

LONG-RUN GROUNDWATER RESERVES UNDER UNCERTAINTY*

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ABSTRACT

In this paper the long-run effects of demand and recharge uncertainty on the socially optimal management of groundwater reserves are studied. Demand uncertainty is modeled in a dynamic framework by letting the demand function shift randomly but continuously through time according to a random shock that follows a stochastic process. Likewise, uncertainty about natural recharge is characterized by identifying that variable as a stochastic process. The results show how the effects on long-run groundwater reserves depends crucially on the properties of the demand function. Basically, the long-run groundwater reserves increase (decrease) with an increase in uncertainty if the demand function is convex (concave). These effects occur because the random changes in demand and natural recharge cause an *expected increase (decrease) in price* depending on the curvature of the demand function.

KEY WORDS: Groundwater management, demand and recharge uncertainty, stochastic control.
JEL CLASSIFICATION: Q25, D90.

RESUMEN

En este artículo se estudian los efectos a largo plazo de la incertidumbre de recarga y de demanda sobre la gestión socialmente óptima de las reservas de aguas subterráneas. La incertidumbre de demanda se trata en un marco dinámico dejando que la función de demanda cambie aleatoriamente pero continuamente a lo largo del tiempo de acuerdo con un shock aleatorio que sigue un proceso estocástico. Asimismo, la incertidumbre sobre la recarga natural se caracteriza identificando esa variable con un proceso estocástico. Los resultados muestran como los efectos sobre las reservas de aguas subterráneas a largo plazo dependen crucialmente de las propiedades de la función de demanda. Básicamente las reservas de aguas subterráneas a largo plazo aumentan (disminuyen) con un aumento en la incertidumbre si la función de demanda es convexa (cóncava). Estos efectos ocurren porque los cambios aleatorios en la recarga natural y en la demanda provocan un *aumento (disminución) esperado en el precio* dependiendo de la curvatura de la función de demanda.

PALABRAS CLAVE: Gestión de reservas de aguas subterráneas, incertidumbre de recarga y de demanda, control estocástico.

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1. INTRODUCTION

In this paper we examine the long-run effects of uncertainty in demand and natural recharge on the socially optimal management of groundwater reserves. This problem has been present on the agenda of environmental and resource economists at least from the sixties. Thus, one of the earliest applications of stochastic dynamic programming to the theory of resource management, as Conrad and Clark (1987) have pointed out, is related to the optimal temporal allocation of groundwater when natural recharge is uncertain, see Burt (1964a, 1967b).

In Burt (1964a) an approximately optimal feedback control policy is derived from a functional equation obtained applying the methods of dynamic programming when natural recharge is a random variable. But in his solution, long-run equilibrium storage depends only on the expected value of the recharge in both the first degree approximation and the second degree approximation he tests¹. In Burt (1967b) the 'certainty equivalent' problem which results when recharge is replaced by its expected value is solved, and some analysis of the probability distribution of groundwater reserves is made. Another interesting work by Burt is his 1964b paper where a procedure is presented for estimation of the value of groundwater reserves as a contingency against uncertain future supplies. This subject has been later developed by Bredehoeft and Young (1983), Tsur (1990) and Tsur and Graham-Tomasi (1991)².

Tsur and Graham-Tomasi (1991) show that the buffer value of groundwater is positive when it is utilized with stochastic surface water supplies. In their model natural recharge is deterministic and supply of surface water is a independent and identically distributed random variable. They conclude that if the third derivative of the water revenue function is nonnegative the groundwater reserves in the uncertain regime fall short of the steady-state reserves of the certain regime.

¹ Later, Burt and Cummings (1977) and Kolberg (1992) have explored the practical value of these approximate decision rules.

² Two more papers were published by this author in the sixties on optimal control of groundwater under uncertainty: Burt (1966) and (1967a). In Burt (1966) an extension of the maximization problem of the expected present value of net benefit is presented by including variance of net benefit in the decision criterion. The other paper is mainly a straightforward exposition, using economic reasoning and geometry, of the results obtained in Burt (1964a).

More recently Provencher and Burt (1993), Knapp and Olson (1995) and Tsur and Zemel (1995) have studied different topics related to the optimal management of groundwater reserves considering uncertainty. Provencher and Burt define a risk externality associated with the common property exploitation of groundwater which appears when surface water supply is stochastic and firms are risk-averse. The cause of this externality is that the distribution of a firm's future income depends on present pumping decisions. Knapp and Olson study the groundwater management with stochastic surface flows and artificial recharge, and Tsur and Zemel focus on optimal exploitation of groundwater resources when extraction affects the probability of occurrence of an irreversible event.

In our paper we reexamine the original problem proposed by Burt in a more *general* setting and apply a different approach than the one used in the previous literature. Unlike above mentioned studies in which natural recharge or surface water supply are a random variable each time a pumping decision is made, we assume uncertainty over future natural recharge. Moreover, we introduce in the model of socially optimal groundwater extraction another source of uncertainty that had not been previously studied: random demand shocks. In this way we extend the analysis of the optimal management of groundwater to incorporate the uncertainty associated with economic and demographic phenomena affecting water demand. We model *demand uncertainty* in a dynamic framework by letting the demand function shift randomly but continuously through time according to a random shock that follows a stochastic process. This means that present demand is known, but future changes in demand are unknown and have a variance that increases as time passes. Likewise, we characterize uncertainty over natural recharge by identifying that variable as a stochastic process. Thus, our approach focuses more on uncertainty over the future evolution of demand and natural recharge than on uncertainty about the present values of those variables.

