OPTIMAL GROWTH AND LAND PRESERVATION*

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ABSTRACT

A model of optimal economic growth with a constant population subject to a constraint on the availability of land is presented. It takes account of the dual character of land as a production factor and as a consumption good (environmental amenities) by determining the optimal intertemporal allocation of land between productive and recreational uses. An extension of the analysis for the case of a growing population with endogeneous growth based on human capital accumulation shows that if the rate of discount is not very low then there exists a set of balanced growth paths compatible with a constant allocation of land.

KEY WORDS: Optimal growth, intertemporal land allocation, environmental preservation, population growth, endogenous growth, human capital.

RESUMEN

En este trabajo se presenta un modelo de crecimiento económico óptimo con una población constante sujeto a una restricción sobre la disponibilidad de tierra. En el modelo se tiene en cuenta el carácter dual de la tierra como factor productivo y como bien de consumo para usos recreacionales y se determina cual es la asignación intertemporal óptima de la tierra entre estos dos usos. En la segunda parte del trabajo, se presenta una extensión del análisis para el caso de una población creciente con crecimiento endógeno basado en la acumulación de capital humano y se demuestra que si la tasa de descuento no es muy pequeña existe un conjunto de sendas de crecimiento equilibrado compatibles con una asignación constante de la tierra.

PALABRAS CLAVE: Crecimiento óptimo, asignación intertemporal de la tierra, preservación medioambiental, crecimiento de la población, crecimiento endógeno, capital humano.

1 Introduction

Traditionally, economic analysis has focused on land as a factor of production. Yet everyday experience shows that land is equally important as a consumption good. This fact has been recognized by economists applying location theory to the organization of a city, where land serves exclusively as a terrain for urban purposes (see Solow and Vickery (1971) and Riley (1973)). Likewise, the dual purpose of land is considered in the economics of land development and wilderness protection where the issues of uncertainty and irreversibility are incorporated (see Arrow and Fisher (1974), Henry (1974), Hodge (1984), Kennedy (1987), and more recently Clarke and Reed (1990)). The aspect of environmental preservation has also been analyzed by Krautkraemer (1985), Olson (1990) and Barrett (1992) within the framework of optimal economic growth models.

Krautkraemer's paper focuses on the effects of technological progress and resource amenities on economic growth and on the extraction of a non-renewable resource, using the remaining non-renewable resource stock as a proxy of the flow of resource amenities. However, we think that this approach might not be suitable to analyze *land* preservation because the rate of depletion is considered as a production factor. If we identify land with a stock of natural capital, it seems more natural to use developed land as a production factor and not the rate at which land is developed.

Barrett's paper can be seen as an extension of the first model presented in Krautkraemer's article. He considers land as a non-renewable stock of natural capital and developed land as a production factor. Consumption depends on the rate of depletion and on the output produced by employing developed land. However, he does not take account of a productive capital stock in the production function[1].

Olson's approach seems to be more appropriate to analyze land preservation. He presents a model where land is developed for productive purposes or remains in a natural pristine state[2]. The total available land is fixed and finite and it

[1]The tropical rain forests is an example used by Barrett to justify the utilization of the productive rate of depletion. At the same time, however, he recognizes (Barrett (1992, p.291)) that it would be more realistic to assume that consumption is independent of resource development, or to allow for investment.

[2]Recently, López et al. (1994) emphasized the fact that significant demand for land in general arises from the desire to enjoy environmental amenities. However, their analysis deter-

must be allocated between these two alternative uses in the framework of a two time period optimal growth model with a productive capital stock. However, he focuses on the effects of irreversibility and learning on land preservation and analyzes neither the optimal intertemporal allocation of land nor the existence and properties of the steady state.

This paper follows Olson's approach to the problem of land preservation. However, we consider time as continuous within an infinite horizon and suggest that land development in general is not an irreversible process. Depending on the time horizon under consideration, the majority of the development processes can normally be reversed. Thus, we propose to model land as another *control* variable along with consumption in an optimal economic growth model. In particular we focus attention on the optimal intertemporal allocation of productive and recreational land[3] and on the existence and properties of the steady state of the economy, following the methodology of optimal growth models and in particular that of Krautkraemer's and Barrett's papers.

The results indicate that for an economy with a *constant* population and without technical progress a unique steady state exists, given by a saddle point. Moreover, the necessary conditions suggest that land is used for production until the value of the marginal product, defined as the marginal utility of consumption times the marginal product, is equal to the marginal cost given by the marginal utility of recreational land. In particular, we show that the specification of the preferences plays a critical role in land preservation. Only for a society with a high degree of ecological consciousness it is guaranteed that a positive amount of land is permanently devoted to recreational uses. This result, based on a more general model by incorporating a productive capital stock and by allowing a wider range of utility functions compared to Barrett, generalizes proposition 1' of his paper (Barrett (1992, p.292))[4]. Moreover, we define *sufficient conditions* based on the properties of the utility function to have some positive level of land preservation at the steady state that do not appear in Barrett's results and, therefore, we show that land preservation is *not* guaranteed at the steady state as Barrett concludes.

We extend the previous analysis by developing a model of *endogenous* growth with human capital and *increasing* population based on a Cobb-Douglas technology and Cobb-Douglas utility function. We establish that the solution of the

mined only the static optimal allocation of land and the amenity benefits of agricultural land exclusively.

[3]The term "recreational land" is understood as a broad aggregate for non productive land which may yield a positive utility for an individual in various ways.

[4]He assumes an additive utility function with a constant elasticity of the marginal social utility of consumption.

model can be characterized by a per capita consumption growing at a constant rate and a *constant* allocation of land, provided that the rate of discount is not too low. We refer to this kind of solution a *sustainable balanced growth path*. Finally, we evaluate the effects of parameter variations on the optimal allocation of land. Based on this analysis we emphasize that an increase in the discount rate, leading to a rise of the marginal costs of capital, has a positive effect on preserving open space. This result is contrasted with our findings with respect to our first model where a comparative static analysis together with a comparative dynamic analysis show that an increase in the discount rate has negative effect on the steady state level of land preservation. The diametric result, however, can be explained by different assumptions with respect to population growth and the 'mechanics' of economic growth in our second model.

The paper is organized as follows. In section two a model of economic growth subject to a land constraint is analyzed where the size of the population is constant. In the subsections of section two we state the model, analyze its stability in the state-costate phase plane, discuss the optimal trajectories for consumption, productive and recreational land, and conduct a comparative static as well as comparative dynamic analysis. Section three considers the case of an exponentially growing population along with the introduction of endogenous growth through the accumulation of human capital. The final section closes out the paper with conclusions and proposals for further research.

2 Optimal economic growth with a constant population

2.1 The model

We will begin with the definition of variables and characterization of the functions of the model. Let the state, $K(t)$, denote the stock of capital. Three control variables are employed, namely land utilized for recreational purposes, $L_R(t)$; land allocated to production, $L_P(t)$, and consumption $C(t)$. To simplify the notation, the argument t of the 'variables' K, L_R, L_P, C and of the other 'variables' to be introduced later, will be suppressed, unless it is necessary for an unambiguous notation. We assume that the production function $F(N, K, L_P)$ is jointly strictly concave, homogeneous of degree one and twice continuously differentiable. The size of the population, N, identical to the labor supply, however, is constant. Hence, the production can be represented by a function of capital stock and the productive land alone and it shows decreasing returns to scale. The production factors are considered essential for production, i.e. $F(0, L_P) = F(K, 0) = 0$, and complementary in the sense that $F_{KL_p} = F_{L_PK} > 0$, where the subscript indicates the partial derivative with respect to the variable. Additionally, it is assumed that the production function satisfies the following other properties: $F_K > 0, \lim_{K\to 0} F_K = +\infty, \lim_{K\to+\infty} F_K = 0, F_{L_p} > 0, \lim_{L_P\to 0} F_{L_p} = +\infty$ and $\lim_{L_P\to+\infty} F_{L_P} = 0$. Finally, the change of the capital stock is given by the state equation and reads as

$$\dot{K} = F(K, L_P) - C - \delta K, \tag{1}$$

where the dot denotes the operator d/dt and δ the rate of depreciation of capital stock.

With respect to the consumer, we assume that the preferences are well defined for the two goods: consumption and recreational land. Furthermore we assume that the corresponding utility function $U(C, L_R)$ is jointly strictly concave and twice continuously differentiable[5]. Consumption is understood to be vital for the survival of each single individual. Consequently, it is stipulated that $U(0, L_R) = U_{L_R}(0, L_R) = 0$ and $\lim_{C\to 0} U_C = +\infty$. The utilization of recreational land as well as its intrinsic value varies considerably among the individuals. Hence, it is not conceived as indispensable for the survival of the individual which suggests that $U(C, 0) \geq 0, U_C(C, 0) \geq 0$ and $\lim_{L_R\to 0} U_{L_R} \leq +\infty$ [6]. Moreover, it is supposed that both goods increase the utility derived from the other good, $U_{CL_R} = U_{L_RC} > 0$ and $\lim_{C\to+\infty} U_C = 0$.

In our model, land is supplied by nature without any costs and can either be utilized in the production process or as a recreational good. In any case, the entire available land, $\bar{L}$, is limited, and we need to impose the following restriction on control variables L_P and L_R:

$$\bar{L} \geq L_R + L_P \tag{2}$$

With ρ denoting the constant rate of time preference we define the objective functional as the present value of the total utility stream

$$\int_0^\infty e^{-\rho t} U(C, L_R) dt \tag{3}$$

[5]The constant size of the population allows us to write the aggregate utility of the society as a function of the total consumption and of the entire recreational land. Yet, it is implicitly assumed that the aggregate utility of the society is given by a representative individual's utility that depends on consumption and recreational land per capita times the number of individuals.

