

SEASONAL UNIT ROOTS IN TRADE VARIABLES

Carol Alexander and Manuel Cantavella-Jordá*

WP-EC 97-13

Corresponding author: Manuel Cantavella-Jordá,
Departamento de Economía, Universitat Jaume I,
Campus Riu Sec, 12080 Castellón, SPAIN,
Phone: 34-964-345780 ext (1919); Fax: 34-964-345849
E-mail: mcantave@eco.uji.es

Editor: Instituto Valenciano de Investigaciones Económicas, S.A.
First Edition October 1997
ISBN: 84-482-1586-9
Depósito Legal: V-3844-1997

IVIE working-papers offer in advance the results of economic research under way in order to encourage a discussion process before sending them to scientific journals for their final publication.

* C. Alexander: University of Sussex; M. Cantavella-Jordá: Universitat Jaume I and Instituto de Economía Internacional.

SEASONAL UNIT ROOTS IN TRADE VARIABLES

Carol Alexander and Manuel Cantavella-Jordá

ABSTRACT

In this paper we examine the presence of seasonal unit roots in trade variables for Germany, France, the United Kingdom, and Italy, using the procedure developed by Hylleberg, Engle, Granger, and Yoo (1990) [HEGY]. Both quarterly and monthly data reject the presence of unit roots at most seasonal frequencies, more frequently in quarterly than in monthly data. This has important implications for econometric modeling of trade balance, exchange rates and income in European Union (EU) countries.

JEL: C22

Key words: seasonal unit root, HEGY procedure, cycle, period, frequency, filter

RESUMEN

En este trabajo se examina la posible presencia de raíces unitarias estacionales en variables de comercio para Alemania, Francia, Reino Unido e Italia utilizando la metodología de Hylleberg, Engle, Granger y Yoo (1990) [HEGY]. Tanto las series mensuales como las trimestrales rechazan la presencia de raíces unitarias en la mayor parte de las frecuencias estacionales, de manera más frecuente en datos trimestrales que en datos mensuales. Esto puede tener implicaciones interesantes en la modelización econométrica de variables de comercio como balanza comercial, tipos de cambio y renta, por lo menos para países de la Unión Europea (EU).

JEL: C22

Palabras clave: raíz unitaria estacional, metodología HEGY, ciclo, periodo, frecuencia, filtro

I. INTRODUCTION

Economic relationships between trade variables have been examined within the context of the 'imperfect substitutes' model.¹ This model is based on a partial equilibrium analysis which combines the elasticities approach of Marshall (1923), Lerner (1944), and Robinson (1947) with the absorption approach of Alexander (1952). Instead of having a structural model of import and export equations in the imperfect substitutes model, a reduced-form approach may be adopted (Bahmani-Oskooee 1985, Rose and Yellen 1989). This reduced form has the trade balance as dependent variable and exchange rates, domestic income, and foreign income as explanatory variables.

Most empirical validation of the imperfect substitutes model employs seasonal data which have been seasonally adjusted prior to the analysis (Rose 1991, Backus et al. 1994). If we work with seasonally adjusted data, one might think that seasonality could be ignored. In principle, those data should have the seasonal pattern removed. However, caution has to be taken when working with seasonally adjusted data. Although a standardised procedure may have been necessary for the OECD original data source, reporting hundreds of series, the procedure might not be best when modeling a single series. Parameter estimates would be biased if a seasonal pattern remains in the data, and inefficient if unnecessary de-seasonalization has been applied.²

When econometric models are built on seasonal data there are certain modelling issues which need to be resolved at the outset. For example, should seasonality be modelled by deterministic or stochastic seasonals, and if they are stochastic what is the appropriate stationarity transform for these data. Are there any common seasonalities in the variables, and if so what are the implications for the econometric specification of multivariate models? We have to answer these questions first, to ensure that economic models are applied to trade variables in a well specified form.

The conventional treatment of seasonality in seasonal unadjusted data employs dummy variables which represent deterministic seasonals. However the use of deterministic seasonal dummy variables is not appropriate if the observed seasonality is generated by a stochastic process (Barsky and Miron, 1989). And if this stochastic process is non-stationary, it is important that

¹ For a broad literature review on international trade models see Goldstein and Khan 1985.

² Moreover, according to Maravall (1996) when using seasonally adjusted series in univariate models which exhibit some seasonality they will become noninvertible. In this case, testing for unit roots on those series may be affected by a misspecification error.

we investigate the presence of unit roots at all frequencies, not just the long run, because in this case it would be necessary to use seasonal rather than annual differencing to attain stationarity. If the annual differencing stationarity filter is inappropriately applied the true dynamic relations between the variables may not be revealed, and consequent forecasting performance will be poor (Frances, 1991). Moreover, according to Beaulieu and Miron (1993), statistical properties of different seasonality models are distinct and the imposition of one type when another is present can result in serious biases or loss of information. Therefore it is important to establish whether seasonal unit roots exist, and at which frequencies, before implementing econometric models of trade variable data.

