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Abstract

This paper proposes a model in which identical sellers of a homogenous product compete in both prices and price frames (i.e., ways to present price information). We model price framing by assuming that firms' frame choices affect the comparability of their price offers: consumers may fail to compare prices due to frame differentiation, and due to frame complexity. In the symmetric equilibrium the firms randomize over both price frames and prices, and make positive profits. This result is consistent with the observed coexistence of price and price frame dispersion in the market. We also show that (i) the nature of equilibrium depends on which source of consumer confusion dominates, and (ii) an increase in the number of firms can increase industry profits and harm consumers.

Keywords: bounded rationality, framing, frame dispersion, incomplete preferences, price competition, price dispersion
JEL classification: D03, D43, L13

1 Introduction

Suppliers often use different ways to convey price information to consumers. Supermarkets frame differently price promotions using, for instance, a direct price reduction, a percentage discount, or a quantity discount. Some restaurants and online booksellers offer a single all-inclusive price, while others divide the price into pieces (e.g., quoting the shipping fee, the table service, or the VAT separately). In insurance, gas

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and electricity, financial services, and mortgage markets, firms commonly use different price presentation modes such as differentiated multi-part tariffs.[1] Frequently, markets with cross-sectional differences in the price frame also exhibit price dispersion.[2] Despite the prevalence of price framing, the practice has received little attention in the economic literature. There is no clear explanation why different firms employ different price frames or why the same firm changes its price frame over time. If the firms use different price presentation modes to compete better for consumers, industry-specific pricing schemes whose terms allow for better comparisons should emerge. In contrast, the persistence of much variation and complexity of price framing seems more likely to confuse consumers and harm competition.

To address this issue, we develop a model in which firms compete to supply a homogeneous product by simultaneously choosing both price frames and prices. We suppose that price framing can confuse the consumers and as a result they fail to compare some prices in the market. An examination of the price tags for groceries and household supplies reveals variations in both price and frames. To buy a 50 ml whitening toothpaste one can choose between Macleans (white and shine) sold at £2.31 with a "buy one get one free" offer, Aquafresh (extreme clean whitening) which "was £1.93 now is £1.28 saves 65p" and Colgate (advanced white) which costs £1.92, amongst others.[3] In these example, each price presentation mode is not particularly involved. However, the variation in the price frame may make it harder for consumers to make price comparisons. In other markets (e.g. telecommunications, mortgages) the pricing scheme usually includes many elements or pieces of information. Table 1 illustrates some prices of fixed-rate mortgages as listed on a British comparison website in July, 2009. Tables 3 in section 5 presents in greater detail the involved pricing schemes used in mobile telephony. In these markets, even if firms adopt the same type of price frame (e.g., a tariff with the same elements), complexity might still make it difficult for consumers to compare prices correctly.

Table 1: Fixed-rate mortgages as listed on www.confused.com (July 31, 2009)

Lender	Initial rate	Subsequent rate	Overall cost	Max% LTV	Lender fee
Leek	3.39%	5.19%	5.2%	75%	£1495
Britannia	4.34%	4.24%	4.5%	60%	£599
Chelsea	4.80%	5.79%	5.5%	65%	£995

[1] Although the existence of multiple tariffs within a firm is mainly driven by demand heterogeneity, the variation in tariff structures across firms is more likely to aim to obstruct consumers' price comparisons.

[2] In fact, many markets are not only characterized by cross-sectional price presentation dispersion, but also by temporal dispersion. Suppliers change both their price frames and the related prices over time. For example, supermarkets change the discounting method from time to time.

[3] These are actual prices collected from a leading supermarket in London on August 11, 2009.

Our model examines the two sources of confusion illustrated above, frame differentiation and frame complexity, in a unified framework.[4] More specifically, we consider two possible (categories of) frames—one relatively simple and the other one (potentially) complex. The consumers might be confused either by frame differentiation (i.e., they fail to compare prices in different frames) or by frame complexity (i.e., they fail to compare prices in the complex frame).

The observed price framing raises the following questions: Is there a relationship between frame dispersion and price dispersion? Do market outcomes depend on whether frame differentiation or frame complexity is more confusing? In the presence of price framing, does an increase in the number of competitors reduce market complexity and enhance consumer welfare? In this research, we show that in the symmetric equilibrium of the price-frame competition model, the firms randomize over *both* prices and price frames, and the features of the equilibrium depend on which source of confusion dominates. We analyze the impact of a decrease in market concentration on firms' equilibrium strategies and on consumer welfare. We find that greater competition tends to increase the price complexity in the market and, under certain conditions, it harms consumers.

Section 2 illustrates the frame and price dispersion result in a duopoly market. For simplicity, we discuss here a version where only frame differentiation is confusing. That is, all consumers accurately compare prices if the firms use the same frame, and all are confused and shop randomly if the firms use different frames. If both firms used the same frame at equilibrium, then they could only price at marginal cost. But, a unilateral deviation to the other frame yields positive profits on the random shoppers. If the firms used different frames, they could only choose the monopoly price. But, a unilateral deviation to rival's frame and a slightly lower price allows to serve the whole market. So, in any equilibrium the firms must mix on frames. As the firms face both price aware buyers (who compare prices perfectly) and confused buyers (who shop at random) with positive probability, they must also mix on prices in equilibrium.

Although there is frame and price dispersion even only with confusion stemming from frame differentiation, the nature of the equilibrium depends on which source of confusion dominates. If frame complexity is more confusing than frame differentiation, then the more complex frame is always associated with higher prices. In contrast, if frame differentiation is more confusing than complexity, there is no clear monotonic relationship between the prices associated with different frames. (With two firms, the pricing strategy is actually independent of the framing strategy at equilibrium, though

[4] The marketing literature points to the fact that consumers have difficulties in comparing prices in different frames (prices which are presented differently) or prices in complex frames (prices which are complicated). See, for instance, Morwitz et al. (1998), Estelami (1997), and Thomas and Morwitz (2009).

this result does not hold in larger oligopolies.) Moreover, the relative importance of these two confusion sources plays an important role in how concentration affects market outcomes.

Section 3 presents the oligopoly model and generalizes the two-dimensional dispersion result. We first analyze two polar cases (i.e., *all* consumers are confused by frame differentiation in one case, and by frame complexity in the other.) These two cases highlight the differences introduced by the consideration of more than two firms in a relatively simple way. We then derive conditions under which the polar case equilibria hold for more general parameter values. The oligopoly model allows us to examine the impact of an increase in the number of firms on market outcomes. We find that, when competition becomes fiercer, the ability of frame differentiation to reduce price competition decreases, and firms rely more on frame complexity. In particular, in large oligopolies, firms use the more complex frame almost all the time. A consequence of this is that industry profits are always bounded away from zero regardless of the number of competitors. In addition, an increase in the number of firms might improve industry profits and harm the consumers under certain market conditions. Therefore, when firms compete in both prices and frames, a naive competition policy which simply increases the number of firms might have undesired effects.

Section 4 extends our model by considering more than two frames, but focuses on the tractable case in which all frames are symmetric and only frame differentiation causes consumer confusion. We show that the availability of more frames softens price competition and improves profits. Section 5 discusses the robustness of our results to alternative modelling approaches and the empirical relevance of our findings. It also explores an alternative interpretation of our model in which confusion stems from product framing (i.e., labeling, packaging, presentation) rather than price framing.

An emerging economic literature documents and investigates price complexity and firms' intentional attempts to degrade the quality of information to the consumers. Ellison and Ellison (2008) provide empirical evidence on retailers's use of obfuscation strategies in online markets. They show, for instance, that retailers deliberately create more confusing websites to make it harder for the consumers to figure out the total price. On the theoretical side, one stream of literature adopts the standard information search framework (Carlin (2009), and Ellison and Wolitzky (2008)) and builds on the fact that it is more costly for consumers to assess more complex prices. An increase in price complexity will reduce consumers' incentive to gather information and, then, weaken price competition.[5] Another stream of literature regards price complexity

[5]The underlying channels in these two papers are, however, very different. Ellison and Wolitzky (2008) introduce a convex search cost function in a sequential search model *a la* Stahl (1989). If a firm increases its in-store search cost (say, by making its price more complex), it makes further search more costly and, therefore, more unlikely. Carlin (2009) takes a more reduced-form approach. He

as a device to exploit boundedly rational consumers. In Spiegler (2006), consumers are assumed to just sample one random dimension of each available complex (multi-dimensional) prices, and buy from the firm with the lowest sampled fee. As a result the firms have incentives to introduce variation across different price dimensions.

Our model also considers price complexity. However, unlike the aforementioned studies, it provides a unified framework which combines the effects of price frame differentiation and price frame complexity. In our setting, frame differentiation is also a source of "market complexity", albeit different from frame complexity. In effect, this study disentangles the relative effects of frame differentiation and complexity on the market outcomes. The inability of boundedly rational consumers to compare framed prices leads to equilibrium frame dispersion in our model. As such, our work also contributes to a growing literature on bounded rationality in industrial organization (see Ellison (2006) for a review).

A feature of our model is that some consumers have to choose from a "partially ordered set" since some offers in the market are incomparable due to price framing. To deal with this consumer choice issue, our model draws on the literature on incomplete preferences (see, for example, Aumann (1962), and Eliaz and Ok (2006)) and uses a *dominance-based choice rule*. Whenever there is confusion, consumers first rule out offers which are dominated by other comparable offers in the market, and then buy from undominated ones according to a stochastic rule. In this sense, this article incorporates consumer incomplete preferences in an oligopoly pricing model.

This paper introduces "framing effects" in market competition. Research in psychology and behavioral economics has long recognized the significance of framing effects in decision making (see Tversky and Kahneman (1981), for instance). Often, people's responses to essentially the same decision problem are systematically different when the problem is framed in different ways. Here we focus on framing as a price presentation mode and on its ability to cause confusion by limiting price comparability. In a related independent article, Piccione and Spiegler (2009) also examine price-frame competition. They allow for a general frame structure by using the random graph theory, but restrict attention to a duopoly model. The relationship between equilibrium properties and the frame structure is central to their analysis. In contrast to our work, their study focuses on a default-bias interpretation. That is, consumers are initially randomly assigned to the firms, and they switch suppliers only if they find a comparable and better deal. Although our dominance-based choice rule (with uniformly

assumes that if a firm makes its price more complex, then consumers will perceive a more complex market environment and so a more costly information gathering process, which will make them more likely to remain uninformed and shop randomly. See Wilson (2008) for an alternative model in which firms differentiate their price complexities (e.g., one firm obfuscates and the other does not) in order to soften price competition.

random purchase) and their default-bias models are consistent in the duopoly case, they diverge when there are more than two firms. We discuss this difference further in Subsection 5.1.

Finally, our study is related to a vast literature on price dispersion (see Baye et al., 2006 for a survey). However, in our model, firms randomize over two dimensions, and price dispersion is rather a by-product of price frame dispersion. Carlin (2009) characterizes a two-dimensional equilibrium similar to ours when frame complexity dominates, though his modelling approach is different. (See Section 5.1 for more details.) In a model with quality choice and inattentive consumers (who care for the price, but ignore the quality), Armstrong and Chen (2009) also derive a similar equilibrium. In their setting, firms randomize over a high and a low quality, and the high quality product is always associated with higher prices.

2 A Duopoly Example

This section introduces a model of competition in prices and price frames and presents some of our main insights in a two-firm example. Consider a market for a homogeneous product with two identical suppliers, firms 1 and 2. Suppose that there are two possible price presentation modes, referred to as frames A and B. The constant marginal cost of production is normalized to zero. There is a unit mass of consumers, each demanding at most one unit of the product and willing to pay at most $v = 1$. Both firms and consumers are risk neutral. The firms simultaneously and noncooperatively choose price frames and prices p_1 and p_2. Each firm can choose just one of the two frames.

Price framing is assumed to affect consumer choice in the following way: (i) If both firms choose frame A, all consumers can perfectly compare the two prices and buy the cheaper product as long as it offers positive net surplus. Formally, in this case firm i's demand is

$$q_i(p_i, p_j) = \begin{cases} 1, & \text{if } p_i < p_j \text{ and } p_i \leq 1 \\ 1/2, & \text{if } p_i = p_j \leq 1 \\ 0, & \text{if } p_i > p_j \text{ or } p_i > 1 \end{cases} \qquad \text{for } i, j \in \{1, 2\} \text{ and } i \neq j. \tag{1}$$

(ii) If the two firms adopt different frames, a fraction $\alpha_1 > 0$ of consumers get confused and are unable to compare the two prices. In this case, we assume that they shop at random (whenever $p_i \leq v$, $\forall i$).[6] The remaining $1 - \alpha_1$ fraction of consumers are still able to accurately compare prices. (iii) If both firms choose frame B, a fraction $\alpha_2 \geq 0$ of the consumers get confused and shop at random (whenever $p_i \leq v$, $\forall i$).

[6] We also assume that even confused consumers are still able to judge if it is worth to buy the product. Alternatively, we could assume that the consumers have a budget constraint at one. Hence, in equilibrium no firm charges a price above $v = 1$.

The following table presents the fraction of confused consumers for all possible frame profiles, where z_i is the frame chosen by firm i and z_j is the frame chosen by firm j.

Table 2: Confused consumers

$z_i \setminus z_j$	A	B
A	$\alpha_0 = 0$	α_1
B	α_1	α_2

Then, firm i's profit is

$$\pi_i(p_i, p_j, z_i, z_j) = p_i \cdot [\alpha_{z_i,z_j}/2 + q_i(p_i, p_j)(1 - \alpha_{z_i,z_j})],$$

where α_{z_i,z_j} is presented in Table 2 and $q_i(p_i, p_j)$ is given by (1).

Without loss of generality, we assume that frame A is the relatively simpler frame. The supposition that nobody gets confused when both firms use frame A is only for expositional simplicity. Our main results hold qualitatively for a positive α_0 if $\alpha_0 \leq \alpha_2$ and $\alpha_0 \neq \alpha_1$.

Note that there are two sources of confusion in our model: one is *frame differentiation* (measured by α_1) and the other is the *complexity of frame B* (measured by α_2). When $\alpha_1 > \alpha_2$, frame differentiation is more confusing than frame complexity. In particular, if $\alpha_2 = \alpha_0 = 0$, the two frames are symmetric. In this case, consumers are confused only by frame differentiation (for instance, frame A is "Price incl. VAT" and frame B is "Price excl. VAT"). Conversely, when $\alpha_1 < \alpha_2$, frame complexity dominates frame differentiation in confusing consumers. For example, frame A is an all-inclusive price and frame B is a multi-dimensional price (e.g., a multi-part tariff).

When consumers get confused, we assume that their choices are entirely independent of the two firms' prices. This is a tractable way to model the idea that confusion in price comparison reduces consumers' price sensitivity. For simplicity, in this section we also assume that confused consumers shop (uniformly) randomly. However, the oligopoly model in Section 3 allows for a more general stochastic purchase rule.

Let us characterize now the equilibrium in the duopoly model. We first show that there is no pure-strategy equilibrium. All proofs missing from this section are relegated to Appendix A.

Lemma 1 *If $\alpha_1 \neq \alpha_2$, there is no equilibrium in which both firms choose deterministic price frames.*

Proof. (a) Suppose both firms choose frame A for sure. Then, the unique candidate equilibrium entails marginal-cost pricing and zero profit. But, if firm i unilaterally deviates to frame B and a positive price (no greater than one), it makes a positive profit. A contradiction.

(b) Suppose both firms choose frame B for sure. For clarity, consider two cases. (b1) If $\alpha_2 = 1$ (and so $\alpha_1 < \alpha_2$), at the unique candidate equilibrium $p_i = 1$ and $\pi_i = 1/2$ for all i. But, if firm i unilaterally deviates to frame A and price $p_i = 1-\varepsilon < 1$, it earns $(1-\varepsilon)[\alpha_1/2 + (1-\alpha_1)] > 1/2$ for any $\varepsilon < \varepsilon_0$ where $\varepsilon_0 = (1-\alpha_1)/(2-\alpha_1) > 0$. (b2) If $\alpha_2 < 1$, the unique candidate equilibrium dictates mixed strategy pricing according to a cdf on $[p_0, 1]$ as in Varian (1980), and each firm's expected profit is $\alpha_2/2 = p_0(1-\alpha_2/2)$.[7] If $\alpha_1 > \alpha_2$, firm i can make a higher profit $\alpha_1/2 > \alpha_2/2$ by deviating to frame A and price $p_i = 1$. If $\alpha_1 < \alpha_2$, firm i can make a higher profit $p_0(1-\alpha_1/2) > p_0(1-\alpha_2/2)$ by deviating to frame A and price $p_i = p_0$. Both (b1) and (b2) lead to a contradiction.

(c) Suppose firm i chooses frame A and firm j chooses B. Again consider two cases. (c1) If $\alpha_1 = 1$, the unique candidate equilibrium entails $p_i = 1$ and $\pi_i = 1/2$ for all i. But, then, firm j is better-off by deviating to frame A and $p_j = 1-\varepsilon$, in which case its profit is $1-\varepsilon > 1/2$ for any $\varepsilon < 1/2$. (c2) If $\alpha_1 < 1$, then the unique candidate equilibrium is again of Varian type and dictates mixed strategy pricing according to a cdf on $[p_0, 1]$, with each firm earning $\alpha_1/2 = p_0(1-\alpha_1/2)$. But if firm j deviates to frame A and price $p_j = p_0$, it makes a higher profit p_0. Both (c1) and (c2) lead to a contradiction. This completes the proof.[8] ■

If both firms use the same simple frame (that is, A or, when $\alpha_2 = 0$, could also be B), they compete *a la* Bertrand and make zero profits. A unilateral deviation to a different frame supports positive profits as some consumers are confused by "frame differentiation" and shop at random. For $\alpha_2 > 0$, Lemma 1 also shows that at equilibrium, the firms cannot rely on only one source of confusion. Otherwise, a firm which uses frame B would have a unilateral incentive to deviate to the simpler frame A to steal business.

