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Closed Form Solution for Dynamics of Sustainable Tourism

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Abstract

The attention to environmental conditions of the planet drives many scientists to study and to analyze the externalities of the economic activities and their relapses on nature. The issue is quite complex because of the non-linear interactions between human and natural phenomena. Our intention is to study the particular case of tourist activities. Starting from the specification of the concept of sustainable development, using a simple model we characterize the conditions for which there exists an optimal equilibrium between nature and tourism. Then, trough several simulations we study which policies are able to guarantee the better synergies between economy and environmental quality.

1 Introduction

Like every human activities, the tourism is related to the environment. Just like any form of industrial development, it produces effects on nature; obviously, to have profitable activity, a high tourism flow must visit the same destinations to consume the tourist product. Therefore it is inevitable and this causes an impact on the environment of the sites (at the worldwide impact of the tourism on the environment is relevant and can not be neglected, because it is considered one of the major economic activities in the world). It should be noted that the main resource of the tourism sector is the environmental quality, therefore the depletion of nature has also direct consequences on the economic performances. The issues will be addressed on the basis of the fundamental concept of the sustainability (carrying capacity). As shown by the temporal dynamics, the interrelation between tourism development and the environment appears complex enough to require advanced mathematical tools such as Optimal Control Theory. Remembering one of the first studies about the relationship between environment and tourism, (Budowski (1976)) , three forms of relationships between economic activity in tourism and the environment are the following:

1)THE CO-EXITENCE, when the two activity(attention to the environmental quality and flow of the tourists) can be distinguished, without any contacts with few negligible connections;

2)CONFLICT, if the actions of the tourists make up obvious environmental damage;

3)THE SYMBIOTIC RELATIONSHIP, is the relationships in which both the tourism activity and the environment quality receive benefits.

Notice that this three forms can coexist simultaneously in the same location, with different types of the tourism.

In 1987, the World Futures Commission, "The Brundtland Report on the Future Common", proposed the official definition of the sustainable development:

"Sustainable development is one that meets the needs of the present generation without compromising the ability of future generations to meet to turn their own needs." (WCED, 1987).

In other words, "we do not receive as an inheritance the environment from our fathers, but we borrowed from our children" (Murphy, 1994). The tourism must lead to management of all resources in such a way that economic, social and aesthetic needs can be fulfilled while maintaining cultural integrity, essential ecological processes, biological diversity and life support systems. In addition they describe the development of sustainable tourism as a process which meets the needs of present tourists and host communities whilst protecting and enhancing needs in the future (World Tourism Organization 1996).

Several recent papers studies the problem of relationship between the tourism flow with environment pollution, but few of them use systematic mathematical approach to solve and discuss the issue. The results obtained are often only qualitative and aren't sufficient to represent the real problems.

On the other hand it is interesting to note the path represented of the following articles, starting with "Lewis and Schmalense (1982)", and continuing with "Tahvonen and Salo (1996)", and yet with "Tahvonen and Withagen (1996)", up to recent papers of "Brock and Starrett (1999)", "Dechert and Block (1999)", "Maler (2000) and Rondeadu (2001)". All these articles deal emphasize the possibility of the existence of multiple equilibriums associated to the thresholds in models of optimal renewable resource extraction. The common feature of all these papers is that connect the existence of thresholds to the multiple equilibriums and these equilibriums are only steady states; instead in our model we have not steady states. In the present article is our purpose to define a zone of the cartesian plane, in which one can to find the states of the dynamic equilibrium, doing change two parameters, that we will indicate later with r and β, and this we will obtain in following to the study of sign of the functions:

$\frac{dE(t)}{dt} > 0$	$\frac{dK(t)}{dt} > 0$

where $E(t)$ is the environmental quality, and $K(t)$ is the tourist facilities. The our approach to the problem is that to consider a dynamic model in which the players are the governance of the local population and the tourists, supposing to see the problem from the governance point of view that wants maximize its utility function.

