A NOTE ON THE EQUAL-LOSS PRINCIPLE FOR BARGAINING PROBLEMS*

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A B S T R A C T

Some shortcomings of the equal-loss solution for bargaining problems are noticed: in general, it is not individually rational, and in case it is, then turns out a selection of the YU^{∞} solutions. Finally, a new way of proving characterization results is provided.

1. INTRODUCTION

The "equal-loss principle" in bargaining problems is introduced by Chun (1988), and then he proposes a new bargaining solution, which he calls *equal-loss solution*. Additionally, he provides us with two alternative characterizations of the aforementioned solution.

In this paper we present some considerations on the equal-loss solution for bargaining problems. Two main shortcomings are noticed: first, we prove (by means of an example) that, in general, the equal-loss solution is not individually rational; secondly, we prove that, in those cases in which the equal-loss solution is individually rational, then it turns out a selection of the set of YU^{∞} solutions to the bargaining problem. Finally, we comment on some problems appearing in the way of proving Chun's characterization results of the equal-loss solution.

Vector inequalities will be $\geq$, $>$, $\gg$.

2. PRELIMINARIES

Following Nash (1950), a n-person bargaining problem is a pair (S,d), where S is a subset of $\mathbb{R}^n$, and d is a point of S. $\mathbb{R}^n$ is the utility space, S is the feasible set and d is the disagreement point. The intended interpretation of (S,d) is as follows: the agents can achieve any point of S if they unanimously agree on it. Otherwise, they end up at d.

Given a class of n-person bargaining problems, a solution is a function F which associates to every problem (S,d) in the class a point F(S,d) in S, representing the agreement made by the agents.

Let Σ^n be the class of bargaining problems (S,d) such that $S \subset \mathbb{R}^n$ is convex, closed and comprehensive (if $x \in S$, and $y \leq x$, then $y \in S$), and such that there exists $x \in S$, with $x \gg d$. Whenever $(S,d) \in \Sigma^n$, we shall call IR(S,d) the set of individually rational points, and WPO(S) the set of weakly Pareto Optimal elements, i.e., $IR(S,d) = \{ x \in S \mid x \geq d\}$, and $WPO(S) = \{ x \in S \mid \text{if } y \gg x, \text{ then } y \notin S\}$.

By considering $a_i(S,d) = \max \{x_i \mid x \in S, x \in IR(S,d)\}$, $i = 1,\dots,n$, we construct the *ideal point* a(S,d), such that for every i, gives the maximal obtainable utility levels of each agent subject to the condition that all agents achieve at least the utility levels of the disagreement point.

For $A \subset \mathbb{R}^n$, we shall denote by Co(A) the convex hull of set A, and by Com(A) the comprehensive hull of set A. CoCom(A) is simply the convex-comprehensive hull of set A.

When looking for solutions to the bargaining problem within the class Σ^n, some axioms are usually considered:

(WPO) *Weak Pareto Optimality*. For all $(S,d) \in \Sigma^n$, $F(S,d) \in WPO(S)$.

(SY) *Symmetry*. For all $(S,d) \in \Sigma^n$ and for all permutations $\pi:\{1,\dots,n\} \to \{1,\dots,n\}$, if $S = \pi(S)$ and $d = \pi(d)$, then $F_i(S,d) = F_j(S,d)$ for all i,j.

(TI) *Translation Invariance*. For all $(S,d) \in \Sigma^n$ and for all $t \in \mathbb{R}^n$, $F(S+\{t\}, d+t) = F(S,d) + t$.

(IR) *Individual Rationality*. For all $(S,d) \in \Sigma^n$, $F(S,d) \in IR(S,d)$.

(CONT) *Continuity*. For each sequence $\{(S^k,d^k)\} \subset \Sigma^n$, and every $(S,d) \in \Sigma^n$, if S^k converges to S in the Hausdorff Topology, and $d^k=d\ \forall\ k$, then $F(S^k,d^k)$ converges to F(S,d).

WPO requires that there be no feasible alternative in which all agents are better off than they are at the solution outcome; SY says that if the only information available on the conflict situation is contained in the mathematical description of (S,d), and (S,d) is a symmetric problem, then there is no ground for favoring one agent at the expense of other agent. TI requires that the choice of origin for the utility functions does not matter. IR says that the solution outcome guarantees every agent to enjoy at least the utility levels they get at the disagreement point, and, finally, CONT implies that small variations in the opportunity set without changes in the disagreement point conveys to small variations in the solution.