Of course, this approach has already been applied by others. Here we basically follow the methodology of Pindyck (1982) in his study of a firm's optimal investment under uncertainty, with adjustment costs. Later the same author, Pindyck (1984), presented an analysis of *stock uncertainty* in the theory of renewable resource extraction using a stochastic process to describe the stock dynamics. More recently Clarke and Reed (1989) and Reed and Clarke (1990) have made use of stochastic differential equations to study the tree cutting problem in a stochastic environment, distinguishing between size-dependent and age-dependent stochastic growth. However, in the literature of groundwater management, only Conrad (1994) has proposed a stochastic version of a bang-bang control problem, where a stochastic differential equation drives the groundwater-reserves dynamics. In this paper we develop that model

assuming no linearity of the net social benefit function with respect to the rate of extraction and incorporating in the analysis the demand uncertainty.

Our results show how the effects of such uncertainties depend crucially on the properties of the demand function. When demand shifts stochastically and there is no uncertainty about the natural recharge, the long-run groundwater reserves *increase (decrease)* with an increase in uncertainty if demand function is *convex (concave)* with respect to the stochastic process representing the demand shocks. Stochastic fluctuations in natural recharge, when demand is certain, have the same effect on long-run reserves if demand function is convex (concave) with respect to the rate of extraction. Furthermore, this result holds for the general case (demand and natural recharge fluctuations) whether the correlation coefficient for the two stochastic processes is zero and whether uncertainty enters into demand function additively or multiplicatively. We find that these effects occur because the random changes in demand and natural recharge cause an *expected increase (decrease) in price* depending on the curvature of the demand function, so that an increase in uncertainty can give rise to different effects for demand functions with different properties. In this way we succeed in presenting a *complete* characterization of the effects of uncertainty on long-run groundwater reserves.

These results allow us to determine the sign of the error made when the 'certainty equivalent' problem is used to define the optimal management of groundwater reserves considering uncertainty. For a *convex (concave)* demand function we have an *overexploitation (subexploitation)* of the reserves when the 'certainty equivalent' approach is followed. So that, in each case, an adequate correction of the extraction rate is required if that approach is used: for a convex (concave) demand function the rate of extraction would have to be less (larger) than the rate of extraction obtained from the 'certainty equivalent' problem³.

We have also developed an example to get a first quantitative approximation of the effects of an increase in uncertainty on long-run groundwater reserves⁴. For this example we deduce that when the standard deviation is in the interval (0,1), 13 of the 16 calculated values for the groundwater reserves give relative variations with respect to the certainty case that are below 5 percentage points.

³ For the linear case our result is consistent with the one obtained by Burt (1967b): the uncertainty about future recharge and demand has no effects on the 'certainty equivalent' steady state.

⁴ This example is based on hydraulic data from the 23 aquifer that extends through Ciudad Real, Cuenca and Albacete.

In this example we have also found that a decrease in the rate of interest increases the impact on reserves of an increase in uncertainty. The following section presents the model developed in this paper. Section 3 examines the exploitation of groundwater reserves in a deterministic setting. This makes it easier to understand the stochastic solution. Section 4 considers the case where natural recharge and demand are stochastic and water management authority is risk-neutral. In section 5 we present the effects of uncertainty on long-run groundwater reserves and develop an example to illustrate quantitatively our results. Some concluding remarks close the paper.

2. OPTIMAL CONTROL IN GROUNDWATER PUMPING

Let the objective of a management authority be to maximize the present value of net social benefit. The net benefit is defined as gross consumer's surplus (B) minus costs (c). The gross consumer's surplus or social benefit is given by

$$B(q,u) = \int_0^q p(s,u) ds \quad [1]$$

where $p(q,u)$ is the water demand (inverse) function, q is the rate of extraction (pumping) and u represents a demand random shock following a stochastic process of the form

$$du = \sigma_1 u dz_1, \quad u(0) = u_0 > 0 \quad [2]$$

z_1 being a standard Brownian motion process⁵.

⁵ For these types of stochastic processes, the increments are independent, with $E[du] = 0$ and $\text{Var}[du] = \sigma_1^2 u^2 dt$, and u is lognormally distributed and always positive. Observe that (2) does not contain a deterministic drift component; although some deterministic component of demand growth might be anticipated we are interested only in the consequences of stochastic fluctuations on demand. For an introduction to the stochastic processes and differential equations, see Malliaris (1982), and more recently, Dixit and Pindyck (1994).

For given u , p is a continuous, strictly decreasing function of q , taking the finite value $p(0,u) \leq p < \infty$, and with $\int p(s,u) ds$ bounded uniformly in u and q . We assume that p

is continuous and increasing in u , so that an increase in u means a shift to the right of the demand curve. Equation (2) implies that current demand is precisely known; uncertainty about future demand increases with the passage of time, and discontinuities do not occur in demand fluctuations.

The cost of pumping is linear with respect to the rate of extraction and decreasing with respect to the stock of water, x , but at a decreasing rate⁶:

$$c = c(x)q, \quad c'(x) < 0, \quad c''(x) \geq 0 \quad [3]$$

The natural recharge rate is modeled as exogenously determined. It is not stock dependent. This is a reasonable approximation for water levels of interest⁷. Then the water stock's dynamic constraint is given by the following differential equation

$$dx = (R - q)dt, \quad x(0) = x_0 > 0 \quad [4]$$

here R is the natural recharge rate⁸. Equation (4) says that the change in groundwater reserves

⁶ Moreover, the average cost is defined with respect to a specific unit of measurement of the extraction rate (e.g., pts/Hm³) so that arbitrary changes in the unit of measurement of the extraction rate are not allowed.

