gap and the share of constrained producers in economic activity. The productivity gap is largely driven by the value of a^H as well as the ‘downpayment tax’ $\tau(\theta)$. It is independent of the level of output at any individual firm. When we add the effects of leverage (again realistically calibrated to match US data), we get substantial re-allocation of collateral between high and low productivity entrepreneurs as asset prices fluctuate. So Cordoba and Ripoll’s conditions for amplification are satisfied and this explains why our constrained economy is so much more volatile relative to the ‘first best’. Our results are similar to those in Vlieghe (2005) who found something very similar in a version of Kiyotaki (1998) with nominal rigidities. In his model (which also featured constant returns to all factors) amplification was very substantial showing the potential of the framework to propagate shocks.

In addition to the amplification of aggregate fluctuations, leverage concentrates the aggregate risk in the hands of only a small subset of agents in the economy. When capital is largely held by high productivity entrepreneurs who finance their capital holdings using simple debt, risk sharing between the two groups deteriorates. We can see this in Table 6 below which shows the variance of the aggregate consumption of the two groups. This difference grows as credit constraints are relaxed due to the increasing collateralisability of capital.

Table 6: Consumption volatility for the workers and entrepreneurs⁷

Capital Requirements	$\tilde{\theta} = 0.80$	$\tilde{\theta} = 0.90$	$\tilde{\theta} = 1.00$	1st best
σ_{cH}	2.48	3.12	5.57	1.37
σ_{cL}	1.33	1.34	1.52	1.37
σ_{cW}	1.45	1.50	1.65	1.37

Note: σ_{cH} is the unconditional standard deviation of the log of the consumption of high productivity entrepreneurs, σ_{cL} is the unconditional standard deviation of the log of the consumption of low productivity entrepreneurs, σ_{cW} is the unconditional standard deviation of the log of the consumption of workers.

This result is not surprising. The low productivity entrepreneurs hold largely riskless debt and small positions in risky capital. In contrast, high productivity entrepreneurs hold leveraged positions in risky capital. This asymmetry in the asset holdings of the two groups leads to a concentration of the aggregate risk in the economy into the hands of very few (high productivity) individuals whose consumption fluctuates very substantially. Our results are in line with the findings of Vissing-Jorgensen and Parker (2009) who find that the aggregate risk is borne by a small fraction of high consumption/high income households. Tightening firms' access to borrowing reduces this asymmetry in the riskiness of different individuals' portfolios and consequently reduces the volatility in their relative consumption levels over the business cycle.⁸

5.5 Discussion

In this section we examined the quantitative significance of four ways in which the credit constrained economy is distorted relative to the first best. These distortions, however, do

⁷Note that these consumption volatilities refer to the standard deviation of the consumption of all agents who happen to be high or low productivity at a given point in time. They are not the expected standard deviation of the consumption of the people who are high or low productivity at time 0 when the policy is decided upon.

Nevertheless we think these numbers are informative of the kind of consumption volatility caused by aggregate uncertainty. It illustrates the fact that high productivity entrepreneurs face a lot of risks to their net worth because of leverage and this causes their consumption to be much more volatile ex post.

⁸In the limit, when no borrowing is allowed and all production is entirely net worth financed, both types of agents hold identical portfolios (only productive projects) and risk sharing is perfect.

not necessarily imply that the economy is constrained inefficient. As long as the government cannot do anything directly about borrowing constraints, many of these distortions will be an unavoidable consequence of credit market imperfections.

For example, any deviations of the economy's steady state from first best would be constrained efficient. The trade off between productive efficiency and consumption smoothing is identical for private individuals and for the government. Private borrowers with good productive opportunities choose to borrow up to the limit and experience a steeply sloped consumption path because the rates of return they can earn on productive projects are much better compared to the cost of debt. The government will make an identical decision because it can redistribute capital holdings between the two groups and compensate the low productivity firms for their lost output while still making the high productivity borrowers better off. The only constraint on this redistribution is the collateral constraint, which binds for the government in the same way as it binds for the *laissez faire* economy.

In a stochastic environment, the efficiency properties of the competitive equilibrium change. The collateral amplification mechanism of Kiyotaki and Moore (1997) introduces feedback effects between asset prices, the net worth of leveraged borrowers and the tightness of borrowing constraints. When aggregate productivity switches from high to low, asset prices fall and this has a disproportionately negative effect on the net worth of leveraged high productivity borrowers. Because part of the capital purchase and the whole of the intermediate input purchase is non-collateralisable, borrowers need their own net worth in order to produce on a large scale. Therefore the fall in the net worth of high productivity borrowers reduces the amount of capital they can invest in production and forces them to scale down their capital holdings. The low productivity agents absorb the capital sold by the high productivity ones but only at lower prices. But this fall in the price of capital further damages the net worth of leveraged firms and forces them to cut their capital holdings even further. This completes the 'credit cycle', amplifying and propagating small shocks into larger fluctuations in output, TFP and asset prices.

Where does the inefficiency of private leverage come from? As identified in Lorenzoni (2008) and Korinek (2009), when collateral constraints bind, the pecuniary externalities we usually consider harmless from an economic efficiency point of view, begin to interfere with

the allocative efficiency of the economy. The forced sales of leveraged borrowers depress asset prices and tighten the credit constraints of all other constrained borrowers, forcing them to sell assets themselves⁹.

6 The Model Economy under Capital Requirements

In this section we turn to the main question of this research: are private leverage decisions optimal from a social point of view? From the work of Lorenzoni (2008) and Korinek (2009) we know that, qualitatively, the answer is ‘no’. Here we examine whether, quantitatively, the inefficiency is large or small.

We assume that capital requirements are chosen by a benevolent government who maximises a social welfare function which weights the values of all agents in the economy. The government is subject to the same collateral and budget constraints facing private agents. So any differences in private and social leverage choices are due to the market price externality discussed above.

6.1 The Government’s Problem

The government optimises the coefficient on a simple state contingent capital requirement rule

$$\tilde{\theta}_t = \min \left[\exp \left(\chi_0^i + \chi_1^i \ln d_t + \chi_2^i \ln Z_t \right), \theta \right] \quad (21)$$

in order to maximise the following social welfare function

$$\Omega_0 = \max_{\{\chi^i\}} E_0 \left[\sum_i \varsigma_E^i \sum_{t=0}^{\infty} \beta^t \ln c_t^i \right] + \varsigma_W \sum_{t=0}^{\infty} \beta^t \ln \left(C_t^W - \varkappa \frac{(H_t)^{1+\omega}}{1+\omega} \right) \quad (22)$$

⁹But although such pecuniary externalities exist they are not always quantitatively significant. For example, Guerrieri (2007) examines the constrained efficiency of a competitive labour market search model with private information and limited commitment. In her model, workers take the value of the outside unemployment option as given while the planner recognises that it is endogenous because the expected value of job matches affects the continuation value of the unemployed. Although Guerrieri (2007) identifies this very interesting source of inefficiency of the competitive equilibrium, she finds that, quantitatively, the externality in question is very small.

where ς_E^i is the Pareto-Negishi weight on entrepreneur i while ς_W is the Pareto-Negishi weight on the workers. We do not consider any other policy instruments.¹⁰ Note that the capital requirements $\tilde{\theta}_t$ is constrained by the exogenously given limit θ .

$$\tilde{\theta}_t \leq \theta$$

In other words the government has no advantage in enforcing debt repayment over the private sector and therefore it cannot choose looser capital requirements than the market. The policy rule (21) allows the capital requirement to undergo mean shifts as the aggregate productivity state changes. Capital requirements also can respond to changes in the other aggregate state variables - total wealth w_t and the share of wealth held by high productivity people d_t . Once the government has chosen capital requirements, the collateral constraint in the regulated economy becomes:

$$b_t \leq \tilde{\theta}_t E_t q_{t+1} k_t \tag{23}$$

Private agents then perform exactly the same maximisation problem as in the unregulated economy, but the collateral constraint they now face may be tighter if $\tilde{\theta}_t < \theta$ in some states of the world.

In Appendix C we show that the value function of the two types of entrepreneurs at time 0 depends on the net present value of future expected log rates of return on wealth as well

¹⁰We do not solve a social planning problem because the collateral constraints in our economy depend on prices and these do not admit to a simple closed form solution in the same way as in Lorenzoni (2008) and Korinek (2009).

In future work, we intend to solve for the full Ramsay problem. We do not do this here because it complicates the solution of the model. At the same time the policy we consider does capture a lot of intuitive features about the way capital requirement policy may be implemented. It is fully state contingent and it is conducted under commitment because the government chooses the χ^i coefficients at the beginning of time and sticks to them for ever.

Our policy rule is, therefore, similar to the ‘Optimal non-inertial plan’ popularised by Woodford (2003) because it is conducted under commitment (the central bank optimises its coefficients in a once and for all fashion) but without responding to lagged variables (which is what the optimal Ramsay commitment policy does).

as the logarithm of current financial wealth.

$$V^i(X_0) = \varphi^i(X_0) + \frac{\ln z_0}{1 - \beta}, i = H, L$$

where $\varphi^i(X_0)$ is the net present value of future rates of return on wealth and z_0 is time 0 financial wealth.

$$\varphi^i(X_t) = \ln(1 - \beta) + \frac{\beta \ln \beta}{1 - \beta} + \max_{x_t, k_t, h_t, b_t} \beta E_t \left[\frac{\ln(R_{t+1}^i)}{1 - \beta} + \varphi^i(X_{t+1}) \right]$$

We assume a particular initial wealth distribution in which all high and all low productivity entrepreneurs have an initial level of wealth equal to the group average in the ‘no regulation’ steady state. This allows us to consider the following social welfare function which weights the utilities of the three groups by the inverse of their marginal utility of consumption evaluated at the initial wealth distribution (more details in Appendix K):

$$\Omega_0 = \max_{\{x^i\}} E_0 \left[\varsigma^H \left(\varphi^H(X_0) + \frac{\ln Z_0^H}{1 - \beta} \right) + \varsigma^L \left(\varphi^L(X_0) + \frac{\ln Z_0^L}{1 - \beta} \right) + \varsigma^W V^W(X_0) \right] \quad (24)$$

where $\varphi^H(X_0)$ and $\varphi^L(X_0)$ are the NPVs of future expected log rates of return on wealth for the two groups of entrepreneurs while Z_0^H and Z_0^L are the initial wealth levels of the high and low productivity entrepreneurs.

