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A Reduced Form Model of Default Spreads with Markov Switching Macroeconomic Factors

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Abstract: An important research area of the corporate yield spread literature seeks to measure the proportion of the spread explained by factors such as the possibility of default, liquidity or tax differentials. We contribute to this literature by assessing the ability of observed macroeconomic factors and the possibility of changes in regime to explain the proportion in yield spreads caused by the risk of default in the context of a reduced form model. For this purpose, we extend the Markov Switching risk-free term structure model of Bansal and Zhou (2002) to the corporate bond setting and develop recursive formulas for default probabilities, risk-free and risky zero-coupon bond yields. The model is calibrated out of sample with consumption, inflation, risk-free yield and default data over the 1987-1996 period. Our results indicate that inflation is a key factor to consider for explaining default spreads during our sample period. We also find that the estimated default spreads can explain up to half of the 10 year to maturity Baa zero-coupon yield in certain regime with different sensitivities to consumption and inflation through time.

Keywords: Credit spread, default spread, Markov switching, macroeconomic factors, reduced form model of default

JEL Classification: G12, G13

Résumé: Un domaine de recherche important de la littérature sur les écarts de taux des obligations privées consiste à mesurer la proportion des écarts de taux expliquée par des facteurs comme la possibilité de défaut, la liquidité et les différences de taxes. Nous contribuons à cette littérature en vérifiant comment des facteurs macroéconomiques observables et des changements possibles de régime peuvent expliquer la proportion des écarts de taux causée par le risque de défaut dans un modèle à forme réduite. À cette fin, nous proposons une extension du modèle de changement de régime Markovien appliqué à la structure à terme des taux sans risque de Bansal et Zhou (2002) aux obligations privées et développons des formules récursives pour les probabilités de défaut, le taux sans risque et les rendements des obligations privées zéro-coupon. Le modèle est calibré hors échantillon sur la consommation, l'inflation, le rendement sans risque et les données de défaut sur la période 1987-1996. Les résultats indiquent que l'inflation est un facteur clé pour expliquer les écarts de taux dus au défaut durant notre période d'estimation. Nous obtenons également que les écarts de taux dus au défaut peuvent expliquer jusqu'à 50 % des écarts de rendement des obligations Baa dans certains régimes avec des variations temporelles sensibles à l'inflation et à la consommation.

Mots clés : Écart de taux lié au risque de crédit, écart de taux lié au risque de défaut, facteur macroéconomique, modèle de défaut à forme réduite, modèle de changement de régime Markovien.

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Abstract

An important research area of the corporate yield spread literature seeks to measure the proportion of the spread explained by factors such as the possibility of default, liquidity or tax differentials. We contribute to this literature by assessing the ability of observed macroeconomic factors and the possibility of changes in regime to explain the proportion in yield spreads caused by the risk of default in the context of a reduced form model. For this purpose, we extend the Markov Switching risk-free term structure model of Bansal and Zhou (2002) to the corporate bond setting and develop recursive formulas for default probabilities, risk-free and risky zero-coupon bond yields. The model is calibrated out of sample with consumption, inflation, risk-free yield and default data over the 1987-1996 period. Our results indicate that inflation is a key factor to consider for explaining default spreads during our sample period. We also find that that the estimated default spreads can explain up to half of the 10 year to maturity Baa zero-coupon yield in certain regime with different sensitivities to consumption and inflation through time.

1 Introduction

Several empirical studies have been recently performed on corporate yield spreads, measured as the difference between the corporate and treasury yield to maturity. These studies attempt to explain some of the observed features of corporate spreads through time. In this article we blend two research directions recently explored in this literature. We investigate if a reduced form model with observed macroeconomic risk factors following a Markov Switching process, can help in explaining the spread behavior through time. The model and the empirical study proposed here can also be seen as an extension of the Elton et al. (2001) study to a risk averse setting. In Elton et al. (2001), an assumption of risk neutrality was needed to justify the use of objective default probabilities in a bond pricing model specified under the risk neutral measure. The model developed here is entirely specified under the objective measure in a risk averse setting, avoiding the need for such a strong assumption.

The motivation for examining macroeconomic fundamentals as drivers of the spread behavior comes from the link between interest rates and output from firms and the macroeconomy. These variables, which should influence yield spreads, fluctuate over the business cycle. It should thus be anticipated that macroeconomic fundamentals play a role in explaining the spread behavior through time. Recently, some attempts have been made to tie macroeconomic activity with the spreads in the context of structural models. For example, Pesaran et al. (2006) examine an econometric model linking credit risk and macroeconomic variables in a Merton-type structural model. Chen et al. (2006) examine how a structural model using pricing kernels that are successful in solving the equity premium puzzle performs to explain the spread. David (2007) looks at how investors learning from inflation helps in generating realistic credit spread levels. To our knowledge, in the context of reduced form models, few attempts have been made apart from Amato and Luisi (2006) where a model of credit spread with both latent and observed macro variables is examined. Further work on the reduced form type models and the macroeconomy is thus an interesting addition to the literature as these models often require fewer inputs in the calibration stage.

Another distinctive feature of the model examined here is the Markov switching environment. The motivation for examining the influence of macroeconomic variables in such a framework comes from empirical evidences suggesting that switching regimes are better descriptions of these variables and risk-free interest behavior than single regime models. See for example Evans (2003), Ang, Bekeart and Wei (2007), Bansal and Zhou (2002) and Dai, Singleton et Yang (2007). Because the possibility of changes in regime might influence macroeconomic factors and risk-free interest rates, it is only natural to assume that this might also affect the corporate yield spreads.

To introduce macroeconomic factors and the possibility of changes in regime in a reduced form spread model, we extend the switching regime risk-free term structure model of Bansal and Zhou (2002) to the risky corporate setting. Starting from the first order condition of the intertemporal consumption problem with a power utility function and state dependant utility parameters, we assume that consumption and inflation dynamics are governed by two independent Markov chains. Using a log linear approximation we derive closed form recursive formulas for risk-free and risky bond yields as well as for default probabilities which are all functions of the growth rates of our two observed factors. We then measure the default spread generated by this approach by calibrating the model with aggregate consumption, inflation, risk-free rate and default data.

The calibration proceeds in three steps. First, we estimate the Markov switching parameters with aggregate consumption and inflation data. Using the parameters obtained in the first step, we then extract, for each quarter, implied utility parameters enabling us to produce realistic theoretical term structure of risk-free rates. In a third step, with the parameter values obtained in the first two steps, we calibrate the parameters linking our theoretical default probabilities with the macroeconomic risk factors to match the observed default probabilities obtained from default data. The default yield spreads implied by our model can then be computed and analyzed.

Our results show that the default spread exhibit different sensitivities to consumption and inflation depending on the different possible regimes. Inflation is found to be an important factor to explain the spreads during the first half of our sample period. We also find that the model can reproduce out of sample some key properties of observed spreads, such as for example, the sharp increase observed at the end of 1990. This result is interesting because it indicates that, in certain regime, the spread level is sensitive to a macroeconomic market wide undiversifiable risk. Such a result is supported by recent studies such as Farnsworth and Li (2007) who provide evidences about the presence of systematic factors associated with default risk. We also find that risk aversion does not influence much the proportion of the spread caused by the risk of default. This result can be, in part, attributed to the low volatility of consumption growth and inflation during the studied period which are used as the sole factors in the model. Finally, we obtain estimates of default spread proportions varying through the different regimes. For example, these proportions range from 21% to 48% of the 5 year Baa spreads and 29% to 47% for the 10 year Baa spread.

Section 2 presents our theoretical models and formulas for the risk-free zero-coupon bonds, the risky zero-coupon bonds and the default probabilities. Section 3 presents our estimation results and calibration procedures. Section 4 analyses the estimated default spreads for industrial Baa bonds. Section 5 concludes.

2 Models

The model developed here starts from the well known first order condition of the intertemporal consumption problem as described in, for example, Cochrane (2005). Because we attempt to model nominal bond prices, we account for the future growth rates of the price level and real consumption. We assume that the future evolution of these variables is well described by a Markov Switching process.

Let C_t denote the real personal consumption expenditures per capita at time t, and Π_t the ratio of nominal over real consumptions (consumption price index) at time t with $t \in \mathcal{N}$. Here, the time variable is expressed in quarters and the continuously compounded quarterly growth rates are defined as $c_t = \ln C_t - \ln C_{t-1}$ and $\pi_t = \ln \Pi_t - \ln \Pi_{t-1}$. We assume that c_t and π_t follow an autoregressive model with switching regimes

$$c_t = a^c_{s^c_t} + b^c_{s^c_t} c_{t-1} + e^c_t \tag{1a}$$

$$\pi_t = a^\pi_{s^\pi_t} + b^\pi_{s^\pi_t} \pi_{t-1} + e^\pi_t \tag{1b}$$

where $s^c_t \in \{1, 2\}$ is the state of consumption at time t and $s^\pi_t \in \{1, 2\}$ is the state of inflation at time t. The error terms e^c_t and e^π_t are i.i.d. Gaussian noises with zero mean, standard deviations $\sigma^c_{s^c_t}$ and $\sigma^\pi_{s^\pi_t}$ and covariance $\rho_{s_t}\sigma^c_{s^c_t}\sigma^\pi_{s^\pi_t}$ with $s_t = \{s^c_t, s^\pi_t\}$ i.e. $s_t \in \{(1, 1), (1, 2), (2, 1), (2, 2)\}$. The states of consumption growth and inflation are assumed to follow two independent Markov chains with transition matrices

$$\phi^c = \begin{pmatrix} \phi^c_{11} & 1 - \phi^c_{11} \\ 1 - \phi^c_{22} & \phi^c_{22} \end{pmatrix}, \qquad \phi^\pi = \begin{pmatrix} \phi^\pi_{11} & 1 - \phi^\pi_{11} \\ 1 - \phi^\pi_{22} & \phi^\pi_{22} \end{pmatrix}.$$

These Markov chains are also assumed to be independent of past values of c and π.

Define the $\sigma-$field $\mathcal{G}_t = \sigma(C_u, \Pi_u, s_u : u \in \{0, 1, ..., t\})$. It may be interpreted as the information available at time t if one observes the evolution of consumption growth, inflation, and the state of consumption and inflation up to time t. Using the first order condition of the intertemporal consumption problem with the assumption of a power utility function, the time t value of a security worth X_{t+1} at time $t+1$ is given by

$$\mathrm{E}_t \left[\beta_{s_{t+1}} \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma_{s_{t+1}}} \frac{\Pi_t}{\Pi_{t+1}} X_{t+1} \right] = \mathrm{E}_t \left[M_{t,t+1} X_{t+1} \right] \tag{2}$$

where

$$M_{t,t+1} = \exp\left(\ln \beta_{s_{t+1}} - \gamma_{s_{t+1}} c_{t+1} - \pi_{t+1} \right) \tag{3}$$

is the nominal discount factor or the pricing kernel for the time period $]t, t+1]$, $\beta_{s_{t+1}}$ and $\gamma_{s_{t+1}}$ are, respectively, the impatience coefficient and risk aversion coefficient in state s_{t+1}. $\mathrm{E}_t[\bullet]$ is a short hand notation for $\mathrm{E}[\bullet |\mathcal{G}_t]$, the conditional expectation with respect to available information at time t. Note that we use a power utility function instead of the log specification of Bansal and Zhou

(2002). Equation (2) thus proposes a pricing kernel which is a function of the consumption, inflation and Markov chain processes. This pricing kernel must take into account regime shifts uncertainty and we assume that this uncertainty should affect the preference parameters. We are thus allowing the preference parameters to be different in the different possible regimes. We also assume that γ and β depend of $t+1$ instead of t in order to incorporate the regime shifts uncertainty related to the conditional distribution of X_{t+1}. The same argument of future regime shifts uncertainty is also used to justify why parameters a and b in equations (1a) and (1b) are functions of t instead of $t-1$.

2.1 Risk-free zero-coupon bond

An exact formula for the time t value of a default-risk-free zero-coupon bond paying one dollar at time T can be obtained using the framework described above. However, such a solution is not practical. For example, with quarterly time steps, the value of a zero-coupon bond maturing in 40 quarters would roughly contain 4^{40} terms to compute. This would make the numerical implementation of the exact solution unmanageable. For this reason, we instead rely on an analytical approximation developed in Bansal and Zhou (2002) for the price of a risk-free zero-coupon bond with n periods to maturity:

$$P\left(t, n, s_t\right) = \exp\left(A^{p}_{n,s_t} - B^{p,c}_{n,s_t} c_t - B^{p,\pi}_{n,s_t} \pi_t\right) \tag{4}$$

where $s_t = \{s^c_t, s^\pi_t\}$ and expressions for A^p_{n,s_t}, $B^{p,c}_{n,s_t}$, and $B^{p,\pi}_{n,s_t}$ are given in Appendix A. The pricing formula is a function of our observed factors and the states of the Markov chains. The sensitivities to the factors are given by the $B(\bullet)$ functions which are determined recursively using backward induction and the terminal condition $A^p_{0,s_T} = B^{p,c}_{0,s_T} = B^{p,\pi}_{0,s_T} = 0$. These expressions are functions of the Markov switching parameters and the actual states of consumption and of inflation s^c_t and s^π_t. At each point in time, four different bond prices can thus be computed since four different states are possible. Because the state of the economy is unknown at a particular point in time, we will define the theoretical zero-coupon bond price as the expected bond price, with the expectation computed over the possible states of the Markov chain whose probability can be conveniently estimated. Section 3.4 provides further details about this procedure. The factor sensitivities are also functions of the time to maturity, $n = T - t$, and the utility function parameters.