However, if $\alpha_1 = \alpha_2 > 0$, there is an equilibrium in which both firms use frame B. The rest of this paper focuses on the general case with $\alpha_1 \neq \alpha_2$.

By Lemma 1, in any candidate equilibrium there is a positive probability that firms compete for fully aware consumers, *and* also a positive probability that they have bases of confused consumers who cannot compare prices at all. The conflict between the incentives to fully exploit confused consumers and to vigorously compete for the aware consumers leads to the inexistence of pure strategy pricing equilibria. The proof of the following result is standard and therefore omitted.

Lemma 2 *There is no equilibrium in which both firms charge deterministic prices.*

[7] See Baye et al. (1992) for the proof of the uniqueness in the two-firm case.

[8] Although parts (a) and (c) used the fact that consumers can compare prices perfectly when both firms use frame A, our result still holds even if $\alpha_0 > 0$ provided that $\alpha_0 \neq \alpha_1$ (the logic in (b) applies).

Lemmas 1 and 2 show that our duopoly model has only equilibria which exhibit dispersion in *both* price frames and prices.

In continuation, we focus on the *symmetric mixed-strategy equilibrium* (λ, F_A, F_B) in which each firm assigns probability $\lambda \in (0,1)$ to frame A and $1-\lambda$ to frame B and, when a firm uses frame $z \in \{A, B\}$, it chooses its price randomly according to a cdf F_z which is strictly increasing on its connected support $S_z = [p_0^z, p_1^z]$.[9] We first show that F_z is continuous (except when $\alpha_2 = 1$).

Lemma 3 *In the symmetric mixed-strategy equilibrium (λ, F_A, F_B), the price distribution associated with frame A (F_A) is always atomless and that associated with frame B (F_B) is atomless whenever $\alpha_2 < 1$.*

Denote by

$$x_z(p) \equiv 1 - F_z(p)$$

the probability that a firm with frame z charges a price higher than p. Suppose firm j is using the equilibrium strategy. Then, if firm i uses frame A and charges $p \in [p_0^A, p_1^A]$, its expected profit is

$$\pi(A, p) = p\left\{\lambda x_A(p) + (1-\lambda)\left[\alpha_1/2 + (1-\alpha_1)x_B(p)\right]\right\}. \tag{2}$$

With probability λ, the rival is also using A such that the firms compete a la Bertrand. With probability $1-\lambda$, the rival is using B, such that a fraction α_1 of the consumers are confused (by frame differentiation) and shop at random and the firms compete a la Bertrand for the remaining $1-\alpha_1$ fully aware consumers.

If instead firm i uses B and charges $p \in [p_0^B, p_1^B]$, its expected profit is

$$\pi(B, p) = p\left\{\lambda\left[\alpha_1/2 + (1-\alpha_1)x_A(p)\right] + (1-\lambda)\left[\alpha_2/2 + (1-\alpha_2)x_B(p)\right]\right\}. \tag{3}$$

With probability λ, the rival is using A such that a fraction α_1 of the consumers are confused (by frame differentiation) and shop at random and the firms compete a la Bertrand for the remaining $1-\alpha_1$ fully aware consumers. With probability $1-\lambda$, the rival is using B such that a fraction α_2 of the consumers are confused (by frame complexity) and shop at random and the firms compete a la Bertrand for the remaining $1-\alpha_2$ fully aware consumers.[10]

The nature of the equilibrium depends on which source of confusion dominates. When $\alpha_2 > \alpha_1$, if a firm switches from frame A to B, then more consumers get confused regardless of its rival's frame. In this case, each firm charges on average

[9] A symmetric mixed-strategy equilibrium can also be expressed as $(F(p), \lambda(p))$ in which $F(p)$ is the price distribution and $\lambda(p)$ is the probability of adopting frame A conditional on price p. These two expressions are equivalent to each other.

[10] The profit functions apply for any price p as $F_z(p) = 0$ for $p < p_0^z$ and $F_z(p) = 1$ for $p > p_1^z$.

higher prices when it uses frame B than when it uses frame A. For $\alpha_2 < \alpha_1$, when a firm switches from frame A to B, more consumers get confused if its rival is using A, while fewer consumers get confused if its rival is using B. In this case, there is no obvious monotonic relationship between the prices associated with A and B. The remainder of this section analyses these two cases separately.

- Frame differentiation dominates frame complexity: $0 \leq \alpha_2 < \alpha_1$

The unique symmetric equilibrium in this case dictates $F_A(p) = F_B(p)$ and $S_A = S_B = [p_0, 1]$. That is, a firm's price is *independent* of its frame. The proof of this result is relegated to Appendix A.2. To characterize the equilibrium, we use the profit functions (2) and (3). Let $F(p)$ be the common price distribution and $x(p) \equiv 1 - F(p)$. From the indifference condition $\pi(A, p) = \pi(B, p)$, one can derive $\lambda\alpha_1 = (1-\lambda)(\alpha_1 - \alpha_2)$, or

$$\lambda = \frac{\alpha_1 - \alpha_2}{2\alpha_1 - \alpha_2} \in (0, \frac{1}{2}]. \tag{4}$$

Let π be a firms' equilibrium profit. Since all prices on $[p_0, 1]$ should lead to the same profit, we can calculate π from, for example, $\pi(A, 1) = (1-\lambda)\alpha_1/2$. Then $F(p)$ solves

$$\pi(A,p) = p\{\lambda x(p) + (1-\lambda)[(1-\alpha_1)x(p) + \alpha_1/2]\} = \pi. \tag{5}$$

Finally, p_0 solves $F(p_0) = 0$:

$$p_0 = \frac{\alpha_1^2}{2(2\alpha_1 - \alpha_2) - \alpha_1^2} \in (0, 1). \tag{6}$$

Proposition 1 *When $0 \leq \alpha_2 < \alpha_1$, there is a unique symmetric mixed-strategy equilibrium in which λ is given by (4) and $F_A = F_B = F$ is defined by (5) on $[p_0, 1]$, where p_0 is given by (6). Each firm's equilibrium profit is $\pi = \alpha_1^2/[2(2\alpha_1 - \alpha_2)]$.*

The economic intuition of the price-frame independence result lies in the equilibrium λ-condition. Rewritten as $(1-\lambda)\alpha_1 = \lambda\alpha_1 + (1-\lambda)\alpha_2$, it requires the (expected) number of confused consumers to be the same when a firm uses frame A (the left-hand side) and when it uses frame B (the right-hand side). Given that in duopoly there are only two types of consumers (the confused and the fully aware), it also implies that the expected number of fully-aware consumers is the same. Therefore, the expected market composition along the equilibrium path does not depend on a firm's frame choice. Then, the pricing is also independent of the frame choice. This is because the pricing balances the incentives to extract all surplus from the confused and to compete for the fully aware and so is determined by the market composition.

Let us briefly analyze the impact of α_1 and α_2 on the equilibrium outcome. (i) When the confusion caused by frame complexity becomes more important, firms use the complex frame B more often (i.e., $1-\lambda$ increases with α_2). In particular, if frame B is also a simple frame ($\alpha_2 = 0$), then $\lambda = 1/2$, in which case firms just maximize the probability of frame differentiation $2\lambda(1-\lambda)$. (ii) When the confusion caused by frame differentiation becomes more important, firms use the simple frame A more often in order to increases the probability of frame differentiation. (iii) Equilibrium profit π increases with both α_1 and α_2. That is, confusion (regardless of its source) always improves firms' payoffs and harms consumers. (In effect, one can check that the price distributions for higher α_1 or α_2 first-order stochastically dominate those for lower α_1 or α_2.)

Finally, notice that the equilibrium price dispersion is driven by firms' obfuscation effort through random framing but *not* necessarily by the coexistence of price aware and confused consumers. This is best seen in the polar case with $\alpha_1 = 1$ and $\alpha_2 = 0$, where consumers are always homogeneous both ex-ante and ex-post (i.e., once a pair of frames is realized, either all consumers are confused or all of them are fully aware), but price dispersion still persists.

- Frame complexity dominates frame differentiation: $0 < \alpha_1 < \alpha_2$

In this case, Appendix A.3 shows that the unique symmetric equilibrium dictates adjacent supports $S_A = [p_0^A, \hat{p}]$ and $S_B = [\hat{p}, 1]$.[11] That is, frame B is always associated with higher prices than frame A. We can characterize the equilibrium by plugging $x_B(p) = 1$ in $\pi(A,p)$ in (2) and $x_A(p) = 0$ in $\pi(B,p)$ in (3). First, from the indifference condition $\pi(A,\hat{p}) = \pi(B,\hat{p})$, one can derive

$$\lambda = 1 - \frac{\alpha_1}{\alpha_2}. \tag{7}$$

Second, a firm's equilibrium profit π can be calculated from $\pi(B,1) = [\lambda\alpha_1 + (1-\lambda)\alpha_2]/2$. Then $F_A(p)$ solves

$$\pi(A,p) = p\left[\lambda x_A(p) + (1-\lambda)(1-\alpha_1/2)\right] = \pi, \tag{8}$$

while $F_B(p)$ solves

$$\pi(B,p) = p\left\{\lambda\alpha_1/2 + (1-\lambda)\left[(1-\alpha_2)x_B(p) + \alpha_2/2\right]\right\} = \pi. \tag{9}$$

Finally, the boundary conditions, $F_A(p_0^A) = 0$ and $F_A(\hat{p}) = 1$, define

$$p_0^A = \frac{\alpha_1(2\alpha_2 - \alpha_1)}{2\alpha_2 - \alpha_1^2} < \hat{p} = \frac{2\alpha_2 - \alpha_1}{2 - \alpha_1} \le 1. \tag{10}$$

In particular, when $\alpha_2 = 1$, frame B is associated with the deterministic price $\hat{p} = 1$.

[11] When $\alpha_2 = 1$, they become $S_A = [p_0^A, 1)$ and $S_B = \{1\}$.

Proposition 2 *(i) When $\alpha_1 < \alpha_2 < 1$, there is a unique symmetric mixed-strategy equilibrium in which λ is given by (7), $F_A(p)$ is defined on $S_A = [p_0^A, \hat{p}]$ and solves (8), and $F_B(p)$ is defined on $S_B = [\hat{p}, 1]$ and solves (9). Each firm's equilibrium profit is $\pi = \alpha_1 (2\alpha_2 - \alpha_1) / (2\alpha_2)$, and the boundary prices, p_0^A and $\hat{p}$, are given by (10).*

(ii) When $\alpha_1 < \alpha_2 = 1$, the equilibrium has the same form except that F_B is a degenerate distribution on $S_B = \{1\}$, and F_A is defined on $S_A = [p_0^A, 1)$.

The economic intuition underlying the separating equilibrium for $\alpha_2 > \alpha_1$ is the following. When a firm switches from frame A to B, more consumers get confused regardless of its rival's frame such that the firm has an incentive to charge higher prices. Unlike the previous case, when $\alpha_2 > \alpha_1$ the probability of using the simple frame (λ) decreases with α_1 and increases with α_2. This is mainly because, when frame complexity becomes a more important confusion source, the prices associated with frame B rise (i.e., a rival which uses frame B becomes a softer competitor), which makes the strategy of using the simple frame A and charging relatively high prices (though still lower than $\hat{p}$) become more attractive. But, the equilibrium profit increases with both α_1 and α_2, as before.

Observe that, in both Propositions 1 and 2, as $\alpha_2 \to \alpha_1$, we have $\lambda \to 0$ (i.e., the firms will always use frame B), and the price distributions associated with frame B in the two cases coincide. Therefore, when $\alpha_1 = \alpha_2 > 0$, there is a unique symmetric equilibrium in which both firms use frame B.

3 The Oligopoly Model

In this section we analyze an oligopoly model in which firms simultaneously choose prices and frames. Our main objective is to investigate the impact of an increase in the number of firms on the market outcomes.

Consider a homogeneous product market with $n \geq 2$ identical suppliers and, as before, two (categories of) frames A and B. Let A be a simple frame such that all prices in this frame are comparable (i.e., $\alpha_0 = 0$). Frame B may involve some complexity and prices in this frame are incomparable with probability $\alpha_2 \geq 0$. Consumers are confused by frame differentiation and so unable to compare prices in different frames with probability $\alpha_1 > 0$. Like in the duopoly example, framing can still lead to two types of consumer confusion: one caused by frame differentiation and the other caused by frame complexity. However, in contrast to the duopoly model, when $n \geq 3$, the simple taxonomy of consumers into confused and fully aware for any realized frame profile does no longer apply. To see why, consider the following example.

Example 1 *Let $n = 3$. Suppose firm 1 uses frame A and charges price p_1, and firms 2 and 3 use frame B and charge prices p_2 and p_3, respectively. Let $\alpha_0 = \alpha_2 = 0$ and*

$\alpha_1 = 1$. *Then, the consumer can accurately compare* p_2 *with* p_3*, but cannot compare* p_1 *with either* p_2 *or* p_3.

In this example, there are no fully aware consumers because $\alpha_1 = 1$, and there are also no fully confused consumers because $\alpha_2 = 0$ and the consumers can always compare the two prices in frame B.[12] As we shall see, this difference underlies the impossibility of price-frame independence for $n \geq 3$.

Example 1 illustrates the fact that with more than two firms the consumer might have to choose an alternative from a "partially ordered set". This is reminiscent of incomplete preferences.[13] From consumers' viewpoint, a combination of frame and price, say (z, p), is an *alternative*. When there is confusion in the market, the consumers behave as if they had *incomplete preferences* over the set of alternatives.

Consumer choice rule. Building on the literature of incomplete preferences (see, for example, Aumann (1962), and Eliaz and Ok (2006)), we adopt a *dominance-based* choice rule to deal with the issue of consumer choice from a partially ordered set. The basic idea is that consumers will only choose, according to some stochastic rule, from the "maximal" elements which are not dominated by any other comparable element.

Definition 1 *Firm* i *which offers alternative* $(z_i, p_i) \in \{A, B\} \times [0, 1]$ *is* ***dominated*** *if there exists firm* $j \neq i$ *which offers alternative* $(z_j, p_j < p_i)$ *and the two offers are comparable.*[14]

Then our *dominance-based* choice rule can be formally stated as follows:

1. Consumers never buy from a dominated firm.

2. Consumers purchase from the undominated firms according to the following stochastic purchase rule (which is independent of prices):[15] (i) if all these firms use the same frame, they share the market equally; (ii) if among them $n_A \geq 1$ firms use frame A and $n_B \geq 1$ firms use frame B, then each undominated A firm

[12] In a variant of Example 1 with $\alpha_i \in (0, 1)$ for $i = 1, 2$, the firms would face four types of consumers: fully aware, totally confused, (partially) confused by frame complexity, and (partially) confused by frame differentiation. When $\alpha_1 = 1$ and $\alpha_2 = 0$, only the last group survives.

[13] In a duopoly, the two offers in the market are either comparable or incomparable so that the use of incomplete preferences and a dominance-based choice rule are not necessary. However, in larger oligopolies, they are a useful tool to formalize the outcome of price framing.

[14] In our model, the comparability of two offers is independent of their comparability with other available offers. This excludes transitivity of comparability. If a consumer can compare offers in different frames, but cannot compare offers in frame B, then the presence of an offer in frame A (which is comparable with any of the B offers) will *not* make the consumer able to compare offers in frame B.

[15] The set of undominated firms is not empty. For example, the firm which charges the lowest price in the market will never be dominated.

is chosen with probability $\phi(n_A, n_B)/n_A$ and each undominated B firm is chosen with probability $[1 - \phi(n_A, n_B)]/n_B$, where $\phi(\cdot) \in (0,1)$ is non-decreasing in n_A and non-increasing in n_B and $\phi(n_A, n_B) \geq n_A/(n_A + n_B)$.

Note that there is no difference among undominated firms which use the same frame. For this reason, both 2(i) and 2(ii) assume that the consumers do not discriminate among them. However, 2(ii) allows the consumers to favor one frame over the other. Specifically, $\phi(n_A, n_B) \geq n_A/(n_A + n_B)$ means that undominated firms which use the simple frame A might be favored.[16] (Notice that $\phi(n_A, n_B) = n_A/(n_A + n_B)$ corresponds to the uniformly random purchase rule, which is used in our duopoly example in Section 2.) The monotonicity assumption in 2(ii) means that the presence of more undominated firms with one frame increases the (overall) probability that consumers buy from them.

To illustrate this choice rule, let us consider the following example.

Example 2 *In Example 1, let $p_2 < p_3$. As $\alpha_2 = 0$, all prices in frame B are comparable. Then, the consumer will eliminate firm 3's offer since it is dominated by firm 2's offer. But, as the consumer cannot compare prices in different frames ($\alpha_1 = 1$), none of the remaining offers will be dominated. Hence, the consumer will buy from firm 1 with probability $\phi(1,1)$ and from firm 2 with probability $1 - \phi(1,1)$.*

For the rest of the paper, let

$$\phi_k \equiv \phi(1, k)$$

denote the probability that a consumer buys from the A firm when there are k undominated B firms and one undominated A firm to choose from. Then, 2(ii) implies that $\{\phi_k\}_{k=1}^{n-1}$ is a non-increasing sequence: when more B firms survive, the undominated A firm has less demand, and $\phi_k \geq 1/(1+k)$.

