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On Debt Service and Renegotiation when Debt-holders Are More Strategic

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Abstract: The contingent claims analysis of the firm financing often presents a debt renegotiation game with a passive bank which does not use strategically its capability to force liquidation, contrary to what is observed in practice. The first purpose of this paper is to introduce more strategic bank behaviour into the continuous-time model developed by Mella-Barral and Perraudin (1997) and Hackbarth, Hennessy, and Leland (2007). Its second purpose is to account for variations in the information obtained by the parties during the contract period. We show that with public information and private debt only, the optimal probability of debt renegotiation is fixed by the firm's anticipated liquidation value. When we add public debt and asymmetric information, the good-type firm may be tempted to mimic the bad-type to reduce its debt service. We show that to deter such mimicking, banks may sometimes refuse to renegotiate with strong firms having a low liquidation value. Our results are in line with the empirical observation that recovery rate at emergence of bankruptcy is function of the share of private debt in all the firm's debt and is relatively low.

Keywords: Debt service, debt renegotiation, recovery rate, strategic bank, bankruptcy, contingent claim

JEL Classification: G13, G32, G33, D81

Résumé: L'analyse contingente du financement des entreprises présente souvent un jeu de renégociation de la dette avec une banque passive qui n'emploie pas stratégiquement ses possibilités de forcer la liquidation, contrairement à ce qui est observé dans la pratique. Le premier but de ce document est de présenter un comportement plus stratégique de la banque dans le modèle développé par Mella-Barral et Perraudin (1997) et Hackbarth, Hennessy, et Leland (2007). Son deuxième but est de prendre en compte l'asymétrie dans l'information obtenue par les parties au cours du temps. Nous montrons qu'avec information publique et dette privée seulement, la probabilité optimale de renégociation de la dette est fixée par la valeur anticipée de liquidation de l'entreprise. Quand nous ajoutons de la dette publique et une information asymétrique, le bon type d'entreprise (forte valeur de liquidation) peut être tenté d'imiter le mauvais type pour réduire son service de la dette. Nous montrons que pour décourager une telle imitation, les banques vont parfois refuser de renégocier avec les sociétés ayant une valeur de liquidation faible. Nos résultats sont en conformité avec l'observation empirique que le taux de recouvrement au moment de la faillite est fonction de la part de la dette bancaire dans la dette totale de l'entreprise et que ce taux est relativement bas.

Mots clés: Service de la dette, renégociation de la dette, taux de recouvrement, banque stratégique, faillite, analyse contingente

1 Introduction

There are many contributions on contingent claims analysis that propose elegant models for the financing of the firm, but they fail to fit important facts observed in financial markets. For example, the Anderson-and-Sundaresan (1996, AS) as well as the Mella-Barral-and-Parraudin (1997, MP) models of costless debt renegotiation never obtain (inefficient) liquidations at equilibrium, whereas such liquidations do occur in many markets even after renegotiation.¹ Moreover, in these models, private debt holders always renegotiate, while, in practice, banks may decide not to renegotiate. The borrower who relies exclusively on bank debt always avoids costly bankruptcy. It would seem that the setting of the bankruptcy game is not broad enough to introduce the bank's capacity for strategic renegotiation and liquidation, since the passive bank always agrees to renegotiate (Anderson, Sundaresan, and Tychon, 1996).

Hackbarth, Hennessy, and Leland (2007, HHL) propose a renegotiation game, similar to that of MP, with take-it or leave-it offers from the strong firm that has full bargaining power in the process of restructuring private debt. In this type of renegotiation, the bank has no incentive to declare the strong firm in default. The authors use this framework to explain the mix of markets versus non-market debt and the specification of debt priority. This is the first trade-off-theory contribution on debt structure to analyze the effect of bargaining power on the optimal debt mix. They show that introducing market debt distorts bilateral bargaining between the bank and the firm, inducing inefficient bankruptcies. By doing so they can reconcile debt-structure theory with some but not all of the empirical facts, since banks liquidate strong firms in many markets or countries. In fact, their model cannot explain the empirical result that recovery rate at emergence of bankruptcy is related to the share of private debt in the total firm's debt (Carey and Gordy, 2007, and Hamilton and Carty, 1999.)

In HHL, inefficient liquidations are explained by the presence of public debt. Public debt introduces inefficient bankruptcies, since the parties fail to internalize the positive externality accruing to lenders when the bank grants interest rate concessions. Public debt complements the private debt capacity by providing tax-shield benefits exceeding those attainable with bank debt alone. Since public debt introduces bankruptcy costs or inefficient bankruptcies, the optimal debt structure implies equalizing the marginal bankruptcy cost linked to market debt with the marginal tax benefits, as in the trade-off literature (Leland, 1994).

HHL focus on a parsimonious pricing model featuring a tax-shield/bankruptcy-cost tradeoff. The model approximates the situation facing large and mature corporations that are likely to be in a strong bargaining position *ex post*. Banks always accept the firm's renegotiation offer and never liquidate strong firms armed with renegotiation

¹See also Fan and Sundaresan (2000) for a model of strategic interactions between debtholders and equity-holders that accommodates varying bargaining powers.

power. One consequence of their model where banks are flexible (or not strategic enough) is that renegotiation limits the firm's capacity to borrow private funds. This explains why strong firms must raise significant debt capital via public debt to increase their benefits from all potential tax shields. Finally, as in Mella-Barral and Perraudin (1997), banks always renegotiate, no matter how the information on the debt contract conditions evolves over time: A scenario which does not necessarily correspond to the reality. Indeed, the firm may be better informed than the bank at the renegotiation date.

The first purpose of this paper is to introduce more strategic bank behavior into the continuous-time model developed by MP and HHL. Its second purpose is to account for variations in the information obtained by the contractual parties during the contract period. The bank can indeed refuse to renegotiate private debt with a given probability (mixed strategy, Fudenberg and Tirole, 1991) resulting in early liquidation of firms as is observed in the banking credit activity. Moreover, we consider that, during the contractual period, the strong firm may be in a position to assess its true value more readily than the bank. So there may be asymmetric information between the strong firm and the bank at the contract renegotiation date.

In our framework the strong firm is always the leader and the bank plays a mixed strategy where it may refuse the renegotiation proposal from the strong firm with a probability q . In absence of renegotiation, it will liquidate the firm immediately. We derive the values of the different claims on the firm depending on the bank's mixed strategy using an extended zero-coupon approach. Interestingly, the firm and the stock values (and the public debt value, if any) exhibit a jump at the renegotiation threshold, either downward (the firm is liquidated) or upward (the bank agreed to renegotiate the debt service). Our main results are summarized in Table 1 where BD stands for Bourgeon and Dionne. We first show, in the pure exogenous private debt framework of AS and MP, that the mixed strategy increases the value of the bank debt. In that first case (BD1), we obtain that the bank never renegotiates at equilibrium ($q^* = 1$), so inefficient bankruptcies can be observed with only private exogenous debt.

However, when the level of debt is endogenous, the more strategic bank has to account for the fact that the firm may react to the risk of liquidation by reducing its debt level. Hence, the bank may prefer to have the reputation of accepting to renegotiate under certain circumstances ($q^* < 1$). A high probability of debt renegotiation reduces the firm's private (bank) debt capacity but a low probability increases the liquidation rate. So the strong firm faces the usual tax-benefit/bankruptcy-cost tradeoff even in the absence of public debt. In fact, in this second case (BD2), we show that the optimal probability of debt renegotiation depends on the firm's anticipated liquidation value or its anticipated Loss Given Default (LGD) in the bankruptcy state, for a given tax rate (see Basel Committee, 2005, for the definition of LGD.) High LGDs (higher than a threshold determined by the tax rate) or low liquidation values eliminate

Contribution	Bank strategy and market environment	Probability of renegotiation refusal	Private debt capacity
MP AS	Non-strategic Private debt only	$q^* = 0$	Exogenous
BD1 Lemma 2	Strategic Private and public debt	$q^* = 1$	Exogenous
BD2 Proposition 4	Strategic Private debt only	$q^* = 0$ with low LGD $q^* = 1$ with high LGD	Minimal Higher
HHL	Non-Strategic Private and public debt	$q^* = 0$	Minimal
BD3 Proposition 5	Strategic Private and public debt	$q^* = 0$	Minimal
BD4 Proposition 6	Private information, strategic, private and public debt or private debt only	$q^* = 0$ with low LGD $q^* > 0$ with high LGD	Endogenous

The first column identifies the contributions: MP is for Mella-Barral and Perraudin (1997), AS for Anderson and Sudaresan (1996), HHL for Hackbarth, Hennessy and Leland (2007) and BD for Bourgeon-Dionne. The second column identifies the different cases analyzed while the third and fourth columns summarize the main results in terms of renegotiation probability and private debt capacity. The first two lines show how introducing more strategic behavior on the part of the bank affects the renegotiation probabilities when private debt is exogenous. The third line shows that, in presence of endogenous private debt, the introduction of more strategic behavior on the bank's part determines the optimal renegotiation probability as a function of the anticipated LGD. The two following lines show how the presence of public debt eliminates inefficient liquidation in the presence of perfect information. The last line reintroduces inefficient liquidation by considering asymmetric information on firm's value at the renegotiation date.

Table 1: Summary of Results.

renegotiation possibility because they correspond to low debt values. Hence, when the resale value of the firm's assets is low, the firm has no incentive to renegotiate because it would be forced into liquidation. So the bank is able to increase the debt capacity of the firm that has a low anticipated liquidation value or a high LGD. It is well known that LGDs vary in function of economic cycles and debt seniority (Gupton et al., 2000; Schuermann, 2004): LGD increases in time of recession and decreases as debt seniority increases. So our contribution explains why banks renegotiate more often in expansion cycles and more often when they are senior. This also explains the variation of private debt capacity in function of anticipated LGD.

Case BD2 considers only private debt. Once the bank has agreed to renegotiate, the firm never goes bankrupt. This is not the case in HHL where public debt is not renegotiable which allows the firm to increase the total debt value, and thus its total debt capacity. We also investigate this case assuming that the bank is strategic and we show that with public debt, the strategic bank always renegotiates ($q^* = 0$, BD3) at the renegotiation date under public information. This result corresponds to the assumption of a passive bank as supposed by HHL. So HHL do not have to assume that the bank is flexible to obtain their optimal debt structure under public information.