⁷ Here we follow the approach adopted by Burt with respect to the dynamic constraint. See Burt (1964a) or Burt (1966) and, more recently, Provencher and Burt (1993). The idea is that recharge would be constant except at very high levels of storage i.e., for many groundwater cases, recharge could be treated as a constant within the economically relevant range of groundwater reserves. See Tsur and Graham Tomasi (1991) for a different approach.

⁸ Shallow aquifers are recharged by local percolation of surface water and discharged by trees that 'pump' the water out of the ground and transpire it into the air. But the great aquifers run deep and are recharged by rain and melting snow from mountains. So for simplification, we do not take into account the local percolation and discharge in the dynamics constraint of water stock. Anyway to introduce a return flow coefficient into equation (4) would not qualitatively change the results obtained in this paper. See Gisser and Sánchez (1980) for a model of optimal control in groundwater pumping with constant natural recharge and return flow coefficient.

can be exactly predicted from the current recharge and extraction rate. But, as a matter of fact, the recharge rate is random, so that the dynamics of groundwater reserves are partly random. So, we write

$$dR = \sigma_2 R dz_2, \quad R(0) = R_0 > 0 \quad [5]$$

This equation implies that current resource stock is known with certainty, but instantaneous change in stock is (in part) random. Given that u is a stochastic process related to economic and demographic phenomena (growth of population and income, changes in the way of life and consumption standards, and, especially, changes in the demand and supply of agricultural produce that use water intensively) whereas natural recharge, R , is a geological phenomenon, we expect that correlation between the two processes is zero, $E[dz_1 dz_2] = 0$, i.e., we assume that they are two independent processes.

In this context, for a risk-neutral water management authority, the problem is to choose $q(t)$ to maximize the expected present value of net social benefit,

$$\max_{\{q\}} E_0 \int_0^{\infty} \left[\int_0^q p(s, u) ds - c(x)q \right] e^{-rt} dt \quad [6]$$

where r is the social rate of discount, subject to the ordinary differential equation (4), the stochastic differential equations (2) and (5), and the usual non-negativity constraint on control variable $q \geq 0$.

⁹ We assume implicitly that the authority has rational expectations. In other words, it knows that demand and groundwater reserves evolve in accordance with equations (2), (4) and (5) and knows the value of σ_1 and σ_2 . We also assume that the non-negativity constraint over state variable is held. On the other hand, E_0 denotes the expectation calculated using the information available at the initial moment.

3. PUMPING UNDER CERTAINTY

We begin by reviewing the characteristics of the solution for the deterministic case ($\sigma_1 = \sigma_2 = 0$), i.e., when u and R are constant. In this case the solution to the optimal control problem (6) requires the maximization of the corresponding current-value Hamiltonian function,

$$H = H(x, q, \lambda) = \int_0^q p(s) ds - c(x)q + \lambda(R - q) \quad [7]$$

where λ is the adjoint variable. Then, the necessary conditions for an interior optimum are

$$p(q) = c(x) + \lambda \quad [8]$$

$$d\lambda/dt = r\lambda + c'(x)q \quad [9]$$

where equation (8) is the optimality condition: price equal to marginal extraction cost plus rent¹⁰.

Developing (8) and (9) we obtain, using differential equation (4), the following differential equation for the rate of extraction

$$p_q(q)(dq/dt) = r[p(q) - c(x)] + c'(x)R \quad [10]$$

that, with the dynamic constraint of water reserves, constitutes the synthesized dynamic system of necessary conditions for optimal management of the aquifer.

¹⁰ As the problem is an infinite horizon autonomous control problem, the transversality condition can be replaced by the stationary conditions ($dx/dt = dq/dt = 0$) that must hold in the long-run. See Seierstad and Sydsacter (1987, Ch. 3, Sections 7-9) for transversality and sufficient conditions and existence theorems.

If we focus on an interior solution and we look for the steady states, we have two equations that have to be satisfied

$$R - q = 0 \quad [11]$$

$$r[p(q) - c(x)] + c'(x)R = 0, \quad [12]$$

as equation (11) does not depend on x we can find the steady state extraction rate directly from this equation: $q^* = R$.

On the other hand, we have from equation (12)

$$\left. \frac{dq}{dx} \right|_{(dq/dt)=0} = \frac{rc'(x) - Rc''(x)}{rp_q(q)} > 0 \quad [13]$$

since $c' < 0$, $c'' \geq 0$ and $p_q < 0$. According to this, there exists a unique steady state (R, x^*) , with $x^* > 0$, if

$$\lim_{x \rightarrow 0} \left[c(x) - \frac{c'(x)R}{r} \right] > p(R) \quad [14]$$

This condition guarantees that the isocline $dq/dt = 0$ cuts the isocline $dx/dt = 0$. The uniqueness of the solution is immediately derived because $dq/dt = 0$ is increasing with respect to x ¹¹.

¹¹ Notice that $p(R) > 0$ is a necessary condition for the existence of a steady state. If the demand function is asymptotic to the axis of quantities, then this condition will be always guaranteed. Moreover if $\lim_{x \rightarrow 0} c'(x) = \infty$ the steady state reserves will be always positive. The *physical* exhaustion of groundwater reserves is not economically feasible and the constant recharge assumption becomes likely. The increase in extraction costs when groundwater reserves fall prevents reserves entering the range of values for which recharge depends on groundwater reserves.

