

**STRATEGIC BEHAVIOR AND EFFICIENCY
IN A GROUNDWATER PUMPING DIFFERENTIAL GAME***

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ABSTRACT

In this paper socially optimal and private exploitation of a common property aquifer are compared. Open-loop and feedback equilibria in nonlinear strategies have been computed to characterize the private solution. The use of these two equilibrium concepts allows us to distinguish between cost and strategic externalities. The open-loop solution captures only the cost externality, whereas the feedback solution captures both externalities. The results show that strategic behavior increases the overexploitation of the aquifer compared to the open-loop solution. However, if the groundwater storage capacity is large, the difference between the socially optimal and private exploitation, characterized by a feedback equilibrium, is negligible and can be ignored for practical purposes.

Key Words: Groundwater exploitation, Common property resources, Strategic externality, Differential games, Feedback solution, Nonlinear strategies.

RESUMEN

En este trabajo se comparan la explotación privada y socialmente óptima de un acuífero de propiedad común. Para caracterizar la solución privada se han calculado los equilibrios 'open-loop' y 'feedback' en estrategias no lineales. El uso de estos dos conceptos de equilibrio nos ha permitido distinguir entre efectos externos estratégicos y de coste. La solución 'open-loop' captura solamente el efecto externo de los costes mientras que la solución 'feedback' captura ambos efectos externos. Los resultados muestran que el comportamiento estratégico aumenta la sobreexplotación del acuífero comparado con la solución 'open-loop'. Sin embargo, si la capacidad de almacenamiento del acuífero es grande, la diferencia entre la explotación privada y la socialmente óptima, caracterizada por un equilibrio 'feedback', es despreciable y puede ignorarse para propósitos prácticos.

Palabras Clave: Explotación de aguas subterráneas, Recursos de propiedad común, Efecto externo estratégico, Juegos diferenciales, Solución 'feedback', Estrategias no lineales.

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1 Introduction

Groundwater has always been regarded as a common property resource where entry is restricted by land ownership and private exploitation is inefficient. Traditionally, two sources of inefficiency have been pointed out: the first one is a pumping *cost externality* and the second one a *strategic externality*. The cost externality appears because the pumping cost increases with pumping lift, so that withdrawal by one farmer lowers the water table and increases the pumping costs for all farmers operating over the aquifer. The strategic externality arises from the competition among the farmers for appropriating groundwater through pumping since property rights over the resource are not well defined.

In 1980, Gisser and Sánchez presented a first estimation of this inefficiency, comparing the socially optimal exploitation with private (competitive) exploitation, using data from the Pecos River Basin, New Mexico. In that paper the private exploitation of the aquifer is characterized assuming that farmers are *myopic* and choose their rate of extraction to maximize their current profits, whereas the optimal exploitation is obtained through the maximization of the present value of the stream of aggregate profits. For a model with linear water demand, average extraction cost independent of the rate of extraction and linearly decreasing with respect to the water table level, they found that if the storage capacity of the aquifer is relatively large, the difference between the two systems is so small that it can be ignored for all practical purposes. This result has been called the Gisser-Sánchez rule by

Nieswiadomy (1985).¹

Since the publication of this paper, a series of empirical works have been published, comparing optimal exploitation with competition: see Feinerman and Knapp (1983), Nieswiadomy (1985), Worthington, Burt and Brustkern (1985), Kim et al. (1989) and Knapp and Olson (1995). The main conclusion we can reach from this literature is that when it is assumed that average extraction cost decreases linearly with respect to the water table level as in the Gisser and Sánchez model, percentage differences in present value are small although nominal differences can be important. However, it seems that regulation of groundwater exploitation is unlikely to be beneficial even when uncertainty about surface water supply is taken into account, as happens in Knapp and Olson's paper.²

Nevertheless, at the beginning of the eighties the hypothesis of myopic behavior had already been replaced by the hypothesis of rationality in the analysis of private exploitation of common property resources by authors such as Levhari and Mirman (1980), for the analysis of a restricted access fishery, and Eswaran and Lewis (1984), for a common property nonrenewable

¹Two more papers were published by Gisser at the beginning of the eighties on the comparison between the optimal and private exploitation of groundwater, Gisser (1983) and Allen and Gisser (1984). In this last paper it is shown that the Gisser-Sánchez rule also works for the case of an isoelastic demand function.

²The optimal exploitation of groundwater under uncertainty conditions has been recently addressed by Tsur and Graham-Tomasi (1991), Provencher and Burt (1993), Tsur and Zemel (1995) and Rubio and Castro (1996).

resource.³ This approach was finally adopted by Negri (1989) for the analysis of the common property aquifer. In Negri's groundwater pumping differential game, open-loop and feedback equilibria are compared and it is shown that the open-loop solution captures only the pumping cost externality whereas the feedback solution captures both externalities, the pumping cost externality and the strategic externality, and exacerbates the inefficient exploitation of the aquifer compared to the open-loop solution. This paper has two weak points: first, the existence and uniqueness of the feedback solution are assumed and, second, the comparison between the different solutions, including the optimal solution, is made in terms of the steady state groundwater reserves because the equilibrium pumping paths cannot be explicitly derived in his general formulation of the game.

In Provencher and Burt (1993) optimal and feedback equilibria, computed using discrete-time dynamic programming, are compared. The authors explore dynamic inefficiencies via Kuhn-Tucker conditions. They conclude that concavity of the value function is a sufficient condition for strategic behavior to increase the inefficiency of private groundwater exploitation, and that the steady state groundwater reserves attained when firms use decision rules strategies are bounded from below by the steady state arising when firms are myopic and from above by the steady state arising from optimal exploitation.

In this paper we adapt the model defined by Gisser and Sánchez to study

³Hartwick (1980), Berck and Perloff (1984) and Van der Ploeg (1987) are other examples in the fishery literature and McMillan and Sinn (1984) and Reinganum and Stokey (1985) in the nonrenewable resource literature.

the effects of strategic behavior on the efficiency of private groundwater exploitation. In particular, we investigate whether the Gisser-Sánchez rule still holds when it is assumed that firms are rational and the effects of strategic behavior are taken into account. To do this we follow Negri's approach and evaluate the impact of the strategic externality as the difference between the open-loop and feedback solutions of a groundwater pumping differential game.

It has been usual in the differential game literature to resort to *linear* strategies to obtain feedback equilibria (see, for instance, Levhari and Mirman (1980), Eswaran and Lewis (1984), Reynolds (1987) and Fershtman and Kamien (1987)). However, since the publication of Tsutsui and Mino's (1990) paper calculation of *nonlinear* strategies has become more frequent.⁴ Tsutsui and Mino examine, for a differential game of duopolistic competition with sticky prices, whether it is possible to construct a more efficient feedback equilibrium using nonlinear strategies. They conclude that it is not possible to construct a feedback equilibrium which supports the cooperative or collusive price, in other words, it is not possible to get a result equivalent to the Folk theorem in repeated games.⁵ Nevertheless, they find that there exist feedback equilibria which approach the cooperative solution more than

⁴See, in the framework of environmental economics, Dockner and Long (1993), Wirl (1994) and Wirl and Dockner (1995), where nonlinear strategies are used to evaluate the benefits of international cooperation in pollution control.

⁵To be precise, they show that, as the discount rate approaches zero, there exists a steady state feedback equilibrium that asymptotically approaches the steady state cooperative or collusive price.

the open-loop equilibrium.

In the context of environmental economics literature Dockner and Long (1993) have obtained results identical to the ones obtained by Tsutsui and Mino for a symmetric differential game of international pollution control with two countries, and Wirl (1994) and Wirl and Dockner (1995) have shown that cooperation between an energy cartel and a consumers' government is not necessary to reach the efficient long-run concentration of CO₂ in the atmosphere.

These precedents have led us to compute the feedback equilibria of our groundwater pumping differential game resorting to nonlinear strategies, with the aim of examining whether strategic behavior plays *against* the efficiency of the solution, as has been established by Negri and Provencher and Burt, or *for* the efficiency, as seems to happen in Tsutsui and Mino, Dockner and Long and Wirl's papers.

Our results show that the difference between the socially optimal and private exploitation of groundwater, this last characterized by a feedback equilibrium, decreases with the storage capacity of the aquifer so that if this is large enough the two equilibria are identical for all practical purposes. This conclusion confirms the applicability of the Gisser-Sánchez rule. Moreover, we find that strategic behavior plays against the efficiency of private exploitation, supporting Negri's results. However, the applicability of the Gisser and Sánchez rule reduces the practical scope of this result. In other words, strategic behavior exacerbates the overexploitation of the aquifer but

if the storage capacity of the aquifer is relatively large the impact of the strategic externality is negligible.⁶ These results establish that the potential benefits coming from the regulation of the resource will be relatively small.

In the next section we present our formulation of the differential game and we derive the open-loop Nash equilibrium and the stationary Markov feedback equilibrium in the subsequent two sections, respectively. In Section 5 we characterize the stationary Markov feedback equilibrium and compare it with the open-loop Nash equilibrium and the optimal solution, and in Section 6 we use Gisser and Sánchez (1980) and Nieswiadomy (1985) data to compute the different equilibria and thus illustrate quantitatively our results. Some concluding remarks close the paper.

2 The model

In this paper we adapt the model developed by Gisser and Sanchez (1980) to the study of strategic behavior effects on groundwater pumping.