In the following section we introduce asymmetric information on the LGD value at the renegotiation date in order to explain why banks do not renegotiate with the strong firm under certain circumstances, as observed in many economies. We consider private and public debt. At the date of debt emission (when the bank and the firm agree on the private debt level), the bank and the firm do not know the LGD value, so both use the ex ante expected value that set the anticipated ex post renegotiation possibility. After the signature of the debt contract, the firm learns its private LGD while the bank does not observe it, so there is asymmetric information on this parameter at the renegotiation stage. Two scenarios are considered and they are discussed with respect to the BD2 case. When we consider an economy where the tax rate is high enough to obtain $q^* = 1$ and a corresponding high debt capacity, the bank has no incentive to fix a q^* lower than one for any firm type because this would reduce debt value, and thus the firm's debt capacity. However, when the tax rate is sufficiently low, the bank may find it beneficial to use a self-selection mechanism to separate the firm types and increase its debt capacity. We show that a liquidation probability q^* strictly between zero and one for the bad-type (lower liquidation value) allows the bank to deter the good-type from mimicking the bad-type in order to obtain better renegotiation conditions (sooner and with a lower debt service).

When we add public debt, the information asymmetry is still a problem for the bank. Indeed, in the presence of public debt, bad-type firms are liquidated later than good-type ones. Hence, the risk of being liquidated is not a deterrent for the good-type firms and, in general, the presence of asymmetric information implies that the bank will not always renegotiate with strong firms having a high LGD or a low liquidation

value (BD4). Our model can be used to explain the variation of the recovery rate on face value (R) (Schönbucher, 2003) as a function of the private debt share in the firm's debt. Indeed, we obtain that R is increasing in the share of private debt, a result consistent with the recent empirical literature (Carey and Gordy, 2007).

The rest of the paper is organized as follows. In section 2, we extend the EBIT continuous-time model of Goldstein, Ju and Leland (2001) to the case of a bank's mixed strategy and show how strategic denial affects equity and debt values with exogenous private debt. In section 3, we derive the optimal bank strategy with endogenous private debt and public debt. Section 4 introduces asymmetric information on LGD and discusses the empirical implications of our results. Section 5 concludes.

2 The model with bank's mixed strategy

In this section, we extend the model developed by Mella-Baral and Perraudin (1997) in order to introduce a more strategic bank behavior at the renegotiation stage. Consider a firm contemplating investment in a project with date- t EBIT flow starting from a_0 and following the geometric Brownian motion

$$da_t/a_t = \delta dt + \sigma dW_t,$$

where drift δ and volatility σ are constant over time, and W_t is a standard Brownian motion. With r the default-free interest rate, the expected net value (before tax) of the firm's assets at date t is given by a_t/μ where $\mu = r - \delta$ corresponds to the expected gross earnings rate.

In order to undertake this investment project, the firm must raise external funds. The structure of the financial underpinning of the venture depends on the expected tax benefits associated with debt and on the characteristics of the different debts the firm may issue. We reckon that the firm may issue two classes of perpetual debt: Private-bank debt and public-market debt. Compared to the public debt, which cannot be renegotiated unless bankruptcy is declared, the bank debt can be renegotiated in the course of a costless private workout.² In the following, we shall consider the two cases of a firm that has (does not have) access to public debt. We assume that the bank is senior, which means that the full liquidation value is paid to the bank.

When the firm renegotiates, say, once the EBIT flow has plummeted to a_R , the threshold that triggers default and negotiation, we suppose that the strong firm can

²There are several reasons (in addition to administrative or legal requirements) why public debt cannot be renegotiated outside the formal bankruptcy process: firstly, coordination costs for widely dispersed creditors are likely to be prohibitive (Hart and Moore, 1995.) Secondly, there is the free-riding problem: Hoping that the other creditors will concede a reduction in their debt payments, it is tempting to refuse to renegotiate one's own (see HHL for a longer discussion).

make take-it-or-leave-it offers of a reduced debt service to the bank. As the bank can always liquidate the firm and resell its assets, the firm's offer must at least match this resale value which is given by $\gamma a_t/\mu$ at any date t , where $1 > \gamma \geq 0$. Denoting by BD_P the value of the bank debt once the bank has agreed to renegotiate, any offer to the bank must satisfy

$$BD_P(a) \geq \gamma a/\mu \quad (1)$$

for all $a \in [a_L, a_E]$, where a_L corresponds to the threshold that triggers liquidation and a_E the EBIT flow level that ends the reduced debt service period, $a_E \geq a_R$. In fact, $(1 - \gamma)a_L/\mu$ can be interpreted as the product of the LGD \times EAD, where LGD = $1 - \gamma$ corresponds to the loss given default and EAD = a_L/μ is the exposure at default. Hence, a period of reduced debt service begins when a falls to a_R and lasts until a climbs to a_E or falls to a_L , in which case the firm is liquidated. Obviously, if shareholders know that the bank always agrees to any offer that satisfies (1), this condition is binding for all $a \in [a_L, a_E]$. The debt service is thus given by

$$s(a) = \begin{cases} \gamma a & a \in (a_L, a_E] \\ b & a > a_E \end{cases} \quad (2)$$

where γa corresponds to the reduced debt service proposed by the firm to the bank once the firm's revenue falls to a_R , leading to a debt value equal to $\gamma a/\mu$ for all $a \in (a_L, a_E]$.

However, satisfaction of (1) does not guarantee that the bank will always agree to renegotiate. Suppose instead that the bank adopts a mixed strategy, i.e., it accepts any renegotiation offer that satisfies (1) with probability $1 - q$ and liquidates the firm with probability q whatever the offer. The bank's expected revenue at the time of renegotiation still corresponds to the liquidation value of the firm's assets, but shareholders take the risk of losing the firm if it defaults. As shown below, accounting for the possibility that the bank may liquidate the defaulting firm gives a situation which differs significantly from the one where the bank always agrees to an offer that satisfies (1). To detail these effects, it is useful to derive the value of zero-coupon contingent claims that are exercised only at a given probability.

Consider first \hat{Z}_t , the current value of a zero-coupon claim that pays \$1 with probability 1 should the asset flow fall to a given threshold $a_L < a_0$, and is worthless afterward. Assuming financial market equilibrium under risk neutrality, it must satisfy

$$r\hat{Z}_t = E_t[d\hat{Z}_t/dt]|_{dt=0}. \quad (3)$$

As \hat{Z}_t depends on t through a_t only, we can write $\hat{Z}_t = Z(a_t)$ and using Itô's lemma, rewrite (3) as

$$rZ(a) = a\delta Z'(a) + \sigma^2 a^2 Z''(a)/2. \quad (4)$$

The general solution of this second-order linear differential equation is given by

$$Z(a) = K_1 a^\beta + K_2/a^\alpha \quad (5)$$

where β and α are two positive parameters satisfying $Q(-\alpha) = Q(\beta) = 0$ with

$$Q(x) = x[\delta + (x - 1)\sigma^2/2] - r, \quad (6)$$

and K_1 and K_2 are two constants determined by boundary conditions. A first condition, called the “value matching condition,” comes from the very definition of the claim which must pay \$1 should the asset flow fall down to a_L . We thus must have $Z(a_L) = 1$. The second one, the so-called “no-bubbles in asset prices condition,” requires that the value of this contingent claim must tend to 0 when the value of the firm’s assets is very large, since the prospect of reaching a_L is low. We thus must have $\lim_{a \rightarrow +\infty} Z(a) = 0$, which implies that $K_1 = 0$ in (5). From the condition $Z(a_L) = 1$, we then get $K_2 = a_L^\alpha$. Consequently, (5) reduces to

$$Z(a) = (a_L/a)^\alpha \quad (7)$$

for all $a \geq a_L$. Consider now the two contingent claims A and P with values and instantaneous rewards denoted by Z_k and z_k respectively, $k = A, P$, and satisfying³

$$z_A(a, x, y) = \begin{cases} 0 & a > a_R \\ qx + (1 - q)Z_P(a, x, y) & a = a_R \end{cases}$$

and

$$z_P(a, x, y) = \begin{cases} Z_A(a_E, x, y) & a = a_E \\ 0 & a \in (a_L, a_E) \\ y & a = a_L \end{cases}$$

where $a_E \geq a_R > a_L$ and $q \in [0, 1]$. Hence, A and P are zero-coupon claims but with probability q , A pays x when a falls to threshold a_R (a passage from above) and is worthless afterward, and with probability $1 - q$ it pays nothing but is transformed to claim P . This latter pays y at the first passage of a to a_L from above and is worthless afterward, or is converted back to claim A at the first passage of a to a_E from below, whichever event occurs first.

Over their relevant supports, $(a_R, +\infty)$ for claim A and (a_L, a_E) for claim P , the values of these claims must satisfy (4) and thus expressions similar to (5). Observing that for $q = 0$ we must have $Z_P(a, x, y) = Z_A(a, x, y)$ for all $a \in [a_R, a_E]$, and using the “no-bubbles” condition $\lim_{a \rightarrow +\infty} Z_A(a) = 0$, it is easy to obtain that the values of these claims satisfy

$$Z_A(a, x, y) = \begin{cases} K/a^\alpha & a > a_R \\ qx + (1 - q)Z_P(a_R, x, y) & a = a_R \end{cases}$$

and

$$Z_P(a, x, y) = \begin{cases} Z_A(a_E, x, y) & a = a_E \\ q(K_1 a^\beta + K_2/a^\alpha) + (1 - q)K/a^\alpha & a \in (a_L, a_E) \\ y & a = a_L \end{cases}$$

³Throughout the paper, A and P are mnemonics for *ex ante* (before renegotiation) and *ex post* (once renegotiation is agreed by the bank) respectively.

where the constants K , K_1 and K_2 are determined by the value matching conditions $Z_P(a_E^-, x, y) = Z_A(a_E, x, y)$, $Z_A(a_R^+, x, y) = Z_A(a_R, x, y)$ and $Z_P(a_L, x, y) = y$. This leads to the following result

Proposition 1 *The value of the zero-coupon contingent claims A and P are given by*

$$Z_A(a, x, y) = \omega(a_L)x(a_R/a)^\alpha + [1 - \omega(a_L)]y(a_L/a)^\alpha \quad a \geq a_R, \quad (8)$$

and

$$Z_P(a, x, y) = y(a_L/a)^\alpha - [\omega(a_L) - \omega(a)] \{y(a_L/a)^\alpha - x(a_R/a)^\alpha\} \quad a \in [a_L, a_E], \quad (9)$$

where

$$\omega(a) = \frac{q(a_E^{\alpha+\beta} - a^{\alpha+\beta})}{q(a_E^{\alpha+\beta} - a_L^{\alpha+\beta}) + (1-q)(a_E^{\alpha+\beta} - a_R^{\alpha+\beta})} \quad a \in [a_L, a_E].$$

Proof: See the appendix.

