

Patents and Research Tools in a Schumpeterian Growth Model with Sequential Innovation

Silvia Galli*

University of Rome "La Sapienza"

In the standard quality-ladder growth models, R&D firms undertake independent innovation processes to discover ideas whose value immediately transfers into tradeable applications. Here the standard multisector neo-Schumpeterian growth theory is extended by decomposing product innovation into a two-stage uncertain research activity.

I compare the general equilibrium innovative performance of an economy where early-stages scientific results are patentable with the general equilibrium innovative performance with unpatentable basic ideas freely disseminated by public research institutions (universities). I show that the widely documented increasing complexity experienced in applied R&D magnifies the public basic R&D inefficiencies and suggests the patentability of research tools. [JEL Classification: O31, O34, O41]

1. - Introduction

A wide literature on the economics of innovation recognizes sequentiality as distinctive characteristic of the innovative process. «As Susanne Scotchmer (1991) argues:

Sir Isaac Newton himself acknowledged, "If I have seen far, it

* <silvia.galli@tele2.it>. The Author is very grateful to Prof. Guido Cozzi for his precious help on this work. She wishes to thank Prof. Domenico Tosato for suggestions during the thesis period and Prof. Vincenzo Denicolò for very useful comments. The Author also wishes to thank seminar participants at the University of Rome "La Sapienza" and three anonymous referees whose remarks contributed to the improvement of this work. Possible deficiencies that might remain are responsibility of the Author.

is by standing on the shoulders of giants". Most innovators stand on the shoulders of giants, and never more so than in the current evolution of high technologies, where almost all technical progress builds on a foundation provided by earlier innovators. For example, most molecular biologists use the basic technique for inserting genes into bacteria that was pioneered by Herbert Boyer and Stanley Cohen in the early 1970s, (...). In pharmaceuticals, many drugs like insulin, antibiotics, and anti-clotting drugs have been progressively improved as later innovators bettered previous technologies».¹

Yet, the formal literature on standard Schumpeterian growth theory (Grossman and Helpman, 1991; Aghion e Howitt, 1992 and 1998; Segerstrom, 1998; Howitt, 1999) misses to take the sequential and cumulative nature of ideas at pre-commercial stages of development into the appropriate consideration. In these models, the innovation process is governed by a Poisson process, by which an idea is instantaneously conceived and implemented into a new product to be sold to the marketplace. Of course, the Schumpeterian growth theory acknowledges the intertemporal spillovers and the sequentiality for marketable products. However, in many real world cases, R&D firms have experience multiple-stage research activities before developing tradeable applications of basic scientific ideas.

Several examples of the sequential behaviour of research activity come from biotechnology field². Think about the human embryonic stem cell discovery, patented in 1998 by Wisconsin University, whose exclusive license for experimental use (regarding three cell types) was granted to Geron to develop new organic materials useful for the treatment of important diseases. Could one state that Wisconsin University's scientific achievement was lacking economic value just because further research was required to render it applicable for commercial purpose? Such a statement would prove restrictive and simplistic in an age in which a large

¹ SCOTCHMER S. (1991, pag. 29).

² In 1980, in the *Diamonds v. Chakrabarty* case, the Supreme Court of United States ruled that microorganisms produced by genetic engineering could be patented. The Supreme Court's decision arrived two years before the introduction of the first commercial product, human insulin, obtained with recombinant DNA techniques.

part of the international academic community, jointly with a large part of the R&D business community, expresses the need for an appropriate intellectual property design to take into account both the sequential and cumulative nature of ideas and the change in the technological paradigm determined by the biotechnology industry³. The possibility, regarding a number of countries, and in particular the United States, to patent scientific findings with no consideration of their commercial value and, above all, with no consideration of their status of research tools, motivated a lot of theoretical and empirical microeconomics works⁴.

Basic R&D activity is traditionally characterized by a strong presence of the public sector. Universities have always been the main investor in basic R&D in the United States, where, in 2004, they accounted for 55% of domestic basic research. An important reason for relatively low private contribution to basic R&D is often found in the high degree of uncertainty that this activity involves in terms of future commercial application and success. Certainly, the patentability of early-stage ideas, by granting monopolistic rents on the applied R&D market to the research tool patent holders, shifts the debate on the merits of public basic research over private basic research from appropriability concerns to other issues, as, for instance, the organization and the objectives of research activity and the monetary and non monetary incentives guiding research activity in the two systems (Lach and Shankerman, 2003; Aghion, Dewatripont and Stein, 2005).

Jensen and Thursby (2001) study the licensing practices of 62 US universities. The authors find that over 75% of the inventions licensed were in such an embryonic state of development, that no one could estimate their commercial potential and the inventor's cooperation was required to get a successful commercial development.

A first theoretical attempt to grasp the sequential nature of

³ According to GAMBARDELLA A. (1995) and HENDERSON R. - ORSENIGO L. - PISANO G. (1999), the pharmaceutical research industry is now more influenced by prior scientific discoveries respect to the past.

⁴ For shortness, just a few of such contributions will be cited in this paper. For a more complete and significative survey, see NATIONAL RESEARCH COUNCIL (2003) and (2004).

the innovation process — i.e. the lack of coincidence of the conception of an idea and its implementation in a tradeable good — by an economic growth model is Guido Cozzi (2001). The author develops a Schumpeterian growth model with industrial espionage. By introducing in the Aghion and Howitt's, (1992 and 1998) basic Schumpeterian model the possibility for an idea, or for an essential part of it, to be stolen and afterward used by agents distinct from the inventor, Cozzi demonstrates that an increase in the number of researchers entails an increasing incentive to industrial espionage and sets an upper bound to the steady state equilibrium per-capita growth rate.

This paper is intended to highlight the opportunity of studying the sequential nature of the innovation process in a dynamic general equilibrium framework, in order to evaluate the conditions under which an intellectual property design is effectively able to promote economic growth. In particular, by taking the R&D sequentiality into the Schumpeterian paradigm, it allows to investigate the relation between the cumulative uncertainty involved in the two-stages innovation process and the concept of increasing complexity in the R&D activity (Kortum 1993 and 1997; Segerstom, 1998).

The assumption of infinitely-lived patents, for the sake of simplicity often made within the contest of the Schumpeterian literature and in the industrial organization models as well, has often been rightly criticised because of its lack of realism. In fact, although in most countries the statutory patent life is twenty years, several studies on this subject show that the effective patent life is substantially shorter. Besides, the enforcement of the exclusive rights that patents virtually attribute to inventors is often denied in the case of declaratory judgement for patent infringement. This is true not only for the circumstance that the declaratory process may be costly and risky⁵, but also because patented scientific

⁵ In fact, patents involved in litigations are frequently invalidated by the courts.

As emphasized by some authors, during the last twenty years, there was an expansion of the patent breadth (a patent breadth specifies a set of products that no other firm can produce without permission from the patent-holder in the form of a licensing agreement), thus increasing more and more the probability of a lawsuit for patent infringement.

knowledge can be lawfully imitated even before the twenty years old patent expires. Also for this reasons, some authors (Cohen et al, 2000) highlight that the US data point out a situation in which, in most cases, R&D firms prefer to rely on trade secrets, rather than to patent, as a protection mechanism for their innovative ideas.

Now, in order to evaluate the option of trade secrets as an alternative to patents we need to consider, from the perspective of static social welfare, that, if the expected duration of the secret equals the patent life⁶, the social deadweight loss due to the monopolistic distortion is the same regardless of the circumstance that the innovative firms rely on either protection mechanisms; in both cases, in fact, firms are perfectly able to fix their price above the marginal cost, apart from considering the origin, legal or not, of the actual monopoly. Still, from a social planner's perspective, who has the collective innovation level at heart, secrecy entails the drawback of cancelling out the scientific knowledge spillovers that are the other natural face of the legal monopoly.

Besides, within the contest of sequential innovation we want to analyse, secrecy as a protection mechanism for the innovative ideas, may impede important commercial developments for such ideas. If, in fact, the difficulty of selling legally unprotected ideas is widely recognised (see Arrow, 1962; Arora and Ceccagnoli, 2006 and Denicolò, 2007), from the other side, it is necessary to

O'DONOGHUE T. - ZWEIMÜLLER J. (2004) analyse the effect on the aggregate R&D of several patent policies directed to strengthen the patent generated inventor's monopoly power. In a general economic equilibrium model, O'Donoghue and Zweimüller find that the circumstance that different industries adopt patent protection can imply that output distortions created by patents are moderated by the positive effects on the aggregate output. In particular, according O'Donoghue and Zweimüller, in particular both patentability requirement and leading breadth, by influencing the characteristics of new products, could counteract the tendencies of the economy towards suboptimally-sized innovations.

JAFFE A.B. - LERNER J. (2004) identify with the lowering of the patentability requirement (in particular referring to the novelty and non-obviousness requirements), and the resulting proliferation of bad or technically dubious patents, the increasing in the costs connected with the defence of patents. Besides, the authors add, the weakening of the patentability requirement, far away from determining an increase in the number of product innovations, mainly caused a lowering in the number of innovations (new products on the market) to patents ratio.

⁶ Here, I am following DENICOLÒ V. (2007)'s argument.

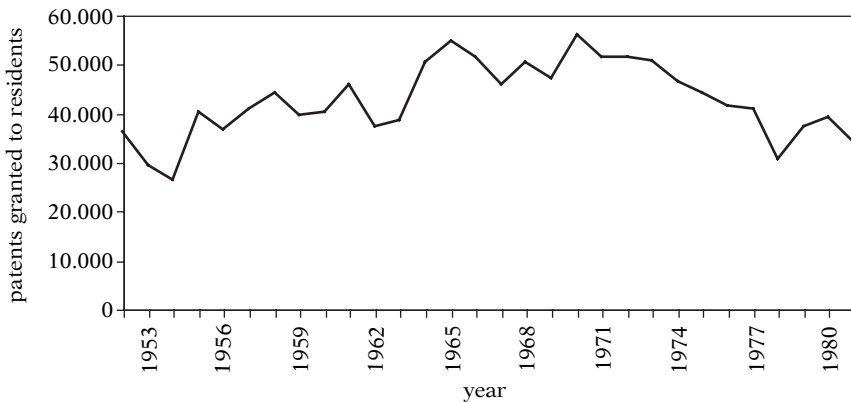
consider that, behind the limits of a theoretical model, in the more general case of n R&D steps needed in order to achieve one single product innovation, as n increases the probability of disclosing the secret to external individuals increases proportionally.