6.2 When is private leverage excessive?

The benevolent government chooses and commits to a time invariant capital requirement function $\tilde{\theta}_t$ which maximises social welfare (24). The government cares about three things in (24). It wants to maximise the Pareto weighted average of the net present expected value of log returns on wealth for the two types of entrepreneurs. These are the φ_0^H and φ_0^L terms in the social welfare function. But it also wants to maximise the welfare of workers which depends on the average level and volatility of real wages. Finally, the government cares about the current financial wealth of entrepreneurs too. It knows that any policy announcement will immediately be reflected in the capital price, impacting on the wealth of the two groups and it takes this into account when designing the optimal policy. In the next section we will compute numerically how these determinants of the welfare of the three groups change as

we vary capital requirements. Then we will see whether the government can increase welfare relative to the market.

Here however we try to add a little more intuition by considering how the capital requirement choices of the government differ from those of private individuals in more detail. We do this by looking at what choices the government would make if allowed to choose $\tilde{\theta}_t$ in order to maximise the log expected portfolio return of the two groups of entrepreneurs as well as the log wage rate of workers. We compute the government's first order condition for each group's portfolio problem and evaluating them at private leverage choices l_t^m . This exercise will be useful for two reasons. First of all it identifies any sources of re-distribution between the two groups as capital requirements are tightened. But secondly, it pinpoints where the externalities discussed by Lorenzoni (2008) and Korinek (2009) might occur in our framework.

6.2.1 High productivity entrepreneurs

Starting with the portfolio problem of high productivity entrepreneurs we find how R_{t+1}^{H*} is affected by tightening collateral requirements around the private optimum l_t^m

$$\frac{\partial R_{t+1}^{H*}(\tilde{\theta}_t = l_t^m)}{\partial \tilde{\theta}_t} \approx \frac{\partial \varpi_t^H}{\partial \tilde{\theta}_t} (E_t \rho_{t+1}^H - 1 - \varpi_t^H \sigma_{R_{t+1}}^2) - \frac{(\varpi_t^H)^2}{2} \frac{\partial (\sigma_{R_{t+1}}^2)}{\partial \tilde{\theta}_t} + \left(\frac{\partial E_t \rho_{t+1}^H}{\partial \tilde{\theta}_t} + \frac{\partial \ln R_t}{\partial \tilde{\theta}_t} \right) \quad (25)$$

Here ρ_{t+1}^H is the excess return on leveraged production for high productivity entrepreneurs, which was defined in equation (11). The value of (25) depends strongly on whether borrowing constraints bind or not in the current period. When borrowing constraints bind, the entrepreneur's portfolio hits the constraint and the private first order condition with respect to the share of the risky asset (equation (12)) holds with inequality:

$$E_t \rho_{t+1}^H - 1 - \varpi_t^H \sigma_{R_{t+1}}^2 > 0$$

But the government takes an additional amplification effect into account. This is the $\frac{(\varpi_t^H)^2}{2} \frac{\partial (\sigma_{R_{t+1}}^2)}{\partial \tilde{\theta}_t}$ term in equation (25). It takes into account the endogeneity of the variance of the portfolio rate of return for high productivity entrepreneurs. The more they borrow to invest into risky assets, the larger the impact of capital price shocks on their rates of

return on wealth. And this is where the amplification mechanism generates the externality identified in Lorenzoni (2008) and Korinek (2009). When capital prices fall, leveraged entrepreneurs make low returns on wealth and this forces them to sell capital because they no longer have the net worth to purchase the non-collateralised inputs needed to support a large capital input into production. The capital sales can only be absorbed by low productivity firms at lower prices, leading to another round of forced capital sales by credit constrained entrepreneurs.

But the government also recognises the fact that its policy instrument has its costs. Raising the downpayment requirement on capital acts like a tax on high productivity entrepreneurs, which reduces their excess return on production: $\frac{\partial E_t \rho_{t+1}^H}{\partial \tilde{\theta}_t} > 0$. So when capital requirements are tightened, the excess return on high productivity projects is reduced due to their distorted input mix. Partially offsetting that, the risk free rate increases when the government tightens credit limits: $\frac{\partial \ln R_t}{\partial \tilde{\theta}_t} < 0$. But overall, tighter capital requirements leads to a lower rate of return on wealth for high productivity entrepreneurs. Finally, high productivity entrepreneurs have substantial capital positions which depreciate in value when regulation is introduced. This has a negative effect on their welfare.

6.2.2 Low productivity entrepreneurs

Moving on to the portfolio of low productivity entrepreneurs we have the following first order condition, which determine the way the capital requirements for high productivity entrepreneurs impact on the log rate of return on wealth for the low types:

$$\frac{\partial \ln R_{t+1}^{L*} \left(\tilde{\theta}_t = l_t^m \right)}{\partial \tilde{\theta}_t} \approx \frac{\partial \varpi_t^L}{\partial \tilde{\theta}_t} \left(E_t \rho_{t+1}^L - 1 - \varpi_t^L \sigma_{rt+1}^2 \right) - \frac{(\varpi_t^L)^2}{2} \frac{\partial (\sigma_{rt+1}^2)}{\partial \tilde{\theta}_t} + \frac{\partial E_t \rho_{t+1}^L}{\partial \tilde{\theta}_t} + \frac{\partial \ln R_t}{\partial \tilde{\theta}_t} \quad (26)$$

Capital requirements will affect low productivity types indirectly because they will reduce the available supply of the risk free asset and force them to invest more of their net worth in production. This is the first term in (26). But in addition, the volatility of the aggregate economy will decline and this will reduce the variance of the returns on the risky asset $\left(\frac{(\varpi_t^L)^2}{2} \frac{\partial (\sigma_{rt+1}^2)}{\partial \tilde{\theta}_t} \right)$. The excess return on the risky asset for low productivity types will also change $\left(\frac{\partial E_t \rho_{t+1}^L}{\partial \tilde{\theta}_t} \right)$ depending on whether the overall portfolio has become riskier or safer as a

result of the policy change. Finally risk free rates will change as the economy becomes more regulated.

For unconstrained low productivity entrepreneurs most of the terms in (26) are zero. $E_t \rho_{t+1}^L - 1 - \varpi_t^L \sigma_{rt+1}^2 = 0$ from optimal portfolio choice. Because the low productivity type prices assets in our economy, any change in the volatility of returns will be reflected in the excess returns demanded in equilibrium. This means that $-\frac{(\varpi_t^L)^2}{2} \frac{\partial(\sigma_{rt+1}^2)}{\partial \tilde{\theta}_t} + \frac{\partial \rho_{t+1}^L}{\partial \tilde{\theta}_t} = 0$: more volatile returns will be accompanied by a higher excess return leaving the welfare of low productivity entrepreneurs unaffected.

There is an interesting difference between the way the government treats the portfolios problems of the two groups. In the case of the high productivity agents, the government was concerned with the welfare consequences of the market price externality which increased the value of σ_{Rt+1}^2 - the variance of the log rate of return on the risky asset for high productivity entrepreneurs. But in this case changes in σ_{rt+1}^2 - the variability of the log excess return on the risky asset for the low types - did not represent any allocative inefficiency.

This difference arises because low productivity entrepreneurs are always unconstrained in their portfolio choice so, on the margin, any increase in the volatility of capital prices due to the excessive leverage of other entrepreneurs is compensated in equilibrium by higher excess returns. For the low productivity types the behaviour of the productive types represents a pure pecuniary externality with no consequences for allocative efficiency. In contrast, high productivity entrepreneurs are borrowing constrained (at least in some states of the world) and the tightness of the borrowing constraint depends on the level of asset prices. So the pecuniary externalities caused by the forced capital sales by leveraged entrepreneurs in downturns do have consequences for the allocative efficiency of the economy. By tightening borrowing constraints for everyone else, forced sales exert an externality the benevolent government should be concerned with correcting.

Because most of the terms in (26) drop out, the expected net present value of future returns for low productivity types is driven largely by what is happening to the log of risk free rates.

$$\frac{\partial \ln R_{t+1}^{L*} \left(\tilde{\theta}_t = l_t^m \right)}{\partial \tilde{\theta}_t} \approx \frac{\partial \ln R_t}{\partial \tilde{\theta}_t}$$

Because tightening collateral requirements in the Kiyotaki (1998) model reduces aggregate TFP and pushes down on capital prices, the lower user cost of capital increases the rate of return on production for low productivity types and, by arbitrage, increases the risk free rate. This effect raises the welfare of low productivity entrepreneurs.

But there are other factors which reduce the welfare of low productivity entrepreneurs. First of all, the continuation value of low productivity agents φ_t^L partly depends on the value of a possible future high productivity opportunity φ_t^H and as we have seen in the previous subsection, this can be reduced by regulation. But secondly, as capital regulation is tightened, this depresses capital prices which form a part of all entrepreneurs' portfolios. So the wealth terms of (24) will fall. Overall, the welfare of the unproductive will rise if they do not hold much capital (hence the loss of wealth from lower prices is small) and if they are not very likely to transit to the high productivity state (hence the fall in the value of productive opportunities does not affect them much).