Recall that in the duopoly example the type of market equilibrium depends on whether frame differentiation or frame complexity creates more confusion (that is, $\alpha_1 < \alpha_2$ or $\alpha_2 < \alpha_1$). The same is true in the oligopoly model. Subsections 3.1 and 3.2 analyze the corresponding symmetric equilibrium and the impact of greater competition when $\alpha_1 < \alpha_2$ and $\alpha_2 < \alpha_1$, respectively.

Before we proceed with the analysis, let us summarize two of our main findings. First, when $\alpha_2 > 0$ (i.e., when frame B is more complex), greater competition tends to induce firms to use frame B more often. In particular, when there is a large number of firms, they use frame B almost surely. Second, when $\alpha_2 > 0$, greater competition can increase industry profit and harm consumers (who may actually pay more in a more competitive market). Industry profit is bounded away from zero even when n converges to infinity.

[16] The following analysis only requires that $\phi(1, n-1) \geq 1/n$.

3.1 Frame differentiation dominates frame complexity ($\alpha_1 > \alpha_2$)

This part deals with a situation in which consumers are more likely to be confused by frame differentiation than by the complexity of frame B. For simplicity, this analysis mainly focuses on the polar case with $\alpha_1 = 1$ (i.e., prices in different frames are always incomparable). We then discuss how the main results can be extended to the case with $\alpha_1 < 1$. All proofs missing from the text are relegated to Appendix B.

We first show that there can only be mixed-strategy equilibria whenever $\alpha_2 > 0$.

Lemma 4 *In the oligopoly model with* $0 < \alpha_2 < \alpha_1 = 1$*, there is no equilibrium in which all firms adopt deterministic frames.*

If $\alpha_2 = 0$ (i.e., if the two frames are totally symmetric) and $n \geq 4$, there are always asymmetric pure-strategy equilibria in which each frame is used by more than one firm and all firms charge a price equal to zero. (In Section 4 we deal with symmetric frames.) Nevertheless, the symmetric mixed-strategy equilibrium presented below applies to this case, too.

A symmetric mixed-strategy equilibrium. We now characterize a symmetric mixed-strategy equilibrium (λ, F_A, F_B), where λ is the probability of using frame A and F_z is a price cdf associated with frame $z \in \{A, B\}$. Let $[p_0^z, p_1^z]$ be the support of F_z. As in Lemma 3, it is straightforward to show that F_z is atomless everywhere (as now $\alpha_2 < 1$). For the rest of the paper,

$$P_{n-1}^k \equiv C_{n-1}^k \lambda^k (1-\lambda)^{n-k-1}$$

denotes the probability that k firms among $n-1$ ones adopt frame A at equilibrium, where C_{n-1}^k stands for combinations of n taken k. Recall that $x_z(p) = 1 - F_z(p)$.

Along the equilibrium path, if firm i uses frame A and charges price p, its profit is:

$$\pi(A, p) = \sum_{k=0}^{n-2} P_{n-1}^k x_A(p)^k \left[\alpha_2 \phi_{n-k-1} + (1-\alpha_2)\phi_1\right] + \lambda^{n-1} x_A(p)^{n-1}. \qquad (11)$$

If k other firms also use frame A, firm i has a positive demand only if all other A firms charge prices higher than p. This happens with probability $x_A(p)^k$. Conditional on that, if there are no B firms in the market (i.e., if $k = n-1$), then firm i serves the whole market. The last term in $\pi(A,p)$ follows from this. Otherwise, firm i's demand depends on whether the consumer can compare offers from the B firms. If she is confused by frame complexity and, therefore, unable to compare (which happens with probability α_2), all B firms are undominated (since no comparison between A and B is possible), and so firm i's demand is ϕ_{n-k-1}. If she is not confused by frame complexity and, therefore, is able to compare (which happens with probability $1-\alpha_2$), only one B firm is undominated and so firm i's demand is ϕ_1.[17]

If instead, along the equilibrium path, firm i uses B and charges price p, its profit is:

$$\begin{aligned}\pi(B,p) &= p(1-\lambda)^{n-1}\left[\alpha_2/n+(1-\alpha_2)x_B(p)^{n-1}\right] \\ &\quad +p\sum_{k=1}^{n-1}P_{n-1}^{k}\left[\alpha_2\frac{1-\phi_{n-k}}{n-k}+(1-\alpha_2)(1-\phi_1)x_B(p)^{n-k-1}\right]. \qquad (12)\end{aligned}$$

The first term gives the expected profit when there are no A firms in the market: the consumers who are confused by frame complexity purchase randomly among all B firms, while the shoppers buy from the lowest price B frame firm. When $k \geq 1$ firms use frame A (note that only one of them is undominated), if the consumer is confused by frame complexity (i.e., unable to compare prices in frame B), all B firms are undominated and have demand $1-\phi_{n-k}$ in total. Firm i shares equally this residual demand with other B firms. If the consumer is able to compare prices in frame B (she is not confused by complexity), to face a positive demand, firm i must charge the lowest price in group B (this happens with probability $x_B(p)^{n-k-1}$), in which case it gets the residual demand $1-\phi_1$.

Since price competition can only take place among firms which use the same frame (as $\alpha_1 = 1$), the expressions for $\pi(A,p)$ and $\pi(B,p)$ also hold even if firm i charges an out-of-equilibrium price. Then, in the symmetric mixed-strategy equilibrium, the upper bound of the price cdf's is frame-independent (i.e., $p_1^A = p_1^B = 1$). Otherwise any price greater than p_1^z would lead to a higher profit. We can pin down λ from the frame indifference condition at $p=1$, $\pi(A,1) = \pi(B,1)$. Dividing each side by $(1-\lambda)^{n-1}$ and rearranging the equation we obtain

$$\alpha_2\left(\phi_{n-1}-\frac{1}{n}\right)+(1-\alpha_2)\phi_1 = \alpha_2\sum_{k=1}^{n-2}\frac{C_{n-1}^{k}(1-\phi_{n-k})}{n-k}\left(\frac{\lambda}{1-\lambda}\right)^{k}+(1-\phi_1)\left(\frac{\lambda}{1-\lambda}\right)^{n-1}. \qquad (13)$$

The right-hand side of (13) increases in λ on $[0,1]$ from zero to infinity, and the left-hand side is positive for any $\alpha_2 \in [0,1)$ as $\phi_{n-1} \geq 1/n$. Hence, (13) has a unique solution in $(0,1)$. Each firm's equilibrium profit is

$$\pi = \pi(A,1) = (1-\lambda)^{n-1}[\alpha_2\phi_{n-1}+(1-\alpha_2)\phi_1]. \qquad (14)$$

The price distributions F_A and F_B are implicitly given by $\pi(z,p) = \pi$ and well defined. The boundary prices $p_0^z < 1$ are determined by $\pi(z,p_0^z) = \pi$. Deviations to $(z, p < p_0^z)$ are not profitable since they only result in a price loss and do not increase demand.

Therefore, we obtain the following result.

[17] Notice that $F_z(p)$ is continuous and therefore the probability of a tie at a price p is zero.

Proposition 3 *For $n \geq 2$ and $\alpha_2 < \alpha_1 = 1$, there is a symmetric mixed-strategy equilibrium (λ, F_A, F_B) in which (i) λ solves (13), and (ii) F_z is defined on $[p_0^z, 1]$ and implicitly determined by $\pi(z, p) = \pi$, where $\pi(z, p)$, $z \in \{A, B\}$, are given by (11) and (12) and π is each firm's equilibrium profit given by (14).*

When $n = 2$ and $\phi_1 = 1/2$, this equilibrium coincides with the one in the duopoly case for $\alpha_1 = 1$. Recall that the duopoly equilibrium in Proposition 1 is characterized by price-frame independence (i.e., $F_A(p) = F_B(p)$). We discuss below whether this independence result still holds in the symmetric oligopoly equilibrium presented above.

The (im)possibility of price-frame independence in oligopoly. Does Proposition 3 dictate $F_A = F_B$ in general? The following graph illustrates the equilibrium price distributions $F_A(p)$ (the solid line) and $F_B(p)$ (the dashed line) in an example with $n = 3$ and $\alpha_2 = 0.5$.

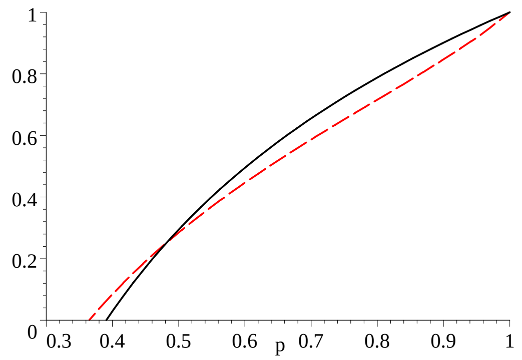

Figure 1: Price distributions with $n = 3$, $\alpha_1 = 1$ and $\alpha_2 = 0.5$

Clearly, $F_A \neq F_B$ in this example (and also notice $p_0^B < p_0^A$). The following result shows that, for $n \geq 3$, equilibrium price-frame independence holds only under very special conditions.

Proposition 4 *In the oligopoly model with $\alpha_2 < \alpha_1 = 1$,*
(i) for $n = 2$, the symmetric equilibrium in Proposition 3 dictates $F_A = F_B$ only if $\phi_1 = 1/2$.
(ii) for $n \geq 3$, the symmetric equilibrium in Proposition 3 dictates $F_A = F_B$ only if $\phi_1 = 1/2$ and $\alpha_2 = 0$, or if there exists a nonincreasing sequence $\{\phi_k\}_{k=1}^{n-1}$ which solves[18]

$$(1 - \phi_1)(1 - \alpha_2)^{2k/(n-1)} = \alpha_2 \phi_k + (1 - \alpha_2)\phi_1 \text{ for } k = 1, \cdots, n-2, \tag{15}$$

[18] When $n \geq 3$, although $\{\phi_k\}_{k=1}^{n-2}$ solved from (15) is a decreasing sequence, ϕ_{n-1}, which is solved from (13), may not be lower than ϕ_{n-2}. For example, when $n = 3$, one can check

$$\phi_1 = \frac{1 - \alpha_2}{2 - \alpha_2} < \phi_2 = \frac{\phi_1 + 1/3 + \sqrt{1 - \alpha_2}}{1 + \sqrt{1 - \alpha_2}}.$$

and (13) with

$$\frac{\lambda}{1-\lambda} = (1-\alpha_2)^{1/(n-1)}. \tag{16}$$

Notice first that the price-frame independent equilibrium in the duopoly model (see Proposition 1) depends on the assumption that consumers use the uniformly random purchase rule when they are confused by frame differentiation. The previous result shows that even a slight consumer favoritism toward one frame would overturn price-frame independence at equilibrium.

Second, the uniformly random purchase rule (i.e., $\phi_k = 1/(1+k)$) does not satisfy (13)–(16). Hence, even under this rule, there is no price-frame independent equilibrium when $n \geq 3$ and $\alpha_2 > 0$. This differs from the duopoly case where the uniformly random purchase rule guarantees price-frame independence.

Let us give intuition on what is different when there are more than two firms. Recall that in the duopoly case with $\alpha_2 < \alpha_1$, equilibrium λ ensures that, regardless of its frame choice, a firm faces the same expected number of confused (who purchase randomly) and fully aware consumers in the market (i.e., it faces the same market composition). When $n \geq 3$, though condition (13), which follows from $\pi(A,1) = \pi(B,1)$, still requires the (expected) number of consumers who are insensitive to a firm's price to be the same regardless of this firm's frame choice, this is no longer equivalent with saying that the market composition is the same. The reason is that the market composition in the oligopoly model with $n \geq 3$ is more complex. The same consumer might be a "shopper" among some firms (say, those using frame A), but be confused among others (e.g., between A and B). Hence, consumers can no longer be simply divided into fully confused and fully aware. This makes it impossible, in general, for a firm to face the same market composition when it switches from one frame to the other, and so its pricing needs to adjust to different environments.

The impact of greater competition. We now study the impact of an increase in the number of firms on the equilibrium framing strategies, and on profits and consumer surplus. Our analysis is based on the equilibrium characterized in Proposition 3. We first consider markets with many suppliers.

Proposition 5 *When there is a large number of firms in the market,*

$$\lim_{n\to\infty} \lambda = \begin{cases} 1/2, & \text{if } \alpha_2 = 0 \\ 0, & \text{if } \alpha_2 > 0 \end{cases} \quad \text{and} \quad \lim_{n\to\infty} n\pi = \begin{cases} 0, & \text{if } \alpha_2 = 0 \\ > 0, & \text{if } \alpha_2 > 0 \end{cases}.$$

When frame B is also a simple frame, the only way to reduce price competition is through frame differentiation. This is why in a sufficiently competitive market λ tends to $1/2$, which just maximizes frame differentiation. However, the ability of frame differentiation alone to weaken price competition is limited. As the industry profit result for $\alpha_2 = 0$ indicates, the impact of frame differentiation actually becomes negligible when there are a large number of firms in the market.

When frame B is complex, the impact of framing on market competition changes completely. In a sufficiently competitive market, firms use frame B almost surely: they rely heavily on frame complexity to soften price competition. (This is true even if frame B is only slightly more complex than frame A.) The reason is that, in a large market, frame differentiation is not effective, while frame complexity always has the ability to reduce price competition.[19] Moreover, due to the complexity of frame B, competition never drives market prices to the marginal cost.[20]

The analysis for large n suggests that, when the number of firms increases, frame B's complexity tends to become a relatively more significant anti-competitive device. Is it then possible that, in the presence of a complex frame B, greater competition can even increase market prices? The answer to this question, in general, depends on the parameter values. But, we show below that, when α_2 is sufficiently large, greater competition can actually harm consumers and increase industry profit. Therefore, in the market with price framing, competition policy which leads to an increase in the number of competitors, might have undesired effects.

For tractability, let us consider the uniformly random purchase rule (i.e., $\phi_k = 1/(1+k)$). Then, (13) becomes

$$1 - \alpha_2 = 2\alpha_2 \sum_{k=1}^{n-2} \frac{C_{n-1}^{k}}{n-k+1} \left(\frac{\lambda}{1-\lambda}\right)^{k} + \left(\frac{\lambda}{1-\lambda}\right)^{n-1}, \tag{17}$$

and industry profit is

$$n\pi = n(1-\lambda)^{n-1}\left(\frac{\alpha_2}{n} + \frac{1-\alpha_2}{2}\right).$$

Proposition 6 *In the oligopoly model with $\alpha_2 < \alpha_1 = 1$ and the random purchase rule $\phi_k = 1/(1+k)$,*
(i) when n increases from 2 to 3, both λ and industry profit $n\pi$ go down for any $\alpha_2 > 0$;
(ii) for any $n \geq 3$, there exists $0 < \alpha' < \alpha'' < 1$ such that for $0 < \alpha_2 < \alpha'$, both λ and industry profit $n\pi$ decrease from n to $n+1$, while for $\alpha_2 > \alpha''$, λ decreases, but industry profit $n\pi$ increases from n to $n+1$.

Beyond the limit results, numerical simulations (based on the uniformly random purchase rule) suggest that λ tends to always decrease in n, and industry profit can increase in n for relatively large α_2. The graph below describes how industry profit varies with n when $\alpha_2 = 0.9$.

[19] Notice that, if all firms employ frame B surely, we have a variant of Varian (1980) and, then industry profit is always α_2, regardless of the number of firms in the market.

[20] In effect, using (13) one can further show that $\lim_{n\to\infty} n\pi \in (\alpha_2(1-\phi_1), \alpha_2)$. The technical details are available upon request.

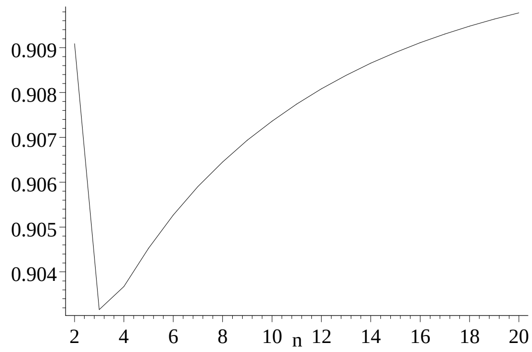

Figure 2: Industry profit and n when $\alpha_1 = 1$ and $\alpha_2 = 0.9$

The case with $\alpha_2 < \alpha_1 < 1$. The related analysis is more tedious and its details are presented in Appendix D.1. We find that if a symmetric mixed-strategy equilibrium exists, then it must hold that $p_1^A = p_1^B = 1$. Equilibrium price-frame independence requires even more stringent conditions than in the polar case $\alpha_1 = 1$. Finally, numerical simulations show that greater competition can still have undesired effects (for example, when α_1 is large and α_2 is close to α_1).

3.2 Frame complexity dominates frame differentiation ($\alpha_2 > \alpha_1$)

This part analyzes the case in which consumers are more likely to be confused by the complexity of frame B than by frame differentiation. For simplicity, we mainly focus on the polar case with $\alpha_2 = 1$ (i.e., prices in frame B are always incomparable). We then discuss the robustness of our main results to the case with $\alpha_2 < 1$. All the proofs missing from the text are relegated to Appendix B.

There is no pure-strategy equilibrium in this case either.

Lemma 5 *In the oligopoly model with $0 < \alpha_1 < \alpha_2 = 1$, there is no equilibrium in which all firms use deterministic frames.*

A symmetric mixed-strategy equilibrium. We then characterize a symmetric mixed-strategy equilibrium (λ, F_A, F_B) in which λ is the probability of using frame A, F_A is defined on $S_A = [p_0^A, 1)$ and is atomless, and F_B is degenerate on $S_B = \{1\}$.