[6]If $U(C, 0) = U_C(C, 0) = 0$ we say that recreational land is an essential good for the individuals.

Given the initial state $K(0) = K_0 > 0$ we are facing the optimal control problem: maximize (3) subject to (1), (2) and the control constraints $C, L_R, L_P \geq 0$. For simplicity, we do not impose $K \geq 0$ as a state constraint but as a terminal condition: $\lim_{t\to\infty} K \geq 0$.

In the following we shall only consider the solutions in the set of control variables defined by the conditions $C > 0, L_R \geq 0, L_P > 0$ and $\bar{L} = L_R + L_P$. Admissible solutions with $C = 0, L_P = 0$ and $\bar{L} > L_R + L_P$ can be excluded from optimality by the assumptions: $\lim_{C\to 0} U_C = +\infty, \lim_{L_P\to 0} F_{L_P} = +\infty$ and $\lim_{L_P\to+\infty} F_{L_P} = 0$. For simplicity in exposition we express L_R by $(\bar{L} - L_P)$ and the condition $\bar{L} \geq L_R + L_P$ is captured by $\bar{L} \geq L_P$. Thus, the maximum conditions can be written as

$$U_C(C, \bar{L} - L_P) = \lambda \tag{4}$$

$$\lambda F_{L_P}(K, L_P) = U_{L_R}(C, \bar{L} - L_P) + w \tag{5}$$

$$\bar{L} - L_P \geq 0, \quad w \geq 0 \quad \text{and} \quad w(\bar{L} - L_P) = 0, \tag{6}$$

where λ is the adjoint function associated with differential equation (1) and w the Lagrange multiplier related to restriction $\bar{L} \geq L_P$. Moreover, the adjoint function λ satisfies the differential equation

$$\dot{\lambda} = \lambda(\rho + \delta - F_K(K, L_P)). \tag{7}$$

From (4) and (5) we conclude that $U_C F_{L_P} = U_{L_R} + w$. Hence, the value of marginal productivity of land, given by the marginal productivity times the marginal utility of consumption, must be equal to its marginal costs which are given by the marginal utility of recreational land plus the shadow price of land, w, or economic rent. If the available land is not completely utilized for production the rent will be zero[7].

Having discussed the necessary conditions, we will now turn to the steady state analysis which is defined by $\dot{K} = \dot{\lambda} = 0$. The differential equations in the state and costate variables read as

$$\dot{K} = 0 = F(K^\infty, L_P^\infty) - C^\infty - \delta K^\infty \tag{8}$$

$$\dot{\lambda} = 0 = \lambda^\infty(\rho + \delta - F_K(K^\infty, L_P^\infty)), \tag{9}$$

[7]The sufficient conditions for the maximization of the Hamiltonian are satisfied since $\lambda > 0$ and the production and utility functions are strictly concave.

where the superscript ∞ denotes the evaluation of the variable at the steady state. The interpretation of these conditions are well known from the theory of economic growth. Therefore, we continue with the analysis of the steady state existence. Let us first assume that $L_p^\infty < \bar{L}$ implying that $w = 0$. Condition (9) allows us to define an implicit function $K(L_P^\infty)$ since it requires that $\rho+\delta = F_K(K^\infty, L_P^\infty)$ for $\lambda^\infty > 0$. Applying the implicit function theorem we obtain that $dK/dL_P = -F_{KL_P}/F_{KK} > 0$. Then, substituting K^∞ in (8) we obtain $C = F(K(L_P^\infty), L_P^\infty) - \delta K(L_P^\infty)$. Finally, using (4) we can write condition (5) as

$$\begin{aligned} &U_C(F(K(L_P^\infty), L_P^\infty) - \delta K(L_P^\infty), \bar{L} - L_P^\infty)\, F_{L_P}(K(L_P^\infty), L_P^\infty) \\ &\quad = U_{L_R}(F(K(L_P^\infty), L_P^\infty) - \delta K(L_P^\infty), \bar{L} - L_P^\infty), \end{aligned} \tag{10}$$

where the left-hand side represents the value of the marginal productivity of land (VMP_{L_P}) and the right-hand side the marginal costs of productive land (MC_{L_P}). Their derivatives results in

$$\begin{aligned} \frac{dVMP_{L_P}}{dL_P} &= U_{CC}F_{L_P}((F_K - \delta)\frac{dK}{dL_P} + F_{L_P}) - U_{CL_R}F_{L_P} \\ &\quad + U_C(F_{L_PL_P} - \frac{F_{KL_P}^2}{F_{KK}}) < 0 \end{aligned} \tag{11}$$

$$\frac{dMC_{L_P}}{dL_P} = U_{L_RC}((F_K - \delta)\frac{dK}{dL_P} + F_{L_P}) - U_{L_RL_R} > 0, \tag{12}$$

since the production function is jointly strictly concave, $F_K - \delta = \rho$ is positive in the steady state, and $U_{CL_R} > 0$.

These derivatives show that VMP_{L_P} results in a monotonically decreasing function on the interval $[0, \bar{L}]$ with $\lim_{L_P \to 0} VMP_{L_P} = +\infty$ and $VMP_{L_P}(\bar{L}) \geq 0$, while MC_{L_P} is a monotonically increasing function on the same interval with $MC_{L_P}(0) = 0$ and $\lim_{L_P \to \bar{L}} MC_{L_P} \leq +\infty$. This analysis indicates that $U_C(C, 0) = 0$ and $\lim_{L_R \to 0} U_{L_R} = +\infty$ are *sufficient conditions* for a solution $0 < L_P^\infty, L_R^\infty < \bar{L}$, whereas $VMP_{L_P}(\bar{L}) < MC_{L_P}(\bar{L})$ is a necessary and sufficient condition for an interior solution. In the case in which $VMP_{L_P}(\bar{L}) \geq MC_{L_P}(\bar{L})$, the steady state is a corner solution in the sense that $L_R^\infty = 0$ and $L_P^\infty = \bar{L}$, and all available land will be used for production. Consequently the shadow price of land, w^∞, will be positive and equal to the difference between VMP_{L_P} and MC_{L_P}. Therefore irrespective of whether $L_P^\infty = \bar{L}$ or $L_P^\infty < \bar{L}$ a unique steady state exists[8].

[8] The terminal condition $\lim_{t \to \infty} K \geq 0$ will also be satisfied in the steady state since K^∞ has to be positive as is implied by (9), since $\lim_{K \to 0} F_K = +\infty$.

2.2 Stability analysis in the state-costate phase plane

We proceed by an analysis of the (K, λ) phase plane, and visualize the maximum conditions (4)-(6) for $L_P < \bar{L}$ solved for $(C, L_P) = (C(K, \lambda), L_P(K, \lambda))$. Applying the implicit function theorem, we obtain a set of four linear equations

$$\begin{bmatrix} 0 & -1 \\ \lambda F_{L_PK} & F_{L_P} \end{bmatrix} + \begin{bmatrix} U_{CC} & -U_{CL_R} \\ -U_{CL_R} & \lambda F_{L_PL_P} + U_{L_RL_R} \end{bmatrix} \begin{bmatrix} \frac{\partial C}{\partial K} & \frac{\partial C}{\partial \lambda} \\ \frac{\partial L_P}{\partial K} & \frac{\partial L_P}{\partial \lambda} \end{bmatrix} = 0, \tag{13}$$

which can easily be solved by using Cramer's rule. The results are

$$\frac{\partial C}{\partial K} = -\frac{1}{\triangle}\lambda F_{L_PK}U_{CL_R} < 0 \tag{14}$$

$$\frac{\partial C}{\partial \lambda} = \frac{1}{\triangle}(\lambda F_{L_PL_P} + U_{L_RL_R} - U_{CL_R}F_{L_P}) < 0 \tag{15}$$

$$\frac{\partial L_P}{\partial K} = -\frac{1}{\triangle}\lambda F_{L_PK}U_{CC} > 0 \tag{16}$$

$$\frac{\partial L_P}{\partial \lambda} = \frac{1}{\triangle}(-U_{CC}F_{L_P} + U_{CL_R}) > 0, \tag{17}$$

where

$$\triangle = \lambda U_{CC}F_{L_PL_P} + U_{CC}U_{L_RL_R} - U_{CL_R}^2 \tag{18}$$

is positive due the strict concavity assumptions with respect to the production and utility functions. The results of (14) and (16) provide plausible economic interpretation; if the stock of capital increases, the marginal productivity of capital decreases. Hence, the marginal productivity will be below the constant marginal cost given by $\rho+\delta$. In order to satisfy condition (9) it is therefore necessary that the productive land increases since F_{KL_P} is positive. However, with an increase in productive land, equivalent with a decrease in recreational land, the marginal utility with respect to consumption will decrease since U_{CL_R} is positive. According to condition (4) the price of the consumption goods λ, which is assumed to be constant when the capital stock is changing, is not equal to the marginal utility of consumption anymore. In order to satisfy (4), U_C has to be raised by a decrease in consumption since U_{CC} is negative. The interpretation of the signs of (15) and (17) is also straightforward from conditions (5) and (8).