A non-seasonal time series may be well described by a deterministic process, a stationary stochastic process, an integrated process, or by a simple combination of these. A seasonal time series has much more structure: In order to model a seasonal component it is common practice to use a stationary form (Banerjee et al. 1993). However, when seasonal components are found to be integrated it is suitable to allow the model of the seasonal component to drift significantly over time. This possibility is implied in the modus operandi of seasonal differencing (Box and Jenkins 1970), whereby a process observed s times per year could be made stationary by transforming to its s -period difference $y_t - y_{t-s}$. The application of this stationarity filter assumes that the process contains an integrated seasonal component, with s unit roots given by the s -th roots of unity.³

It is our purpose to analyse whether unit roots at frequencies other than the long run exist in seasonally unadjusted monthly and quarterly trade variable data for some European Union countries. The methodology that we use for the examination of seasonal unit roots is based on that of Hylleberg, Engle, Granger, and Yoo (1990) also known as HEGY.

Section 2 deals with the construction of data variables and their sources. Section 3 explains the methodology for investigating seasonal integration given by Hylleberg et al. (1990) and provides empirical evidence on quarterly data. Section 4 examines similar issues following the method proposed by Beaulieu and Miron (1993) for monthly data. Empirical evidence on stochastic seasonality for monthly data is within the same section. Finally, section 5 comments on the implications of the results presented.

³ That is, the roots of $1 - L^s = 0$. (L denotes the usual lag operator: $L^n y_t = y_{t-n}$).

II. SEASONAL INTEGRATION WITH QUARTERLY DATA

Theory: HEGY test procedure

This test is based on the same idea as the Dickey-Fuller tests (1979 and 1981) for a unit root at the long-run frequency. It is also easy to implement from an OLS regression. The HEGY procedure starts by specifying a general autoregressive model parameterised so as to introduce the seasonal and long run unit roots:

$$A(L)y_t = \varepsilon_t \quad (1)$$

where $\varepsilon_t \sim \text{i.i.d.}(0, \sigma^2)$ and $A(L)$ is a fourth order lag polynomial.

The process is said to be stationary if all the roots of the polynomial $A(L)$ lie outside the unit circle. To test the hypothesis that the roots of $A(L)$ lie on the unit circle against the alternative that they lie outside the unit circle Hylleberg et al. (1990) use a polynomial expansion of $A(L) = 1 - L^4$. They show that the polynomial in the lag operator on the left-hand side may be expanded around the roots $1, -1, i$, and $-i$ (quarterly data).⁴ Hence, the quarterly seasonal unit root process has four roots of modulus unity. Each of these roots correspond to different cycles in the time domain. For quarterly data, Number of Cycle per year = $4/\text{period}$; Period = $2\pi/\text{frequency}$; Frequency = $2\pi\theta$, where $\theta = 0, 1/4, 1/2, 3/4$ of a cycle.⁵

Thus, for $\theta = 0$ corresponds a 0 frequency, a ∞ period and a 0 cycle. This would be the long run cycle. For $\theta = 1/4$ corresponds a $\pi/2$ frequency, a 4 period, and a 1 cycle (annual) per year. For $\theta = 1/2$ corresponds a π frequency, a 2 period, and 2 cycles per year. Finally, for $\theta = 3/4$ corresponds a $3/2\pi$ frequency, a $4/3$ period and 3 cycles per year.⁶ In order to relate

⁴ The HEGY test is based on quarterly data. For quarterly data, the seasonal difference operator $1 - L^4$ can be factorized to show all the possible roots in the generating process as follows:

$1 - L^4 = (1-L)(1+L+L^2+L^3) = (1-L)(1+L)(1+L^2) = (1-L)(1+L)(1-iL)(1+iL)$, so that the seasonal process has four possible roots: $1, -1, i$, and $-i$.

⁵ The frequency is the angle of the root in polar coordinates; A period which is usually measured by the length of a cycle is expressed as $p = 2\pi/f$.