2 The tourism that exploit the natural resources

Before studying the issue with use of the mathematical model, let us consider the relationship between the tourism and natural resource, in order to classify the kinds of natural resources and the kinds of pollution. We define natural resources like the wealth does not produced with human work, but let as an inheritance from nature. This assumption does not exclude the possibility that the natural resources are physically produced, but this process is a natural process. The natural resources can be classified according to various aspects, which are important from an economic point of view:

a) subject to availability, natural resources can be distinguished in exhaustible and inexhaustible;

b) according to the natural possibility of reinstatement of the resources, these can be distinguished in reproducible and not reproducible;

The classification of natural resources according to their decay due to the use and according to their reproducibility gives rise to four possibilities, each of which carries with oneself a particular problem of exploitation of the tourism activity. The idea is to use an index $E(t)$ by which to express the state of the natural resource over time t, whose evolution will be described by the differential equation of constraint that will be illustrated below.

2.1 Natural resources indestructible and not reproducible

A natural resource is indestructible and not reproducible if the use does not alter its future availability. An example of this type of resource is the solar energy, but we can also think of the sea or coast for seaside tourism (a city of the sea, remains a city of the sea regardless of the quality of the sea). If $E(t)$ is the state of the resource, the idea of indestructibility and not reproducibility is expressed by the following differential equation:

$$\frac{dE(t)}{dt} = 0 \qquad (1)$$

which defines the status of a resource that does not change over time.

2.2 Natural resources not reproducible and exhaustible

A resource is not reproducible and is exhaustible if due to its use, it ends. The classic example is that of the mines which are exhausted by their exploitation. In the case of the tourism, one can make the hypothesis that the depletion of the resource is done by tourists, according to a function $F(R) = \vartheta R$ that, for simplicity, we considered linear, where R is the part of the natural resources that we suppose to be used to tourism activity. Therefore the state of the resource can be described by the following differential equation:

$$\frac{dE(t)}{dt} = -\vartheta R(t) \qquad 0 < \vartheta < 1 \qquad (2)$$

which expresses the value of $E(t)$ that decreases as a function of a intensity proportional to the exploitation of tourists. Indeed, the exhaustible resources show a generational problem: more tourists for the present generation undermine the wealth of future generations. The optimal use of exhaustible resource is treated as a social problem between generations, rather than strictly individual.

2.3 Natural resources reproducible and exhaustible

Some natural resources are able to regenerate themselves, for example one can consider aquatic species or even the woods and more. The status of these resources may be represented with the following linear differential equation:

$$\frac{dE(t)}{dt} = G(E(t)) = \alpha(\overline{E} - E(t)) \qquad 0 < \alpha < 1 \qquad (3)$$

where $\overline{E}$ is the natural steady state of environmental with no resource exploitation, i.e. it represents an equilibrium state of the environmental and α is the speed by which the natural resources regenerate themselves.

In our model, that we show in next paragraph, we can induce the value of $E(t)$ choosing the appropriate value of the control variables C and R. It is immediately clear that no every state of the resource, represented of the value of $E(t)$ can be maintained:

a) if the initial condition $E(0)$ is less than the desired value E^*, it is sufficient to use values for R and C such that $\beta > r$, so that increases the natural reproduction, up to become equal to E^*;

b) if the initial condition $E(0)$ is greater than the desired E^*,it is sufficient to exploit the natural resources by increasing values of R and C, such that $\beta < r$ in order to decrease the value of E until you reach E^*.

2.4 Natural resources reproducible and inexhaustible

The case of the resources reproducible and inexhaustible is an ideal case and we do not consider it as a real case. Infact, in this particular situation the governance could to do indifferent strategies for activity of the tourism and due to this reason, we do not consider this case is an interessant case to discus.

3 Mathematical Model

Suppose to be in presence of a tourist resort, in which the principal economic activity is the tourism. The local population wants to manage the touristic flow to do not damage environment. The topic we face is to describe the interaction dynamics between Tourism and environmental quality, in presence of " reproducible and exhaustible resources ". Assume that the presence of tourists is a variable positively conditioned by the environmental quality $E(t)$ and from the presence of tourist facilities $K(t)$. Consider a continuos-time model in which R(t) is the part of natural resources that the local governance decides to offer to the tourists. Let $K(t)$ and $E(t)$ be respectively the stock of tourist facilities and the stock of a renewable natural resources. Those will be considered such as substitutable inputs in the aggregate production function of tourism activity. The natural question arising is, how to control in optimal way the tourists without damage resources. The mathematical model that we propose to answer above question needs to use an aggregate production function of the form:

$$Y = \sqrt{KR} \tag{4}$$

which output can be consumed or invested. Let C denote consumption and I denote investment. Then:

$$C = \sqrt{KR} - I \tag{5}$$

Furthermore, assume there is no depreciation of touristic facilities, then we can consider:

$$\frac{dK}{dt} = I \tag{6}$$

Let G(E(t)) be the natural growth function of the resources. We assume a linear specification $G(E(t))$:

$$G(E) = \alpha(\overline{E} - E(t)) \tag{7}$$

where $\alpha \in [0, 1]$.
There exist two interesting cases when: $E \leq \overline{E}$ and $E > \overline{E}$. Our goal is to study the dynamics of the problem in both cases. The net growth rate of the resource stock is:

$$\frac{dE}{dt} = G(E) - R \tag{8}$$

We formulate the social planet problem, we assume that: the instantaneous utility function depends on C and E, in order to simplify the analysis we consider the following specification:

$$U(C) = \sqrt{C} + \gamma E \qquad \gamma \in [0, 1] \tag{9}$$

where γ measure the importance of the environmental quality for the local population. Notice that the linearity in E could be justified for two reasons: 1)it leads in very elegant way to a closed form solution of the problem;
2)by the no clear relationship between the environmental and utility function, it could be considered as the more plausible assumption.

If we suppose to be in presence of a local poor population, the governance deals do not much of the environmental quality; thus it is corrected to think that γ assumes a value much much small. Instead, if the local population is sufficiently rich, it is correct to think that γ assumes a value nearly 1. Therefore we will see that the key role of the model is given by the relative magnitude of α and γ. The objective of the local population and in particular of the local governance, is to maximize the integral of the discounted utility:

$$max \int_0^T e^{-rt}[\sqrt{C} + \gamma E]dt \qquad \gamma << 1 \tag{10}$$

The maximization is subject to the constraints:

$$\frac{dK}{dt} = \sqrt{KR} - C, \qquad \frac{dE}{dt} = G(E) - R \tag{11}$$

with boundary conditions $K(0) = K_0 > 0, E(0) = E_0 > 0$.

3.1 Steady States

To start, we define the Hamilton's Function as follows(see Clark 1976):

$$H(C, K, R, E, \lambda_1, \lambda_2) = \sqrt{C} + \gamma E + \lambda_1[\sqrt{KR} - C] + \lambda_2[G(E) - R] \tag{12}$$

where λ_1 is the shadow price of touristic facilities and λ_2 is the shadow price of the renewable resource. The necessary conditions, for Pontryagin's theorem, are:

$$\frac{\partial H}{\partial C} = \frac{1}{2\sqrt{C}} - \lambda_1 = 0 \tag{13}$$

$$\frac{\partial H}{\partial R} = \frac{1}{2}\lambda_1\sqrt{\frac{K}{R}} - \lambda_2 = 0 \tag{14}$$

$$\dot{\lambda_1} = \lambda_1(r - \frac{1}{2}\sqrt{\frac{R}{K}}) \tag{15}$$

$$\dot{\lambda_2} = \lambda_2(r - (\gamma - \alpha)), \qquad where \quad \alpha > \gamma \tag{16}$$

to simplify the notation we introduce $\beta \equiv \gamma - \alpha$, therefore we can rewrite:

$$\dot{\lambda_2} = \lambda_2(r - \beta), \tag{17}$$

from above equations (13) and (14), follows that λ_1 and λ_2 are always greater zero, furthermore, $\dot{\lambda_2}$ is equal zero only for $r = \beta$; we can write too:

$$(r - \beta) - (r - \frac{\partial F}{\partial K}) = \frac{\dot{\lambda_2}}{\lambda_2} - \frac{\dot{\lambda_1}}{\lambda_1} = \frac{1}{\frac{\partial F}{\partial R}}\dot{\frac{\partial F}{\partial R}} \tag{18}$$