Although the formula in appendix is complex, it is possible to get some intuition by looking at the one period case, rewritten in terms of an annualized yield to maturity:

$$y_p\left(t,1,s_t\right)=4\sum_{i=1}^{2}\sum_{j=1}^{2}\phi_{s_t^c,i}^{c}\phi_{s_t^\pi,j}^{\pi}\left[\begin{array}{c}-\ln\beta_{i,j}+\gamma_{i,j}\left(a_i^c+b_i^c c_t\right)+\left(a_j^\pi+b_j^\pi\pi_t\right)\\ -\frac{1}{2}\left(\sigma_j^\pi\right)^2-\frac{1}{2}\gamma_{i,j}^2\left(\sigma_i^c\right)^2-\gamma_{i,j}\rho_{i,j}\sigma_i^c\sigma_j^\pi\end{array}\right]$$

with the term inside brackets is the expression for the yield to maturity, in state i,j, of a one period risk-free bond within the power utility lognormal framework. The one period bond yield in state s_t is the conditional expected value of the bond yields in the different possible states where the ϕ's are the conditional probabilities. The various terms forming the bond yield in state i,j are then interpreted the usual way.

The first term of the expression within brackets is a function of the impatience coefficient. A smaller impatience coefficient (more impatient investor) is associated with higher yields since the impatient investor prefers consumption to saving. The second term, $\gamma_{i,j}\left(a_i^c+b_i^c c_t\right)$, is the risk aversion parameter multiplied by the conditional expected growth of consumption in state i,j. Given positive a_i^c and b_i^c, higher values of these coefficients will lead to higher expected consumption growth and higher yields. The risk aversion parameter $\gamma_{i,j}>0$ makes the yield more or less sensitive to the expected consumption growth rate. The sum of the third and fourth term, $\left(a_j^\pi+b_j^\pi\pi_t\right)-\frac{1}{2}\left(\sigma_j^\pi\right)^2$, is the portion of yield rewarding the investor for the expected loss in real purchasing power on the nominal one dollar bond payoff at maturity and where the variance of inflation appears because of the convexity of the bond pricing function. The fifth term, $\frac{1}{2}\gamma_{i,j}^2\left(\sigma_j^\pi\right)^2$, is the precautionary savings effect brought by the volatility of consumption. An increase in the volatility of consumption brings more extreme low and high paths of future consumption. Because investors worry more about the low consumption states than they are pleased by the high ones, a demand for savings is created which drives down the yield on the bond. Finally, the last term, $\gamma_{i,j}\rho_{i,j}\sigma_i^c\sigma_j^\pi$, is the inflation risk premia. A negative correlation will obtain a positive risk premia because inflation decreases the real nominal bond payoff in states where the investor needs it the most. For example, a future low consumption state would likely be associated with a high inflation path and low real value for the nominal payoff.

2.2 Risky zero-coupon bond and default spread

We consider a risky zero-coupon bond paying one dollar at T if it has not defaulted before. In case of default, the bondholder receives at the default time τ, a fraction of its market value if it had not defaulted. In this well studied context (see Duffie and Singleton 1999), the time t value of the survived risky zero-coupon bond is

$$\widetilde{V}(t,n) = \mathrm{E}_t\left[M_{t,t+1}\left(1 - Lh_{t+1}\right)\widetilde{V}(t+1,n-1)\ \right] \tag{5}$$

where $L = (1-\rho)$ is the loss given default (LGD) and ρ is the recovery rate assumed constant. Here, $h_{t+1} = \Pr_{\mathcal{G}_{t+1}}[\tau = t+1 \mid \tau > t]$ represents the conditional probability that the default arises within the next period of time knowing that the firm as survived at time t and having the information available at time $t+1$. Since default probabilities are usually small, it is reasonable to use a first order Taylor expansion to approximate $1 - Lh_{t+1}$ by $\exp(-Lh_{t+1})$. Hence

$$\widetilde{V}(t,n) \cong \mathrm{E}_t\left[M_{t,t+1}\exp\left(-Lh_{t+1}\right)\widetilde{V}(t+1,n-1)\right]. \tag{6}$$

We also assume that the conditional default probability h_{t+1} is approximated by an affine function of c_{t+1} and π_{t+1}, that is,

$$h_{t+1} \cong \alpha_{s_{t+1}} + \alpha^c_{s_{t+1}}c_{t+1} + \alpha^\pi_{s_{t+1}}\pi_{t+1} \tag{7}$$

where $\alpha_{s_{t+1}}, \alpha^c_{s_{t+1}}$ and $\alpha^\pi_{s_{t+1}}$ are parameters. Note that the specification (7) can produce negative probabilities as well as probabilities larger than one. Using these assumptions and those required by the approach of Bansal and Zhou (2002), Appendix B develops the following analytical approximation for the prices of risky zero-coupon bonds :

$$V(t,n,s_t) = \exp\left(A^v_{n,s_t} - B^{v,c}_{n,s_t}c_t - B^{v,\pi}_{n,s_t}\pi_t\right). \tag{8}$$

As shown in Appendix B, the coefficients A^v_{n,s_t}, $B^{v,c}_{n,s_t}$ and $B^{v,\pi}_{n,s_t}$ are obtained recursively starting with $A^v_{0,s_T} = B^{v,c}_{0,s_T} = B^{v,\pi}_{0,s_T} = 0$. The resulting pricing formula is very similar to the risk-free case developed earlier. It is a function of the current values of our two observed factors with the loadings given by the $B(\bullet)$ functions. These quantities are function of the Markov switching model parameters, the actual states of consumption and of inflation s^c_t and s^π_t, the utility function parameter values and the time to maturity. Unlike the risk-free case, however, we find the the additional $L\alpha_{i,j}, L\alpha_{i,j}^{c}$ and $L\alpha_{i,j}^{\pi}$ terms appearing because of the possibility of default.

Using the above analytical approximation and the earlier approximation for the risk-free yield, an expression for the annualized default spread, defined as the difference between the risky yield to maturity and the risk-free yield to maturity, is given by:

$$DS\left(t,n,s_t\right) = \frac{A_{n,s_t}^{p} - A_{n,s_t}^{v} + \left(B_{n,s_t}^{v,c} - B_{n,s_t}^{p,c}\right)c_t + \left(B_{n,s_t}^{v,\pi} - B_{n,s_t}^{p,\pi}\right)\pi_t}{n/4}. \tag{9}$$

Again, to get some intuition about the role of the different parameters on the spread, it is interesting to look at the one period case :

$$\begin{aligned} DS\left(t,1,s_t\right) = {} & 4\sum_{i=1}^{2}\sum_{j=1}^{2}\phi_{s_t^c,i}^{c}\phi_{s_t^\pi,j}^{\pi} \\ & \times L\left[\begin{array}{c} \alpha_{i,j} + \alpha_{i,j}^{c}\left(a_i^c + b_i^c c_t\right) + \alpha_{i,j}^{\pi}\left(a_j^\pi + b_j^\pi \pi_t\right) \\ -\frac{1}{2}L\left(\left(\alpha_{i,j}^{c}\sigma_i^c\right)^2 + \left(\alpha_{i,j}^{\pi}\sigma_j^\pi\right)^2 + 2\alpha_{i,j}^{\pi}\alpha_{i,j}^{c}\rho_{i,j}\sigma_i^c\sigma_j^\pi\right) \\ -\alpha_{i,j}^{\pi}\left(\sigma_j^\pi\right)^2 - \gamma_{i,j}\left(\alpha_{i,j}^{c}\left(\sigma_i^c\right)^2 + \left(\alpha_{i,j}^{\pi} + \alpha_{i,j}^{c}\right)\rho_{i,j}\sigma_i^c\sigma_j^\pi\right) \end{array}\right] \end{aligned}$$

where the term inside brackets is the expression for the default spread, in state i,j, for a one period risk-free bond. The default spread in state s_t is the conditional expected value of the bond yield spreads in the different possible states next period.

The first line of the term between brackets can be interpreted as one of the portions forming the expected loss next period in state i,j. In the context of a one period bond, L represents the loss given default. This quantity is multiplied by the conditional expected default probability in state i,j that is $\alpha_{i,j} + \alpha_{i,j}^{c}\left(a_i^c + b_i^c c_t\right) + \alpha_{i,j}^{\pi}(a_j^\pi + b_j^\pi \pi_t)$. The signs of the $\alpha_{i,j}, \alpha_{i,j}^{c}$ and $\alpha_{i,j}^{\pi}$ will determine the influence of consumption and inflation on this portion of the spread. The second and third lines are the additional impacts of potential losses brought by the convexity of our recovery factor model. Again, the sign of these terms will depend on the signs of the α's. For example, the effect of a change in consumption volatility is not clear as it depends on the magnitude and signs of the α's and the correlation. Hence, given $\rho_{i,j} > 0$ with a negative and large $\alpha_{i,j}^{c}$ (relatively to the $\alpha_{i,j}^{\pi}$), an increase in consumption volatility will increase the spread. Finally, it is interesting to note that the risk aversion parameter interacts only with the squared volatilities and covariance of the factors. Hence, risk aversion is here a second order effect whose magnitude will be determined by the relative importance of the volatilities, covariance term and the magnitudes and signs of the $\alpha_{i,j}^{c}$ and $\alpha_{i,j}^{\pi}$ parameters. In the context of this model, this proportion caused by risk aversion can be conveniently assessed. One can first compute the portion of the spread which is caused by the actuarial loss. This is the default spread a risk neutral investor would be satisfied with. This quantity, that we label the *default risk spread*, can be computed by setting $\gamma_{i,j} = 0$ in the default spread equation i.e.

$$DR(t,n,s_t) = DS\left(t,n,s_t \mid \gamma = 0\right) \tag{10}$$

with $\gamma = \{\gamma_{1,1}, \gamma_{1,2}, \gamma_{2,1}, \gamma_{2,2}\}$. The portion of the spread caused by risk aversion, which we label the *default premium spread*, can then be computed by difference with

$$DP(t,n,s_t) = DS\left(t,n,s_t\right) - DR(t,n,s_t). \tag{11}$$

This is the spread a risk averse investor would ask for in addition of the default risk spread. In the context of a one period bond, this quantity becomes

$$DP(t,1,s_t) = 4L \times \sum_{i=1}^{2}\sum_{j=1}^{2} \phi_{s_t^c,i}^{c}\phi_{s_t^\pi,j}^{\pi}\left[-\gamma_{i,j}\left(\alpha_{i,j}^{c}\left(\sigma_i^c\right)^2 + \left(\alpha_{i,j}^{\pi} + \alpha_{i,j}^{c}\right)\rho_{i,j}\sigma_i^c\sigma_j^\pi\right)\right].$$

2.3 Survival probability

A final theoretical quantity obtained within the framework of this model is the term structure of survival probabilities. This quantity will be used to calibrate our model to match the default probabilities that are observed for the sample period examined in this study.

The survival probability at t, $\Pr_{\mathcal{G}_t}[\tau > t+n \mid \tau > t]$, is the probability that the default occur in more than n periods from t knowing that the firm has not defaulted at time t in a given state of our macro factors at t. Because this probability is usually small, we use the approximation $e^{-h_u} \cong 1 - h_u$ to write

$$\Pr_{\mathcal{G}_t}\left[\tau > t+n \mid \tau > t\right] = \mathrm{E}_t\left[\prod_{u=t+1}^{t+n}\left(1-h_u\right)\right]$$

as

$$\Pr_{\mathcal{G}_t}\left[\tau > t+n \mid \tau > t\right] \cong \mathrm{E}_t\left[\exp\left(-\sum_{u=t+1}^{t+n}\left(\alpha_{s_u} + \alpha_{s_u}^{c}c_u + \alpha_{s_u}^{\pi}\pi_u\right)\right)\right]$$

from our assumption given in equation (7). As shown in Appendix C, an analytical approximation for this expected value is given by:

$$q\left(t,n,s_t\right) = \exp\left(-A^{q}_{n,s_t} - B^{q,c}_{n,s_t}c_t - B^{q,\pi}_{n,s_t}\pi_t\right). \tag{12}$$

As in the previous cases, the coefficients A^{q}_{n,s_t}, $B^{q,c}_{n,s_t}$ and $B^{q,\pi}_{n,s_t}$ are obtained recursively starting with $A^{q}_{0,s_T} = B^{q,c}_{0,s_T} = B^{q,\pi}_{0,s_T} = 0$. These coefficients are function of the maturity n, the Markov switching parameters, the unobserved state s_t and the unknown parameters linking the one period default probability h_t with the real consumption growth and inflation.