⁶ According to Hylleberg et al. (1990), the effects of the two complex roots are indistinguishable.

these cycles and roots in an intuitive way, let us consider the application of the particular factor (corresponding to the root of interest) of the difference operator to the series y_t , observing this way the number of periods needed for the process to return to its original value. For example, consider the deterministic process $A(L)y_t = 0$, now, if $A(L) = (I+L)$ for root $-I$, then $y_{t+1} = -y_t$ and $y_{t+2} = -(y_{t+1}) = y_t$, Thus the process returns to its original value on a two-period cycle. If $A(L) = (1-iL)$ for root i , then $y_{t+1} = iy_t$; $y_{t+2} = i(y_{t+1}) = -y_t$; $y_{t+3} = i(y_{t+2}) = -iy_t$; and $y_{t+4} = i(y_{t+3}) = y_t$. The process in this case returns to its original value on a cycle of four periods. Zero frequency is given, obviously, when $A(L) = (I-L)$ in which case the unit root is positive ($+I$) and $y_{t+1} = y_t$.

Testing for a unit root at a seasonal frequency has much in common with testing for unit roots at zero frequency (Banerjee et al. 1993). In order to test the null hypothesis that the roots of $A(L)$ lie on the unit circle against the hypothesis that they lie outside Hylleberg et al. (1990) define three positive parameters δ_1 , δ_2 , and δ_3 so that,

$$A(L) = (1 - \delta_1 L)(1 + \delta_2 L)(1 + \delta_3 L^2) \quad (2)$$

For δ_j close to unity, the expansion of the series can be carried out by using a Taylor approximation around the four roots. This polynomial yields terms that correspond to each of the roots plus a remainder (R) as,⁷

$$A(L) = -\pi_1(1 + L + L^2 + L^3)(L) - \pi_2(1 - L + L^2 - L^3)(-L) - (\pi_4 + \pi_3 L)(-L)(1 - L^2) + R \quad (3)$$

The polynomial $A(L)$ will have a root at I , $-I$, i , and $-i$ if the corresponding terms in π_i are zero. The factorization of (3) yields,

$$(1 - L^4) y_t = \pi_1 y_{1t-1} + \pi_2 y_{2t-1} + \pi_3 y_{3t-2} + \pi_4 y_{3t-1} - \varepsilon_t \quad (4)$$

where the polynomials in the lag operator,

$$y_{1t} = (1 + L + L^2 + L^3) y_t$$

$$y_{2t} = -(1 - L + L^2 - L^3) y_t$$

$$y_{3t} = -(1 - L^2) y_t$$

⁷ See Hylleberg et al. 1990 for the approximation theorem.

represent different filters. Thus, filter y_{1t} will adjust any series for all roots except for that corresponding to the factor $1-L$; filter y_{2t} will adjust any series for all roots except for that corresponding to the factor $1+L$; and filter y_{3t} will adjust any series for all roots except for those corresponding to the two complex roots in the factor $1+L^2$.

Equation 4 is amenable to a test where the maintained model includes an intercept, a time trend, and some seasonal dummies. According to Perron (1988), the inclusion of seasonal dummies is due to the fact that the critical values might be influenced by starting values of the dependent variable in their simulations. Equation 4 can be estimated by OLS, with additional lags of the dependent variable $\Delta_4 y_t = (1-L^4)y_t$, to avoid serial correlation in the errors. In order to test the null hypothesis that there is a unit root at zero frequency one tests $\pi_1 = 0$. To test the null for a unit root of -1 (half-yearly frequency) one tests $\pi_2 = 0$. For the complex roots of $\pm i$ (annual frequency) one applies a joint test of $\pi_3 = 0$ and $\pi_4 = 0$. The alternative hypotheses are that there will be no seasonal unit roots if π_2 and either π_3 or π_4 are different from zero and no long run unit root if π_1 is also different from zero. Rejection of all the aforementioned null hypotheses indicates stationarity of the process.

The critical values for these tests can be obtained from those of Dickey and Fuller (1979) for π_1 and π_2 , and from Dickey, Hasza, and Fuller (1984) for π_3 if π_4 is assumed to be zero. These values are tabulated by Hylleberg et al. (1990) through Monte Carlo simulations.

Empirical evidence

The Dickey-Fuller regression for quarterly data is of the form,

$$(1-L^4)y_t = \alpha + \sum_{k=1}^3 m_k M_{kt} + \sum_{i=1}^p \phi_i \Delta_4 y_{t-i} + \pi_1 y_{1t-1} + \pi_2 y_{2t-1} + \pi_3 y_{3t-2} + \pi_4 y_{3t-1} + \varepsilon_t \quad (5)$$

Given the absence of deterministic trend in our data (tested but results not shown),⁸ only intercept (α) and seasonals (M) are included as deterministic part of equation 5. Sufficient lagged dependent variables are added in order to whiten the residuals at the cost of the minimum number

⁸ We used the D-H tests of Durbin and Hausman (Choi 1992) - results available from authors on request.

of parameters.

Table 1 reports the results of seasonal unit root test for quarterly data. The first column indicates the roots whose corresponding frequencies are shown in *Note* below *Critical values*. The first row specifies the variables for each country which are fully displayed underneath the table, in *Note*. The quarterly data variables under consideration are: trade balance, real effective exchange rate, GDP, all these three variables for Germany, France, the UK, and Italy, plus OECD GDP. The numbers in the cells express the *ADF* statistics on the coefficients.