Along the equilibrium path, if firm i uses frame A and charges $p \in [p_0^A, 1)$, its profit is given by

$$\pi(A,p) = p \sum_{k=0}^{n-1} P_{n-1}^k x_A(p)^k (\alpha_1 \phi_{n-k-1} + 1 - \alpha_1). \tag{18}$$

This expression follows from the fact that, when k other firms also use frame A, firm i has a positive demand only if all other A firms charge prices higher than p. Conditional on that, with probability α_1, the consumer is confused by frame differentiation and buys from firm i with probability ϕ_{n-k-1} (since all $n-k-1$ firms which use B are undominated); with probability $1-\alpha_1$, the consumer can compare A and B and, because all B firms charge price $p_B = 1 > p$ and consequently are dominated, she only buys from firm i.

A firm's equilibrium profit is equal to

$$\pi = \lim_{p \to 1} \pi(A, p) = (1-\lambda)^{n-1} \left(\alpha_1 \phi_{n-1} + 1 - \alpha_1\right). \tag{19}$$

Then the expression for $F_A(p)$ follows from $\pi(A, p) = \pi$, and p_0^A satisfies $\pi(A, p_0^A) = \pi$. Both of them are well defined.

If firm i uses B and charges $p = 1$, then its profit is

$$\pi(B, 1) = \frac{(1-\lambda)^{n-1}}{n} + \alpha_1 \sum_{k=1}^{n-1} P_{n-1}^k \frac{1-\phi_{n-k}}{n-k}. \tag{20}$$

Notice that firm i has a positive demand only if all other firms also use frame B or there are A firms but the consumer is unable to compare prices in different frames.

The equilibrium condition $\pi(B, 1) = \lim_{p \to 1} \pi(A, p)$ pins down λ:

$$\frac{1-1/n}{\alpha_1} + \phi_{n-1} - 1 = \sum_{k=1}^{n-1} \frac{C_{n-1}^k \left(1-\phi_{n-k}\right)}{n-k} \left(\frac{\lambda}{1-\lambda}\right)^k. \tag{21}$$

The left-hand side of (21) is positive given $\phi_{n-1} \geq 1/n$, and the right-hand side is increasing in λ from zero to infinity. Hence, for any given $n \geq 2$ and $\alpha_1 \in (0, 1)$, equation (21) has a unique solution in $(0, 1)$. In the Appendix, we also show that there is no profitable deviation.

Proposition 7 *For $n \geq 2$ and $0 < \alpha_1 < \alpha_2 = 1$, there exists a symmetric mixed-strategy equilibrium (λ, F_A, F_B) in which (i) λ solves (21), (ii) F_A is defined on $S_A = [p_0^A, 1)$ and implicitly given by $\pi(A, p) = \pi$, where π is each firm's equilibrium profit given by (19), and (iii) F_B is a degenerate cdf on $S_B = \{1\}$.*

It is straightforward to check that, when $n = 2$ and $\phi_1 = 1/2$, this equilibrium coincides to that in the duopoly model with $\alpha_2 = 1$ (see Proposition 2). In continuation, we analyze the impact of greater competition on market outcomes using the equilibrium in Proposition 7.

The impact of greater competition. When there are many suppliers in the market, our results in Proposition 5 for $\alpha_2 > 0$ also hold in this case. That is, $\lim_{n \to \infty} \lambda = 0$ and $\lim_{n \to \infty} n\pi > 0$.[21] The same intuition applies: in a sufficiently com-

[21] This follows from the observation that the right-hand side of (21) is greater than

$$\frac{1-\phi_1}{n} \sum_{k=1}^{n-1} C_{n-1}^k \left(\frac{\lambda}{1-\lambda}\right)^k = \frac{1-\phi_1}{n} \left[\frac{1}{(1-\lambda)^{n-1}} - 1\right],$$

while the left-hand side of (21) is bounded, and so $\lim_{n \to \infty} n(1-\lambda)^{n-1} > 0$.

petitive market, firms resort to the complexity of frame B to reduce price competition since the ability of frame differentiation to weaken price competition is now negligible.

The following result shows that greater competition always improves industry profit and so decreases consumer surplus when α_1 is sufficiently small. The reason is that, when α_1 is small, frame B is more effective in reducing price competition, which makes the frequency of using frame B increase more rapidly with the number of firms.

Proposition 8 *In the oligopoly model with* $0 < \alpha_1 < \alpha_2 = 1$, *for any* $n \geq 2$, *there exists* $\hat{\alpha} \in (0,1)$ *such that for* $\alpha_1 < \hat{\alpha}$, λ *decreases while industry profit* $n\pi$ *increases from* n *to* $n+1$.

Beyond this limit case, numerical simulations (based on the uniformly random purchase rule) suggest that λ tends to always decrease with n, and for a small α_1 industry profit can increase in n when n is relatively small.[22] The following graph describes how industry profits vary with n when $\alpha_1 = 0.01$.

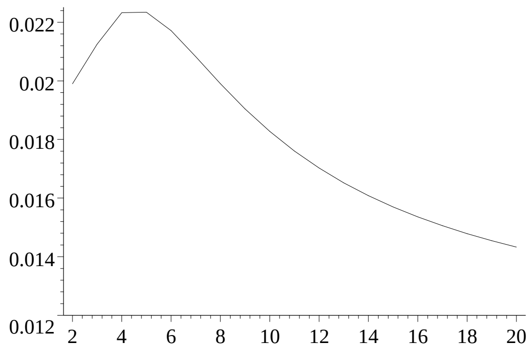

Figure 3: Industry profits and n when $\alpha_1 = 0.01$ and $\alpha_2 = 1$

The case with $\alpha_1 < \alpha_2 < 1$**.** We report here two main findings and relegate a more detailed analysis to Appendix D.2. First, a symmetric separating equilibrium with $S_A = [p_0^A, \hat{p}]$ and $S_B = [\hat{p}, p_1^B]$, resembling the one in Proposition 7, exists for $n \geq 3$ only under some parameter restrictions (when α_1 is not too close to $\alpha_2 < 1$). Second, for fixed $\alpha_2 < 1$, if α_1 is sufficiently small, greater competition still increases industry profit and harms consumers. (Note that the symmetric separating equilibrium always exists in this limit case.)

4 More frames

To surpass the difficulties of a general oligopoly model with arbitrary number of frames, we first focus on the relatively simpler case with $m \geq 2$ completely symmetric frames

[22] Given the random purchase rule, we can actually show that industry profit always decreases with n for sufficiently large α_1. The details are available upon request.

$\{A_1, \cdots, A_m\}$. Specifically, we assume that (i) consumers are able to perfectly compare prices in the same frame, (ii) but they are totally confused between any two prices in different frames. (This is a generalization of the two-frame case with $\alpha_0 = \alpha_2 = 0$ and $\alpha_1 = 1$.) We still use the dominance-based choice rule, but consider only the uniformly random purchase rule among undominated firms. (Notice that in this case a firm can only be dominated by firms which use the same frame.)

Our first result explores pure-strategy equilibria.

Proposition 9 *In the oligopoly model with m completely symmetric frames,*
(i) if $n \geq 2m$, there exist asymmetric pure-strategy equilibria where each frame is used by more than one firm and all firms charge zero price;
(ii) if $n < 2m$, there is no equilibrium in which all firms adopt deterministic frames.

Proof. The proof of (i) is straightforward and so omitted. We prove (ii) by discussing the following two cases:

(a) If $\exists\ A_k$ which is chosen by more than one firm, then these firms must earn zero profit in any possible equilibrium. Since $n < 2m$, there must exist another frame $A_l \neq A_k$ which has been chosen by at most one firm. But then, it is profitable for any firm which is using A_k to deviate to A_l and an appropriate positive price. (If no firm uses A_l then any positive price $p \leq 1$ supports the deviation; if one firm already uses A_l, this firm must be charging $p = 1$ in any possible equilibrium, and so any price below one would work.)

(b) If $\nexists\ A_k$ which is chosen by more than one firm (i.e., each firm chooses a distinct frame), the only possible equilibrium entails monopoly pricing $p = 1$ and each firm earns $1/n$. Then any firm can earn a higher profit close to $1/(n-1)$ by deviating to one rival's frame and a price slightly lower than one. ■

Therefore, if the set of available frames is large enough, there can only be equilibria in which firms randomize over frames. However, even when fewer frames are available ($n \geq 2m$), as we show below, there also exists a mixed-strategy equilibrium in which firms randomize over frames and make positive profits.

A symmetric mixed-strategy equilibrium. Let us characterize the symmetric mixed-strategy equilibrium in which each firm adopts each price frame with probability $1/m$ and charges a random price according to a continuous cdf $F(p)$ defined on $[p_0, p_1]$. Notice that, in such an equilibrium, $p_1 = 1$.

Along the equilibrium path, if firm i adopts frame A_j and charges a price $p \in [p_0, 1]$, its profit depends on the number of firms (including itself) using frame A_j and the number of distinct frames in the market. If there are k firms in group A_j and $l \geq 1$ distinct frames in total in the market, then firm i's expected demand is

$$\frac{1}{l}\left[1 - F(p)\right]^{k-1}.$$

Firm i has a positive demand only if it offers the lowest price in group A_j and, when it does so, it shares the market equally with all winners from other groups. Notice that, when k firms use frame A_j, the number of other distinct frames in the market cannot exceeds $m-1$ and $n-k$. So

$$l \leq \min\{m, n-k+1\} \equiv J(k).$$

Let $\Pr(k,l)$ be the probability that there are k firms in group A_j and l distinct frames in total in the market conditional on the fact that firm i has chosen A_j. (See Appendix C for details on the calculation of $\Pr(k,l)$.) Then firm i's expected profit is

$$\pi(A_j, p) = p \sum_{k=1}^{n} [1-F(p)]^{k-1} \underbrace{\left(\sum_{l=1}^{J(k)} \frac{\Pr(k,l)}{l} \right)}_{a_k}. \tag{22}$$

At equilibrium, each firm earns $\pi(A_j, 1) = a_1$. The expression for $F(p)$ is then implicitly given by the equation $\pi(A_j, p) = a_1$. Clearly, $F(p)$ is well defined. From $\pi(A_j, p_0) = a_1$, we can solve $p_0 = a_1 / \sum_{k=1}^{n} a_k < 1$. It is also clear that any deviation to a price below p_0 is not profitable. This establishes the following result:

Proposition 10 *In the oligopoly model with m completely symmetric frames, there is a symmetric mixed-strategy equilibrium in which firms assign probability $\lambda = 1/m$ to each available frame, and price according to a common cdf (F) which is defined on $[p_0, 1]$ and solves $\pi(A_j, p) = a_1$. Each firm's equilibrium profit is $\pi = a_1$.*

The impact of greater competition and more frames. Let us now investigate how profits vary with n and m at the mixed-strategy equilibrium in Proposition 10. We first consider two simple cases. (i) With only two frames, if firm i chooses one frame and charges $p = 1$, it has a positive demand only if all other firms use the other frame. This happens with probability $(1/2)^{n-1}$ and, in this case, firm i's market share is $1/2$. So its profit is $a_1 = (1/2)^n$. Hence, when $m = 2$, both individual and industry profit decrease with n. (ii) When there are only two firms, if firm i chooses one frame and price $p = 1$, it has a positive demand only if the other firm chooses a different frame. The probability of this event is $1 - \frac{1}{m}$. Hence, when $n = 2$, we have $a_1 = (1 - \frac{1}{m})/2$. Clearly, both individual and industry profits increase with m.

In general,

$$a_1 = \sum_{l=1}^{\min\{m,n\}} \frac{\Pr(1,l)}{l}$$

does not have a concise expression. Numerical simulations suggest that industry profit (na_1) decreases with n for fixed m and increases with m for fixed n. The graph below reports several examples, in which starting from the bottom m equals 3, 10, and 20, respectively.

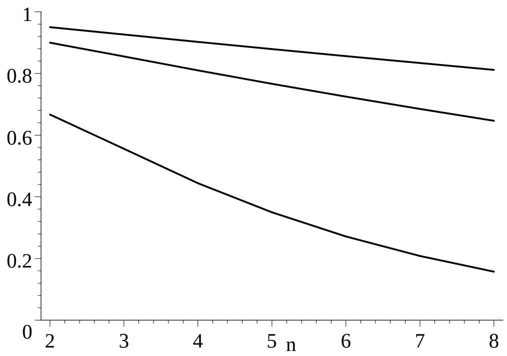

Figure 4: Competition and industry profit with symmetric frames

When consumers are confused only by frame differentiation, greater competition seems to benefit consumers. Intuitively, when there are more firms, differentiating from other firms by using other frames becomes more difficult, such that firms compete more aggressively in prices. Figure 4 also suggests that industry profit increases in the number of frames. This is because when more frames are available it becomes easier for firms to differentiate frames and avoid price competition.[23]

The consideration of a general frame structure for $m \geq 3$ brings about significant technical complications. Although we do not deal here with this general setting, let us briefly comment on some possibilities for future work. An oligopoly model with a general frame structure could (i) assign to each frame a complexity index—the probability that the consumer gets confused among prices in the frame; (ii) assign to each pair of frames a differentiation index—the probability that the consumer gets confused between prices in the two different frames. In this case, the dominance-based choice rule and an appropriately modified stochastic purchase rule (e.g., the uniformly random one) can apply. We conjecture that our main insights would still apply in this framework.

5 Conclusion and Discussion

This paper has presented a model of competition in both prices and price frames where price framing can obstruct consumers' price comparisons. We characterized the symmetric mixed-strategy equilibrium in which firms randomize over both frames and prices, and examined how the degree of competition affects firms' strategies, profits,

[23]When there is a large number of frames, industry profit tends to $\lim_{m\to\infty} na_1 = \lim_{m\to\infty} \Pr(1,n) = 1$.

and consumer welfare.

In the remainder of this section, (i) we discuss alternative consumer choice rules and interpretations of consumer confusion; and (ii) we present more examples of differentiated and complex frames and argue that our results match observed market outcomes.

5.1 Robustness, modelling and interpretations

Alternative consumer choice rules.

(1) *More restrictive consideration sets.* In our dominance-based choice rule, consumers' "consideration set" includes all available options. The consumers make correct comparisons among *all* pairs of comparable alternatives, rule out the dominated alternatives, and then select from the set of undominated ones. Alternatively, confused consumers may restrict their consideration set at the outset (to save time and effort, for instance). The following example illustrates such choice heuristics.

Example 3 *When $n_A \geq 1$ firms use frame A and $n_B \geq 1$ firms use frame B, a consumer, if she cannot compare B options, will restrict attention to a consideration set which consists of the A firm(s) with the lowest price and $k \leq n_B$ randomly chosen B firms. She then applies the dominance-based choice rule to this restricted consideration set.*

Our benchmark choice procedure corresponds to $k = n_B$ and is the most sophisticated one in this class in the sense that it eliminates all identifiable dominated options. When $k < n_B$, some dominated options may survive: when a consumer can compare A to B, she would fail to eliminate the A option(s) if the B option(s) which dominate it were not included in her consideration set. It can be shown that (at least) for $k = 1$, our main results hold qualitatively.

(2) *A default-bias choice rule.* The dominance-based choice rule embeds a *simultaneous* assessment of competing offers, and a consumer's choice outcome is not affected by the particular sequence of pairwise comparisons. This "simultaneous search" feature is more suitable in a market where the consumers are not influenced by their previous experiences (or, are newcomers). Piccione and Spiegler (2009) consider a default-bias duopoly model in which consumers are initially randomly attached to one brand (their default option), and they switch to another brand only if that is comparable to their default and better than it. In this case, due to the sequential comparison, a consumer's final choice will depend on her default option.

In the duopoly case, the default-biased model is actually equivalent to our simultaneous assessment model (with the random purchase rule for confused consumers).[24]

[24] More precisely, the equivalence requires that the probability that a consumer is confused by two

This is because, if the two firms' offers are comparable, in both models the better offer wins all consumers; while if they are incomparable, in both models the firms share the market equally. However, when there are more than two firms, the two approaches diverge. In fact, with more than two firms, the default-bias model calls for further structure on the choice rule. To see why, consider the following example.

Example 4 *There are three firms in the market. Let $\alpha_2 = 1$ and $\alpha_1 = 0$ (i.e., the only confusion source is frame complexity). Suppose that firm 1 adopts frame A and charges a price p_1, while firms 2 and 3 adopt frame B and charge prices p_2 and p_3, respectively, with $p_2 < p_1 < p_3$.*

The dominance-based rule implies that consumers will purchase only from firm 2 since firm 3 is dominated by firm 1 and firm 1 is dominated by firm 2. Now consider the default-bias model. If a consumer is initially attached to firm 2, she will not switch. If she is initially attached to firm 1, she will switch to firm 2. However, if she is initially attached to firm 3, she will switch to firm 1, but whether she will further switch to firm 2 depends on what the choice rule of the default-biased consumer dictates. Such rule should specify if she will assess firm 2's offer from the perspective of her default option or from the perspective of her new choice. In contrast, the dominance-based rule applies equally well regardless of the number of firms in the market.[25]

Both a more restrictive consideration set and a default bias add another dimension of bounded rationality on top of consumer confusion caused by framing. In this sense, our framework is the minimum deviation from the standard Bertrand competition model.