6.2.3 Workers

Workers' period welfare is determined by the log of the real wage.

$$\frac{\partial \ln w_t \left(\tilde{\theta}_t = l_t^m \right)}{\partial \tilde{\theta}_t}$$

As the results in Table 5 above showed, tightening capital requirements in relatively well developed financial systems (with a high value of $\tilde{\theta}_t$) resulted in slightly higher real wages and higher welfare for workers. However, tightening collateral requirements in a less well developed financial system resulted in lower wages for workers.

To summarise. We can see that introducing capital requirements may improve the welfare entrepreneurs and workers although this is by no means guaranteed. When collateral requirements are already binding at the time of capital requirement reform, such a reform may not be welfare increasing despite the existence of externalities. This is because the binding collateral constraint makes the policy instrument (tightening collateral constraints even further in some states of the world) a very distortionary one. In order for the government to distort an already distorted economy even further, two things have to be true: the collateral

amplification mechanism must be very powerful and/or private individuals must care very much about consumption volatility. We now proceed to check whether numerically this is the case or not in our economy.

7 Optimal Collateral Requirements and Welfare

7.1 Numerical Results

In this section we use numerical simulations to compare the market and the government's choices of the collateral requirements on capital. We do this under different states of the financial system as measured by θ - the fraction of capital which is collateralisable¹¹. This is done in Table 7 below. The first row of the table shows that firms always choose to invest up to the debt limit in the competitive equilibrium. The second row shows the government's choice of capital requirement as it tries to maximise the social welfare function (24). The capital requirement turns out to be invariably equal to the privately permissible maximum leverage and, unsurprisingly, private agents borrow the same amount as they do in the unregulated economy (shown in the third row of the table). This is the main result of this paper - when credit constraints bind tightly due to a substantial productivity differential between the two types of entrepreneurs in our economy, the government wants to encourage investment all the way to the incentive-compatibility determined borrowing limit θ .

Table 7: The government's collateral requirement choices

	$\theta = 0.80$	$\theta = 0.90$	$\theta = 1.00$
$E(l_t^m)$	0.80	0.90	1.00
$E(\tilde{\theta}_t)$	0.80	0.90	1.00
$E(l_t^g)$	0.80	0.90	1.00

Note: $E(l_t^m)$ is the average private choice of debt as a fraction of tangible assets in the Laissez Faire economy, $E(\tilde{\theta}_t)$ is the average capital requirement in the regulated economy and $E(l_t^g)$ is the average

¹¹In each we recalibrated the model to match the target discussed in the calibration section. These are (1) aggregate leverage, (2) leverage of the most indebted decile of firms, (3) the ratio of tangible assets to GDP, (4) the fraction of time spent working and (5) the standard deviation of real GDP.

private choice of debt as a fraction of tangible assets in the regulated economy.

Table 8 below tries to delve a little deeper into the determinants of welfare for individual groups as well as the aggregate economy in order to see how they they are affected by changes in capital requirements. The table looks at the change in a number of measures of welfare from the imposition of a capital requirement $\tilde{\theta}_t = \theta - 0.01$ in all states of the world. Because we are interested in how the initial state of the financial system affects the incentives of the government to regulate leverage, we repeat our exercise for several financial systems, represented by different values of the maximum collateral limit θ .

So for example, the first column of the table takes an economy where the state of the financial system can collateralise up to a 0.8 fraction of capital values. To see the local incentives for the government to regulate we consider the welfare effects of the imposition of a capital requirement $\tilde{\theta}_t = 0.79$.

Table 8: Capital requirements and welfare

	$\theta = 0.80$	$\theta = 0.90$	$\theta = 1.00$
Welfare of high productivity entrepreneurs			
$100\Delta \ln \varphi_0^H$	-0.33	-0.32	-0.23
$100\Delta \ln Z_0^H$	-1.06	-1.91	-4.15
$100\Delta \ln V_0^H$	-0.33	-0.58	-1.15
Welfare of high productivity entrepreneurs			
$100\Delta \ln \varphi_0^L$	0.37	0.58	1.11
$100\Delta \ln Z_0^L$	-0.14	-0.32	-0.71
$100\Delta \ln V_0^L$	0.05	0.03	0.04
Workers' welfare			
$100\Delta \ln V_0^W$	-0.09	-0.02	0.14
Aggregate welfare			
$100\Delta \ln V_0$	-0.18	-0.23	-0.33

Note: All variables in the table measure the percentage change in the relevant component of welfare from a tightening of collateral requirements by 0.01 (or 1% of the value of tangible assets). $\Delta \ln \varphi_0^H$ is the change in the net present value of future expected log rates of return on wealth for high productivity entrepreneurs,

$\Delta \ln \varphi_0^L$ is the change in the net present value of future expected log rates of return on wealth for low productivity entrepreneurs, $\Delta \ln w_0^H$ is the wealth change for high productivity entrepreneurs, $\Delta \ln w_0^L$ is the wealth change for low productivity entrepreneurs, $\Delta \ln V_0^H$ is the welfare change for high productivity entrepreneurs, $\Delta \ln V_0^L$ is the welfare change for low productivity entrepreneurs, $\Delta \ln V_0$ is the aggregate welfare change.

Starting with the baseline case of $\theta = 1$ (the third column of the table) we can see that changing capital requirements a little in the neighbourhood of the competitive equilibrium reduces aggregate welfare by 0.3%. But this masks a number of different competing effects on welfare. Starting with the high productivity entrepreneurs, the second row of the table shows that the expected net present value of future log returns on wealth (φ_0^H) decreases by just under 0.2%. There is also a 4% decline in wealth (Z_0^H) and causes a 1% drop in the welfare of high productivity entrepreneurs (V_0^H). Further down the $\theta = 1$ column we have the components of welfare for low productivity entrepreneurs. The expected net present value of future rates of return (φ_0^L) increases by around 1% driven by the higher safe rate of return. Lower asset prices depress the wealth of this group (Z_0^L) which falls by 0.7%. The effect of higher rates of return on wealth dominates, leading to a 0.04% increase in welfare (V_0^L). Workers' welfare (V_0^W) also rises by a small amount driven by a small rise in the real wage and a decline in the volatility of real wages.

The cases of $\theta = 0.9$ and $\theta = 0.8$ (the first and second column of the Table) are qualitatively similar to the baseline case though all the magnitudes get progressively smaller in absolute value as the economy gets more and more distorted at lower levels of financial development. Appendix L contains a number of other sensitivity checks we performed in order to be sure of the robustness of the 'no regulation' result. We found that our results were robust to different values of the productivity differential a^H as well as to the form of the borrowing constraint.

7.2 Discussion

Our numerical results show that the capital requirement is a very blunt instrument, which is best left unused in the context of our model and calibration. The main losers from tighter

regulation of private leverage are the high productivity entrepreneurs who find that their access to borrowing is reduced with detrimental effects on their steady state consumption and welfare. On the positive side, as the results in the penultimate row of Table 8 shows, the volatility in their consumption declines very sharply. This is because reduced leverage improves both consumption smoothing over the idiosyncratic productivity cycle as well as risk sharing over the business cycle. But the beneficial impact of greater consumption stability are insufficient to generate a welfare improvement.

Low productivity entrepreneurs also lose out though by a smaller margin. For them, capital regulation represents a finer balance. On the one hand they gain because the reduced access to credit reduces capital prices and boosts the rate of return they earn on their own production. The consumption is also smoother due to the reduced volatility of consumption over the productivity cycle as well as the business cycle. But these gains are relatively small because low productivity entrepreneurs are not leveraged and their consumption is already smooth. On the other hand, lower wealth due to poorer borrowing opportunities and lower asset prices affects them too.

Taken as a whole, the economy is made worse off by capital requirements. This is because the productivity reducing effect of regulation turns out to have a larger impact on welfare compared to its impact in terms of greater macro-economic stability. This suggests that one reason for the surprising result of this paper is that private agents value average consumption a lot more than they value consumption stability. One simple way to test this hypothesis is to examine the premium on risky assets in our economy. This is done in Table 9 below, which shows the difference between the expected return on the risky asset for low productivity entrepreneurs and the risk free rate. We focus on low productivity entrepreneurs because they are unconstrained and therefore they price assets in our economy.

Table 9: The risk premium under different financial systems

	$\theta = 0.80$	$\theta = 0.90$	$\theta = 1.00$
$100 (E_t r_{t+1}^k - R_t)$	0.0010	0.0015	0.0020

Note: 0.01 denotes 1 basis point.

The table shows that the risk premium is very small - less than 1bp for the calibration we

consider. Put another way, low productivity entrepreneurs strongly prefer excess returns to smooth returns. It is therefore clear why the government finds that it cannot improve on the competitive allocation. The pecuniary externality results in excessive volatility of consumption and asset prices while the policy response we consider has its own costs in terms of the level of output and consumption. Consumers in this model do not find such a trade off advantageous.

Again, note that an absence of amplification in the Kiyotaki (1998) model is not the reason for this result. Contrary to the findings of Cordoba and Ripoll (2004) we find that there is substantial amplification with the standard deviation of output and TFP around 40% higher than the ‘first best’ and the standard deviation of consumption and asset prices more than 80% higher than the ‘first best’. This shows that the model can magnify the effects of shocks but consumers do not care sufficient about this to be willing to pay the costs of the regulation.

There are at least three reasons for this. First of all, the assumption of log utility limits entrepreneurs’ risk aversion and the amount of steady state consumption they are willing to give up in order to have a smooth consumption profile over time. This reduces the costs of weak risk sharing and consumption smoothing in our economy and therefore makes regulation (which improves both risk sharing and consumption smoothing) less desirable. Secondly, aggregate shocks are small. The high productivity state alternates between values 0.6% above or below steady state. This is consistent with aggregate fluctuations in developed economies during the recent ‘Great Moderation’ period. It remains to be seen whether the volatility of technology shocks picks up following the 2008 Lehmans Crisis.