Table 1. Tests for seasonal unit roots (HEGY), quarterly data

	GETB 1 lag	GEXR 1 lag	GEY 1 lag	FRTB 1 lag	FRXR 1 lag	FRY 1 lag	UKTB 1 lag	UKXR 1 lag	UKY 1 lag	ITTB 1 lag	ITXR 1 lag	ITY 1 lag	Y 1 lag
Π_1	1.76*	0.006*	0.17*	0.60*	0.90*	-0.27*	0.68*	-0.20*	0.05*	0.82*	0.19*	-0.28*	-0.12
Π_2	-4.23	-5.80	-2.94*	-4.89	-6.35	-5.87	-4.02	-4.28	-4.65	-5.09	-5.40	-5.8	-5.21
Π_3	-6.95	-6.74	-3.24	-6.13	-4.88	-5.48	-5.38	-4.46	-7.17	-7.10	-3.68	-3.71	-5.10
Π_4	-4.05	-4.98	-3.75	-4.07	-5.06	-5.36	-3.65	-6.94	-2.6	-3.60	-3.92	-8.91	-6.01

Critical values

Π_1 Π_2 Π_3 Π_4

5% (Intercept, Seasonal Dummies, No trend)

-2.95 -2.94 -3.44 -1.96

1%

-3.55 -3.60 -4.06 -2.78

Note: Π (upper case π) denotes the root at each frequency. Thus, Π_1 corresponds to frequency 0 (long run trend and no cycles); Π_2 to frequency $\pi/2$ (annual cycle); Π_3 to frequency π (two cycles per year); Π_4 to frequency $3/2 \pi$ (three cycles per year).

- GETB Stands for Germany trade balance, GEXR for Germany real effective exchange rates, GEY for Germany income (GDP), analogously for the rest of the countries (France, The United Kingdom, and Italy); whereas Y stands for OECD GDP.
- An asterisk indicates the existence of a unit root at the corresponding frequency.
- All variables are in logs.
- Estimation has been implemented in TSP 4.2a

In the first place it is worth noting that the null hypothesis for every series at zero frequency (root Π_1) cannot be rejected. This indicates that there exists a unit root in the long run for every variable. However, for seasonal unit roots it seems that there is little evidence. Only seasonal unit roots at frequency $\pi/2$ for Germany GDP (GEY) cannot be rejected, but at 1 per cent significance level. The most striking aspect in quarterly data, then, is the fact that practically in all cases no seasonal pattern emerges at any frequency at 5 per cent significance level. For all countries trade balances, real effective exchange rates, and GDPs, unit roots at the 5 per cent level and at all the seasonal frequencies are rejected.⁹

⁹ In Beaulieu and Miron (1993) and Alexander and Barrow (1994), quarterly data on different variables also reject the presence of unit roots at most seasonal frequencies. The same results apply in Beaulieu and Miron

III. SEASONAL INTEGRATION WITH MONTHLY DATA

Theory

The same procedure can be applied for monthly data. Frances (1991) and Beaulieu and Miron (1993) adapt the HEGY methodology for monthly data. Considering the same autoregressive model (1) but *with* $A(L) = 1-L^{12}$, Beaulieu and Miron (1993) show that the seasonal difference operator $1-L^{12}$ is factorized as,

$$(1-L^{12}) = (1-L)(1+L)(1+L^2)(1+L+L^2)(1-L+L^2)(1+\sqrt{3}L+L^2)(1-\sqrt{3}L+L^2)$$

and so the unit roots are,

$$\pm 1; \pm i; -1/2(1 \pm i\sqrt{3}); 1/2(1 \pm i\sqrt{3}); -1/2(\sqrt{3} \pm i); 1/2(\sqrt{3} \pm i)$$

Hence, the monthly seasonal unit root process has twelve roots of modulus unity. All these roots correspond to ∞ , 6, 3, 9, 8, 4, 2, 10, 7, 5, 1, and 11 cycles per year, respectively (see Table 3 in Appendix for relation of these roots to respective cycles and frequencies).

The aim is to know whether the polynomial in the lag operator $A(L)$ has roots equal to one at the zero or seasonal frequencies. In other words, one has to test hypotheses about a particular unit root without taking a stance on whether other seasonal or zero frequency unit roots are present. Again, the process y_t will be stationary if all the roots of the polynomial in the lag operator $A(L)$ lie outside the unit circle. The hypothesis testing is supported through a polynomial expansion of the series that uses a Taylor approximation, now, around the twelve roots. Thus, the new model is (see Appendix for corresponding filters),

$$(1-L^{12})y_t = \sum_{k=1}^{12} \pi_k y_{k,t-1} + \varepsilon_t \tag{6}$$

(1993) with monthly periodicity.