Hence

$$\frac{\partial F}{\partial K} = \beta + \frac{1}{\frac{\partial F}{\partial R}}\dot{\frac{\partial F}{\partial R}} \qquad\qquad \frac{\dot{C}}{2C} = \frac{\partial F}{\partial K} - r \tag{19}$$

which is the Ramsey-Euler Rule: "The proportional rate consumption growth, multiplied by the elasticity of marginal utility, must be equated to the difference between the rate of interest $\frac{\partial F}{\partial K}$ and the utility discounted rate, r."
Now, it is convenient to define a new variable x as follows:

$$x = \frac{K(t)}{R(t)} \tag{20}$$

with this variable we measure the capital(touristic facilities) intensity of the production process at time t. We want to find, if they exist stationary solutions, but before this, we show that x satisfies the following differential equation:

$$-\frac{1}{2}\frac{1}{\sqrt{x}} - \frac{1}{2}\frac{\dot{x}}{x} = -\beta \tag{21}$$

This is shown from the necessary condition (16):

$$-\frac{\dot{C}}{2C} = \frac{\dot{\lambda_1}}{\lambda_1} \tag{22}$$

but we have from (18):

$$\frac{\dot{\lambda}_1}{\lambda_1} = r - \frac{1}{2}\frac{1}{\sqrt{x}} \tag{23}$$

From (17) we get the relationship between λ_1 and x:

$$\frac{\partial H}{\partial R} = \frac{1}{2}\lambda_1\sqrt{x} - \lambda_2 = 0 \tag{24}$$

so that

$$\frac{\dot{\lambda}_2}{\lambda_2} = \frac{\dot{\lambda}_1}{\lambda_1} + \frac{1}{2}\frac{\dot{x}}{x} \tag{25}$$

therefore, substituting we obtain:

$$-\frac{1}{2}\frac{1}{\sqrt{x}} - \frac{1}{2}\frac{\dot{x}}{x} = -\beta \tag{26}$$

Using equation $-\frac{\dot{C}}{2C} = 0$, we can to deduce the stationary solution for x:

$$x = \left(\frac{1}{2r}\right)^2 \tag{27}$$

Substituting this value into (26), and noting that $\lambda_2 > 0$, we can conclude that the steady state requirement for $-\frac{\dot{C}}{2C} = 0$ implies that at the steady state, $\frac{\dot{\lambda}_2}{\lambda_2} = 0$. But this is possible only if $r = \beta$, and this proves, that there are no steady states.

3.2 Dynamic of the variables of State and of the Control variables

3.2.1 The time path of capital/ratio

The time path of capital/ratio, input of the aggregate production function, is expressed by the variable $x(t)$, which dynamic is the solution of last differential equation (27). Multiplying each side of (27) by $\sqrt{x}$ we reach:

$$\frac{-1}{2} - \frac{1}{2}\frac{\dot{x}}{\sqrt{x}} = -\beta\sqrt{x} \tag{28}$$

let $y = \sqrt{x}$

$$-\frac{1}{2} + \dot{y} = -\beta y \tag{29}$$

the solution can be written in the form:

$$y(t) = \left(y_0 - \frac{1}{2}\right) e^{-\beta t} + \frac{1}{2\alpha} \tag{30}$$

or

$$y(t) = \left(y_T - \frac{1}{2\beta}\right) e^{-\beta(t-T)} + \frac{1}{2\beta} \tag{31}$$

where $y(T) = y_T$, therefore we have:

$$x(t) = \left(\left(\sqrt{x_0} - \frac{1}{2\beta}\right) e^{-\beta t} + \frac{1}{2\beta}\right)^2 \tag{32}$$

or

$$x(t) = \left(\left(\sqrt{x_T} - \frac{1}{2\beta}\right) e^{-\beta(t-T)} + \frac{1}{2\beta}\right)^2 \tag{33}$$