From equation (12) we can obtain the optimality condition that characterizes the steady state,

$$p(R) = c(x^*) - \frac{c'(x^*)R}{r} \quad [15]$$

This condition establishes that the steady state equilibrium rent is the capitalized value of future increases in cost resulting from a reduction in groundwater reserves equal to the natural recharge of the aquifer.

To determine the stability properties of the first order differential equation system, we examine the characteristic roots of the linearized system. On the basis of the two-equation system (4) and (10) we form the Jacobian matrix and evaluate it at the steady state point $E = (x^*, R)$

$$J_E = \begin{bmatrix} \frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial q} \\ \frac{\partial \dot{q}}{\partial x} & \frac{\partial \dot{q}}{\partial q} \end{bmatrix}_{(x^*, R)}$$

The four partial derivatives¹², when evaluated at E , turn out to be

$$\begin{aligned} \frac{\partial \dot{x}}{\partial x} &= 0, & \frac{\partial \dot{x}}{\partial q} &= -1, \\ \frac{\partial \dot{q}}{\partial x} &= \frac{1}{P_q} [-rc'(x^*) + c''(x^*)R] < 0, \\ \frac{\partial \dot{q}}{\partial q} &= r > 0, \end{aligned}$$

from which we obtain the Jacobian determinant

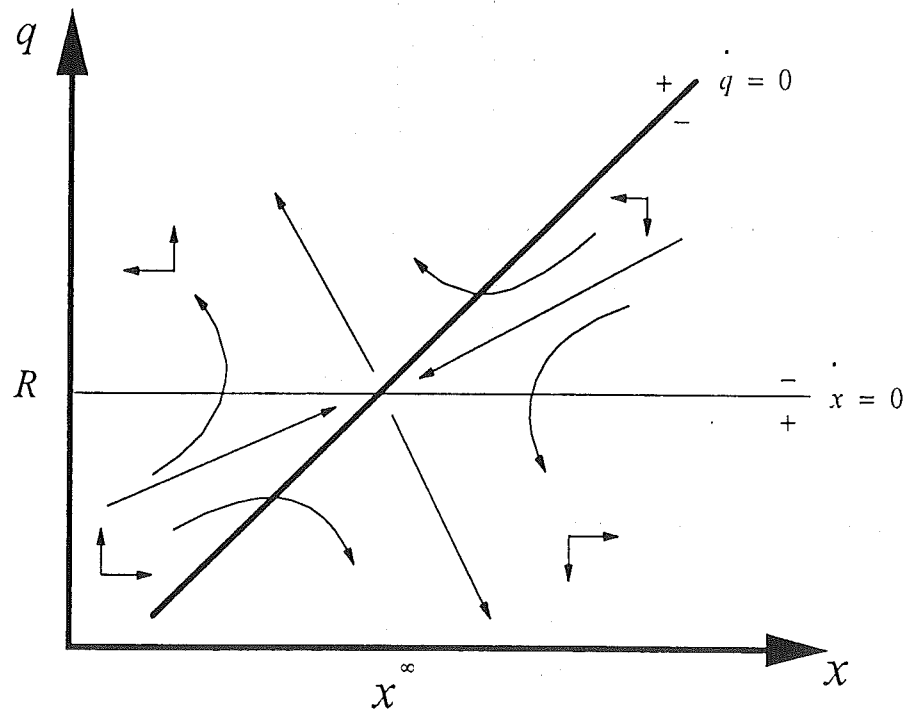
$$|J_E| = \frac{1}{P_q} [-rc'(x^*) + c''(x^*)R] < 0, \quad [16]$$

and as the Jacobian determinant is equal to the product of the characteristic roots, (16) implies

¹² Where $\dot{x} = dx/dt$ and $\dot{q} = dq/dt$

that the two roots have opposite signs, which establishes that the critical point of the linearized system is locally a saddle point. For this kind of critical point there are two stable branches leading toward E in the phase plane, so there exists an optimal path that leads to steady state (x^*, R) as is illustrated in Fig. 1¹³.

FIGURE 1



¹³ For a detailed analysis of the model of optimal control of groundwater reserves under certainty see Rubio, Martínez and Castro (1994).

4. PUMPING UNDER UNCERTAINTY

Here, we solve the stochastic control problem that we have just posed in section 2 by using the stochastic version of Bellman equation for dynamic programming¹⁴. Let J be the value of water reserves assuming q is chosen optimally, so that

$$J(x, u, R) = \max_{\{q\}} E_t \int_t^{\infty} \pi(x, q, u) e^{-r(\tau-t)} d\tau \quad [17]$$

where $\pi(x, q, u) = \int p(s, u) ds - c(x)q$ is the social benefit coming from the exploitation of the aquifer.