Using (14)-(17), the Jacobian matrix of the system (1) and (7), with $C = C(K, \lambda)$ and $L_P = L_P(K, \lambda)$ can be determined by

$$\frac{\partial \dot{K}}{\partial K} = F_K - \delta + F_{L_P}\frac{\partial L_P}{\partial K} - \frac{\partial C}{\partial K} > 0 \tag{19}$$

$$\frac{\partial \dot{K}}{\partial \lambda} = F_{L_P}\frac{\partial L_P}{\partial \lambda} - \frac{\partial C}{\partial \lambda} > 0 \quad (20)$$

$$\frac{\partial \dot{\lambda}}{\partial K} = -\lambda(F_{KK} + F_{KL_P}\frac{\partial L_P}{\partial K})\begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \quad (21)$$

$$\frac{\partial \dot{\lambda}}{\partial \lambda} = -\lambda F_{KL_P}\frac{\partial L_P}{\partial \lambda} < 0. \quad (22)$$

An evaluation at the equilibrium or stationary point ($\dot{K} = \dot{\lambda} = 0$) where $F_K - \delta = \rho > 0$ yields a positive sign for (19). To determine the sign of (21), equation (16) and the jointly strictly concavity of the production and utility functions are employed

$$F_{KK} + F_{KL_P}\frac{\partial L_P}{\partial K} = F_{KK} - \frac{\lambda F^2_{L_PK}U_{CC}}{\lambda U_{CC}F_{L_PL_P} + U_{CC}U_{L_RL_R} - U^2_{CL_R}}$$
$$= \frac{\lambda U_{CC}(F_{KK}F_{L_PL_P} - F^2_{L_PK}) + F_{KK}(U_{CC}U_{L_RL_R} - U^2_{CL_R})}{\lambda U_{CC}F_{L_PL_P} + U_{CC}U_{L_RL_R} - U^2_{CL_R}} < 0. \quad (23)$$

Moreover, the results of (19) - (22) show that

$$\det J = \frac{\partial \dot{K}}{\partial K}\frac{\partial \dot{\lambda}}{\partial \lambda} - \frac{\partial \dot{K}}{\partial \lambda}\frac{\partial \dot{\lambda}}{\partial K} < 0. \quad (24)$$

Hence, the system of differential equations (1) and (7) has a unique stationary point which is a saddle point.

The sign of the slope of the two isoclines ($\dot{K} = 0$ and $\dot{\lambda} = 0$) can be specified by applying the implicit function theorem,

$$\left.\frac{d\lambda}{dK}\right|_{\dot{K}=0} = -\frac{\partial \dot{K}}{\partial K}\Big/\frac{\partial \dot{K}}{\partial \lambda}\begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \quad (25)$$

$$\left.\frac{d\lambda}{dK}\right|_{\dot{\lambda}=0} = -\frac{\partial \dot{\lambda}}{\partial K}\Big/\frac{\partial \dot{\lambda}}{\partial \lambda} > 0. \quad (26)$$

The sign of the first derivative remains undetermined since $F_K - \delta > 0$ only when $\dot{\lambda} > 0$. Then we have that (25) is negative when $\dot{\lambda} \leq 0$ and it has an undetermined sign when $\dot{\lambda} > 0$. Thus, it can be concluded that the isocline $\dot{K} = 0$ is downward sloping in the (K, λ) phase plane when it is above or cuts the isocline $\dot{\lambda} = 0$. In this case, the stable branch representing the optimal solution for an infinite horizon is downward sloping and converges towards the equilibrium[9] (see

[9] As long as there exists an unique stationary point, (24) implies for (25) and $\dot{\lambda} > 0$ that (26) is greater than (25). However, Fig. 1 must be *carefully* interpreted. Since the results obtained are only valid within a neighborhood of the steady state, we have that $d\lambda/dK < 0$.

Fig.1), given the differentiability of isocline (25).
The stability analysis indicates that for a low stock of capital the associated shadow price, identical with the price of the consumption goods, is comparatively high resulting in a low level of consumption. In the case of a high stock of capital, an additional unit of capital is far less valuable. As time tends to infinity, capital, K, and its associated shadow price, λ, monotonically approach their steady state values. Hence, the stable branch represents the optimal solution provided that appropriate sufficiency conditions hold. By Arrow's sufficiency theorem for infinite horizon problems (see Arrow and Kurz (1970, Prop. 8, p. 49)) the concavity of the maximized (derived) Hamiltonian $\mathcal{H}(K, \lambda)$ in the state variable K along with the satisfied transversality condition provide that the stable path is optimal[10]. It has been shown by Hartl (1983, p.289) that the maximized Hamiltonian is concave in K, if the matrix $A = \partial^2 H/\partial(K, C, L_P)^2$ is negative semidefinite, where H denotes the Hamiltonian. The evaluation of the principal minors of the matrix A is straightforward and shows that A is negative definite. Hence, it can be concluded that Arrow's sufficiency theorem applies.

2.3 Optimal consumption and productive land

In the previous section we derived the properties of the optimal solution in the state-costate phase plane. Now, our interest is to discuss the trajectories of the optimal path for consumption and productive land, and to evaluate their changes when the restriction, $L_P \leq \bar{L}$, is operative in the steady state, i.e. $L_P^\infty = \bar{L}$. For this purpose we differentiate $\partial H/\partial C = 0$ and $\partial H/\partial L_P = 0$ with respect to time and obtain the following system of equations linear in $(\dot{C}, \dot{L}_P)$

$$U_{CC}\dot{C} - U_{CL_R}\dot{L}_P - \dot{\lambda} = 0$$
$$-U_{L_RC}\dot{C} + (U_{L_RL_R} + \lambda F_{L_PL_P})\dot{L}_P + \lambda F_{L_PK}\dot{K} + F_{L_P}\dot{\lambda} = 0$$

Thus, $\dot{C}$ and $\dot{L}_P$ are given by

$$\dot{C} = \frac{1}{\Delta}\{-\lambda U_{CL_R}F_{L_PK}\dot{K} + (U_{L_RL_R} + \lambda F_{L_PL_P} - U_{CL_R}F_{L_P})\dot{\lambda}\} \quad (27)$$

$$\dot{L}_P = \frac{1}{\Delta}\{-\lambda U_{CC}F_{L_PK}\dot{K} + (U_{CL_R} - U_{CC}F_{L_P})\dot{\lambda}\} \quad (28)$$

where Δ, $\dot{K}$ and $\dot{\lambda}$ are given by (18), (1) and (7) respectively. Solving the equations $\partial H/\partial C = 0$ and $\partial H/\partial L_P = 0$ for $\lambda = \lambda(K, C)$ and $L_P = L_P(K, C)$

[10] The transversality condition established by Arrow and Kurz takes the form $\lim_{t\to\infty} e^{-\rho t}\lambda \geq 0$ and $\lim_{t\to\infty} e^{-\rho t}\,\lambda K = 0$.

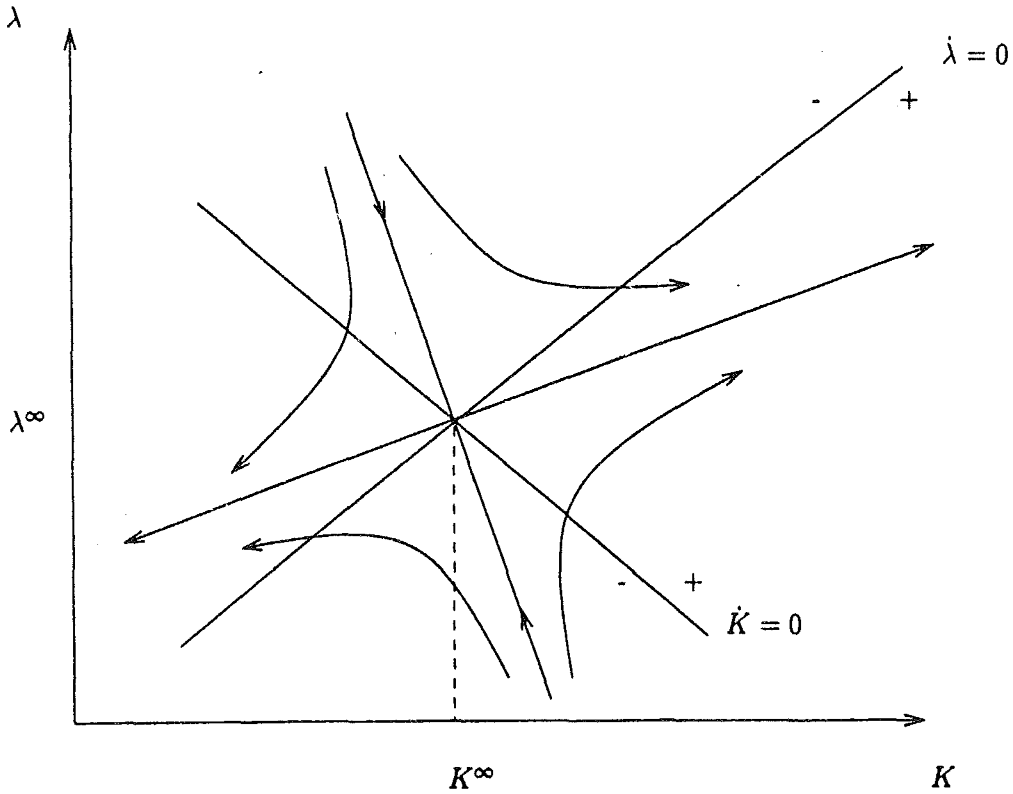

Figure 1: Phase diagram in the $K - \lambda$ plane

allows us to analyze the stability of the system of differential equations given by (1) and (27) in the variables K and C. Likewise, $\partial H/\partial C = 0$ and $\partial H/\partial L_P = 0$ can be solved for $\lambda = \lambda(K, L_P)$ and $C = C(K, L_P)$ to evaluate the stability of the system (1) and (28) in the variables K and L_P. Hartl (1983, Th.2) analyzed a nonlinear economic control problem with two control variables. Based on his work it is possible to conclude that the determinant of the Jacobian matrix of the system of differential equations (1) and (27) in (K, C) and that of (1) and (28) in (K, L_P) have the same value of the Jacobian determinant as system (1) and (7) in (K, λ). Consequently, the steady states of the different systems demonstrate the saddle point property. Thus, it is easy to describe the trajectories of the optimal paths for C and L_P by applying the results of section 2.2 to (27) and (28).