We assume that demand for irrigation water is a negatively sloped linear

⁶The different results concerning the effects of strategic behavior on the efficiency of private solution can be explained by the different *nature* of the existing strategic interdependence in each game. For duopolistic firms there exists a potential gain associated with cooperation, whereas in a groundwater pumping differential game firms compete for the appropriation of a *finite* common property resource. Nevertheless, if the resource and the number of firms are large the competition is feeble and the strategic externality practically disappears.

function

$$W = g + kP, \quad k < 0 \quad (1)$$

where W is pumping and P is the price of water. We also assume that farmers sell their production in competitive markets so that the price of water is equal to the value of water marginal product, and moreover that the agricultural production function is constant returns to scale and that factors other than water and land are optimized conditional on the rate of water extraction.

Access to the aquifer is restricted by land ownership and consequently the number of farmers is fixed and finite over time. In the model all farmers are identical. This symmetry assumption allows us to resolve the game analytically and thus to obtain some initial results on the effects of strategic behavior on private groundwater pumping. Moreover, it also makes feasible the study of the effects of changes in property structure on private solution efficiency. By symmetry we can write the aggregate rate of extraction as $W = Nw_i$, where N is the number of farmers and w_i the rate of extraction of the representative farmer. Then, the individual demand functions are

$$w_i = \frac{1}{N}(g + kP), \quad i = 1, \dots, N \quad (2)$$

and the revenues of the i th farmer

$$\int p(w_i)dw_i = \frac{N}{2k}w_i^2 - \frac{g}{k}w_i. \quad (3)$$

The total cost of extraction depends on the quantity of water extracted and the depth of the water table

$$C(H, W) = (c_0 + c_1H)W, \quad c_1 < 0, \quad (4)$$

where H is the water table elevation above sea level, c_0 is the maximum average cost of extraction and $H^m = -c_0/c_1$ represents the maximum water table elevation that we associate with the natural hydrologic equilibrium of the aquifer. Then, as the marginal and average costs do not depend on the rate of extraction, the individual farmer's extraction costs are

$$C_i(H, w_i) = \frac{1}{N}(c_0 + c_1 H)W = (c_0 + c_1 H)w_i. \quad (5)$$

Costs vary directly with the pumping rate and inversely with the level of the water table. Marginal and average costs increase with the pumping lift and are independent of the extraction rate. We are implicitly assuming that changes in the water level are transmitted instantaneously to all users. This assumption clearly exaggerates the degree of common property. Moreover, the symmetry assumption requires that the groundwater basin has parallel sides with a flat bottom.

The differential equation which describes the dynamics of the water table is obtained as the difference between natural recharge and net extractions

$$AS \dot{H} = R + (\gamma - 1)W, \quad 0 < \gamma < 1 \quad (6)$$

where R is natural recharge, γ is return flow coefficient, and AS is area of the aquifer times storativity. We assume that the rate of recharge is constant and deterministic and, although artificial recharge of the aquifer is feasible in this specification, we focus on the case where the resource is being depleted.⁷

⁷See Knapp and Olson (1995) for a groundwater management model with stochastic surface flows and artificial recharge.

Finally, we assume that the interactions among the agents are completely noncooperative and rational, then the i th farmer faces the following dynamic optimization problem:

$$\begin{aligned} \max_{\{w_i\}} \int_0^\infty e^{-rt} \left[\frac{N}{2k} w_i^2 - \frac{g}{k} w_i - (c_0 + c_1 H) w_i \right] dt \quad (7) \\ \text{s.t. } \dot{H} = \frac{1}{AS} \left[R + (\gamma - 1) \sum_{i=1}^N w_i \right], \quad H(0) = H_0 > 0 \end{aligned}$$

where r is the discount rate. We implicitly assume the nonnegativity constraint on the control variable and we do not impose $H \geq 0$ as a state constraint but as a terminal condition: $\lim_{t \rightarrow \infty} H(t) \geq 0$ for simplicity.⁸

3 Open-loop Nash equilibrium

In the open-loop Nash equilibrium, farmers commit themselves at the moment of starting to an entire temporal path of water extraction that maximizes the present value of their stream of profits given the extraction path of rival farmers.⁹ Then for every given path $w_j(t)$ of farmer j , $j = 1, \dots, N - 1$, farmer i faces the problem of maximizing (7) given $w_j(t)$. A similar problem faces the other players j . An equilibrium of the game are N open-loop strategies that solve the N optimization problems simultaneously. Forming the current value Hamiltonian in the standard way, the necessary conditions

⁸To simplify the notation, the t argument of the variables has been suppressed. It will be used only if it is necessary for an unambiguous notation.

⁹For a formal definition of strategy space and equilibrium concepts used in this paper see Fershtman and Kamien (1987) and Tsutsui and Mino (1990). By extension they can easily be adapted to our game.

for an interior open-loop equilibrium are

$$\frac{N}{k}w_i - \frac{g}{k} - (c_0 + c_1H) + \lambda \frac{\gamma - 1}{AS} = 0, \quad i = 1, \dots, N \quad (8)$$

$$\dot{\lambda}_i = r\lambda_i + c_1w_i, \quad i = 1, \dots, N, \quad (9)$$

the transversality conditions being:

$$\lim_{t \rightarrow \infty} e^{-rt}\lambda_i(t) \geq 0, \quad \lim_{t \rightarrow \infty} e^{-rt}\lambda_i(t)H(t) = 0, \quad i = 1, \dots, N. \quad (10)$$

Assuming the marginal extraction cost of the last unit of water, c_0 , is higher than the maximum value of marginal product, $-g/k$, ($c_0 \geq -g/k$) eliminates the possibility of a corner solution in which $H \leq 0$.¹⁰ On the other hand, assuming symmetric farmers simplifies the solution. With symmetry, $w_i = w_j = w$ and $\lambda_i = \lambda_j = \lambda$ and therefore the $2N$ equations defined by (8) and (9) reduce to 2.

Differentiating (8) with respect to t and substituting $\dot{\lambda}$ and λ in (9) yields

$$\dot{w} = \frac{kc_1R}{ASN} - \frac{kr}{N} \left(\frac{g}{k} + c_0 \right) + rw + \frac{(\gamma - 1)kc_1(N - 1)}{ASN} w - \frac{kr c_1}{N} H. \quad (11)$$

¹⁰See Rubio, Martínez and Castro (1994) for a complete characterization of all possible long-term equilibria (steady states), including the physical exhaustion of water reserves in a finite or infinite time. In fact, the condition for an interior solution given above can be relaxed, as it is shown in Rubio, Martínez and Castro (1994), Prop. 2), since it must be also taken into account to define it the steady state user cost. However, the analysis of this issue in the framework of the differential game presented in Section 2 is outside of the scope of this paper.

Taking into account that at the steady state $\dot{H} = \dot{w} = 0$, we can use equations (6) and (11) to find the stationary equilibrium, given by

$$H_{OL}^* = -\frac{R}{kc_1(\gamma - 1)} + \frac{R}{rASN} - \frac{1}{c_1} \left(\frac{g}{k} + c_0 \right) \quad (12)$$

and

$$w^* = -\frac{R}{(\gamma - 1)N}. \quad (13)$$

All this can be summarized as:

Proposition 1 *There exists a unique stationary open-loop Nash equilibrium for the game. The water table at this equilibrium is given by (12) and the rate of extraction by (13).*

Observe that in this game as the dynamics of water table H does not depend on H , the stationary equilibrium extraction rate is independent of the equilibrium concept used to resolve the game. On the other hand, equation (8) implies that at every moment each player follows the policy $Nw/k - g/k = c_0 + c_1H - \lambda(\gamma - 1)/AS$. This rule is the well-known price equal to marginal cost, but in this case marginal cost presents two components: the marginal extraction cost and the user cost, $-\lambda(\gamma - 1)/AS$.¹¹ At the steady-state, λ is equal to the capitalized value of the increase in cost resulting from a one-unit reduction in the water table for an extraction rate equal to its stationary value so that the user cost at the steady-state is equal to $-c_1R/rASN$.

¹¹The necessary conditions for optimal groundwater pumping have been established in the literature a long time ago. See Negri (1989) for an interpretation of λ .

To evaluate the efficiency of this equilibrium we need the *socially optimal or efficient* equilibrium. That equilibrium can easily be obtained as a particular case of the open-loop Nash equilibrium making N equal to one.

$$H_{SO}^* = -\frac{R}{kc_1(\gamma - 1)} + \frac{R}{rAS} - \frac{1}{c_1} \left(\frac{g}{k} + c_0 \right). \quad (14)$$

Now we can compare the stationary values of the water table for the two equilibria

$$\Delta H^* = H_{SO}^* - H_{OL}^* = \frac{R}{ASr} \left(1 - \frac{1}{N} \right) > 0. \quad (15)$$

This difference represents the impact of the *pumping cost externality* on the stationary value of the water table. If we now make a comparative statics analysis of this difference we get

$$\begin{aligned} \frac{\partial(\Delta H^*)}{\partial r} &= -\frac{R}{ASr^2} \left(1 - \frac{1}{N} \right) < 0 \\ \frac{\partial(\Delta H^*)}{\partial N} &= \frac{R}{ASrN^2} > 0. \end{aligned}$$

These results allow us to present the following proposition:

Proposition 2 *The socially optimal stationary equilibrium water table is higher than the stationary open-loop Nash equilibrium water table and the difference declines as the discount rate increases or the number of farmers decreases.*

The effect a discount rate variation has on the difference between the two stationary values is explained by the different impact that a variation of the discount rate has on the user cost in each case. As $\lambda_{OL}^* N = \lambda_{SO}^*$ we find that $|\partial\lambda_{OL}^*/\partial r| < |\partial\lambda_{SO}^*/\partial r|$. Thus an increase in the discount rate decreases the

user cost in both cases but by a larger amount in the optimal solution, so that, although the two stationary values decrease because of the reduction in the user cost, the decrease in the optimal value is higher than that in the inefficient value, resulting in a decrease of the difference between the two steady-states. On the other hand, the effect a variation in the number of farmers has on the difference is clear, if one notices that the socially optimal equilibrium is independent of the number of farmers. Thus an increase in the number of farmers reduces the user cost of the resource and, consequently, the stationary open-loop Nash equilibrium water table, causing an increase in the difference between the two equilibria.