3 Calibration and estimation

3.1 Empirical yield curves

To measure the capacity of our model to generate realistic Baa default spread levels through time, estimates of the credit yield spread curves of Baa zero-coupon bonds are required. These yield-spread curves are obtained by first estimating risk-free term structures of zero-coupon yield from risk-free bonds with coupons using the Nelson and Siegel (1987) curve fitting approach. The Baa zero-coupon bond yields are then obtained with the same approach. The yield spreads are measured as the difference between these term structures of yields at each point in time. The data comes from the Lehman Brothers Fixed Income Database (Warga, 1998). We choose this data to enable comparisons with other articles in this literature using the same database. The data contains information on monthly prices (quote and matrix), accrued interest, coupons, ratings, callability, and returns on all investment-grade corporate and government bonds for the period from January 1987 to December 1996. All bonds with matrix prices and options were eliminated; bonds not included in Lehman Brothers' bond indexes and bonds with an odd frequency of coupon payments were also dropped. All bonds with a pricing error higher than \$5 were dropped. We then repeated this estimation and data removal procedure until all bonds with a pricing error larger than \$5 have been eliminated. Using this procedure, 695 bonds were eliminated out of a total of 12,849 bonds found in the Baa industrial sector, which is the focus of this study. For government bonds, four bonds were eliminated out of a total of 13,552. Table 1 presents the average zero-coupon yield spreads for industrial Baa for maturities of 1 to 10 years during the 1987-1996 period.

3.2 Markov Switching parameters

This section describes how the parameters of the Markov Switching model are estimated. Let θ denote the set of parameters associated with the growth rate equations that is $\theta = (a_1^c,\ a_2^c,\ b_1^c,\ b_2^c,\ a_1^\pi,\ a_2^\pi,\ b_1^\pi,\ b_2^\pi,\ \sigma_1^c,\ \sigma_2^c,\ \sigma_1^\pi,\ \sigma_2^\pi,\ \rho_{1,1},\ \rho_{1,2},\ \rho_{2,1},\ \rho_{2,2})$ and the transition probabilities parameters $\phi = (\phi_{11}^c,\ \phi_{22}^c,\ \phi_{11}^\pi,\ \phi_{22}^\pi)$. From the time series of consumptions $C_0, ..., C_T$ and price index $\Pi_0, ..., \Pi_T$ from which we create the sample $c_1, ...c_T, \pi_1, ..., \pi_T$, we define $v_t = (x_1, ..., x_t)$ as the set of observed data point up to time t and $x_t = (c_t, \pi_t)$ as the set of observed consumption growth and inflation at t. The log-likelihood function based on the observed sample v_T up to time T is then computed with

$$\mathcal{L}\left(\theta,\phi;v_T\right) = \sum_{t=2}^{T} \ln f\left(x_t \mid v_{t-1};\theta,\phi\right) \tag{13}$$

where

$$f\left(x_t \mid v_{t-1};\theta,\phi\right) = \eta_t' \times \xi_{t|t-1}$$

represents the conditional likelihood function of x_t given the observed sample v_{t-1}. The 4×1 vector η_t contains the likelihood value of x_t conditional on states i, j and the observed sample v_{t-1}. The 4×1 vector $\xi_{t|t-1}$contains the probability of being in state i, j at time t conditional on the observed sample v_{t-1}. Appendix D describes how these quantities can be computed. The maximization of the log-likelihood function $\mathcal{L}\left(\theta,\phi;v_T\right)$ is done numerically using a hill climbing algorithm.

The data series used here are the growth rate of non-durable personal consumption expenditures per capita (real) from the first quarter of 1957 to the last quarter of 1996 and the growth rate of consumption price index for the same period (160 quarters). The data comes from the U.S. Department of Commerce: Bureau of Economic Analysis. The data period contains seven recessions according to the NBER and many of them should be important enough to generate regime shifts.[1] Figure 1 illustrates the temporal evolution of the two growth rates.

The results of the estimation procedure are presented in Table 2. Many parameters are statistically different from zero but are not necessarily different two by two, as shown in Table 3. For consumption, regime switching seems to appear only in the volatility. For inflation, the autore-

[1] The official recession periods during our research period, according to the NBER, are: 1957-III to 1958-II, 1960-II to 1961-I, 1969-IV to 1970-IV, 1973-IV to 1975-I, 1980-I to 1980-III, 1981-III to 1982-IV, and 1990-III to 1991-I.

gressive parameters as well as the volatility parameters are modified with the regime shifts. We also observe that ρ_{12} and ρ_{22} are negative and statistically different from zero, which confirms a negative empirical correlation between consumption and inflation implying a positive risk premium for inflation. As reported in Table 3, it seems reasonable to assume that $a_1^c = a_2^c = a^c$, $b_1^c = b_2^c = b^c$, $a_1^\pi = a_2^\pi = a^\pi$ which is also true for the correlation coefficients $\rho_{1,1} = \rho_{2,1}$, $\rho_{1,2} = \rho_{2,2}$ and $\rho_{1,1} = \rho_{1,2}$.

3.3 States of consumption and inflation

Within the context of our regime switching model, two different conditional probabilities are of interest. The ex-ante probability, $\xi_{t|t}$, is useful in forecasting future inflation and consumption rates based on an evolving information set. The smoothed probability, $\xi_{t|T}$, estimated using the entire information set available, is of interest for the determination of the prevailing regime at each time point within the sample period. To estimate $\xi_{t|T} = f\left(s_t \mid \upsilon_T; \theta, \phi\right)$ for $s_t \in \{(1,1), (1,2), (2,1), (2,2)\}$, we use the following algorithm developed in Kim (1994):

$$\xi_{t|T} = \xi_{t|t}\,(\times)\left[\Phi\left(\xi_{t+1|T}\,(\div)\,\xi_{t+1|t}\right)\right]$$

where $(\times)$ and $(\div)$ means element-by-element multiplication and division respectively and where the quantity Φ is described in appendix D.

We apply the estimation procedure to the same time series as for the estimation of the parameters of the Markov Switching process but restricted our analysis to the period 1987-I- to 1996-IV which contains 40 quarters. This period corresponds to the data period we have regarding our risky bond information. Note that we used the estimates of θ and ϕ from Table 2.

The results of the estimation procedure are presented in Figure 2. There are only two quarters for which there is not one of the four values of the mass function clearly dominating the others. Indeed, at the third quarter of 1990, we obtain 0.5046 for $s_t = (1,2)$ and 0.4950 for $s_t = (2,2)$. At the second quarter of 1993, the mass function is 0.5439 for $s_t = (1,1)$ and 0.4511 for $s_t = (2,1)$. The estimated states $\widehat{s}_t$ at time t is the one for which the estimated probability in vector $\xi_{t|T}$ is the highest among all the possible states. The results are reported in Figure 3.

The interpretation of the estimated states are as follows: $s_t = (1,1)$ corresponds to a state of low consumption volatility combined with low level and low volatility of inflation; $s_t = (1,2)$ is the state of low volatility of consumption with high and volatile inflation; $s_t = (2,1)$ corresponds to the high volatility of consumption rates combined with low level and low volatility of inflation; finally $s_t = (2,2)$ is for high volatility of consumption rates with high and more volatile inflation. A detailed examination of the results reveals that the estimated state of consumption is 1 for two distinct time periods: 1987-I to 1990-III (15 quarters) and for 1993-II to 1996-IV (15 quarters). In between, the consumption's estimated state is 2 for the period 1990-IV to 1993-I (10 quarters). For inflation, we note only one change of regime. Indeed, the state of inflation is estimated to 2 for the time period 1987-I to 1990-IV (16 quarters) and becomes 1 for the time period 1991-I to 1996-IV (24 quarters). If we consider the system globally, the estimated state is (1,2) during the 15 first quarters (1987-I to 1993-III), it switches to the state (2,2) for only one quarter (1990-IV), goes to state (2,1) for 9 quarters (1991-I to 1993-I) and ends in state (1,1) for 15 quarters (1993-II to 1996-IV).

It is interesting to note that the observed average consumption growth rate and volatility are 0.39% and 0.34% during the two periods 1987-I to 1990-III and 1993-II to 1996-IV which correspond to $\widehat{s}_t^c = 1$ while they are -0.07% and 0.91% during the 1990-IV to 1993-I period corresponding to $\widehat{s}_t^c = 2$. The observed average inflation growth rate and volatility are 0.32% and 0.33% during the 1991-I to 1996-IV period corresponding to $\widehat{s}_t^\pi = 1$ and 1.21% and 0.68% during the 1987-I to 1990-IV period for $\widehat{s}_t^\pi = 2$. Figure 4 illustrates the changes of regime behavior for the growth rate of personal consumption expenditures per capita and for the growth rate of price index respectively. The regimes are well related to the business cycles during that period. The consumption rate and inflation clearly exhibits different behavior in each regime.

3.4 Preference parameters

In this section, we explain how the impatience coefficients $\beta = (\beta_{11},\ \beta_{12},\ \beta_{21},\ \beta_{22})$ and the risk aversion coefficients $\gamma = (\gamma_{11},\ \gamma_{12},\ \gamma_{21},\ \gamma_{22})$ are estimated. We assume that the parameters θ and ϕ of the Markov Switching processes are known and are set to their estimated value.

As argued in Dai, Singleton and Yang (2006), because the state of the economy is unknown at a particular point in time, we define the theoretical zero-coupon bond price as the expected bond price, with the expectation computed over the possible states of the Markov chain. Using $\widehat{\theta}$ and $\widehat{\phi}$, the estimated parameters for the Markov chain, the zero-coupon risk-free bond price at time t is defined as

$$\overline{P}\left(t,n,\beta,\gamma\right)=\sum_{k=1}^{4}\hat{\xi}_{t|t}(k)\times P\left(t,n,s_t(k)\right)$$

where $\hat{\xi}_{t|t}(k)$ is the estimated ex-ante probability of being in one of the four possible states at time t, $s_t(k)$ denotes the kth possible value of s_t and $P(\bullet)$ is the risk-free zero-coupon bond price computed with equation (4). Notice that this price is a function of the estimated Markov switching model parameters $\widehat{\theta}$ and $\widehat{\phi}$ and the preference parameters. The estimates of the preference parameters are obtained by minimizing the following objective function with respect to β and γ :

$$Q\left(t,\beta,\gamma\right)=\sum_{t}\sum_{n=1}^{40}\left(-\frac{\ln\overline{P}\left(t,n,\beta,\gamma\right)}{n/4}-y_g\left(t,n\right)\right)^2. \tag{14}$$

with the constraints that $\beta_{i,j}>0$ and where $y_g\left(t,n\right)$ is the yield to maturity of a zero-coupon government bond estimated with the Nelson and Siegel (1987). We use maturities up to ten years.

This calibration procedure obtains estimates of $\widehat{\gamma}=\{0.6409,\ 0.1342,\ 0.1413,\ 6.1032\}$ and $\widehat{\beta}=\{0.9890,\ 1.0000,\ 0.9964,\ 1.0000\}$. To study how good the model fits the data, we report the root mean squared error (*rmse*), the average absolute error (*aae*) and the average error (*ae*) in Table 4. The average errors are large and in many cases larger than the observed spread itself. The top graph in Figure 5 illustrates the evolution of the observed and fitted 10 years to maturity yield from which we can visualize the large errors. A detailed examination of the fitted and observed risk-free yield curves shows that in many cases, the slope and curvature do not agree.

Because these large errors in our fitted risk-free yield might affect the quality of the computed spread values, we use an alternative calibration procedure which fits different values of β and γ through time. At each quarter of our sample, we estimate a set of preference parameters by minimizing the following objective function with respect to β and γ :

$$\widetilde{Q}\left(t,\beta,\gamma\right)=\sum_{n=1}^{40}\left(-\frac{\ln\overline{P}\left(t,n,\beta,\gamma\right)}{n/4}-y_g\left(t,n\right)\right)^2 \tag{15}$$

This procedure allows us to obtain a calibrated model which can accurately replicate the level, slope and curvature of the risk-free term structures at each time point of our sample. The preference parameters estimated with this calibration procedure are presented in Figure 6 and 7. The average estimates of the γ's are 1.2894, 2.2819, 1.7819, and 1.2569 while they are 0.9950, 0.9917, 0.9910, and 0.9983 for the β's. Table 5 reports the fit of this calibration procedure which is, as expected, more accurate than with the earlier procedure with small root mean squared errors. The bottom graph in Figure 5 illustrates the evolution of the observed and fitted 10 years to maturity yield from which are nearly identical. A detailed examination of the results shows that for each period the level slope and curvature of our risk free term structures are fitted very closely.

3.5 Conditional default probability parameters

We describe here the calibration procedure for the conditional default probability parameters $\alpha = (\alpha_{1,1},\ \alpha_{1,2},\ \alpha_{2,1},\ \alpha_{2,2},\ \alpha^c_{1,1},\ \alpha^c_{1,2},\ \alpha^c_{2,1},\ \alpha^c_{2,2},\ \alpha^\pi_{1,1},\ \alpha^\pi_{1,2},\ \alpha^\pi_{2,1},\ \alpha^\pi_{2,2})$ required by our corporate bond pricing and credit spread model.