3. YU AND EQUAL-LOSS SOLUTIONS

A class of solutions for this kind of problems was proposed by Yu (1973), sharing the idea of looking for *the closest point in* IR(S,d) *to the ideal point.* Then, by means of considering a particular family of distances in $\mathbb{R}^n$, namely, $d_p(x,y) = [\sum_{i=1}^{n} |(x_i - y_i)|^p]^{1/p}$, $1 \le p < \infty$, and $d_\infty(x,y) = \max_i |x_i - y_i|$, he gets a family of solutions to the bargaining problem, as those minimizing the adequate distance to the ideal point, and he called them YU^p, $1 \le p \le \infty$.

Strictly speaking, neither YU^1, nor YU^∞ are solutions to the bargaining problem, since the associated norms $\|.\|_1$, $\|.\|_\infty$ are not strictly convex, and therefore both YU^1, YU^∞ can be multi-valued [see Freimer & Yu (1976)].

It is worth mentioning that $YU^p(S,d) \in WPO(S) \cap IR(S,d)$, for $1 < p < \infty$, and $YU^q(S,d) \subset WPO(S) \cap IR(S,d)$, for $q = 1, \infty$.

Moreover, as was pointed out by Chun (1988), in case $n = 2$, one of the elements in $YU^\infty(S,d)$ corresponds to that point x in $WPO(S) \cap IR(S,d)$ such that $|a_1(S,d) - x_1| = |a_2(S,d) - x_2|$. Taking this idea into account, Chun proposed a new solution concept, as a variant of the YU^∞ solution, as that point $EL(S,d) = y$ in S such that $|a_i(S,d) - y_i| = |a_j(S,d) - y_j|$, $\forall$ i,j, and this common difference is minimum. Chun's defence of its proposed solution is made on the grounds of the *equal loss principle*, namely, equalization across agents the losses from the ideal point, in a similar spirit of the egalitarian solution [Kalai (1977)], which equalizes across agents the gains from the disagreement point.

Even though the equal-loss principle has received attention in the literature on bankrupcy and taxation [cf. Aumann & Maschler (1985) or Young (1987) (1988)], its introduction in bargaining problems presents some difficulties. Let us consider the following example:

Consider, in Σ^3, the bargaining problem (S,d) such that d = (0,0,0), and S = CoCom{ (6,0,0); (0,2,0); (0,0,10)}. The ideal point is a = (6,2,10). In this problem, the equal-loss solution, EL(S,d) = (9/4,-7/4,25/4), with common losses from the ideal point of 15/4. Obviously, EL(S,d) is not individually rational, since agent 2 is worse off at EL(S,d) than she is at the disagreement point d. It is easy to see that there is no point in IR(S,d) such that it equalizes losses across agents from the ideal point. Let us now consider the possibility (9/4,0,25/4). It is easy to check that (9/4,0,25/4) $\in$ $YU^{\infty}(S,d)$. Obviously, (9/4,0,25/4) is both individually rational and Pareto optimal. Moreover, (9/4,0,25/4) dominates (in the Pareto sense) EL(S,d).

In this example, two main shortcomings of EL(S,d) are showed: (i) EL(S,d) is not individually rational; (ii) If S = Com[IR(S,d)], then there exists another feasible outcome [in $YU^{\infty}(S,d)$], individually rational in which those individuals which gain from their status-quo situation are at the same utility level than in EL(S,d).

The relationship between $YU^{\infty}(S,d)$ and $EL(S,d)$ is contained in the following proposition:

Proposition 1.- In case EL(S,d) is individually rational, then EL(S,d) belongs to $YU^{\infty}(S,d)$.

Proof:

Without loss of generality, we can assume $d = 0$, since, as was pointed out in Chun (1988), $EL(S,d)$ satisfies TI.

Let us call $x := EL(S,d)$, and suppose $x \in IR(S,d)$, but $x \notin YU^{\infty}(S,d)$. In this case, there exists $y \in IR(S,d) \cap WPO(S)$, such that $\|a-y\|_{\infty} < \|a-x\|_{\infty}$, i.e, $y \in \text{int } B_{\infty}(a, \|a-x\|)$, where $B_{\infty}(a,r) = \{z \in \mathbb{R}^n; \|a-z\|_{\infty} \leq r\}$. Now, since S is a comprehensive set, there exists a vertix in $B_{\infty}(a, \|a-y\|)$ in $IR(S,d)$. Contradiction. ■

Chun (1988) noticed that previous result holds for 2-person bargaining problems. Notice that in case $n = 2$, $EL(S,d)$ is always individually rational. So, $n=2$ is a particular case of Proposition 1.