Finally, we want to comment on how these results affect Gisser and Sánchez's conclusions. The first thing that we can point out is that our results confirm their rule, so that we predict, like them, that if the storage capacity of the aquifer is relatively large, the two equilibria would be very close; in fact, identical for all practical purposes.¹² The second remark is that Gisser and Sanchez's estimations are *overvalued* because these authors assume that the farmers are *myopic*.¹³ However, if one assumes, as we do, that the farmers are rational, their private evaluation of the user cost will be positive and price will consequently exceed marginal extraction costs (see

¹²In Section 6 we use the data from Gisser and Sánchez (1980) to illustrate this result for an open-loop Nash equilibrium.

¹³In fact, as they are assuming a constant marginal extraction cost, they are implicitly using an equilibrium concept equivalent to the *open access* long-run equilibrium for common property resources ($P = MC = AC$).

(8)). With rational farmers the myopic solution applies only asymptotically, that is, when the number of farmers approaches infinity,

$$\lim_{N \rightarrow \infty} H_{OL}^* = H_{MY}^* = -\frac{R}{kc_1(\gamma-1)} - \frac{1}{c_1} \left(\frac{g}{k} + c_0 \right),$$

which implies that

$$\lim_{N \rightarrow \infty} (H_{SO}^* - H_{OL}^*) = (H_{SO}^* - H_{MY}^*) = \frac{R}{ASr},$$

where the right-hand side is equal to the difference found by Gisser and Sánchez (see Gisser and Sanchez (1980, p. 641)) in their model, which is higher than the difference (15).

4 Stationary Markov feedback equilibrium

In an economic environment in which binding commitments are not feasible because of undefined property rights and where all players can have access to current information on water table elevation, strategies that depend only on time cannot be credible. As is well known this requirement of credibility is fulfilled by a stationary Markov feedback equilibrium which is derived by the dynamic programming approach. In Markov feedback equilibria farmers adopt decision rules that depend on the water table, taking as given the decision rules of their rivals.

In this section we demonstrate, following Tsutsui and Mino (1990) and Dockner and Long (1993), that our groundwater pumping game admits non-linear Markov feedback equilibria. As noted above, a stationary Markov

feedback equilibrium must satisfy the dynamic programming equation

$$rV_i(H) = \max_{\{w_i\}} \left\{ \left[\frac{N}{2k} w_i^2 - \frac{g}{k} w_i - (c_0 + c_1 H) w_i \right] + V_i'(H) \left[\frac{1}{AS} \left(R + (\gamma - 1) \sum_{i=1}^N w_i \right) \right] \right\} \quad (16)$$

where $i = 1, \dots, N$. Using the maximization condition $V_i'(H) = (AS/(\gamma-1))((g/k) + c_0 + c_1 H - (Nw_i/k))$ and the symmetry assumption we have

$$rV(H) = \left[\frac{N}{2k} w^2 - \left(\frac{g}{k} + c_0 + c_1 H \right) w \right] + \left(\frac{R}{\gamma-1} + Nw \right) \left(\frac{g}{k} + c_0 + c_1 H - \frac{N}{k} w \right). \quad (17)$$

Assuming a zero discount rate, the Bellman equation (17) becomes the quadratic equation in w

$$0 = -N \left(\frac{2N-1}{2k} \right) w^2 + \left[(N-1) \left(\frac{g}{k} + c_0 + c_1 H \right) - \frac{NR}{k(\gamma-1)} \right] w + \frac{R}{\gamma-1} \left(\frac{g}{k} + c_0 + c_1 H \right) \quad (18)$$

with two solutions

$$w_{1,2} = \frac{k}{N(2N-1)} \left\{ (N-1) \left(\frac{g}{k} + c_0 + c_1 H \right) - \frac{NR}{k(\gamma-1)} \pm \left[\left((N-1) \left(\frac{g}{k} + c_0 + c_1 H \right) - \frac{NR}{k(\gamma-1)} \right)^2 + \frac{2N(2N-1)R}{k(\gamma-1)} \left(\frac{g}{k} + c_0 + c_1 H \right) \right]^{\frac{1}{2}} \right\}. \quad (19)$$

Based on this result for $r = 0$ we propose a nonlinear strategy for the case with $r > 0$ given by

$$w(H) = \frac{1}{N(2N-1)} \left[k(N-1) \left(\frac{g}{k} + c_0 + c_1 H \right) - \frac{NR}{(\gamma-1)} \right] + f(H) \quad (20)$$

where $f(H)$ is a nonlinear function in H . Working with (20) and the Bellman equation we obtain a set of stationary Markov strategies implicitly defined by the equation¹⁴

$$K = \left\{ w - \frac{1}{N(2N-1)} \left[k(N-1) \left(\frac{g}{k} + c_0 + c_1 H \right) - \frac{NR}{(\gamma-1)} \right] - \left(H + \frac{G}{F} \right) y^a \right\}^{\xi_1}$$

$$\left\{ w - \frac{1}{N(2N-1)} \left[k(N-1) \left(\frac{g}{k} + c_0 + c_1 H \right) - \frac{NR}{(\gamma-1)} \right] - \left(H + \frac{G}{F} \right) y^b \right\}^{\xi_2} \quad (21)$$

where K is an arbitrary constant, and

$$F = -\frac{k(N-1)^2 c_1^2}{N(2N-1)} + \frac{NASc_1 r}{(\gamma-1)(2N-1)} > 0 \quad (22)$$

$$G = \frac{F}{c_1} \left(\frac{g}{k} + c_0 \right) - \frac{Nk(\gamma-1)c_1 - NASr}{k(2N-1)(\gamma-1)^2} R < 0 \quad (23)$$

$$y^{a,b} = \frac{1}{2} \left\{ \frac{rAS}{(\gamma-1)(2N-1)} \pm \left[\left(\frac{rAS}{(\gamma-1)(2N-1)} \right)^2 - \frac{4kF}{N(2N-1)} \right]^{\frac{1}{2}} \right\} \quad (24)$$

where $\xi_1 = \frac{y^a}{y^b - y^a} < 0$ and $\xi_2 = \frac{-y^b}{y^b - y^a} < 0$.

The set of solution curves given by (21) includes two linear stationary Markov strategies corresponding to the case of $K = 0$ given by

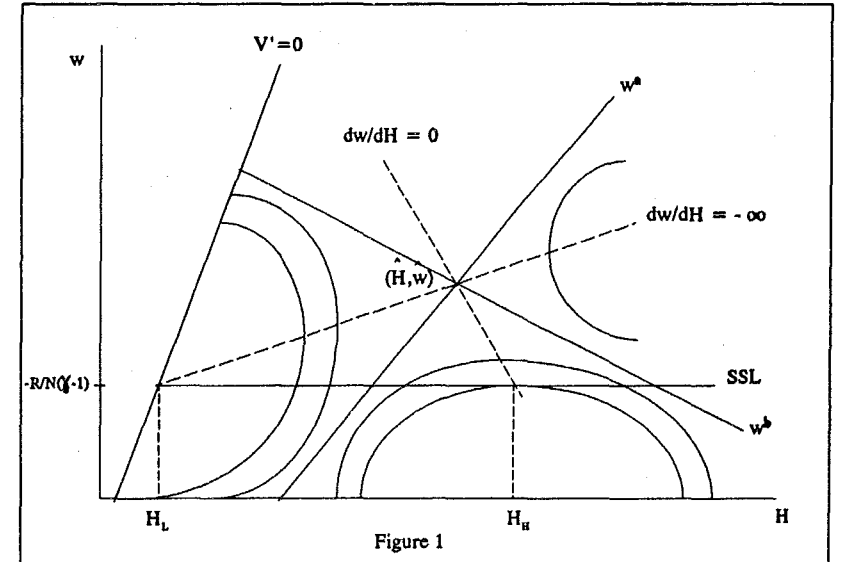
$$w^a = \frac{k(N-1)}{N(2N-1)} \left(\frac{g}{k} + c_0 \right) - \frac{R}{(\gamma-1)(2N-1)} + \frac{G}{F} y^a + \left[\frac{k(N-1)c_1}{N(2N-1)} + y^a \right] H \quad (25)$$

$$w^b = \frac{k(N-1)}{N(2N-1)} \left(\frac{g}{k} + c_0 \right) - \frac{R}{(\gamma-1)(2N-1)} + \frac{G}{F} y^b$$

¹⁴The derivation of this equation follows step by step the one presented by Dockner and Long (1993) and will not be repeated in this paper.

$$+ \left[\frac{k(N-1)c_1}{N(2N-1)} + y^b \right] H. \quad (26)$$

In Fig. 1, we note that each solution curve is only well defined in the region of nonnegative marginal value, that is, on the right of the line defined by $V' = 0$, and the set of solution curves consists of two straight lines and a family of hyperbolic curves. The two straight lines $w^a(H)$ and $w^b(H)$ correspond to the singular solutions (25) and (26). The first is positively sloping and the second negatively and they go through (\hat{H}, \hat{w}) . The steep dotted line is the locus $dw/dH = -\infty$, whereas the dotted line with negative slope is the locus $dw/dH = 0$, for Eq. (21).