Z_A given by (8) is the weighted sum of the values of two simple contingent claims, the first one paying x when a reaches a_R (the x -claim) and the second paying y when a reaches a_L (the y -claim), with respective weights $\omega(a_L)$ and $1 - \omega(a_L)$. Using $a_E \geq a_R > a_L$, one can verify that $\omega(a_L) > q$ when $q \in (0, 1)$, hence the weight on the x -claim is greater than the probability q that A effectively pays up x at the first passage of a through a_R . This is due to the fact that a may cross a_R several times whereas it can cross a_L only once. $\omega(a_L)$ corresponds to the corrected probability of pocketing x and accounts for the fact that once a has crossed a_R and claim A is transformed to claim P , a has to exceed a_E before P is converted back to A . Z_P given by (9) also involves a weighted sum of the same two simple claim values. We have

$$Z_P(a, x, y) - Z_A(a, x, y) = \omega(a) [ya_L^\alpha - xa_R^\alpha] / a_L^\alpha$$

for all $a \in [a_R, a_E]$, and thus $Z_P > Z_A$ if $xa_R^\alpha < ya_L^\alpha$, i.e., when the value of the x -claim is lower than the value of the y -claim. This case is depicted in Figure 1. Because $\omega(a)$ is decreasing, Z_P is lower than the value of the y -claim for all $a > a_L$. The term

$$\omega(a_L) - \omega(a) = \frac{q(a^{\alpha+\beta} - a_L^{\alpha+\beta})}{q(a_E^{\alpha+\beta} - a_L^{\alpha+\beta}) + (1-q)(a_E^{\alpha+\beta} - a_R^{\alpha+\beta})}$$

corresponds to the corrected probability of pocketing x given $a < a_E$ and accounts for the fact that, if the earnings flow crosses threshold a_E , contingent claim P is converted to A , and thus may yield x the next time a falls to a_R .

2.1 Strategic denial and claim values

We can now analyze the effects of the bank's mixed strategy on the values of the different claims on the firm. As the probability of liquidating the firm is given by q whatever the shareholders' offer, (1) holds as an equality during the renegotiation period. Hence, the bank debt can be considered as a contingent claim that pays coupon b as long as the firm continues to operate, yielding the scrap value of the firm's assets at the time of liquidation. The value of the bank debt before renegotiation is thus given by

$$BD_A(a) = b/r - [b/r - \gamma a_L/\mu](a_L/a)^\alpha.$$

The value of the bank debt contains two terms: The face value of the debt, i.e. the discounted value of the coupon flow b/r and the option value associated with the irreversible decision to liquidate the firm and sell its assets. This latter term involves the difference between the discounted value of the coupon flow which the bank loses in case of bankruptcy, and the resale value of the firm's assets at liquidation threshold a_L . This liquidation revenue is discounted by $(a_L/a)^\alpha$, the current value of a zero-coupon claim that pays \$1 should the asset value fall to a_L .

Denote by c the perpetual coupon of the market debt. Assuming that the full liquidation value is paid to the bank and that this value is lower than the face value of the bank debt, i.e. $b/r \leq \gamma a_R/\mu$, the value of the market debt is given by

$$MD_k(a) = c/r - Z_k(a, c/r, c/r)$$

where $k = A, P$. With effective tax rate τ , the value of the firm is given by⁴

$$V_k(a) = (1 - \tau)a/\mu + \tau[BD_k(a) + MD_k(a)] - [DBC_k(a) + IBC_k(a)],$$

$k = A, P$, where DBC and IBC denote direct and indirect bankruptcy costs respectively. DBC corresponds to the loss due to the liquidation of the firm's assets, while IBC is the loss of the tax-sheltering value of interest payments. More precisely, DBC is equivalent to a security that pays no coupon but has a value equal to $(1 - \tau - \gamma)\tilde{a}/\mu$ should bankruptcy occur when the assets flow falls to $\tilde{a} \in \{a_L, a_R\}$. Bankruptcy is triggered with probability 1 when $\tilde{a} = a_L$ and with probability q when $\tilde{a} = a_R$. Hence, this security is equivalent to the mix of the two contingent claims A and P investigated above, with $x = (1 - \tau - \gamma)a_R$ and $y = (1 - \tau - \gamma)a_L$, and we have⁵

$$DBC_k(a) = Z_k(a, a_R, a_L)(1 - \tau - \gamma)/\mu$$

⁴The effective tax rate τ is a compound deduced from the tax rate on corporate profits τ_c and the tax rate on dividends τ_d following the relation $\tau = 1 - (1 - \tau_c)(1 - \tau_d)$. To simplify the presentation, we do not consider interest income taxes.

⁵According to Graham's (2000) results, $\tau_c = 31\%$ and $\tau_d = 10\%$, which gives $\tau = 38\%$. So, to obtain $1 - \tau - \gamma \geq 0$, γ must be lower than 62%, which is a reasonable upper value for γ . See Miao (2005) for other useful parameter values.

$k = A, P$. Indirect bankruptcy costs correspond to the tax benefits that are lost once the firm is liquidated. Should liquidation occur when EBIT reaches \tilde{a} , these losses are given by $\tau\gamma\tilde{a}/\mu$. We thus have

$$IBC_k(a) = Z_k(a, a_R, a_L)\tau\gamma/\mu$$

$k = A, P$. The total bankruptcy costs, the sum of the $DBC_k(a)$ and the $IBC_k(a)$, are given by $(1 - \tau)BC_k(a)$ where

$$BC_k(a) = Z_k(a, a_R, a_L)(1 - \gamma)/\mu.$$

The value of the firm is thus given by

$$V_k(a) = (1 - \tau)[a/\mu - BC_k(a)] + \tau[BD_k(a) + MD_k(a)].$$

The value of equity is deduced from the firm value by

$$\begin{aligned} S_k(a) &= V_k(a) - [BD_k(a) + MD_k(a)] \\ &= (1 - \tau)[a/\mu - BC_k(a) - BD_k(a) - MD_k(a)]. \end{aligned}$$

The following proposition summarizes these results:

Proposition 2 *When the bank liquidates a defaulting firm with probability $q \in (0, 1)$ whatever the offer of reduced debt service, the equity, debt, and firm values are given by*

$$\begin{aligned} S_A(a) &= (1 - \tau)\{a/\mu - (c + b)/r + [b/r - \gamma a_R/\mu] (a_R/a)^\alpha - Z_A[a, g(a_R), g(a_L)]\} \\ BD_A(a) &= b/r - [b/r - \gamma a_R/\mu] (a_R/a)^\alpha \\ MD_A(a) &= c/r - Z_A(a, c/r, c/r) \\ V_A(a) &= (1 - \tau)\{a/\mu - c/r - Z_A[a, g(a_R), g(a_L)]\} \\ &\quad + \tau\{(c + b)/r - [b/r - \gamma a_R/\mu] (a_R/a)^\alpha + Z_A(a, c/r, c/r)\} \end{aligned} \tag{10}$$

before renegotiation (where $a \geq a_R$), and by

$$\begin{aligned} S_P(a) &= (1 - \tau)\{g(a) - Z_P[a, g(a_R), g(a_L)]\} \\ BD_P(a) &= \gamma a/\mu \\ MD_P(a) &= c/r - Z_P(a, c/r, c/r) \\ V_P(a) &= (1 - \tau)\{a/\mu - c/r - Z_P[a, g(a_R), g(a_L)]\} \\ &\quad + \tau\{\gamma a/\mu + c/r - Z_P(a, c/r, c/r)\} \end{aligned} \tag{11}$$

if the bank agrees to renegotiate and until either liquidation or return to normal debt service ($a \in (a_L, a_E)$), where

$$g(a) = (1 - \gamma)a/\mu - c/r. \tag{12}$$

(12) corresponds to the expected value of the equity before tax once the bank has agreed to renegotiate when the EBIT flow is equal to a . It is equal to the value of the firm's asset, a/μ , minus the amounts corresponding to the renegotiated debt service $\gamma a/\mu$ and to the public debt c/r . The *ex post* value of equity given by (11) also accounts for the liquidation option $Z_P[a, g(a_R), g(a_L)]$ that must also be removed from $g(a)$ and the tax payments. The *ex ante* value of equity given by (10) presents two option values: The option to renegotiate the debt-service, which brings in $b/r - \gamma a_R/\mu$ in the event the asset flow falls to a_R and the option $Z_A[a, g(a_R), g(a_L)]$ to liquidate which is only partly under shareholders' control, because the bank may refuse to renegotiate and force liquidation at a_R . Using $Z_P(a_L, x, y) = y$, one can verify that we have $S_P(a_L) = MD_P(a_L) = 0$. Also, since the bank only agrees with probability $(1 - q)$ to any renegotiation offer and liquidates the firm with probability q , we must have $S_A(a_R) = (1 - q)S_P(a_R)$ and $MD_A(a_R) = (1 - q)MD_P(a_R)$. This is easily verified using the expressions given in (10) and (11). Indeed, as

$$Z_A[a_R, x, y] = qx + (1 - q)Z_P[a_R, x, y]$$

we have

$$\begin{aligned} S_A(a_R) &= (1 - \tau)\{g(a_R) - Z_A[a, g(a_R), g(a_L)]\} \\ &= (1 - \tau)(1 - q)\{g(a_R) - Z_P[a_R, g(a_R), g(a_L)]\} \\ &= (1 - q)S_P(a_R) \end{aligned}$$

and

$$MD_A(a_R) = c/r[1 - Z_A(a_R, 1, 1)] = c/r(1 - q)[1 - Z_P(a_R, 1, 1)] = (1 - q)MD_P(a_R).$$

As a consequence, with $q > 0$, we have $S_A(a_R) < S_P(a_R)$ and $MD_A(a_R) < MD_P(a_R)$. Graphically, we observe in Figure 2 a jump at a_R from $S_A(a_R)$ to $S_P(a_R)$. This is not the case when the debt service resumes to plain coupon b , i.e. when a has climbed to a_E , and we have $S_P(a_E) = S_A(a_E)$ and $MD_A(a_E) = MD_P(a_E)$. Observe also that the firm's value satisfies

$$\begin{aligned} V_P(a) - V_A(a) &= Z_A[a, g(a_R), g(a_L)] - Z_P[a, g(a_R), g(a_L)] \\ &= \omega(a) \{g(a_R)a_R^\alpha - g(a_L)a_L^\alpha\} / a^\alpha > 0 \end{aligned}$$

for all $a \in [a_R, a_E]$. Hence, the firm's value still benefits from the agreement once the firm has (partly) recovered. Since $\omega(a)$ is decreasing, this benefit diminishes when the value of the assets increases, and we have $V_P(a_E) = V_A(a_E)$.