Just as in the case of quarterly data, the maintained model could include an intercept, time trend and seasonal dummies. In order to test hypotheses about various unit roots, one has to estimate equation 6 by OLS : For frequencies 0 (filtered series y_{1t}) and π (filtered series y_{2t}), one simply has to test the corresponding t -statistic for $\pi_k = 0$ ($k=1,2$ resp.) against the alternative that $\pi_k < 0$ (stationarity). For the rest of the frequencies,

$$(\pm\pi / 6, \pm\pi / 3, \pm\pi / 2, 2\pi / 3, \pm5\pi / 6)$$

the corresponding roots come in complex conjugate pairs and therefore one has to implement joint tests one for each pair. The critical values are tabulated in Beaulieu and Miron (1993).

Empirical evidence

Equation 6 is modified to allow for a deterministic component to enter the regression. This deterministic part includes an intercept, seasonal dummies, and a trend. Moreover, the introduction of lagged dependent variables is to avoid serial correlation in the error term of equation 7. In general, one lag of the dependent variable (y_{13t}) has been sufficient in each regression to whiten the errors, so $p=1$ in (7).

We run Dickey-Fuller regressions of the form,

$$(1 - L^{12})y_t = \alpha + \sum_{k=1}^{11} m_k M_{kt} + \delta t + \sum_{i=1}^p \phi_i \Delta_{12} y_{t-i} + \sum_{k=1}^{12} \pi_k y_{k,t-1} + \varepsilon_t \quad (7)$$

where the deterministic part includes, an intercept (α), seasonal dummies (M), and a trend (t). The outcomes of the formal test for monthly data are reported in Table 2. The first column indicates the roots whose corresponding frequencies are shown below the table. The first row expresses the variables for each country. The variables are: trade balance, real effective exchange rate and industrial production, for Germany, France, the UK, and Italy, plus industrial production for the OECD. The numbers in the cells express the *ADF* statistics on the coefficients. All series are in *log* levels, and the sample period is from 1975 (January) to 1993 (December). Unlike Table 1, the critical values for monthly data also depend on whether the auxiliary regression includes a time trend or not. The trend is only included in those equations whose variables have been proved to

have a deterministic trend. We found that trade balances (GETB, FRTB, UKTB, ITTB) and Italy industrial production (ITY) in monthly data are trend stationary processes according to the results of the Durbin-Hausman test (Choi 1992). The rest of the equations do not include a trend. The critical area for π_k (k even and larger than 2) is two-sided because these coefficients can be positive or negative under the alternative.¹⁰ Except for Π_1 and Π_2 the rest of the statistics have to be evaluated in pairs in order to reject or not the null hypothesis.

Table 2. Tests for seasonal unit roots (HEGY), monthly data

	GETB 1 lag	GEXR 1 lag	GEY 1 lag	FRTB 1 lag	FRXR 1 lag	FRY 3 lags	UKTB 2 lags	UKXR 1 lag	UKY 1 lag	ITTB 1 lag	ITXR 1 lag	ITY 1 lag	Y 1 lag
Π_1	-0.35*	0.18*	-0.17*	0.84*	0.26*	-1.21*	1.59*	0.07*	-0.43*	0.99*	0.69*	-0.48*	0.23*
Π_2	-2.53*	-5.08	-4.56	-5.64	-4.92	-5.46	-4.57	-4.95	-3.83	-2.09*	-1.88*	-2.25*	-4.23
Π_3	-2.18	-5.94	-1.52	-1.75	-4.56	-1.77	-2.03	-5.51	0.28	-2.17	-6.64	-1.9	-0.55
Π_4	-7.75	-2.85	-5.39	-5.16	-2.95	-4.75	-4.83	-2.78	-4.41	-5.21	1.12	-2.85	-6.78
Π_5	-1.82	-5.91	-3.40	-0.92	-5.63	-0.99	-1.65	-5.38	-2.43	-0.45	-6.19	-2.66	-3.91
Π_6	3.84	0.60	4.73	3.79	1.90	2.87	4.68	-0.66	7.11	3.21	1.18	2.82	4.76
Π_7	0.35	-3.31	0.05	2.57	-6.02	-0.39	-0.08	-3.55	-0.43	0.46	-3.91	-0.23	-0.48
Π_8	-3.36	-4.93	-4.24	-6.20	-4.79	-3.19	-5.13	-3.22	-3.91	-4.18	-3.18	-4.96	-5.34
Π_9	-5.43	-7.08	-5.41	-2.81	-5.54	-3.63*	-5.92	-7.05	-4.64	-4.67	-5.49	-3.66*	-5.81
Π_{10}	3.77	1.15	1.51	4.60	-1.04	1.55*	2.63	1.59	1.77	2.52	0.81	0.34*	1.74
Π_{11}	6.12	-0.31	1.49	2.58	-1.50	0.04	0.32	-3.31	1.31	4.28	-1.25	0.60	-0.08
Π_{12}	-9.84	-6.95	-4.51	-4.43	-6.41	-5.77	-6.87	-4.88	-9.51	-7.86	-3.64	-8.47	-7.63