In a first step we estimate a term structure of empirical survival probabilities via credit rating transition matrices. These rating transitions are estimated from the generator of the Markov chain underlying the rating migration, as in Lando and Skodeberg (2002) and Christensen, Hansen and Lando (2004). These studies suggest estimating a Markov-process generator rather than a one-year transition matrix. Lando and Skodeberg (2002) have shown that this continuous-time analysis of rating transitions using generator matrices improves the estimates of rare transitions even when they are not observed in the data, a result that cannot be obtained with the discrete-time analysis of Carty and Fons (1993) and Carty (1997). A continuous-time analysis of defaults permits estimates of default probabilities even for cells that have no defaults. The rating transition histories used to estimate the generator are taken from the January, 09, 2002 version of Moody's Corporate Bond Default Database. A precise description of the data used to obtain the transition estimates is given in Appendix E. Using default data from 1987 to 1996, a generator matrix $\mathbf{G}$ is estimated. The estimated generator matrix is presented in Table 6. With this generator, the transition matrix for a horizon of t periods and the corresponding default probability can be obtained by computing

$$\exp\left(\frac{t}{4}\mathbf{G}\right) = \sum_{i=0}^{\infty} \frac{\left(\frac{t}{4}\mathbf{G}\right)^i}{i!}.$$

The term structure of empirical survival probabilities is then extracted from these computed matrices obtained with $t = 1$ to 40.

The estimate for α is got by minimizing the squared errors between our theoretical and the empirical survival probabilities. Again, as in the case of the theoretical risk-free bond prices, we define the survival probability as the expected survival probability, with the expectation taken over the regime of the Markov chain. More formally, the expected survival probability is defined as :

$$\overline{q}(t,n,\alpha)=\sum_{k=1}^{4}\hat{\xi}_{t|t}(k)\times q(t,n,s_t(k))$$

where $\hat{\xi}_{t|t}(k)$ is the estimated ex-ante probability of being in one of the four possible state at time t , $s_t(k)$ denotes the kth possible value of s_t and $q(\bullet)$ is the survival probability computed with equation (12). The sum of squared error function is then defined as:

$$R(\alpha)=\frac{1}{40}\sum_{t}\sum_{n=1}^{40}\left(q^{\mathrm{emp}}\left[\tau>t+n\mid\tau>t\right]-\overline{q}(t,n,\alpha)\right)^2$$

where $q^{\mathrm{emp}}\left[\tau>t+n\mid\tau>t\right]$ is the empirical survival probability obtained from the generator with $t\in\{4,8,...,36,40\}$. The minimization of the above function is done numerically under the constraint that the one-period conditional default probability is non-negative. Figure 8 shows the empirical and fitted term structure of default probabilities. The estimated parameters of the conditional default probability are shown in Table 7. We can observe that most parameters linking the consumption growth are negative at the exception of the one for the low consumption volatility and low inflation state while inflation is loading positively for all states. The magnitude of the coefficients of the state of high consumption volatility and high inflation is large when compared with the other parameters. This indicates that the probability will be sensitive to our factor in this state.

Figure 9 reproduces the estimated one period conditional default probability computed as $1-\overline{q}(t,1,\widehat{\alpha})$ along with the consumption growth and inflation. It is interesting to note that the conditional default probability jumps during the high volatility of consumption and high level and high volatility of inflation. This period is within the brief economic recession which occurred in our sample as specified by the NBER (1990-III to 1991-I). Table 8 reports the correlation between consumption and inflation as well as their correlations with the estimated default probabilities. The estimated probabilities are negatively correlated with the real consumption growth rate with an estimated correlation of -0.5271 over the 1987-1996 period. The sign of the correlation is also constant across all states except for the state of low consumption volatility and low inflation. In this state, we can observe that the link between consumption and inflation has changed as they are positively correlated. Hence, apart for this subperiod, when real consumption increases the default probability is expected to decrease. This can be explained by the positive relation between the firms cash flows and the consumption level. The conditional default probability is positively correlated with the inflation rate (0.5174 over the period 1987-1996). The sign of the correlations are also constant across the different states.

4 Default Spread in Baa Corporate Yield Spread

Having calibrated the model to consumption, inflation, risk-free yields and default data, we examine in this section the properties of the default spreads generated by our approach. It is important to notice that our default spread estimates are entirely computed out of sample i.e. without using yield spreads or corporate yields. The recovery rate ρ is assumed constant through time and fixed at the average recovery rate during the 1987-1996 period, that is 36.67% for industrials Baa bonds as documented by Moody's in 2005.

Figure 10 plots a two scale graph showing the evolution of the default spread for five and ten years to maturity Baa zero-coupon bonds in conjunction with the observed yield spread. As shown in these graphs, the estimated default spreads shows some similarities with the observed yields spread. For example, the sharp increase at the end of 1990 is well captured (out of sample) by the model.

Table 9 presents some statistics about the estimated default spread. This table also reports the statistics across the different states of consumption and inflation. The estimated default spread represents, on average, 36% and 40% of the 5 and 10 years yield spreads. This proportion however varies in the different sub periods. For example, in the low consumption and high inflation state, $s_t = (1, 2)$, which is roughly the first half of the sample, the proportions jump to 48% and 47% and go down to 21% and 29% for the high consumption volatility and low inflation state, $s_t = (2, 1)$. It can also be noticed that the volatility of our theoretical default spreads are small when compared to the yield spread volatility for the whole sample and in all subperiods. Our estimated default spreads are positively related to the yields spreads with correlations of 0.33 and 0.46 for the 5 and

10 years to maturity cases. Across the different regimes, these correlations are typically positive and strong around 0.5 except for the low consumption and high inflation state $s_t = (1, 2)$ which shows a small and negative correlation. This indicates that an increase in default spread is not typically linked to an increase in the overal spread during this state.

The correlation of the default spreads with consumption growth is overall negative and low around 0.2. When conditioning on the possible states, we observe that this link with consumption is not constant across the different regimes. In most regimes the correlation remains negative at the exception of the low volatility of consumption and low inflation state $s_t = (1, 1)$ which shows positive correlations of 0.6 and 0.5 for the 5 and 10 year cases.

The correlation of the default spread with inflation is overall positive and strong around 0.8. Conditioning on the different states reveals that, again, the sign of the correlations can change from high and positive in the low volatility of consumption and high inflation state $s_t = (1, 2)$ to high and negative in the high consumption volatility and low inflation state $s_t = (2, 1)$. The table also reports the correlations of changes in risk-free yield with changes in yield spread as well as changes in default spreads. The signs of these correlations generally agree in the different subperiods for the 5 and 10 year cases except for $s_t = (2, 1)$ in the five year case.

The portion of the default spread associated with the default premium is estimated to be of small magnitude indicating that our estimated default spread are mostly caused by the actuarial risk of default and that risk aversion plays a minor role. As shown by the expression for the one period yield spread, risk aversion impacts on the spreads through the squared volatilities and covariance and is a second order effect brought by the convexity of our pricing kernel and approximate recovery factor. A possible explanation for the low default premia spread obtained here is thus the low volatility of consumption growth and inflation. For reasonable risk aversion parameters, these low volatilities have difficulties to produce high default premia.

Finally, Table 10 presents the credit spreads and proportions computed with the constant set of preference parameters estimated in section 3.4. As shown in this table, the results presented in Table 9 regarding the computed spreads are robust to this alternative calibration procedure.

5 Conclusion

We proposed here an approach for estimating the default spread component of corporate yield spreads. Our model uses observed macroeconomic factors in a reduced form framework and is built on the objective measure. We use a pricing kernel function of discrete regime shifts in consumption growth and inflation. The parameters of consumption, inflation and conditional default probability variations over time are also functions of the discrete regime shifts. Using consumption, inflation, risk-free yield and default data, the model is calibrated over the 1987-1996 period.

Our results indicate that the proportion of default spreads in yield spreads explained by aggregate consumption growth and inflation varies across the different regimes. This proportion is the greatest during the states of low volatility of consumption growth and high and volatile inflation. This proportion ranges from 29% in states of high consumption growth volatility and low inflation to 47% in the states of low consumption growth volatility and high inflation for the case of 10 years Baa corporate zero-coupon bonds. We also find that the correlation between the default spread and inflation is positive and strong during the states of high inflation but negative during the states of low inflation. Consumption growth has a negative impact on the spreads in general but its effect can become positive during the periods of low consumption growth volatility and low inflation. Finally, we find that a large fraction of the estimated default spread is explained by the default risk while a small fraction is due to the default risk premium. This finding is explained by the low volatility of the consumption growth and inflation which are the main drivers of the default risk premium in this model.

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Appendices

A Risk-free zero-coupon bond price analytical approximation

In this section, we derive functions A^p_{n,s_t}, $B^{p;c}_{n,s_t}$, and $B^{p;\pi}_{n,s_t}$ appearing in the analytical approximation formula of the zero-coupon risk-free bond price $P(t,n,s_t)$. Note that for the derivation in this appendix, for notational convenience, we drop the p superscript from the A and B functions. The derivation is based on two approximations: (i) the true time t value of the zero-coupon bond given the actual states of consumption and inflation, $\widetilde{P}(t,n)$, is well approximated by an exponential function (instead of a sum of exponential functions) and (ii) the function $\exp(x)$ may be replaced by its Taylor expansion around zero truncated after the second term, that is, $\exp(x) \cong 1+x$. Starting from equation (2), we have

$$1 = \mathrm{E}_t\left[M_{t,t+1}\frac{\widetilde{P}(t+1,n,s_{t+1})}{\widetilde{P}(t,n,s_t)}\right] \cong \mathrm{E}_t\left[M_{t,t+1}\frac{P(t+1,n,s_{t+1})}{P(t,n,s_t)}\right].$$

Substituting $M_{t,t+1}$, $P(t,n,s_t)$ and $P(t+1,n-1,s_{t+1})$ using equations (3) and (4), rewriting s_{t+1} as $\left\{s^c_{t+1},s^\pi_{t+1}\right\}$, applying embedded conditional expectation rule $\mathrm{E}_t[\bullet] = \mathrm{E}_t[\mathrm{E}_t[\bullet|\, s_{t+1}]]$, and using the fact that for a Gaussian random variable z, $\mathrm{E}[\exp z] = \exp\left(\mathrm{E}[z] + \frac{1}{2}\mathrm{Var}[z]\right)$, we get

$$\sum_{i=1}^{2}\sum_{j=1}^{2}\phi^c_{s^c_t,i}\phi^\pi_{s^\pi_t,j}\exp\left(\begin{array}{c} \ln\beta_{i,j} + A_{n-1,i,j} - A_{n,s^c_t,s^\pi_t} + B^c_{n,s^c_t,s^\pi_t}c_t + B^\pi_{n,s^c_t,s^\pi_t}\pi_t \\ -\left(B^c_{n-1,i,j}+\gamma_{i,j}\right)\left(a^c_i + b^c_i c_t\right) - \left(B^\pi_{n-1,i,j}+1\right)\left(a^\pi_j + b^\pi_j\pi_t\right) \\ +\frac{1}{2}\left(B^c_{n-1,i,j}+\gamma_{i,j}\right)^2\left(\sigma^c_i\right)^2 + \frac{1}{2}\left(B^\pi_{n-1,i,j}+1\right)^2\left(\sigma^\pi_j\right)^2 \\ +\left(B^c_{n-1,i,j}+\gamma_{i,j}\right)\left(B^\pi_{n-1,i,j}+1\right)\rho_{i,j}\sigma^c_i\sigma^\pi_j \end{array}\right) \cong 1.$$

Because $\exp(x) \cong 1+x$ for x in the neighborhood of zero and since $\sum_{i=1}^{2}\sum_{j=1}^{2}\phi^c_{s^c_t,i}\phi^\pi_{s^\pi_t,j} = 1$,

$$\begin{aligned} 0 \cong\; & -A_{n,s^c_t,s^\pi_t} + \sum_{i=1}^{2}\sum_{j=1}^{2}\phi^c_{s^c_t,i}\phi^\pi_{s^\pi_t,j}G^A_{n-1,i,j} \\ & + \left(B^c_{n,s^c_t,s^\pi_t} - \sum_{i=1}^{2}\sum_{j=1}^{2}\phi^c_{s^c_t,i}\phi^\pi_{s^\pi_t,j}b^c_i\left(B^c_{n-1,i,j}+\gamma_{i,j}\right)\right)c_t \\ & + \left(B^\pi_{n,s^c_t,s^\pi_t} - \sum_{i=1}^{2}\sum_{j=1}^{2}\phi^c_{s^c_t,i}\phi^\pi_{s^\pi_t,j}b^\pi_j\left(B^\pi_{n-1,i,j}+1\right)\right)\pi_t \end{aligned}$$

where

$$\begin{aligned} G^A_{n-1,i,j} =\; & A_{n-1,i,j} + \ln\beta_{i,j} - a^c_i\left(B^c_{n-1,i,j}+\gamma_{i,j}\right) - a^\pi_j\left(B^\pi_{n-1,i,j}+1\right) \\ & + \frac{1}{2}\left(B^c_{n-1,i,j}+\gamma_{i,j}\right)^2\left(\sigma^c_i\right)^2 + \frac{1}{2}\left(B^\pi_{n-1,i,j}+1\right)^2\left(\sigma^\pi_j\right)^2 \\ & + \left(B^c_{n-1,i,j}+\gamma_{i,j}\right)\left(B^\pi_{n-1,i,j}+1\right)\rho_{i,j}\sigma^c_i\sigma^\pi_j. \end{aligned}$$