2.2 Renegotiation and liquidation thresholds

To completely specify the claim values, it remains to determine the thresholds a_E , a_R and a_L . Let us first discuss threshold a_E , which triggers the end of the reduced

debt service. Contrary to the renegotiation and liquidation thresholds a_R and a_L , which are optimally determined by the firm, a_E is constrained by the possibility that the bank has to liquidate the firm. Indeed, once the bank has agreed to the reduced debt service, we have $b/r - [b/r - \gamma a_R/\mu] (a_R/a)^\alpha > \gamma a/\mu$ as long as a belongs to (a_R, a_E) , meaning that with a normal debt service, the value of the debt would be greater than the resale value of the firm, which is also the current value of the debt. But for $a > a_E$, the bank would be better-off liquidating the firm than receiving the normal debt service b , i.e., $b/r - [b/r - \gamma a_R/\mu] (a_R/a)^\alpha < \gamma a/\mu$. Threshold a_E is thus given by $BD_A(a_E) = \gamma a_E/\mu$. The bank debt values are depicted in Figure 3. Observe that, contrary to the equity value, there is no jump at a_R since we have $BD_A(a_R) = BD_P(a_R)$.

Thresholds a_R and a_L are optimally chosen by the firm and thus maximize S_A and S_P given in (10) and (11). Equivalently, these thresholds must satisfy the smooth-pasting conditions

$$S'_A(a_R^+) = (1 - q)S'_P(a_R) \quad (13)$$

$$S'_P(a_L) = 0 \quad (14)$$

which can be written as

$$\frac{\partial Z_P}{\partial a}[a_L, g(a_R), g(a_L)] = g'(a_L), \quad (15)$$

and

$$\begin{aligned} \frac{\partial Z_A}{\partial a}[a_R, g(a_R), g(a_L)] &= qg'(a_R) + (1 - q)\frac{\partial Z_P}{\partial a}[a_R, g(a_R), g(a_L)] \\ &\quad - [\alpha b/r - (1 + \alpha)\gamma a_R/\mu]/a_R \end{aligned} \quad (16)$$

using (10) and (11). Using $\partial Z_A/\partial a = -\alpha Z_A/a$ and $\partial Z_P/\partial a = -\alpha Z_P/a + \omega'(a) \{ya_L^\alpha - xa_R^\alpha\}/a^\alpha$, (15) and (16) lead to the following result

Lemma 1 *Optimal thresholds a_L and a_R satisfy*

$$g'(a_L) + \alpha g(a_L)/a_L = \omega'(a_L) \{g(a_L)a_L^\alpha - g(a_R)a_R^\alpha\}/a_L^\alpha \quad (17)$$

when $a_L > 0$, and

$$\frac{\alpha b}{a_R r} - (1 + \alpha)\frac{\gamma}{\mu} - q \left[g'(a_R) + \alpha \frac{g(a_R)}{a_R} \right] = (1 - q)\omega'(a_R) \{g(a_L)a_L^\alpha - g(a_R)a_R^\alpha\}/a_R^\alpha. \quad (18)$$

To interpret (17) and (18), consider the two extreme cases $q = 1$ (the bank refuses any renegotiation offer and always liquidates the firm) and $q = 0$ (the bank always agrees to renegotiate). The LHS of (17) reflects the effect of a marginal increase in

a_L on the value of the liquidation option. When $q = 0$, the RHS (17) is equal to 0, and the optimal liquidation threshold is given by

$$a_C = \mu \frac{\alpha}{1 + \alpha} \frac{c/r}{1 - \gamma} \quad (19)$$

which is equivalent to condition (32) in HHL when interest income tax is nil. Hence the firm is liquidated when the expected value of the firm's assets, a_C/μ , is lower than the break-even level $c/[r(1 - \gamma)]$ given the face value of the (non-renegotiable) public debt, c/r : Given the low resale value of the firm's assets during the renegotiation period, it is optimal to let the firm operate below its break-even level in the hope that recovery will occur. With $q > 0$, as a_C maximizes $g(a)a^\alpha$ and $\omega'(a) < 0$ for all a , (17) implies $a_L > a_C$. This early liquidation rule is due to the fact that, if the firm recovers sufficiently to come back to a normal debt service, it may be forced into liquidation the next time it defaults; thus it is not worthwhile to wait until a reaches a_C . This is the effect reflected by the RHS of (17), where $\omega'(a_L)$ is the marginal decrease in the probability the firm will be liquidated at a_R in place of a_L , and $g(a_R)a_R^\alpha - g(a_L)a_L^\alpha$ the corresponding loss. When $q = 1$, liquidation occurs when the firm defaults and we have $a_R = a_L$. Liquidation rule (17) is then not relevant, but (18) allows us to determine the optimal liquidation threshold, given by

$$a_B = \frac{\alpha}{1 + \alpha} \frac{c + b}{r} \mu. \quad (20)$$

This result can be directly obtained observing that we have $Z_A[a, g(a_R), g(a_L)] = g(a_L)(a_L/a)^\alpha$ when $q = 1$, which corresponds to the value of another liquidation option. Hence, in that case, the firm possesses only a liquidation option, equal to $[(c + b)/r - a_L/\mu](a_L/a)^\alpha$, which is maximized at $a_L = a_B$ given by (20). More generally, the LHS of (18) corresponds to the effect of a marginal increase in a_R on the value of the sum of the renegotiation and liquidation options, the latter being weighted by q , i.e., $[b/r - \gamma a_R/\mu - qg(a_R)]a_R^\alpha$. Denote by $T(q)$ the threshold that maximizes this sum, i.e.,

$$T(q) = \frac{\alpha}{1 + \alpha} \frac{(qc + b)/r}{\gamma + q(1 - \gamma)} \mu. \quad (21)$$

Hence, (21) would be the optimal renegotiation threshold were the RHS of (18) nil. This term corresponds to the marginal increase in the loss of liquidating the firm at a_R in the future assuming that the bank agrees to renegotiate today. As this term is positive, we have $a_R \leq T(q)$. Of course, when $q = 0$, the renegotiation threshold is at its maximum level, given by $\bar{a} = T(0)$.⁶ The next proposition summarizes these results.

⁶The reader can verify that we have $T'(q) < 0$ when $b/c > \gamma/(1 - \gamma)$, which we assume in this section. Observe that if $b/c < \gamma/(1 - \gamma)$, the liquidation threshold is greater than the renegotiation threshold when $q = 0$, meaning that the bank debt is so small that the firm would not even bother renegotiating its private debt, even though the bank always agrees to do so.

Proposition 3 *When the bank liquidates a defaulting firm with probability $q \in (0, 1)$ whatever the offer made by the firm, we have $a_L > a_C$ and $a_R < T(q)$.*

It is easy to see from (10) that the bank debt decreases with the renegotiation threshold (we have $dBD_A(a)/da_R < 0$ if $a_R < T(0)$, which is always the case). For a given coupon b , as a_R decreases with q (indeed, one can verify using $T(q)$ as a proxy for a_R , that we have $T'(q) < 0$ provided $c/b < (1 - \gamma)/\gamma$), the bank would be better-off if it had a reputation of never accepting to renegotiate the debt service with the firm. This is formally stated in the following lemma

Lemma 2 *For a given coupon b , the bank debt is maximum when $q = 1$.*

Obviously, this is not the whole story: If the bank has the reputation of never renegotiating, it may end up with a relatively low coupon b and finally with a low debt value. Hence, when determining its renegotiation strategy, the bank has to consider the effect of its reputation on the firm's demand for private and public funds. This is the question that we discuss in the following section.

3 Capital structure and the choice of q .

To investigate the question of optimal leverage, let us first consider the case without public debt. As $c = 0$, the optimal liquidation threshold is $a_L = 0$. The optimal debt level maximizes the value of the leveraged firm, a choice made at $t = 0$, the period of the borrowing agreement. Ultimately, this problem comes down to determining the value of the coupon, b , anticipating that the firm will eventually renegotiate the debt service should the asset flow fall to the optimally chosen threshold a_R . Consequently, the optimal coupon b satisfies the following condition⁷

$$\tau \left\{ \frac{\partial BD(a_0; b, a_R)}{\partial b} + \frac{\partial BD(a_0; b, a_R)}{\partial a_R} \frac{da_R}{db} \right\} - (1 - \tau) \frac{\partial BC(a_0; a_R)}{\partial a_R} \frac{da_R}{db} = 0 \quad (22)$$

As shown above, the problem of determining the optimal renegotiation threshold is equivalent to maximizing the (weighted) sum of the renegotiation and liquidation options. This can also be written as $\max_{a_R} S(a; b, a_R)$, leading to the condition

$$\frac{\partial BD(a; b, a_R)}{\partial a_R} + \frac{\partial BC(a; a_R)}{\partial a_R} = 0 \quad (23)$$

⁷With a slight abuse of notation.

which holds for all $a \geq a_R$. Plugging (23) in (22) allows us to simplify the latter to

$$\tau \frac{\partial BD(a_0; b, a_R)}{\partial b} = - \frac{\partial BD(a_0; b, a_R)}{\partial a_R} \frac{da_R}{db} \quad (24)$$

Using q as a proxy for ω , (23) gives⁸

$$a_R = \frac{\alpha}{1 + \alpha} \frac{\mu}{\gamma + q(1 - \gamma)} \frac{b}{r} \quad (25)$$

which can also be derived from (21) by setting $c = 0$. Use of (24) allows us to obtain

$$a_R = a_0 \left(1 + \frac{\alpha q(1 - \gamma)}{\tau[\gamma + q(1 - \gamma)]} \right)^{-1/\alpha}. \quad (26)$$

When $q = 0$, we have $a_R = a_0 = \bar{a}$, leading to an optimal debt level equal to $\gamma a_0 / \mu$: As observed by HHL, the firm is constrained by the debt capacity, which is equal to the resale value of its assets at the time of emission. Hence, the firm will try to renegotiate the debt service as soon as the contract has been signed. When $q > 0$, we have $a_0 > a_R$ and, more generally, one can derived from (26) that a_R decreases when q increases. Straightforward computations allow us to obtain that the firm and the debt values at $t = 0$ are given by

$$V_0(q) = \frac{a_0}{\mu} \left\{ 1 - \tau + [\tau\gamma + q(\tau + \alpha)(1 - \gamma)] \left[\frac{a_R}{a_0} \right]^{1+\alpha} \right\} \quad (27)$$

and

$$BD_0(q) = \frac{a_0}{\mu} [\gamma + q(1 + \alpha)(1 - \gamma)/\tau] \left[\frac{a_R}{a_0} \right]^{1+\alpha}. \quad (28)$$