Thirdly, the nature of borrowing in this model is entirely constrained efficient. The flow of funds between borrowers and lenders serves to boost productivity and benefit everyone. There is no misallocation of resources such as might arise if lenders or borrowers make mistakes in allocating credit; there are no defaults and no bankruptcy costs associated with default. So perhaps it is unsurprising that regulation cannot help in this environment: we have made its task relatively difficult.

These considerations introduce many possible avenues for future work. Examining the robustness of the ‘no regulation’ result to different preferences is one obvious extension I am

already working on. But examining other economic environments is also a promising avenue in studying the question of whether and how capital requirements can improve social welfare.

8 Conclusions

This paper aims to assess quantitatively the extent to which private leverage choices are inefficient from a social point of view. We found that, to a very close approximation, these choices are efficient. In the Kiyotaki-Moore framework credit constraints bind because limited commitment makes the financing of productive opportunities more difficult. Thus although leverage introduces a certain degree of financial fragility into the economy, it also allows the funding of high value added activities which, on average, allow society to enjoy a higher level of output and consumption.

So we find that regulation has a number of costs and benefits for economic agents. The main benefits involve reducing the inefficient volatility of output and consumption which arises from the workings of the collateral amplification mechanism. In the *laissez faire* equilibrium individual borrowers decide to borrow up to the debt limit in order to take advantage of attractive productive opportunities. They know that when aggregate shocks hit, leverage will magnify the effect of asset prices on balance sheets and force them to sell productive assets at a time when the price is already low. But atomistic agents take the low price in downturns as given even though the amount of asset sales and the size of the price fall are closely linked. The more assets are sold by leveraged high productivity entrepreneurs the more the price falls because the only buyers are the unleveraged low productivity types. This exerts downward pressure on the aggregate efficiency of the economy and depresses asset prices even further tightening credit constraints even more. It is binding borrowing constraints that make the usually harmless pecuniary externalities between different agents important for allocative efficiency.

But regulation has substantial costs too. When borrowing constraints bind, not enough funds flow from low to high productivity entrepreneurs and this reduces average TFP and consumption over time. Imposing tighter collateral requirements further squeezes the flow of credit and further reduces its average productive efficiency even though it makes it more

stable as a result.

The social choice between the level and the volatility of consumption is largely driven by the preferences of economic agents as well as the marginal rate of transformation between the level and volatility of consumption. In our calibration we find that economic agents do not care about volatility as much as they care about the level of consumption. This is clearly demonstrated by the low premium on risky assets (below 1bp). In addition, capital requirements reduce volatility at too high a cost in terms of average efficiency. Consequently, the benevolent government chooses not to regulate finance in our model economy.

In future research I want to explore the robustness of this result. One obvious extension is to change the structure of the model in order to generate a more realistic equity risk premium, for example by incorporating the Epstein-Zin-Weil preferences and an environment of long run consumption risk. A high equity premium indicates that private investors are very concerned about risk. So an environment with a high equity premium is more likely to be one in which the imposition of capital requirements is optimal.

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A Solving for the consumption function

Suppose the entrepreneur has optimally chosen her capital, labour and intermediate inputs and purchases/short sales of the risk free security. This means that she can earn a state contingent rate of return on invested wealth of R_{t+1} . The first order condition for optimal consumption then becomes:

$$\frac{1}{c_t} = \beta E_t \left(R_{t+1} \frac{1}{c_{t+1}} \right)$$

We guess that the entrepreneur consumes a fixed fraction of her available resources:

$$c_t = (1 - \beta) z_t$$

where z_t is the entrepreneur’s wealth. This means that

$$z_{t+1} = \beta R_{t+1} z_t$$

Substituting into the consumption Euler equation we have:

$$\begin{aligned} \frac{1}{(1 - \beta) z_t} &= \beta E_t \left(R_{t+1} \frac{1}{(1 - \beta) z_{t+1}} \right) \\ &= \beta E_t \left(R_{t+1} \frac{1}{(1 - \beta) \beta R_{t+1} z_t} \right) \\ &= \frac{1}{(1 - \beta) z_t} \end{aligned}$$

This confirms our initial guessed consumption function.

B Solving for the rate of return on wealth of a high productivity entrepreneur

We start with the flow of funds constraint of the agent.

$$c_t + w_t h_t + x_t + q_t k_t - \frac{b_t}{R_t} = y_t + q_t k_{t-1} - b_{t-1}$$

From the conditions for optimal production (4) and (5) we know that

$$x_t = (1 - \alpha) u_t^H k_t / \alpha$$

and

$$w_t h_t = (1 - \alpha - \eta) u_t^H k_t / \alpha$$

Then if entrepreneurs borrow $l_t \leq \theta$ of the expected value of collateral, this allows us to solve for their debt choice:

$$b_t \leq l_t E_t q_{t+1} k_t$$

The entrepreneur's total saving is given by:

$$w_t h_t + x_t + q_t k_t - \frac{b_t}{R_t} = \left(q_t + (1 - \alpha) u_t^H / \alpha - \frac{l_t E_t q_{t+1}}{R_t} \right) k_t$$

This will deliver the following level of wealth in the following period:

$$\begin{aligned} w_{t+1} &= y_{t+1} + q_{t+1} k_t - b_t \\ &= \left[(A_{t+1} \alpha^H / \alpha) w_t^{\alpha+\eta-1} (u_t^H)^{1-\alpha} + q_{t+1} - l_t E_t q_{t+1} \right] k_t \end{aligned}$$

The entrepreneur's rate of return on total wealth invested is given by:

$$\begin{aligned} R_{t+1}^H &= \frac{y_{t+1} + q_{t+1} k_t - b_t}{x_{t+1} + q_t k_t - \frac{b_t}{R_t}} \\ &= \frac{(A_{t+1} \alpha^H / \alpha) w_t^{\alpha+\eta-1} (u_t^H)^{1-\alpha} + q_{t+1} - l_t E_t q_{t+1}}{q_t + (1 - \alpha) u_t^H / \alpha - (l_t / R_t) E_t q_{t+1}} \end{aligned}$$

C Solving for the value function

The value function of an entrepreneur is:

$$V(z_t, a_t, X_t) = \max_{x_t, k_t, h_t, b_t, c_t} \{ \ln c_t + \beta E_t V(z_{t+1}, a_{t+1}, X_{t+1}) \} \quad (27)$$

$$= \max_{x_t, k_t, h_t, b_t} \{ \ln(1 - \beta) + \ln w_t + \beta E_t V(z_{t+1}, a_{t+1}, X_{t+1}) \} \quad (28)$$

Guess that the solution is of the form

$$V(z_t, a_t, X_t) = \varphi(a_t, X_t) + \frac{\ln z_t}{1 - \beta}$$

This implies that:

$$\begin{aligned} \varphi(a_t, X_t) + \frac{\ln z_t}{1 - \beta} &= \max_{x_t, k_t, h_t, b_t} \left\{ \ln(1 - \beta) + \ln z_t + \beta E_t \left[\varphi(a_{t+1}, X_{t+1}) + \frac{\ln z_{t+1}}{1 - \beta} \right] \right\} \\ &= \max_{x_t, k_t, h_t, b_t} \left\{ \ln(1 - \beta) + \ln z_t + \frac{\beta}{1 - \beta} E_t [\varphi(a_{t+1}, X_{t+1}) + \ln(\beta R_{t+1} z_t)] \right\} \\ &= \left\{ \ln(1 - \beta) + \frac{\beta \ln \beta}{1 - \beta} + \frac{\beta}{1 - \beta} E_t \left[\max_{x_t, k_t, h_t, b_t} [\ln(R_{t+1})] + \varphi(a_{t+1}, X_{t+1}) \right] + \frac{\ln z_t}{1 - \beta} \right\} \end{aligned}$$

Equating coefficients we get the expression for the intercept of the value function:

$$\varphi(a_t, X_t) = \ln(1 - \beta) + \frac{\beta \ln \beta}{1 - \beta} + \frac{\beta}{1 - \beta} \max_{x_t, k_t, h_t, b_t} E_t [[\ln(R_{t+1})] + \varphi(a_{t+1}, X_{t+1})]$$

The above expression shows that the agent has to choose productive inputs and borrowing so as to maximise the expected log rate of return on wealth in each period.

So the value of an entrepreneur in our economy depends on the net present value of expected log returns on the optimal portfolio as well as the log of current financial wealth.

D Solving for the rate of return on wealth of a low productivity entrepreneur

We start with the flow of funds constraint of the agent.

$$c_t + w_t k_t + x_t + q_t k_t + \frac{b_t}{R_t} = y_t + q_t k_{t-1} + b_{t-1}$$

From the condition for optimal production (4) and (5) we know that

$$x_t = (1 - \alpha) u_t^L k_t / \alpha$$

and

$$w_t h_t = (1 - \alpha - \eta) u_t^L k_t / \alpha$$

The entrepreneur's total saving is given by:

$$w_t h_t + x_t + q_t k_t + \frac{b_t}{R_t} = (q_t + (1 - \alpha) u_t^L / \alpha) k_t + \frac{b_t}{R_t}$$

This will deliver the following level of wealth in the following period:

$$\begin{aligned} w_{t+1} &= y_{t+1} + q_{t+1} k_t + b_t \\ &= \left[(A_{t+1} / \alpha) w_t^{\alpha+\eta-1} (u_t^L)^{1-\alpha} + q_{t+1} \right] k_t + b_t \end{aligned}$$

The entrepreneur's rate of return on total wealth invested is given by:

$$\begin{aligned} R_{t+1}^L &= \frac{y_{t+1} + q_{t+1} k_t + b_t}{w_t h_t + x_t + q_t k_t + \frac{b_t}{R_t}} \\ &= \frac{\left[(A_{t+1} / \alpha) w_t^{\alpha+\eta-1} (u_t^L)^{1-\alpha} + q_{t+1} \right] k_t + b_t}{(q_t + (1 - \alpha) u_t^L / \alpha) k_t + b_t} \end{aligned}$$

Imposing market clearing in the capital and debt markets and recognising that all low productivity entrepreneurs chose the same portfolio, we get the following equilibrium rate of return on wealth for the low type:

$$R_{t+1}^L = \frac{\left[(A_{t+1} / \alpha) w_t^{\alpha+\eta-1} (u_t^L)^{1-\alpha} + q_{t+1} \right] (1 - K_t) + l_t E_t q_{t+1} K_t}{(q_t + (1 - \alpha) u_t^L / \alpha) (1 - K_t) + l_t E_t q_{t+1} K_t / R_t}$$

where K_t is the aggregate capital-holding of the high productivity entrepreneurs.