Each curve in Fig. 1 corresponds to a nonlinear stationary Markov feedback equilibrium and as in our game the set of solution curves covers the entire $H - w$ plane we see that for each point on the steady-state line, SSL, (defined by $w^* = -R/N(\gamma - 1)$) it is always possible to find some strategy satisfying the stationarity condition: $\dot{H} = 0$. However, the existence of a stationary point does not necessarily mean that there exists a path, $H^*(t)$, that converges to it. For that reason we are interested in the stable stationary points.

Taking into account that the rate of extraction is given by the nonlinear stationary Markov strategies implicitly defined by (21), the state equation (6) can be written as¹⁵

$$\dot{H} = \frac{1}{AS} [R + (\gamma - 1)Nw(H)].$$

Linearizing this equation around the steady-state gives the stability condition: $dw/dH > 0$, that implies that a stationary water table H^* is *locally* stable when the slope of $w(H)$ is positive at the intersection point with SSL. Graphically this means that set of locally stable stationary points is defined by the interval (H_L, H_H) (see Fig. 1). The limits of this interval can be calculated as the intersection points of the lines $V' = 0$ and $dw/dH = 0$ with

¹⁵Remember, in order to avoid confusion resulting from the term stationary in this section, that stationary Markov strategies describe decision rules that prescribe an extraction rate as a function exclusively of the observed water table, and are, consequently, independent of time.

SSL

$$H_L = H_{MY}^* = -\frac{R}{kc_1(\gamma - 1)} - \frac{1}{c_1} \left(\frac{g}{k} + c_0 \right) \quad (27)$$

$$H_H = H_{OL}^* = \frac{R}{kc_1(\gamma - 1)} + \frac{R}{rASN} - \frac{1}{c_1} \left(\frac{g}{k} + c_0 \right). \quad (28)$$

The derivation of H_L is immediate. To calculate H_H we have to solve for

$$f'(H) = -\frac{k(N-1)c_1}{N(2N-1)}, \quad (29)$$

which has been obtained from (20) for $dw/dH = 0$.

Substituting (20) into the Bellman equation yields

$$rV(H) = \frac{1}{2kN(2N-1)} \left[-k(N-1) \left(\frac{g}{k} + c_0 + c_1H \right) + \frac{NR}{\gamma-1} \right]^2 + \frac{R}{\gamma-1} \left(\frac{g}{k} + c_0 + c_1H \right) - \frac{(2N-1)N}{2k} f(H)^2.$$

Differentiating with respect to H and substituting V' again using Eq. (20) for $w(H)$ yields

$$\begin{aligned} & r \left\{ \left[\frac{ASN}{(\gamma-1)(2N-1)} \right] \left(\frac{g}{k} + c_0 + c_1H \right) + \frac{ASNR}{(2N-1)k(\gamma-1)^2} \right. \\ & \left. - \frac{ASN}{k(\gamma-1)} f(H) \right\} \\ & = \frac{(N-1)c_1}{(2N-1)N} \left[k(N-1) \left(\frac{g}{k} + c_0 + c_1H \right) - \frac{NR}{\gamma-1} \right] \\ & + \frac{Rc_1}{\gamma-1} - \frac{(2N-1)N}{k} f(H)f'(H), \end{aligned}$$

which upon rewriting results in

$$f'(H) = \frac{k}{(2N-1)Nf(H)} \left[\frac{ASNr}{k(\gamma-1)} f(H) - FH - G \right]. \quad (30)$$

On the other hand, we know that at the steady-state $w^* = -R/N(\gamma - 1)$, then using Eq. (20) one more time we have

$$f(H^*) = -\frac{k(N-1)}{N(2N-1)} \left(\frac{g}{k} + c_0 + c_1 H^* + \frac{R}{(\gamma-1)k} \right), \quad (31)$$

and we can use (29), (30) and (31) to obtain H_H .¹⁶ The next proposition summarizes these results.

Proposition 3 Any water table level in the interval (H_L, H_H) is a locally stable steady-state, where H_L is the stationary water table for the myopic solution and H_H is the stationary water table for the open-loop solution.

In our groundwater pumping game, more can be said about the stability of a steady-state. Specifically, we can identify the domain of initial water table values from which $H^*(t)$ converges to H^* , the stationary point. But first, we need to introduce more notation. As we have already pointed out, the set of solution curves consists of two straight lines and a family of hyperbolic curves. This family is divided into six types. Let $g^n(H)$ be a solution curve of type n and G^n the set of $g^n(H)$ ($n = 1, \dots, 6$) (see Fig. 2). Each G^n contains an uncountable number of hyperbolic curves. From Fig. 2 and equation (21), it is clear that each $g^n(H) \in G^n$ including the linear ones, $n = a, b$, is well defined and continuously differentiable on the domain $D^n \equiv [x^n, \bar{x}^n]$,

¹⁶As $H_L < H_H$ the SSL must be below the intersection point of linear strategies in Fig. 1. If SSL were above (\hat{H}, \hat{w}) then $H_L > H_H$ since H_L is defined by the intersection point of the $dw/dH = -\infty$ line with SSL, and H_H is defined by the intersection point of the $dw/dH = 0$ line with SSL.

where x^n represents the lower bound of the domain and \bar{x}^n the upper bound. Let $B(H^*)$ denote a reachable initial water table set for H^* . To find $B(H^*)$ we use the fact that at any point above (below) SSL in the $H - w$ plane, $dH/dt < (>) 0$. Hence, for some H_0 , if the point $(H_0, g^n(H_0))$ is above (below) SSL, then $H^*(t)$ decreases (increases).

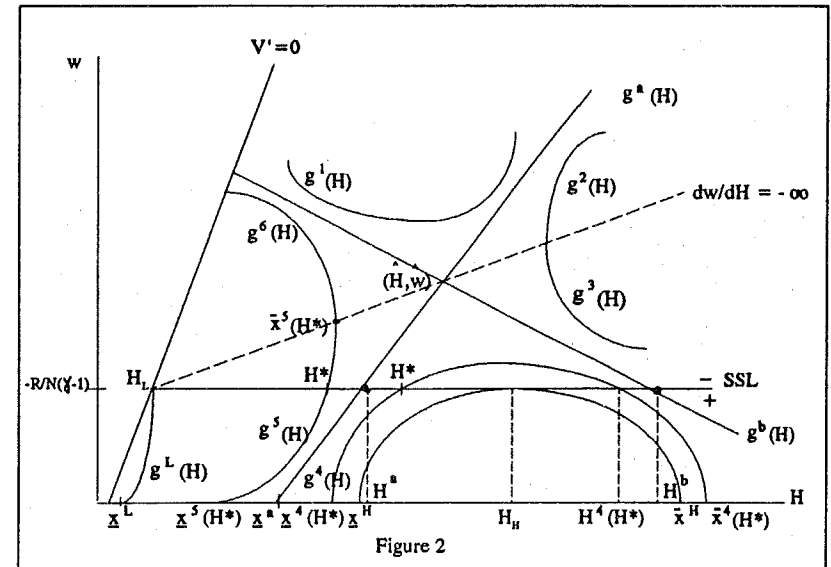


Figure 2

Now we consider four cases for H^* . (i) If $H^* = H_H$, the corresponding $g^H(H)$ is tangent to SSL at H_H and defined on $D^H = [\underline{x}^H, \bar{x}^H]$. It is easily checked graphically that $B(H_H) = [\underline{x}^H, H_H] \subsetneq D^H$. In this case the difference between the domain of the strategy and the B set is clearly seen. (ii) If $H^* \in (H^a, H_H)$, there exists some $g^4(H) \in G^4$ which supports H^* (see Fig. 2). $g^4(H)$ is defined on the H^* -dependent domain $D(H^*) = [\underline{x}^4(H^*), \bar{x}^4(H^*)]$. From the hyperbolic property of $g^4(H)$, it can be seen that there exists $H^4(H^*) \in D(H^*)$ such that $H^4(H^*) > H^*$ and $g^4(H)$ intersects SSL again at $H^4(H^*)$. So that $B(H^*) = [\underline{x}^4(H^*), H^4(H^*)] \subsetneq D(H^*)$. (iii) If $H^* = H^a$, the corresponding $g^a(H)$ is defined on $D^a = [\underline{x}^a, H^m]$, where H^m is the water table level associated with the maximum capacity or natural hydrologic equilibrium of the aquifer. For each $H_0 \in D^a$, $H^*(t)$ converges to H^a . Thus, $B(H^a) = D^a$. (iv) If $H^* \in (H_L, H^a)$, there exists some $g^5(H) \in G^5$ which supports H^* (see Fig. 2). $g^5(H)$ is defined on the H^* -dependent domain $D(H^*) = [\underline{x}^5(H^*), \bar{x}^5(H^*)]$. For each $H_0 \in D(H^*)$, $H^*(t)$ converges to H^* . Hence $B(H^*) = D(H^*)$. These relationships are summarized in the next proposition.