It is shown in the appendix that $V_0(q)$ is a convex function of q , and more precisely that $V_0'(q) > 0$ iff

$$q > \bar{q} \equiv \frac{\gamma(1 - \tau)}{(1 - \gamma)(\tau + \alpha)}.$$

Hence, starting from $q = 0$, the firm's value first diminishes until q reaches \bar{q} , and then increases as q increases. This is generally also the case for the debt value since we have $BD_0'(0) = 0$ and $BD_0'(q) > 0$ for $q > 0$ if

$$q > \frac{\gamma(1 - 2\tau)}{(1 - \gamma)(\tau + \alpha)},$$

⁸Whereas $T(q)$ is an upper bound for a_R which is reached only when $q = 1$ or $q = 0$, simulations show that $T(q)$ constitutes a good proxy for a_R for intermediate values of q . More generally, we will replace $\omega(a_\ell)$ by q and use $T(q)$ (and a_C) as the optimal renegotiation (liquidation) threshold to demonstrate our results in the following.

a condition that is satisfied for all $q > 0$ when $\tau > 1/2$. In that case, $BD_0(0)$ is a minimum, and the debt value increases with q . For lower tax rates (and more reasonable ones, see footnote 5), i.e. $\tau < 1/2$, $BD_0(0)$ is a local maximum. Hence, we can expect that the optimal probability level for debtholders and shareholders is either $q = 0$ or $q = 1$. It is demonstrated in the appendix that we have $V_0(1) > V_0(0)$ if

$$\tau > \bar{\tau} \equiv \frac{\alpha(1-\gamma)\gamma^{1/\alpha}}{1-\gamma^{1/\alpha}} \quad (29)$$

which also ensures that $BD_0(1) > BD_0(0)$. We can thus claim that

Proposition 4 *When the firm does not have access to public debt, both shareholders and the bank expected revenues are maximum at $q = 1$ when γ is low (LGD is high).*

The intuition of this result is the following. In the situation where the bank always agrees to renegotiate the debt service, an increase in the debt level allows an increase in the firm's value without affecting bankruptcy costs. It is thus optimal to have as much debt as possible in order to benefit from the tax-shield. However, the firm's debt capacity is relatively small because the bank expects the firm to renegotiate the debt service frequently. By credibly committing to liquidate the firm in the event shareholders decide to default, the bank deters the firm from renegotiating prematurely. This increases the debt value for a given coupon. Expecting this, shareholders could then reduce the debt level. However, the bank's strategy is also beneficial to shareholders when γ is low: It allows the firm to increase its debt capacity. The strategy is thus optimal for both the bank and the firm.

Let us now consider the case with private and public debt. Similar computations (see Proof of Proposition 5 in the appendix) show that the optimal renegotiation threshold is still given by (26) and that the optimal liquidation threshold satisfies

$$a_L = a_0 \left[\frac{\tau}{\tau + \alpha} \right]^{1/\alpha}. \quad (30)$$

The optimal level of (total) debt at $t = 0$ can be written as

$$TD_0(q) = \frac{a_0}{\mu} \left\{ \gamma \left(\frac{a_R}{a_0} \right)^{1+\alpha} + \frac{(1+\alpha)(1-\gamma)}{\tau} \left[q \left(\frac{a_R}{a_0} \right)^{1+\alpha} + (1-q) \left(\frac{a_L}{a_0} \right)^{1+\alpha} \right] \right\} \quad (31)$$

and thus contains an additional term compared to (28) which is decreasing when q increases. As a result, we have $TD'_0(q) < 0$ for all q : The higher the probability the bank will refuse to renegotiate, the lower the debt value. The value of the firm, given

by

$$V_0(q) = \frac{a_0}{\mu} \left\{ 1 - \tau + \tau\gamma \left(\frac{a_R}{a_0}\right)^{1+\alpha} + (1-\gamma)(\tau + \alpha) \left[q \left(\frac{a_R}{a_0}\right)^{1+\alpha} + (1-q) \left(\frac{a_L}{a_0}\right)^{1+\alpha} \right] \right\} \quad (32)$$

also exhibits a similar additional term compared to (27). It is shown in the appendix that $V_0(q)$ is also generally convex but, more importantly, that $V_0(0) > V_0(q)$ for all $q > 0$. Hence we have the following result

Proposition 5 *With public debt, the firm value is maximum for $q = 0$.*

This result is easily understood: Without public debt the bank can increase the debt capacity of firms with a high LGD by liquidating systematically a defaulting firm. However, this is exactly the situation that a firm faces if it has only issued public debt. Hence, when there is already public debt, the only valuable contribution from a bank is offering the firm a renegotiable debt. So, our result is similar to HHL in the presence of public debt, even when we allow a more strategic behavior.

4 Asymmetric information on LGD

The previous section showed that the bank follows an unambiguous decision rule regarding the liquidation of the firm (we either have $q = 1$ when LGD is large and the firm has no public debt, or $q = 0$ in the other cases). However, if $q = 0$ is optimal when the bank has perfect information about the LGD of the firm, this may not always be the case when the information on LGD is not equally shared by debtholders and equity-holders. We investigate this question in this section and we show how a strictly positive value of q may explain the observed recovery rates.

4.1 Renegotiation strategy under asymmetric information

Suppose that at $t = 0$ (the date debt is issued), neither the firm nor the bank and the financial markets know the exact value of γ . They all know that γ can take two values, $\bar{\gamma}$ and $\underline{\gamma}$ with $1 > \bar{\gamma} > \underline{\gamma} > 0$, and the probability of either case. So, all parties use $E[\gamma]$ when debt is issued. Suppose then that the firm learns its γ only after the emission of debt and that this information is not public. When the bank's strategy is always to renegotiate (i.e. $q(\underline{\gamma}) = q(\bar{\gamma}) = 0$), $\bar{\gamma}$ -type firms may be tempted to renegotiate their debt with the bank as a $\underline{\gamma}$ -type firm would. Indeed, the corresponding debt service is lower ($\underline{\gamma}a$ compared to $\bar{\gamma}a$) and renegotiation occurs earlier (using (21) we have $a_R(\underline{\gamma}) > a_R(\bar{\gamma})$). Besides, as (19) indicates, we have $a_L(\underline{\gamma}) < a_L(\bar{\gamma})$ and thus $\bar{\gamma}$ -type

firms do not face the risk of being liquidated prematurely. To avoid such behavior, the bank may threaten to liquidate firms with probability q whenever they try to renegotiate at $a_R(\underline{\gamma})$. We first investigate this question by considering first the case $c = 0$, i.e., firms only have private debt. In that case, we have $a_L(\underline{\gamma}) = a_L(\bar{\gamma}) = 0$. Of course, the asymmetric information problem is relevant in that case only if the optimal perfect information strategy for the bank is to always agree to renegotiate the debt whatever γ , which corresponds to a situation where $\underline{\gamma}$ is high enough (low LGD and/or low tax rate).⁹

At $a_R(\underline{\gamma})$, the stock value of a $\bar{\gamma}$ -type firm that decides to renegotiate the debt like a $\underline{\gamma}$ -type firm is equal to

$$S(a_R(\underline{\gamma}); \underline{\gamma}, \bar{\gamma}) = (1 - \tau)[1 - q(\underline{\gamma})](1 - \underline{\gamma})a_R(\underline{\gamma})/\mu,$$

i.e., the stock value of a $\underline{\gamma}$ -type firm, which accounts for the risk that the firm will be liquidated by the bank with probability $q(\underline{\gamma})$. The firm is deterred from mimicking a $\underline{\gamma}$ -type firm if $S(a_R(\underline{\gamma}); \underline{\gamma}, \bar{\gamma}) \leq S(a_R(\underline{\gamma}); \bar{\gamma}, \bar{\gamma})$, i.e. if

$$[1 - q(\underline{\gamma})](1 - \underline{\gamma})\frac{a_R(\underline{\gamma})}{\mu} \leq \frac{a_R(\underline{\gamma})}{\mu} - \frac{b}{r} + \left[\frac{b}{r} - \frac{\bar{\gamma}a_R(\bar{\gamma})}{\mu} \right] \left[\frac{a_R(\bar{\gamma})}{a_R(\underline{\gamma})} \right]^\alpha.$$

Using $a_R(\bar{\gamma})/\mu = \alpha b/[r\bar{\gamma}(1 + \alpha)]$ and

$$a_R(\underline{\gamma}) = \frac{\alpha}{1 + \alpha} \frac{\mu}{\underline{\gamma} + q(\underline{\gamma})(1 - \underline{\gamma})} \frac{b}{r},$$

we obtain that we must have $a_R(\bar{\gamma}) \geq a_R(\underline{\gamma})$, which leads to

$$q(\underline{\gamma}) \geq \frac{\bar{\gamma} - \underline{\gamma}}{1 - \underline{\gamma}} > 0. \quad (33)$$

It is shown in the appendix (Proof of Proposition 6) that if firms have issued both public and private debts, it is still necessary to liquidate $\underline{\gamma}$ -type firms with a strictly positive probability to deter $\bar{\gamma}$ -type firms from mimicking $\underline{\gamma}$ -type. Indeed, with $q(\underline{\gamma}) = q(\bar{\gamma}) = 0$, we have $a_L(\underline{\gamma}) < a_L(\bar{\gamma})$ and thus $\bar{\gamma}$ -type firms extract from the information asymmetry an additional advantage: Once the private debt has been renegotiated with the low service flow of a $\underline{\gamma}$ -type firm, they are able to put off liquidation longer than they otherwise could. Hence, public debt does not help to mitigate the private information problem. We thus have the following result

Proposition 6 *With asymmetric information on γ , the $\bar{\gamma}$ -type firm is deterred from mimicking the $\underline{\gamma}$ -type firm only if the probability of liquidating the $\underline{\gamma}$ -type firm is sufficiently high. This result holds with or without public debt.*

⁹In the other case, we have $q(\underline{\gamma}) = 1$ and the firm is immediately liquidated.