These remarks, far away from entering a detailed analysis on the opportunity for a single firm to rely on trade secrets rather than patent their scientific findings, explain why, besides the greater availability and reliability of the data, here I have chosen the number of patents granted to residents as an indicator for the innovative performance of the United States from 1953 to 1982⁷.

Graph 1 shows the time series of the patents granted to residents in the United States from 1953 to 1992. By simply observing the data we infer that the time series does not exhibit

GRAPH 1

PATENTS GRANTED TO RESIDENTS USA 1953-1982



Source: WORLD INTELLECTUAL PROPERTY ORGANIZATION.

⁷ Patent statistics are available for wider samples (JAFÉ A.B. - LERNER J., 2004). The choice of this sample has been determined in view of the consideration that during the early Eighties, in the United States the change occurred in the jurisprudential orientation that led to the patentability of the research tools. Therefore, it seems appropriate, in order to understand the relation between the cumulative uncertainty involved in the two-stages innovation process and the increasing complexity in the applied R&D activity, to depict the innovative performance in the country, as it was in a pre-Eighties patent regime.

any trend. The growth rate of the number of patents granted to US residents in the period points out a pattern oscillating around a null mean, showing, at least at a first look, the distinctive characteristics of a weakly stationary stochastic process⁸.

Graph 3 shows that in thirty years the applied R&D expenditure in the US quadrupled, from 7065 million dollars in 1953 to 29142 million dollars in 1982 (year 2000 dollars).

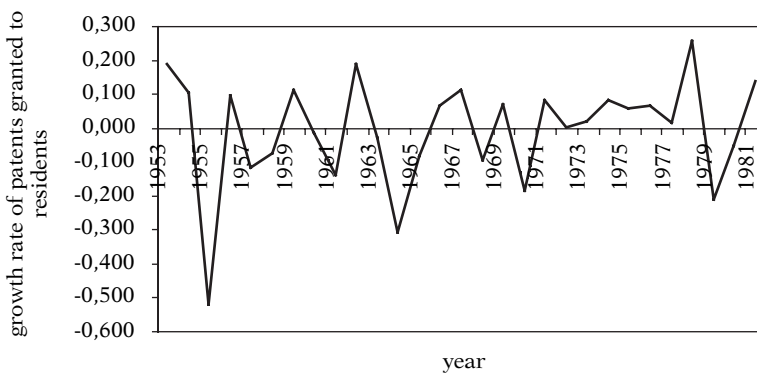
These data point out that in thirty years in the US applied R&D expenditure considerably increased, whereas the number of patents granted each year to residents maintained a constant range of variation.

By theoretically exploring the multiple-stages uncertain nature of research activity within the Schumpeterian framework, we cast some light on the incentives that firms have to engage a follow-on discovery R&D. I hope this might help understanding new ways of providing incentives for the commercialization of new technologies and for the promotion economic growth.

The rest of this paper is organized as follows. Section 2 sets

GRAPH 2

GROWTH RATE OF PATENTS GRANTED TO RESIDENTS
USA 1953-1982

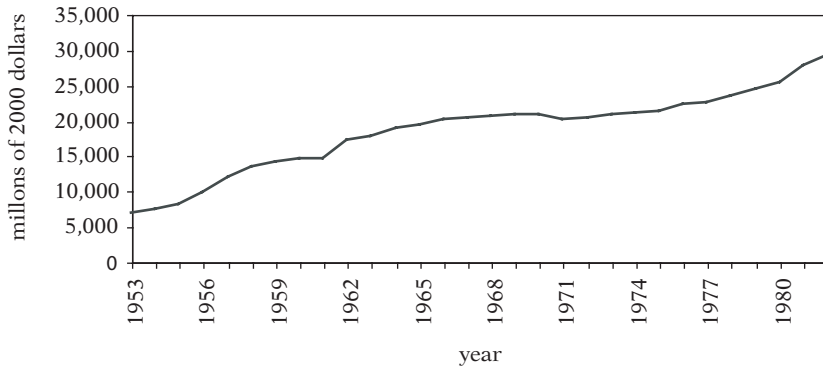


Source: WORLD INTELLECTUAL PROPERTY ORGANIZATION.

⁸ A stochastic process is said to be weakly stationary (or covariance stationary) if its first and second order moments are time independent. For a formal definition of a weakly stationarity, see HAMILTON J.D. (1994, pages 45-46).

GRAPH 3

INFLATION ADJUSTED APPLIED R&D EXPENDITURE
USA 1953-1982 (Year 2000 *constant dollars*)



Source: NATIONAL SCIENCE BOARD (2006, Vol. 2, Appendix 4-12).

up a Schumpeterian model with sequential innovation in two different patent policy scenarios. In the first scenario (case A) basic R&D achievements are patented and, afterwards, developed into tradable applications within a completely privatized economy. In the second scenario (case B) basic research findings are conceived and put into the public domain, and subsequently embodied into marketable products by a large number of perfectly competitive private R&D firms. In Section 3 we try to assess, on the basis of the analytical structure provided by the model, the relative advantages and disadvantages of granting no intellectual protection to half-ideas. Finally, the main results are summarized in Section 5.

2. - The Model

2.1 Overview

Consider an economy made up of a final good sector and a research and development (R&D) sector, along the lines of Grossman and Helpman (1991), where product improvements

occur in consumption goods. Within each industry, firms are distinguished by the quality of final good they produce. When the state-of-the-art quality product in an industry $\omega \in [0,1]$ is $j_t(\omega)$, research firms compete in order to learn how to produce $j_t(\omega) + 1$ st quality product. This learning process involves a two-stages innovation path, so first a firm catches a glimpse of the innovation through the $j_t(\omega) + 1/2$ th inventive half-idea and then other firms engage in a patent race to implement it in the $j_t(\omega) + 1$ st quality product.

In what follows we refer to the term “quality leader” to denote the firm that produces the current state-of-the-art quality product. We will call “half-idea follower”, or simply the follower⁹, the R&D firm that owns the first half-idea invented in order to introduce, in the second stage, the product’s innovation that pushes up the quality ladder. Finally, we let the term “outsider” denote the R&D firm who tries to invent a new first half idea in the basic research sector (i.e. tries to become the new follower). We assume that firms are able to instantaneously patent both the inventive half-idea and the product innovation. Then, patent protection determines a monopolistic position both in the applied R&D sector and in the final good sector, and the winner of the final patent R&D race become the sole producer of a $j_t(\omega) + 1$ st quality consumption product. Research firms rationally choose on which stage of the innovation process to settle in by simply comparing the expected benefits from the total R&D effort and its costs.

Time is continuous with an unbounded horizon and there is a continuum infinitely-lived dynasties of households with identical intertemporally additive preferences. Heterogeneous labour, skilled and unskilled, is the only factor of production and the whole endowment of labour is constant over time. Let M denote the unskilled labour amount, given at the aggregate level. Let L denote the economy-wide skilled labour endowment. Both labour markets are assumed perfectly competitive. In the final good sectors

⁹The terminology “leader”, here adopted, is somehow intended to recall in the reader’s mind the expression “follow on R&D activity” often used within the literature on the economics of innovation to denote sequential and cumulative R&D activity.

$\omega \in [0,1]$, monopolistically competitive firms produce differentiated consumption goods by combining skilled and unskilled labour, whereas research firms employ only skilled labour.

2.2 Households

The representative household's preferences are represented by the following intertemporal utility function:

$$(1) \quad U = E_0 \left[\int_0^{\infty} e^{-\rho t} u(t) dt \right]$$

where $\rho > 0$ is the subjective discount rate and E_0 denotes the expectation operator at time $t=0$. Instantaneous utility $u(t)$ is defined as:

$$(2) \quad u(t) = \int_0^1 \ln \left[\sum_j \gamma^j d_{jt}(\omega) \right] d\omega$$

where $d_{jt}(\omega)$ is the quantity consumed of a good of quality j (that is, a product that has experienced with j quality jumps) and produced in industry ω at time t . Assume that j is forced to assume integer values¹⁰. Parameter $\gamma > 1$ measures the size of quality upgrades (i.e., the magnitude of innovations). This formulation, the same of Grossman and Helpman (1991) and Segerstrom (1991), assumes that each consumer prefers higher quality products.

The representative consumer is endowed with $L > 0$ units of skilled labor and $M > 0$ units of unskilled labor. Since labour bears no disutility it will be inelastically supply for any level of non negative wages. Population is assumed constant and it is normalized to 1. Hence L and M will also equal, in equilibrium, the aggregate supply of skilled, respectively, unskilled labour.

¹⁰ This assumption is common in the quality-ladder endogenous growth literature; still, in our framework, it has the meaning of explicitly stating that half-ideas discoveries do not affect consumer's utility.

Unskilled labor can only be employed in the final goods production. Skilled labour is the most versatile, being also able to perform R&D activities.

In the first step of the consumer's dynamic maximization problem, she selects the set $J_t(\omega)$ of the existing quality levels with the lowest quality-adjusted prices. Then, at each instant, the households allocate their income to maximize the instantaneous utility (2), taking product prices as given in the following expenditure constraint equation:

$$(3) \quad E(t) = \int_0^1 \sum_{j \in J_t(\omega)} p_{jt}(\omega) d_{jt}(\omega) d\omega$$

Here $E(t)$ denotes consumption expenditure and $p_{jt}(\omega)$ is the price of a product of quality j produced in industry ω at time t .