(3) *Noisy price comparisons.* For the sake of tractability, we have assumed in our consumer choice rule that confused consumers' choice from the set of undominated alternatives is entirely *independent* of the prices. Alternatively, confusion might only lead to noisy price comparisons, such that consumers' choice is still influenced to some extent by prices. For instance, in the duopoly case, when the price difference between firms 1 and 2 is $p_1 - p_2$, the consumer might misperceive it as being $p_1 - p_2 + \delta$, where δ is a frame-profile dependent random variable. If all δ have symmetric distributions around zero, then our result that in symmetric equilibrium, the firms randomize in both frames and prices carries over. However, unless we restrict attention to a duopoly case where confusion stems only from frame differentiation, we cannot characterize the symmetric mixed-strategy equilibrium in this setting.

frames is independent of which one is the default option.

[25] The fact that these two choice rules may lead to different choice outcomes can also be seen from the following example: following Example 4, now suppose $\alpha_2 = 0$ and $\alpha_1 = 1$, and $p_1 < p_2 < p_3$. Our approach (with the uniform purchase rule) predicts that firms 1 and 2 will share the market equally; while the default-bias rule predicts that firm 1 has demand $\frac{1}{3}$ and firm 2 has demand $\frac{2}{3}$.

Costly information processing. Price comparisons in the presence of framing might require costly information processing. Then, consumer confusion could be the result of consumers' rational decision to opt out of information processing when its cost is too high or its expected benefit is too low. Therefore, our model could be interpreted also as one of costly information processing with rational consumers, and not only as one of bounded rationality.

However, rational consumers should eventually be able to infer prices from frames (*if* they can distinguish between frames). In this case, a separating equilibrium where the complex frame is associated with higher prices (as the ones in Propositions 2 and 7) would not survive, since the consumers should then choose simple-frame products.[26] (This is not an issue in Propositions 1 and 3, where the complex frame is not necessarily associated with higher prices.)

However, our separating equilibrium still makes sense if (i) consumers are yet to understand the market equilibrium or they purchase the product infrequently (see Subsection 5.2), or (ii) there is a non-trivial mass of naive consumers who choose randomly when they are confused.[27]

Carlin (2009) considers a model similar to our case with $\alpha_2 > \alpha_1$. In his model, each firm chooses its price complexity level and consumers decide whether to become knowledgeable of *all* prices in the market by incurring in a search cost. In equilibrium, higher complexity is also associated with higher prices. Carlin avoids the inference problem by exogenously assuming that consumers can only observe the aggregate market complexity index, but not each firm's price complexity.

5.2 Examples and empirical relevance

In a unified framework, we analyze how both price frame differentiation and complexity, as sources of consumer confusion, affect price competition. The predictions of our model depend on which source of confusion dominates. Below, we present further examples of differentiated and complex frames, provide evidence that price framing creates confusion, and discuss the empirical relevance of our results.

In grocery stores or supermarkets, the prices of otherwise homogeneous products are often presented in different ways: a discount can be specified in monetary terms, or in percentage, or might be implicit in a "buy one, get one free" offer. High street retailers offer store cards with terms such as "10% off first shop if opened online or 10% for first week if opened in store" or "500 bonus points on first order" or "£5

[26] In this sense, our assumption that consumers weakly favor the simple frame (i.e., $\phi(n_A, n_B) \geq n_A/(n_A + n_B)$) partially reflects such sophistication.

[27] In the case of $\alpha_2 > \alpha_1$, suppose $\gamma < 1$ of consumers are rational and understand the market equilibrium, and $1 - \gamma$ are naive (like in our model) and choose (uniformly) randomly from the undominated alternatives. Then, we still have the separating equilibrium with $\phi_k = \gamma + (1-\gamma)\frac{1}{1+k}$.

voucher after first purchase". Some online book retailers quote an all-inclusive price, while others quote separately shipping and handling charges. Some restaurants quote separately the VAT or the tip, while others quote the total price. The booking fee charged by airlines or travel agencies to a customer for the use of a debit or credit card are often presented in different ways. For example, Wizz charges a flat £4 (per person), while Virgin Atlantic charges 1.3% percent of the total booking.[28]

In these examples, frame differentiation dominates frame complexity. Each of these price presentation modes is not particularly involved, but they are likely to make it more difficult or costly for the consumers to compare the prices of close substitutes. Consumers with high cognitive costs or less time available to make a decision are more likely to make errors. The literature on psychology and behavioral economics presents evidence that consumer choice is not "description invariant".[29] Marketing research suggests that framed pricing (e.g., partitioned pricing such as price plus VAT) might affect consumers' recalled total cost. The manner in which a discount is framed (i.e., as a percentage or a flat reduction) might also influence consumers' assessment of the partitioned price (see Morwitz, Greenleaf and Johnson, 1998).

Our model predicts that, in markets where frame differentiation is a dominant source of consumer confusion, there is no clear ranking (on average) among prices associated with different frames. For example, there should be no significant price differences across different discounting methods. This is an empirically testable result and seems consistent with casual observations in the markets discussed above. In addition, notice that frame differentiation seems to prevail in markets where the consumers purchase with relatively high frequency. If some frames were associated with higher prices, even consumers with high cognitive costs would be able to figure it out over time and avoid buying these products.

On the other hand, in markets for financial services, automobile leasing, insurance, or mobile telephony, complex pricing, such as multi-part tariffs, seems to be prevalent. In these cases, the pricing scheme includes many elements or pieces of information which need to be aggregated together in order to accurately assess the total cost. For instance, the price of a mortgage or a car lease comprises various elements (see Table 1). The price of a mobile telephone is linked to different terms and conditions: the monthly cost, the number of free minutes and the number of free texts. Table 3 in the end of this section gives details of mobile phone price plans in the UK. In these examples, the involved pricing might obstruct consumers' price comparisons even if

[28] See "Calls for airline charges clean-up" on BBC News on July 17, 2009 (*http://news.bbc.co.uk*).

[29] Research in psychology and marketing often focuses on the specific heuristics used by the consumers to assess framed (partitioned) prices. A cost/benefit analysis suggests that the costs (time and effort) of fully and accurately evaluating framed prices are high, such that consumers might actually use lower effort heuristics (see Payne, Bettman and Luce, 1996). Then, some consumers are likely to be indecided or make judgement errors.

the firms adopt the same price frame (e.g., a tariff with the same elements).

There is mounting evidence that in such markets consumers do not understand well complex prices. For instance, a EU study of European mortgage markets states that "consumers do not necessarily have all the information that they require in order to make a decision and even if consumers do have the relevant information, they do not necessarily understand it".[30] Similarly, in a research report on the gas and electricity market in UK, a consumer organization called *Which?* says that "complex tariff structures made it very difficult for consumers to understand what type of deal they were on and how to reduce energy use and costs".[31]

In markets where frame complexity is a dominant source of confusion, our model predicts that the more complex frame is always associated with higher prices. For example, Woodward (2003) provides evidence that, in the mortgage market, the deals with the service fees rolled in the interest rate are on average better than the deals with separate fees. Our model also predicts that, in markets with frame complexity (even if it does not dominate frame differentiation), an increase in the number of firms can increase prices and harm consumers. Hortaçsu and Syverson (2004) show that in the S&P index fund market, a decrease in concentration between 1995-99 triggered an increase in the average price. Notice that price complexity is mostly common in markets where the consumers participate infrequently and therefore do not have the opportunity to infer prices from presentation modes.

Finally, although our study has been motivated by price framing, it also applies to situations where *product* framing reduces buyers' ability to compare sellers' offers. For instance, the way in which nutritional information is presented might frame differently essentially identical food products. A label indicating an "improved recipe" or a "British meal" might spuriously differentiate a ready meal from close substitutes.[32] Differences in package size or quantity premia also make it harder to compare products. On the same shelf toothpastes come in tubes of 50, 75 or 100 ml, mouthwashes in bottles of 250 or 500 ml, tea boxes might offer 50% extra free (that is, 240 softbags instead of rivals' 160). Frequently, pack size variation is also accompanied by price presentation variations. In addition, Betrand et al. (2009) and Choi et al. (2008) document evidence that in the personal finance market, providing some payoff irrelevant information (e.g., a female photo in the loan advertising letter or the information concerning mutual fund historical returns) can significantly influence consumers' choices.[33]

[30] See the "White Paper on the Integration of EU Mortgage Credit Markets" (2007).

[31] See "Customers confused by energy tariffs" on *http://www.which.co.uk/news* on May 7, 2009.

[32] The reportage "What's really in our food?" broadcast on BBC One on July 14, 2009 stressed this point. For instance, interviewed customers confess to being misled by a ready food made with imported meat and labeled as "British meal". Aslo, buyers seem to have a poor understanding of what labels such as "free range" really mean.

[33] For example, the first paper shows that the effect of including a female photo in the loan ad-

Our main insights also apply to this kind of product-framing situations.

Table 3: UK Mobile Phone Price Plans[34]

(18-month contract, monthly payment $\leq$ £35, June 15, 2009)

	Monthly Cost	Free Minutes	Free Texts
Vodafone	15	100	500
	20a	100	500
	20b	300	Unlimited
	25a	100	500
	25b	300	Unlimited
	25c	600	Unlimited
	30a	300	Unlimited
	30b	600	Unlimited
	35a	200	1000
	35b	600	Unlimited
T-Mobile	15	100	200
	20	200	400
	25a	300	600
	25b	800	Unlimited
	30	700	Unlimited
	31.5	800	Unlimited
Orange	29.36	600	Unlimited
	34.25	900	Unlimited
	34.25	900	500
O2	19.58	75	250
	24.48	200	400
	29.38	400	1000
	34.26	600	1000

A Appendix: Proofs in the Duopoly Case

A.1 Proof of Lemma 3

Suppose that equilibrium F_z has a mass point at some price $p \in S_z$. Then there is a positive probability that both firms use frame z and tie at p. Given $\alpha_2 < 1$, there is always a positive measure of price aware consumers regardless of z, such that for any

vertising letter on increasing customers' loan take-up is as strong as a 25% reduction in the interest rate.

[34]Different price plans are also usually paired with different free phones. For example, Vodafone assigns more expensive phones to those apparently "dominated" options.

firm is more profitable to offer $(z, p - \varepsilon)$ (for some small $\varepsilon > 0$) than (z, p). This leads to a contradiction.

A.2 Proof of Proposition 1

The proposed configuration is indeed an equilibrium since no deviation to $p < p_0$ is profitable and we show now that it is the unique symmetric mixed-strategy equilibrium with F_z strictly increasing on its support. Recall that, by Lemma 3, when $\alpha_2 < 1$, in any symmetric mixed-strategy equilibrium F_z is continuous on S_z. Our proof consists of the following steps.

Step 1: $S_A \cap S_B \neq \emptyset$. Suppose $p_1^A < p_0^B$. Then if a firm uses frame A and charges p_1^A, its profit is

$$\pi(A, p_1^A) = p_1^A (1 - \lambda) [(1 - \alpha_1) + \alpha_1/2] .$$

The firm has positive demand only if the other firm is using frame B, In which case, it sells to all price aware consumers and to half of the confused consumers. Clearly, this firm can do better by charging a price slightly higher than p_1^A. A contradiction. Similarly, we can also rule out the possibility of $p_1^B < p_0^A$.

Step 2: $\max\{p_1^A, p_1^B\} = 1$. Suppose $p_1^z = \max\{p_1^A, p_1^B\} < 1$. Then, p_1^z is dominated by $p_1^z + \varepsilon$ (for some small $\varepsilon > 0$).

Step 3: $S_A = S_B = [p_0, 1]$. Suppose $p_1^A < p_1^B = 1$. Then, along the equilibrium path, if firm i uses frame A and charges $p \in [p_1^A, 1]$, its profit is

$$\pi (A, p) = p (1 - \lambda) [(1 - \alpha_1) x_B (p) + \alpha_1/2] ,$$

since it faces a positive demand only if firm j uses frame B. If firm i uses frame B and charges the same price p, its profit is

$$\pi (B, p) = p \{\lambda\alpha_1/2 + (1 - \lambda) [(1 - \alpha_2) x_B (p) + \alpha_2/2]\} ,$$

which should be equal to the candidate equilibrium profit. Since Step 1 implies that $p_1^A \in S_B$, the indifference condition requires $\pi(A, p_1^A) = \pi(B, p_1^A)$ or, equivalently:

$$(1 - \lambda) (\alpha_1 - \alpha_2) - \lambda\alpha_1 = 2 (1 - \lambda) (\alpha_1 - \alpha_2) x_B(p_1^A).$$

However, if this equation holds, $\pi (A, p) > \pi (B, p)$ for $p \in (p_1^A, 1]$ as $\alpha_1 > \alpha_2$ and x_B is strictly decreasing on S_B. This is a contradiction. Similarly, we can exclude the possibility of $p_1^B < p_1^A = 1$. Therefore, it must be that $p_1^A = p_1^B = 1$.

Then, from $\pi (A, 1) = \pi (B, 1)$, it follows that

$$(1 - \lambda) (\alpha_1 - \alpha_2) = \lambda\alpha_1. \tag{23}$$

Now suppose $p_0^A < p_0^B$. Then

$$\pi(A, p_0^B) = p_0^B \left[\lambda x_A(p_0^B) + (1-\lambda)(1-\alpha_1/2)\right] \text{ and}$$
$$\pi(B, p_0^B) = p_0^B \left\{\lambda \left[(1-\alpha_1) x_A(p_0^B) + \alpha_1/2\right] + (1-\lambda)(1-\alpha_2/2)\right\}.$$

Since Step 1 implies $p_0^B \in S_A$, we need $\pi(A, p_0^B) = \pi(B, p_0^B)$, or equivalently

$$2x_A(p_0^B) = 1 + \frac{1-\lambda}{\lambda}\frac{\alpha_1 - \alpha_2}{\alpha_1}.$$

The left-hand side is strictly lower than 2 given that x_A is strictly increasing on S_A and $p_0^A < p_0^B$. While (23) implies that the right-hand side is equal to 2. A contradiction. Similarly, we can exclude the possibility of $p_0^A < p_0^B$. Therefore, it must be that $p_0^A = p_0^B$.

Step 4: $F_A = F_B$. For any $p \in [p_0, 1]$, the indifference condition requires $\pi(A, p) = \pi(B, p)$. Using (2) and (3), we get

$$\lambda\alpha_1 [x_A(p) - 1/2] = (1-\lambda)(\alpha_1 - \alpha_2)[x_B(p) - 1/2]$$

for all $p \in [p_0, 1]$. Then (23) implies $x_A = x_B$ (or $F_A = F_B$).

A.3 Proof of Proposition 2

(1) Let us first prove the result for $\alpha_2 < 1$.

(1-1) A deviation to $(A, p < p_0^A)$ is obviously not profitable. A deviation to $(A, p > \hat{p})$ generates a profit equal to

$$p(1-\lambda)[(1-\alpha_1)x_B(p) + \alpha_1/2].$$

One can easily check that this deviation profit is lower than $\pi(B, p)$ by using (7). One last possible deviation is $(B, p < \hat{p})$ which results in a profit equal to

$$p\{\lambda[(1-\alpha_1)x_A(p) + \alpha_1/2] + (1-\lambda)(1-\alpha_2/2)\}.$$

One can also check that this deviation profit is lower than $\pi(A, p)$ by using (7).

(1-2) Let us now prove the uniqueness. As in the proof of Proposition 1, we can show that $S_A \cap S_B \neq \emptyset$ and $\max\{p_1^A, p_1^B\} = 1$. Then, the following two steps complete the proof.

Step 1: It must be that $S_A \cap S_B = \{\hat{p}\}$ for some $\hat{p}$. Suppose to the contrary that $S_A \cap S_B = [p', p'']$ with $p' < p''$. Then for any $p \in [p', p'']$, it must be that $\pi(A, p) = \pi(B, p)$, where the profit functions are given by (2) and (3). This indifference condition requires

$$\lambda\alpha_1 [x_A(p) - 1/2] = (1-\lambda)(\alpha_1 - \alpha_2)[x_B(p) - 1/2]$$

for all $p \in [p', p'']$. Since $\alpha_2 > \alpha_1$ and F_z is strictly increasing on S_z, the left-hand side is a decreasing function of p while the right-hand side is an increasing function. So this condition cannot hold for all $p \in [p', p'']$. A contradiction.

Step 2: $p_1^B = 1$. Suppose $p_1^B < 1$. Then Step 1 and $\max\{p_1^A, p_1^B\} = 1$ imply that $p_1^A = 1$ and $p_1^B = p_0^A = \hat{p} < 1$. Then each firm's equilibrium profit should be equal to $\pi(A, 1) = (1 - \lambda)\alpha_1/2$ since the prices associated with B are lower than one. However, if a firm chooses frame B and $p = 1$, its profit is $[\lambda\alpha_1 + (1 - \lambda)\alpha_2]/2$ since it sells to half of the confused consumers. This deviation profit is greater than $\pi(A, 1)$ given that $\alpha_2 > \alpha_1$. A contradiction.

Therefore, in equilibrium, it must be that $S_A = [p_0^A, \hat{p}]$ and $S_B = [\hat{p}, 1]$.

(2) The equilibrium with $\alpha_2 = 1$ is just the limit of the equilibrium in (1) as $\alpha_2 \to 1$. But now, $S_A = [p_0^A, 1]$ and $S_B = \{1\}$.

B Appendix: Proofs in the Oligopoly Model

B.1 Proof of Lemma 4

(a) In any possible equilibrium in which firms use deterministic frames, at most one firm uses frame A. Suppose to the contrary that at least two firms use frame A. Then they must all earn zero profit at any putative equilibrium. But then any of them has a unilateral incentive to deviate to frame B and a positive price, which will bring a positive profit as $\alpha_2 > 0$. A contradiction.