E Approximating the optimal portfolio problem as a mean variance utility problem

The entrepreneur's portfolio problem involves maximising the log return on his portfolio of assets. The portfolio can be written as the weighted sum of the return on the risky asset and the rate of return on the safe asset

$$R_{t+1}^i = \varpi_t^i \left[\frac{(A_{t+1} a^i / \alpha) w_t^{\alpha+\eta-1} (u_t^i)^{1-\alpha} + q_{t+1}}{q_t + (1 - \alpha) u_t^i / \alpha} \right] + (1 - \varpi_t^i) R_t \quad (29)$$

Let

$$R^{i*} = \max_{\varpi_t^i} E_t \ln R_{t+1}^i$$

denote the maximum value of the expected log portfolio return. Using the approximation

$$\ln E_t x \approx E_t \ln x - \frac{1}{2} \text{var}(\ln x) \quad (30)$$

we can write the portfolio problem as a mean-variance utility maximisation problem.

E.1 High Productivity Entrepreneurs

For high productivity entrepreneurs the (29) expression above can be written as follows:

$$\begin{aligned} R_{t+1}^H &= \varpi_t^H \left[\frac{(A_{t+1} a^H / \alpha) w_t^{\alpha+\eta-1} (u_t^H)^{1-\alpha} + q_{t+1}}{q_t + (1-\alpha) u_t^H / \alpha} \right] + (1 - \varpi_t^H) R_t \\ &= R_t + \varpi_t^H \left[\frac{(A_{t+1} a^H / \alpha) w_t^{\alpha+\eta-1} (u_t^H)^{1-\alpha} + q_{t+1}}{q_t + (1-\alpha) u_t^H / \alpha} - R_t \right] \\ &= R_t \left\{ 1 + \varpi_t^H \left[\frac{(A_{t+1} a^H / \alpha) w_t^{\alpha+\eta-1} (u_t^H)^{1-\alpha} + q_{t+1}}{q_t + (1-\alpha) u_t^H / \alpha} / R_t - 1 \right] \right\} \\ &\equiv R_t \{ 1 + \varpi_t^H [\rho_{t+1}^H - 1] \} \end{aligned}$$

where

$$\rho_{t+1}^H = \frac{(A_{t+1} a^H / \alpha) w_t^{\alpha+\eta-1} (u_t^H)^{1-\alpha} + q_{t+1}}{q_t + (1-\alpha) u_t^H / \alpha} / R_t$$

Taking logs and using the approximation $\ln(1+x) \approx x$ for small x we have

$$\ln R_{t+1}^H \approx \ln R_t + \varpi_t^H [\rho_{t+1}^H - 1]$$

Applying the approximation (30) we have:

$$\begin{aligned} R^{H*} &\approx \max_{\varpi_t^H} \left[\ln E_t R_{t+1}^H - \frac{1}{2} \text{var}(\ln R_{t+1}^H) \right] \\ &= \max_{\varpi_t^H} \left[\ln R_t + \ln(1 + \varpi_t^H (E_t \rho_{t+1}^H - 1)) - \frac{1}{2} \text{var}(\ln R_t + \ln(1 + \varpi_t^H (\rho_{t+1}^H - 1))) \right] \\ &\approx \max_{\varpi_t^H} \left[\ln R_t + \varpi_t^H (E_t \rho_{t+1}^H - 1) - \frac{1}{2} \text{var}(\ln R_t + \varpi_t^H (\rho_{t+1}^H - 1)) \right] \\ &\approx \max_{\varpi_t^H} \left[\ln R_t + \varpi_t^H (E_t \rho_{t+1}^H - 1) - \frac{(\varpi_t^H)^2}{2} \sigma_{R_{t+1}}^2 \right] \end{aligned}$$

Define

$$\psi_{t+1}^H = (a^H/\alpha) (u_t^H)^{1-\alpha}$$

as output per efficiency unit of capital at time $t + 1$ for high productivity entrepreneurs.

Then the variance of the risky asset's rate of return is given by:

$$\begin{aligned} \sigma_{R_{t+1}}^2 &= \left(\frac{\psi_{t+1}^H}{q_t + (1-\alpha) u_t^H/\alpha} \right)^2 \sigma_A^2 + \left(\frac{\psi_{t+1}^H}{q_t + (1-\alpha) u_t^H/\alpha} \right) \sigma_{Aq_{t+1}} + \left(\frac{1}{q_t + (1-\alpha) u_t^H/\alpha} \right)^2 \sigma_{q_{t+1}}^2 \\ &= \frac{(\psi_{t+1}^H)^2 \sigma_A^2 + \psi_{t+1}^H (q_t + (1-\alpha) u_t^H/\alpha) \sigma_{Aq_{t+1}} + \sigma_{q_{t+1}}^2}{(q_t + (1-\alpha) u_t^H/\alpha)^2} \end{aligned}$$

where σ_A^2 is the variance of the technology shock, $\sigma_{Aq_{t+1}}$ is the conditional covariance of the technology shock and the capital price and $\sigma_{q_{t+1}}^2$ is the conditional variance of the capital price.

E.2 Low Productivity Entrepreneurs

Analogously with the previous subsection we learn that the log rate of return on wealth for low productivity agents can be approximated by:

$$\ln R_{t+1}^L \approx \ln R_t + \varpi_t^L [\rho_{t+1}^L - 1]$$

where

$$\rho_{t+1}^L = \frac{(A_{t+1}/\alpha) (u_t^L)^{1-\alpha} + q_{t+1}}{q_t + (1-\alpha) u_t^L/\alpha} / R_t$$

Then we can approximate the expected log rate of return on wealth of low productivity agents by the following expression:

$$\begin{aligned} R^{L*} &\approx \max_{\varpi_t^L} \left[\ln E_t R_{t+1}^L - \frac{1}{2} \text{var} (\ln R_{t+1}^L) \right] \\ &\approx \max_{\varpi_t^L} \left[\ln R_t + \varpi_t^L (E_t \rho_{t+1}^L - 1) - \frac{(\varpi_t^L)^2}{2} \sigma_{rt+1}^2 \right] \end{aligned}$$

Define

$$\psi_{t+1}^L = (1/\alpha) (u_t^L)^{1-\alpha}$$

as output per efficiency unit of capital at time $t + 1$ for low productivity entrepreneurs. Then the variance of the risky asset's rate of return is given by:

$$\begin{aligned}\sigma_{rt+1}^2 &= \left(\frac{\psi_{t+1}^L}{q_t + (1 - \alpha) u_t^L / \alpha} \right)^2 \sigma_A^2 + \left(\frac{\psi_{t+1}^L}{q_t + (1 - \alpha) u_t^L / \alpha} \right) \sigma_{Aqt+1} + \left(\frac{1}{q_t + (1 - \alpha) u_t^L / \alpha} \right)^2 \sigma_{qt+1}^2 \\ &= \frac{(\psi_{t+1}^L)^2 \sigma_A^2 + \psi_{t+1}^L (q_t + (1 - \alpha) u_t^L / \alpha) \sigma_{Aqt+1} + \sigma_{qt+1}^2}{(q_t + (1 - \alpha) u_t^L / \alpha)^2}\end{aligned}$$

Again, just like in the previous subsection, the variance of the risky rate of return for the low productivity is driven by σ_A^2 - the variance of the technology shock, σ_{Aqt+1} - the conditional covariance of the technology shock and the capital price and σ_{qt+1}^2 - the conditional variance of the capital price.

F The Frictionless Benchmark

F.1 The Problem of Entrepreneurs

Entrepreneurs solve the following problem

$$\max_{c_t, x_t} E_t \sum_{t=0}^{\infty} \beta^t \ln c_{t+s}^e$$

subject to the resource constraint:

$$c_t^e + x_t + u_t + w_t h_t^d + \sum_s \frac{b_t^s}{R_{t+1}^s} = \frac{a^H A_t}{\alpha^\alpha} \left(\frac{x_{t-1}}{\eta} \right)^\eta \left(\frac{h_{t-1}}{1 - \alpha - \eta} \right)^{1 - \alpha - \eta} + b_{t-1}$$

Here we have already taken into account the fact that only high productivity entrepreneurs will produce in equilibrium and the entire capital supply will be used in production. $\sum_s \frac{b_t^s}{R_{t+1}^s}$ are the entrepreneurs' net purchases (or sales) of Arrow securities at price $1/R_{t+1}^s$ from workers. The first order conditions are as follows:

(1) Investment

$$x_t = \frac{\eta}{\alpha} u_t$$

(2) Labour demand

$$w_t h_t^d = \frac{1 - \alpha - \eta}{\alpha} u_t$$

(3) Arbitrage between production and Arrow securities

$$R_{t+1}^s = \frac{a^H A_{t+1}^s}{\alpha^\alpha} \left(\frac{x_{t-1}}{\eta} \right)^\eta \left(\frac{h_{t-1}}{1 - \alpha - \eta} \right)^{1 - \alpha - \eta}$$

(4) Entrepreneurs' consumption function

$$c_t^e = (1 - \beta) \left[\frac{a^H A_t}{\alpha^\alpha} \left(\frac{x_{t-1}}{\eta} \right)^\eta \left(\frac{h_{t-1}}{1 - \alpha - \eta} \right)^{1 - \alpha - \eta} + b_{t-1} \right]$$