If the time horizon is infinite and time appears only through the discount factor, the Bellman equation for this problem can be written as

$$rJ = \max_{\{q\}} \left[\int_0^q p(s, u) ds - c(x)q + (1/df) E_t dJ \right] \quad [18]$$

Since u and R are stochastic processes, we can use Itô's Lemma to write

$$dJ = J_x dx + J_u du + J_R dR + \frac{1}{2} J_{uu} (du)^2 + J_{uR} du dR + \frac{1}{2} J_{RR} (dR)^2 \quad [19]$$

¹⁴ See Kamien and Schwartz (1991, Sect. 22) and Dixit and Pindyck (1994, Ch. 4) for the statement of the stochastic version of the Bellman equation. See also Fleming and Rishel (1975, Ch. VI, Sect. 6) for existence theorems for stochastic optimal control.

substituting for dx , du , dR , $(du)^2$ and $(dR)^2$ we obtain

$$dJ = (R-q)J_x dt + \sigma_1 u J_u dz_1 + \sigma_2 R J_R dz_2 + \frac{1}{2}(\sigma_1 u)^2 J_{uu} (dz_1)^2 + \sigma_1 \sigma_2 u R J_{uR} dz_1 dz_2 + \frac{1}{2}(\sigma_2 R)^2 J_{RR} (dz_2)^2$$

which reduces to

$$dJ = (R-q)J_x dt + \sigma_1 u J_u dz_1 + \sigma_2 R J_R dz_2 + \frac{1}{2}(\sigma_1 u)^2 J_{uu} dt + \sigma_1 \sigma_2 u R J_{uR} dz_1 dz_2 + \frac{1}{2}(\sigma_2 R)^2 J_{RR} dt \quad [20]$$

since $(dz_i)^2$ is equal to dt , from the definition of a Wiener process. Applying the differential operator $(1/dt)E_t$ to (20) and considering that $E_t[dz_i] = 0$, again from the definition of a Wiener process, and $E_t[dz_1 dz_2] = 0$, since we have assumed that $\rho_{12} = 0$ the Bellman equation can be written as

$$rJ = \max_{q'} \left[\int_0^q p(s,u) ds - c(x)q + (R-q)J_x + \frac{1}{2}(\sigma_1 u)^2 J_{uu} + \frac{1}{2}(\sigma_2 R)^2 J_{RR} \right] \quad [21]$$

Maximizing with respect to q we have the optimality condition

$$p(q,u) = c(x) + J_x \quad [22]$$

that is equivalent to condition (8) of the deterministic maximization problem with $J_x = \lambda$, and has the same interpretation¹⁵.

¹⁵ From now, in order to simplify the notation, we omit the independent variables of demand and cost functions.

To examine how uncertainty affects long-run groundwater reserves, we shall need to derive an expression for the expected dynamics of extraction rate, thus making the transition from the Bellman equation to solution of the stochastic control problem defined in section 2 in terms of one stochastic differential equation for the control variable, the rate of extraction, and the differential equations (2), (4) and (5). It will then be possible to use that system of equations to characterize a long-run (steady state) stochastic equilibrium and evaluate the effects of changes in variance on long-run groundwater reserves¹⁶.

Differentiating equation (21) with respect to x , and taking into account that the optimal value of q is given by equation (22), we obtain

$$rJ_x = (p-c-J_x)q_x - c'q + (R-q)J_{xx} + \frac{1}{2}(\sigma_1 u)^2 J_{uux} + \frac{1}{2}(\sigma_2 R)^2 J_{RRx} \quad [23]$$

Since the terms in the first set of parentheses sum to zero, J_x is given by (22), and $(1/dt)E_t dJ_x = (R-q)J_{xx} + \frac{1}{2}(\sigma_1 u)^2 J_{uux} + \frac{1}{2}(\sigma_2 R)^2 J_{RRx}$ by Itô's Lemma, equation (23) can be written as

$$(1/dt)E_t dJ_x = r(p-c) + c'q \quad [24]$$

Applying the differential operator to (22) and equating it to (24), we get

$$(1/dt)E_t d(p-c) = r(p-c) + c'q \quad [25]$$

¹⁶ This approach is applied by Pindyck (1982) to study the optimal investment of the firm under uncertainty with adjustment costs, and Rubio (1992) to analyze optimal investment in an extractive industry. Basically, it is an extension of the comparative statics analysis used in deterministic control theory to evaluate changes in steady state values caused by variations in the model's parameters.

Developing the left-hand side of (25), we can obtain the expected dynamics of extraction rate. From Itô's Lemma

$$dp = p_q dq + p_u du + \frac{1}{2} p_{qq} (dq)^2 + p_{qu} dq du + \frac{1}{2} p_{uu} (du)^2 \quad [26]$$

Considering that $q^* = q(x, u, R)$ along the optimal path, using Itô's Lemma again,

$$dq = q_x dx + q_u du + q_R dR + \frac{1}{2} q_{uu} (du)^2 + q_{uR} du dR + \frac{1}{2} q_{RR} (dR)^2 \quad [27]$$

and by substitution of dx , du , dR , $(du)^2$ and $(dR)^2$,

$$dq = (R-q)q_x dt + \sigma_1 u q_u dz_1 + \sigma_2 R q_R dz_2 + \frac{1}{2} (\sigma_1 u)^2 q_{uu} dt + \sigma_1 \sigma_2 u R q_{uR} dz_1 dz_2 + \frac{1}{2} (\sigma_2 R)^2 q_{RR} dt \quad [28]$$

The implied expression for $(dq)^2$ is greatly simplified by neglect of terms in higher powers of dt as dt goes to zero, so that we are left with

$$(dq)^2 = (\sigma_1 u q_u)^2 dt + 2\sigma_1 \sigma_2 u R q_{uR} dz_1 dz_2 + (\sigma_2 R q_R)^2 dt \quad [29]$$