Since this relation must be true for any c_t and π_t, we set the coefficients in front of c_t and π_t and the remaining term equal to zero to obtain:

$$\begin{aligned}
A_{n,s_t^c,s_t^\pi} &= \sum_{i=1}^{2}\sum_{j=1}^{2}\phi^c_{s_t^c,i}\phi^\pi_{s_t^\pi,j}G^A_{n-1,i,j}, \\
B^c_{n,s_t^c,s_t^\pi} &= \sum_{i=1}^{2}\sum_{j=1}^{2}\phi^c_{s_t^c,i}\phi^\pi_{s_t^\pi,j}b^c_i\left(B^c_{n-1,i,j}+\gamma_{i,j}\right), \\
B^\pi_{n,s_t^c,s_t^\pi} &= \sum_{i=1}^{2}\sum_{j=1}^{2}\phi^c_{s_t^c,i}\phi^\pi_{s_t^\pi,j}b^\pi_j\left(B^\pi_{n-1,i,j}+1\right).
\end{aligned}$$

B Risky zero-coupon bond price analytical approximation

In this section, we derive the functions A^v_{n,s_t}, $B^{v,c}_{n,s_t}$, and $B^{v,\pi}_{n,s_t}$ appearing in the analytical approximation formula of the zero-coupon risky bond price $V(t,T,s_t)$. Note that for the derivation in this appendix, for notational convenience, we drop the v superscript from the A and B functions. The derivations are based on our assumption of affine default probability in consumption growth and inflation and three approximations: (i) the time t value of the zero-coupon bond given the actual states of consumption and inflation, $\widetilde{V}(t,T,s_t)$, is well approximated by an exponential function and (ii) the function $\exp(x)$ may be replaced by its Taylor expansion around zero truncated after the second term, that is, $\exp(x) \cong 1+x$; (iii) $\exp(-Lh_{t+1}) \simeq 1-Lh_{t+1}$. Starting from equations (6) and (7) we obtain:

$$1 \cong \mathrm{E}_t\left[\exp\begin{pmatrix} \left(\ln\beta_{s_{t+1}} - L\alpha_{s_{t+1}}\right) \\ -\left(\gamma_{s_{t+1}} + L\alpha^c_{s_{t+1}}\right)c_{t+1} \\ -\left(1+L\alpha^\pi_{s_{t+1}}\right)\pi_{t+1}\end{pmatrix}\frac{V(t+1,n-1,s_{t+1})}{V(t,n,s_t)}\right].$$

Substituting $V(t,n,s_t)$ and $V(t+1,n-1,s_{t+1})$ using equation (8), we use the same solution techniques as in Appendix A to obtain

$$\begin{aligned}
A_{n,s_t^c,s_t^\pi} &= \sum_{i=1}^{2}\sum_{j=1}^{2}\phi^c_{s_t^c,i}\phi^\pi_{s_t^\pi,j}G^A_{n-1,i,j}, \\
B^c_{n,s_t^c,s_t^\pi} &= \sum_{i=1}^{2}\sum_{j=1}^{2}\phi^c_{s_t^c,i}\phi^\pi_{s_t^\pi,j}b^c_i\left(L\alpha^c_{i,j}+B^c_{n-1,i,j}+\gamma_{i,j}\right), \\
B^\pi_{n,s_t^c,s_t^\pi} &= \sum_{i=1}^{2}\sum_{j=1}^{2}\phi^c_{s_t^c,i}\phi^\pi_{s_t^\pi,j}b^\pi_j\left(L\alpha^\pi_{i,j}+B^\pi_{n-1,i,j}+1\right),
\end{aligned}$$

with

$$\begin{aligned}
G^A_{n-1,i,j} = {} & A_{n-1,i,j} + \ln\beta_{i,j} - L\alpha_{i,j} - a^c_i\left(L\alpha^c_{i,j}+B^c_{n-1,i,j}+\gamma_{i,j}\right) - a^\pi_j\left(L\alpha^\pi_{i,j}+B^\pi_{n-1,i,j}+1\right) \\
& + \frac{1}{2}\left(L\alpha^c_{i,j}+B^c_{n-1,i,j}+\gamma_{i,j}\right)^2\left(\sigma^c_i\right)^2 + \frac{1}{2}\left(L\alpha^\pi_{i,j}+B^\pi_{n-1,i,j}+1\right)^2\left(\sigma^\pi_j\right)^2 \\
& + \left(L\alpha^c_{i,j}+B^c_{n-1,i,j}+\gamma_{i,j}\right)\left(L\alpha^\pi_{i,j}+B^\pi_{n-1,i,j}+1\right)\rho_{i,j}\sigma^c_i\sigma^\pi_j.
\end{aligned}$$

C Survival probability analytical approximation

We assume that $\Pr_{\mathcal{G}_t}[\tau > t+n \mid \tau > t] \cong q(t,n,s_t)$ where

$$q(t,n,s_t) = \exp\left(-A^q_{n,s_t} - B^{q,c}_{n,s_t}c_t - B^{q,\pi}_{n,s_t}\pi_t\right).$$

The coefficients A^q_{n,s_t}, $B^{q,c}_{n,s_t}$ and $B^{q,\pi}_{n,s_t}$ are obtained recursively starting with $A^q_{0,s_T} = B^{q,c}_{0,s_T} = B^{q,\pi}_{0,s_T} = 0$. Indeed, since

$$\begin{aligned}
\Pr_{\mathcal{G}_t}[\tau > t+n \mid \tau > t] &\cong \mathrm{E}_t\left[\exp\left(-h_{t+1}\right)\mathrm{E}_t\left[\exp\left(-\sum_{u=t+2}^{t+n} h_u\right)\right]\right] \\
&\cong \mathrm{E}_t\left[\exp\left(-h_{t+1}\right)\Pr_{\mathcal{G}_{t+1}}[\tau > t+n \mid \tau > t+1]\right]
\end{aligned}$$

where $\sum_{u=t+2}^{t+1} h_u$ is set to zero whenever it happens, then

$$\begin{aligned}
1 &\cong \mathrm{E}_t\left[\exp\left(-h_{t+1}\right)\frac{\Pr_{\mathcal{G}_{t+1}}[\tau > t+n \mid \tau > t+1]}{\Pr_{\mathcal{G}_t}[\tau > t+n \mid \tau > t]}\right] \\
&\cong \mathrm{E}_t\left[\exp\left(-h_{t+1}\right)\frac{q\left(t+1, n-1, s_{t+1}\right)}{q\left(t, n, s_t\right)}\right] \\
&\cong \mathrm{E}_t\left[\exp\left(\begin{array}{c} -\alpha_{s_{t+1}} - A^q_{n-1,s_{t+1}} + A^q_{n,s_t} \\ +B^{q,c}_{n,s_t} c_t - \left(\alpha^c_{s_{t+1}} + B^{q,c}_{n-1,s_{t+1}}\right) c_{t+1} \\ +B^{q,\pi}_{n,s_t} \pi_t - \left(\alpha^\pi_{s_{t+1}} + B^{q,\pi}_{n-1,s_{t+1}}\right) \pi_{t+1} \end{array}\right)\right]
\end{aligned}$$

where the last line is obtained by replacing h_{t+1} by the approximation (7). Using the same solution technique as in Appendix A we get

$$\begin{aligned}
A^q_{n,s^c_t,s^\pi_t} &= \sum_{i=1}^{2}\sum_{j=1}^{2} \phi^c_{s^c_t,i}\phi^\pi_{s^\pi_t,j} G^A_{n-1,i,j}, \\
B^{q,c}_{n,s^c_t,s^\pi_t} &= \sum_{i=1}^{2}\sum_{j=1}^{2} \phi^c_{s^c_t,i}\phi^\pi_{s^\pi_t,j} b^c_i \left(\alpha^c_{i,j} + B^{q,c}_{n-1,i,j}\right), \\
B^{q,\pi}_{n,s^c_t,s^\pi_t} &= \sum_{i=1}^{2}\sum_{j=1}^{2} \phi^c_{s^c_t,i}\phi^\pi_{s^\pi_t,j} b^\pi_j \left(\alpha^\pi_{i,j} + B^{q,\pi}_{n-1,i,j}\right),
\end{aligned}$$

with

$$\begin{aligned}
G^A_{n-1,i,j} = {} & A^q_{n-1,i,j} + \alpha_{i,j} + a^c_i\left(\alpha^c_{i,j} + B^{q,c}_{n-1,i,j}\right) + a^\pi_j\left(\alpha^\pi_{i,j} + B^{q,\pi}_{n-1,i,j}\right) \\
& - \frac{1}{2}\left(\alpha^c_{i,j} + B^{q,c}_{n-1,i,j}\right)\left(\sigma^c_i\right)^2 - \frac{1}{2}\left(\alpha^\pi_{i,j} + B^{q,\pi}_{n-1,i,j}\right)\left(\sigma^\pi_j\right)^2 \\
& - \left(\alpha^c_{i,j} + B^{q,c}_{n-1,i,j}\right)\left(\alpha^\pi_{i,j} + B^{q,\pi}_{n-1,i,j}\right)\rho_{i,j}\sigma^c_i\sigma^\pi_j.
\end{aligned}$$

D Log-likelihood function of the Markov Switching model

The terms of the log likelihood function are computed as follows. Let the conditional likelihood be rewritten using Bayes rule as:

$$
\begin{aligned}
f\left(x_t \mid v_{t-1};\theta,\phi\right) &= \frac{f\left(x_t, v_{t-1};\theta,\phi\right)}{f\left(v_{t-1};\theta,\phi\right)} \\
&= \frac{\sum_{s_t} f\left(x_t, s_t, v_{t-1};\theta,\phi\right)}{f\left(v_{t-1};\theta,\phi\right)} \\
&= \sum_{s_t} f\left(x_t \mid s_t, v_{t-1};\theta\right) \times f\left(s_t \mid v_{t-1};\theta,\phi\right) \\
&= \eta_t' \times \xi_{t|t-1}.
\end{aligned}
$$

where the sum over s_t is performed for values of $s_t \in \{(1,1),(1,2),(2,1),(2,2)\}$ and

$$
\eta_t = \begin{pmatrix} f\left(x_t \mid (1,1), v_{t-1};\theta\right) \\ f\left(x_t \mid (1,2), v_{t-1};\theta\right) \\ f\left(x_t \mid (2,1), v_{t-1};\theta\right) \\ f\left(x_t \mid (2,2), v_{t-1};\theta\right) \end{pmatrix} \quad \text{and} \quad \xi_{t|t-1} = \begin{pmatrix} f\left((1,1) \mid v_{t-1};\theta,\phi\right) \\ f\left((1,2) \mid v_{t-1};\theta,\phi\right) \\ f\left((2,1) \mid v_{t-1};\theta,\phi\right) \\ f\left((1,2) \mid v_{t-1};\theta,\phi\right) \end{pmatrix}.
$$

The components of η_t are computed analytically using the bivariate Gaussian density function. Indeed, from the Markov Switching model, the likelihood function of $x_t = (c_t, \pi_t)$ given $x_{t-1} = (c_{t-1}, \pi_{t-1})$ and the actual states of consumption and inflation $s_t = (s_t^c, s_t^\pi)$ is

$$
f\left(x_t \mid s_t, v_{t-1};\theta\right) = \frac{1}{2\pi} \frac{\exp\left(-\frac{1}{2}\left(z_t^c\right)^2 - \frac{1}{2}\left(z_t^\pi\right)^2 + \rho_{s_t^c, s_t^\pi} z_t^c z_t^\pi\right)}{\sigma_{s_t^c}^c \sigma_{s_t^\pi}^\pi \sqrt{1-\rho_{s_t^c, s_t^\pi}^2}}
$$

with

$$
z_t^c = \frac{c_t - a_{s_t^c}^c - b_{s_t^c}^c c_{t-1}}{\sigma_{s_t^c}^c \sqrt{1-\rho_{s_t^c, s_t^\pi}^2}}
$$

and

$$
z_t^\pi = \frac{\pi_t - a_{s_t^\pi}^\pi - b_{s_t^\pi}^\pi \pi_{t-1}}{\sigma_{s_t^\pi}^\pi \sqrt{1-\rho_{s_t^c, s_t^\pi}^2}}.
$$