(b) In any possible pure-strategy framing equilibrium, at least one firm uses frame A. Suppose to the contrary that all firms use frame B. Then with probability α_2 consumers shop randomly, and with probability $1 - \alpha_2$ they buy from the cheapest firm. This is a version of Varian (1980), and each firm earns α_2/n.[35] But then any firm can earn more by deviating to frame A and the price $p = 1$, which generates a profit of at least $\phi_{n-1} \geq 1/n$. This is because at most $n - 1$ B firms can survive and the deviator will never be dominated as $\alpha_1 = 1$.

(c) Finally, suppose that one firm uses A and all other firms use B.[36] Suppose such an equilibrium exists. First of all, the A firm must charge the price $p = 1$ given that $\alpha_1 = 1$ and make a profit at least equal to ϕ_{n-1}. Second, each B firm must also earn at least ϕ_{n-1}. Suppose some B firm earns $\pi_B < \phi_{n-1}$. Then if it deviates to frame A and a price $1 - \varepsilon$, it will dominate the original A firm and earn at least $(1 - \varepsilon)\phi_{n-2}$

[35] For $n \geq 3$, there are both symmetric and asymmetric mixed-strategy equilibria in the Varian model, but all of them are outcome equivalent. (See Baye et al., 1992).

[36] This part of the proof is different from that in the duopoly case since it is hard to directly characterize the pricing equilibrium when one firm uses frame A and other $n - 1 \geq 2$ firms use frame B.

(which is greater than π_B for a sufficiently small ε since $\phi_{n-2} \geq \phi_{n-1}$). If the sum of all firms' profits exceeds one (e.g., when $\phi_{n-1} > 1/n$), this candidate equilibrium collapses since industry profit is bounded by one. If the sum of all firms' profits is just one, then each firm should earn $1/n$. This also means that all firms charge the monopoly price $p = 1$.[37] But then any B firm has an incentive to deviate to a price slightly below one given that $\alpha_2 < 1$. A contradiction.

B.2 Proof of Proposition 4

At equilibrium, each firm's demand can actually be decomposed into two parts: the consumers who are insensitive to its price, and the consumers who are sensitive. Explicitly, we have

$$\pi(A,p)/p = \pi(A,1) + \lambda^{n-1} x_A(p)^{n-1} + \sum_{k=1}^{n-2} P_{n-1}^k x_A(p)^k \left[\alpha_2 \phi_{n-k-1} + (1-\alpha_2)\phi_1\right],$$

and

$$\pi(B,p)/p = \pi(B,1) + (1-\alpha_2)(1-\lambda)^{n-1} x_B(p)^{n-1} + (1-\alpha_2)(1-\phi_1)\sum_{k=1}^{n-2} P_{n-1}^k x_B(p)^{n-k-1}.$$

Suppose now $x_A = x_B = x$, and its support is $[p_0, 1]$. At equilibrium, it should hold that $\pi(A,p) = \pi(B,p)$ for any $p \in [p_0, 1]$. In particular, $\pi(A,1) = \pi(B,1)$ determines λ.

(i) For $n = 2$, the last term in each expression disappears. So $\pi(A,p) = \pi(B,p)$ for $p < 1$ requires

$$\frac{\lambda}{1-\lambda} = 1 - \alpha_2.$$

Meanwhile, $\pi(A,1) = \pi(B,1)$ or (13) requires

$$\frac{\lambda}{1-\lambda} = \frac{\phi_1 - \alpha_2/2}{1-\phi_1}.$$

These two conditions hold simultaneously if and only if $\phi_1 = 1/2$.

(ii) For $n \geq 3$, $\pi(A,p) = \pi(B,p)$ for $p < 1$ requires

$$\begin{aligned} & \lambda^{n-1} + \sum_{k=1}^{n-2} P_{n-1}^{n-k-1} x(p)^{-k} \left[\alpha_2 \phi_k + (1-\alpha_2)\phi_1\right] \\ = \ & (1-\alpha_2)(1-\lambda)^{n-1} + (1-\alpha_2)(1-\phi_1)\sum_{k=1}^{n-2} P_{n-1}^k x(p)^{-k} \end{aligned}$$

[37] If some firm would charge prices lower than one with a positive probability, then at these prices its demand must be positive (otherwise its equilibrium profit would be zero, which contradicts the fact that each firm earns $1/n$). But then consumer surplus would be positive. A contradiction.

by using $\pi(A,1) = \pi(B,1)$, dividing each side by $px(p)^{n-1}$, and relabelling k in $\pi(A,p)$ by $n-k-1$. It is further equivalent with

$$\sum_{k=1}^{n-2} b_k x(p)^{-k} = (1-\alpha_2)(1-\lambda)^{n-1} - \lambda^{n-1} \tag{24}$$

where

$$b_k \equiv P_{n-1}^{n-k-1}\left[\alpha_2\phi_k + (1-\alpha_2)\phi_1\right] - P_{n-1}^{k}(1-\alpha_2)(1-\phi_1).$$

The left-hand side of (24) is a polynomial of $1/x(p)$. Since $x(p)$ is a decreasing function, (24) holds for all $p \in [p_0, 1]$ only if $b_k = 0$ for $k = 1, \cdots, n-2$ and the right-hand side is also zero. That is,

$$\left(\frac{\lambda}{1-\lambda}\right)^{n-1} = 1-\alpha_2$$

and

$$\left(\frac{\lambda}{1-\lambda}\right)^{n-2k-1} = \frac{(1-\alpha_2)(1-\phi_1)}{\alpha_2\phi_k + (1-\alpha_2)\phi_1} \quad \text{for } k = 1, \cdots, n-2.$$

They are equivalent to (15) and (16). For $\alpha_2 = 0$, both of them and (13) will hold if $\phi_1 = 1/2$ (in which case, $\lambda = 1/2$). Beyond this special case, (15) pins down a decreasing sequence $\{\phi_k\}_{k=1}^{n-2}$ uniquely. Substituting it and (16) into (13), we can obtain ϕ_{n-1}. This means that, if $n \geq 3$ and $\alpha_2 > 0$, the price-frame independent equilibrium can only hold for a particular sequence of ϕ_k.

B.3 Proof of Proposition 5

When $\alpha_2 = 0$ (i.e., when frame B is also a simple frame), (13) becomes

$$\frac{\lambda}{1-\lambda} = \left(\frac{\phi_1}{1-\phi_1}\right)^{1/(n-1)}.$$

It follows that λ tends to $1/2$ as $n \to \infty$.[38] Then industry profit $n\pi = n\phi_1(1-\lambda)^{n-1}$ must converge to zero.[39]

Now consider $\alpha_2 > 0$. The left-hand side of (13) is bounded, so it must be that $\lim_{n\to\infty} \lambda < 1/2$ (otherwise the right-hand side would tend to infinity). Since $\{\phi_k\}_{k=1}^{n-1}$ is a non-increasing sequence, the right-hand side of (13) is greater than

$$\frac{\alpha_2(1-\phi_1)}{n} \sum_{k=1}^{n-2} C_{n-1}^{k} \left(\frac{\lambda}{1-\lambda}\right)^{k} = \frac{\alpha_2(1-\phi_1)}{n}\left[\frac{1-\lambda^{n-1}}{(1-\lambda)^{n-1}} - 1\right].$$

[38] In this case, the sign of $\partial\lambda/\partial n$ depends on the value of ϕ_1. If $\phi_1 > 1/2$, $\partial\lambda/\partial n < 0$; if $\phi_1 < 1/2$, $\partial\lambda/\partial n > 0$; and if $\phi_1 = 1/2$, $\partial\lambda/\partial n = 0$ (as $\lambda = 1/2$).

[39] Each firm's profit, when $\alpha_2 = 0$, is $\pi = \phi_1(1-\lambda)^{n-1} = (1-\phi_1)\lambda^{n-1}$, which must fall with n no matter how λ varies with n. However, industry profit $n\pi$ can rise with n when n is not too large and ϕ_1 is sufficiently large or small. For example, when $\phi_1 = 0.95$ or 0.05, from $n = 2$ to 3, industry profit $n\pi$ increases from 0.095 to about 0.099.

So it must be that $\lim_{n\to\infty} n\,(1-\lambda)^{n-1} > 0$, otherwise the right-hand side of (13) would tend to infinity (given $\lim_{n\to\infty}\lambda < 1/2$). This result implies that λ must converge to zero and industry profit $n\pi = n\,(1-\lambda)^{n-1}\left[\alpha_2\phi_{n-1} + (1-\alpha_2)\,\phi_1\right]$ is bounded away from zero as $n\to\infty$.

B.4 Proof of Proposition 6

(i) For $n=2$, we have $\lambda = \frac{1-\alpha_2}{2-\alpha_2}$. For $n=3$, we have $\lambda = \frac{x}{1+x}$ with

$$x = \sqrt{\frac{4\alpha_2^2}{9} + 1 - \alpha_2} - \frac{2\alpha_2}{3}.$$

Then one can show that λ, π and $n\pi$ all decrease from $n=2$ to 3.

(ii) As $\alpha_2 \to 1$, (17) implies $\lambda \to 0$. Let $\alpha_2 = 1-\varepsilon$ with $\varepsilon \approx 0$, and use the second-order approximation $\lambda \approx k_1\varepsilon + k_2\varepsilon^2$. For $n \geq 4$, the right-hand side of (17) can be approximated as

$$2\,(1-\varepsilon)\left[a_1\left(\lambda+\lambda^2\right) + a_2\left(\lambda+\lambda^2\right)^2\right]$$

by using $\lambda/\,(1-\lambda) \approx \lambda + \lambda^2$, where $a_k = C_{n-1}^k/\,(n-k+1)$. By discarding all terms of order higher than ε^2 in the square bracket, we can further approximate it as

$$\begin{aligned}
& 2\,(1-\varepsilon)\left[a_1\lambda + (a_1+a_2)\,\lambda^2\right] \\
\approx\; & 2\,(1-\varepsilon)\left[a_1k_1\varepsilon + \left(a_1k_2 + (a_1+a_2)\,k_1^2\right)\varepsilon^2\right] \\
\approx\; & 2\left[a_1k_1\varepsilon + \left(a_1\,(k_2-k_1) + (a_1+a_2)\,k_1^2\right)\varepsilon^2\right].
\end{aligned}$$

Since the left-hand side of (17) is ε, we can solve

$$k_1 = \frac{n}{2\,(n-1)}; \quad k_2 = k_1 - \frac{n^2-2}{2\,(n-1)}k_1^2.$$

For $n=3$, the right-hand side of (17) is $(4\alpha_2/3)\lambda/(1-\lambda) + (\lambda/(1-\lambda))^2$. Using a similar approximation procedure, one can verify that the same expressions for k_1 and k_2 apply. It follows that λ decreases with n as k_1 decreases with n.

Industry profit (for $n \geq 3$) becomes

$$\begin{aligned}
n\pi &= (1-\lambda)^{n-1}\left[1 + (n/2-1)\varepsilon\right] \\
&\approx \left[1-(n-1)\,\lambda + C_{n-1}^2\lambda^2\right]\left[1+(n/2-1)\varepsilon\right] \\
&\approx \left\{1-(n-1)\,k_1\varepsilon + \left[C_{n-1}^2k_1^2 - (n-1)\,k_2\right]\varepsilon^2\right\}\left[1+(n/2-1)\varepsilon\right] \\
&\approx 1-\varepsilon + \left[C_{n-1}^2k_1^2 - (n-1)\,k_2 - (n-1)\,(n/2-1)\right]\varepsilon^2 \\
&= 1-\varepsilon + \frac{(n-2)\,n^2}{8\,(n-1)^2}\varepsilon^2,
\end{aligned}$$

which increases with n. (The first-order approximation of λ is not sufficient to tell how $n\pi$ varies with n.) It is also clear that π decreases with n.

Now consider the limit case of $\alpha_2 \to 0$. Under the random purchase rule, $\lambda = 1/2$ (for any n) when $\alpha_2 = 0$. In that case, industry profit is just $n/2^n$, which decreases in n. Hence, for α_2 sufficiently close to zero, the same result will hold.

To see how λ changes with n, let $\alpha_2 = \varepsilon \approx 0$ and approximate λ by $1/2 - \theta\varepsilon$, where θ is yet to be determined. First, notice that

$$\frac{\lambda}{1-\lambda} \approx \frac{1/2 - \theta\varepsilon}{1/2 + \theta\varepsilon} \approx 1 - 4\theta\varepsilon.$$

Then the right-hand side of (17) can be approximated as

$$2K\varepsilon + 1 - 4(n-1)\theta\varepsilon,$$

by discarding all terms of order higher than ε, where $K = \sum_{k=1}^{n-2} C_{n-1}^k / (n-k+1)$. Since the left-hand side is just $1 - \varepsilon$, it follows that

$$\theta = \frac{K}{2(n-1)}.$$

Note that $K = \frac{(2^n-1)(n-1)}{n(n+1)} - \frac{1}{2}$ and, consequently, θ increases with n. Thus, λ decreases with n for α_2 close to zero.

B.5 Proof of Lemma 5

(a) First, in any equilibrium with pure strategy framing, at most one firm uses frame A. Suppose to the contrary that at least two firms use frame A. Then, they must all earn zero profit at any putative equilibrium. But then any of them has a unilateral incentive to deviate to frame B and a positive price. A contradiction.

(b) Second, in any equilibrium with pure strategy framing, at least one firm uses frame A. Suppose to the contrary that all firms use frame B. The only candidate equilibrium entails monopoly pricing $p = 1$ and each firm earns $1/n$. But then if one firm deviates to frame A and price $1 - \varepsilon$, it will earn $(1-\varepsilon)\left(\alpha_1\phi_{n-1} + 1 - \alpha_1\right)$. The reason is that, if the consumer is unable to compare prices in different frames (which happens with probability α_1), the deviator's demand is ϕ_{n-1}; if the consumer is able to compare prices in different frames (which happens with probability $1 - \alpha_1$), the deviator serves the whole market (because all other firms charge $p = 1$ and so are dominated options). As $\phi_{n-1} \geq 1/n$, the deviation profit is greater than $1/n$ for a sufficiently small ε and any $\alpha_1 \in (0, 1)$.

(c) The final possibility is that one firm uses A and all other firms use B. Suppose such an equilibrium exists. Let π_A be the A firm's profit and π_B^j be the profit of a B firm indexed by j. (Notice that the B firms may eventually use different pricing strategies and make different profits). Let p_A be the lowest price on which the A firm puts positive probability (it might be a deterministic price). (i) Suppose that, at equilibrium, $\pi_A > \min\{\pi_B^j\}$. Then, if the B firm which earns the least deviates to frame A and a price $p_A - \varepsilon$, it will replace the original A firm and have a demand no less than the original A firm's demand since it now charges a lower price *and* faces fewer competitors.[40] So this deviation will be profitable at least when ε tends to zero. A contradiction. (ii) Suppose now that, at equilibrium, $\pi_A \leq \min\{\pi_B^j\}$. Notice that $\pi_A \geq 1/n$, otherwise the A firm would deviate to frame B and a price $p = 1$, and make profit $1/n$. As industry profit cannot exceed one, all firms must earn $1/n$ at the candidate equilibrium and consumer surplus is zero. This also implies that all firms must be charging the monopoly price. But then any B firm has an incentive to deviate to frame A and price $1 - \varepsilon$, in which case it makes profit $(1-\varepsilon)\left(\alpha_1\phi_{n-2} + 1 - \alpha_1\right) > 1/n$ for a sufficiently small ε. A contradiction.

B.6 Proof of Proposition 7

We only need to rule out profitable deviations from the proposed equilibrium. Consider two possible deviations with frame A first: (i) a deviation to $\left(A, p < p_0^A\right)$ is not profitable as the firm does not gain market share but loses on prices; (ii) a deviation $(A, p = 1)$ is not profitable either, since the deviator's profit is $(1-\lambda)^{n-1}\phi_{n-1} < \pi$.

Let us now consider a deviation to $\left(B, p \in [p_0^A, 1)\right)$. Deviator's profit is

$$\hat{\pi}(B,p) = p\pi(B,1) + p(1-\alpha_1)\sum_{k=1}^{n-1} P_{n-1}^k x_A(p)^k.$$

This expression captures the fact that when $n-1$ other firms also use B, or when $k \geq 1$ firms use A and the consumer is confused between A and B, firm i's demand does not depend on its price and so is equal to $\pi(B,1)$. When $k \geq 1$ firms use A and the consumer is not confused between A and B, all other B firms (which charge price $p = 1$) are dominated by the cheapest A firm, and the consumer buys from firm i only if the cheapest A firm charges a price greater than p. Notice that, from $\pi(A,p) = \pi$ for $p \in [p_0^A, 1)$, the second term in $\hat{\pi}(B,p)$ is equal to

$$\pi - p\pi - p\alpha_1 \sum_{k=1}^{n-1} P_{n-1}^k x_A(p)^k \phi_{n-k-1}.$$

Then,

$$\hat{\pi}(B,p) < p\pi + \pi - p\pi = \pi.$$

The deviation to $\left(B, p < p_0^A\right)$ results in a lower profit. This completes the proof.

[40] When the consumer is unable to compare prices in different frames, the deviator's demand is ϕ_{n-2} which is (weakly) greater than ϕ_{n-1}, the original A firm's demand in this case. When the consumer is able to compare prices in different frames, the deviator is more likely to dominate the remaining B firms (and so has a higher expected demand) than the orginal A firm.