Consequently, in order to discipline $\bar{\gamma}$ -type firms, the bank must refuse to renegotiate with $\underline{\gamma}$ -type firms with a strictly positive probability. If the bank chooses to do so (i.e., if the fraction of $\bar{\gamma}$ -type firms is large) and if there is only private debt, then condition (33) is binding. Hence, both types of firms approach the bank at the same time (when a_t reaches $a_R(\bar{\gamma}) = a_R(\underline{\gamma})$), but $\underline{\gamma}$ -type firms ask for a lower debt service than $\bar{\gamma}$ -type firms and take the risk of being liquidated. The renegotiation offer of a $\bar{\gamma}$ -type firm, which corresponds to an higher debt service, is always accepted by the bank.

4.2 Recovery rate on firm's total debt

Following Carey and Gordy (2007), one can compute the recovery rate on the total debt face value when the bank forces the liquidation of the firm with probability $q > 0$. We have

$$R = \frac{\gamma a_R / \mu}{(c + b) / r} = \frac{\alpha}{1 + \alpha} \frac{\gamma}{\gamma + q(1 - \gamma)} \frac{qc + b}{c + b} \quad (34)$$

using (21). It is interesting to observe that R is increasing in b as tested empirically by Carey and Gordy (2007). When the bank is less strategic as in HHL, i.e., when $q \equiv 0$, R becomes

$$R = \frac{\alpha}{1 + \alpha} \frac{1}{1 - \gamma} \frac{c}{c + b} \quad (35)$$

using (19), which is decreasing in b . When $q \equiv 0$, the bank always agrees to renegotiate and liquidation never occurs at renegotiation. Liquidation only occurs when shareholders decide to stop refinancing the firm's losses. When $q > 0$, bankruptcy may occur at renegotiation since this is the bank's choice to do so. Here, the bank has more incentive to liquidate the firm having a low liquidation value. This behavior is also in line with the empirical result that observed recovery rates are lower than those predicted by the structural model in absence of asymmetric information.

5 Conclusion

In this paper we present a debt renegotiation model where the bank adopts a more strategic approach than in the previous literature. As shown by Hackbarth, Hennessy, and Leland (2007), optimal debt structure is linked to whether it is firms or debt-holders that control ex post bargaining power. In this paper, we assign the debt renegotiation power to strong firms but also allow banks to bargain more strategically. This research is motivated by the empirical observation that banks do not always renegotiate and that costly bankruptcies are observed in many markets or countries.

The paper's main results call for discussion regarding the information available on the firm's value at the renegotiation date. We limit this discussion to the more general environment, where the private debt capacity is endogenous. When both parties have the same information and if there is no public debt, we show that the more strategic bank does not always renegotiate with the firm. This behavior makes it possible to increase the private debt capacity. In the presence of public debt, the bank will always renegotiate, seeing that the firm can turn to other sources of funding to increase its total debt capacity.

If, at the renegotiation date, the information on the firm's liquidation value is asymmetrically distributed between the bank and the firm, the results will be less dependent on the presence of public debt. We show that, to deter firms to renegotiate their debt services at level lower than their perfect information level, the bank will not always accept the renegotiation proposal. We also verify that the recovery rate on the firm's total debt face value is increasing in the share of private debt in total debt, a result that cannot be obtained when the bank is less strategic.

Our contribution can be extended in several directions. We discuss two of them here. The explicit consideration of ex ante asymmetric information can be an important ingredient in the detailed analysis of the optimal debt structure. In this research we are limited to asymmetric information at the renegotiation stage, but asymmetric information can also be present when the debt is first negotiated. As a rule, banks and private-market debt issuers use risk classification (credit ratings) when issuing debt. This reduces the ex ante information gap but does not eliminate it, so there is still room for renegotiation under asymmetric information at the renegotiation date. One rationale for ex ante risk classification is the high cost of obtaining information on all the firms that need external financing. It is therefore probably optimal to wait for an opportunity to renegotiate with a subset of firms with financial difficulties, so as to reduce the total information cost. But the whole picture may be more complicated because credit ratings may affect capital structure (Kisgen, 2006, Bourgeon and Dionne, 2007).

Another extension would be to do an empirical analysis of how differences in bankruptcy laws affect debt structure. Various studies have undertaken empirical measurements of market reactions to financial distress in different legislative settings (Gutiérrez, Ollala and Olmo, 2005). They report that the valuation of firms' securities will depend on the bankruptcy laws applicable in the country where it is actually done. These regulations can be interpreted so as to assign the renegotiation power either to debt-holders (creditor protection) or to shareholders (shareholder protection). It would be interesting to document how these different bankruptcy regimes treat the ex ante acquisition of information and how they redistribute the renegotiation power between debt-holders and shareholders under asymmetric information.

Appendix

A Proof of Proposition 1

Using $Z_P(a_E^-, x, y) = Z_A(a_E, x, y)$, we get $K_2 = K - K_1 a_E^{\alpha+\beta}$, and thus Z_P simplifies to

$$Z_P(a, x, y) = \begin{cases} Z_A(a_E, x, y) & a = a_E \\ [K - qK_1(a_E^{\alpha+\beta} - a^{\alpha+\beta})]/a^\alpha & a \in (a_L, a_E) \\ y & a = a_L \end{cases}$$

Condition $Z_A(a_R^+, x, y) = Z_A(a_R, x, y)$ leads to

$$K = xa_R^\alpha + (1 - q)K_1(a_R^{\alpha+\beta} - a_E^{\alpha+\beta})$$

while $Z_P(a_L, x, y) = y$ implies

$$K = ya_L^\alpha - qK_1(a_L^{\alpha+\beta} - a_E^{\alpha+\beta})$$

Identifying and rearranging terms, we come to

$$K_1 = \frac{xa_R^\alpha - ya_L^\alpha}{q(a_E^{\alpha+\beta} - a_L^{\alpha+\beta}) + (1 - q)(a_E^{\alpha+\beta} - a_R^{\alpha+\beta})}$$

and

$$\begin{aligned} K &= \frac{q(a_E^{\alpha+\beta} - a_L^{\alpha+\beta})xa_R^\alpha + (1 - q)(a_E^{\alpha+\beta} - a_R^{\alpha+\beta})ya_L^\alpha}{q(a_E^{\alpha+\beta} - a_L^{\alpha+\beta}) + (1 - q)(a_E^{\alpha+\beta} - a_R^{\alpha+\beta})} \\ &= [1 - \omega(a_L)]ya_L^\alpha + \omega(a_L)xa_R^\alpha \end{aligned}$$

We also have

$$q(a_E^{\alpha+\beta} - a^{\alpha+\beta})K_1 = \omega(a)(xa_R^\alpha - ya_L^\alpha)$$

and thus

$$\begin{aligned} K - qK_1(a_E^{\alpha+\beta} - a^{\alpha+\beta}) &= [1 - \omega(a_L)]ya_L^\alpha + \omega(a_L)xa_R^\alpha - \omega(a)(xa_R^\alpha - ya_L^\alpha) \\ &= ya_L^\alpha - [\omega(a_L) - \omega(a)][ya_L^\alpha - xa_R^\alpha] \end{aligned}$$

which gives Z_P .

B Proof of Proposition 4

Using (28), we have $BD_0(0) < BD_0(1)$ if

$$\gamma < [(a_R/a_0)|_{q=1}]^{1+\alpha} [(1 + \alpha)(1 - \gamma)/\tau + \gamma]$$

which can also be written as

$$\frac{\tau\gamma + (1 + \alpha)(1 - \gamma)}{\tau} [(a_R/a_0)|_{q=1}]^{1+\alpha} > \gamma$$

Using

$$\begin{aligned} \tau\gamma + (1 + \alpha)(1 - \gamma) &= \tau + \alpha(1 - \gamma) + (1 - \gamma)(1 - \tau) \\ &> \tau + \alpha(1 - \gamma) \end{aligned}$$

and

$$[(a_R/a_0)|_{q=1}]^\alpha = \frac{\tau}{\tau + \alpha(1 - \gamma)},$$

a sufficient condition for $BD_0(0) < BD_0(1)$ is thus given by

$$\frac{\tau}{\tau + \alpha(1 - \gamma)} \geq \gamma^{1/\alpha}$$

hence $\tau \geq \bar{\tau}$ defined in (29). Differentiating (28), we obtain

$$\begin{aligned} BD'_0(q) &= \frac{a_0}{\mu} [q(1 + \alpha)(1 - \gamma)/\tau + \gamma] (1 + \alpha) \left[\frac{a_R}{a_0} \right]^\alpha \frac{d}{dq} \left[\frac{a_R}{a_0} \right] \\ &\quad + \frac{a_0}{\mu} (1 + \alpha)(1 - \gamma)/\tau \left[\frac{a_R}{a_0} \right]^{1+\alpha} \end{aligned}$$

where, using (25),

$$\frac{d}{dq} \left[\frac{a_R}{a_0} \right] = \frac{-\tau(1 - \gamma)\gamma}{[\tau\gamma + q(1 - \gamma)(\tau + \alpha)]^2} \left[\frac{a_R}{a_0} \right]^{1-\alpha} \quad (36)$$

Consequently

$$BD'_0(q) = q \frac{a_R}{\mu} (1 + \alpha)(1 - \gamma)^2 \frac{q(1 - \gamma)(\tau + \alpha) - \gamma(1 - 2\tau)}{[\tau\gamma + q(1 - \gamma)(\tau + \alpha)]^2},$$

and we have $BD'_0(0) = 0$ and $BD'_0(q) > 0$ for $q > 0$ if

$$q > \frac{\gamma(1 - 2\tau)}{(\tau + \alpha)(1 - \gamma)}.$$

Hence, if $\tau > 1/2$, $BD_0(0)$ is a minimum. Otherwise, for $\tau < 1/2$, $BD_0(0)$ is a local maximum. Similarly, using (27), we have

$$\begin{aligned} V_0(1) - V_0(0) &= \frac{a_0}{\mu} \{ [(a_R/a_0)|_{q=1}]^{1+\alpha} [\tau + \alpha(1 - \gamma)] - \tau\gamma \} \\ &= \frac{a_0}{\mu} \tau \{ (a_R/a_0)|_{q=1} - \gamma \} \end{aligned}$$

and $V_0(1) > V_0(0)$ if $\tau > \bar{\tau}$. As

$$V_0'(q) = \frac{a_R}{\mu} \tau (1 - \gamma) \frac{q(1 - \gamma)(\tau + \alpha) - (1 - \tau)\gamma}{\tau\gamma + q(1 - \gamma)(\tau + \alpha)}$$

we have $V_0'(q) > 0$ iff

$$q > \frac{(1 - \tau)\gamma}{(1 - \gamma)(\tau + \alpha)}.$$

C Proof of Proposition 5

The optimal *ex post* choices of the firm satisfy

$$\frac{\partial BC}{\partial a_R} + \frac{\partial BD}{\partial a_R} + \frac{\partial MD}{\partial a_R} = 0 \quad (37)$$

and

$$\frac{\partial BC}{\partial a_L} + \frac{\partial MD}{\partial a_L} = 0 \quad (38)$$

while the optimal *ex ante* choices of the firm satisfy

$$\tau \left\{ \frac{\partial BD}{\partial b} + \left[\frac{\partial BD}{\partial a_R} + \frac{\partial MD}{\partial a_R} \right] \frac{da_R}{db} \right\} - (1 - \tau) \frac{\partial BC}{\partial a_R} \frac{da_R}{db} = 0 \quad (39)$$

and

$$\tau \left\{ \frac{\partial MD}{\partial c} + \frac{\partial MD}{\partial a_L} \frac{da_L}{dc} + \left[\frac{\partial BD}{\partial a_R} + \frac{\partial MD}{\partial a_R} \right] \frac{da_R}{dc} \right\} - (1 - \tau) \left[\frac{\partial BC}{\partial a_R} \frac{da_R}{dc} + \frac{\partial BC}{\partial a_L} \frac{da_L}{dc} \right] = 0 \quad (40)$$