Imposing the transversal condition that at the time T, the variable x calculated in $t = T$, takes the following value:

$$x_T = \left(\frac{1}{2r}\right)^2 \tag{34}$$

which is its steady state value. It results that the capital intensity $x(t) = \dfrac{K(t)}{R(t)}$ is decreasing over time for $0 < r < \beta$ and is increasing for $r > \beta$. In order to prove the above statement, could be useful to compute $\dfrac{\dot{x}}{x}$:

$$\frac{\dot{x}}{x} = -2\beta \frac{\left(\sqrt{x_T} - \frac{1}{2\beta}\right) e^{-\beta(t-T)}}{\left(\left(\sqrt{x_T} - \frac{1}{2\beta}\right) e^{-\beta(t-T)} + \frac{1}{2\beta}\right)} \tag{35}$$

$$= -2\beta \left(1 - \frac{1}{\left(\left(\frac{\beta}{r} - 1\right) e^{-(\beta(t-T))}\right) + 1}\right) \tag{36}$$

so that being for $\left(\dfrac{\beta}{r} - 1\right) > 0$ results $\dfrac{\dot{x}}{x} < 0$ and for $\left(\dfrac{\beta}{r} - 1\right) < 0$ results $\dfrac{\dot{x}}{x} > 0$.

3.2.2 The time path of λ_1 and λ_2

The path of λ_1 is given by the following differential equations:

$$\frac{\dot{\lambda}_2}{\lambda 2} = \frac{\dot{\lambda}_1}{\lambda 1} + \frac{1}{2}\frac{\dot{x}}{x} \tag{37}$$

with

$$\dot{\lambda_2} = \lambda_2(r - \beta) \tag{38}$$

such that

$$\frac{\dot{\lambda_1}}{\lambda 1} = (r - \beta) - \frac{1}{2}\frac{\dot{x}}{x} \tag{39}$$

the integration gives:

$$\lambda_1(t) = \lambda_1(T)\frac{\sqrt{x(T)}}{\sqrt{x(t)}}e^{(r-\beta)(t-T)} \tag{40}$$

Thus we have, for $r < \beta$

$$\frac{d\lambda_1(t)}{dt} > 0. \tag{41}$$

and for $r > \beta$

$$\frac{d\lambda_1(t)}{dt} < 0. \tag{42}$$

3.2.3 The time path of control variable C

The path of consumption is given by the following equation, which has been obtained using the previous equations:

$$\left(\frac{1}{2\lambda_1}\right)^2 = C \tag{43}$$

that is

$$C(t) = \frac{C(T)\left(\left(\sqrt{x(T)} - \frac{1}{2\beta}\right)e^{-\beta(t-T)} + \frac{1}{2\beta}\right)^2 e^{-2(r-\beta)(t-T)}}{x(T)} \tag{44}$$

The evolution of the consumption path is given by:

$$\frac{\dot{C}}{C} = -2\frac{\dot{\lambda_1}}{\lambda_1} < 0 \tag{45}$$

Thus we have that, $\frac{\dot{C}}{C}$ is negative for $r < \beta$ and positive for $r > \beta$.

3.2.4 The time path of control variable R

From the definition of $x = \dfrac{K}{R}$ we have:

$$\dot{K} = \dot{R}x + \dot{x}R \tag{46}$$

and

$$\dot{K} = \sqrt{KR} - C = R\sqrt{x} - C \tag{47}$$

so

$$\dot{R}x + \dot{x}R = R\sqrt{x} - C \tag{48}$$

either

$$\dot{R} = R\left(\frac{1}{\sqrt{x}} - \frac{\dot{x}}{x}\right) - \frac{C}{x} \tag{49}$$

using (23) we have:

$$\frac{-1}{2} - \frac{1}{2}\frac{\dot{x}}{\sqrt{x}} = -\beta\sqrt{x} \tag{50}$$

so

$$\dot{R} = 2\beta R - \frac{C(t)}{x(t)} \tag{51}$$

where C(t) is given by (44) and x(t) is given by (33) and so that we can write:

$$\dot{R(t)} = 2\beta R(t) - \frac{C(T)e^{-2\beta(r-\beta)(t-T)}}{x(T)} \tag{52}$$