B.7 Proof of Proposition 8

From (21), it follows that $\lambda \to 1$ as $\alpha_1 \to 0$. Let $\alpha_1 = \varepsilon$ with $\varepsilon \approx 0$, and $\lambda = 1 - \delta$ with $\delta \approx 0$. Then the right-hand side of (21) can be approximated as

$$\left(\frac{1-\delta}{\delta}\right)^{n-1}(1-\phi_1) \approx \frac{1-\phi_1}{\delta^{n-1}}.$$

This is because only the term with $k = n-1$ matters when $\delta \approx 0$. Hence, from (21), we can solve

$$\delta \approx \left(\frac{1-\phi_1}{\frac{1}{\varepsilon}\left(1-\frac{1}{n}\right)+\phi_{n-1}-1}\right)^{1/(n-1)} \approx \left(\frac{(1-\phi_1)\, n\varepsilon}{n-1}\right)^{1/(n-1)}.$$

One can show that $\ln\delta$ increases with n (and so λ decreases with n). Each firm's profit is

$$\pi = \delta^{n-1}\left[1+\left(\phi_{n-1}-1\right)\varepsilon\right] \approx \frac{(1-\phi_1)\, n\varepsilon}{n-1}.$$

We have discarded the term of ε^2. Clearly, π decreases with n, but $n\pi$ increases with n.

C Appendix: The formula for $\Pr(k,l)$

Notice that

$$\Pr(k,l) = C_{n-1}^{k-1}\left(\frac{1}{m}\right)^{k-1}\left(1-\frac{1}{m}\right)^{n-k} w(n-k, l-1),$$

where the product of the first three terms is the probability that $k-1$ firms among $n-1$ ones are also using frame A_j given that firm i has already chosen A_j, and $w(n-k, l-1)$ is the conditional probability that $n-k$ firms outside group A_j adopt $l-1$ other distinct frames in total. In fact, $w(n-k, l-1)$ is the probability that $n-k$ balls are thrown at random into $l-1$ boxes among $m-1$ ones. (In particular, we let $w(n-k, 0) = 0$ for $n-k > 0$, and $w(0,0) = 1$).

Now suppose that $l \geq 2$. If we let 1 to $l-1$ be the targeted "boxes" and E^0 be the event that the remaining $m-l$ boxes are empty, then

$$\begin{aligned} w(n-k, l-1) &= C_{m-1}^{l-1} \cdot \Pr\left(E^0\right) \cdot \Pr\left(\text{all targeted boxes are nonempty} | E^0\right) \\ &= C_{m-1}^{l-1}\left(\frac{l-1}{m-1}\right)^{n-k}\left[1-\Pr\left(\bigcup_{i=1}^{l-1} H_i\right)\right], \end{aligned}$$

where H_i is the event that box $i \in \{1, \cdots, l-1\}$ is empty conditional on the fact that all $n-k$ balls are thrown at random towards the targeted $l-1$ boxes, and so

$$\Pr\left(\bigcup_{i=1}^{l-1} H_i\right) = \sum_{h=1}^{l-1}(-1)^{h-1} C_{l-1}^{h} \Pr\left(H_1 \cdots H_h\right)$$

with

$$\Pr\left(H_1 \cdots H_h\right) = \left(1 - \frac{h}{l-1}\right)^{n-k}.$$

Using the above formula, we have

$$\begin{aligned}
\frac{\Pr(1,l)}{l} &= \frac{1}{l}\left(1-\frac{1}{m}\right)^{n-1} C_{m-1}^{l-1}\left(\frac{l-1}{m-1}\right)^{n-1}\left[1-\sum_{h=1}^{l-1}(-1)^{h-1} C_{l-1}^{h}\left(1-\frac{h}{l-1}\right)^{n-1}\right] \\
&= \frac{(l-1)^{n-1} C_m^l}{m^n}\left[1-\sum_{h=1}^{l-1}(-1)^{h-1} C_{l-1}^{h}\left(1-\frac{h}{l-1}\right)^{n-1}\right].
\end{aligned}$$

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D Appendix: Extensions of the Oligopoly Model

The oligopoly model in the main text focuses on two polar cases: $\alpha_2 < \alpha_1 = 1$ and $\alpha_1 < \alpha_2 = 1$. In this part we discuss the general oligopoly model with α_1 and α_2 strictly smaller than 1. When frame differentiation is more confusing than frame complexity ($\alpha_2 < \alpha_1 < 1$), we show that, if an equilibrium exists, it resembles the one in the polar case (with $\alpha_1 = 1$). When frame complexity is more confusing than frame differentiation ($\alpha_1 < \alpha_2 < 1$), we derive a condition under which, like in the polar case (with $\alpha_2 = 1$), there is an equilibrium where the complex frame is always associated with higher prices. In both cases, an increase in the number of firms can still harm consumers.

D.1 The case with $\alpha_2 < \alpha_1 < 1$

This part deals with the oligopoly model with $\alpha_2 < \alpha_1 < 1$. We focus on the symmetric mixed-strategy equilibrium (λ, F_A, F_B), in which λ is the likelihood that each firm employs frame A and F_z is the continuous price distribution associated with frame $z \in \{A, B\}$. Let $S_z = [p_0^z, p_1^z]$ be the support of F_z. As before, let $P_{n-1}^k \equiv C_{n-1}^k \lambda^k (1-\lambda)^{n-k-1}$ and $x_z(p) \equiv 1 - F_z(p)$.

We first derive a firm's profit given that the other firms use the equilibrium strategy. If firm i employs frame A and prices at p, its profit is equal to

$$\begin{aligned}\pi(A,p) &= p\lambda^{n-1}x_A(p)^{n-1} + \\ &\quad p\sum_{k=0}^{n-2} P_{n-1}^k x_A(p)^k[(1-\alpha_1)x_B(p)^{n-k-1} + \alpha_1(\alpha_2\phi_{n-k-1} + (1-\alpha_2)\phi_1)].\end{aligned}$$

Notice that when k other firms also use frame A, firm i has a positive market share only if it is undominated in group A, which happens with probability $x_A(p)^k$. The first term in $\pi(A,p)$ captures the fact that if $k = n-1$, then firm i serves the whole market. The second term deals with $k < n-1$. (i) If the consumer is able to compare A and B, firm i serves the whole market whenever it prices below all the B firms (see the first term in the square bracket). (ii) If the consumer is unable to compare A and B, then firm i's demand depends on consumer's ability to compare prices in frame B. If she cannot compare prices in frame B, then no B firm is dominated so that firm i's demand is ϕ_{n-k-1}. If the consumer can compare prices in frame B, only one B firm is selected from the B group and so firm i's demand is ϕ_1.

If firm i uses frame B and charges price p, its profit is

$$\begin{aligned}\pi(B,p) &= p(1-\lambda)^{n-1}\left[\frac{\alpha_2}{n} + (1-\alpha_2)x_B(p)^{n-1}\right] + \\ &\quad p\sum_{k=1}^{n-1} P_{n-1}^k \left(\begin{array}{c}(1-\alpha_2)x_B(p)^{n-k-1}\left[\alpha_1(1-\phi_1) + (1-\alpha_1)x_A(p)^k\right] + \\ \alpha_2\left[\alpha_1\frac{1-\phi_{n-k}}{n-k} + (1-\alpha_1)H_k(p)\right]\end{array}\right).\end{aligned}$$

The first term captures the situation in which all the other firms also use frame B. Then, if the consumer is confused, she shops at random and chooses firm i with probability $1/n$, and if the consumer can compare prices, she chooses firm i only if it offers the best deal. The summation term captures the case in which $k \geq 1$ firms use frame A. (i) If the consumer can compare prices in frame B (which happens with probability $1-\alpha_2$), firm i has a positive demand only if it offers the lowest price in group B, the probability of which is $x_B(p)^{n-k-1}$. If the consumer is unable to compare A and B, firm i's demand is $1-\phi_1$ since only one firm is undominated in group A; if the consumer is able to compare A and B, firm i serves the whole market when all A firms charge prices higher than p (that is, with probability $x_A(p)^k$). (ii) If the consumer is unable to compare prices in frame B or prices in different frames (which happens with probability $\alpha_1\alpha_2$), firm i has a demand $\frac{1-\phi_{n-k}}{n-k}$ since all B firms are undominated. (iii) If the consumer is unable to compare prices in frame B but is able to compare prices in different frames (that is, with probability $\alpha_2(1-\alpha_1)$), firm i's demand is

$$H_k(p) \equiv \sum_{l=1}^{n-k} \frac{C_{n-k-1}^{l-1}}{l} \int_p^{p_1^A} F_B(x)^{l-1} \left[1-F_B(x)\right]^{n-k-l} dG_k(x),$$

where $G_k(x) \equiv 1-\left[1-F_A(x)\right]^k$ is the distribution function of the minimum price in group A of cardinality k. In this case, to have a positive demand, firm i must price below the minimum price (let it be x) in group A. (That is why we integrate over x from p to p_1^A.) Conditional on that, firm i's market share depends on how many other B firms survive. Given the minimum price x in group A, the probability that exactly $l-1$ other B firms survive is $C_{n-k-1}^{l-1} F_B(x)^{l-1}\left[1-F_B(x)\right]^{n-k-l}$. When l firms from group B (including firm i) survive, firm i's market share is $1/l$. Noting that $C_{n-k-1}^{l-1}/l = C_{n-k}^{l}/(n-k)$ and using the binomial formula, $H_k(p)$ becomes

$$H_k(p) = \frac{1}{n-k} \int_p^{p_1^A} \frac{1-\left[1-F_B(x)\right]^{n-k}}{F_B(x)} dG_k(x).$$

We now show that, if a symmetric equilibrium with continuous price distributions exists, it specifies $p_A^1 = p_B^1 = 1$ under certain conditions.

Claim 1 *If the symmetric equilibrium with continuous F_z exists, and if $\phi_k \geq 1/(1+k)$ (i.e., if the frame A is always weakly favored), then at equilibrium it must hold that $p_A^1 = p_B^1 = 1$.*

Proof. First, as in the duopoly case, it is easy to show that $\max\{p_1^A, p_1^B\} = 1$ and there is no gap between S_A and S_B (i.e., $S_A \cap S_B \neq \emptyset$). Then we rule out the possibility of having only one frame associated with $p_1^z = 1$.

(i) It cannot be that $p_1^A < p_1^B = 1$. Suppose, to the contrary, that this is true at equilibrium. Then the indifference condition requires $\pi(A, p_1^A) = \pi(B, p_1^A)$ since $p_1^A \in S_B$. For any $p \in [p_1^A, 1]$, $x_A(p) = 0$ and so we have

$$\pi(A,p)/p = (1-\lambda)^{n-1}[(1-\alpha_1)x_B(p)^{n-1} + \alpha_1\alpha_2\phi_{n-1} + \alpha_1(1-\alpha_2)\phi_1],$$

and

$$\begin{aligned}\pi(B,p)/p &= (1-\lambda)^{n-1}[\frac{\alpha_2}{n} + (1-\alpha_2)x_B(p)^{n-1}] \\ &\quad + \alpha_1 \sum_{k=1}^{n-1} P_{n-1}^k \left[\alpha_2 \frac{1-\phi_{n-k}}{n-k} + (1-\alpha_2)(1-\phi_1)x_B(p)^{n-k-1}\right].\end{aligned}$$

From $\alpha_1 > \alpha_2$, it follows that $\pi(B,p)/p$ decreases with $p \in [p_1^A, 1]$ faster than $\pi(A,p)/p$. Thus, $\pi(A, p_1^A) = \pi(B, p_1^A)$ implies $\pi(A,p) > \pi(B,p)$ for $p \in (p_1^A, 1]$. But this is a contradiction, since the latter is equilibrium profit.

(ii) It cannot be that $p_1^B < p_1^A = 1$. A similar logic applies. Suppose, to the contrary, that this is true at equilibrium. Then $\pi(A, p_1^B) = \pi(B, p_1^B)$ since $p_1^B \in S_A$. For any $p \in [p_1^B, 1]$, $x_B(p) = 0$ and so we have

$$\begin{aligned}\pi(A,p)/p &= \lambda^{n-1}x_A(p)^{n-1} + \alpha_1 \sum_{k=0}^{n-2} P_{n-1}^k x_A(p)^k [\alpha_2\phi_{n-k-1} + (1-\alpha_2)\phi_1] \\ &= \lambda^{n-1}x_A(p)^{n-1} + (1-\lambda)^{n-1}\alpha_1\left[\alpha_2\phi_{n-1} + (1-\alpha_2)\phi_1\right] \\ &\quad + \alpha_1 \sum_{k=1}^{n-2} P_{n-1}^k x_A(p)^k \left[\alpha_2\phi_{n-k-1} + (1-\alpha_2)\phi_1\right],\end{aligned}$$

and

$$\begin{aligned}\pi(B,p)/p &= (1-\lambda)^{n-1}\frac{\alpha_2}{n} + \lambda^{n-1}(1-\alpha_2)\left[\alpha_1(1-\phi_1) + (1-\alpha_1)x_A(p)^{n-1}\right] \\ &\quad + \alpha_2 \sum_{k=1}^{n-1} P_{n-1}^k \left[\alpha_1 \frac{1-\phi_{n-k}}{n-k} + (1-\alpha_1)H_k(p)\right] \\ &= (1-\lambda)^{n-1}\frac{\alpha_2}{n} + \lambda^{n-1}\left[\alpha_1(1-\phi_1) + (1-\alpha_1)x_A(p)^{n-1}\right] \\ &\quad + \alpha_2 \sum_{k=1}^{n-2} \frac{P_{n-1}^k}{n-k}\left[\alpha_1\left(1-\phi_{n-k}\right) + (1-\alpha_1)x_A(p)^k\right],\end{aligned}$$

where we have used the fact that for $p \in [p_1^B, 1]$,

$$\begin{aligned}H_k(p) &= \frac{1}{n-k}\int_p^1 \frac{1-(1-F_B(x))^{n-k}}{F_B(x)} dG_k(x) \\ &= \frac{1}{n-k}[G_k(1) - G_k(p)] \\ &= \frac{1}{n-k}x_A(p)^k.\end{aligned}$$

It follows that the coefficient of $x_A(p)^{n-1}$ in $\pi(A,p)/p$ is λ^{n-1}, which is greater than $\lambda^{n-1}(1-\alpha_1)$, the counterpart in $\pi(B,p)/p$, and for $k \leq n-2$, the coefficient of $x_A(p)^k$ in $\pi(A,p)/p$ is

$$\alpha_1 P_{n-1}^k \left[\alpha_2\phi_{n-k-1} + (1-\alpha_2)\phi_1\right] \geq \alpha_1 P_{n-1}^k \phi_{n-k-1},$$

which is also greater than the counterpart $\alpha_2(1-\alpha_1) P_{n-1}^k/(n-k)$ in $\pi(B,p)/p$ given that $\alpha_1 > \alpha_2$ and $\phi_{n-k-1} \geq 1/(n-k)$. That is, on $[p_1^B, 1]$, $\pi(A,p)/p$ decreases in p faster than $\pi(B,p)/p$. Therefore, $\pi(A,p_1^B) = \pi(B,p_1^B)$ implies $\pi(B,p) > \pi(A,p)$ for $p \in (p_1^B, 1]$. Since the latter is equilibrium profit, we have reached a contradiction.

Hence, (i) and (ii) imply that it can only be that $p_A^1 = p_B^1 = 1$. ■

From $p_1^z = 1$ it follows that each firm's equilibrium profit (π) should be equal to $\pi(A,1) = \pi(B,1)$. Specifically, we have

$$\pi(A,1) = \alpha_1(1-\lambda)^{n-1}[\alpha_2\phi_{n-1} + (1-\alpha_2)\phi_1], \tag{25}$$

and

$$\pi(B,1) = (1-\lambda)^{n-1}\frac{\alpha_2}{n} + \lambda^{n-1}\alpha_1(1-\phi_1) + \alpha_1\alpha_2\sum_{k=1}^{n-2} P_{n-1}^k \frac{1-\phi_{n-k}}{n-k}.$$

From $\pi(A,1) = \pi(B,1)$, we can pin down λ:

$$\begin{aligned} &\alpha_2\phi_{n-1} + (1-\alpha_2)\phi_1 - \frac{\alpha_2}{n\alpha_1} \\ = \quad &\alpha_2 \sum_{k=1}^{n-2} \frac{C_{n-1}^k\left(1-\phi_{n-k}\right)}{n-k}\left(\frac{\lambda}{1-\lambda}\right)^k + (1-\phi_1)\left(\frac{\lambda}{1-\lambda}\right)^{n-1}. \end{aligned} \tag{26}$$

Since $\phi_1 \geq \phi_{n-1} \geq 1/n$ and $\alpha_1 > \alpha_2$, the left-hand side is positive. The right-hand side rises with λ from zero to infinity. Then, (26) has a unique solution in $(0,1)$. Therefore, if there exists a symmetric equilibrium with continuous F_z's, the expression for the equilibrium profit and the condition which determines λ resemble those in the polar case with $\alpha_1 = 1$.

Existence of equilibrium. A symmetric mixed-strategy equilibrium with continuous F_z exists if the system of equations $\pi(z,p) = \pi$ for $z = A, B$ has a well defined solution (F_A, F_B). Notice that for $\alpha_2 > 0$, $\pi(B,p) = \pi$ is a functional equation due to the presence of $H_k(p)$. Proving existence in this case is therefore difficult. However, when $\alpha_2 = 0$, $\pi(B,p) = \pi$ degenerates to an ordinary polynomial equation, and existence of equilibrium is not difficult to be established. In continuation, we assume that the symmetric equilibrium with continuous F_z exists for arbitrary $\alpha_2 < \alpha_1 < 1$.