Critical values:		Π_1	Π_2	odd	even
5%	1) I,SD,NT ^a :	-2.76	-2.76	-3.25	-1.86
1%		-3.32	-3.28	-3.83	-2.60
		Π_1	Π_2	odd	even
5%	2) I,SD,T ^b :	-3.28	-2.75	-3.24	-1.86
1%		-3.83	-3.31	-3.79	-2.57

Note:

^aAuxiliary regression with intercept, seasonal dummies (11), and no trend.(applied to all variables except trade balances and Italy income).

^bAuxiliary regression with intercept, seasonal dummies (11) and trend (applied to all trade balances and Italy income).

- Π (upper case π) denotes the root at each frequency. Thus, Π_1 corresponds to frequency 0 (long run trend and no cycles);

Π_2 to frequency π (six cycles per year); Π_3 and Π_4 to frequency $\pm \pi/2$ (three cycles per year); Π_5 and Π_6 to frequency

$\pm 2\pi/3$ (four cycles per year); Π_7 and Π_8 to frequency $\pm \pi/3$ (two cycles per year); Π_9 and Π_{10} to frequency

$\pm 5\pi/6$ (five cycles per year); Π_{11} and Π_{12} to frequency $\pm \pi/6$ (annual cycle).

- All variables are in logs.

-GETB Stands for Germany trade balance, GEXR for Germany real effective exchange rates, GEY for Germany income (industrial production), analogously for the rest of the countries (France, The United Kingdom, and Italy); finally, Y stands for foreign income (OECD GDP).

- An asterisk indicates the existence of a unit root at the corresponding frequency.

- Estimation has been implemented in TSP 4.2a.

¹⁰ In the case that the null $\pi_k = 0$ is not rejected, then $\pi_{k-1} = 0$ is tested against $\pi_{k-1} < 0$ (see Beaulieu and Miron 1993).

As expected from the results on quarterly data unit roots at zero frequency exist in all time series. For trade balances, we reject unit roots at the 5 per cent level at all the seasonal frequencies (see critical values for auxiliary regression with trend) except for Germany (GTTB) and Italy (ITTB) at frequency π (root Π_2).¹¹ For real effective exchange rates, we reject unit roots at every seasonal frequency except for Italy (ITXR) at frequency π (see now critical values of the auxiliary regression without trend). Industrial production for Italy is integrated of order I at frequency π (5 per cent significance level) and $\pm 5\pi/6$ (1 per cent significance level) and for France at frequency $\pm 5\pi/6$ (1 per cent significant level). Finally, for the OECD industrial production, seasonal unit roots are rejected at 5 per cent and at every frequency (except for the long run).

Summarising, there is weak evidence of seasonal unit roots on monthly series. In the German trade balance, and French industrial production series there is weak evidence of seasonal integration. The UK does not present seasonal unit roots at all. Italy is the only country that exhibits seasonal unit roots in all its three variables and at the same frequency, π .

IV. CONCLUSIONS

In this paper we present an introductory analysis which can serve as motivation for understanding seasonality and our ability to capture it in an econometric model of trade balances. Results derived from these kind of tests should be taken cautiously since they depend on multiple factors such as, previous tests on the integration order of the lagged endogenous variable, the presence of outliers, the inappropriate inclusion or omission of determinist components, asymptotic validation, so on and so forth.

Even though our empirical research on the presence of seasonal unit roots at different frequencies show that they are more evident on monthly than quarterly data, we cannot say that there is strong seasonal integration in trade variables of Germany, France, the UK and Italy.

¹¹ Recall that ADF statistics for roots Π_k where $k > 2$ have to be examined by pairs, that is, both statistics have to agree about rejecting the null hypothesis of seasonal unit root against the alternative of no seasonal unit root at the corresponding frequency. The test is two-sided.

The demonstration presented in this paper indicates that unit roots are often absent at some or all of the seasonal frequencies. Therefore we have learned that empirical research should be implemented for their presence rather than imposing them a priori. The annual differencing filter assumes the existence of unit roots at all seasonal frequencies, and since we have found this not to be the case, any model which employs this stationarity filter will be misspecified. In fact very few seasonal unit roots were indicated by our tests, so we have shown that the seasonality for most of the series does not apparently change over time. Therefore, the use of dummy variables for these stationary seasonal components such as in the *first differences seasonal dummies* model of Frances (1991) may be appropriate.