F.2 The Problem of Workers

Workers have the following preferences

$$\max_{c_t^w, h_t} E_t \sum_{s=0}^{\infty} \beta^s \ln \left(c_t^w - \varkappa \frac{h_t^{1+\omega}}{1+\omega} \right)$$

subject to the resource constraint:

$$c_t^w + \sum_s \frac{b_t^s}{R_{t+1}^s} = b_{t-1} + w_t h_t$$

First order conditions are given by:

$$\frac{1}{c_t^w - \varkappa \frac{h_t^{1+\omega}}{1+\omega}} = \beta \pi^s \frac{R_{t+1}^s}{c_{t+1}^{ws} - \varkappa \frac{(h_{t+1}^s)^{1+\omega}}{1+\omega}}$$

and

$$w_t = \varkappa h_t^\omega \tag{31}$$

We can derive the consumption function of the workers as follows. Define:

$$\begin{aligned} \tilde{c}_t &= c_t^w - \varkappa \frac{h_t^{1+\omega}}{1+\omega} \\ &= c_t^w - \varkappa \frac{(w_t/\varkappa)^{\frac{1+\omega}{\omega}}}{1+\omega} \end{aligned}$$

and

$$\begin{aligned} \tilde{w}_t &= w_t h_t - \varkappa \frac{h_t^{1+\omega}}{1+\omega} \\ &= w_t (w_t/\varkappa)^{\frac{1}{\omega}} - \varkappa \frac{(w_t/\varkappa)^{\frac{1+\omega}{\omega}}}{1+\omega} \end{aligned}$$

Then redefine the inter-temporal budget constraint using \tilde{c}_t and \tilde{w}_t :

$$\tilde{c}_t + \sum_s \frac{b_t^s}{R_{t+1}^s} = b_{t-1} + \tilde{w}_t$$

and the Euler equation:

$$\frac{1}{\tilde{c}_t} = \beta \pi^s \frac{R_{t+1}^s}{\tilde{c}_{t+1}}$$

This problem now looks like the standard consumption-savings problem with log utility. The consumption function is:

$$\tilde{c}_t = (1 - \beta) (H_t + b_{t-1})$$

where

$$H_t = \tilde{w}_t + E_t \left(\frac{H_{t+1}}{R_{t+1}} \right)$$

is the human wealth of the worker. The workers' aggregate consumption function is therefore given by:

$$c_t^w = (1 - \beta) (H_t + b_{t-1}) + \varkappa \frac{(w_t/\varkappa)^{\frac{1+\omega}{\omega}}}{1 + \omega}$$

Aggregate consumption in the economy is given by:

$$c_t^w + c_t^e = (1 - \beta) (H_t + q_t + Y_t) + \varkappa \frac{(w_t/\varkappa)^{\frac{1+\omega}{\omega}}}{1 + \omega}$$

F.3 The full set of aggregate equilibrium conditions

Aggregate output

$$Y_t = \frac{a^H A_t}{\alpha} u_t^{1-\alpha} w_t^{\alpha+\eta-1}$$

Market clearing

$$(1 - \beta) (H_t + q_t + Y_t) + \varkappa \frac{(w_t/\varkappa)^{\frac{1+\omega}{\omega}}}{1 + \omega} + \frac{\eta}{\alpha} u_t = \frac{a^H A_t}{\alpha} u_t^{1-\alpha} w_t^{\alpha+\eta-1}$$

Human wealth

$$H_t = w_t (w_t/\varkappa)^{\frac{1}{\omega}} - \varkappa \frac{(w_t/\varkappa)^{\frac{1+\omega}{\omega}}}{1 + \omega} + E_t \left(\frac{H_{t+1}}{R_{t+1}} \right)$$

Price of capital

$$q_t = u_t + E_t \left(\frac{q_{t+1}}{R_{t+1}} \right)$$

Arrow security price

$$R_{t+1} = a^H A_t u_t^{-\alpha} w_t^{\alpha+\eta-1}$$

Labour demand

$$w_t (w_t/\varkappa)^{\frac{1}{\omega}} = \frac{1 - \alpha - \eta}{\alpha} u_t$$

G Data Definitions and Sources

G.1 Computing the share of capital in private value added

We compute the share of capital in private value

$$\tilde{\alpha} \equiv \frac{\alpha}{1 - \eta}$$

added following the method in Cooley and Prescott (1995). We define unambiguous capital income (Y^U) as the sum of \square and ambiguous capital income (Y^A) as Proprietors income. We assume that the share of capital in ambiguous capital income is equal to its share in total national income. All series are obtained from the BEA national accounts. Then the share of capital in total income (Y) is defined as the sum of unambiguous capital income and the capital share of ambiguous capital income:

$$\tilde{\alpha}Y = Y^U + \tilde{\alpha}Y^A$$

Hence

$$\tilde{\alpha} = \frac{Y^U}{Y - Y^A}$$

G.2 Computing η the share of intermediate inputs in gross output

We use the BEA Industrial Accounts to compute this parameter. The Industrial Accounts produces sector by sector input output tables, showing the value added and gross output of each sector. This allows us to compute the share of intermediate inputs for each sector. The aggregate share of intermediate inputs can be obtained by averaging across all the sectors. Weighting different sectors by their weight in aggregate gross output gave almost identical results.

G.3 Computing the ratio of tangible assets to GDP

We compute the economy's stock of tangible assets by adding the nominal value of tangible assets of the Household (Table B.100, FL152010005), Corporate Non-Financial sector (Table B.102, FL102010005) and Non-corporate Non-Financial sector (Table B.103, FL112010005) from the September 2009 release of the US Flow of Funds. GDP is nominal GDP excluding the value added of the Government sector (Table 1.1.5, Line 1-Line 21). Data is for the period 1952-2008. The model counterparts to the ratio of tangible assets to GDP is defined as follows:

$$\frac{q}{Y^H + Y^L - X^H - X^L}$$

G.4 Computing aggregate corporate leverage

We use corporate (Table B.102, FL102000005) and non-corporate (Table B.103, FL112000005) total assets. This includes both tangible and financial assets on firms' books. For corporate net worth we use the market value of corporate equity (Table B.102, FL103164003). For non-corporate net worth we use the net worth data in Table B.103, FL112090205. Leverage is computed as (Assets-Net Worth)/Assets.

The model counterpart to aggregate corporate leverage is defined as follows:

$$L^A = \frac{\theta q K}{q + (Y^H + Y^L) / R}$$

G.5 Computing the second moments in the data

Our measure of GDP is private sector value added (Table 1.1.5, Line 1-Line 21). Consumption is the sum of non-durable goods and services consumption. The value of the firm is proxied by the S&P 500. All series have been deflated by the non-durable goods deflator to convert them into real terms (non-durables consumption goods). All data is annual and the data sample is 1929-2008. Total employment in hours is obtained from the Bureau of Labour Statistics. The sample is 1964 - 2008. We convert the monthly data into annual averages. All data is detrended using the HP filter. Following Uhlig and Ravn (2001) we use a smoothing parameter of 2.06 for annual data

H Deriving the tax wedge formulation for steady state

$$u_t^H$$

The user cost of capital for low productivity entrepreneurs is

$$u_t^L = q_t - \frac{q_{t+1}}{R_t}$$

while that for high productivity entrepreneurs is

$$u_t^H = q_t - \left[\frac{\theta}{R_t} + \frac{1-\theta}{R_{t+1}^H} \right] q_{t+1}$$

We can re-write the u_t^H expression in terms of the excess return on wealth for high productivity entrepreneurs

$$\begin{aligned} u_t^H &= q_t - \frac{q_{t+1}}{R_t} + (1-\theta) q_{t+1} \left(\frac{1}{R_t} - \frac{1}{R_{t+1}^H} \right) \\ &= u_t^L + (1-\theta) \frac{q_{t+1}}{R_t} \left(1 - \frac{1}{\rho_{t+1}} \right) \end{aligned}$$

where

$$\rho_{t+1} = \frac{R_{t+1}^H}{R_t}$$

is the excess return. We can use the user cost expression to substitute out the expected future price in terms of ‘ex-dividend’ value of capital:

$$\begin{aligned} u_t^H &= u_t^L + (1-\theta) (q_t - u_t^L) \left(1 - \frac{1}{\rho_{t+1}} \right) \\ &= u_t^L \left[1 + (1-\theta) \left(\frac{q_t}{u_t^L} - 1 \right) \left(1 - \frac{1}{\rho_{t+1}} \right) \right] \\ &= u_t^L [1 + \tau_t] \end{aligned}$$

This completes our derivation of the downpayment ‘tax wedge’:

$$\tau_t = (1-\theta) \left(\frac{q_t}{u_t^L} - 1 \right) \left(1 - \frac{1}{\rho_{t+1}} \right)$$

I Deriving the level of TFP in steady state

The level of TFP is given by the following expression:

$$TFP_t = \frac{Y_t^H + Y_t^L}{\left(\frac{1}{\alpha}\right)^\alpha \left(\frac{H_{t-1}^H + H_{t-1}^L}{\eta}\right)^\eta \left(\frac{X_{t-1}^H + X_{t-1}^L}{1-\alpha-\eta}\right)^{1-\alpha-\eta}}$$