Recall that $-\lambda U_{CL_R} F_{L_P K} < 0$, $U_{L_R L_R} + \lambda F_{L_P L_P} - U_{CL_R} F_{L_P} < 0$, $-\lambda U_{CC} F_{L_P K} > 0$ and $U_{CL_R} - U_{CC} F_{L_P} > 0$ by assumptions with respect to the utility and production functions. Hence, we obtain $\dot{C}/\dot{K} = \partial C/\partial K < 0$, and it follows immediately from (27) that $\dot{C} > 0$ when the trajectories of K and λ pursue the unstable path of Figure 1 to the left of the steady state. When the trajectories follow the unstable path of Figure 1 to the right of the steady state, $\dot{K}$ and $\dot{\lambda}$ will be positive. Since $\dot{C} < 0$, given by (27), we obtain $\dot{C}/\dot{K} = \partial C/\partial K < 0$ for the unstable path in the state-control phase plane. Since the slopes of the stable and unstable paths show opposite signs as a result of the saddle point property, we can conclude that any trajectory of the optimal solution, which is a downward sloping curve in the (K, λ) phase diagram is upward sloping in the (K, C) diagram. Likewise, for (28) we obtain that the stable path is downward sloping in the (K, L_P) diagram. Thus, the optimal solution for the problem can be described by a feedback rule: $C = G(K)$ and $L_P = J(K)$ where $G' > 0$ and $J' < 0$.

2.4 Comparative statics of the steady state

Now, we turn to a sensitivity analysis of the steady state values with respect to a change in the parameters of the model. In order to simplify the calculations we first use system (1) and (7) evaluated at $\dot{K} = \dot{\lambda} = 0$. Additionally (C, L_P) are substituted by $(C(K, \lambda), L_P(K, \lambda))$ which are obtained from the necessary conditions (4) - (6). Thus it is possible to determine the effects on the state and costate variables resulting from a change in a parameter. Then, we use (14) - (17) to calculate the response of the control variables with respect to a variation in the parameters. Applying the implicit function theorem to system (1) and (7) yields a system of four equations

$$\begin{bmatrix} -K^{\infty} & 0 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} \frac{\partial \dot{K}}{\partial K} & \frac{\partial \dot{K}}{\partial \lambda} \\ \frac{1}{\lambda^{\infty}}\frac{\partial \dot{\lambda}}{\partial K} & \frac{1}{\lambda^{\infty}}\frac{\partial \dot{\lambda}}{\partial \lambda} \end{bmatrix} \begin{bmatrix} \frac{\partial K^{\infty}}{\partial \delta} & \frac{\partial K^{\infty}}{\partial \rho} \\ \frac{\partial \lambda^{\infty}}{\partial \delta} & \frac{\partial \lambda^{\infty}}{\partial \rho} \end{bmatrix} = 0.$$

Solved by Cramer's rule we obtain

$$\frac{\partial K^{\infty}}{\partial \delta} = \frac{\lambda^{\infty}}{\det J}\left(\frac{K^{\infty}}{\lambda^{\infty}}\frac{\partial \dot{\lambda}}{\partial \lambda} + \frac{\partial \dot{K}}{\partial \lambda}\right)\left\{\begin{matrix} > \\ = \\ < \end{matrix}\right\} 0 \quad (29)$$

$$\frac{\partial K^{\infty}}{\partial \rho} = \frac{\lambda^{\infty}}{\det J}\frac{\partial \dot{K}}{\partial \lambda} < 0 \quad (30)$$

$$\frac{\partial \lambda^{\infty}}{\partial \delta} = -\frac{\lambda^{\infty}}{\det J}\left(\frac{\partial \dot{K}}{\partial K} + \frac{K^{\infty}}{\lambda^{\infty}}\frac{\partial \dot{\lambda}}{\partial K}\right) > 0 \quad (31)$$

$$\frac{\partial \lambda^{\infty}}{\partial \rho} = -\frac{\lambda^{\infty}}{\det J}\frac{\partial \dot{K}}{\partial K} > 0. \quad (32)$$

where $\partial \dot{K}/\partial K$, $\partial \dot{K}/\partial \lambda$, $\partial \dot{\lambda}/\partial K$ and $\partial \dot{\lambda}/\partial \lambda$ are given by (19)- (22)

Variations in the rate of depreciation, δ, affect the capital stock in an undetermined way, while an increase in this parameter has a positive effect on the price of consumption goods. Given that an increase in the rate of depreciation leads to higher marginal opportunity cost for capital and the marginal productivity of capital is decreasing with respect to the capital stock one would expect a negative sign for (29). However, according to (17) productive land increases with the price of the consumption goods which in turn increases with the rate of depreciation. Hence, the marginal productivity function of capital will move to the northeast because productive land has risen. The initial reduction in the capital stock caused by an increase in the rate of depreciation may thus be over compensated by the shift of the function resulting finally in an increase in the capital stock.

On the other hand, a rise of the rate of time preference, ρ, has a negative effect on the capital stock in the steady state and a positive effect on the price of consumption goods which is equivalent with the shadow price of the capital stock. Evaluating the response of the control variables with respect to the variations in the parameters δ and ρ, we refer to the partial derivatives of the control variables given by (14) - (17). Additionally using (30) and (32) we obtain that an increase in the rate of time preference reduces the level of steady state consumption. The calculations of the sign yield

$$\frac{\partial C^{\infty}}{\partial \rho} = \frac{\partial C}{\partial K}\frac{\partial K^{\infty}}{\partial \rho} + \frac{\partial C}{\partial \lambda}\frac{\partial \lambda^{\infty}}{\partial \rho} = \frac{\lambda}{\det J}\left(-\frac{\partial C}{\partial \lambda}(F_K - \rho) - F_{L_P}\left(\frac{-\lambda F_{L_P}}{\Delta}\right)\right) < 0, \quad (33)$$

since $\partial C/\partial \lambda$, $\det J$ are negative and $F_K - \delta$ is positive in the steady state.

Variations in the rate of depreciation have an ambiguous effect on control variables attributed to their undetermined effect on the capital stock. The effect on productive land resulting from an increase in ρ, however, cannot be determined within the comparative static analysis. Yet, in the next section it is shown that the sign of $\frac{\partial L_P^{\infty}}{\partial \rho}$ can be determined by a comparative dynamic analysis. The results of this analysis indicate that productive land increases and recreational land decreases with an increase in the rate of time preference. Thus, the new steady state is characterized by an increase in productive land substituted for capital and by a lower preservation of land.

2.5 Comparative dynamics of the optimal paths

In the previous section we discussed the sensitivity of the steady state with respect to changes in the parameters. Now, we extend our sensitivity analysis of the parameters along the optimal path of K and λ. Currently there are three approaches to conducting comparative dynamic analysis, see Caputo (1990). The first approach is limited to infinite horizon problems that are autonomous in present-value or current-value terms. Moreover, this type of comparative dynamic analysis only reflects the alteration of the optimal paths of the state, costate and control variables within a limited neighborhood of the steady state. The second approach introduced by Caputo (1990) pertains to how changes in the system parameter affect the entire optimal path of the state, costate and control variables by analyzing their aggregate effect on a value function evaluated at the optimal values for the control, state and costate variables. Its particular strength lies in the fact that it is easily applicable even if there are more than one state variable. However, this approach does not indicate how the paths of the state, costate or control variables change individually. The third approach introduced into the economic literature by Oniki (1973) is based on a system of variational differential equations. Its solution reveals the comparative dynamics over the entire time horizon for each state, costate and control variable. Using the phase diagram in the perturbed variables makes this approach readily applicable for models with just one state variable, and thus, it is applied in this paper.

The necessary conditions (4) and (5) for $L_P < \bar{L}$ allow us to solve for (C, L_P)

given as $(C(K,\lambda), L_P(K,\lambda))$. Upon substitution in (1) and (7), the differential equations and the boundary conditions become

$$\begin{aligned}
\dot{\lambda} &= \lambda(\rho + \delta - F_K(K, L_P(K,\lambda))) && (34)\\
\dot{K} &= F(K, L_P(K,\lambda)) - C(K,\lambda) - \delta K && (35)\\
K(0) &= K_0 && (36)\\
\lim_{t\to\infty} K &= K^\infty, \qquad \lim_{t\to\infty} \lambda = \lambda^\infty. && (37)
\end{aligned}$$

Section 2.1 shows that a unique solution for the optimal control problem exists for given values of ρ and δ. The optimal paths for the state and costate variables are given by $K(t;\beta^0)$ and $\lambda(t;\beta^0)$ where the parameter vector $\beta = (\rho,\delta)$ is specified and denoted by β^0.