The vectors $\xi_{2|1}, ..., \xi_{T|T-1}$ are obtained recursively using

$$
\xi_{t|t-1} = \Phi' \times \frac{\eta_{t-1}' \,(\times)\, \xi_{t-1|t-2}}{\eta_{t-1}' \, \xi_{t-1|t-2}}
$$

where $(\times)$ denotes element-by-element multiplication and Φ is the transition matrix

$$
\Phi = \begin{pmatrix}
\phi_{1,1}^c \phi_{1,1}^\pi & \phi_{1,1}^c \phi_{1,2}^\pi & \phi_{1,2}^c \phi_{1,1}^\pi & \phi_{1,2}^c \phi_{1,2}^\pi \\
\phi_{1,1}^c \phi_{2,1}^\pi & \phi_{1,1}^c \phi_{2,2}^\pi & \phi_{1,2}^c \phi_{2,1}^\pi & \phi_{1,2}^c \phi_{2,2}^\pi \\
\phi_{2,1}^c \phi_{1,1}^\pi & \phi_{2,1}^c \phi_{1,2}^\pi & \phi_{2,2}^c \phi_{1,1}^\pi & \phi_{2,2}^c \phi_{1,2}^\pi \\
\phi_{2,1}^c \phi_{2,1}^\pi & \phi_{2,1}^c \phi_{2,2}^\pi & \phi_{2,2}^c \phi_{2,1}^\pi & \phi_{2,2}^c \phi_{2,2}^\pi
\end{pmatrix}.
$$

To initialize the recursion, we let $\eta_1 = (1,1,1,1)'$ and $\xi_{1|0}$ is set to the stationary distribution of the Markov chain associated with Φ. Using the independence between the evolution of the state of consumption and the state of inflation, the stationary distribution is obtained as the product of the stationary distribution of s^c and s^π:

$$\xi_{1|0} = \begin{pmatrix} \frac{1-\phi^c_{2,2}}{1-\phi^c_{1,1}+1-\phi^c_{2,2}} \times \frac{1-\phi^\pi_{2,2}}{1-\phi^\pi_{1,1}+1-\phi^\pi_{2,2}} \\ \frac{1-\phi^c_{2,2}}{1-\phi^c_{1,1}+1-\phi^c_{2,2}} \times \frac{1-\phi^\pi_{1,1}}{1-\phi^\pi_{1,1}+1-\phi^\pi_{2,2}} \\ \frac{1-\phi^c_{1,1}}{1-\phi^c_{1,1}+1-\phi^c_{2,2}} \times \frac{1-\phi^\pi_{2,2}}{1-\phi^\pi_{1,1}+1-\phi^\pi_{2,2}} \\ \frac{1-\phi^c_{1,1}}{1-\phi^c_{1,1}+1-\phi^c_{2,2}} \times \frac{1-\phi^\pi_{1,1}}{1-\phi^\pi_{1,1}+1-\phi^\pi_{2,2}} \end{pmatrix}.$$

E Data for default probability estimation

We considered only issuers domiciled in United States and having at least one senior unsecured estimated rating. We started with 5,719 issuers (in all industry groups) with 46,305 registered debt issues and 23,666 ratings observations. For each issuer we checked the number of default dates in the Master Default Table (Moody's, January, 09, 2002). We obtained 1,041 default dates for 943 issuers in the period 1970-2001. Some issuers (91) had more than one default date. In the rating transition histories, there are 728 withdrawn ratings that are not the last observation of the issuer. Theses irrelevant withdrawals were eliminated and so we obtained 22,938 ratings observations. The most important and difficult task was to get a proper definition of default. In order to compare our results with recent studies, we treated default dates as did Christensen *et al.* (2004). First, all the non withdrawn-rating observations up to the date of default have typically been unchanged. However, the ratings that occur within a week before the default date were eliminated. Rating changes observed after the date of default were eliminated unless the new rating reached the B3 level or higher and the new ratings were related to debt issued after the date of default. In theses cases we treated theses ratings as related to a new issuer. It is important to emphasize that the first rating date of the new issuer is the latest date between the date of the first issue after default and the first date we observe an issuer rating higher than or equal to B3. The same treatment is applied for the case of two and three default dates. Finally, few issuers have a registered default date before the first rating observation in the Senior Unsecured Estimated Rating Table (Moody's, January, 09, 2002). In theses cases, we considered that there was no default. With this procedure we got 5821 issuers with 965 default dates. We aggregated all rating notches and so we got the nine usual ratings Aaa, Aa, A, Baa, Ba, B, Caa-C, Default and NR (Not Rated) with 15,564 rating observations. The final data set corresponds to that without entries and right censoring in Dionne et al. (2005) which is more in line with the standard data set used by Moodys'.

Table 1: Average treasury spot rates and Baa spreads 1987-1996

Maturity (years)	Treasuries (%)	Baa spreads (%)
1	6.187	1.386
2	6.472	1.163
3	6.731	1.191
4	6.941	1.216
5	7.106	1.225
6	7.236	1.224
7	7.338	1.217
8	7.420	1.206
9	7.486	1.192
10	7.540	1.176

This table reports the average treasury yields and corporate yield spreads of industrial Baa for maturities from one to ten years. The treasury and corporate spot rates are computed for the 1987-1996 period using the Nelson and Siegel (1987) approach. Corporate yield spreads are calculated as the difference between the corporate spot rates and treasury spot rates for a given maturity.

Table 2: Parameter estimates for the Markov Switching model

	a_1^c	a_2^c	b_1^c	b_2^c	σ_1^c	σ_2^c
Point estimate	0.00286	0.00277	0.16172	0.32978	0.00360	0.00910
Standard deviation	0.00068	0.00109	0.12809	0.10973	0.00043	0.00090
p-value	0.00%	1.10%	20.68%	0.27%	0.00%	0.00%

	a_1^π	a_2^π	b_1^π	b_2^π	σ_1^π	σ_2^π
Point estimate	0.00397	0.00627	0.06904	0.59119	0.00396	0.00736
Standard deviation	0.00062	0.00152	0.09976	0.08191	0.00035	0.00065
p-value	0.00%	0.00%	48.89%	0.00%	0.00%	0.00%

	$\rho_{1,1}$	$\rho_{1,2}$	$\rho_{2,1}$	$\rho_{2,2}$
Point estimate	-0.13308	-0.37932	-0.12396	-0.58733
Standard deviation	0.23721	0.17541	0.19834	0.11809
p-value	57.48%	3.06%	53.20%	0.00%

	$\phi_{1,1}^c$	$\phi_{2,2}^c$	$\phi_{1,1}^\pi$	$\phi_{2,2}^\pi$
Point estimate	0.87495	0.88528	0.96932	0.95610
Standard deviation	0.06494	0.07384	0.02566	0.03237
p-value	0.00%	0.00%	0.00%	0.00%

This table reports the point estimates and estimated standard deviations for the parameters of the Markov Switching model given by $c_t = a_{s_t^c}^c + b_{s_t^c}^c c_{t-1} + e_t^c$ and $\pi_t = a_{s_t^\pi}^\pi + b_{s_t^\pi}^\pi \pi_{t-1} + e_t^\pi$ with $\phi_{ij}^c = \Pr(s_t^c = i \mid s_{t-1}^c = j)$ and $\phi_{ij}^\pi = \Pr(s_t^\pi = i \mid s_{t-1}^\pi = j)$ and ρ_{ij} the correlation between e_t^c and e_t^π in state i, j. The last line of each panel reports the p-value associated with the test of a zero parameter value. These estimates have been obtained with the growth rate of non-durable personal consumption expenditures per capita (real) from the first quarter of 1957 to the last quarter of 1996 and the growth rate of non-durable consumption price index for the same period (160 quarters).

Table 3: Tests on parameter equality

Consumption		Inflation	
Parameter	p-values	Parameter	p-values
$a_1^c = a_2^c$	94.44%	$a_1^\pi = a_2^\pi$	15.15%
$b_1^c = b_2^c$	35.48%	$b_1^\pi = b_2^\pi$	0.01%
$\sigma_1^c = \sigma_2^c$	0.00%	$\sigma_1^\pi = \sigma_2^\pi$	0.00%
$\rho_{1,1} = \rho_{2,1}$	97.96%	$\rho_{1,1} = \rho_{1,2}$	41.43%
$\rho_{1,2} = \rho_{2,2}$	34.84%	$\rho_{2,1} = \rho_{2,2}$	5.60%

This table reports tests on the equality of estimated parameters of the Markov Switching model given by $c_t = a_{s_t^c}^c + b_{s_t^c}^c c_{t-1} + e_t^c$ and $\pi_t = a_{s_t^\pi}^\pi + b_{s_t^\pi}^\pi \pi_{t-1} + e_t^\pi$ with $\phi_{ij}^c = \Pr(s_t^c = i \mid s_{t-1}^c = j)$ and $\phi_{ij}^\pi = \Pr(s_t^\pi = i \mid s_{t-1}^\pi = j)$ and ρ_{ij} the correlation between e_t^c and e_t^π in state i, j. Column *Parameter* indicates the null hypotheses beeing tested.

Table 4: Fit of the risk-free zero-coupon bond pricing model: constant preference parameters

Maturity ($n/4$)	1	3	5	8	10
rmse (%)	1.195	0.960	0.801	0.755	0.770
aae (%)	0.991	0.768	0.674	0.645	0.662
ae (%)	0.461	0.292	0.047	-0.172	-0.256
avg fitted (%)	6.648	7.023	7.153	7.248	7.285
avg obs. (%)	6.187	6.731	7.106	7.420	7.540

rmse is the root-mean-squared error for a given maturity computed over our 40 quarters sample with $\varepsilon_{t,n} = -\frac{\ln \overline{P}(t,n,\beta,\gamma)}{n/4} - y_g(t,n)$, the difference between our fitted theoretical risk-free yield and the risk-free Nelson Siegel yield to maturity. *aae* is the absolute average error while *ae* is the average error. *avg obs.* and *avg fitted* are, respectively, the average Nelson and Siegel yield and average fitted yield for a given maturity.

Table 5: Fit of the risk-free zero-coupon bond pricing model: time varying preference parameters

Maturity ($n/4$)	1	3	5	8	10
rmse (%)	0.092	0.065	0.042	0.033	0.068
aae (%)	0.053	0.041	0.027	0.019	0.047
ae (%)	0.034	0.024	-0.011	-0.009	0.007
avg fitted (%)	6.221	6.755	7.096	7.411	7.547
avg obs. (%)	6.187	6.731	7.106	7.420	7.540

rmse is the root-mean-squared error for a given maturity computed over our 40 quarters sample with $\varepsilon_{t,n} = -\frac{\ln \overline{P}(t,n,\beta,\gamma)}{n/4} - y_g(t,n)$, the difference between our fitted theoretical risk-free yield and the Nelson Siegel risk-free yield to maturity. *aae* is the absolute average error while *ae* is the average error. *avg obs.* and *avg fitted* are, respectively, the average Nelson and Siegel yield and average fitted yield for a given maturity.

Table 6: Estimated generator for 1987 to 1996

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	-0.0757	0.0729	0.0000	0.0000	0.0028	0.0000	0.0000	0.0000
AA	0.0103	-0.1146	0.1019	0.0008	0.0000	0.0016	0.0000	0.0000
A	0.0000	0.0142	-0.0761	0.0555	0.0047	0.0017	0.0000	0.0000
BBB	0.0004	0.0000	0.0593	-0.1247	0.0553	0.0064	0.0012	0.0020
BB	0.0005	0.0005	0.0031	0.0672	-0.1907	0.1037	0.0010	0.0146
B	0.0009	0.0000	0.0009	0.0027	0.0641	-0.2661	0.0614	0.1362
CCC	0.0000	0.0000	0.0164	0.0000	0.0327	0.0818	-1.1783	1.0474
D	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Estimated generator for the 1987 to 1996 period using the approach described in Lando and Skodeberg (2002) with Moody's Corporate Bond Default Database (January, 09, 2002).

Table 7: Parameter estimates for the conditional survival probabilities

	$s_t = (1,1)$	$s_t = (1,2)$	$s_t = (2,1)$	$s_t = (2,2)$
$\alpha_{i,j}$	0.000078	0.000026	0.000712	0.002405
$\alpha^c_{i,j}$	0.003624	-0.002526	-0.052830	-0.224830
$\alpha^\pi_{i,j}$	0.022580	0.004172	0.066810	0.403282

The table reports the parameter estimates for the conditional probability function $h_t = \alpha_{s_t} + \alpha^c_{s_t} c_t + \alpha^\pi_{s_t} \pi_t$ obtained by minimizing the distance between our model generated theoretical average term structure of survival probabilities and the empirical survival probabilities obtained from a generator matrix got from default data.

Table 8: Correlations of the estimated default probability with consumption growth and inflation.

	All	$s_t = (1,1)$	$s_t = (1,2)$	$s_t = (2,1)$	$s_t = (2,2)$
Nobs	40	15	15	9	1
Corr. c_t and π_t	-0.2593	0.1341	-0.5582	-0.0652	n.a.
Corr. h_t and c_t	-0.5796	0.0833	-0.4927	-0.9513	n.a.
Corr. h_t and π_t	0.5174	0.8463	0.7828	0.3234	n.a.

The table reports the correlation between the estimated conditional probability h_t and consumption growth and inflation. Column All report results for the full sample i.e. 1987:I to 1996:IV. Columns $s_t = (1,1)$, $s_t = (1,2)$, $s_t = (2,1)$ and $s_t = (2,2)$ report the statistics computed over the sample periods for which the given state is prevailing. For $s_t = (2,2)$, because there is only one data point, the correlation is non available (n.a.)