Proposition 4 (a) $B(H^{*'}) \subset B(H^*)$ for any $H^*, H^{*'}$ such that $H^a < H^* < H^{*'}$ $\leq H_H$. (b) $B(H^*) \rightarrow [\underline{x}^a, H^b]$, as $H^* \rightarrow H^a$ from above. (c) $B(H^*) \rightarrow [\underline{x}^a, \hat{H}]$, as $H^* \rightarrow H^a$ from below.

(a) shows that if there are two stationary water tables H^* and $H^{*'}$ with $H^* < H^{*'}$, then the domain of the reachable initial water table for $H^{*'}$ is smaller than the one for H^* . Whereas (b) and (c) imply that $B(H^*)$ is

discontinuous at $H^* = H^a$, since $B(H^*)$ is a set-valued function.

To conclude this section we show that the above constructed stationary Markov strategies define a stationary Markov feedback equilibrium.

Proposition 5 For each $w(H)$ given by (21) the function $J(H)$ defined by

$$J(H) = \frac{1}{r} \left[-\frac{N(2N-1)}{2k} w(H)^2 + \left((N-1) \left(\frac{g}{k} + c_0 + c_1 H \right) - \frac{NR}{k(\gamma-1)} \right) w(H) + \frac{R}{\gamma-1} \left(\frac{g}{k} + c_0 + c_1 H \right) \right] \quad (32)$$

is a twice differentiable value function that generates stationary Markov feedback equilibria that support any stationary point, H^* , in the interval (H_L, H_H) , if $H_0 \in B(H^*)$.

Proof. See Appendix.

One important characteristic of this result is the nonuniqueness of the stationary Markov feedback equilibrium. In this case we find that not only a stationary water table H^* is indeterminate but for a given initial value of the water table, different stationary Markov feedback equilibria can be reached. So it looks interesting to wonder which equilibrium is the most efficient or generates the highest payoff.¹⁷

¹⁷See Tsutsui and Mino (1990, p. 153) for an explanation of the indeterminacy of the solution. From a mathematical point of view this is caused by the incomplete transversality condition.

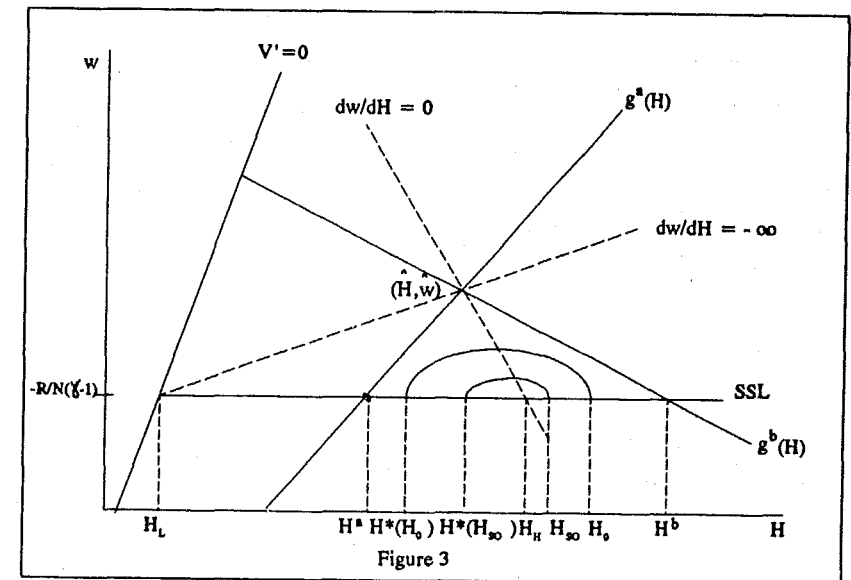
5 Characterization of stationary Markov feedback equilibrium

As we have established in Proposition 5 the stationary Markov feedback equilibria in the interval (H_L, H_H) that can be supported by a stationary Markov strategy depend on the initial water table level. In this paper we assume that the initial water table level is equal to its *natural hydrologic equilibrium*, corresponding to the maximum water table elevation at which the water reserves coincide with the storage capacity of the aquifer, and that the human activity, justified by economic parameters, consists of mining the aquifer until an *economic hydrologic equilibrium* has been reached. The difference between the two equilibria is that the first depends exclusively on hydrologic parameters whereas the second is explained by hydrologic and economic parameters. This assumption has a clear consequence: the socially optimal equilibrium water table, H_{SO}^* , will be lower than the initial water table, H_0 , and consequently we can establish the following relationship: $H_H < H_{SO}^* < H_0$, since H_H , defined by (28), is lower than H_{SO}^* , defined by (14).¹⁸

Now we can say more about the possible equilibria that can be reached

¹⁸Notice that if we assume that the initial value is lower than the optimal stationary equilibrium water table and, consequently, lower than the natural hydrologic equilibrium, this means that there existed a previous phase of resource exploitation from the natural hydrologic equilibrium whose steady-state value would now be the initial value we are assuming in our model. In this paper we focus exclusively on the mining of the aquifer and we do not consider this case.

through a stationary Markov strategy. Specifically, we can identify the locally stable equilibria that can be reached from the initial water table value. Let $E(H_0)$ denote a stable stationary point set reachable from H_0 . To find $E(H_0)$ we use the fact that any point above SSL in the $H - w$ plane, $dH/dt < 0$. Hence, for a certain H_0 , if the point $(H_0, g(H_0))$ is above SSL, then $H^*(t)$ decreases.



Now we consider two cases for H_0 . (i) If $H_0 \geq H_b$, the only strategy that leads to a stable stationary point is the linear one, $g^a(H)$ and then $E(H_0) = [H^a]$, that is, the only stationary water table that can be supported as a stationary Markov feedback equilibrium is H^a . (ii) If $H_0 < H_b$, there exists a set of $g^A(H) \in G^A$, including linear strategy $g^a(H)$, which support different stable stationary points, H^* . From the hyperbolic property of $g^A(H)$, one can see that there exists one solution curve and one stable stationary point $H^*(H_0)$ such that $H^*(H_0) < H_H < H_0$ and $g^A(H)$ intersects SSL at $H^*(H_0)$ and H_0 , so that $H^*(H_0)$ defines an upper bound for the stable stationary points which can be reached from the initial value, H_0 .¹⁹ On the other hand, the lowest stable stationary point which can be supported as a stationary Markov feedback equilibrium from H_0 is given by the intersection of the linear strategy, $g^a(H)$, with SSL. Thus, the reachable stable stationary point set for H_0 , $E(H_0)$, is given by the H_0 - dependent interval $[H^a, H^*(H_0))$. These results are summarized in the next proposition.

Proposition 6 (a) $E(H'_0) \subset E(H_0)$ for any H_0, H'_0 such that $H_{SO}^* < H_0 < H'_0 < H^b$. (b) $E(H_0) \rightarrow [H^a, H^*(H_{SO}^*))$, as $H_0 \rightarrow H_{SO}^*$.

(a) shows that for two different initial water tables H_0 and H'_0 with $H_0 < H'_0$ the stable stationary point set reachable from H'_0 is smaller than the one from H_0 , and (b) implies that the highest stable stationary water table that can be supported by a stationary Markov strategy is lower than

¹⁹Notice that the solution curve that defines the point $H^*(H_0)$ cannot be used to reach that point because it cuts the SSL at H_0 .

H_H since H_{SO}^* is higher than H_H and consequently $H^*(H_{SO}^*)$ is on the left of H_H (see Fig. 3). This comparison allows us to conclude that the *open-loop Nash equilibrium* water table is **higher** than the highest stationary water table that can be reached as a *stationary Markov feedback equilibrium* from $H_0 > H_{SO}^*$ since $H_{SO} = H_{OL}^*$. In other words, although the stationary water table H^* remains indeterminate, this result establishes that the *strategic externality* that arises from the competition among farmers to capture ground-water reserves exacerbates the overexploitation of the aquifer compared to the open-loop solution.²⁰ Finally, we want to point out that another corollary of this proposition is that the linear stationary Markov feedback equilibrium is *global*; that is, it can be reached from any initial value; whereas the nonlinear stationary Markov strategies can only be used when the initial value of the state variable is lower than the upper bound defined by H^b . However, in that case, nonlinear strategies support a stationary water table that is closer to the open-loop Nash equilibrium water table than the stationary water table supported by the linear strategy, and consequently closer to the socially optimal solution.

Now if we want to evaluate the impact of the strategic externality on the stationary value of the water table it seems necessary to investigate if it is possible to resolve the indetermination of the stationary Markov feedback

²⁰Notice that independently of which is the initial value, H_0 , always provided that $H_C < H_0$, we have that $H^*(H_0) < H^*(H_{SO}) < H_H = H_{OL}$, (see Fig. 3), and consequently the stationary water table for the feedback solution has to be lower than the one for the open-loop solution since $H^*(H_0)$ is the upper extreme of $E(H_0)$.

equilibrium. The way to do that is to study if a positive relationship between the stationary water table level and the payoff of the game can be established, because if that kind of relationship exists, then we could conclude that the stationary equilibrium water table is that which generates the highest payoff. For this point, we have the following proposition.