The exact solution is:

$$R(t) = \frac{C(T)}{2x(T)r}e^{2\beta(r-\beta)(t-T)} + \left(R(T) - \frac{C(T)}{2x(T)r}\right) \tag{53}$$

with

$$x(T) = \left(\frac{1}{2r}\right)^2, \qquad C(T) = \beta\overline{E}\left(\frac{1}{2r}\right) \tag{54}$$

and $R(T) = \beta\overline{E}$. Thus, we have:

$$R(t) = \alpha\overline{E}e^{(\beta-r)(t-T)}, \tag{55}$$

$$\dot{R(t)} = 2(\beta - r)\beta\overline{E}e^{(\beta-r)(t-T)} \tag{56}$$

which is positive for $r < \beta$ and negative for $r > \beta$

3.2.5 The time path of capital K

We now turn to the capital:

$$K = xR \tag{57}$$

$$\frac{\dot{K}}{K} = \frac{\dot{x}}{x} + \frac{\dot{R}}{R} = \frac{\dot{x}}{x} + 2(\beta - r) \tag{58}$$

using (58) and (23) yields

$$\frac{\dot{K}}{K} = -2\beta + \frac{1}{\sqrt{x}} + 2(\beta - r) = \frac{1}{\sqrt{x}} - 2r \tag{59}$$

since $\dot{x} < 0$ results that $\frac{d\sqrt{x}}{dt} > 0$ with $X(T) = (2r)^{-1}$ and therefore $\frac{1}{\sqrt{x}} - 2r < 0$ for all t¡T and thus we can write as follows:

$$\frac{\dot{K}}{K} = \frac{1}{\sqrt{x}} - 2r < 0 \tag{60}$$

through the appropriate substitution we have:

$$K = xR = \beta\overline{E}e^{2(\beta-r)(t-T)}\left(\left(\sqrt{x(T)} - \frac{1}{2\beta}\right)e^{-\beta(t-T)} + \frac{1}{2\beta}\right)^2 \tag{61}$$

and

$$K(0) = K_0 = \beta\overline{E}e^{-2(\beta-r)(T)}\left(\left(\frac{1}{2r} - \frac{1}{2\beta}\right)e^{\beta(T)} + \frac{1}{2\beta}\right)^2 \tag{62}$$

$$\frac{dK_0}{dT} = \beta\overline{E}\frac{d\left(\left(e^{-(\beta-r)(T)}\left(\left(\frac{1}{2r} - \frac{1}{2\beta}\right)e^{\beta(T)} + \frac{1}{2\beta}\right)\right)^2\right)}{dT} \tag{63}$$

that is positive for $r < \beta$ and negative for $r > \beta$.

3.2.6 The time path of the resource E

The behaviour of the environmental resources is out line by following differential equation:

$$\dot{E} = \beta E - R \tag{64}$$

substituting R from (57)

$$\dot{E} = \beta E - \beta\overline{E}e^{2\beta(\beta-r)(t-T)} \tag{65}$$

$$E(T) = \overline{E} \tag{66}$$

exact solution is:

$$E(t) = \overline{E}\left(\frac{\beta}{-\beta+2r}e^{2(\beta-r)(t-T)} + 2\frac{-\beta+r}{-\beta+2r}e^{(\beta)(t-T)}\right) \tag{67}$$

$$\dot{E}(t) = 2\beta(\beta-r)e^{\beta(t-T)}\overline{E}\left(\frac{e^{(\beta-2r)(t-T)}-1}{(-\beta+2r)}\right) \tag{68}$$

The initial quality of environmental is given by $E(0) = E_0$

$$E(0) = E_0 = \overline{E}(T)\left(\frac{\beta}{-\beta+2r}e^{-2(\beta-r)T} + \frac{e^{(\beta-2r)T}-1}{(-\beta+2r)}\right) \tag{69}$$

so we have

$$\frac{dE_0}{dT} = 2\beta(\beta-r)\overline{E}e^{-\beta T}\left(\frac{1-e^{-(\beta-2r)T}}{-\beta+2r}\right) \tag{70}$$

that assume positive value for $\frac{\beta}{2} < r < \beta$ and negative value for $r < \frac{\beta}{2} \bigcup r > \beta$. Remembering that $\frac{dK_0}{dT} > 0$ for $r < \beta$ and $\frac{dK_0}{dT} < 0$ for $r > \beta$, we can study the sign of the ratio of the variations $\frac{dK_0}{dE_0}$, obtaining an important result to understand, in order to make decisions what strategies take to manage the resources of the nature E(t) and the flow of tourism attracted by the tourism facilities(else capital K(t)) and by the environmental. So, we have the following pattern:

$\frac{dK_0}{dE_0} > 0$	$\frac{\beta}{2} < r < \beta$
$\frac{dK_0}{dE_0} < 0$	$r < \frac{\beta}{2} \bigcup r > \beta$

in the above table is possible to read that exists an interval for r in which K and E crease together, but outside of this, we have that if K crease, E decrease and if K decrease, E crease.