Only a good with the lowest quality-adjusted price is consumed, since there is no demand for any other good $j \notin J_t(\omega)$. We also assume, as usual, that if two products have the same quality-adjusted price, consumers will buy the higher quality product although they are formally indifferent between the two products, because the quality leader can always slightly lower the price of its product and drive the others out of the market.

Let us define $j_t^*(\omega) \equiv \max\{j : j \in J_t(\omega)\}$. The solution to this maximization problem yields the static demand function:

$$(4) \quad d_{jt}(\omega) \begin{cases} E(t) / p_{jt}(\omega) & \text{for } j = j_t^*(\omega) \\ 0 & \text{otherwise} \end{cases}$$

By using the instantaneous optimization results, we can rewrite (2) as:

$$(5) \quad u(t) = \int_0^1 \ln \left[\gamma^{j_t^*(\omega)} E(t) / p_{j_t^*(\omega)t}(\omega) \right] d\omega =$$

$$(6) \quad = \ln[E(t)] + \ln(\gamma) \int_0^1 j_t^*(\omega) d(\omega) - \int_0^1 \ln \left[p_{j_t^*(\omega)t}(\omega) \right] d\omega$$

Therefore, given the independent and — in equilibrium and by the law of large number — deterministic evolution of the quality jumps and prices, the consumer will only choose the piecewise continuous expenditure trajectory, $E(\cdot)$, that maximizes the following functional:

$$(7) \quad U = \left[\int_0^{\infty} e^{-\rho t} \ln[E(t)] dt \right]$$

Assume that all consumers possess equal shares of all firms at time $t=0$. Letting $A(0)$ denote the present value of human capital plus the present value of asset holdings at $t=0$ each household's intertemporal budget constraint is:

$$(8) \quad \int_0^{\infty} e^{-R(t)} E(t) dt \leq A(0)$$

where $R(t) = \int_0^t r(s) ds$ represents the equilibrium cumulative real interest rate up to time t .

Finally, the representative consumer chooses the time pattern of consumption expenditure to maximize (7) subject to the intertemporal budget constraint (8). The optimal expenditure trajectory satisfies the Euler equation:

$$(9) \quad \dot{E}(t) / E(t) = r(t) - \rho$$

where $r(t) = \dot{R}(t)$ is the instantaneous market interest rate at time t .

Euler equation (9) implies that a constant (steady state) per-capita consumption expenditure is optimal when the instantaneous market interest rate equals the consumer's subjective discount rate. Since preferences are homothetic, in each industry aggregate demand is proportional to the representative consumer's one, so in what follows we use E to denote the aggregate consumption spending and d to denote the aggregate demand.

2.3 Production

In this section we examine the production side of the economy. We assume constant returns to scale technologies in the (differentiated) manufacturing sectors represented by the following production functions:

$$(10) \quad y(\omega) = x^\alpha(\omega) M^{1-\alpha}(\omega), \quad \text{for all } \omega \in [0, 1]$$

where $\alpha \in (0, 1)$, $y(\omega)$ denotes output flow per unit time, $x(\omega)$ and $M(\omega)$ are, respectively, skilled and unskilled labour employment flows in industry $\omega \in [0, 1]$.

Letting w_s and w_u denote skilled and unskilled wage rates, in each industry, the quality leader seeks to minimize its total cost flow $C = w_s x(\omega) + w_u M(\omega)$ subject to constraint (10). For $y(\omega) = 1$, the solution to this minimization problem yields the conditional unskilled (11) and skilled (12) labour demands (i.e. the per-unit labour requirements):

$$(11) \quad M(\omega) = \left(\frac{1-\alpha}{\alpha} \right)^\alpha \left(\frac{w_s}{w_u} \right)^\alpha$$

$$(12) \quad x(\omega) = \left(\frac{\alpha}{1-\alpha} \right)^{1-\alpha} \left(\frac{w_u}{w_s} \right)^{1-\alpha}$$

Thus the (minimum) cost function $C(w_s, w_u, y)$ is:

$$(13) \quad C(w_s, w_u, y) = c(w_s, w_u) y$$

where $c(w_s, w_u)$ is the per-unit cost function:

$$(14) \quad c(w_s, w_u) = \left[\left(\frac{1-\alpha}{\alpha} \right)^{-(1-\alpha)} \left(\frac{\alpha}{1-\alpha} \right)^{-\alpha} \right] w_s^\alpha w_u^{1-\alpha}$$

Since unskilled labour is uniquely employed in the final

good sectors and all price variables (including wages) are assumed to instantaneously adjust to their market clearing values, unskilled labour aggregate demand $= \int_0^1 M(\omega) d\omega$, is equal to its aggregate supply, M , in any instant. Since industries are symmetric and their number is normalized to 1, in equilibrium¹¹ $M(\omega) = M$.

Letting $w_u = 1$, from equations (11) and (12) we get the firm's skilled labour demand negatively depending on skilled (/unskilled) wage (ratio):

$$(15) \quad x(w_s) = \frac{1}{w_s} \left(\frac{\alpha}{1-\alpha} \right) M$$

In each industry, at each instant, firms compete in prices. Given equation (4), within each industry product innovation is non-drastic¹², hence the quality leader will fix its (limit) price p by charging a mark-up γ over the unit cost (remember that parameter γ measures the size of product quality jumps):

$$(16) \quad p = \gamma c(w_s, 1) \Rightarrow d = \frac{E}{\gamma c(w_s, 1)}$$

Hence each monopolist earns a flow of profit:

$$(17) \quad \pi = \frac{\gamma-1}{\gamma} E = (\gamma-1) \frac{w_s x}{\alpha}$$

$$\pi = (\gamma-1) \frac{1}{1-\alpha} M$$

¹¹ In general, if there were a mass $N > 0$ of final good industries, in equilibrium $M(\omega) = (M/N)$.

¹² We are following AGHION P. - HOWITT P. (1992) and (1998) definition of drastic innovation as generating a sufficiently large quality jump to allow the new monopolist to maximize profits without risking the re-entry of the previous monopoly. Given the unit elastic demand, here the unconstrained profit maximizing price would be infinitely high: that would induce the previous incumbent to re-enter.

From eq. (17) follows:

$$(18) \quad \frac{\gamma-1}{\gamma} E = (\gamma-1) \frac{1}{1-\alpha} M \Rightarrow E = \frac{\gamma}{1-\alpha} M$$

Interestingly, eq. (18) implies that in equilibrium total expenditure is always constant.

2.4 R&D Sectors

In each industry, R&D activity is conceived as a step-by-step process through which, first a new idea is invented and then it is used to find the way to introduce a higher quality product. The concept of half-idea of basic R&D activity provided by the National Science Board (2006):

«The objective of basic research is to gain more comprehensive knowledge or understanding of the subject under study without specific applications in mind. In industry, basic research is defined as research that advances scientific knowledge but does not have specific immediate commercial objectives, although it may be performed in fields of present or potential commercial interest».¹³

A) Patentable Research Tools

We assume that once the half-idea is invented it is immediately protected by an infinitely-lived patent. With this assumption we impose the publicity about the state-of-the-art of research activity in each industry and exclude the possibility for an outsider firm to lie announcing a false half-idea finding in order to discourage its competitors.

Imagine that the half-idea patent holder can only grant an exclusive license. The licensee of the patent for the research tool is the only one who can use it to invent a completed new product.

¹³ NATIONAL SCIENCE BOARD (2006, Vol. 1, Chap. 4, page 8).

Following Aghion and Howitt (1998, Ch. 7), assume that each R&D firm faces an U-shaped unit cost function. Let $i=F,O$ denote a follower and an outsider R&D firm respectively, N_i , with $i=F,O$, indicates the mass of follower and, respectively, outsider firms in each R&D sector. Since we assumed that the half-idea patent holder can only grant an exclusive license, in equilibrium we have $N_F = 1$ ¹⁴.

The individual firm's Poisson process probability intensity to succeed in inventing a half-idea or completing one (i.e. introducing the product innovation) is $\theta_i(z_i - \phi, N_i)$, increasing and concave, depending on the R&D effort z_i , in excess of the fixed cost, in terms of labour input, $\phi > 0$, that each firm has to pay per-unit time in order to engage in the R&D race. In particular, we specify the per-unit time Poisson probability intensity to succeed for an outsider and a follower firm respectively as:

$$(19) \quad \theta_O(z_O - \phi, N_O) \equiv \lambda_0 \sqrt{\max(z_O - \phi, 0)} N_O^{-a}$$

$$(20) \quad \theta_F(z_F - \phi, N_F) \equiv \lambda_1 \sqrt{\max(z_F - \phi, 0)} N_F^{-a}$$

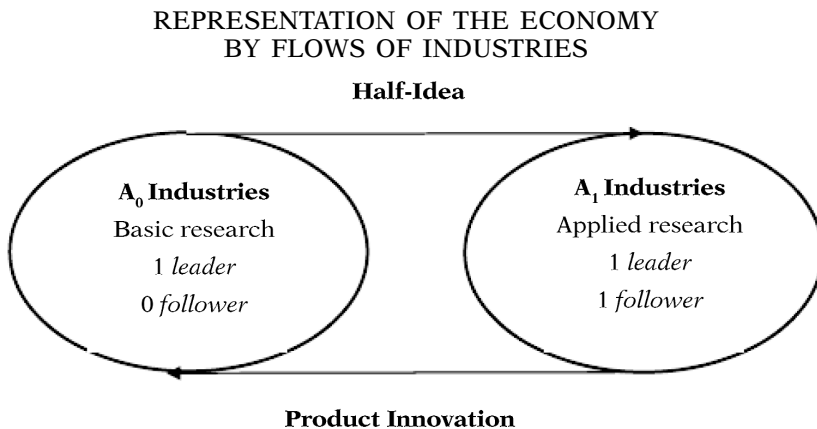
where $\lambda_k > 0$, $k = 0, 1$, are R&D laboratory productivity constants; N_i ($i = O, F$) represent the number of laboratories in each industry and constant $a > 0$ is an inter-firm intra-sectoral congestion parameter, capturing the risk of R&D duplications, knowledge theft and other diseconomies of fragmentation in the R&D.