Moreover we have that $dqdu = (\sigma_1 u)^2 q_{uu} dt + \sigma_1 \sigma_2 u R q_{uR} dz_1 dz_2$, then substituting in equation (26) for du , $(dq)^2$, $dqdu$ and (dt) , and applying the differential operator $(1/dt)E_t dt$ we get the differential equation for the expected dynamics of price

$$\left(\frac{1}{dt} \right) E_t dp = p_q \left(\frac{1}{dt} \right) E_t dq + \frac{1}{2} p_{qq} [(\sigma_1 u q_u)^2 + (\sigma_2 R q_R)^2] + p_{qu} (\sigma_1 u)^2 q_u + \frac{1}{2} p_{uu} (\sigma_1 u)^2 \quad [30]$$

Finally, developing in the left-hand side of (25) the differential operator for cost we have

$$\left(\frac{1}{dt} \right) E_t dc = c'(R-q) \quad [31]$$

Using (30) and (31) equation (25) can be written as

$$\begin{aligned} p_q \left(\frac{1}{dt} \right) E_t dq + \frac{1}{2} p_{qq} [(\sigma_1 u q_u)^2 + (\sigma_2 R q_R)^2] + \\ + p_{qu} (\sigma_1 u)^2 q_u + \frac{1}{2} p_{uu} (\sigma_1 u)^2 - c'(R-q) = \\ = r(p-c) + c'/q \end{aligned}$$

ordering terms, we obtain the desired expression for the expected dynamics of extraction rate

$$\begin{aligned} p_q \left(\frac{1}{dt} \right) E_t dq = r(p-c) + c'/R - \frac{1}{2} p_{qq} [(\sigma_1 u q_u)^2 + (\sigma_2 R q_R)^2] \\ - p_{qu} (\sigma_1 u)^2 q_u - \frac{1}{2} p_{uu} (\sigma_1 u)^2 \end{aligned} \quad [32]$$

that is equivalent, under uncertainty conditions, to equation (10) of section 3.

Equation (32), together with (4), describes the expected dynamics of q and x , and determines the long-run values of these variables given a current value of u and R . Moreover, we can use them to illustrate how these long-run values change as σ_1 and σ_2 increase, even if there is no guarantee that a stochastic equilibrium exists in the sense of convergence to a

long-run stationary distribution for q and x ¹⁷. That means that although the system may evolve (stochastically) in various ways, we can determine how, given a particular state of the world, the long-run values of the rate of extraction and groundwater reserves will vary with σ_1 and σ_2 in a way which depends on the properties of the demand function.

5. THE EFFECTS OF UNCERTAINTY

In this section we use equation (32) to evaluate the effects of uncertainty on long-run groundwater reserves. Applying the differential operator to (4) and equating to zero we get $R = q(x, u, R)$, then $q_R = 1$ and $q_u = 0$, and equation (32) can be written as

$$p(R, u) = c - \frac{c'R}{r} + \frac{1}{2r} p_{qq}(R, u)(\sigma_2 R)^2 + \frac{1}{2r} p_{uu}(R, u)(\sigma_1 u)^2 \quad [33]$$

when $(1/dt)E_dq = 0$ ¹⁸.

From this equation it is immediately evident that the effects of uncertainty on groundwater reserves depend on the curvature of demand function with respect to u and R . If demand function is strictly *convex (concave)* with respect to these two variables then the

¹⁷ In Pindyck (1984) this author analyzes the existence of a long-run stationary distribution for three examples of renewable resource management. A more extensive analysis of the conditions for the existence of a steady-state distribution in a one-sector neoclassical model of the Solow-type can be found in Merton (1975).

¹⁸ Given that there is not a functional relationship between R and u , R does not depend on u and vice versa, then we have

$$\frac{\partial q}{\partial u} = \frac{dR}{du} = 0$$

long-run reserves *increase (decrease)* with respect to σ_1 and σ_2 for a given state of the world¹⁹.

This relationship can be obtained from (33) by differentiation

$$dx = \frac{1}{Rc'' - rc'} (p_{qq} R^2 \sigma_2 d\sigma_2 + p_{uu} u^2 \sigma_1 d\sigma_1) \quad [34]$$

If demand function is strictly convex with respect to one variable and strictly concave with respect to the other one, the signs of the effects of uncertainty on the optimal value of groundwater reserves are undetermined. The net effect depends on the relationship between the absolute values of $p_{qq} R^2 \sigma_2$ and $p_{uu} u^2 \sigma_1$ ²⁰.

To explain these effects, assume $p_{qq} > 0$ and $p_{uu} > 0$. Then an increase in σ_1 and σ_2 has the same effect as an increase in marginal user cost or rent in equation (33), and consequently, for a given price, an increment in σ_1 and σ_2 will increase the long-run value of groundwater reserves²¹. The deviation from the certainty case is a consequence of the simultaneous random changes in u and R . Random changes in u that balance each other out lead to movements in price, but with $p_{uu} > 0$, increases in u raise price more than decreases in u lower it. Therefore, under demand uncertainty, there exists a new rent component equal to the *expected increase in price* coming from the random changes in u . Obviously, if $p_{uu} < 0$, i.e., if the inverse demand function is concave with respect to u , there exists a new marginal income component or a reduction in the user cost equal to the absolute value of the expected decrease in price.

Simultaneously, we have random changes in R which cause random changes in $c'R/r$ that balance each other out and the same changes in q as long as $q = R$ when $(1/dt)E_dq = 0$. These fluctuations in q imply fluctuations in price, but with $p_{qq} > 0$, increases in q lower price less than decreases in q raise it. So, under recharge uncertainty, there also exists a new user cost or rent

¹⁹ We are assuming that σ_1 and σ_2 change in the same direction. This is very clear when we are comparing the certainty case ($\sigma_1 = \sigma_2 = 0$) with the uncertainty case ($\sigma_1, \sigma_2 > 0$).