Substituting, we get

$$\tau \frac{\partial BD}{\partial b} + \left[\frac{\partial BD}{\partial a_R} + \frac{\partial MD}{\partial a_R} \right] \frac{da_R}{db} = 0$$

and

$$\tau \left\{ \frac{\partial MD}{\partial c} - \frac{\partial BD}{\partial b} \frac{da_R}{dc} \Big/ \frac{da_R}{db} \right\} + \frac{\partial MD}{\partial a_L} \frac{da_L}{dc} = 0$$

Solving for a_L and a_R gives (26) and (30). Using (19) and (21), we thus have

$$\frac{c}{r} = \frac{a_0}{\mu} \frac{1 + \alpha}{\alpha} (1 - \gamma) \left[\frac{\tau}{\tau + \alpha} \right]^{1/\alpha}$$

and

$$\frac{b}{r} = \frac{a_0}{\mu} \frac{1 + \alpha}{\alpha} \left\{ \frac{\tau^{1/\alpha} [\gamma + q(1 - \gamma)]^{1+1/\alpha}}{[\tau\gamma + q(1 - \gamma)(\tau + \alpha)]^{1/\alpha}} - q(1 - \gamma) \left[\frac{\tau}{\tau + \alpha} \right]^{1/\alpha} \right\}$$

Defining

$$\begin{aligned} TD_0(q) &\equiv BD_0(q) + MD_0(q) \\ &= \left(\frac{b}{r} + q\frac{c}{r}\right) \left[1 - \left(\frac{a_R}{a_0}\right)^\alpha\right] + \gamma\frac{a_R}{\mu} \left(\frac{a_R}{a_0}\right)^\alpha + (1-q)\frac{c}{r} \left[1 - \left(\frac{a_L}{a_0}\right)^\alpha\right] \end{aligned}$$

where

$$\frac{b}{r} + q\frac{c}{r} = \frac{1+\alpha}{\alpha} \frac{a_R}{\mu} [\gamma + q(1-\gamma)]$$

and

$$\frac{c}{r} \left[1 - \left(\frac{a_L}{a_0}\right)^\alpha\right] = \frac{a_0}{\mu} \frac{1+\alpha}{\tau} (1-\gamma) \left(\frac{a_L}{a_0}\right)^{1+\alpha}$$

we arrive at

$$\begin{aligned} TD_0(q) &= \frac{a_0}{\mu} \left(\frac{a_R}{a_0}\right)^{1+\alpha} \left\{ \frac{1+\alpha}{\alpha} [\gamma + q(1-\gamma)] \left[\left(\frac{a_0}{a_R}\right)^\alpha - 1 \right] + \gamma \right\} \\ &\quad + (1-q) \frac{a_0}{\mu} \frac{1+\alpha}{\tau} (1-\gamma) \left(\frac{a_L}{a_0}\right)^{1+\alpha} \end{aligned}$$

Rearranging terms gives (31). Differentiating, we have

$$TD'_0(q) = -\frac{a_0}{\mu} \frac{(1+\alpha)(1-\gamma)}{\tau} \left\{ \frac{a_R}{a_0} \frac{\tau\gamma[(1+\alpha)q(1-\gamma) + \tau\gamma]}{\tau\gamma + q(1-\gamma)(\tau + \alpha)} + \left(\frac{a_L}{a_0}\right)^{1+\alpha} \right\} < 0.$$

The value of the firm is given by

$$V_0(q) = (1-\tau) \left[\frac{a_0}{\mu} - BC_0(q) \right] + \tau TD_0(q)$$

where

$$\begin{aligned} BC_0(q) &= (1-\gamma) \{ q(a_R/a_0)^\alpha a_R/\mu + (1-q)(a_L/a_0)^\alpha a_L/\mu \} \\ &= (1-\gamma) \frac{a_0}{\mu} \left\{ q \left(\frac{a_R}{a_0}\right)^{1+\alpha} + (1-q) \left(\frac{a_L}{a_0}\right)^{1+\alpha} \right\} \end{aligned}$$

which gives (32). Using (36), we arrive at

$$V'_0(q) = \frac{a_0}{\mu} (1-\gamma) \tau \left\{ \left[\frac{\tau[\gamma + q(1-\gamma)]}{\tau\gamma + q(1-\gamma)(\tau + \alpha)} \right]^{1/\alpha} \frac{q(1-\gamma)(\tau + \alpha) - (1-\tau)\gamma}{\tau\gamma + q(1-\gamma)(\tau + \alpha)} - \left[\frac{\tau}{\tau + \alpha} \right]^{1/\alpha} \right\}$$

We have $V'_0(q) > 0$ if $\phi(q) > \psi(q)$ where

$$\phi(q) \equiv \frac{q(1-\gamma)(\tau + \alpha) - (1-\tau)\gamma}{\tau\gamma + q(1-\gamma)(\tau + \alpha)}$$

and

$$\psi(q) \equiv \left[\frac{\tau\gamma + q(1-\gamma)(\tau + \alpha)}{(\tau + \alpha)[\gamma + q(1-\gamma)]} \right]^{1/\alpha}$$

As $\phi(0) < 0 < \psi(0)$ and

$$\psi'(q) = \frac{\alpha\gamma}{(\tau + \alpha)^2[\gamma + q(1-\gamma)]^2} < \frac{\gamma}{[\tau\gamma + q(1-\gamma)(\tau + \alpha)]^2} = \phi'(q)$$

if there exists $\bar{q} > 0$ such that $\phi(\bar{q}) = \psi(\bar{q})$ we have $V_0'(q) < 0$ iff $q < \bar{q}$. Hence, $V_0(q)$ reaches its minimum at $\bar{q} \leq 1$ with $\bar{q} < 1$ if $\phi(1) > \psi(1)$. As

$$\phi(1) = \frac{(1-\gamma)(\tau + \alpha) - (1-\tau)\gamma}{\tau\gamma + (1-\gamma)(\tau + \alpha)} = 1 - \frac{\gamma}{\tau\gamma + (1-\gamma)(\tau + \alpha)}$$

and

$$\psi(1) = \left[\frac{\tau\gamma + (1-\gamma)(\tau + \alpha)}{\tau + \alpha} \right]^{1/\alpha} = \left[1 - \frac{\alpha\gamma}{\tau + \alpha} \right]^{1/\alpha}$$

we may have $\bar{q} < 1$ if α is large. In any case, it suffices that $V_0(0) > V_0(1)$ to have $q = 0$ optimal. As we have

$$\begin{aligned} V_0(1) &= \frac{a_0}{\mu} \left\{ 1 - \tau + \left(\frac{\tau}{\tau + \alpha(1-\gamma)} \right)^{\frac{1+\alpha}{\alpha}} [(1-\gamma)(\tau + \alpha) + \tau\gamma] \right\} \\ &= \frac{a_0}{\mu} \left\{ 1 - \tau \left\{ 1 - \left[\frac{\tau}{\tau + \alpha(1-\gamma)} \right]^{\frac{1}{\alpha}} \right\} \right\} \end{aligned}$$

and

$$\begin{aligned} V_0(0) &= \frac{a_0}{\mu} \left\{ 1 - \tau + \tau\gamma + (1-\gamma)(\tau + \alpha) \left(\frac{a_L}{a_0} \right)^{1+\alpha} \right\} \\ &= \frac{a_0}{\mu} \left\{ 1 - \tau(1-\gamma) \left\{ 1 - \left[\frac{\tau}{\tau + \alpha} \right]^{\frac{1}{\alpha}} \right\} \right\}, \end{aligned}$$

we have $V_0(0) > V_0(1)$ if

$$(1-\gamma) \left\{ 1 - \left[\frac{\tau}{\tau + \alpha} \right]^{\frac{1}{\alpha}} \right\} < 1 - \left[\frac{\tau}{\tau + \alpha(1-\gamma)} \right]^{\frac{1}{\alpha}}$$

hence if $\gamma > \xi(\gamma)$ where

$$\xi(\gamma) \equiv \frac{\left[\frac{\tau}{\tau + \alpha(1-\gamma)} \right]^{\frac{1}{\alpha}} - \left[\frac{\tau}{\tau + \alpha} \right]^{\frac{1}{\alpha}}}{1 - \left[\frac{\tau}{\tau + \alpha} \right]^{\frac{1}{\alpha}}}$$

As $\xi(0) = 0$, $\xi(1) = 1$ with

$$\xi'(\gamma) = \frac{\tau^{\frac{1}{\alpha}}}{\{1 - [\frac{\tau}{\tau+\alpha}]^{\frac{1}{\alpha}}\}[\tau + \alpha(1 - \gamma)]^{\frac{1}{\alpha}+1}} > 0$$

and $\xi''(\gamma) > 0$, we have $\gamma > \xi(\gamma)$ for all $\gamma \in (0, 1)$ and thus $q = 0$ is optimal.