At the beginning of the comparative dynamic analysis, the solutions of (34) - (37), the functions $K(t,\beta)$ and $\lambda(t,\beta)$, are inserted back into (34) - (37) and yield the following identities

$$\begin{aligned}
\dot{\lambda}(t;\beta) &\equiv \lambda(t;\beta)(\rho+\delta) - \lambda(t;\beta)F_K(K(t;\beta),\lambda(t;\beta))) && (38)\\
\dot{K}(t;\beta) &\equiv F(K(t;\beta), L_P(K(t;\beta),\lambda(t;\beta))) - C(K(t;\beta),\lambda(t;\beta)) \\
&\quad -\delta K(t;\beta) && (39)\\
K(0;\beta) &\equiv K_0 && (40)\\
\lim_{t\to\infty} K(t;\beta) &\equiv K^\infty(t;\beta) \qquad \lim_{t\to\infty} \lambda(t;\beta) \equiv \lambda^\infty(t;\beta) && (41)
\end{aligned}$$

The variational differential equations are obtained by differentiating (38) - (41) with respect to ρ or δ, evaluated at β^0. Hence, the variational differential equations for perturbed ρ are[11]

$$\begin{aligned}
\dot{\lambda}_\rho &= a_{11}\lambda_\rho + a_{12}K_\rho + \lambda(t;\beta^0) && (42)\\
\dot{K}_\rho &= a_{21}\lambda_\rho + a_{22}K_\rho && (43)\\
K_\rho(0) &= 0 && (44)\\
\lim_{t\to\infty} K_\rho^\infty(\beta^0) &< 0, \qquad \lim_{t\to\infty} \lambda_\rho^\infty(\beta^0) > 0, && (45)
\end{aligned}$$

where

[11]Note that we assume that $K(t;\beta)$ and $\lambda(t;\beta)$ converge uniformly on an interval I, $\beta \in I$. Hence the interchange of the two limit processes, $\lim_{t\to\infty}$ and $\lim_{\beta\to\beta_0} \frac{K(t;\beta)-K(t;\beta_0)}{\beta-\beta_0}$ as well as $\lim_{t\to\infty}$ and $\lim_{\beta\to\beta_0} \frac{\lambda(t;\beta)-\lambda(t;\beta_0)}{\beta-\beta_0}$ in equation (45) is admissible.

$$\begin{aligned}
a_{11}(t;\beta^0) &= \rho+\delta - F_K(t;\beta^0) - \lambda(t;\beta^0)F_{KL_P}(t;\beta^0)\frac{\partial L_P(t;\beta^0)}{\partial\lambda} \left\{ \begin{matrix} > \\ = \\ < \end{matrix} \right\} 0 && (46)\\
a_{12}(t;\beta^0) &= -\lambda(t;\beta^0)\left(F_{KK}(t;\beta^0) + F_{KL_P}(t;\beta^0)\frac{\partial L_P(t;\beta^0)}{\partial K}\right) > 0 && (47)\\
a_{21}(t;\beta^0) &= -\frac{\partial C(t;\beta^0)}{\partial\lambda} + F_{L_P}(t;\beta^0)\frac{\partial L_P(t;\beta^0)}{\partial\lambda} > 0 && (48)\\
a_{22}(t;\beta^0) &= F_K(t;\beta^0) - \delta + F_{L_P}(t;\beta^0)\frac{\partial L_P(t;\beta^0)}{\partial K} - \frac{\partial C(t;\beta^0)}{\partial K} \left\{ \begin{matrix} > \\ = \\ < \end{matrix} \right\} 0 && (49)
\end{aligned}$$

The signs of (46) - (49) were determined by using (14) - (18) and (23).

An analysis of the movement of $K_\rho$and λ_ρ yields that an increase in ρ results in

$$\begin{aligned}
&K_\rho(t;\beta) \le 0 \\
&\lambda_\rho(t;\beta) < 0 \quad \text{for} \quad t\in[0,t_1], \quad t_1 > 0 && (50)\\
&\lambda_\rho(t;\beta) > 0 \quad \text{for} \quad t\in(t_1,\infty].
\end{aligned}$$

To verify the signs of (50) the motions of K_ρ and λ_ρ are evaluated in the (K_ρ, p_ρ) phase plane, as illustrated in Fig. 2.
The analysis starts by assuming particular signs for λ_ρ and K_ρ, which determine, along with the signs of $a_{ij}(t;\beta)$, $i,j = 1,2$, the signs of $\dot{\lambda}_\rho$ and $\dot{K}_\rho$. For example, if $K_\rho = c$, $c > 0$ and $\lambda_\rho = 0$, then by (42) - (43), $\dot{\lambda}_\rho > 0$ and $\dot{K}_\rho \begin{smallmatrix}\le\\=\\>\end{smallmatrix} 0$. Hence, the arrows in the K_ρ, λ_ρ plane emanating from the point $(c,0)$ are directing northeast and northwest. This reflects the fact that λ_ρ is increasing but K_ρ may be increasing or decreasing. Completing this analysis yields the motion of K_ρ and λ_ρ in all four quadrants. From the perturbed initial condition $K_\rho(0) = 0$, the optimal path must begin along the λ_ρ-axis. Intuitively this is quite clear since the initial capital stock is fixed and thus it is independent of the social discount rate. The perturbed boundary conditions suggest that the optimal paths for λ_ρ and K_ρ lie in the second quadrant. Therefore, any time path starting along the positive λ_ρ-axis cannot be optimal because the second quadrant cannot be reached. Consequently the optimal time path begins along the negative λ_ρ-axis, and the boundary condition requires this path to end in the second quadrant. Fig. 2 illustrates three possible paths satisfying the perturbed boundary conditions. Note that the path may intersect itself since the system (42)-(43) is non autonomous. There exist many more possible optimal perturbed trajectories.

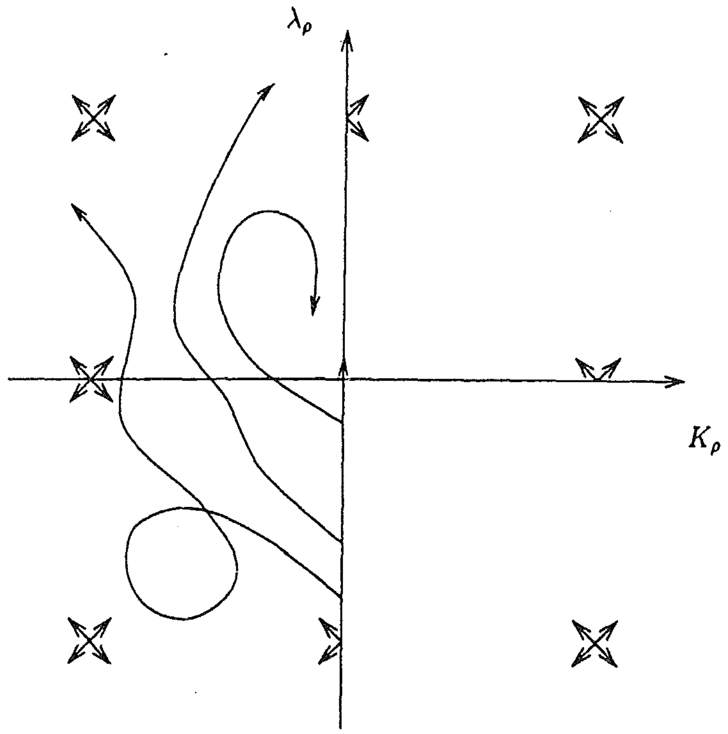

Figure 2: The motion of K_ρ and λ_ρ

However, they are all confined to the second and third quadrant. Fig. 2 indicates that an increase in the social discount rate leads to a lower capital stock over the entire time horizon. It confirms economic intuition that an increase in the social discount rate provides less incentive to accumulate capital. For some initial time period the lower capital stock is accompanied by a decrease in the price of the consumption good λ. However, as t tends to infinity the price of the consumption good will rise as a result of an increase in the social discount rate. An evaluation of an increase in δ yields

$$\begin{aligned} &K_\delta(t;\beta) \lesseqgtr 0 \\ &\lambda_\delta(t;\beta) \lesseqgtr 0 \quad \text{for} \quad t \in [0, t_1], \quad t_1 \geq 0 \\ &\lambda_\delta(t;\beta) \geq 0 \quad \text{for} \quad t \in (t_1, \infty]. \end{aligned} \tag{51}$$

These signs are derived from the evaluation of variational differential equations for perturbed δ given by

$$\dot{\lambda}_\delta = a_{11}\lambda_\delta + a_{21}K_\delta + \lambda(t;\beta^0) \tag{52}$$

$$\dot{K}_\delta = a_{12}\lambda_\delta + a_{22}K_\delta - K(t;\beta) \tag{53}$$

$$K_\rho(0) = 0 \tag{54}$$

$$\lim_{t\to\infty} K_\delta^\infty(\beta^0) \lesseqgtr 0, \qquad \lim_{t\to\infty} \lambda_\delta^\infty(\beta^0) > 0. \tag{55}$$

Now, in the same way as before it is possible to construct a phase diagram in the $(K_\delta, \lambda_\delta)$ plane to determine the motion of λ_δ and K_δ. In Fig. 3 three possible perturbed trajectories are given satisfying the perturbed boundary conditions. The economic interpretation suggests that an increase in the rate of depreciation provides less incentives to invest capital for future consumption. Hence, an increase in the depreciation rate tends to decrease the capital stock. Yet, Fig. 3 indicates that an increase in the capital stock cannot be excluded as a result of an increase in δ, either for some closed time interval of positive length or over the entire time horizon. An increase in δ, however, will never lead to an increase in the capital stock accompanied by a decrease in the price of the consumption good.

Finally an increase in ρ results in

$$C_\rho(t;\beta) \leq 0, \quad \text{and} \quad L_{P_\rho}(t;\beta) \geq 0. \tag{56}$$

These signs follow immediately by recalling the results of section 2.3, in particular the functions $C = G(K)$ and $L_P = J(K)$, and employing the previous results of this section.