²⁰ In the numerical example developed at the end of this section we find that the component with negative sign always dominates the component with positive sign resulting in a decrease in the long-run reserves when uncertainty increases.

²¹ Equation (33) is equivalent, under uncertainty conditions, to equation (15) which is the optimality condition that characterizes the steady state equilibrium in the certainty case.

component to take into account equal to the *expected increase in price* coming from the random changes in R ²².

Thus, under uncertainty, user cost on the right-hand side of equation (33) presents two components: the capitalized value of future increases in cost resulting from a reduction in groundwater reserves equal to the natural recharge of the aquifer, and the expected increase (reduction) in price associated with the random changes in u and R ; the third component on the right-hand side being the marginal cost of extraction.

Based on these results, we can determine the sign of the deviation resulting when the 'certainty equivalent' problem (CEP) is applied to define the optimal exploitation of groundwater under uncertainty. For a *convex (concave)* demand function we have that the optimal long-run groundwater reserves are *higher (less)* than those obtained from the CEP, so that the application of this approach causes an *overexploitation (subexploitation)* of the reserves. Thus, the rate of extraction path associated with the CEP is *above (below)* the optimal path. Therefore an adequate correction of the extraction rate is necessary if the CEP is used: for a *convex (concave)* demand function the rate of extraction would have to be *lower (higher)* than the rate of extraction derived from the CEP.

For the linear case ($p_{qq} = p_{uu} = 0$) equation (33) is equal to equation (15) and the long-run values of q and x will be the same as in the certainty case. When the demand function is linear with respect to q and u , fluctuations in price balance each other out and the expected variation in price associated with the random changes in u and R is zero. Thus, uncertainty about future recharge and demand has no effect on the certain steady state²³. Notice that when the demand function is linear, the net social benefit is quadratic with respect to the extraction rate, and then we face a 'certainty equivalent' problem. In this case optimal control of groundwater

²² The effects of recharge and demand uncertainty are easily obtained from (34) as particular cases: ($\sigma_1 = 0, \sigma_2 > 0$) and ($\sigma_1 > 0, \sigma_2 = 0$).

²³ Notice that from this result we know that for the two following specifications: $p(q, u) = u(a-bq)$, multiplicative uncertainty and $p(q, u) = u + a-bq$, additive uncertainty, an increase in variance has no effect on the long-run groundwater reserves. However, since we do not have a solution for the optimal groundwater reserves path, we cannot determine the effect of changes in σ , on current reserves. This means that the way in which the stochastic process u enters the demand function could play a critical role in the determination of optimal groundwater reserves and extraction rate paths. In equation (32) if $P_{qq} = P_{uu} = 0$ the expected dynamics of extraction rate depends on P_{qu} which takes different values depending on whether demand uncertainty is additive or multiplicative.

reserves depends only on the first moment of the probability distribution of u and R , and the expected long-run equilibrium of groundwater reserves can be calculated based on the deterministic optimal control problem when u and R are replaced by their expected values. This approach was developed by Burt (1967b) for the case of recharge uncertainty, and our conclusions here are consistent with this earlier result.

5.1. EXAMPLE

Next we develop an example to illustrate numerically the results of the previous section and obtain a first quantitative evaluation of the effects of an increase in uncertainty on long-run reserves of an aquifer.

For this example we have chosen a variation of the linear case with additive uncertainty: $p(q, u) = a-bq^\eta + u^\gamma$, with $\eta, \gamma > 0$. For this specification we have that if $\eta < (>) 1$ the function is convex (concave) with respect to q and if $\gamma > (<) 1$ the function is convex (concave) with respect to u . So then, giving the parameters the appropriate values, we can study the four kinds of possible cases. With $\eta = 0.5$ and $\gamma = 1.5$ we have that the demand function is convex with respect to the two mentioned variables. If $\eta = 0.5$ and $\gamma = 0.5$ or $\eta = 1.5$ and $\gamma = 1.5$, the demand function is convex with respect to one variable and concave with respect to the other one; and, finally, with $\eta = 1.5$ and $\gamma = 0.5$ we have that the function is concave with respect to q and u .

On the other hand, we propose a linear extraction cost function: $c(x, q) = (c_0 - c_1 x)q$, where the marginal and average cost is decreasing with respect to the reserves, $c_{qx} = -c_1 < 0$.

For the hydraulic variables we use the available data from 23 aquifer that extends through Ciudad Real, Cuenca and Albacete and feeds the Tablas de Daimiel wetlands²⁴.

The parameter values of the demand and cost functions have been adjusted to obtain from the model a solution equal to the present level of reserves when the rate of interest is equal to

²⁴ The source is the study 'La quimera del agua presente y futuro de Daimiel y La Mancha occidental' written by Mario Gaviria y Juan Serna. See *El País*, May 2, 1995. The natural recharge is calculated at around 320 Hm³/year and the maximum reserves between 10,000 and 12,500 Hm³. At present (1995) it is estimated that the reserves only reach 5,000 Hm³. In the example, we have taken as expected value of the recharge the amount of 200 Hm³/year and we have placed the maximum reserves at 10,000 Hm³.

0.1, resulting in the following values: $c_0 = 350$, $c_1 = 0.035$, $a = 244$ and $b = 0.001$. Finally, we have considered it interesting to make the calculations for two interest rates, 0.05 and 0.1, with the aim of presenting a minimum sensitivity analysis of the results. We show in Tables I and II the optimal values of the reserves for an expected value of natural recharge of 200 Hm³/year and of u variable equal to one, and for different values of the standard deviation, from 0, the certainty case, to 2.