4. CHARACTERIZATIONS OF THE EQUAL-LOSS SOLUTION

Besides those properties presented in Section 2, Chun considers some additional properties:

(W.MON) *Weak Monotonicity*. For every (S^1,d^1), $(S^2,d^2) \in \Sigma^n$, if $S^1 \subset S^2$, and $d^1=d^2$, and $a(S^1,d^1) = a(S^2,d^2)$, then $F(S^1,d^1) \leq F(S^2,d^2)$.

(S.MON*) *Strong Monotonicity other than ideal point*. For all (S^1,d^1), $(S^2,d^2) \in \Sigma^n$, if $S^1 \subset S^2$ and $a(S^1,d^1) = a(S^2,d^2)$, then $F(S^1,d^1) \leq F(S^2,d^2)$.

(ID.MON) *Ideal point monotonicity*. For all (S^1,d^1), $(S^2,d^2) \in \Sigma^n$, and for all i, if $S^1 = S^2$, and $a_i(S^1,d^1) \leq a_i(S^2,d^2)$ and for all $j \neq i$, $a_j(S^1,d^1) = a_j(S^2,d^2)$, then $F_i(S^1,d^1) \leq F_i(S^2,d^2)$.

W.MON was introduced by Kalai & Smorodinski (1975) for two person bargaining problems, and was extended to n-person bargaining problems by Roth (1979). This property says that, if the feasible set expands in such a way that neither the disagreement point nor the ideal point change, then no agent may be worse off.

S.MON was introduced by Chun (1988), and requires that, if the feasible set expands and the disagreement point changes without affecting the ideal point, then no agent should loose. It is a variant of the requirement of strong monotonicity [Luce & Raiffa (1957)] which requires that if the feasible point spands while the disagreement point remains fixed, then no agent should loose. It can be interpreted as a fairness condition: all agents should benefit from expanding opportunities.

ID.MON, introduced also by Chun (1988) requires that an increase of an agent's utility level at the ideal point, while the feasible set remains fixed, would not hurt her. For $n = 2$ it is essentially equivalent to the axiom of disagreement point monotonicity [Thomson (1987)], which requires that an increase of an agent's utility level at the disagreement point, *ceteris paribus*, would not hurt her.

Chun (1988) provides two characterization results:

Theorem 1.- The equal loss solution is the only solution satisfying WPO,SY,T.Inv and S.Mon*.

Theorem 2.- The equal-loss solution is the only solution satisfying WPO, SY, T.Inv., Cont., W.Mon. and Id.Mon.

In proving both theorems Chun starts by considering a generic problem $(S,d) \in \Sigma^n$ such that WPO{IR(S,d)} = PO{IR(S,d)}. By T.Inv., he assumes $a(S,d) = (1,\ldots,1)$ and he denotes by $x^* = EL(S,d)$. In order to prove uniqueness, he takes a solution F satisfying the adequate axioms and constructs auxiliar bargaining problems in order to apply properties of F and get the solution. Among the auxiliar problems he considers (S^1,d) and (S^2,d), where $S^1=\text{Com}\{IR(S,d)\}$, $S^2=\{y \in H\_(x^*) | y \leq a(S,d)\}$, p being the normal vector to a supporting hyperplane of S at x^*, and $H\_(x^*) = \{y \in \mathbb{R}^n | py \leq px^*\}$.

Whenever he gets $F(S^2,d) = x^*$, he claims

(a) In Th.1, by applying S.MON* on (S^2,d), (S^1,d), it follows $F(S^1,d)=x^*$.

(b) In Th.2, by applying W.MON on (S^2,d), (S^1,d), it follows $F(S^1,d)=x^*$.

But notice that if we take (S,d) such that $EL(S,d) \notin IR(S,d)$ and $EL(S,d) \notin S^1$, we can conclude neither in (a) nor in (b) $F(S^1,d) = x^*$, since in such a case, $x^* \notin S^1$ [as an example, take $(S,d) \in \Sigma^3$ such that $d=(0,0,0)$, and $S = CoCom\{(15,-3,0),(0,-3,25),(0,2,0)\}$].

Nevertheless, Theorems 1 and 2 are correct, as can be seen by considering the following modification in the proofs:

Start by taking $(S,d) \in \Sigma^n$ such that $WPO(A) = PO(A)$, where $A = \{x \in S \mid x_i \geq \min(x^*_i, d_i)\}$, and $x^* = EL(S,d)$. By T.INV, take $a(S,d) = (1,\ldots,1)$. Now construct S^1, S^2 and S^3 as in Chun (1988), but taking A instead of IR(S,d). Then, everything follows.