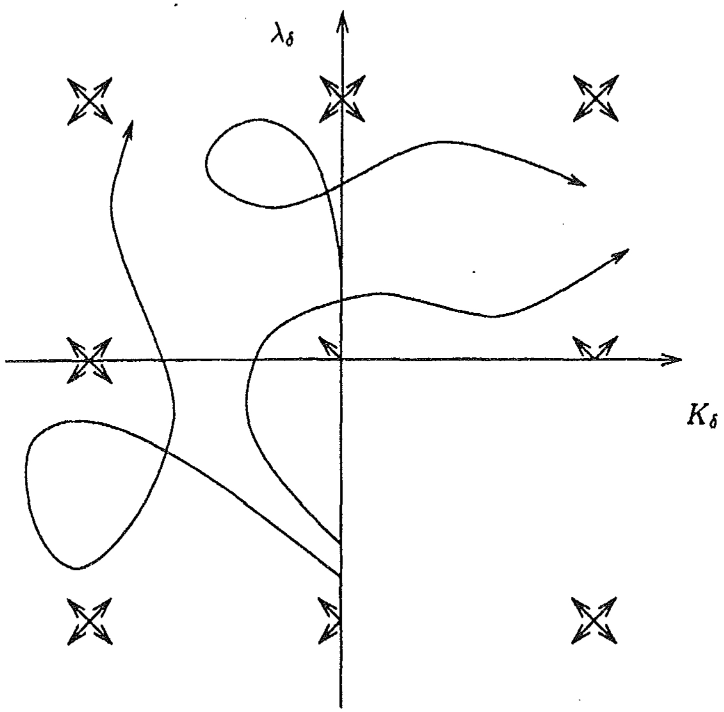

Figure 3: The motion of K_δ and λ_δ

It shows that an increase in ρ leads to a decrease in K as well as in C, contrasted by an increase in productive land. In other words capital is replaced by productive land since the intertemporal opportunity cost for capital has risen. It implies also that the amount of recreational land is lower along the new optimal path. The sign of the changes of C, L_P as a result of an increase in δ are ambiguous, and therefore little economic interpretation can be offered.

3 Optimal economic growth with a growing population

3.1 The model

In this section we extend our analysis to the case of a growing population. Now we assume that the population is growing exponentially at a rate π. For this case it is intuitively clear that a steady state where per capita recreational land and consumption constantly increase does not exist. For this reason we propose a first approach to the problem based on a Cobb-Douglas, technology and utility function.

The preferences of the representative consumer are given by the strictly concave utility function $U(c, l_r) = c^a l_r^b$ where $c = C/N$, $l_r = L_R/N$, $a, b \in (0,1)$ and $a + b < 1$. This specification of the preferences places a high weight on the land utilized for recreational purposes. Notice that for $l_r = 0$ the total and the marginal utility for consumption are zero. Therefore we consider this case as one representing a society with a high degree of ecological consciousness. Given these preferences, the objective function of the optimal control problem can be written as follows, $\int_0^\infty e^{-rt} C^a L_R^b \, dt$, where $r = \rho - \pi(1 - a - b)$ with ρ denoting the social discount rate. Additionally we set $N(0) = 1$.

The technology is represented by a Cobb-Douglas production function with constant returns to scale: $F(K, L_P, N^e) = AK^\alpha L_P^\beta (N^e)^\gamma$where $N^e$is the efficiency units of labor, $\alpha + \beta + \gamma = 1$ and the technology level $A$is assumed to be constant. Each consumer in the economy owns one unit of nonleisure time per period. If he devotes the fraction u of his or her nonleisure time to work and the efficiency per unit of labor supplied is h, then $N^e = uhN$. The remaining $1 - u$ is devoted to accumulating human capital through schooling. With respect to the accumulation of human capital we assume that $\dot{h} = h\epsilon(1 - u)$,where $\epsilon > 0$ is the maximal growth rate of human capital[12].

Given the properties of the utility and production function the optimal values for the control and state variables are strictly positive, if a solution for the problem exists. Moreover, the land restriction is satisfied as a strict equality ($\bar{L} = L_R + L_P$). Thus, the optimal control problem can be simplified and written as

$$\max_{\{C, L_P, u\}} \int_0^\infty e^{-rt} \, C^a(\bar{L} - L_P)^b \, dt \tag{57}$$

$$s.t. \; \dot{K} = AK^\alpha L_P^\beta (uhN)^\gamma - C - \delta K, \quad K(0) = K_0 \tag{58}$$

$$\dot{h} = h\epsilon(1-u), \quad h(0) = h_0 \tag{59}$$

$$\bar{L} \geq L_P, \; C, L_P, \geq 0, \; u \in [0,1] \tag{60}$$

The current-value Hamiltonian H for an interior solution, with costate variables $\lambda_1$and λ_2, is given by

$$\begin{aligned} & H(K, h, \lambda_1, \lambda_2, C, L_P, u, t) \\ = \; & C^a(\bar{L} - L_P)^b + \lambda_1[AK^\alpha L_P^\beta (uhN)^\gamma - C - \delta K] \\ & + \lambda_2[h\epsilon(1-u)], \end{aligned}$$

and the necessary conditions are

$$\frac{a(\bar{L} - L_P)^b}{C^{1-a}} = \lambda_1 \tag{61}$$

$$\lambda_1 \beta AK^\alpha L_P^{\beta-1} (uhN)^\gamma = \frac{bC^a}{(\bar{L} - L_P)^{1-b}} \tag{62}$$

$$\lambda_1 \gamma AK^\alpha L_P^\beta u^{\gamma-1} (hN)^\gamma = \lambda_2 \epsilon h. \tag{63}$$

On the margin, goods must be equally valuable in their two uses: consumption and capital accumulation (61), land must be equally valuable in its two uses:

[12]This model of endogenous growth with human capital is based on Luca's model (1988). Our main departure from Lucas lies in the modeling of the production sector. We consider a linearly homogeneous Cobb-Douglas production function, whereas Lucas considers a Cobb-Douglas production function with external effects in the production of the physical good. The kind of model we present has been used, among others, by Caballé and Santos (1993) to study the stability conditions of the accumulation process and by Gradus and Smulders (1993) and Bovenberg and Smulders (1995) to analyze the relations between environmental quality and economic growth.

recreation and production (62) and time also must be equally valuable in its two uses: production and human capital accumulation (63).

Moreover, we obtain for the costate variables

$$\dot{\lambda}_1 = \lambda_1(r + \delta - \alpha AK^{\alpha-1} L_P^\beta (uhN)^\gamma) \tag{64}$$

$$\dot{\lambda}_2 = r\lambda_2 - \lambda_1 \gamma AK^\alpha L_P^\beta h^{\gamma-1} (uN)^\gamma - \lambda_2 \epsilon(1-u). \tag{65}$$

Thus, equations (58),(59) and (61)-(65), together with the transversality conditions

$$\lim_{t \to \infty} e^{-rt} \lambda_1 K = 0, \qquad \lim_{t \to \infty} e^{-rt} \lambda_2 h = 0, \tag{66}$$

implicitly describe the optimal path of $K$and h for any initial conditions given for these two different types of capital.

Similar to the Lucas model, the easiest way to characterize optimal paths is to look for *sustainable balanced growth paths* of the system that we define as an optimal solution $\{K, h, \lambda_1, \lambda_2, C, L_P, u\}$ to the optimization problem for some initial conditions $K(0) = K_0$and $h(0) = h_0$, such that the rates of growth of $K, h, \lambda_1, \lambda_2, C$ are constant, L_P and u are constant, and the output/capital ratio is also constant. We refer to this kind of path as a sustainable path because in this case, growth is compatible with a positive preservation of land for recreational uses[13]. Let κ denote the rate of growth of consumption $\dot{C}/C$. Then from (61), we have $\dot{\lambda}_1/\lambda_1 = -(1-a)\kappa$, where $1 - a > 0$ is the elasticity of the marginal utility. Next from (64), we obtain

$$\alpha AK^{\alpha-1} L_P^\beta (uhN)^\gamma = r + \delta + (1-a)\kappa. \tag{67}$$

Thus, along the balanced path, the marginal product of capital is equal to its constant opportunity cost defined by $r + \delta + (1-a)\kappa$. For the Cobb-Douglas production function, the marginal product of capital is equal to α times the average product, so that dividing the state equation for physical capital through by K and applying (67) we have

[13]Note that we are assuming an interior solution for the optimization problem, so $L_P < \bar{L}$. Later on we show that this is the optimal solution given the specifications of the utility and production functions. This kind of solution is a straightforward extension of the solution proposed by Lucas which has been extensively used in the literature of optimal growth.

$$\frac{\dot{K}}{K}+\frac{C}{K}=\frac{r+\delta+(1-a)\kappa}{\alpha}-\delta. \tag{68}$$

By definition of a balanced path, $\dot{K}/K$ is constant so (68) implies that C/K is constant. Hence differentiating with respect to time we obtain that

$$\frac{\dot{K}}{K}=\frac{\dot{C}}{C}=\kappa, \tag{69}$$

implying that capital and consumption grow at the same rate κ. To calculate the common growth rate of consumption and capital we differentiate (67) with respect to time, resulting in

$$\kappa=\frac{\gamma}{\gamma+\beta}(v+\pi), \tag{70}$$

where v stands for the human capital rate of growth $\dot{h}/h$. Note that this rate is less than the capital rate of growth in Luca's model without external effects: $v+\pi$. This means that the existence of a fixed factor in the technology reduces the growth potential of the economy but it is not an obstacle for the economy to attain a path of balanced growth. This result also establishes that a balanced growth is compatible with a production function with decreasing returns to scale, given that the constant returns to scale of the utilized Cobb-Douglas technology become decreasing returns when productive land is fix and stays constant along the accumulation process. Finally, we like to point out in respect of this result that the rate of growth is decreasing with respect to the partial elasticity of land productivity, β, so that a low productivity of this factor places the growth rate of capital near to the rate of the human capital accumulation model $v+\pi$.