From the Poisson process properties, the probability of simultaneously inventing two half-ideas in a tiny interval of time of duration Δt is a zero of order higher than the first. If instead an outsider R&D firm invents a second half-idea after the inventor of a first half-idea has already been granted a patent, the patent office will not grant a second patent on that half-idea, based on

¹⁴ However, in general, the production technology in each R&D sector is expressed by equations (19)-(20), in which N_F and N_O must be determined endogenously within the general economic equilibrium conditions of the model.

the legal principle of patentability requirement (O'Donoghue 1998, O'Donoghue and Zweimüller, 2004), because the second half-idea does not promise to generate further utility gains to the consumer. Therefore no second R&D firm will ever invest resources in inventing a half-idea on which it will not be able to claim intellectual property rights. As a result, in any instant of time, no industry will have more than one follower at any moment in time and the whole set of industries $\omega \in [0,1]$, gets partitioned into two sets of industries: industries $\omega \in A_0$, (temporarily) with no half-ideas and, therefore, with one quality leader (the final product patent holder), no followers and a mass of outsider firms, and the industries $\omega \in A_1 = [0,1] \setminus A_0$, with one half-idea and, therefore, one half-idea leader (the final product patent holder) and one follower (the half-idea patent holder). Firms engage in basic R&D only in $\omega \in A_0$ industries and engage in applied R&D activity aimed at a direct product innovation only in A_1 industries. When a quality improvement occurs in an industry $\omega \in A_1$ the half-idea follower becomes the new quality leader and the industry switches from A_1 to A_0 . When an inventive half-idea discovery arises in an industry $\omega \in A_0$ this industry switches to A_1 and the winner of the first-stage patent race is the new follower. Graph 4 illustrates the flow of industries from a condition to the other:

GRAPH 4



Notice that the two sets A_0 and A_1 change over time, even if the economy will eventually admit a steady state. At any instant we can measure the mass of industries without any half-idea as $m(A_0) \in [0, 1]$, and the mass of industries with an uncompleted half-idea as $m(A_1) = 1 - m(A_0)$. Clearly, in a steady state these measures will be constant and equal to each other, as the flows in and out will offset each other.

In light of the definitions so far, we can express the skilled labor market equilibrium as:

$$(L') \quad L = x(w_t) + m(A_0) N_{Oz_O} + m(A_1) N_{Fz_F}$$

stating that, at each date, the aggregate supply of skilled labor, L , finds employment in the manufacturing sectors, $x(w_t)$, in all $[0,1]$, in the A_0 sectors, N_{Oz_O} , and in the A_1 sectors, N_{Fz_F} .

Assuming that the patent holder — the “follower” — can license the research tool to only one R&D firm, we impose $N_F = 1$.

Each Poisson process governing the two-stage innovative process is supposed to be independent. The stock value of all firms is determined by privately arbitraging between risk free consumption loans, firm bonds and equities, viewed as perfect substitutes also due to the ability of financial intermediaries to perfectly diversify portfolios and eliminate risk¹⁵.

Letting V_O , V_F^1 , V_L^0 and V_L^1 , denote respectively the present expected value of being an outsider firm (V_O), a half-idea follower (V_F^1), a quality leader (V_L^0) and a half-idea leader (V_L^1); the following Bellman’s equations must hold in equilibrium:

$$(21a) \quad rV_O = \max_{z_O} \lambda_0 \sqrt{(z_O - \phi, 0)} N_O^{-a} (V_F^1 - V_O) - w_s z_O + \frac{dV_O}{dt}$$

$$(21b) \quad rV_F^1 = \max_{z_F} \lambda_1 \sqrt{(z_F - \phi, 0)} N_F^{-a} (V_L^0 - V_F^1) - w_s z_F + \frac{dV_F^1}{dt}$$

¹⁵ Hence, despite individuals’ being risk averse, average returns will be deterministic, the risk premia will be zero, and agents will only compare expected returns. A usual in this class of models, we invoke the law of large numbers, which allows individuals who invest in a continuum of sectors with idiosyncratic risk to transform probabilities into frequencies.

$$(21c) \quad rV_L^0 = \pi - N_O^{1-a} \lambda_0 \sqrt{(z_O - \phi, 0)} (V_L^0 - V_L^1) + \frac{dV_L^0}{dt}$$

$$(21d) \quad rV_L^1 = \pi - \lambda_1 \sqrt{(z_F - \phi, 0)} V_L^1 + \frac{dV_L^1}{dt}$$

Equation (21a) states that the risk free income deriving from the liquidation of the expected present value of an outsider R&D firm in an A_0 industry, rV_O , is equal to the expected gain from becoming a follower,

$$\lambda_0 \sqrt{(z_O - \phi, 0)} N_O^{-a} (V_F^1 - V_O)$$

i.e. the patent holder on the next half-idea in an A_1 industry, minus the R&D expenditure, $w_s z_O$, plus the gradual stock market appreciation in the case of the half-idea not occurring, dV_O/dt .

Equation (21b) equals the risk free income deriving from the liquidation of the expected present value of follower R&D firm in an A_1 industry, rV_F^1 , and the expected increase in value from becoming a quality leader (*i.e.* completing the product innovation process),

$$\lambda_1 \sqrt{(z_F - \phi, 0)} N_F^{-a} (V_L^0 - V_F^1)$$

minus the relative R&D cost, $w_s z_F$, plus the gradual appreciation in the case of R&D success not arriving, dV_F^1/dt .

Equation (21c) states that the risk free income deriving from the liquidation of the expected present value of a leader in an A_0 industry, V_L^0 , equals the flow of profit π , minus the capital loss from being challenged by a half-idea on a better product in case a follower appears,

$$N_O^{1-a} \lambda_0 \sqrt{(z_O - \phi, 0)} (V_L^0 - V_L^1)$$

plus gradual appreciation in case of such event not occurring within a year, dV_L^0/dt .

Finally, equation (21d) equals the risk free income deriving

from the liquidation of the value function of a leader in an A_1 industry, rV_L^1 , to the relative flow of profit π , minus the expected loss deriving from the follower's success,

$$\lambda_1 \sqrt{(z_F - \phi, 0) V_L^1}$$

plus gradual appreciation in case obsolescence does not occur, dV_L^1/dt .

Each perfectly competitive outsider firm determines the amount of labour devoted to basic research z_O^* by trying to maximize its expected profit flow.

LEMMA 1 *a)* The equilibrium amount of labour employed by each outsider R&D firm is $z_O^* = 2\phi$.

b) The positive R&D equilibrium value of the skilled wage ratio is

$$w_s = \max\left(\frac{\lambda_0 V_F^1 N_O^{-a}}{2\sqrt{\phi}}, 1\right)$$

PROOF (in APPENDIX 1.A)

Interestingly, unlike the usual features of previous quality ladders models¹⁶, in this model, due to decreasing returns at the industry level, there is no indeterminacy in the intersector allocation of R&D labor.

Let us now turn to the follower's problem. The following LEMMA 2 holds:

LEMMA 2. The optimal amount of labour employed in research in each industry A_1 industry is

$$z_F^* = \phi + \left(\frac{\lambda_1 (V_L^0 - V_F^1)}{2w_s}\right)^2$$

¹⁶ See Cozzi G. (2007) for a proof of indeterminacy in quality ladders models.

PROOF (in APPENDIX 1.A).

Equations (21b), (21c), (21d), LEMMA 1a) and LEMMA 2 imply the following:

$$(22a) \quad rV_F^1 = \lambda_1^2 \frac{(V_L^0 - V_F^1)^2}{4w_s} - \phi w_s + \frac{dV_F^1}{dt}$$

$$(22b) \quad rV_L^0 = (\gamma - 1) \frac{1}{1 - \alpha} M - N_O^{1-a} \lambda_0 \sqrt{\phi} (V_L^0 - V_L^1) + \frac{dV_L^0}{dt}$$

$$(22c) \quad rV_L^1 = (\gamma - 1) \frac{1}{1 - \alpha} M - \lambda_1^2 \frac{V_L^0 - V_F^1}{4w_s} V_L^1 + \frac{dV_L^1}{dt}$$

Equations (22a)-(22c) are a first order system of ordinary differential equations describing the dynamics of the stock market value of all non-zero profit firms in this economy. Plugging eq.s (15) and (17) into the expression of the skilled labour wage ratio (LEMMA 1b)), we obtain:

$$(23) \quad x(w_s) = \frac{1}{w_s} \left(\frac{\alpha}{1 - \alpha} \right) M = \min \left(\frac{2\sqrt{\phi}}{\lambda_0 V_F^1 N_O^{-a}}, 1 \right) \left(\frac{\alpha}{1 - \alpha} \right) M$$

Therefore the skilled labor employment in the manufacturing sector is inversely related to the market value of half-patented ideas. Higher valued research tools draw more skilled labor from the manufacturing plants into the basic research laboratories, thereby increasing the manufacturing unskilled/skilled labor ratio and consequently raising skilled labor marginal productivity and real wages. Since the patent on a half-idea does not derive value from the direct production of a marketable good V_F^1 , is in turn pinned down by rV_L^0 . Therefore, the equilibrium value of the unskilled wage is indirectly related to the stream of profits expected from the future commercialization of the product of the completed idea. Unlike the traditional Schumpeterian innovative process, the skilled wage here does not immediately incorporate the discounted expected value of the next commercially fruitful

patent, but it does so only one step ahead: the value of the future monopolist is scaled down to current R&D labor wage by the composition of two innovation probabilities.