APPENDIX

Monthly filters

The corresponding monthly filters are (Beaulieu and Miron 1993)

$$y_{1t} = (1 + L + L^2 + L^3 + L^4 + L^5 + L^6 + L^7 + L^8 + L^9 + L^{10} + L^{11})y_t,$$

$$y_{2t} = -(1 - L + L^2 - L^3 + L^4 - L^5 + L^6 - L^7 + L^8 - L^9 + L^{10} - L^{11})y_t,$$

$$y_{3t} = -(L - L^3 + L^5 - L^7 + L^9 - L^{11})y_t,$$

$$y_{4t} = -(1 - L^2 + L^4 - L^6 + L^8 - L^{10})y_t,$$

$$y_{5t} = -1/2(1 + L - 2L^2 + L^3 + L^4 - 2L^5 + L^6 + L^7 - 2L^8 + L^9 + L^{10} - 2L^{11})y_t,$$

$$y_{6t} = \sqrt{3}/2(1 - L + L^3 - L^4 + L^6 - L^7 + L^9 - L^{10})y_t,$$

$$y_{7t} = 1/2(1 - L - 2L^2 - L^3 + L^4 + 2L^5 + L^6 - L^7 - 2L^8 - L^9 + L^{10} + 2L^{11})y_t,$$

$$y_{8t} = -\sqrt{3}/2(1 + L - L^3 - L^4 + L^6 + L^7 - L^9 - L^{10})y_t,$$

$$y_{9t} = -1/2(\sqrt{3} - L + L^3 - \sqrt{3}L^4 + 2L^5 - \sqrt{3}L^6 + L^7 - L^9 + \sqrt{3}L^{10} - 2L^{11})y_t,$$

$$y_{10t} = 1/2(1 - \sqrt{3}L + 2L^2 - \sqrt{3}L^3 + L^4 - L^6 + \sqrt{3}L^7 - 2L^8 + \sqrt{3}L^9 - L^{10})y_t,$$

$$y_{11t} = 1/2(\sqrt{3} + L - L^3 - \sqrt{3}L^4 - 2L^5 - \sqrt{3}L^6 - L^7 + L^9 + \sqrt{3}L^{10} + 2L^{11})y_t,$$

$$y_{11t} = -1/2(1 + \sqrt{3}L + 2L^2 + \sqrt{3}L^3 + L^4 - L^6 - \sqrt{3}L^7 - 2L^8 - \sqrt{3}L^9 - L^{10})y_t,$$

The data are filtered in twelve different ways to eliminate each of the seasonal roots in turn. For instance the filter,

$$y_{1t} = (1 + L + L^2 + L^3 + L^4 + L^5 + L^6 + L^7 + L^8 + L^9 + L^{10} + L^{11})y_t$$

removes all roots except those at frequency zero. One would proceed analogously with the rest of the filters.

Data definitions and sources






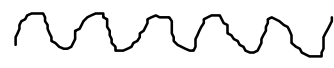

The empirical work considers both monthly and quarterly data and deals with four European Union countries: Germany, France, the United Kingdom, and Italy. Monthly data span the first month of 1975 through the last month of 1993 and quarterly data cover the first quarter of 1975 up to the last quarter of 1993.

The trade balance is defined on an aggregate basis taking into account just visible goods traded around the world. The construction of this variable has been carried out through a ratio of exports over imports. It is measured in 1985 U.S. dollars. The real effective exchange rate is the one used by the International Monetary Fund. This is a weighted index that combines the exchange rates between a currency in particular and the currencies of seventeen other industrial countries (partner and/or competitor countries). It is adjusted for relative movements in national price and expressed on 1985 year base. Industrial production (1985 index) is used as a proxy for domestic income in monthly data whereas gross domestic product (GDP), in 1985 domestic currency units, performs the same function but for quarterly data. Industrial production for the OECD countries (1985 index) is employed as a proxy for foreign income in monthly data. OECD GDP (1985 U.S dollars) represents the same variable for quarterly data. All variables are real and

transformed into natural logarithms. Unfortunately, the OECD industrial production index and the OECD GDP are seasonally adjusted.

Trade balance and real effective exchange rate data are drawn from *International Financial Statistics* 1993 monthly data series CD-ROM published by the IMF. Quarterly data for the same variables are obtained by dealing with monthly data. For instance, trade balance variables are realisable by simply adding their three consecutive monthly observations; real effective exchange rate variables are attainable by calculating the average index of their three consecutive monthly data. Industrial production in monthly data and GDP in quarterly data are collected from *OECD Main Economic Indicators* and *OECD Quarterly National Accounts* respectively, both from various issues.