We know that:

$$\begin{aligned} Y_t^L &= \frac{1}{\alpha} w_{t-1}^{\alpha+\eta-1} (u_{t-1}^L)^{1-\alpha} (1 - K_{t-1}) \\ Y_t^H &= \frac{a^H}{\alpha} w_{t-1}^{\alpha+\eta-1} (u_{t-1}^H)^{1-\alpha} K_{t-1} \\ &= \frac{a^H (1 + \tau(\theta))^{1-\alpha}}{\alpha} w_{t-1}^{\alpha+\eta-1} (u_{t-1}^L)^{1-\alpha} K_{t-1} \end{aligned}$$

where we have used the fact that $u_t^H = (1 + \tau(\theta)) u_t^L$. Aggregate intermediate input investment is given by:

$$\begin{aligned} X_{t-1}^H + X_{t-1}^L &= \frac{\eta}{\alpha} u_{t-1}^L (1 - K_{t-1} + (1 + \tau(\theta)) K_{t-1}) \\ &= \frac{\eta}{\alpha} u_{t-1}^L (1 + \tau(\theta) K_{t-1}) \\ H_{t-1}^H + H_{t-1}^L &= \frac{1 - \alpha - \eta}{\alpha} \frac{u_{t-1}^L}{w_{t-1}} (1 - K_{t-1} + (1 + \tau(\theta)) K_{t-1}) \\ &= \frac{1 - \alpha - \eta}{\alpha} \frac{u_{t-1}^L}{w_{t-1}} (1 + \tau(\theta) K_{t-1}) \end{aligned}$$

Aggregate TFP for our economy is therefore given by:

$$TFP_t = \frac{1 + K_{t-1} [a^H (1 + \tau(\theta))^{1-\alpha} - 1]}{1 + \tau(\theta) K_{t-1}}$$

J Deriving the aggregate state

In setting up the individual maximisation problem, we had assumed that aggregate wealth Z_t and the share of wealth that belongs to productive individuals d_t are the key endogenous state variables. Following the derivation of the conditions for optimal consumption and investment by entrepreneurs, we can see why this is indeed the case. We do this by showing

that our market clearing conditions are functions of current and expected future market prices as well as the state variables in question.

Starting with the bond market clearing condition (15) we can see straight away from the collateral constraint that the gross amount of debt in any given period is given by the condition:

$$B_t = \theta E_t q_{t+1} K_t$$

The aggregate capital holding of high productivity entrepreneurs is given by:

$$K_t = \beta \frac{d_t Z_t}{q_t + (1 - \alpha) u_t^H / \alpha - \theta E_t q_{t+1} / R_t}$$

which implies that debt is a function of market prices and W_t and d_t .

Moving on to the capital market clearing condition (16) we already know that capital demand by high productivity agents is recursive in the aggregate state. The capital demand of low productivity entrepreneurs is:

$$\begin{aligned} (1 - K_t) &= \frac{\beta (1 - d_t) Z_t - B_t / R_t}{(q_t + \frac{1 - \alpha}{\alpha} u_t^L)} \\ &= \frac{\beta (1 - d_t) Z_t - \theta E_t q_{t+1} K_t / R_t}{(q_t + \frac{1 - \alpha}{\alpha} u_t^L)} \end{aligned}$$

This implies that the capital market clearing condition is a function of market prices as well as W_t and d_t .

Finally looking at the goods market clearing condition (17) we can see that because of log utility, consumption is proportional to individual wealth and, consequently, aggregate consumption by entrepreneurs is proportional to aggregate wealth:

$$C_t^E \equiv C_t^H + C_t^L = (1 - \beta) Z_t$$

The consumption of workers is very simple because they do not save in equilibrium:

$$C_t^W = w_t H_t = w_t \left(\frac{w_t}{\chi} \right)^{\frac{1}{\nu}}$$

Due to the Cobb-Douglas production function, the aggregate expenditure on intermediate input in the economy is given by the following expression:

$$X_t^H + X_t^L = \frac{1 - \alpha}{\alpha} (u_t^H K_t + u_t^L (1 - K_t))$$

where we already know that the capital demands of the two groups are recursive in the state. The definition of total wealth implies that:

$$Y_t^H + Y_t^L = W_t - q_t$$

So goods market clearing depends on market prices as well as W_t and d_t .

K Deriving the Social Welfare Function

The government solves the following policy problem.

$$\Omega_0 = \max_{\{\chi^i\}} E_0 \left[\sum_i \varsigma_E^i \sum_{t=0}^{\infty} \beta^t \ln c_t^i \right] + \varsigma_W \sum_{t=0}^{\infty} \beta^t \ln \left(C_t^W - \varkappa \frac{(H_t)^{1+\omega}}{1+\omega} \right) \quad (32)$$

We can represent the net present value of period utilities of the two groups as the sum of Pareto weighted value functions:

$$\Omega_0 = \max_{\{\chi^i\}} E_0 \left[\sum_i \varsigma_E^i V^E(z_0^i, a_0^i, X_0 | \chi^i) + \varsigma^W V^W(X_0 | \chi^i) \right] \quad (33)$$

$$= \max_{\{\chi^i\}} E_0 \left[\sum_i \varsigma^i \left(\varphi(a_0^i, X_0 | \chi^i) + \frac{\ln z_0^i(\chi^i)}{1-\beta} \right) + \varsigma^W V^W(X_0 | \chi^i) \right] \quad (34)$$

Under the assumption that all entrepreneurs hold their group average level of initial wealth and all workers hold zero wealth allows us to re-write the value function (34) as follows:

$$\Omega_0 = \max_{\{\chi^i\}} E_0 \left[d_0 Z_0 \left(\varphi^H(X_0 | \chi^i) + \frac{\ln Z_0^H(\chi^i)}{1-\beta} \right) + (1-d_0) Z_0 \left(\varphi^H(X_0 | \chi^i) + \frac{\ln Z_0^H(\chi^i)}{1-\beta} \right) + \left(\frac{1+\omega}{\omega} w_t \right) V^W \right]$$

L Sensitivity Analysis

L.1 Sensitivity to a^H

We performed extensive sensitivity analysis to check whether the value of a^H affected the results. We found that it did not and the result from the exercise are shown in Table A below. Again, at each value of a^H , the model is recalibrated for each parameter value in order to match our five targets from the data).

The value of a^H has two offsetting effects on the incentives to regulate. A higher value of a^H increases amplification because fluctuations in the share of wealth of high productivity entrepreneurs leads to bigger endogenous fluctuations in TFP and land prices. This would increase the incentive of the government to impose capital requirements in order to dampen the amplification mechanism. But a higher value of a^H also increases the benefits of getting more funds into productive hands so the welfare costs of capital requirements in terms of lower average productivity and consumption also increase. We examined a number of different values of a^H and found that at all of them, the government chose not to regulate.

Table A: Capital requirements and welfare under different values of a^H

	$a^H = 1.05$	<i>Baseline</i>	$a^H = 1.25$
$100\Delta \ln \varphi_0^H$	-0.25	-0.23	0.02
$100\Delta \ln Z_0^H$	-1.07	-4.15	-7.60
$100\Delta \ln V_0^H$	-0.44	-1.15	-2.01
$100\Delta \ln \varphi_0^L$	0.26	1.11	1.97
$100\Delta \ln Z_0^L$	-0.16	-0.71	-1.55
$100\Delta \ln V_0^L$	0.03	0.04	-0.11
$100\Delta \ln V_0^W$	0.06	0.14	0.17
$100\Delta \ln V_0$	-0.10	-0.33	-0.66
$100\Delta \sigma_c$	-0.02	-0.12	-0.21
$100\Delta \sigma_{cW}$	-0.01	-0.03	-0.08
$100\Delta \sigma_{cH}$	-0.22	-0.60	-0.75
$100\Delta \sigma_{cL}$	0.02	-0.06	-0.18

L.2 Sensitivity to the form of the borrowing constraint

L.2.1 'Worst case' borrowing limit

We also experimented with an alternative borrowing constraint of the form:

$$b_t \leq \theta q_{t+1}^l k_t$$

Such a constraint focuses on the value of collateral in the low aggregate productivity state. So it would be equivalent to a ‘worst case’ scenario value of collateral. Such a borrowing constraint also introduces two opposing incentives for the government. The case for higher regulation arises because the externality is much more severe under this constraint. This is because volatility of asset prices now has a first order effect on borrowing constraints. The more volatile land prices are, the more constrained entrepreneurs become because lenders become worried by large falls in the land price. This externality means that capital requirements might be beneficial because they reduce volatility and may even relax borrowing constraints.

But there is another offsetting effect. Suppose entrepreneurs attempt to leverage up and this leads to an increase in land price volatility. This would lead to tighter borrowing limits, stopping the rise in leverage in the first place. So the ‘worst case’ borrowing constraints exhibit a lot self-regulation which is missing in the standard ‘expected value’ borrowing constraints we consider in the main paper. This self-regulation effect makes government regulation unnecessary in equilibrium.

Table B: Capital requirements and welfare under ‘worst case’ borrowing contracts

	$\theta = 0.80$	$\theta = 0.90$	$\theta = 1.00$
$100\Delta \ln \varphi_0^H$	-0.33	-0.31	-0.25
$100\Delta \ln Z_0^H$	-1.06	-1.79	-3.68
$100\Delta \ln V_0^H$	-0.33	-0.55	-1.04
$100\Delta \ln \varphi_0^L$	0.37	0.54	0.96
$100\Delta \ln Z_0^L$	-0.14	-0.30	-0.61
$100\Delta \ln V_0^L$	0.05	0.04	0.05
$100\Delta \ln V_0^W$	-0.09	-0.02	0.11
$100\Delta \ln V_0$	-0.18	-0.34	-0.29
$100\Delta \sigma_c$	-0.02	-0.03	-0.04
$100\Delta \sigma_{cW}$	-0.01	-0.02	-0.01
$100\Delta \sigma_{cH}$	-0.08	-0.13	-0.20
$100\Delta \sigma_{cL}$	0.00	0.01	-0.01

L.2.2 Collateralisable output

In this case the borrowing constraint is of the form:

$$b_t \leq E_t(q_{t+1}k_t + \theta_y y_{t+1})$$

Entrepreneurs can now borrow up to the full value of their capital holdings and also up to a fraction θ_y of their future output. The results are shown in Table C below. Again, looking at the effects of this parameter did not change the basic result that aggregate welfare declined as the result of the imposing tighter capital requirements.