The skilled labor market clearing condition states that in each instant the following relation must hold:

$$(24) \quad m(A_0) N_O z_O^* + (1 - m(A_0)) z_F^* + x = L$$

Hence, since wages are pinned down by the optimal firm size and zero profit perfectly competitive basic R&D labor market, the unique equilibrium per-sector mass of entrant basic R&D firms consistent with skilled labor market clearing (24) is determined by solving equation (24) for N_O :

$$(25) \quad N_O = \frac{L - x - (1 - m(A_0)) \left[\phi + \lambda_1^2 \frac{(V_L^0 - V_F^1)^2}{4w_s^2} \right]}{2\phi m(A_0)}$$

To complete our analysis, let us look more closely at the macroeconomic dynamics depicted by Graph 4. In the set of basic research industries a given number of perfectly competitive (freely entered) outsider firms, N_O , employ a flow of labour input z_O^* to get a flow probability of becoming A_1 followers, while, in the set of innovative industries the only per-industry follower ($N_F = 1$) employs a flow of labour input z_F^* to obtain a flow probability to succeed in implementing the state-of-the-art research level. The flows of probability, i.e. the per-unit time probabilities, for the individual firm to pass from A_0 to A_1 and from A_1 to A_0 , at the aggregate level, become deterministic frequencies of industries flows from A_0 to A_1 and from A_1 to A_0 . Hence the industrial dynamics of this economy is described by the following first order ordinary differential equation:

$$(26) \quad \frac{dm(A_0)}{dt} = (1 - m(A_0))\theta_F(z_F^* - \phi, 1) - m(A_0)N_O^{1-a}\theta_O(z_O^* - \phi, N_O)$$

System — (22a), (22b), (22c) e (26) — jointly with cross equation restrictions ((LEMMA 1b)), (23) and (25) — form a system of four first order ordinary differential equations, whose solution describes the dynamics of this economy for any admissible initial value of the unknown functions of time V_L^0 , V_L^1 , V_F^1 and $m(A_0)$. In a steady state,

$$\frac{dV_L^1}{dt} = \frac{dV_L^0}{dt} = \frac{dV_F^1}{dt} = \frac{dm(A_0)}{dt} = 0$$

Given the analytical complexity of such system we resorted to numerical analysis. In all numerical simulations, the steady state exists, it is unique and it is saddle point stable for any set of parameter values. In particular, in all numerical results¹⁷ we obtain a unique economically meaningful equilibrium, and determinacy of equilibrium is (locally) guaranteed¹⁸ because the linear approximation around the steady state has three unstable manifolds — associated with market variables V_L^0 , V_L^1 , and V_F^1 — and one stable manifold — associated with predetermined variable $m(A_0)$.

¹⁷ The files .mod, used to simulate the model in Matlab, are available from the author on request.

¹⁸ A well known result derived within the analysis of the local stability of a system of two differential equations is that if one solution of the characteristic equation has real part greater than zero and the other solution has real part less than zero, then the steady-state is a saddle-point. In this case, the system is unstable for all initial conditions but the one that corresponds to the eigenvector associated to the eigenvalue whose real part is less than zero. In order to analyze the stability of the steady-state in our model, here we make use of a generalization of the previous result to the case of a system of four differential equations. In fact, in all our computations the Jacobian of the system (22a), (22b), (22c) and (26) computed at the steady state has three eigenvalues with positive real parts (associated with market variables, i.e. with the patent market prices, V_L^0 , V_L^1 , and V_F^1) and one eigenvalue with negative real part (associated with predetermined variable $m(A_0)$). In the economy *a la* Arrow-Debreu we analyze, market prices are represented by the stock market values of each firm in each instant of time. They are free to adjust (jump variables) until the economic system reaches the equilibrium values compatible with the stock of ideas accumulated. The adjustment to such equilibrium is unique because of the properties of the saddle point stability. If, on the contrary, a continuum of paths to the equilibrium compatible with the stock of ideas accumulated existed, clearly, we would be in presence of an indeterminacy. Excellent bibliographical references on this subject are FARMER R.E.A. (1993) and GANDOLFO G. (1997).

Therefore, given an initial condition for $m(A_0)$, there is (locally) only one initial condition for V_L^0 , V_L^1 , and V_F^1 , such that the generated trajectory tends to the steady state vector, and the equilibrium is determinate.

B) Unpatentable Research Tools

In this section we drop the assumption of patentable basic scientific results. Lacking patent protection on half-ideas, the innovative process would need to resort to non-profit motivated R&D firms to start: publicly funded universities and laboratories have often been motivated by the induced scientific spillovers on potentially marketable future technical applications. This section also introduces a singular behavioural rule for public researchers: we assume that public researchers are not perfectly mobile across sectors, so that when in a sector ω that lacked a half-idea, i.e. belonged to A_0 , a half-idea appears, i.e. it becomes A_1 , the public R&D workers keep carrying out basic research in that sector. Given our technological assumptions, this behaviour will likely lead to the discovery of a second half-idea that is redundant from the economic view point. This may represent the case of university researchers who keep investigating along intellectual trajectories even when they know that no private firm will ever profit from adapting to their market the new knowledge they may create. Of course we are not excluding in principle that the efforts of publicly hired scientists allow society to achieve important scientific findings by focusing on the required basic R&D sectors. Still, the assumption that public researchers may not do research with commercial appeal seems to be reasonable: unguided by the invisible hand, researchers will keep devoting their efforts proving that they are able to invent a second, third, ..., n^{th} genial — but socially useless — idea to enrich their *cv* and their academic carrier opportunities. Hence, we will assume from here on that the public researchers are allocated across different industries according to a uniform distribution. In other terms, we are assuming that the efforts of public researchers will be uniformly

distributed across the different basic R&D sectors: A_0 industries, in which half-idea is socially needed, and A_1 industries where half-ideas are redundant. Other assumptions about the public sector are possible, as for example public researchers guided by altruism for society (Cozzi and Galli, 2007a) or that the government uses Kremer's (1998) auctions to finance R&D (Cozzi and Galli, 2007b). The assumption here formulated emphasizes the role of markets to give R&D laboratories the right incentives to divert their resources from the unprofitable sectors and to quickly reallocate them towards more profitable aims.

We also make the assumption that the government chooses to hire a fixed global amount of skilled workers, \bar{L}_G , to be allocated in the heterogenous research activities conducted by universities and other scientific institutions. The government basic R&D expenditure, equal to $\bar{L}_G w_s$, is funded by lump sum per-capita taxes on consumers. The assumption of lump sum taxation guarantees that government R&D expenditure does not imply additional distortions on private decisions. This allows us to use the previous notation and derivations also in the case of a balanced government budget taxing all households in order to transfer the tax proceeds to the basic R&D workers.

The optimizing behaviour of the public sector consists in maximizing the expected flow of half-ideas per sector with respect to the intensity of basic research effort z_G , that is the government chooses the optimal scale for the public laboratories.

The fixed global amount of skilled workers, \bar{L}_G , hired in the basic public R&D is equal to the intensity of basic research effort, z_G , multiplied by the number of public laboratories, N_G , *i.e.*:

$$(27) \quad \bar{L}_G = N_G z_G$$

LEMMA 3. The solution for the public sector maximization problem is

$$z_G^* = 2\phi \frac{1-a}{1-2a}$$

PROOF (in APPENDIX 1.B).

Therefore, solving eq. (27) for N_G and substituting the solution of the government maximization problem, we have:

$$(28) \quad N_G = \frac{\bar{L}_G}{2\phi \frac{1-a}{1-2a}}$$

The financial arbitrage implies the following market valuations of each sector's unchallenged¹⁹ leader firm, V_L^0 , directly challenged leader firm, V_L^1 , and follower R&D firm V_F^1 ²⁰:

$$(29a) \quad rV_L^0 = \pi - \lambda_0 N_G^{1-a} \sqrt{z_G - \phi} (V_L^0 - V_L^1) + \frac{dV_L^0}{dt}$$

$$(29b) \quad rV_L^1 = \pi - \lambda_1 N_F^{1-a} \sqrt{z_F - \phi} V_L^1 + \frac{dV_L^1}{dt}$$

$$(29c) \quad rV_F^1 = \max_{z_F} \lambda_1 N_F^{-a} \sqrt{z_F - \phi} (V_L^0 - V_F^1) - w_s z_F + \frac{dV_F^1}{dt}$$

Plugging eq. (28), and the optimal size of public laboratories as given in LEMMA 3 into (29a) allows us to rewrite the equation of leader's financial arbitrage as:

$$(30) \quad rV_L^0 = \left(\frac{\bar{L}_G}{2\phi \frac{1-a}{1-2a}} \right)^{1-a} \lambda_0 \sqrt{\frac{\phi}{1-2a}} (V_L^0 - V_L^1) + \frac{dV_L^0}{dt}$$

Solving Bellman's eq. (29c) and setting (as a consequence of

¹⁹ We here mean "unchallenged" by a follower. However, a monopolist in an A_0 industry is indirectly challenged by the basic R&D laboratories trying to invent a new half-idea on which future follower firms will work to render it obsolete.