5. FINAL REMARKS

It is true that the equal-loss principle is an appealing one when dealing with problems of bankrupcy, property rigths and taxation, and, as was pointed out by Young (1987), it has been traditionally used in distributive justice problems. Nevertheless, Chun's application to bargaining problems of the equal-loss principle has a main shortcoming, namely, that his proposed solution is not individually rational. It is not clear to us how a non-individually rational solution can be defended on the grounds of agreements among agents, taking into account that, with no agreement, they end up at d.

Notice that, in case n=2, then EL is individually rational. So, previous remark does not apply for n=2.

In order to ensure Pareto Optimality (and not only Weak Pareto Optimality), recently Chun & Peters (1991) presented a variation of the equal-loss solution, namely, the *lexicographic equal-loss solution*, LEL(S,d), in which, by starting with EL(S,d), he constructs a lexicographic extension of EL(S,d), by means of increasing the utility levels of some agents, without damaging the rest, and looking for a maximal element in this way. It is worth pointing out that by means of this modification, the problem of the lack of individual rationality is not solved, as can be seen by considering the following example (which is only a slight modification of that presented in Section 3): Let S: = $\{(x,y,z) \in \mathbb{R}^3: 5x + 15y + 3z \leq 30\}$, and let d = (0,0,0). In this case, a(S,d) = (6,2,10), and EL(S,d) = (70/23,-14/23,170/23), with common losses from the ideal point of 60/23. Clearly, in this example, EL(S,d) $\notin$ IR(S,d). On the other hand, EL(S,d) $\in$ PO(S,d), and therefore, EL(S,d) = LEL(S,d).

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APPENDIX

<u>Proof of Th.1:</u> Obviously, EL satisfies the four axioms. In order to prove uniqueness, suppose F is a solution satisfying those axioms. Let $(S,d) \in \Sigma^n$ such that $a(S,d) = (1,\ldots,1)$ (T.Inv), and denote $x^* = EL(S,d)$. Let now $A = \{x \in S \mid x_i \geq \min(x^*_i, d_i)\}$, and assume that WPO(A) = PO(A). Let now $S^1 = Co(A)$, $p \in \Delta^{n-1}$ the normal vector to the supporting hyperplane of S at x^*, $H_-(x^*) = \{y \in \mathbb{R}^n \mid py \leq px^*\}$, and $S^2 = \{y \in H_-(x^*) \mid y \leq a(S,d)\}$. Now, $\forall i$, let y^i be the WPO maximal point in S^2 such that $y^i_i = a_i(S,d) = 1$, $y^i_j = y^i_k = \alpha^i$, $\forall$ $j,k \neq i$. Let $\alpha^* = \min \alpha^i$, and $\forall i$, let z^i such that $z^i_i = a_i(S,d) = 1$, $z^i_j = \alpha^*$ $\forall$ $j \neq i$. Finally, let $S^3 = \Gamma Co\ \{x^*, z^1, \ldots, z^n\}$. By choosing $d^* = (\alpha^*, \ldots, \alpha^*)$, we have $a(S^3, d^*) = (1,\ldots,1)$, and by WPO ans SY, $F(S^3, d^*) = x^*$. By applying SMON* to (S^3, d^*) and (S^2, d^*), we conclude $F(S^2, d^*) = x^*$, and by SMON* on (S^2, d^*) and (S^2, d), we get $F(S^2, d) = x^*$. Now, by applying SMON* twice on $(S^2,d),(S^1,d)$ and $(S^1,d),(S,d)$, we conclude $F(S,d) = x^*$.

Finally, for an arbitrary element (S,d) in Σ^n, we apply S.MON*. ∎

<u>Proof of Th.2:</u> Obviously, EL satisfies all the six axioms. In order to prove uniqueness, let F be a solution satisfying all the axioms. Let $(S,d) \in \Sigma^n$ such that $a(S,d) = (1,\ldots,1)$ (TINV), and denote $x^* = EL(S,d)$. Construct A as in the proof of Th.1, and assume that WPO(A)=PO(A). Let now S^1, S^2 and S^3 be as in the proof of Th.1.

By Choosing $d^* = (\alpha^*,\ldots,\alpha^*)$, we get $a(S^3,d^*) = (1,\ldots,1)$, and by WPO and SY, $F(S^3,d^*) = x^*$. By applying WMON on (S^3,d^*), (S^2,d^*), we get $F(S^2,d^*) = x^*$, and by ID.MON on (S^2,d^*), (S^2,d), we get $F(S^2,d) = x^*$. Now, by applying WMON twice on $(S^2,d),(S^1,d)$ and $(S^1,d),(S,d)$, it follows that $F(S,d) = x^*$.

For an arbitrary element (S,d) in Σ^n we apply CONT. $\blacksquare$

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