From the results it can be showed up that when the standard deviation is in the interval (0,1) the relative variations (in absolute value) with respect to the certainty case are always below 10 percentage points, and that 13 of the 16 calculated values are below 5 points. This indicates to us that, at least in this interval, obtaining the optimal extraction rate of the aquifer based on the expected values of R and u would not cause a significant deviation from the optimal value of the reserves.

TABLE I
($r=0.05$)

$\sigma_1 = \sigma_2$	$\eta = 0.5$	$\gamma = 1.5$	$\eta = 0.5$	$\gamma = 0.5$
0	7000.4040		7000.4040	
0.5	7054.2280	(0.77)	6982.7994	(-0.25)
1	7215.6999	(3.07)	6929.9856	(-1.00)
1.5	7484.8197	(6.92)	6841.9625	(-2.26)
2	7861.5874	(12.31)	6718.7303	(-4.02)
$\sigma_1 = \sigma_2$	$\eta = 1.5$	$\gamma = 1.5$	$\eta = 1.5$	$\gamma = 0.5$
0	7080.8123		7080.8123	
0.5	6982.8607	(-1.38)	6911.4321	(-2.39)
1	6689.0059	(-5.53)	6403.2916	(-9.57)
1.5	6199.2478	(-12.45)	5556.3907	(-21.53)
2	5513.5866	(-22.13)	4370.7295	(-38.27)

TABLE II
($r=0.10$)

$\sigma_1 = \sigma_2$	$\eta = 0.5$	$\gamma = 1.5$	$\eta = 0.5$	$\gamma = 0.5$
0	5000.4037		5000.4037	
0.5	5027.3156	(0.54)	4991.6013	(-0.17)
1	5108.0514	(2.15)	4965.1943	(-0.70)
1.5	5242.6110	(4.84)	4921.1824	(-1.58)
2	5430.9945	(8.60)	4859.5659	(-2.81)
$\sigma_1 = \sigma_2$	$\eta = 1.5$	$\gamma = 1.5$	$\eta = 1.5$	$\gamma = 0.5$
0	5080.7517		5080.7517	
0.5	5031.8062	(-0.96)	4996.0919	(-1.66)
1	4884.9697	(-3.85)	4742.1126	(-6.66)
1.5	4640.2422	(-8.67)	4318.8136	(-15.00)
2	4297.6237	(-15.41)	3726.1951	(-26.66)

We can observe as well that when the function is concave with respect to some of its arguments, the effect caused by the concavity of the function (an expected decrease in price) dominates over the effect caused by the convexity (an expected increase in price) resulting in a decrease of reserves when the uncertainty increases. This deviation can be above 25% if the demand function is concave with respect to the two variables, and the standard deviation is equal to 2.

Finally, we want to point out that a reduction of the interest rate causes an increase in the optimal value of the reserves, owing to the increase of user cost since this is inversely related to the rate of interest. However, a reduction in the interest rate also increases the impact on reserves of an increase in uncertainty. Thus, it can be observed that for all the considered values of the standard deviation, the bias with respect to the certainty case is larger when the interest rate is smaller. This is explained by the relationship between the user cost and the rate of interest that we have just pointed out (see (33)). If the rate of interest decreases, the additional component of the user cost under uncertainty (the expected increase (decrease) in price) is higher, and then an increase of the standard deviation has a larger effect on long-run water reserves.

6. CONCLUSIONS

In this paper we have analyzed the socially optimal management of groundwater under recharge and demand uncertainty. The uncertainties that we have been concerned with are over future natural recharge and demand shocks with uncertainty growing with respect to time. As we have pointed out in the introduction, this kind of uncertainty is characterized by a demand function and a recharge that shift randomly but continuously through time according to a stochastic process. Furthermore, socially optimal groundwater pumping is affected by these uncertainties owing to the price fluctuations related to changes in recharge or demand function, and therefore the effects depend critically on the curvature of the demand function.

This procedure is different from that used in the previous literature on optimal management of groundwater reserves. On the other hand, our results characterize *completely* the effects of uncertainty for this model of resource management. As they were presented extensively in the introduction, we do not repeat them in these concluding remarks.

Finally, we want to suggest some extensions, mainly related to the limitations of the model. A natural extension in order to complete the analysis would be to study the effects of uncertainty under the risk-aversion assumption. In section 2 we have assumed a risk-neutral *water management authority*. To study the risk-aversion assumption we would have to introduce a concave utility function to evaluate the net social benefit coming from the exploitation of groundwater reserves²⁵. The other extensions are defined by the scope of the approach used in this paper. Our analysis only shows how uncertainty affects the long-run value of groundwater reserves given a particular state of the world, but we have not established what are the effects of uncertainty on *current* value both of the rate of extraction and of the groundwater reserves. Moreover, we do not know if there exists a long-run stochastic equilibrium and what is its nature. These limitations clearly point out the way for future research. A next step could be to resume uncertainty in a stochastic differential equation for groundwater reserves (stock uncertainty) and to investigate the existence and properties of a long-run equilibrium for a more specific net social benefit function.

²⁵ For a risk-averse water management authority the problem would be to choose $q(t)$ to maximize the expected present value of utility

$$\max_q E_0 \int_0^{\infty} U[\pi(x, q, u)] e^{-rt} dt$$

with $U' > 0$ and $U'' < 0$.

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