From (70) we obtain that the rate of growth of per capita consumption and capital is

$$\frac{\dot{k}}{k}=\frac{\dot{c}}{c}=\frac{\gamma v-\pi\beta}{\gamma+\beta}. \tag{71}$$

Therefore, $\beta\pi/\gamma<v$ is a necessary and sufficient condition for an increasing per capita consumption.

Now, to obtain the human capital rate of growth we differentiate (63) with respect to time and substituting for $\dot{\lambda}_1$ and λ_1 we have

$$\frac{\dot{\lambda}_2}{\lambda_2}=(\alpha-(1-a))\kappa-(1-\gamma)v+\gamma\pi. \tag{72}$$

Now using again (63) with (65) we obtain

$$\frac{\dot{\lambda}_2}{\lambda_2}=r-\epsilon. \tag{73}$$

Eliminating $\dot{\lambda}_2/\lambda_2$ between (72) and (73) and substituting for κ, using (70), yields the solution for the human capital growth rate:

$$v=\frac{(\gamma+\beta)(\epsilon-r)+a\gamma\pi}{\gamma(1-a)+\beta}.$$

Substituting for r we have

$$v=\frac{1}{\gamma(1-a)+\beta}[(\gamma+\beta)(\epsilon-\rho)+\pi(\gamma(1-b)+\beta(1-a-b))]. \tag{74}$$

Finally, we need to establish conditions under which the integral, which defines the objective function, converges to a finite value. For a balanced growth path the consumption is growing at a rate equal to κ while recreational land is constant. Hence, the convergence requires that $-r+[a\gamma(v+\pi)/(\gamma+\beta)]<0$ or written as a constraint on v, $v<[(\gamma+\beta)r/a\gamma]-\pi$.

Summarizing our results, the following two conditions on parameter values guarantee the existence of an optimal path with increasing per capita consumption and a constant allocation of land between productive and recreational uses[14]:

$$\frac{\beta}{\gamma}\pi<v<\frac{\gamma+\beta}{a\gamma}r-\pi.$$

These constraints can be written on parameter ϵ, which denotes the productivity of education activities. Substituting for v using (74) and reordering we obtain

$$\rho+\left(\frac{\beta^2-\gamma^2}{\gamma(\gamma+\beta)}+b\right)\pi<\epsilon<\rho\frac{\gamma+\beta}{a\gamma}-\frac{\gamma(1-b)+\beta(1-a-b)}{a\gamma}\pi. \tag{75}$$

Thus, if (75) defines a non-empty interval with positive values we can conclude that there exists a non-empty set for the parameter values of the model which supports a *sustainable balanced growth path* as an optimal solution of the proposed problem. In order to have a positive value for the upper limit of the

[14]Moreover, it is easy to check that the transversality conditions (66) are satisfied provided that the right-hand side of the inequality holds.

interval defined by (75), it is required that

$$\frac{\gamma(1-b)+\beta(1-a-b)}{\gamma+\beta}\pi < \rho. \tag{76}$$

To assure that (75) defines a non-empty interval the difference between the upper and lower limit has to be positive which translate into the imposition of the following inequality

$$\frac{(\gamma+\beta)[\gamma(1-b)+\beta(1-a-b)+ab\gamma]+a(\beta^2-\gamma^2)}{((1-a)\gamma+\beta)(\gamma+\beta)}\pi < \rho. \tag{77}$$

It is easy to check that the left-hand side of (77) is greater than the left-hand side of (76). Hence, whenever inequality (77) is satisfied the inequality (76) is also satisfied. Therefore, constraint (75) is summarized in inequality (77), and the unique constraint on the parameter values of the model, necessary for the definition of a balanced growth path, is the requirement that the rate of discount is not too low. In fact, as the fraction on the left-hand side of (77) is less than one, we can express this constraint by saying that the rate of discount should not fall strongly below the growth rate of the population.

Now, we calculate the optimal value for productive land. Using (61) in (62) we obtain the optimality condition for the allocation of land as the equality between the value of marginal product of land and the marginal cost of productive land given by the marginal utility of recreational land

$$\frac{a(\bar{L}-L_P)^b}{C^{1-a}}\beta AK^\alpha L_P^{\beta-1}(uhN)^\gamma = \frac{bC^a}{(\bar{L}-L_P)^{1-b}}, \tag{78}$$

where $L_R = \bar{L} - L_P$.

From this condition we can conclude that the land is not used entirely for production, because sufficient conditions for a solution $L_P, L_R \in ((0, \bar{L})$, defined in section 2.1, are satisfied. Notice that $U_C(C,0) = 0$ and $\lim_{L_R \to 0} U_{L_R} = +\infty$ where

$$U_C = \frac{a(\bar{L}-L_P)^b}{C^{1-a}}, \quad U_{L_R} = \frac{bC^a}{(\bar{L}-L_P)^{1-b}}.$$

Returning to the calculation of L_P we use (78) and (69). Thus, the differential equation describing the law of motion for physical capital can be written as

$$\kappa K = AK^\alpha L_P^\beta (uhN)^\gamma - \frac{a}{b}(\bar{L}-L_P)\beta AK^\alpha L_P^{\beta-1}(uhN)^\gamma - \delta K. \tag{79}$$

Factoring $AK^\alpha L_P^\beta(uhN)^\gamma$ out on the right-hand side and dividing by K we obtain

$$\kappa = AK^{\alpha-1}L_P^\beta(uhN)^\gamma\left(\frac{a\beta+b}{b} - \frac{a\beta\bar{L}}{bL_P}\right) - \delta. \tag{80}$$

Using (67) to eliminate $AK^{\alpha-1}L_P^\beta(uhN)^\gamma$ and solving for L_P we obtain

$$L_P = \frac{a\beta}{a\beta+b(1-\phi)}\bar{L}, \qquad \text{where } \phi = \frac{\alpha(\delta+\kappa)}{r+\delta+(1-a)\kappa} < 1. \tag{81}$$

The solution for the optimal value of u is straightforward from (59) using (74). Moreover, one can check that its value is positive and less than one if condition (75) holds.

Finally, we would like to point out that equations (70) and (74) describe the asymptotic rates of change for both kinds of capital, but, inherent to the analysis of growth models with two distinct types of capital, there exists a set of balanced growth paths since the levels of the two capitals remains undetermined (see (67)). The multiplicity of sustainable balanced growth paths implies that economies with different initial levels of human and physical capital may grow at a common rate, although with different physical/human capital ratios. However, if the economies only differ by their initial conditions, the model predicts that the economies converge to a state where land is allocated identical, since the optimal level of productive land does not depend on the level of physical and human capital (see (81)). This equation also establishes that differences in the preferences (for instance, in the elasticity of the marginal utility with respect to recreational land, $1-b$), would be sufficient to have different *levels* of land preservation[15]

3.2 Comparative statics analysis

In this last subsection we present a comparative static analysis. We analyze the effects of variations in the parameters on the optimal value of L_P. Due to our approach we have to restrict ourselves to variations which satisfy condition (77).

[15] We would also want to point out that Luca's model without external effects can be obtained from our model as a particular case by setting $\beta = b = 0$.

From (81) we obtain

$$\frac{\partial L_P}{\partial b} = -\frac{a\beta\bar{L}[(1-\phi) - b\frac{\partial\phi}{\partial b}]}{(a\beta + b(1-\phi))^2} < 0, \tag{82}$$

since

$$\frac{\partial\phi}{\partial b} = -\frac{\alpha\pi\left[\frac{\gamma(r+a\delta)}{\gamma(1-a)+\beta} + \delta + \kappa\right]}{(r+\delta+(1-a)\kappa)^2} < 0.$$

The sign of $\partial L_P/\partial a$ is undetermined. Again from (81) we know that

$$\frac{\partial L_P}{\partial q} = \frac{\partial\phi}{\partial q}\frac{ab\beta\bar{L}}{(a\beta + b(1-\phi))^2}, \tag{83}$$

where q is an element of the set $\mathcal{T} = (\rho, \pi, \epsilon, \delta)$. Thus, we can determine the sign of $\partial L_P/\partial q$ by calculating the sign of $\partial\phi/\partial q$, resulting in

$$\frac{\partial\phi}{\partial\rho} = -\frac{\frac{\alpha\gamma(r+a\delta)}{\gamma(1-a)+\beta} + \alpha(\delta+\kappa)}{(r+\delta+(1-a)\kappa)^2} < 0 \tag{84}$$

$$\frac{\partial\phi}{\partial\pi} = \frac{\alpha(1-a-b)\left[\frac{\gamma(r+a\delta)}{\gamma(1-a)+\beta} + \delta + \kappa\right]}{(r+\delta+(1-a)\kappa)^2} > 0 \tag{85}$$

$$\frac{\partial\phi}{\partial\epsilon} = \frac{\alpha\gamma(r+a\delta)}{(\gamma(1-a)+\beta)(r+\delta+(1-a)\kappa)^2} > 0 \tag{86}$$

$$\frac{\partial\phi}{\partial\delta} = \frac{\alpha(r-a\kappa)}{(r+\delta+(1-a)\kappa)^2} > 0. \tag{87}$$

The positive sign of (87) requires that $-r + a\kappa = [-r + a\gamma(v+\pi)]/(\gamma+\beta) < 0$, which is identical to the condition imposed to guarantee the convergence of the objective function to a finite value.