²⁰ Notice that $\lambda_0 N_G^{1-a} \sqrt{z_G - \phi} (V_L^0 - V_L^1)$ captures the expected partial obsolescence of unchallenged leadership in each A_0 sector (29a), $\lambda_1 N_F^{1-a} \sqrt{z_F - \phi} V_L^1$ the expected final obsolescence of directly challenged leadership in each sector A_1 (29b), and $\lambda_1 N_F^a \sqrt{z_F - \phi} (V_L^0 - V_L^1)$ the probability per unit time that a single follower succeeds in each sector A_1 (29c).

free entry) $V_F^1 = 0$, we get the flow of researcher hired by each follower $z_F^* = 2\phi$. Hence, the previous system eq.s (29a)-(29c) can be rewritten as:

$$(31a) \quad rV_L^0 = (\gamma - 1) \frac{1}{1 - \alpha} M - \left(\frac{\bar{L}_G}{2\phi \frac{1-a}{1-2a}} \right)^{1-a} \lambda_0 \sqrt{\frac{\phi}{1-2a}}$$

$$\max(V_L^0 - V_L^1, 0) + \frac{dV_L^0}{dt}$$

$$(31b) \quad rV_L^1 = (\gamma - 1) \frac{1}{1 - \alpha} M - \lambda_1 N_F^{1-a} \sqrt{\phi} V_L^1 + \frac{dV_L^1}{dt}$$

$$(31c) \quad rV_F^1 = \lambda_1 \sqrt{\phi} V_L^0 N_F^{-a} - w_s 2\phi$$

From eq. (31c) we can solve for the skilled/unskilled wage ratio, getting:

$$(32) \quad w_s = \max \left(\frac{\lambda_1 V_L^0}{2\sqrt{\phi}} N_F^{-a}, 1 \right)$$

Let us remember that, from the final production analysis, we have:

$$(33) \quad x = \frac{1}{w_s} \left(\frac{\alpha}{1 - \alpha} \right) M$$

The dynamics of the industries is now described by the following first order ordinary differential equation:

$$(34) \quad \begin{aligned} \frac{dm(A_0)}{dt} &= (1 - m(A_0)) N_F^{1-a} \theta_F (z_F^* - \phi) - m(A_0) N_G^{1-a} \sqrt{z_G^* - \phi} = \\ &= (1 - m(A_0)) N_F^{1-a} \lambda_1 \sqrt{\phi} - m(A_0) \left(\frac{\bar{L}_G}{2\phi \frac{1-a}{1-2a}} \right)^{1-a} \lambda_0 \sqrt{\frac{\phi}{1-2a}} \end{aligned}$$

From the following skilled labor market clearing condition:

$$(35) \quad x + \bar{L}_G + (1 - m(A_0))N_F 2\phi = L$$

Hence the equilibrium mass of per-sector followers is:

$$(36) \quad N_F = \frac{L - \frac{1}{w_s} \frac{\alpha}{1-\alpha} M - \bar{L}_G}{2\phi(1 - m(A_0))}, \quad m(A_0) \in [0,1]$$

In the stationary distribution $dm(A_0)/dt = 0$. Therefore the flow of industries entering the A_0 group must equal the flow of industries entering the A_1 group. Given the complexity of our problem, also in this case we performed numerical simulations in Matlab²¹. In all simulations a unique economically meaningful steady state exists and it is, at least locally, determinate.

3. - Comparing the Different Intellectual Property Regimes

This section draws the implications of the two intellectual property scenarios we have depicted in the previous sections. To compare the three different regimes, we assume that in the scenarios with un-patentable research tools (case B) the government chooses the same amount of basic R&D that the private economy (case A) obtains in equilibrium. This allows us to compare the different inefficiencies after controlling for the different innovative public R&D employment level.

A first analytical result is expressed by the following:

LEMMA 4. As the congestion externality tends to disappear, $a \rightarrow 0$, the equilibrium innovative performance of the private economy with patentable research tools is better than the equilibrium growth performance of the economy with an inefficient public R&D sector.

PROOF (in APPENDIX 1.C).

Even when the congestion externality is present, the economy where research tools are patentable and basic R&D is privately

²¹ See note 17.

carried out can lead to more innovation than an economy in which the public basic research is conducted in an inertial way. The following lemma shows that this will happen for sufficiently low levels of applied R&D productivity parameter λ_1 :

LEMMA 5. As applied R&D productivity parameter λ_1 , becomes very high, the equilibrium innovative performance of the private economy with patentable research tools becomes worse than the equilibrium growth performance of the economy with an inefficient public R&D sector.

PROOF (in APPENDIX 1.C).

If instead it is relatively easy to find economic applications of scientific ideas, most of the sectors will tend to need basic ideas. Therefore the broadly focussed academic research by pure scientists uninterested in economic applications will turn out to be performing a useful service to the private R&D firms, by disclosing a large spectrum of potentially profitable innovations. Though blind to the invisible hand, the publicly run universities would internalize their academic externalities and — if applied R&D is simple enough or if basic research is difficult enough — would allow the economy to reach a stronger innovative performance than if we allowed the patenting of research tools by uncoordinated basic R&D institutions. Table 1, built for constant values of all parameters excepted λ_1 , summarizes the economy functioning for different values of applied R&D productivity in the two different intellectual property regimes: from Table 1 we easily infer that as λ_1 becomes very low the innovation rate in the first scenario overcomes the equilibrium growth performance of the economy with an inefficient public R&D sector. Simulation results also show that, when λ_1 varies, the measure of the A_0 industries tends to be \cup -shaped. Such \cup -shaped curve is reported in Graph 5, where, m_{0Priv} and $m_{0PuIneff}$ denote, respectively, the equilibrium mass of A_0 sectors in the private — i.e. with patentable research tools — basic R&D economy, and the equilibrium mass of A_0 sectors in the inefficiently run public basic R&D economy.

The economic argument at the basis of this numerical result

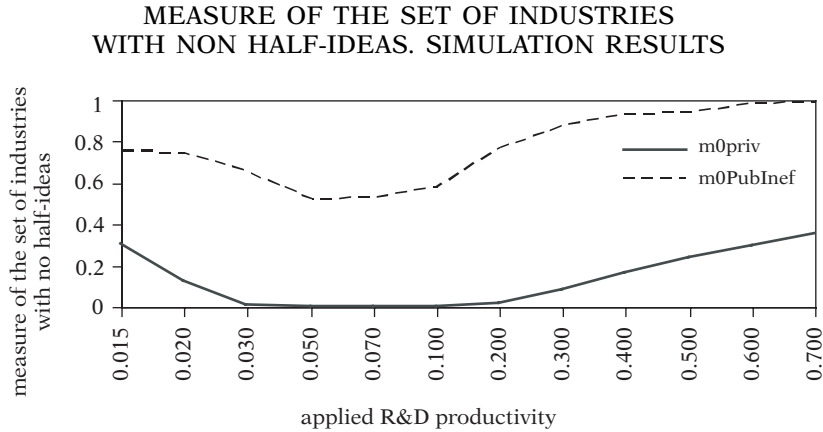
TABLE 1

RESULTS OF THE NUMERICAL SIMULATION OF THE MODEL FOR DIFFERENT VALUES OF APPLIED RESEARCH PRODUCTIVITY λ_1
Simulations parameters $\alpha = 0.3$; $\gamma = 1.3$; $\phi = 2$; $\rho = 0.05$; $L = 15$; $M = 60$; $a = 0.1$; $\lambda_0 = 0.5$

	λ_1	Innovation rate	V_L^0	V_L^1	x	w	m_0	R&D Labour
Public basic research	0.015	0.013342	444.101	244.869	12.13	2.12	0.757472	$L_G = 0.0656994$
Private basic research	0.015	0.0155451	433.95	355.633	11.99	2.14	0.303091	$z = 0.0656994$ $z_F = 2.52354$
Public basic research	0.02	0.024351	380.883	176.932	10.92	2.35	0.744573	$L_G = 0.130671$
Private basic research	0.02	0.0263426	357.619	321.482	11.13	2.31	0.121526	$z = 0.130671$ $z_F = 2.39563$
Public basic research	0.025	0.0357292	330.462	146.297	10.13	2.54	0.71591	$L_G = 0.208992$
Private basic research	0.025	0.0362059	305.135	292.805	10.68	2.41	0.0426887	$z = 0.208992$ $z_F = 2.25307$
Public basic research	0.03	0.0455315	270.52	267.916	9.54	2.69	0.656786	$L_G = 0.317755$
Private basic research	0.03	0.0477921	289.993	135.876	10.37	2.48	0.00972701	$z = 0.317755$ $z_F = 2.12554$
Public basic research	0.05	0.0922992	195.822	105.242	8.31	3.09	0.52505	$L_G = 0.846682$
Private basic research	0.05	0.0795353	198.541	198.466	9.62	2.67	0.000380018	$z = 0.846682$ $z_F = 2.02651$
Public basic research	0.07	0.132201	150.534	77.4736	7.75	3.32	0.531053	$L_G = 1.24626$
Private basic research	0.07	0.111892	158.853	158.797	9.19	2.79	0.000355811	$z = 1.24626$ $z_F = 2.02648$

Public basic research	0.1	0.188006	113.486	51.394	7.29	3.53	0.58252	$L_G = 1.66304$
Private basic research	0.1	0.158728	123.221	123.113	8.82	2.92	0.000875156	$z = 1.66304$
Public basic research	0.2	0.348368	65.9767	16.3938	6.64	3.87	0.770589	$z_F = 2.04303$
Private basic research	0.2	0.303614	72.9173	71.4829	8.27	3.11	0.0197347	$L_G = 2.41835$
Public basic research	0.3	0.476303	49.3521	6.46047	6.29	4.09	0.878811	$z = 2.41835$
Private basic research	0.3	0.421064	55.0364	50.4184	8.06	3.19	0.084679	$z_F = 2.21479$
Public basic research	0.4	0.578104	41.1339	2.81959	6.02	4.27	0.936261	$L_G = 2.95831$
Private basic research	0.4	0.511935	46.3131	38.8966	7.95	3.23	0.162265	$z = 2.95831$
Public basic research	0.5	0.885736	27.5565	1.59002	7.47	3.44	0.945061	$z_F = 2.46593$
Private basic research	0.5	0.584154	41.1211	31.6497	7.88	3.26	0.233863	$L_G = 3.8019$
Public basic research	0.6	0.718418	33.4984	0.575536	5.62	4.57	0.983902	$z = 3.8019$
Private basic research	0.6	0.642993	37.6639	26.6756	7.84	3.28	0.296478	$z_F = 2.85206$
Public basic research	0.7	0.766359	31.5142	0.271197	5.47	4.7	0.991913	$L_G = 4.11967$
Private basic research	0.7	0.691878	35.1924	23.0514	7.81	3.29	0.350665	$z = 4.11967$
Public basic research	0.8	0.805089	30.0794	0.133403	5.35	4.81	0.995822	$z_F = 3.00258$
Private basic research	0.8	0.733149	33.3361	20.2936	7.79	3.30	0.397632	$L_G = 4.38654$
Public basic research	0.9	0.837266	28.9851	0.0686474	5.24	4.9	0.997765	$z = 4.38654$
Private basic research	0.9	0.768459	31.89	18.1249	7.77	3.31	0.438558	$z_F = 3.13608$
Public basic research								$L_G = 4.61333$
Private basic research								$z = 4.61333$
								$z_F = 3.25698$
								$L_G = 4.80823$
								$z = 4.80823$
								$z_F = 3.36814$

GRAPH 5



Source: Simulation parameters $\alpha = 0.3$; $\gamma = 1.3$; $\rho = 0.05$; $L = 15$; $M = 60$; $a = 0.1$; $\lambda_0 = 0.5$.