Table C: Capital requirements and welfare under collateralisable output

	<i>Baseline</i>	$\theta_y = 0.1$	$\theta_y = 0.2$
$100\Delta \ln \varphi_0^H$	-0.23	-0.34	-0.39
$100\Delta \ln Z_0^H$	-4.15	-1.27	-1.78
$100\Delta \ln V_0^H$	-1.15	-0.44	-0.55
$100\Delta \ln \varphi_0^L$	1.11	0.25	0.35
$100\Delta \ln Z_0^L$	-0.71	-0.21	-0.28
$100\Delta \ln V_0^L$	0.04	-0.01	-0.01
$100\Delta \ln V_0^W$	0.14	-0.05	-0.07
$100\Delta \ln V_0$	-0.33	-0.13	-0.14
$100\Delta \sigma_c$	-0.12	-0.06	-0.10
$100\Delta \sigma_{cW}$	-0.03	-0.03	-0.04
$100\Delta \sigma_{cH}$	-0.60	-0.26	-0.42
$100\Delta \sigma_{cL}$	-0.06	-0.04	-0.07

M Solution method

M.1 The Laissez Faire economy

We use the following ‘parameterised expectations’ algorithm in order to solve for the recursive competitive equilibrium of our model economy.

1. Start by guessing parameter values for current and future expected price functions. All equilibrium pricing functions are log linear in the state variables d_t and Z_t .

$$\ln q(X_{t+1}|X_t) = \omega_c(X_{t+1}|X_t) + \omega_d(X_{t+1}|X_t) \ln d_t + \omega_w(X_{t+1}|X_t) \ln Z_t \quad (35)$$

$$\ln q(X_t) = \varphi_c(X_t) + \varphi_d(X_t) \ln d_t + \varphi_w(X_t) \ln Z_t \quad (36)$$

$$\ln r(X_{t+1}|X_t) = \kappa_c(X_{t+1}|X_t) + \kappa_d(X_{t+1}|X_t) \ln d_t + \kappa_w(X_{t+1}|X_t) \ln Z_t \quad (37)$$

where X_t is the aggregate state of the economy.

2. Static portfolio maximisation

Next we find optimal leverage levels. Due to the non-convex choice set we need to compute and compare the value function when the constraint is binding and when it is non-binding. We pick the leverage choices associated with the largest of the two value functions.

(a) The value of the constraint binding is

$$\begin{aligned} R^{H*}(l_{t+1} = \theta) &= E_t \ln R_{t+1}^H \\ &= E_t \ln \left[\frac{(A_{t+1} a^H / \alpha) w_t^{\alpha+\eta-1} (u_t^H)^{1-\alpha} + q_{t+1} - \theta E_t q_{t+1}}{q_t + (1-\alpha) u_t^H / \alpha - (\theta / R_t) E_t q_{t+1}} \right] \end{aligned}$$

where

$$u_t^H = q_t - E_t \left(\frac{q_{t+1}}{R_{t+1}^H} \right) - \theta E_t q_{t+1} E_t \left(\frac{1}{R_{t+1}^L} - \frac{1}{R_{t+1}^H} \right) \quad (38)$$

is the user cost of capital under the binding constraint.

(b) The value of the constraint not binding

$$\begin{aligned} R^{H*}(l_{t+1} < \theta) &= \max_{0 < l_{t+1} < \theta} E_t \ln R_{t+1}^H \\ &= \max_{0 < l_{t+1} < \theta} E_t \ln \left[\frac{(A_{t+1} a^H / \alpha) w_t^{\alpha+\eta-1} (u_t^H)^{1-\alpha} + q_{t+1} - l_t E_t q_{t+1}}{q_t + (1-\alpha) u_t^H / \alpha - (l_t / R_t) E_t q_{t+1}} \right] \end{aligned}$$

where

$$u_t^H = q_t - E_t \left(\frac{q_{t+1}}{R_{t+1}^H} \right)$$

is the user cost when the constraint does not bind. We solve this maximisation problem using the inbuilt Matlab function `fmincon.m`

3. Compute the equilibrium at time t :

We use the latest guess of the q_{t+1} pricing function, the portfolio policy function l_{t+1} as well as the current realisations of the state variables A_t , d_t and W_t .

$$R_{t+1}^L = \frac{\left[A_{t+1} \frac{w_t^{\alpha+\eta-1} (u_t^L)^{1-\alpha}}{\alpha} + q_{t+1} \right] (1 - K_t) + l_{t+1} q (X_{t+1}) K_t}{\left[q_t + \frac{1-\alpha}{\alpha} u_t^L \right] (1 - K_t) + (l_{t+1}/R_t) E_t q_{t+1} K_t} \quad (39)$$

where

$$u_t^L = q_t - E_t \left(\frac{q_{t+1}}{R_{t+1}^L} \right)$$

High productivity entrepreneurs invest the following fraction of their wealth in capital.

$$K_t = \frac{\beta d_t W_t}{q_t + \frac{1-\alpha}{\alpha} u_t^H - (l_{t+1}/R_t) E_t q_{t+1}} \quad (40)$$

Their rate of return is given by:

$$R_{t+1}^H = \frac{(A_{t+1} a^H / \alpha) w_t^{\alpha+\eta-1} (u_t^H)^{1-\alpha} + q_{t+1} - l_t E_t q_{t+1}}{q_t + (1 - \alpha) u_t^H / \alpha - (l_t / R_t) E_t q_{t+1}} \quad (41)$$

when the collateral constraint is slack and

$$u_t^H = q_t - E_t \left(\frac{q_{t+1}}{R_{t+1}^H} \right) - \theta E_t q_{t+1} \left(\frac{1}{R_t} - E_t \left(\frac{1}{R_{t+1}^H} \right) \right)$$

Finally, goods market clearing is:

$$(1 - \beta) W_t + w_t H_t + \frac{1 - \alpha}{\alpha} [u_t^L (1 - K_t) + u_t^H K_t] = W_t - q_t$$

Using the inbuilt Matlab zero-finding routine `fsolve.m`, solve for the values of

$\{R_t, R_{t+1}^L, K_t, q_t, R_{t+1}^H, u_t^H, u_t^L\}$ at which these conditions are satisfied up to an error tolerance level.

4. Use the state evolution equations to compute next period's state vector:

$$W_{t+1} = [d_t R_{t+1}^H + (1 - d_t) R_{t+1}^L] \beta W_t \quad (42)$$

$$d_{t+1} = \frac{(1 - \delta) d_t R_{t+1}^H + n \delta (1 - d_t) R_{t+1}^L}{d_t R_{t+1}^H + (1 - d_t) R_{t+1}^L} \quad (43)$$

5. Repeat steps (1)-(4) for 2000 periods. Using the simulated data (minus a 200 period 'burn in' period), update the price and forecasting function coefficients using linear regression.

6. Re-compute a simulated time series of the endogenous variables in our model economy under the new forecasting rule. Repeat steps (1)-(5) until the coefficients on the forecasting rule have converged up to an error tolerance level.

M.2 The economy with capital requirements

In our government economy, the government chooses state contingent leverage functions $\tilde{\theta}_t$ in order to maximise social welfare

$$\begin{aligned}\Omega &= \max_{\{\chi^i\}} [\varsigma^H n V^H (z_0^H/n, X_0) + \varsigma^L V^L (z_0^L, X_0) + \varsigma^W V^W (X_0)] \\ &= \max_{\{\chi^i\}} \left[\varsigma^H n \left(\varphi^H (X_0) + \frac{\ln(z_0^H/n)}{1-\beta} \right) + \varsigma^L \left(\varphi^L (X_0) + \frac{\ln z_0^L}{1-\beta} \right) + \varsigma^W V^W (X_0) \right]\end{aligned}\quad (44)$$

(1.) Pose a candidate leverage function and make a starting guess on its parameters. In this paper we guess a first order log-linear formulation for each aggregate state $i = h, l$.

$$\ln \tilde{\theta}_t = \chi_0^i + \chi_1^i \ln d_t + \chi_2^i \ln Z_t$$

(2.) Compute the equilibrium quantities of our model economy using steps (1)-(6) in the previous subsection

(3.) Compute the entrepreneurs' value function

$$\varphi(a_t, d_t, Z_t, A_t) = \ln(1-\beta) + \frac{\beta \ln \beta + \beta E_t(\ln R_{t+1}^i)}{1-\beta} + \beta E_t \varphi(a_{t+1}, d_{t+1}, Z_{t+1}, A_{t+1})$$

and the workers' value function

$$V^W(d_t, Z_t, A_t) = \Theta + \frac{\omega}{1+\omega} \ln w_t + \beta E_t V^W(d_{t+1}, Z_{t+1}, A_{t+1})$$

(3.1.) Discretise the space of the continuous state variables d_t and Z_t . We use 10 grid points on each state variable. The value function is almost linear in the direction of both state variables so using more grid points makes very little difference to the results while slowing down the computations considerably.

(3.2.) Use value function iterations to compute the value function at each grid point. When state variables fall in between grid points, we use bi-linear interpolation to approximate the value function.

(4.) Compute social welfare for the candidate leverage function $\tilde{\theta}_t$. This consists of two steps:

(4.1.) Compute the realisation of the capital price in the initial period when the private sector is surprised by the policy change. This allows us to compute the realisations of the aggregate state variables (the vector X_0) when the policy is announced. It also allows us to compute the realisations of the wealth of each group when the policy is announced.

(4.2) Evaluate the the social welfare function (44) at the post regulation reform aggregate state X_0 and individual wealth positions - z_0^H and z_0^L .

(5.) Place steps (1)-(4) above in a function which outputs the value of social welfare for a candidate leverage function and maximise it with respect to the parameters of the leverage function. Because function evaluations are very time consuming we use the inbuilt Matlab routine `fminsearch.m` which uses a Nelder-Meade algorithm.