To interpret these results we make use of the necessary conditions of the problem. From (58) and (61)- (65), we have already obtained

$$\frac{a(\bar{L}-L_P)^b}{C^{1-a}}\beta AK^\alpha L_P^{\beta-1}(uhN)^\gamma = \frac{bC^a}{(\bar{L}-L_P)^{1-b}} \tag{88}$$

$$\alpha AK^{\alpha-1}L_P^\beta(uhN)^\gamma = r + \delta + (1-a)\kappa, \tag{89}$$

and using (61) and (63) yields

$$\frac{a(\bar{L}-L_P)^b}{C^{1-a}}\gamma AK^\alpha L_P^{\beta-1}(uh)^{\gamma-1}N^\gamma = \lambda_2\epsilon. \tag{90}$$

These conditions define the standard optimality condition for factor demand (productive land, L_P, capital, K, and unitary efficient labor, uh): the value of marginal productivity of a factor (left-hand side) must be equal to its marginal cost (right-hand side). In this model the value of marginal productivity is given by the marginal utility of consumption, except for the capital. The consumption is given by

$$C = K^\alpha L_P^\beta(uhN)^\gamma - (\delta+\kappa)K. \tag{91}$$

Equations (88)- (90) suggest that an increase in b, i.e., a reduction in the elasticity of the marginal utility with respect to recreational land, increases the value of marginal productivity of land and unitary efficient labor. This has a direct and indirect positive effect on productive land demand given that the productive factors are complementary. However, an increase in b also increases the marginal cost of productive land and capital, resulting in a negative effect on demand for productive land. Our result establishes that the net effect is negative and the optimal value of productive land decreases with a reduction in the elasticity of the marginal utility with respect to recreational land. Thus, we find that the more inelastic the marginal utility of the recreational land is the stronger is the demand for recreational uses.

An increase in the discount rate, ρ, reduces the amount of land allocated to production and therefore has a positive effect on land preservation. An increase in ρ raises the marginal costs of capital and reduces its employment. Consequently, the marginal productivity of land, being complementary to capital, will also decline, which in turn reduces the amount of land allocated to production and increases the amount of land allocated to recreation. This result differs from the findings for our first model with a constant population where an increase in the rate of time preference leads to a higher employment of productive land and to a reduction of recreational land at the steady state. The diametric results can be explained by two reasons. First, in the model of this section efficient labor is continuously increasing due to human capital accumulation. Hence, the marginal productivity of land will finally be higher than in the model of section 1 where the population is constant. Therefore, a reduction in physical capital will have a *higher effect* on the marginal productivity of land when population is increasing as when it is constant[16]. Second, if population is constant, then consumption is decreasing with respect to capital, and therefore the value of the marginal productivity with respect to land increases with the reduction in consumption while

[16]Remember that in both models population is identical with labor supply. The only difference is that labor can have different levels of skill in the second model.

the marginal cost decreases. The net effect according to the results in this section is an increase in the demand for productive land. However, with increasing population the effect of a variation in the discount rate on consumption is ambiguous. From (91) we see that consumption decreases with capital for a given rate of growth, κ, and increases with κ for a given level of capital, since the physical capital accumulation absorbs less resources when the rate of growth is lower[17]. Thus, even though the net effect results in a reduction in consumption with a corresponding increase in the value of the marginal productivity of land, this effect will be lower for the case of a constant population than for the case of an increasing population. Hence, the *net effect* of an increase of the discount rate on productive land is positive when population is increasing.

An increase in the rate of population growth, π, increases the demand for productive land. This positive effect is explained by the variations in consumption caused by the variations in the growth rate of population. As π is positively related with the rate of growth of capital, consumption decreases with an increase in population growth rate (see again (91). Thus, the value of marginal productivity of land increases and its marginal costs decreases leading to an increase in the share of land used for production.

An increase in the productivity of education activities, ϵ, has the same effect on the demand for land as an increase in π. Moreover, these positive effect over compensate the negative effect on the marginal costs of physical and efficient labor resulting from an increase in ϵ. For variations in the rate of depreciation of physical capital, δ,we have the same effects as for variations in the productivity of education activities except that the rate of depreciation does not affect the marginal cost of efficient labor.

4 Conclusions

A neoclassical growth model is analyzed where land is explicitly considered. The model takes account of land as a production factor as well as a consumption good being utilized for recreational purposes. In this way land enters the aggregate production function and the utility function of a representative consumer. This paper shows that for the case where the size of the population is constant, the

[17]Notice that an increase in the discount rate leads to an increase in the marginal cost of capital ($\partial MC_K/\partial\rho > 0$), but reduces its growth rate ($\partial\kappa/\partial\rho < 0$). On the other hand, although the function defined by (91) presents a maximum with respect to K, the solution for the problem is always in the increasing section of the function. This explains why consumption decreases, other things being equal, when K decreases.

model has a unique steady state solution which can be characterized locally by a saddle point. The optimal allocation of the land is defined by the necessary conditions stating that the value of the marginal product of land must be equal to its marginal costs, where the former corresponds to the product of the marginal utility of consumption times the marginal product of land, and the latter to the marginal utility of recreational land. Based on this optimality condition we define sufficient conditions for a positive level of land preservation at the steady state, showing that preferences (ethical views or normative parameters in Barrett's words) play a critical role in the fate of natural environments. In general, we can say that only if recreational land (land in its 'pristine' state) is an essential good for the individuals, or if the marginal cost of productive land is infinite while recreational land is zero, then it is guaranteed that land will not be *completely* developed.

With this paper we aim at a generalization of Olson's and Barrett's models. In particular, we present an optimality condition (10) which coincides with Barrett's optimality condition (3.2) of his model where consumption does not depend on the rate of depletion (Barrett (1992, p.292)). However, our results show that his Proposition 1' is *only* true if the marginal utility of recreational land (natural capital for Barrett) tends to infinity as recreational land goes to zero, i.e., the available land is completely used for production, or if the marginal utility of consumption is zero given that the entire land is developed[18].

Additionally we want to point out that our results neither depend on the elasticity of substitution between the production factors nor on the initial endowment of capital as in Krautkraemer's paper. This difference is explained by the distinct approach followed in our paper. Whereas Krautkraemer incorporates the rate of depletion of a finite stock of a non renewable resource in the production function of the economy, we follow Olson's and Barrett's approach by considering, that developed land is a production factor. According to our model the elasticity of substitution between capital services and land does not play any decisive role in explaining a permanent land preservation. Actually, it is the substitutability between consumption and recreational land, two consumption goods, that determines the possibility of attaining a steady state with some land devoted to environmental uses. Our results show that if the indifference curves are asymptotic to the axes, i.e., land cannot be completely substituted by consumption on an indifference curve, then it is optimal to preserve land permanently as an environmental amenity, independently of the initial endowment of the productive capital stock[19].

[18]Obviously, Barrett is implicitly assuming the former because he supposes that marginal utility of consumption does not depend on recreational land (see Barrett (1992, p. 291)).

[19]Notice that in Krautkraemer's model it is not possible to define a steady state, given that a *constant* rate of depletion is not compatible with a *finite* stock of a non renewable resource for

Departing from Barrett's and Olson's assumption of zero population growth we extend our analysis by considering the case of an ecologically orientated society whose population grows exponentially in the framework of an endogenous growth model with human capital[20]. It shows that it is possible to have economic growth with increasing per capita consumption based on a constant allocation of land between productive and recreational uses. We establish a sufficient condition for this result in terms of a lower bound for the rate of discount. Thus, if the rate of discount is not too low, it is possible to have increasing per capita consumption together with a fixed production factor (productive land) implying decreasing returns to scale. In fact, there exists a set of sustainable balanced growth paths since the levels of the two different types of capital remain undetermined so that economies with different initial levels of physical and human capital may finally grow at a common rate, however with different physical/human capital ratio. Yet, if the economies only differ with respect to the initial conditions the model predicts that they converge to an identical allocation with respect to land. Nonetheless, a difference in the elasticity of the marginal utility (preferences) with respect to recreational land would be sufficient to have different *levels* of land preservation.

A 'comparative static analysis' shows that, given an increase in the discount rate, the level of land preservation is decreasing for the case of a constant population where it is increasing for the case of a growing population. The latter case, showing that a decrease in the discount rate leads to a deterioration of the environment, has already been pointed out by Fisher and Krutilla (1975). The diametric results, however, can be explained by different assumptions with respect to population and the 'mechanics' of economic growth. The difference can basically be explained by the finding that the rate of discount is a determinant for the rate of growth of the economy when the engine of growth is based on the accumulation of human capital.

Finally, we would like to point out that the existence of an optimal trajectory from the initial capital stock to one of the optimal paths defined in section 3, i.e. the transitional dynamics of the model, remains open for further research[21]. An-

an *infinite* horizon. Thus, the possibility of substitution between production factors is critical to prevent the physical exhaustion of the resource. However, if a production factor is given by developed land, then it is possible to find an optimal but *constant* allocation of land for an *infinite* horizon, even if the initial endowment of land is *finite*.

[20]We understand that recreational land is an 'essential' good for individuals of an ecologically orientated society.

[21]See Caballé and Santos (1993) for an analysis of this subject for a model of endogenous growth with human capital but without external effects in the production of the goods and without land.

other possible line of research could be the analysis of the optimal intertemporal allocation of land with agents strategically acting with different preferences on environmental amenities. All this may help to answer relevant questions such as whether or not sustainable growth is compatible with a growing population and a fixed production factor which can also be used for consumption.

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