4 Conclusions

The present work deals, in mathematical way, of the topic of the Tourism and the problems that this activity produce on the environmental. In principle we classify the kinds of relationship between the human activity and the environmental, giving a definition of sustainability development, and thus of tourism sustainable. We indicate three kinds of constraints, for classify the several problems that one can verify:

1)THE CO-EXITENCE, when the two phenomena can be isolated, without any contact or with few negligible connections;

2)CONFLICT, if the action of the tourists make up obvious environmental damage;
3)THE SYMBIOTIC RELATIONSHIP, is the relationship in which both the tourism that the environment, can to receive benefits.

Considering the tourism as an activity that can to coexist with a good quality of environmental, we decide to be in the third case above indicated. Therefore, we classify the kinds of resources as follows:

1)Natural resources indestructible and not reproducible
2)Natural resources not reproducible and inexhaustible
3)Natural resources reproducible and exhaustible
4)Natural resources reproducible and inexhaustible

In this article we consider the environmental like a resource reproducible and exhaustible. Then, we assume a typical aggregate production function, and suppose that there is no deprecation of the touristic facilities. Furthermore, we choose the growth function of the environmental and indicate this with G(E), in which we have β the rate of growth of the environmental and r the discounted rate for the utility function. At this point, we define the constraints, and we write the Hamilton's function and thus calculate the equations of Hamilton. The solution is that doesn't exist any steady states, and that by study of the dynamic of the variables, of the control variables and of the prices shadow, we obtain the sign of each of them. Thus we can to define the strategies most useful at our objective, i.e. the strategies that allow manage the tourism activity, to protect the environmental quality, and maximize the Utility function defined in former. The strategies of this kind, are strategies that consider the relations between the rate of natural growth with the rate used for discounted value of the utility function. As indicated in the table of the above paragraph, the co-existence between human activity and a good environmental quality is possible, but to obtain an dynamic equilibrium between the two activity, the parameters introduced, r and β, should be nearly equal, but instead, are in general very different between them. Thus the unique way for manage the tourism activity is to switch from the policy for tourism, to the policy doesn't for the tourism in way that the nature can autoregenerate. The gift of the present paper, is to have demonstrated in mathematical language, that is no possible which exists an touristic eternal town, if this base its tourism activity only on the environmental quality.

References

Agarwal, S. 1997. The resort cycle and seaside tourism: an assessment of its applicability and validity. Tourism Management 18:65-73.

Anderies, J. M. 2000. On modelling human behavior and institutions in simple ecological economic systems. Ecological Economics 35:393-412.

Arrow, K., B. Bolin, R. Costanza, P. Dasgupta, C. Folke, C. S. Holling, B. Jansson, S. Levin, K. Mler, C. Perrings, and D. Pimental. 1995. Economic growth, carrying capacity and the environment. Science 268:520- 521.

Beltratti, A. 1997. Growth with natural and environmental resources. Pages 7-42 in C. Carraro and D. Siniscalco, editors. New directions in the economic theory of the environment. Cambridge University Press, Cambridge, UK.

Bender, M. J., G. V. Johnson, and S. P. Simonovic. 1994. Sustainable management of renewable resources: a comparison of alternative decision approach. International Journal of Sustainable Development 1:77-88.

Brock, W. A., and A. G. Malliaris. 1989. Differential equations, stability and chaos in dynamic economics. North-Holland, Amsterdam, The Netherlands.

Brown, K., R. K. Turner, H. Hameed, and I. Bateman. 1996. Tourism and sustainability in environmentally fragile areas: case studies from the development and cultural theory. The International Journal of Sustainable Development and World Ecology 3:1-14.

Bramwell, B., and B. Lane. 1993. Sustainable tourism: an evolving global approach. Journal of Sustainable Tourism 1:6-16.