Proposition 7 *For a given H_0 such that $H_{SO}^* < H_0 < H_b$ there exists more than one stationary Markov feedback equilibrium, $E(H_0) \subset [H^a, H^*(H_{SO}^*)]$. Then the equilibrium which supports the highest stationary water table generates the highest payoff of the game that starts at H_0 . Thus the payoff of a stationary Markov feedback equilibrium increases with the level of its supporting water table.*

Proof. See Appendix.

This result implies that for a given initial value the steady-state equilibrium water table is the highest value of the H_0 -dependent interval $E(H_0)$. Notice that as the interval is open on the right, $H^*(H_0)$ is actually the lowest upper bound of the stationary point. Moreover, this result also establishes that the linear strategy, $g^a(H)$, generates the lowest payoff of the game that starts at H_0 . This implies that, in spite of linear strategy being global, when $H_0 < H^b$ it is dominated by the nonlinear strategies, since these generate a higher payoff. In that case using linear strategies to compute the feedback solution will lead to an overestimation of the *overexploitation* of the aquifer caused by the strategic behavior of the agents.

Returning now to the discussion of the evaluation of the strategic externality effect on the stationary value of the water table, we find that the comparison between the two solutions is not feasible, at least for the non-linear strategies, since these are not defined in an explicit way (see equation (21)). However, it is very easy to find an *upper bound* for this effect using H_L

$$\Delta H = H_{OL}^* - H_L = H_{OL}^* - H_{MY}^* = \frac{R}{ASrN} \quad (33)$$

The comparative statics analysis of this difference results in

$$\begin{aligned} \frac{\partial(\Delta H)}{\partial r} &= -\frac{R}{ASNr^2} < 0 \\ \frac{\partial(\Delta H)}{\partial N} &= -\frac{R}{ASrN^2} < 0. \end{aligned}$$

These results are summarized in the last proposition.

Proposition 8 *(i) The stationary open-loop Nash equilibrium water table is higher than the highest stationary water table that can be supported by a stationary Markov strategy. (ii) The impact of the strategic externality on the stationary value of the water table presents an upper bound given by the difference between the stationary water table for the open-loop solution and the stationary water table for the myopic solution, this difference declines as the discount rate and/or the number of farmers increases.*

This result confirms Negri's conclusion: the competition among users for the appropriation of a finite common property resource increases the overexploitation of the aquifer, compared to the open-loop solution. On the other hand, the effect of the discount rate variation is explained by the different

impacts that the variations of discount rate have on H_{MY}^* and H_{OL}^* . Thus, an increase in the discount rate decreases the user cost for the open-loop solution, but does not have any effect on H_{MY}^* since this value is independent of the discount rate. Moreover, the effect that a variation in the number of farmers has on difference (33) is clear if one notices that H_{MY}^* is also independent of N . Then an increase in N reduces the user cost of the resource and the stationary open-loop Nash equilibrium water table, causing a decrease in the difference between H_{OL}^* and H_{MY}^* .

Finally, we can define using (15), the effect of cost externality, and (33), the effect of strategic externality, an *upper bound* for the dynamic inefficiency associated with the private exploitation of groundwater

$$\bar{\Delta} = H_{SO}^* - H_{OL}^* + H_{OL}^* - H_{MY}^* = \frac{R}{ASr}. \quad (34)$$

This result is consistent with Gisser and Sánchez rule. Thus, we can conclude that the difference between the *socially optimal exploitation* and the *private exploitation* of groundwater, characterized by a stationary Markov feedback equilibrium, decreases with the storage capacity of the aquifer and if this is large enough the two equilibria are identical for all practical purposes. In fact, when we add the two differences (15) and (33) we get the same expression as the one derived by Gisser and Sánchez.²¹ Obviously, this result limits the practical scope of Negri's result, so that the consideration of the

²¹This happens because the upper bound for the strategic externality is defined resorting to the myopic solution. However, it must not be forgotten that this difference is an *upper bound* of the difference between the optimal and private solutions and therefore represents an overestimation of the overexploitation of the aquifer.

strategic externality does not significantly increase the inefficiency of private exploitation.

6 An empirical illustration

Using data from Gisser and Sánchez (1980) corresponding to Pecos Basin, New Mexico and Nieswiadomy (1985) corresponding to the Texas High Plains we have computed the different equilibria studied in this paper: the optimal exploitation, the open-loop Nash equilibrium and the stationary Markov feedback equilibrium in linear strategies. The results obtained allow us to compare the different regimens in terms of steady state values of the water table as well as in terms of present values.

These results establish that the relationship found in Prop. 7 between the payoff of the game and the stationary water table level for the stationary Markov feedback equilibria is also verified when we compare the different equilibria. Thus, we observe that the equilibrium with the highest stationary water table always generates the highest payoff of the game. This allows us to extend the first part of Prop. 8 to the present values associated with each equilibrium, and conclude that the payoff generated by the open-loop Nash equilibrium is higher than the highest present value that can be generated by a stationary Markov strategy. Obviously, the present value associated with the optimal exploitation will be the highest payoff of the game.

TABLE I: COMPARISONS OF ECONOMIC AND HYDROLOGIC PARAMETERS PERTAINING TO THE TEXAS HIGH PLAINS AND THE PECOS BASIN, NEW MEXICO.

Symbol	Description	High Plains	Pecos Basin
k	Decrease in demand for water per \$1 increase in price*	-134,337 ac. ft./yr.	-3,259 ac. ft./yr.
g	Quantity intercept in the demand for water equation	2,401,161 ac. ft./yr.	470,365 ac. ft./yr.
C ₁	Cost of pumping one acre foot of water per foot of lift**	-0.035 dollars/ac. ft./ft. of lift	-0.035 dollars/ac. ft./ft. of lift
C ₀	Intercept of the pumping cost function	125 dollars/ac. ft.	125 dollars/ac. ft.
γ	Return flow coefficient	0.20	0.27
A	Area of aquifer	4,304,640 acres	789,120 acres
S	Scorativity coefficient (feet of water/foot of lift)	0.15	0.17
R	Recharge rate	358,720 ac. ft./yr.	173,000 ac. ft./yr.
H	Initial water table elevation	3,400 ft. above sea level	3,400 ft. above sea level

* The High Plains of Texas estimate is based on 1967 dollars and the Pecos Basin estimate is based on 1968 dollars.

**The pumping cost in the High Plains is assumed to be the same as the Pecos Basin estimate for purposes of comparison.

TABLE II: HIGH PLAINS

Equilibria	r=0.02	r=0.05	r=0.1
Optimal exploitation			
H*	3,183.884	3,167.217	3,161.662
DPV	354,639,301	164,498,988	87,737,545
Open-loop Nash equilibrium			
H* (N=44)	3,156.737	3,156.359	3,156.232
H* (N=174)	3,156.266	3,156.170	3,156.138
H* (N=697)	3,156.146	3,156.122	3,156.114
DPV (N=44)	341,013,024	162,998,748	87,501,920
DPV (N=174)	340,460,232	162,941,604	87,493,290
DPV (N=697)	340,317,917	162,926,538	87,490,925
Stationary Markov feedback equilibrium (linear strategies)			
H* (N=44)	3,156.598	3,156.333	3,156.226
H* (N=174)	3,156.230	3,156.163	3,156.136
H* (N=697)	3,156.137	3,156.120	3,156.114
DPV (N=44)	340,869,408	162,991,488	87,501,348
DPV (N=174)	340,422,474	162,939,690	87,493,116
DPV (N=697)	340,308,159	162,926,538	87,490,925
Externality Evaluation	r=0.02	r=0.05	r=0.1
Cost externality			
H* (N=44)	27.147	10.858	5.430
H* (N=174)	27.618	11.047	5.524
H* (N=697)	27.747	11.097	5.548
DPV (N=44)	13,626,277	1,500,240	235,625
DPV (N=174)	14,179,069	1,557,384	244,255
DPV (N=697)	14,321,384	1,572,181	246,620
Strategic externality			
H* (N=44)	0.139	0.026	0.006
H* (N=174)	0.036	0.007	0.002
H* (N=697)	0.009	0.002	0
DPV (N=44)	143,616	7,260	572
DPV (N=174)	37,758	1,914	174
DPV (N=697)	9,758	0	0
Total externality			
H* (N=44)	27.286 (0.857%)	10.884 (0.344%)	5.436 (0.172%)
H* (N=174)	27.654 (0.869%)	11.054 (0.349%)	5.526 (0.175%)
H* (N=697)	27.756 (0.872%)	11.099 (0.350%)	5.548 (0.176%)
DPV (N=44)	13,769,893 (3.883%)	1,507,500 (0.916%)	236,197 (0.269%)
DPV (N=174)	14,216,827 (4.009%)	1,559,298 (0.948%)	244,429 (0.279%)
DPV (N=697)	14,331,142 (4.041%)	1,572,181 (0.956%)	246,620 (0.281%)

Cost externality: difference between the optimal depletion and the open-loop Nash equilibrium.

Strategic externality: difference between the open-loop Nash equilibrium and the stationary Markov feedback equilibrium in linear strategies.