Table 3. Possible unit roots in monthly data

Frequency	Period	Cycles per year	Filter	Filtered Series
0	∞	0	$(1-L^{12})/(1-L)$	 y_{1t} (Long-run Trend)
$\pm \pi/6$	12	1	$(1-L^{12})/(1-\sqrt{3}L+L^2)$	 $\{y_{11t}, y_{12t}\}$ (Annual Cycle)
$\pm \pi/3$	6	2	$(1-L^{12})/(1-L+L^2)$	 $\{y_{7t}, y_{8t}\}$ (Biannual Cycle)
$\pm \pi/2$	4	3	$(1-L^{12})/(1+L^2)$	 $\{y_{3t}, y_{4t}\}$
$\pm 2\pi/3$	3	4	$(1-L^{12})/(1+L+L^2)$	 $\{y_{5t}, y_{6t}\}$
$\pm 5\pi/6$	12/5	5	$(1-L^{12})/(1+\sqrt{3}L+L^2)$	 $\{y_{9t}, y_{10t}\}$
π	2	6	$(1-L^{12})/(1+L)$	 $\{y_{2t}\}$

REFERENCES

- Alexander, C. O. and Barrow, M. (1994): Seasonality and cointegration of regional house prices in the UK, *Urban Studies*, 31, 1667-1689.
- Alexander, S. S. (1952): Effects of a devaluation on a trade balance, *IMF Staff Papers*, 2, 263-278.
- Backus, D. K., Kehoe, P. J. and Kydland, F. E. (1994): Dynamics of the trade balance and the terms of trade: The J-curve, *American Economic Review*, 84, 84-103.
- Bahmani-Oskooee, M. (1985): Devaluation and the J-curve: Some evidence from LDC., *Review of Economics and Statistics*, 67, 500-504.
- Banerjee, A., Dolado, J., Galbraith, J. W. and Hendry D. F. (1993): *Co-Integration, error correction, and the econometric analysis of non-stationary data*. Oxford University Press.
- Barsky, R. B. and Miron, J. A. (1989): The seasonal cycle and the business cycle, *Journal of Political Economy*, 97, 503-534.
- Beaulieu, J. J. and Miron, J. A. (1993): Seasonal unit roots in aggregate US data, *Journal of Econometrics*, 55, 305-328.
- Box, G. E.P. and Jenkins, G. M. (1970): *Time series analysis forecasting and control*. Holden-day, San Francisco.
- Choi, I. (1992): Durbin-Hausman test for a unit root, *Review of Economics and Statistics*, 54, 289-304.
- Dickey, D. A. and Fuller, W. A. (1981): Likelihood ratio statistics for autoregressive time series with a unit root, *Econometrica*, 50, 1057-1072.
- Dickey, D. A. and Fuller, W. A. (1979): Distribution of the estimators for autorregressive time series with a unit root, *Journal of American Statistical Association*, 74, 427-431.
- Dickey, D. A., Hasza D. P. and Fuller, W. A. (1984): Testing for a unit root in seasonal time series, *Journal of the American Statistical Association*, 79, 355-367.
- Engle, R. F., Granger, C. W. J., Hylleberg, S. and Lee H. S. (1993): Seasonal cointegration: The Japanese consumption function, *Journal of Econometrics*, 55, 275-298.
- Frances P. H. (1991): *Model selection and seasonality in time series*, Tinbergen Institute Research Series.
- Goldstein, M. and Khan M. S. (1985): Income and price effects in foreign trade, in *Handbook of International Economics* (Ed.) Jones R. W. and Kenen P. B., North Holland, Amsterdam.
- Hylleberg, S., Engle, R., Granger, C. W. J. and Yoo, B. S. (1990): Seasonal integration and cointegration, *Journal of Econometrics*, 44, 215-238.
- Lerner, A. P. (1944): *The Economics of control*, Macmillan.
- Maravall, A. (1996): *Unobserved components in economic time series*, Banco de España, Documento de trabajo nº 9609.
- Marshall, A. (1923): *Money, credit, and commerce*, London, Macmillan.

- Perron, P. (1988): Trends and random walks in macroeconomic time series, *Journal of Economic Dynamics and Control*, 12, 333-346.
- Robinson, J. (1947): The Foreign Exchanges, in *The theory of employment* (Ed.) Robinson, J. Essays, Blackwell.
- Rose, A. K. (1991): Does the Marshall-Lerner condition hold ?, *Journal of International Economics*, 30, 301-316.
- Rose, A. K. and Yellen J. L. (1989): Is there a J-curve?, *Journal of Monetary Economics*, 24, 53-68.