Table 9: Statistics on estimated 5 and 10 years to maturity default Spread: time varying preference parameters

	5 years to maturity					10 years to maturity				
	All	$s_t=(1,1)$	$s_t=(1,2)$	$s_t=(2,1)$	$s_t=(2,2)$	All	$s_t=(1,1)$	$s_t=(1,2)$	$s_t=(2,1)$	$s_t=(2,2)$
Nobs.	40	15	15	9	1	40	15	15	9	1
Average YS_t (%)	1.225	0.951	1.228	1.564	2.219	1.176	0.953	1.221	1.326	2.491
Average DS_t (%)	0.425	0.301	0.575	0.329	0.911	0.449	0.374	0.545	0.386	0.719
Average DS_t prop. (%)	36	33	48	21	41	40	40	47	29	29
Stdv. YS_t (%)	0.335	0.212	0.148	0.192	n.a.	0.321	0.089	0.269	0.147	n.a.
Stdv. DS_t (%)	0.159	0.017	0.081	0.010	n.a.	0.097	0.014	0.050	0.006	n.a.
corr. DS_t and YS_t	0.338	0.638	-0.285	0.649	n.a.	0.467	0.423	-0.155	0.734	n.a.
corr. DS_t and c_t	-0.227	0.611	-0.350	-0.060	n.a.	-0.197	0.519	-0.352	-0.008	n.a.
corr. DS_t and π_t	0.821	-0.017	0.838	-0.816	n.a.	0.820	-0.094	0.843	-0.763	n.a.
corr. ΔYS_t and $\Delta y_{g,t}$	-0.397	-0.025	-0.565	-0.526	n.a.	-0.048	-0.323	0.405	-0.261	n.a.
corr. ΔDS_t and $\Delta y_{g,t}$	-0.003	-0.111	0.205	-0.145	n.a.	0.038	-0.172	0.224	-0.082	n.a.
Average DR_t (%)	0.421	0.295	0.570	0.325	0.910	0.443	0.366	0.539	0.381	0.718
Average DR_t prop. (%)	36	32	47	21	41	39	39	47	29	29
Average DP_t (%)	0.005	0.006	0.005	0.004	0.001	0.006	0.008	0.005	0.005	0.001
Average DP_t prop. (%)	0.414	0.551	0.400	0.249	0.059	0.537	0.791	0.417	0.368	0.047

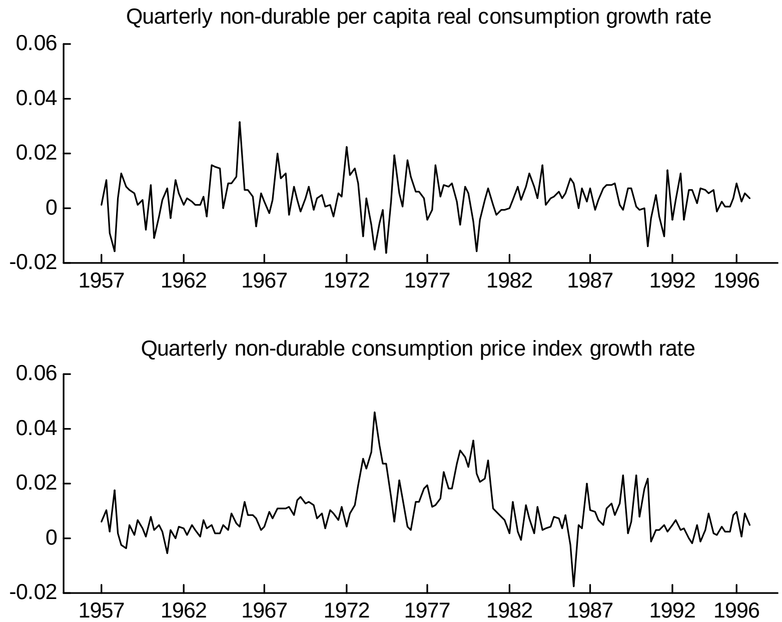
Nobs is the number of observations; YS_t is the yield spread computed as the difference between the industrial Baa and treasury zero-coupon yields estimated with the Nelson and Siegel (1987) approach. DS_t, DR_t and DP_t represent, respectively, the default spread, the defaut risk and the default premia computed with equations (9), (10) and (11) and the calibrated parameters; DR_t *prop* and DP_t *prop* are, respectively, DR_t/YS_t and DP_t/YS_t; *Stdv* are standard deviations and *corr* are correlations; ΔYS_t, ΔDS_t and $\Delta y_{g,t}$ are the first differences in the yield spread, default spread and risk-free zero-coupon yield. Columns All report results for the full sample i.e. 1987:I to 1996:IV. Columns $s_t = (1,1)$, $s_t = (1,2)$, $s_t = (2,1)$ and $s_t = (2,2)$ report the statistics computed over the sample periods for which the given state is prevailing.

Table 10: Statistics on estimated default Spread: constant preference parameters

	5 years to maturity					10 years to maturity				
	All	$s_t = (1,1)$	$s_t = (1,2)$	$s_t = (2,1)$	$s_t = (2,2)$	All	$s_t = (1,1)$	$s_t = (1,2)$	$s_t = (2,1)$	$s_t = (2,2)$
Nobs.	40	15	15	9	1	40	15	15	9	1
Average YS_t (%)	1.225	0.951	1.228	1.564	2.219	1.176	0.953	1.221	1.326	2.491
Average DS_t (%)	0.443	0.309	0.604	0.340	0.962	0.468	0.385	0.571	0.400	0.760
Average DS_t prop. (%)	37	34	50	22	43	41	41	49	31	31
Stdv. YS_t (%)	0.335	0.212	0.148	0.192	n.a.	0.321	0.089	0.269	0.147	n.a.
Stdv. DS_t (%)	0.170	0.014	0.083	0.012	n.a.	0.105	0.008	0.051	0.007	n.a.
corr. DS_t and YS_t	0.333	0.468	-0.303	0.596	n.a.	0.465	0.214	-0.193	0.490	n.a.
corr. DS_t and c_t	-0.227	0.709	-0.337	-0.060	n.a.	-0.199	0.738	-0.339	-0.010	n.a.
corr. DS_t and π_t	0.817	0.142	0.817	-0.827	n.a.	0.818	0.214	0.816	-0.829	n.a.
corr. ΔYS_t and $\Delta y_{g,t}$	-0.397	-0.025	-0.565	-0.526	n.a.	-0.048	-0.323	0.405	-0.261	n.a.
corr. ΔDS_t and $\Delta y_{g,t}$	0.004	0.031	0.220	-0.145	n.a.	0.055	0.080	0.248	-0.079	n.a.
Average DR_t (%)	0.421	0.295	0.570	0.325	0.910	0.443	0.366	0.539	0.381	0.718
Average DR_t prop. (%)	36	32	47	21	41	39	39	47	29	29
Average DP_t (%)	0.022	0.014	0.033	0.015	0.052	0.025	0.019	0.032	0.020	0.042
Average DP_t prop. (%)	1.876	1.493	2.770	0.969	2.349	2.165	2.016	2.745	1.501	1.679

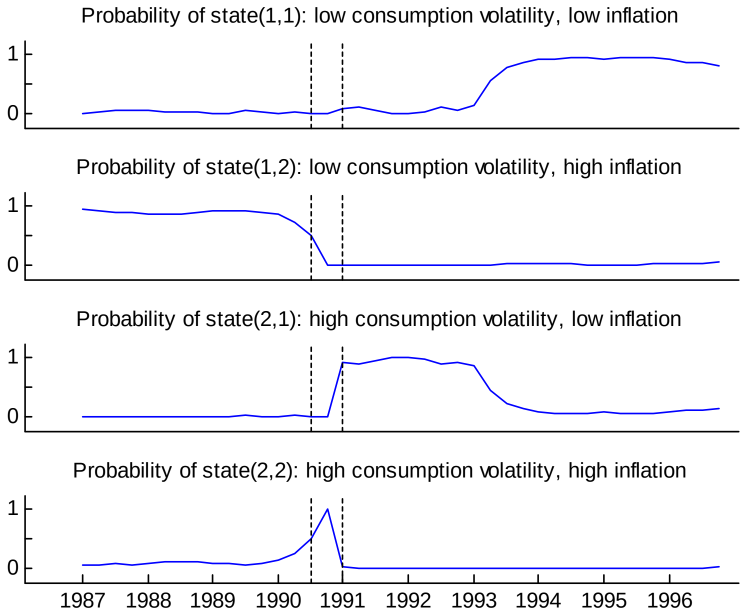
Nobs is the number of observations; YS_t is the yield spread computed as the difference between the industrial Baa and treasury zero-coupon yields estimated with the Nelson and Siegel (1987) approach. DS_t, DR_t and DP_t represent, respectively, the default spread, the defaut risk and the default premia computed with equations (9), (10) and (11) and the calibrated parameters; DR_t *prop* and DP_t *prop* are, respectively, DR_t/YS_t and DP_t/YS_t; *Stdv* are standard deviations and *corr* are correlations; ΔYS_t, ΔDS_t and $\Delta y_{g,t}$ are the first differences in the yield spread, default spread and risk-free zero-coupon yield. Columns All report results for the full sample i.e. 1987:I to 1996:IV. Columns $s_t = (1,1)$, $s_t = (1,2)$, $s_t = (2,1)$ and $s_t = (2,2)$ report the statistics computed over the sample periods for which the given state is prevailing.

Figure 1: Consumption growth and inflation



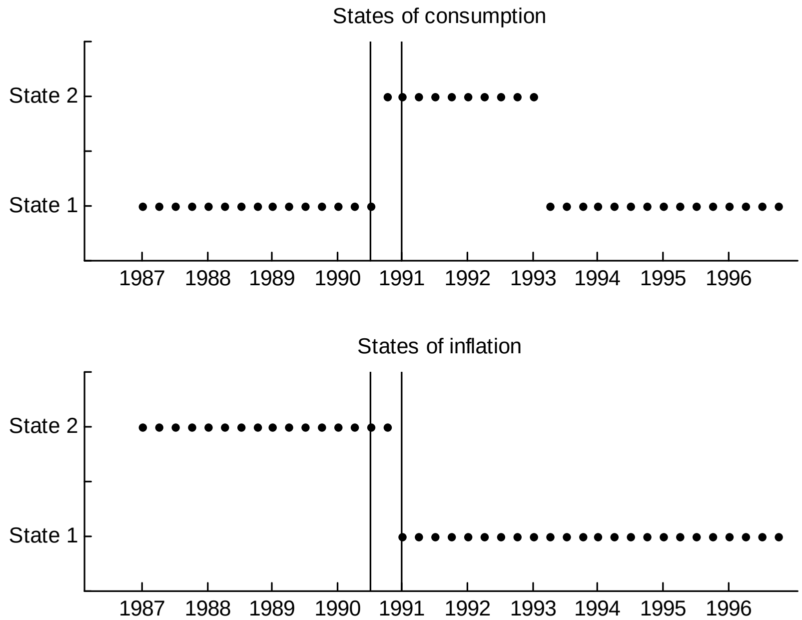
This figure plots the data series of the observed bond pricing factors used for the estimation of the Markov Switching parameters. The data series used here are the growth rate of non-durable personal real consumption expenditures per capita (c_t) from 1957-I to the last quarter of 1996-IV and the growth rate of the consumption price index (π_t) for the same period (160 quarters). Source: U.S. Department of Commerce, Bureau of Economic Analysis.

Figure 2: Smoothed probabilities estimates



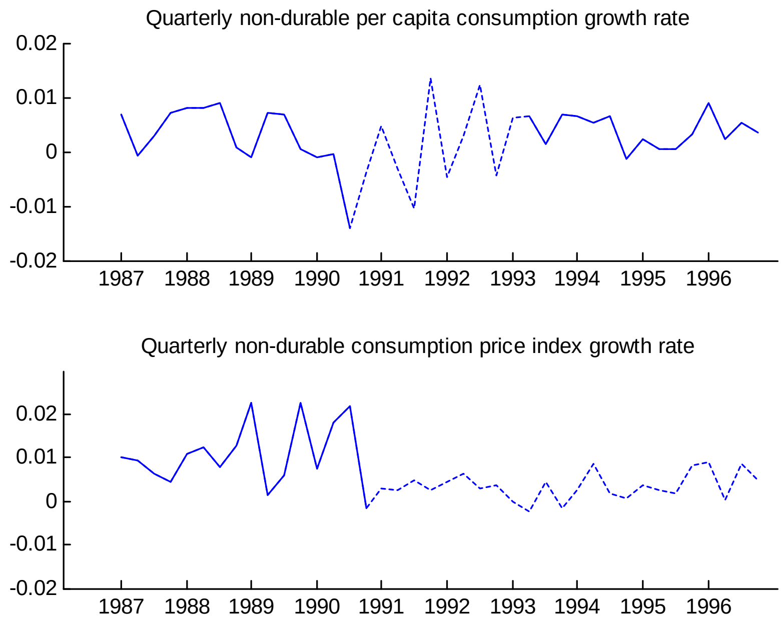
This figure plots the estimated smoothed probabilities $\hat{\xi}_{t|T}$ of being in state $s_t = (i, j)$ for the 1987-I- to 1996-IV period corresponding to the sample period of our corporate bond prices (40 quarters). Vertical lines indicate the official NBER 1990-III to 1991-I recession within our sample period. State (1,1): low consumption volatility growth with low level and volatility of inflation; State (1,2): low volatility of consumption growth and high and volatile inflation; State (2,1): high volatility of consumption growth with low level and low volatility of inflation; State (2,2): high volatility of consumption growth with high and more volatile inflation.