Equilibrium price-frame (in)dependence. We explore the possibility of a symmetric equilibrium with $F_A = F_B = F$. Let $[p_0, 1]$ be the support of F. Then for any $p \in [p_0, 1]$, it should hold that $\pi(A, p) = \pi(B, p)$. Using the procedure in the proof of Proposition 4, we can rewrite this condition as

$$\begin{aligned}
&\sum_{k=1}^{n-2} b_k x(p)^{-k} - \alpha_2(1-\alpha_1) \sum_{k=1}^{n-2} P_{n-1}^k \frac{H_k(p)}{x(p)^{n-1}} \\
= \;& \alpha_1(1-\alpha_2)(1-\lambda)^{n-1} - (\alpha_1 - \alpha_2 + \alpha_1\alpha_2)\lambda^{n-1} - \alpha_2(1-\alpha_1), \qquad (27)
\end{aligned}$$

where

$$b_k \equiv \alpha_1 \left\{ P_{n-1}^{n-k-1} [\alpha_2\phi_k + (1-\alpha_2)\phi_1] - P_{n-1}^k (1-\alpha_2)(1-\phi_1) \right\}.$$

For $n = 2$, it is ready to check that conditions (26) and (27) hold only if $\phi_1 = 1/2$. For $n \geq 3$, they both hold if $\phi_1 = 1/2$ and $\alpha_2 = 0$. Except for these two cases, (27) cannot hold when $\alpha_1 < 1$ since the $H_k(p)$ term is nonzero.

The impact of greater competition. This part of the analysis relies only on (25) and (26). First, the results in Proposition 5 still hold. In particular, we have $\lim_{n\to\infty} \lambda = 1/2$ for $\alpha_2 = 0$ and $\lim_{n\to\infty} \lambda = 0$ for $\alpha_2 > 0$. Hence, even if frame B is only slightly complex, sufficient competition forces the firms to use frame B almost all the time.

For $\alpha_1 = 1$ and the random purchase rule, we have shown that competition has a perverse effect on consumer welfare when α_2 is close to α_1. Note that the limit analysis developed in Proposition 6 for $\alpha_2 \to \alpha_1 = 1$ does not work here. For a given $\alpha_1 < 1$, when α_2 converges to α_1, the left-hand side of (26) tends to

$$\alpha_1\phi_{n-1} + (1-\alpha_1)\phi_1 - \frac{1}{n}.$$

With the random purchase rule, this becomes $(1-\alpha_1)\left(\frac{1}{2} - \frac{1}{n}\right)$ and is *not* equal to zero unless $n = 2$. That is, λ does *not* tend to zero as $\alpha_2 \to \alpha_1 < 1$. This is why the limit analysis for $\alpha_1 = 1$ does not extend to $\alpha_1 < 1$. However, numerical simulations suggest that for a sufficiently high α_1, an increase in the number of firms still has a perverse effect when α_2 approaches α_1.

D.2 The case with $\alpha_1 < \alpha_2 < 1$

This part extends the oligopoly model to the case with $\alpha_2 < 1$. We first derive a condition for the existence of a symmetric equilibrium (λ, F_A, F_B) with $S_A = [p_0^A, \hat{p}]$ and $S_B = [\hat{p}, 1]$.

Claim 2 *In the oligopoly model with $0 < \alpha_1 < \alpha_2 < 1$, there is a symmetric equilibrium (λ, F_A, F_B) with $S_A = [p_0^A, \hat{p}]$ and $S_B = [\hat{p}, 1]$ if and only if*

$$\frac{\alpha_2 - \alpha_1}{\alpha_1(1-\alpha_2)} \frac{1}{1-\phi_1} > \frac{1-\lambda^{n-1}}{(1-\lambda)^{n-1}} - 1, \qquad (28)$$

where $\lambda \in (0,1)$ *solves*

$$\begin{aligned} & \alpha_2 \phi_{n-1} + (1-\alpha_2)\phi_1 + \frac{\alpha_2}{\alpha_1}\left(1 - \frac{1}{n}\right) - 1 \\ = & \sum_{k=1}^{n-1} C_{n-1}^k \left[\alpha_2 \frac{1-\phi_{n-k}}{n-k} + (1-\alpha_2)(1-\phi_1)\right]\left(\frac{\lambda}{1-\lambda}\right)^k . \end{aligned} \tag{29}$$

Proof. Given that the other firms use the equilibrium strategy, if firm i chooses frame A and charges $p \in [p_0^A, \hat{p}]$, its profit is

$$\pi(A,p) = p \sum_{k=0}^{n-2} P_{n-1}^k x_A(p)^k \left[1 - \alpha_1 + \alpha_1 \alpha_2 \phi_{n-k-1} + \alpha_1 (1-\alpha_2)\phi_1\right] + p\lambda^{n-1} x_A(p)^{n-1} .$$

Note first that firm i has a positive market share if it charges the lowest price in group A, which happens with probability $x_A(p)^k$ if there are k other A firms. The last term gives firm i's revenues for $k = n-1$. The summation terms gives firm i's revenues when there are $k < n-1$ other A firms. If the consumer can compare prices in different frames, then firm i serves the whole market since all B firms' equilibrium prices are higher than p. This explains the term $(1-\alpha_1)$ in the square bracket. If the consumer cannot compare A and B, firm i's demand depends on whether the consumer can compare prices in frame B. If she cannot compare them, all B firms survive and firm i's demand is ϕ_{n-k-1}; if she can compare, only one B firm wins in group B and firm i's demand is ϕ_1. (All subsequent profit functions are constructed similarly and, therefore, we omit further explanations.)

When firm i charges $p = \hat{p}$, it has a positive market share only if all other firms use frame B (i.e., only if $k = 0$), so its profit is

$$\pi(A,\hat{p}) = \hat{p}(1-\lambda)^{n-1}\left\{1 - \alpha_1 + \alpha_1\left[\alpha_2 \phi_{n-1} + (1-\alpha_2)\phi_1\right]\right\} .$$

Given that the other firms use the equilibrium strategy, if firm i chooses frame B and charges $p \in [\hat{p}, 1]$, its profit is

$$\begin{aligned} \pi(B,p) &= p \sum_{k=1}^{n-1} P_{n-1}^k \alpha_1 \left[\alpha_2 \frac{1-\phi_{n-k}}{n-k} + (1-\alpha_2)(1-\phi_1) x_B(p)^{n-k-1}\right] \\ & \quad + p(1-\lambda)^{n-1}\left[\alpha_2/n + (1-\alpha_2) x_B(p)^{n-1}\right] . \end{aligned}$$

Notice that, if there are A firms in the market (which are charging prices lower than the B firms at equilibrium), then firm i makes sales only if the consumer cannot compare prices in different frames. In particular, when firm i charges $p = \hat{p}$, $\pi(B,\hat{p})$ is just $\pi(B,p)$ with $x_B(p)$ replaced by $x_B(\hat{p}) = 1$.

At equilibrium, it should hold that $\pi(A,\hat{p}) = \pi(B,\hat{p})$. Dividing each side by $\alpha_1(1-\lambda)^{n-1}$ we obtain equation (29) which determines λ. (One can check that, for

$\alpha_2 = 1$, this equation degenerates to (21) in Proposition 7.) Since ϕ_k is non-increasing in k, the left-hand side of (29) is (weakly) greater than

$$\phi_{n-1} + \frac{\alpha_2}{\alpha_1}\left(1 - \frac{1}{n}\right) - 1,$$

which is positive as $\phi_{n-1} \geq 1/n$ and $\alpha_2 > \alpha_1$. Therefore, (29) has a unique solution in $(0,1)$.

To determine $\hat{p}$, we can use $\pi(B,\hat{p}) = \pi(B,1)$. Note that

$$\pi(B,1) = \lambda^{n-1}\alpha_1(1-\phi_1) + (1-\lambda)^{n-1}\frac{\alpha_2}{n} + \alpha_1\alpha_2\sum_{k=1}^{n-2} P_{n-1}^k \frac{1-\phi_{n-k}}{n-k} \equiv \pi. \tag{30}$$

In continuation, we refer to π as each firm's equilibrium profit. Note that $\hat{p} < 1$ since the demand at $p = \hat{p}$ is greater than that at $p = 1$. In addition, the expressions for F_z, $z \in \{A, B\}$, can be solved from $\pi(z,p) = \pi$, and p_0^A follows from $\pi(A, p_0^A) = \pi$. All of them are well defined.

Let us show that there are no profitable unilateral deviations.

(i) If firm i deviates to $\left(B,\ p \in [p_0^A, \hat{p})\right)$, it makes profit

$$\begin{aligned}
\hat{\pi}(B,p) &= p\sum_{k=1}^{n-1} P_{n-1}^k\left[\alpha_1\alpha_2\frac{1-\phi_{n-k}}{n-k} + \alpha_1(1-\alpha_2)(1-\phi_1) + (1-\alpha_1)x_A(p)^k\right] \\
&\quad + p(1-\lambda)^{n-1}(\alpha_2/n + 1 - \alpha_2) \\
&= \frac{p\pi}{\hat{p}} + p(1-\alpha_1)\sum_{k=1}^{n-1} P_{n-1}^k x_A(p)^k,
\end{aligned}$$

where the second equality follows from $\pi(B,\hat{p}) = \pi$. Notice that from $\pi(A,p) = \pi$ for $p \in [p_0^A, \hat{p}]$, one can check that the second term is actually equal to

$$\pi - \frac{p\pi}{\hat{p}} - M,$$

where

$$M = p\alpha_1\lambda^{n-1}x_A(p)^{n-1} + p\alpha_1\sum_{k=1}^{n-2} P_{n-1}^k x_A(p)^k\left[\alpha_2\phi_{n-k-1} + (1-\alpha_2)\phi_1\right].$$

Since $M > 0$, it is clear that $\hat{\pi}(B,p) < \pi$, so that the deviation to $\left(B, p \in [p_0^A, \hat{p})\right)$ is not profitable. Clearly, deviation to $\left(B,\ p < p_0^A\right)$ result is even lower profit.

(ii) If firm i deviates to $(A,\ p \in (\hat{p}, 1])$, then it makes profit

$$\hat{\pi}(A,p) = p(1-\lambda)^{n-1}\left[\alpha_1\alpha_2\phi_{n-1} + \alpha_1(1-\alpha_2)\phi_1 + (1-\alpha_1)x_B(p)^{n-1}\right].$$

This deviation is not profitable if $\hat{\pi}(A,p) < \pi(B,p)$, where the right-hand side is the equilibrium profit. Dividing each side of this inequality by $p\alpha_1(1-\lambda)^{n-1}$ and using

(29), it follows that the condition holds if and only if

$$\frac{\alpha_2 - \alpha_1}{\alpha_1 (1 - \alpha_2)} \frac{1}{1 - \phi_1} [1 - x_B (p)^{n-1}] > \sum_{k=1}^{n-1} C_{n-1}^k \left(\frac{\lambda}{1 - \lambda} \right)^k [1 - x_B (p)^{n-k-1}].$$

(Notice that the term with $k = n - 1$ in the right-hand side is actually zero.) A necessary condition for the above inequality to hold when $p = 1$ is

$$\frac{\alpha_2 - \alpha_1}{\alpha_1 (1 - \alpha_2)} \frac{1}{1 - \phi_1} > \sum_{k=1}^{n-2} C_{n-1}^k \left(\frac{\lambda}{1 - \lambda} \right)^k = \frac{1 - \lambda^{n-1}}{(1 - \lambda)^{n-1}} - 1. \tag{31}$$

(This is just (28).) Moreover, since $x_B (p)^{n-k-1}$ increases with k, (31) is also a sufficient condition. Therefore, the deviation to $(A, p \in (\hat{p}, 1])$ is not profitable *if and only if* (31) holds. This completes the proof. ■

For $n = 2$, the right-hand side of (28) is zero and the condition always holds. For $n \geq 3$, however, it may fail to hold. For example, for given $n \geq 3$ and $\alpha_2 < 1$, if α_1 is sufficiently close to α_2, the condition fails. This happens because, as $\alpha_1 \to \alpha_2$, λ (derived from 29) is bounded away from zero so that the right-hand side of (28) is also bounded away from zero, but the left-hand side tends to zero. (Notice that this argument does not apply if $\alpha_2 = 1$.)

As condition (28) depends on λ which is endogenous, we explore in continuation more primitive conditions. First, we identify two limit conditions: (i) For fixed n and $\alpha_1 < 1$, (28) holds if α_2 is sufficiently close to one. (ii) For fixed n and $\alpha_2 < 1$, (28) holds if α_1 is sufficiently close to zero. Condition (i) is straightforward, and (ii) is proved in Claim 3 below.

Second, suppose $(1 - \phi_{n-k}) / (n - k)$ decreases with $n - k$ (i.e., the greater the number of undominated B firms, the lower each B firm's demand). Then (29) implies that

$$\begin{aligned} &\alpha_2 \phi_{n-1} + (1 - \alpha_2) \phi_1 + \frac{\alpha_2}{\alpha_1} \left(1 - \frac{1}{n} \right) - 1 \\ > \; &\left[\alpha_2 \frac{1 - \phi_{n-1}}{n - 1} + (1 - \alpha_2)(1 - \phi_1) \right] \sum_{k=1}^{n-1} C_{n-1}^k \left(\frac{\lambda}{1 - \lambda} \right)^k . \end{aligned}$$

Using this inequality, we can derive a sufficient condition for (28):

$$\phi_{n-1} - \frac{1}{n \alpha_1} + \left(\frac{1}{\alpha_2} - 1 \right) \phi_1 < \frac{\alpha_2 - \alpha_1}{(n - 1) \alpha_1 (1 - \alpha_2)} \frac{1 - \phi_{n-1}}{1 - \phi_1}.$$

If we further use the uniformly random purchase rule (i.e., $\phi_{n-1} = 1/n$ and $\phi_1 = 1/2$), this sufficient condition becomes

$$1 - \frac{1}{\alpha_1} + \frac{n}{2} \left(\frac{1}{\alpha_2} - 1 \right) < \frac{2 (\alpha_2 - \alpha_1)}{\alpha_1 (1 - \alpha_2)}.$$

Notice that condition (28) is necessary and sufficient for a symmetric mixed-strategy equilibrium with adjacent supports (like the one identified in the polar case with $\alpha_2 = 1$) to exist. When (28) is violated, a symmetric mixed-strategy equilibrium might still exist, but the supports of the equilibrium price distributions will eventually overlap. Note that with overlapping supports, existence of equilibrium is hard to prove due to the fact that the price distributions are defined by a system involving functional equations (which is similar to the case discussed in D.1.)

The impact of greater competition. We now prove a limit result similar to the one in the polar case in Subsection 3.2: for fixed $\alpha_2 < 1$, greater competition improves industry profit and harms consumers if α_1 is sufficiently small.

Claim 3 *In the oligopoly model with $0 < \alpha_1 < \alpha_2 < 1$, for given n and α_2, there exists $\hat{\alpha} > 0$ such that for $\alpha_1 < \hat{\alpha}$, (i) condition (28) holds, and (ii) industry profit $n\pi$ increases from n to $n+1$.*

Proof. For fixed n and $\alpha_2 < 1$, if α_1 tends to zero, then from (29) it follows that λ tends to one. Let $\alpha_1 = \varepsilon \approx 0$ and $\lambda = 1 - \delta$ with $\delta \approx 0$. Then the right-hand side of (29) can be approximated as

$$\left(\frac{1-\delta}{\delta}\right)^{n-1} (1-\phi_1) \approx \frac{1-\phi_1}{\delta^{n-1}}.$$

This is because only the term with $k = n-1$ matters when $\delta \approx 0$. In addition, the left-hand side can be approximated as $\frac{\alpha_2}{\varepsilon}\left(1 - \frac{1}{n}\right)$. Hence, from (29), we can solve

$$\delta \approx \left(\frac{(1-\phi_1)\, n\varepsilon}{\alpha_2 (n-1)}\right)^{1/(n-1)}.$$

It can be shown that $\ln \delta$ increases (and so λ decreases) in n.

We now show that (28) holds in this limit case. The left-hand side of (28) becomes

$$\frac{\alpha_2/\varepsilon - 1}{(1-\alpha_2)(1-\phi_1)} \approx \frac{\alpha_2/\varepsilon}{(1-\alpha_2)(1-\phi_1)} \tag{32}$$

as $\varepsilon \approx 0$. The right-hand side of (28) is now

$$\frac{1-(1-\delta)^{n-1}}{\delta^{n-1}} - 1 \approx \frac{n-1}{\delta^{n-2}} \approx (n-1)\left(\frac{\alpha_2 (n-1)}{(1-\phi_1)\, n\varepsilon}\right)^{(n-2)/(n-1)},$$

which is lower than (32) if

$$[(1-\alpha_2)(n-1)]^{n-1} \left(\frac{n-1}{n}\right)^{(n-2)} < \frac{\alpha_2}{(1-\phi_1)\,\varepsilon}.$$

For fixed n and α_2, this is always true for $\varepsilon \to 0$.

In this limit case, each firm's profit given in (30) can be approximated as

$$\begin{aligned}\pi &\approx (1-\delta)^{n-1}(1-\phi_1)\,\varepsilon + \frac{\alpha_2}{n}\delta^{n-1} \\ &\approx (1-\phi_1)\,\varepsilon + \frac{1-\phi_1}{n-1}\varepsilon \\ &= \frac{n\,(1-\phi_1)}{n-1}\varepsilon.\end{aligned}$$

The first step follows from the fact that all terms for $k = 1, \cdots, n-2$ in (30) are of higher order than ε. It is ready to see that π decreases while $n\pi$ increases at n. ■