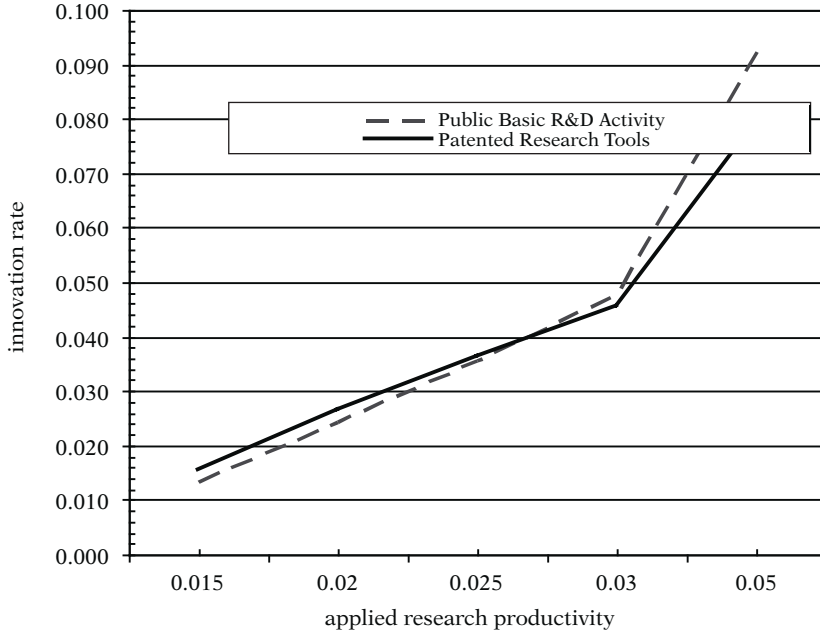
is that as $\lambda_1 \rightarrow 1$, the half-ideas arriving from the basic research are quickly completed, i.e. the A_1 industries quite immediately shift into A_0 . If instead $\lambda_1 \rightarrow 0$, the half-ideas deriving from the basic research will be slowly completed, but the decreasing in the skilled wage level, connected to the increasing difficulty in completing half-ideas, provides a disincentive for the basic R&D activity. Therefore, as the time goes on, half-ideas will be slowly completed and the A_1 industries will gradually tend to disappear.

Graph 6, built on the data provided in Table 1, shows that for sufficiently low levels of applied R&D productivity parameter λ_1 , the innovation rate of the economy where research tools are patentable and basic R&D is privately carried out overcomes the economy with unpatentable research tools.

Graph 7 shows that in the United States the ratio of the patents granted each year to US residents on applied R&D expenditure per year (in year 2000 dollars) decreased by about four fifths from 1953 to 1982. This confirms the existence of an increasing complexity in the application of the basic scientific results for commercial purposes. Such patents to R&D ratio is an index of applied research productivity. We can view it as an

GRAPH 6

COMPARISON OF THE INNOVATIVE
PERFORMANCES, SIMULATION RESULTS



empirical proxy for our parameter λ_1 . In fact, in terms of our model, such ratio can be written as:

$$(37) \quad \sigma = \frac{\lambda_1 \sqrt{z_F - \phi} N_F^{1-a}}{N_F z_F w_s}$$

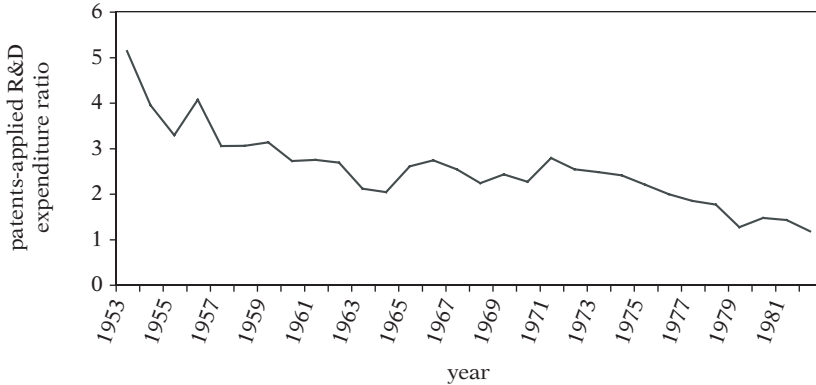
Plugging the flow of researchers employed by each follower $z_F^* = 2\phi$ into (37), and letting $a \rightarrow 0$, we have:

$$(38) \quad \sigma = \frac{\lambda_1}{2\sqrt{\phi} w_s} \propto \lambda_1$$

In the light of the endogenous R&D-driven growth model with

GRAPH 7

PATENTS-APPLIED R&D EXPENDITURE RATIO
(USA, 1953-1982)



Source: WORLD INTELLECTUAL PROPERTY ORGANIZATION and NATIONAL SCIENCE BOARD (2006).

sequential R&D activity developed in the previous sections, these remarks allow us to consider the policy change in favour of the research tools patentability occurred in the United States from the early Eighties as the rational policy result of a decrease in the applied R&D productivity.

4. - Final Remarks

This paper has developed a general equilibrium R&D-driven growth model in which the innovation process is decomposed into two successive innovative stages. The extension of multisector Schumpeterian models to a more realistic dimension allows us to answer a question about the policy shift in the US towards the extension of patentability to research tools and basic scientific ideas. These normative innovations have been modifying the industrial and academic lives in the last two decades. LEMMA 5 may suggest that one of the reasons why they were efficiently introduced was the increased relative difficulty in the marketable applications of basic scientific discoveries.

As the chances of finding profitable economic applications of each scientific result get more and more remote, a larger number of industries need to be endowed with basic ideas to try to build upon, which means that the scientists should focus their research energies in the sectors where important scientific results are still lagging. This social need is less likely met by academic researchers whose carrier proceeds when they show themselves able to discover theoretical results regardless of the potential commercial profits that applied R&D firms may make by building on their inventions.

As documented by several studies, the last decades seem characterized by a higher relative complexity at the applied stage of scientific innovations than at the basic stage. In light of the model results, the aggregate economic innovation rate would benefit from the patent races among profit seeking scientific institutions.

APPENDIX**1.A - PROOF OF LEMMA 1**

a) Consider the outsider profits maximization problem in a A_0 industry (eq. (21a) in text):

$$(39) \quad rV_0 \max_{z_0} \lambda_0 \sqrt{(z_0 - \phi, 0)} N_0^{-a} (V_F^1 - V_0) - w_s z_0 + \frac{dV_0}{dt}$$

The first order condition for a maximum is:

$$(40) \quad \frac{\lambda_0}{2\sqrt{z_0 - \phi}} (V_F^1 - V_0) N_0^{-a} = w_s$$

Since other outsider firms can freely enter each first half-idea patent race, in equilibrium the unskilled labor real wage will adjust such that expected profits will be annihilated at the optimal firm size. Therefore in any equilibrium with positive R&D activity, the stock value of every first half-idea R&D firm, V_0 , will be zero, and the average and marginal product of research will be equal, i.e.

$$(41) \quad V_0 \equiv 0 \text{ implies } \frac{dV_0}{dt} = 0 \text{ and } \lambda_0 \sqrt{z_0 - \phi} V_F^1 N_0^{-a} = z_0 w_s$$

Solving eq. (40) and the last implication of (41) for z_0 , we obtain the equilibrium amount of labour, z_0^* , employed by each outsider R&D firm:

$$(42) \quad z_0^* = 2\phi$$

b) Plugging (42) into (41) allows us to find the positive R&D equilibrium value of the skilled wage ratio as:

$$(43) \quad w_s = \max \left(\frac{\lambda_0 V_F^1 N_O^{-a}}{2\sqrt{\phi}}, 1 \right)$$

In eq. (43), we have constrained the equilibrium skilled wage not to be lower than the unskilled wage, because otherwise the skilled workers would apply for unskilled jobs. Notice that if the skilled wage were lower than

$$\frac{\lambda_0 V_F^1 N_O^{-a}}{2\sqrt{\phi}}$$

there would be excess demand for skilled labor, because the freely entrant basic R&D firms would try to make unboundedly high profits. Hence w_s would immediately increase. If instead the wage was higher than the r.h.s. of eq. (43) no basic R&D would be carried out, and eventually no R&D at all. Unlike Aghion and Howitt (1992) and Grossman and Helpman (1991), such no growth path would never afflict the economy depicted in our model, because of the assumed form of decreasing sectoral returns in each R&D sector.

Q.E.D.

PROOF OF LEMMA 2. Consider the follower's profit maximization problem (eq. (21b) in text):

$$(44) \quad \max_{z_F} \lambda_1 \sqrt{(z_F - \phi, 0)} N_F^{-a} (V_L^0 - V_F^1) - w_s z_F + \frac{dV_F^1}{dt}$$

whose first order condition for a maximum is:

$$(45) \quad \frac{\lambda_1}{2\sqrt{z_F - \phi}} (V_L^0 - V_F^1) = w_s$$

The optimal amount of labour employed in research in each A_1 industry is:

$$(46) \quad z_F^* = \phi + \left(\frac{\lambda_1 (V_L^0 - V_F^1)}{2w_s} \right)^2$$

Q.E.D.