TABLE III: PECOS BASIN

Equilibrium	r = 0.02	r = 0.05	r = 0.1
Optimal exploitation			
H*	1,589.893	1,551.206	1,538.310
DPV	1,516,433,487	616,477,637	309,991,403
Open-loop Nash equilibrium			
H* (N=44)	1,533.474	1,528.638	1,527.026
H* (N=174)	1,527.429	1,526.220	1,525.817
H* (N=697)	1,525.917	1,525.615	1,525.514
DPV (N=44)	1,515,416,496	616,407,680	309,982,440
DPV (N=174)	1,515,180,160	616,391,680	309,980,416
DPV (N=697)	1,515,117,056	616,387,456	309,979,904
Stationary Markov feedback equilibrium (linear strategies)			
H* (N=44)	1,533.261	1,528.603	1,527.017
H* (N=174)	1,527.370	1,526.210	1,525.814
H* (N=697)	1,525.902	1,525.613	1,525.514
DPV (N=44)	1,515,408,736	616,407,464	309,982,424
DPV (N=174)	1,515,177,760	616,391,616	309,980,384
DPV (N=697)	1,515,116,416	616,387,456	309,979,904
Externality Evaluation			
Cost externality			
H* (N=44)	56.419	22.568	11.284
H* (N=174)	62.464	24.986	12.493
H* (N=697)	63.976	25.591	12.796
DPV (N=44)	1,016,991	69,957	8,963
DPV (N=174)	1,253,327	85,957	10,987
DPV (N=697)	1,316,431	90,181	11,499
Strategic externality			
H* (N=44)	0.213	0.035	0.009
H* (N=174)	0.059	0.010	0.003
H* (N=697)	0.015	0.002	0
DPV (N=44)	7,760	216	16
DPV (N=174)	2,400	64	32
DPV (N=697)	640	0	0
Total externality			
H* (N=44)	56.632 (3.562%)	22.603 (1.457%)	11.293 (0.734%)
H* (N=174)	63.054 (3.966%)	24.996 (1.611%)	12.496 (0.812%)
H* (N=697)	63.991 (4.025%)	25.593 (1.649%)	12.796 (0.832%)
DPV (N=44)	1,024,751 (0.068%)	70,173 (0.011%)	8,979 (0.003%)
DPV (N=174)	1,255,727 (0.083%)	86,021 (0.014%)	11,019 (0.004%)
DPV (N=697)	1,317,071 (0.087%)	90,181 (0.015%)	11,499 (0.004%)

Cost externality: difference between the optimal depletion and the open-loop Nash equilibrium.

Strategic externality: difference between the open-loop Nash equilibrium and the stationary Markov feedback equilibrium in linear strategies.

In Tables II and III we have represented the impact of the cost and strategic externalities on the stationary water table and present values for three different values of the number of farmers and the rate of discount. The cost externality has been calculated as the difference between the optimal exploitation and the open-loop Nash equilibrium, and the strategic externality as the difference between the open-loop Nash equilibrium and the stationary Markov feedback equilibrium in linear strategies. The use of linear strategies is justified in this last case because we have checked that the initial value for the water table is higher than the value defined by the intersection of the unstable linear strategy and the steady state line and in that case, as we have pointed out at the beginning of Section 5 (see Fig. 3), the only strategy that leads to a stable stationary point is the linear one with positive slope.

The results show that the cost externality decreases as the discount rate increases and increases as the number of farmers pumping water from the aquifer increases, whereas the strategic externality also decreases as the rate of discount increases but decreases as the number of farmers increases.

The largest cost externality corresponds to the largest number of farmers and the lowest rate of discount (0.02). This externality amounts to 27.747 feet for water table elevation and \$14,321,384 for the present value at Texas High Plains, and 63.976 feet and \$1,316,431 dollars at Pecos Basin, New Mexico. The largest strategic externality corresponds to the lowest number of farmers and the lowest rate of discount. This externality amounts to 0.139 feet and \$143,616 at High Plains and 0.213 feet and \$7,760 at Pecos Basin.

Therefore, the cost externality is greater than the strategic externality for the two cases, so that the largest total externality corresponds to the largest number of farmers and the lowest rate of discount for both percentages and levels. Total externalities reduce by 27.756 feet, 0.872 in percentage, the water table elevation at the steady state and \$14,331,142, 4.041 in percentage, the present value with respect to the optimal exploitation at High Plains; and by 63.991 feet, 4.025 in percentage, the water table elevation and \$1,317,071 dollars, 0.087 in percentage, the present value at Pecos Basin.

The results indicate that the benefits from groundwater management most likely are small, especially relative to any reasonable costs of regulating pumping. For example, for a discount rate of 2%, it would only take an annual regulating cost higher than \$286,623 per year at High Plains and \$26,341 per year at Pecos Basin to make the present value of the costs exceed the present value of the benefits coming from regulation.

7 Conclusions

In this paper we have developed the model defined by Gisser and Sánchez (1980) to study the effects of strategic behavior on the efficiency of private groundwater exploitation. We have followed Negri's (1989) approach and have evaluated the impact of the strategic externality as the difference between the open-loop and feedback solutions. In particular, we have investigated if the Gisser and Sánchez rule still works when it is assumed that agents are rational and the strategic externality is taken into account. To

compute the feedback equilibria we have used nonlinear strategies following Tsutsui and Mino's (1990) procedure.

Our results show that strategic behavior, which arises from the competition among firms to capture the groundwater reserves, increases the inefficiency of private exploitation with respect to the open-loop equilibrium which captures only the pumping cost externality. However, they also show that the difference between the socially optimal exploitation and the private exploitation of the aquifer, represented by a feedback equilibrium, decreases with the storage capacity of the aquifer, and thus if this is relatively large the two equilibria are identical for all practical purposes. A corollary of this result is that the potential benefits associated with the regulation of the resource are relatively small.

Finally, we would like to present some remarks about the scope of this last conclusion. First, as Worthington, Burt and Brustkern (1985) have pointed out in an empirical work using data from a confined aquifer underlying the Crow Creek Valley, Montana, it can happen that the difference between the two regimens is not trivial if the relationship between average extraction cost and the water table level is not linear and there exist significant differences in land productivity. Consequently, we think that further research is necessary in at least two directions before taking a position against regulation of the resource. One would be to undertake more empirical work to test the hypothesis of linearity, and the other to develop more theoretical work to resolve an asymmetric groundwater pumping differential game where the

differences in land productivity were taken into account. To complete the analysis, the comparison between the two regimes would have to be carried out, also assuming uncertainty about recharge or surface water supply.²²

Moreover, we also think that using only the firms' profits to characterize the socially optimal exploitation is problematic when there exists the possibility of irreversible events or irreparable damage to nature. In that case, the water management authority would have to incorporate the water table level into its objective function and postulate some kind of intervention to avoid 'extinction' or the occurrence of irreversible events.²³ This could be another subject for future research.

Another situation that could require some kind of regulation may present itself when groundwater is also used for urban consumption. In that case the water pollution caused by the use of chemical products in agricultural activity alters the quality of water and affects negatively the welfare of urban consumers, generating another externality that would increase the inefficiency of private exploitation of groundwater.

²²As far as we know only Knapp and Olson (1995) have addressed this issue, and they have found that when surface water supply is uncertain the benefits from groundwater management continue to be relatively small.

²³See Tsur and Zemel (1995) for the study of the optimal exploitation of groundwater when extraction affects the probability of occurrence of an irreversible event.

A Proof of Proposition 5

Since (32) is obtained by substitution of the necessary condition for the maximization of the right-hand side of Bellman equation, $V' = AS/(\gamma - 1)((g/k) + c_0 + c_1H - (Nw(H)/k))$ in the Bellman equation, $J(H)$ is a value function that generates stationary Markov strategies such as the ones defined by equation (21). By its construction, it is clear that $J(H)$ is twice differentiable.

Now for each $H^* \in (H_L, H_H)$, we have to show that $J(H)$ is nonnegative on $B(H^*)$.

Except for H^a , we have that all the solution curves on $B(H^*)$ are bounded by

$$0 \leq g(H) \leq \min [g^L(H), w(H) \text{ defined by } dw/dH = -\infty, g^b(H)] \quad (35)$$

for each $H^* \in (H_L, H_H)$, as one can see from the Fig. 2.²⁴ Define

$$h(w, H) = -\frac{N(2N-1)}{2k}w^2 + \left((N-1) \left(\frac{g}{k} + c_0 + c_1H \right) - \frac{NR}{k(\gamma-1)} \right) w + \frac{R}{\gamma-1} \left(\frac{g}{k} + c_0 + c_1H \right).$$

Note that $rJ(H) = h(g(H), H)$. Let us consider the area surrounded by $w = 0$, $w = g^L(H)$,

$$w = \frac{k}{(2N-1)N} \left[(N-1) \left(\frac{g}{k} + c_0 + c_1H \right) - \frac{NR}{k(\gamma-1)} \right] \quad (36)$$

²⁴We suppress the superscript n of the solution curves for notational simplicity when no confusion arises or the argument is independent of n .

defined by $dw/dH = -\infty$ and

$$w^b = g^b(H) = \frac{k(N-1)}{N(2N-1)} \left(\frac{g}{k} + c_0 \right) - \frac{R}{(\gamma-1)(2N-1)} + \frac{G}{F} y^b + \left[\frac{k(N-1)c_1}{N(2N-1)} + y^b \right] H. \quad (37)$$

The intersection between $w = 0$ and $w = g^L(H)$ is $(\underline{x}^L, 0)$; the intersection between $w = g^L(H)$ and (36) is $(H_L, -R/N(\gamma-1))$; the intersection between (36) and (37) is (\hat{H}, \hat{w}) ; finally the intersection between (37) and SSL is $(H^b, -R/N(\gamma-1))$. It is easy to see that $\underline{x}^L < H_L < \hat{H} < H^b$. It is important that except for $w = g^a(H)$, $w = g(H)$ defined on $B(H^*)$ is contained in this area. Thus, if we can show $h(w, H) \geq 0$ in this area, the proof will be completed for $H^* = H^a$.