Butler, R. W. 1980. The concept of a tourist area cycle of evolution: implications for management of resources. Canadian Geographer 24:5-12.

Butler, R. W. 1991. Tourism, environment, and sustainable development. Environmental Conservation 18:201-209.

Carpenter, S., W. Brock, and P. Hanson. 1999a. Ecological and social dynamics in simple models of ecosystem management. Conservation Ecology 3(2):4. *URL : http : //www.consecol.org/vol3/iss2/art4*.

Cheve, M. 2000. Irreversibility of pollution accumulation. Environmental and Resource Economics 16:93-104.

Christaller, W. 1963. Some considerations of tourism location in Europe: the peripheral regions Conservation Ecology 6(1):13.
URL : http : //www.consecol.org/vol6/iss1/art13 underdeveloped countries-recreation areas. Papers of the Regional Science Associations 12:95-105.

Clark, C. W. 1976. Mathematical bioeconomics: the optimal management of renewable resources. John Wiley Sons, New York, New York, USA.

Coccossis, H., and P. Nijkamp, editors. 1995. Sustainable Tourism Development. Avebury, Aldershot,UK.

Goodland, R., and G. Ledec. 1987. Neoclassical economics and principles of sustainable development. Ecological Modelling 38:19-46.

Harrison, D. 1995. Development of tourism in Swaziland. Annals of Tourism Research 22:135-156.

Herfindahl, O. C., and A. V. Kneese. 1974. Economic Theory of Natural Resources. Merrill Publishing Company, Columbus, Ohio, USA.

Holling, C. S. 1986. The resilience of terrestrial ecosystems: local surprise and global change. Pages 292- 317 in W. C. Clark and R. E. Munn, editors. Sustainable development of the biosphere. Cambridge University Press, Cambridge, UK.

Holling, C. S. 1993. Investing in research for sustainability. Ecological Applications 3:552-555.

Keeney, R. L., and H. Raiffa. 1976. Decisions with Multiple Objectives: Preferences and Value Tradeoffs. Wiley, New York, New York, USA.

Lim, C. 1997. Review of international tourism demand models. Annals of Tourism Research. 24:835-849.

Lindberg, K., editor. 1991. Policies for Maximizing Nature Tourism's Ecological and Economic Benefits. World Resources Institute, Washington, DC, USA.

Ludwig, D. 1993. Environmental sustainability: magic, science, and religion in natural resource management. Ecological Applications 3:555-558.

Morley, C. L. 1998. A dynamic international demand model. Annals of Tourism Research 25:70-84.

Pecchenino R. Jhon A. An overlapping generations model of growth and the environment. Econ J 1994; 104: 1393-410.

Rinaldi, S., W. C. Sanderson, and A. Gragnani. 1996. Pollution control policies and natural resources dynamics: a theoretical analysis. Journal of Environmental Management 48:357-373.

Roe, E. M. 1996. Sustainable development Journal of Sustainable Development and World Ecology 3:1-14. Ross, R. 1909. The Prevention of Malaria. Murray, London, UK.

Shah, A. 1995. The Economics of Third World National Parks. Edward Elgar Publishing, Aldershot, UK.

Smith, V. L., and W. R. Eadington, editors. 1992. Tourism Alternatives: Potential and Problems in the Development of Tourism. University of Pennsylvania Press, Philadelphia, Pennsylvania, USA.

Solow, R. M. 1991. Sustainability: an Economist's Perspective. Marine Policy Center, Woods Hole Oceanogravarphic Institution, Massachusetts, USA.

Turner, R. K. 1993. Sustainability: principles and practice. Pages 3-36 in R. K. Turner, editor. Sustainable environmental economics and management. Belhaven, London, UK.

Wall, G. 1997. Is ecotourism sustainable? Environmental Management 21:83-491.

Walker, P. A., R. Reiner, D. McDonald, and V. Lyne. 1998. The Tourism Futures Simulator: a systems thinking approach. Environmental Modelling Software 14:59- 67.

Walters, C. J. 1986. Adaptive Management of Renewable Resources. McGraw Hill, New York, New York, USA.

Wirl F. 2004. Analysis Threshold in concave renewable resource models Ecology Economics 48. (259-267).