1.B - PROOF OF LEMMA 3. From eq. (27) we have:

$$(47) \quad N_G = \frac{\bar{L}_G}{z_G}$$

The public authorities seek to maximize the per sector expected flow of half ideas by choosing the optimal scale for public laboratories:

$$(48) \quad \max_{z_G} \frac{\bar{L}_G}{z_G} \phi_G (z_G - \phi) = \max_{z_G} \left(\frac{\bar{L}_G}{z_G} \right)^{1-a} \lambda_0 \sqrt{z_G - \phi}$$

The solution for the public sector maximization problem (48) is:

$$(49) \quad z_G^* = 2\phi \frac{1-a}{1-2a}$$

Q.E.D.

1.C - PROOF OF LEMMA 4. Letting $m_{0\text{priv}}$ denote the equilibrium mass of A_0 sectors in the private — i.e. with patentable research tools — basic R&D economy, $m_{0\text{PuEff}}$ the equilibrium mass of A_0 sectors in the efficiently run public — i.e. with unpatentable research tools — basic R&D economy, $m_{0\text{PuIneff}}$ the equilibrium mass of A_0 sectors in the inefficiently run public basic R&D economy, the following relationship holds:

$$(50) \quad \bar{L}_G = m_{0\text{Priv}} N_O 2\phi \Rightarrow N_O = \frac{\bar{L}_G}{m_{0\text{Priv}} 2\phi}$$

The innovation rate of the private basic R&D economy is:

$$(51) \quad \text{Innov}_{\text{Priv}} = m_{0\text{Priv}} \left(\frac{\bar{L}_G}{m_{0\text{Priv}} 2\phi} \right)^{1-a} \lambda_0 \sqrt{\phi} = m_{0\text{Priv}}^a \left(\frac{\bar{L}_G}{2\phi} \right)^{1-a} \lambda_0 \sqrt{\phi}$$

The innovation rate of the public inefficient economy is:

$$(52) \quad \text{Innov}_{\text{PuIneff}} = m_{0\text{PuIneff}} \left(\frac{\bar{L}_G}{2\phi \frac{1-a}{1-2a}} \right)^{1-a} 2\lambda_0 \sqrt{\frac{\phi}{1-2a}}$$

By dividing eq. (51) for (52), and letting $a \rightarrow 0$, we get:

$$(53) \quad \frac{\text{Innov}_{\text{Priv}}}{\text{Innov}_{\text{PuIneff}}} \rightarrow \frac{(m_{0\text{Priv}})^a}{m_{0\text{PuIneff}}} \left(\frac{1-a}{1-2a} \right)^{1-a} (1-2a)^{\frac{1}{2}} \xrightarrow{a \rightarrow 0} \frac{1}{m_{0\text{PuEf}}} > 1$$

Q.E.D.

PROOF OF LEMMA 5. Consider the following ratio:

$$(54) \quad \frac{\text{Innov}_{\text{Priv}}}{\text{Innov}_{\text{PuIneff}}} = \frac{(m_{0\text{Priv}})^a}{m_{0\text{PuIneff}}} \left(\frac{1-a}{1-2a} \right)^{1-a} (1-2a)^{\frac{1}{2}}$$

Let us first note that

$$\left(\frac{1-a}{1-2a} \right)^{1-a} (1-2a)^{\frac{1}{2}}$$

is a positive number lower than 1 for all value of a between zero and 1/2. To see this, remind that

$$\left(\frac{1-a}{1-2a} \right)^{1-a} (1-2a)^{\frac{1}{2}} = 1 \quad \text{when } a = 0$$

and that it is positive when $0 < a < 1/2$. Moreover, it is strictly decreasing for $0 \leq a < 1/2$ which is seen in a simple way by derivating its natural logarithm with respect to a , and getting:

$$\ln \left(\frac{1-2a}{1-a} \right)^{1-a} < 0$$

Let us now note that, from the properties of our model the limit, as λ_1 tends to infinity, the measure of A_0 tends to 1 in both

economies, because in the limit the applied R&D has infinite productivity ($\lambda_1 \rightarrow \infty$) and more and more new half-idea tends to be completed, eventually completing all the stock of half-ideas²². Hence:

$$(55) \quad \frac{\text{Innov}_{\text{Priv}}}{\text{Innov}_{\text{PubIneff}}} \xrightarrow{\lambda_1 \rightarrow \infty} \frac{(1-a)^{1-a}}{(1-2a)^{1-a-\frac{1}{2}}} < 1, \quad 0 < a < 1/2$$

Q.E.D.

²² Notice that in this case our model behaves as the standard quality ladder growth model — in which $A_0=[0,1]$ — and no completing half-ideas are required.

BIBLIOGRAPHY

- AGHION P. - DEWATRIPONT M. - STEIN J.C., «Academia, Private-Sector Focus, and the Process of Innovation», *NBER, Working Paper*, no. 11542, 2005.
- AGHION P. - HOWITT P., «A Model of Growth through Creative Destruction», *Econometrica*, vol. 60, no. 2, 1992, pages 323-351.
- — — — —, *Endogenous Growth Theory*, Cambridge (MA), MIT Press, 1992.
- ARORA A. - CECCAGNOLI M., «Patent Protection, Complementary Assets, and Firms' Incentives for Technology Licensing», *Management Science*, vol. 52, no. 2, 2006, pages 293-308.
- ARROW K., «Economic Welfare and the Allocation of Resources for Invention», in NELSON R.R. (eds.), *The Rate and Direction of Inventive Activity: Economic and Social Factors*, Princeton, New Jersey, Princeton University Press, 1962, pages 609-625.
- COZZI G., «Inventing or Spying? Implications for Growth», *Journal of Economic Growth*, vol. 6, no. 1, 2001, pages 55-77.
- — — — —, «Self-fulfilling Prophecies in the Quality Ladders Economy», *Journal of Development Economics*, vol. 84, no. 1, 2007, pages 445-464.
- COZZI G. - GALLI S., *Sequential R&D and Public Researchers Altruism*, Mimeo, 2007.
- — — — —, *Changing the Research Patenting Regime: Schumpeterian Explanation*, Mimeo, 2007.
- DENICOLÒ V., «Do Patents Over-Compensate Innovators?», forthcoming su *Economic Policy*, 2007.
- FARMER R.E.A., *The Macroeconomics of Self-Fulfilling Prophecies*, Cambridge (MA), MIT Press, 1993.
- GAMBARDELLA A., *Science and Innovation: The US Pharmaceutical Industry in the 1980s*, Cambridge, Cambridge University Press, 1995.
- GANDOLFO G., *Economics Dynamics*, Heidelberg, Springer-Verlag, Berlin, 1997.
- GROSSMAN G.M. - HELPMAN E., «Quality Ladders in the Theory of Growth», *Review of Economic Studies*, vol. 58, 1991, pages 43-61.
- HAMILTON J.D., *Time Series Analysis*, New Jersey, Princeton University Press, 1994.
- HENDERSON R. - ORSENIGO L. - PISANO G., «The Pharmaceutical Industry and the Revolution in Molecular Biology: Interactions among Scientific, Institutional and Organizational Change», in MOWERY D.C. - NELSON R.R. (eds.), *Sources of Industrial Leadership: Studies of Seven Industries*, New York, Cambridge University Press, 1999.
- HOWITT P., «Steady Endogenous Growth with Population and R&D Inputs Growing», *Journal of Political Economy*, vol. 107, no. 4, 1999, pages 715-730.
- JAFFE A.B. - LERNER J., *Innovation and Its Discontents: How Our Broken Patent System is Endangering Innovation and Progress, and What to Do About It*, New Jersey, Princeton University Press, 2004.
- JENSEN R. - THURSBY M., «Proofs and Prototypes for Sale: The Licensing of University Inventions», *American Economic Review*, vol. 91, no. 1, 2001, pages 240-259.
- KORTUM S., «Equilibrium R&D and the Decline of the Patent - R&D Ratio: US Evidence», *American Economic Review, Papers and Proceedings*, vol. 65, no. 5, 1993, pages 1389-1419.
- — — — —, «Research, Patenting and Technological Change», *Econometrica*, vol. 83, no. 2, 1997, pages 450-457.

- KREMER M., «Patent Buyouts: A Mechanism for Encouraging Innovation», *Quarterly Journal of Economics*, 1998, pages 1137-1167.
- LACH S. - SHANKERMAN M., «Incentives and Invention in Universities», *CEPR, Discussion Paper*, no. 3916, Londra, 2003.
- NATIONAL RESEARCH COUNCIL, «Patents in the Knowledge-Based Economy», in COHEN W.M. - MERRILL S.A. (eds.), *Committee on Intellectual Property Rights in the Knowledge-Based Economy*, National Research Council, Washington (DC), The National Academies Press, 2003.
- —, «A Patent System for the 21st Century», in MERRILL S.A. - LEVIN R.C. - MYERS M.B. (a cura di), Washington (DC), The National Academies Press, 2004.
- NATIONAL SCIENCE BOARD, *Science and Engineering Indicators 2006*, Two volumes. Arlington, VA, National Science Foundation, 2006.
- O'DONOGHUE T., «A Patentability Requirement for Sequential Innovation», *RAND, Journal of Economics*, vol. 29, no. 4, 1998, pages 654-679.
- O'DONOGHUE T. - ZWEIMÜLLER J., «Patents in a Model of Endogenous Growth», *Journal of Economic Growth*, vol. 9, no. 1, 2004, pages 81-123.
- ROMER P.M., «Endogenous Technological Change», *Journal of Political Economy*, vol. 98, 1990, pages S71-S102.
- SCOTCHMER S., «Standing on the Shoulders of Giants: Cumulative Research and the Patent Law», *Journal of Economic Perspectives*, vol. 5, no. 1, 1991, pages 29-41.
- SEGERSTROM P.S., «Innovation, Imitation and Economic Growth», *Journal of Political Economy*, 1991, pages 807-827.
- —, «Endogenous Growth Without Scale Effects», *American Economic Review*, vol. 88, no. 5, 1998, pages 1290-1310.
- WIPO, «100 Years Protection of Intellectual Property Statistics», disponibile su <http://www.wipo.int/ipstats/en/statistics/patents/>.