(i) We represent the function given by $h(w, H) = 0$ in the $H - w$ plane.

The function $H(w)$ defined by condition $h(w, H) = 0$ has two extremes at the points:

$$H'_1 = \frac{NR((2N-1)^{\frac{1}{2}} - N)}{c_1 k(\gamma-1)(N-1)^2} - \frac{1}{c_1} \left(\frac{g}{k} + c_0 \right) \quad (38)$$

$$w'_1 = -\frac{R(1 - (2N-1)^{-\frac{1}{2}})}{(\gamma-1)(N-1)} \quad (39)$$

and

$$H'_2 = -\frac{NR((2N-1)^{\frac{1}{2}} + N)}{c_1 k(\gamma-1)(N-1)^2} - \frac{1}{c_1} \left(\frac{g}{k} + c_0 \right) \quad (40)$$

$$w'_2 = -\frac{R(1 + (2N-1)^{-\frac{1}{2}})}{(\gamma-1)(N-1)} \quad (41)$$

(H'_1, w'_1) being a local maximum and (H'_2, w'_2) a local minimum. Moreover, the function presents a discontinuity point at $w = -R/(\gamma-1)(N-1)$, so

that

$$\lim_{w \rightarrow -\frac{R}{(\gamma-1)(N-1)}} H(w) = -\infty, \quad (42)$$

and is concave on the right of this discontinuity point and convex on the left.

For $w = 0$, H is $-1/c_1((g/k) + c_0)$ which is the same value defined by the function $V' = 0$ for $w = 0$. It is also easy to check that (H'_1, w'_1) and (H'_2, w'_2) satisfy equation (36) and $H'_1 < H_L$ and $\hat{H} < H'_2$.

On the other hand, for the function $h(w, H)$ we have

$$\frac{dh}{dH} = c_1 \left((N-1)w + \frac{R}{\gamma-1} \right), \quad (43)$$

and in that case

$$\frac{dh}{dH} \begin{cases} > \\ < \end{cases} 0 \quad \text{if} \quad w \begin{cases} < \\ > \end{cases} -\frac{R}{(\gamma-1)(N-1)} \quad (44)$$

and then we can determine the areas where $h(w, H)$ is positive. These results are represented in the Fig. 4

Moreover, as $g^b(H)$ intersects the line $V' = 0$ on the left of the line $dw/dH = 0$ and above the line $-R/(\gamma-1)(N-1)$ (see Fig. 4), we have that $h(g^b(H'), H')$ is strictly positive.

All this shows, as can be seen in Fig. 4, that $w = g(H)$ defined by (35) on $B(H^*)$ gives a positive value for $h(w, H)$.

(iii) If $H^* = H^a$, $g^a(H)$ is defined by $B(H^a) = [x^a, H^m]$. As before, we can show

$$\begin{aligned} h(g^a(H), H) &= \frac{NASr}{2k(\gamma-1)} \left(\frac{kc_1}{2N-1} - y^a \right) H^2 \\ &\quad - \frac{NASr}{k(\gamma-1)} \left[\frac{G}{F} y^a - \frac{k}{2N-1} \left(\frac{g}{k} + c_0 \right) \right. \\ &\quad \left. - \frac{R}{(2N-1)(\gamma-1)} \right] H + \frac{k(N-1)^2}{2N(2N-1)} \left(\frac{g}{k} + c_0 \right)^2 \\ &\quad + \frac{RN}{(\gamma-1)(2N-1)} \left(\frac{g}{k} + c_0 \right) + \frac{NR^2}{2k(\gamma-1)^2(2N-1)} \\ &\quad - \frac{N(2N-1)}{2k} \left(\frac{G}{F} \right)^2 (y^a)^2 \\ &\geq -\frac{k}{2N} \left(\frac{g}{k} + c_0 + c_1 H' \right)^2 \geq 0. \end{aligned} \quad (48)$$

Notice that if $h(g^a(H'), H') = 0$, then the linear strategy would have to be tangent to $h(w, H) = 0$ at H' and would intersect the $V' = 0$ line from above, but as $g^a(H)$ passes through (\hat{H}, \hat{w}) , which is on the right of the $V' = 0$ line, the linear strategy cuts the $V' = 0$ line from below, what means that $h(g^a(H'), H')$ is strictly positive at its minimum. Furthermore, it is easy to confirm using (24) that function (48) is convex.

These three steps complete the proof.

Q.E.D.

B Proof of Proposition 7

Suppose that $H^*, H^{*'} \in E(H_0)$. Let $g(H)$ and $g'(H)$ be the solution curves that support H^* and $H^{*'}$ respectively, and let $J(H)$ and $J'(H)$ be the corresponding value functions. From Proposition 5, we have

$$\begin{aligned} rJ(H_0) &= -\frac{N(2N-1)}{2k} g(H_0)^2 + \left[(N-1) \left(\frac{g}{k} + c_0 + c_1 H_0 \right) \right. \\ &\quad \left. - \frac{NR}{k(\gamma-1)} \right] g(H_0) + \frac{R}{\gamma-1} \left(\frac{g}{k} + c_0 + c_1 H_0 \right), \end{aligned}$$

$$\begin{aligned} rJ'(H_0) &= -\frac{N(2N-1)}{2k} g'(H_0)^2 + \left[(N-1) \left(\frac{g'}{k} + c_0 + c_1 H_0 \right) \right. \\ &\quad \left. - \frac{NR}{k(\gamma-1)} \right] g'(H_0) + \frac{R}{\gamma-1} \left(\frac{g'}{k} + c_0 + c_1 H_0 \right). \end{aligned}$$

The difference $rJ'(H_0) - rJ(H_0)$ can be written as

$$\begin{aligned} &-\frac{N(2N-1)}{2k} (g'(H_0) - g(H_0)) \times \\ &\left\{ g'(H_0) - \frac{k}{N(2N-1)} \left[(N-1) \left(\frac{g}{k} + c_0 + c_1 H_0 \right) - \frac{NR}{k(\gamma-1)} \right] \right. \\ &\left. + g(H_0) - \frac{k}{N(2N-1)} \left[(N-1) \left(\frac{g}{k} + c_0 + c_1 H_0 \right) - \frac{NR}{k(\gamma-1)} \right] \right\}. \end{aligned}$$

Without loss of generality, we assume $H^* < H^{*'}$. Then $g'(H_0) < g(H_0)$ from the property of the resolution curves. For $g(H) \neq g^a(H)$ we know from demonstration of Proposition 5 that $g'(H_0)$ and $g(H_0)$ are lower than (36):

$$w = \frac{k}{N(2N-1)} \left[(N-1) \left(\frac{g}{k} + c_0 + c_1 H_0 \right) - \frac{NR}{k(\gamma-1)} \right],$$

the linear function defined by the condition: $dw/dH = -\infty$, and thus $rJ'(H_0) - rJ(H_0)$ is positive. Suppose now that $g(H) = g^a(H)$. Then as

$g'(H_0) < g^b(H_0)$ we have

$$\begin{aligned} & g'(H_0) - \frac{k}{N(2N-1)} \left[(N-1) \left(\frac{g}{k} + c_0 + c_1 H_0 \right) - \frac{NR}{k(\gamma-1)} \right] \\ & + g^a(H_0) - \frac{k}{N(2N-1)} \left[(N-1) \left(\frac{g}{k} + c_0 + c_1 H_0 \right) - \frac{NR}{k(\gamma-1)} \right] \\ < & g^b(H_0) - \frac{k}{N(2N-1)} \left[(N-1) \left(\frac{g}{k} + c_0 + c_1 H_0 \right) - \frac{NR}{k(\gamma-1)} \right] \\ & + g^a(H_0) - \frac{k}{N(2N-1)} \left[(N-1) \left(\frac{g}{k} + c_0 + c_1 H_0 \right) - \frac{NR}{k(\gamma-1)} \right], \end{aligned}$$

substituting $g^a(H_0)$ and $g^b(H_0)$ using (25) and (26) yields

$$(y^a + y^b) \left(\frac{G}{F} + H_0 \right)$$

where $y^a + y^b$ is negative. The sign of $G/F + H_0$ depends on the slope of the linear function defined by $dw/dH = 0$. If this is negative, then $\hat{H} < H_H < H_{SO}^* < H_0$, and therefore $G/F + H_0$ is positive. If instead it is positive, $H_H < \hat{H}$, and $G/F + H_0$ remains undetermined. If we assume that $G/F + H_0 \leq 0$, then $G \leq -FH_{SO}^*$, substituting (23), (22) and (14) for G , F and H_{SO}^* respectively we obtain

$$0 \leq -\frac{(N-1)^2 c_1 R}{N(\gamma-1)(2N-1)} + \frac{k(N-1)^2 c_1^2 R}{N(2N-1)ASr}$$

which is a contradiction since the right-hand side of the inequality is negative. So we have that $G/F + H_0$ is positive and therefore $(y^a + y^b)(G/F + H_0)$ is negative, resulting in a positive value for the difference $rJ'(H_0) - rJ(H_0)$. To sum up, if $H^* < H'$, $rJ(H_0) < rJ'(H_0)$, which gives Proposition 7. Q.E.D.

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