Figure 3: Estimated states of consumption growth and inflation



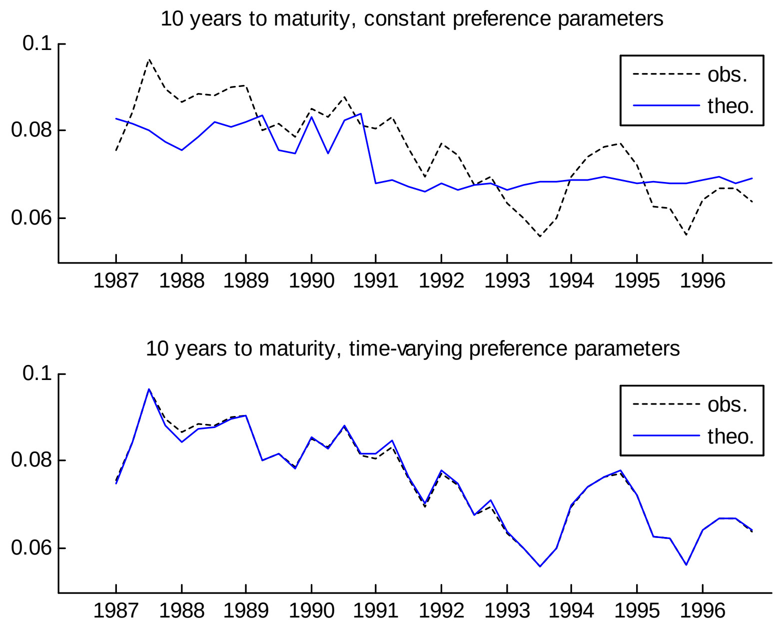
This figure plots the estimated state $\hat{s}_t = (i, j)$ for the 1987-I- to 1996-IV period corresponding to the sample period of our corporate bond prices (40 quarters). Vertical lines indicate the official NBER 1990-III to 1991-I recession within our sample period. State (1,1): low consumption volatility growth with low level and volatility of inflation; State (1,2): low volatility of consumption growth and high and volatile inflation; State (2,1): high volatility of consumption growth with low level and low volatility of inflation; State (2,2): high volatility of consumption growth with high and more volatile inflation.

Figure 4: Consumption growth and inflation - 1987:I to 1996:IV



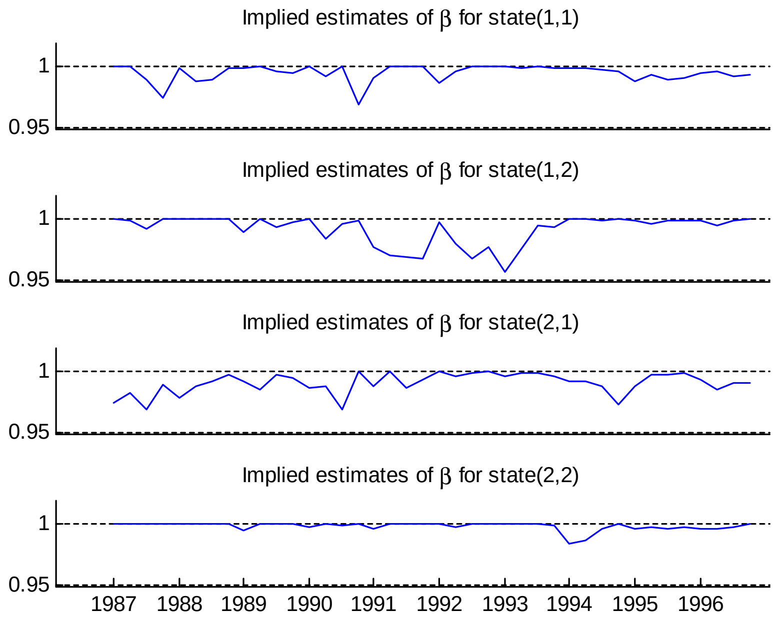
This figure plots the data series of our observed bond pricing factors for the 1987-I- to 1996-IV period corresponding to the sample period of our corporate bond prices (40 quarters). For consumption, the dotted line indicates the periods for which state 2 prevails (high volatility). For inflation, the dotted line indicates state 1 (low level and low volatility.)

Figure 5: Observed and fitted risk-free yield to maturity



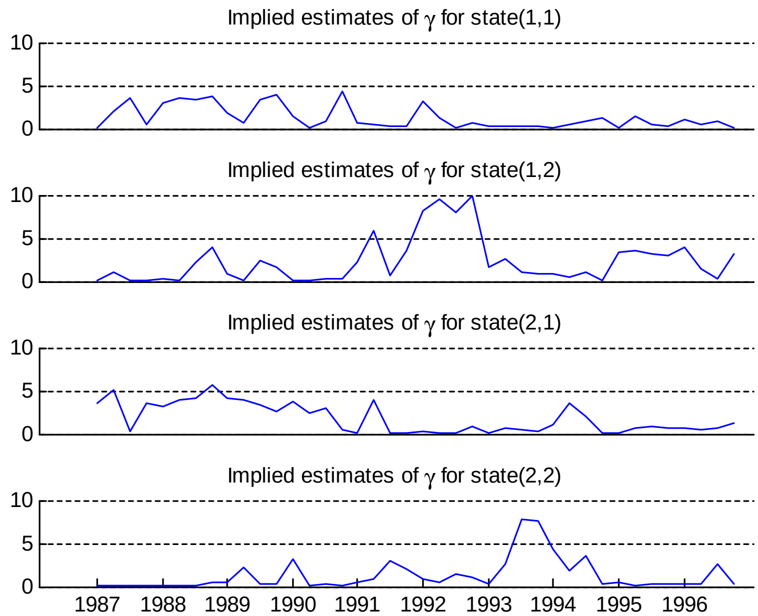
This figure plots the fitted and observed risk-free yield to maturity for the 10 year case. The top graph is obtained by fitting a common set of utility parameters for all quarters while the bottom graph is obtained by fitting a different set of utility parameters each quarter.

Figure 6: Implied estimate of impatience parameters



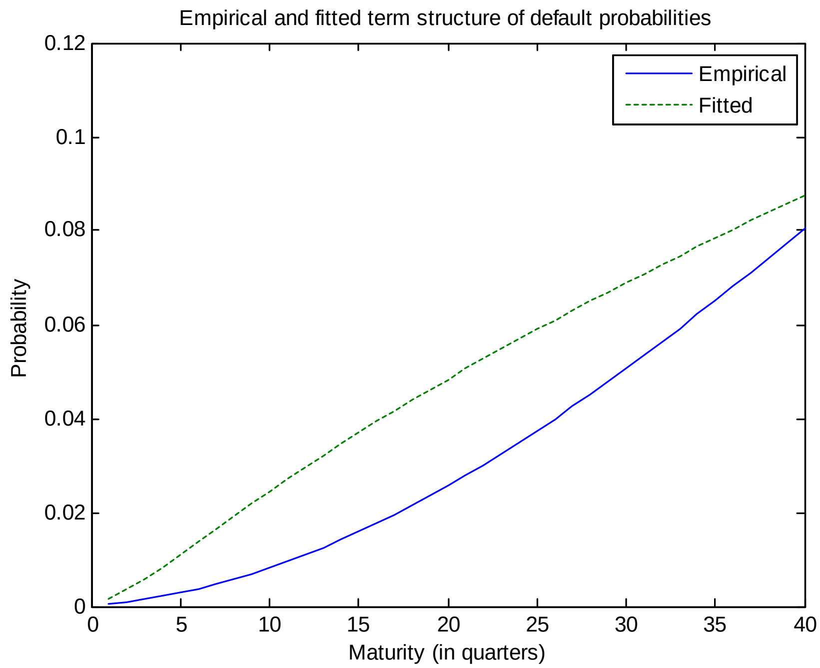
TThis figure plots the implied estimates for the impatience parameter obtained by fitting at each quarter the Nelson-Siegel observed risk-free yield curve of the period with the risk-free bond yield model. State (1,1): low consumption volatility growth with low level and volatility of inflation; State (1,2): low volatility of consumption growth and high and volatile inflation; State (2,1): high volatility of consumption growth with low level and low volatility of inflation; State (2,2): high volatility of consumption growth with high and more volatile inflation.

Figure 7: Implied estimates of risk aversion parameters



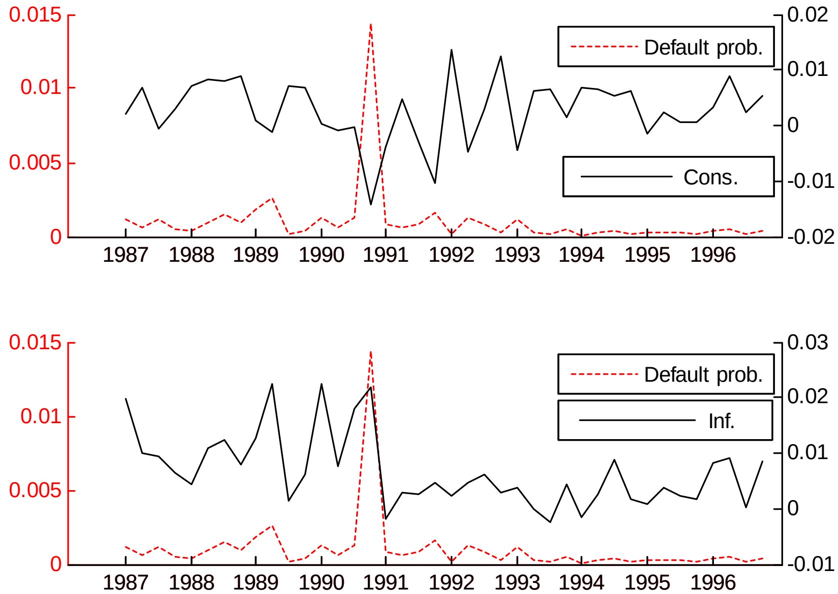
This figure plots the implied estimate for the risk aversion parameter obtained each quarter by fitting the Nelson-Siegel observed risk-free yield curve of the period with the risk-free bond yield model. State (1,1): low consumption volatility growth with low level and volatility of inflation; State (1,2): low volatility of consumption growth and high and volatile inflation; State (2,1): high volatility of consumption growth with low level and low volatility of inflation; State (2,2): high volatility of consumption growth with high and more volatile inflation.

Figure 8: Term structure of default probabilities



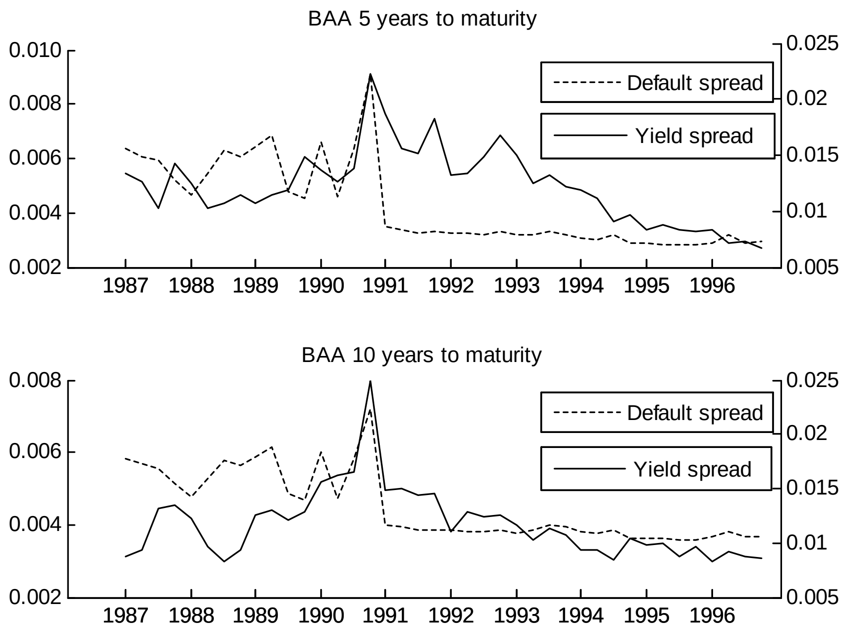
The empirical term structure of default probabilities is obtained from a rating transition generator estimated from Moody's corporate Bond Default Database. The fitted term structure of default probabilities is the average of the 40 theoretical term structures of default probabilities obtained at each point of our sample and computed with the estimated α's.

Figure 9: One period conditional default probability with consumption growth and inflation



This figure plots on a two scale graph the estimated one period conditional default probability computed as $1-\overline{q}\,(t, 1, \widehat{\alpha})$ along with the consumption growth and inflation. The conditional default probability jumps occur during a state of high volatility of consumption and high level and high volatility of inflation. This period is within the economic recession identified by the NBER (1990-III to 1991-I).

Figure 10: Corporate yield spread and estimated default spread



This figure shows on a two scale graph the evolution of the yield spread with the estimated default spread for five and ten years to maturity Baa zero-coupon bonds.