D Proof of Proposition 6

The result is directly derived from the text in the case with private debt only. If the firm has issued both public and private debts, the equity value of a $\bar{\gamma}$ -type firm mimicking a $\underline{\gamma}$ -type firm is given by

$$\begin{aligned} S(a_R(\underline{\gamma}); \underline{\gamma}, \bar{\gamma}) &= (1 - \tau)[1 - q(\underline{\gamma})] \left\{ (1 - \underline{\gamma}) \frac{a_R(\underline{\gamma})}{\mu} - \frac{c}{r} - \left[(1 - \underline{\gamma}) \frac{a_L(\underline{\gamma})}{\mu} - \frac{c}{r} \right] \left[\frac{a_L(\underline{\gamma})}{a_R(\underline{\gamma})} \right]^\alpha \right\} \\ &= S(a_R(\underline{\gamma}); \underline{\gamma}, \underline{\gamma}). \end{aligned}$$

Observe that as $S(a_R(\underline{\gamma}); \bar{\gamma}, \bar{\gamma}) > 0$, the $\bar{\gamma}$ -type firm is deterred from mimicking the $\underline{\gamma}$ -type one when $q(\underline{\gamma}) = 1$. More generally, the incentive condition, $S(a_R(\underline{\gamma}); \underline{\gamma}, \bar{\gamma}) \leq \bar{S}(a_R(\underline{\gamma}); \bar{\gamma}, \bar{\gamma})$, can be written as

$$\begin{aligned} [1 - q(\underline{\gamma})] &\left\{ (1 - \underline{\gamma}) \frac{a_R(\underline{\gamma})}{\mu} - \frac{c}{r} - \left[(1 - \underline{\gamma}) \frac{a_L(\underline{\gamma})}{\mu} - \frac{c}{r} \right] \left[\frac{a_L(\underline{\gamma})}{a_R(\underline{\gamma})} \right]^\alpha \right\} \\ &\leq \frac{a_R(\underline{\gamma})}{\mu} - \frac{b+c}{r} + \left[\frac{b}{r} - \frac{\bar{\gamma} a_R(\bar{\gamma})}{\mu} \right] \left[\frac{a_R(\bar{\gamma})}{a_R(\underline{\gamma})} \right]^\alpha + \left[\frac{c}{r} - (1 - \bar{\gamma}) \frac{a_L(\bar{\gamma})}{\mu} \right] \left[\frac{a_L(\bar{\gamma})}{a_R(\underline{\gamma})} \right]^\alpha \end{aligned}$$

which simplifies to

$$[a_R(\bar{\gamma})]^\alpha \geq [a_R(\underline{\gamma})]^\alpha + \frac{c}{b} \{q(\underline{\gamma}) [a_R(\underline{\gamma})]^\alpha + [1 - q(\underline{\gamma})] [a_L(\underline{\gamma})]^\alpha - [a_L(\bar{\gamma})]^\alpha\} \quad (41)$$

using $a_L(\underline{\gamma})/\mu = \alpha c / [(1 + \alpha)r(1 - \underline{\gamma})]$, $a_R(\bar{\gamma})/\mu = \alpha b / [(1 + \alpha)r\bar{\gamma}]$ and

$$a_R(\underline{\gamma})/\mu = \frac{\alpha}{1 + \alpha} \frac{[b + q(\underline{\gamma})c]/r}{\underline{\gamma} + q(\underline{\gamma})(1 - \underline{\gamma})}.$$

With $q(\underline{\gamma}) = 0$, (41) simplifies to

$$\left[\frac{c}{b} \right]^{1+\alpha} \geq \frac{[1/\underline{\gamma}]^\alpha - [1/\bar{\gamma}]^\alpha}{[1/(1 - \bar{\gamma})]^\alpha - [1/(1 - \underline{\gamma})]^\alpha}$$

As $a_R(\gamma) > a_L(\gamma)$ for $\gamma \in \{\underline{\gamma}, \bar{\gamma}\}$, we must have $c/b < (1 - \bar{\gamma})/\bar{\gamma}$, which leads to the following necessary condition

$$\frac{1 - \bar{\gamma}}{\bar{\gamma}} \geq \frac{[\bar{\gamma}/\underline{\gamma}]^\alpha - 1}{1 - [(1 - \bar{\gamma})/(1 - \underline{\gamma})]^\alpha} \quad (42)$$

Denoting $\bar{\gamma} = \underline{\gamma} + k(1 - \underline{\gamma})$ where $k \in (0, 1)$ we have,

$$\frac{[\bar{\gamma}/\underline{\gamma}]^\alpha - 1}{1 - [(1 - \bar{\gamma})/(1 - \underline{\gamma})]^\alpha} = \frac{[1 + k(1 - \underline{\gamma})/\underline{\gamma}]^\alpha - 1}{1 - (1 - k)^\alpha} \approx \frac{[1 + k(1 - \underline{\gamma})/\underline{\gamma}]^\alpha - 1}{1 - (1 - \alpha k)}$$

For $\alpha \geq 1$, we have

$$\frac{[1 + k(1 - \underline{\gamma})/\underline{\gamma}]^\alpha - 1}{\alpha k} \geq \frac{[1 + \alpha k(1 - \underline{\gamma})/\underline{\gamma}] - 1}{\alpha k} = \frac{1 - \underline{\gamma}}{\underline{\gamma}} > \frac{1 - \bar{\gamma}}{\bar{\gamma}}$$

For smaller α , using the L'Hospital rule, we have

$$\lim_{\alpha \rightarrow 0} \frac{[1 + k(1 - \underline{\gamma})/\underline{\gamma}]^\alpha - 1}{\alpha k} = \frac{\ln [1 + k(1 - \underline{\gamma})/\underline{\gamma}]}{k}$$

For $\underline{\gamma}$ close to $\bar{\gamma}$ (hence, for low k , implying that $k(1 - \underline{\gamma})/\underline{\gamma}$ is small), we have

$$\frac{\ln [1 + k(1 - \underline{\gamma})/\underline{\gamma}]}{k} \approx \frac{1 - \underline{\gamma}}{\underline{\gamma}} > \frac{1 - \bar{\gamma}}{\bar{\gamma}}$$

For $\underline{\gamma}$ close to 0, we have

$$\lim_{\underline{\gamma} \rightarrow 0} \ln [1 + k(1 - \underline{\gamma})/\underline{\gamma}] / k = +\infty$$

Hence, as $\ln [1 + k(1 - \underline{\gamma})/\underline{\gamma}]$ decreases with $\underline{\gamma}$, we have $\ln [1 + k(1 - \underline{\gamma})/\underline{\gamma}] / k > (1 - \bar{\gamma})/\bar{\gamma}$. Consequently, condition (42) is violated, and thus (41) cannot be satisfied with $q(\gamma) = 0$.

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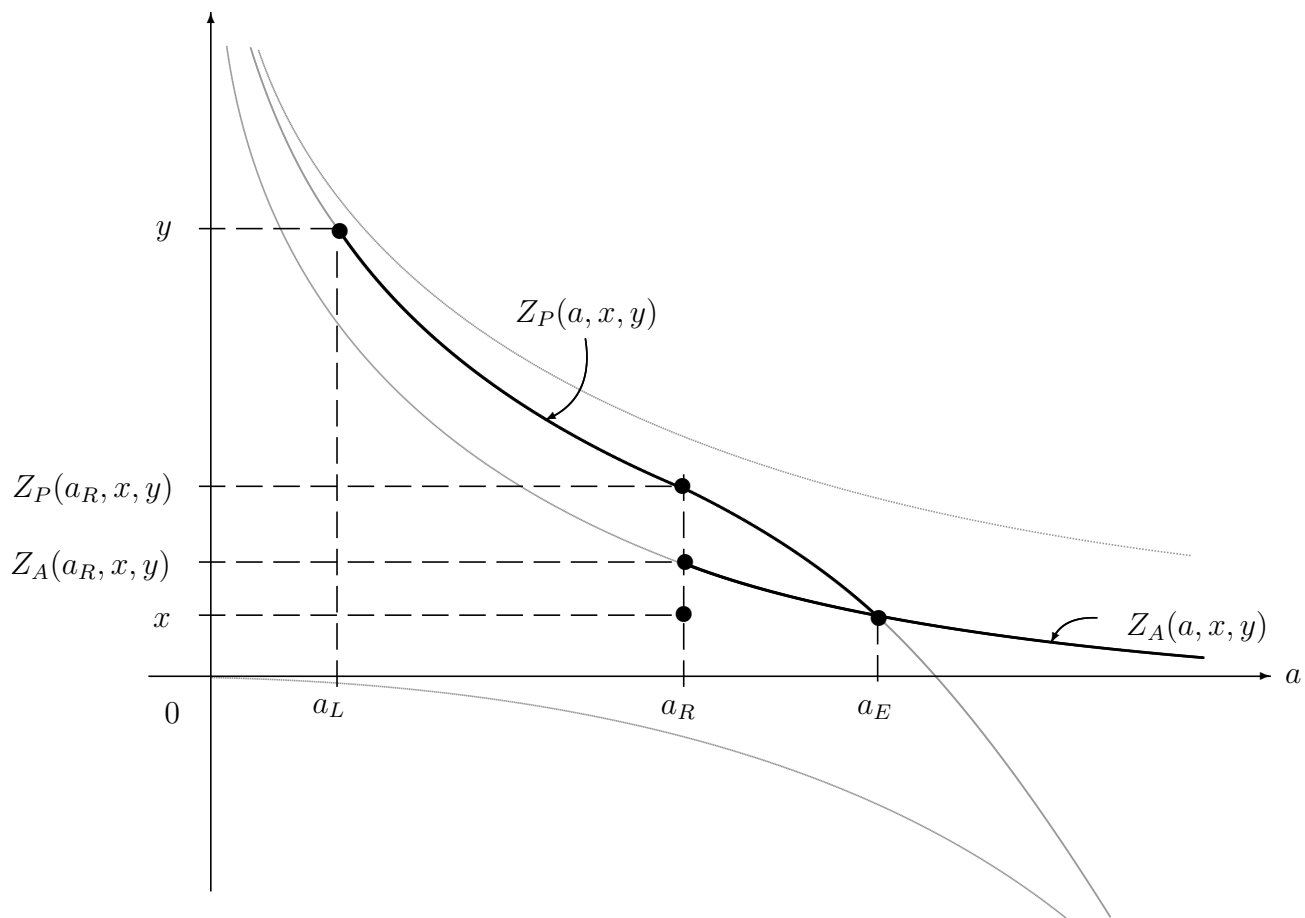


Figure 1: **Value of zero-coupon contingent claims**

Z_A and Z_P are the values of the zero-coupon contingent claims A and P . When EBIT flow a falls to threshold a_R (a passage from above), A either pays x with probability q and is worthless afterward, or it is transformed to claim P with probability $1 - q$. This latter either pays y at the first passage of a to a_L from above and is worthless afterward, or it is converted back to claim A at the first passage of a to a_E from below, whichever event occurs first.

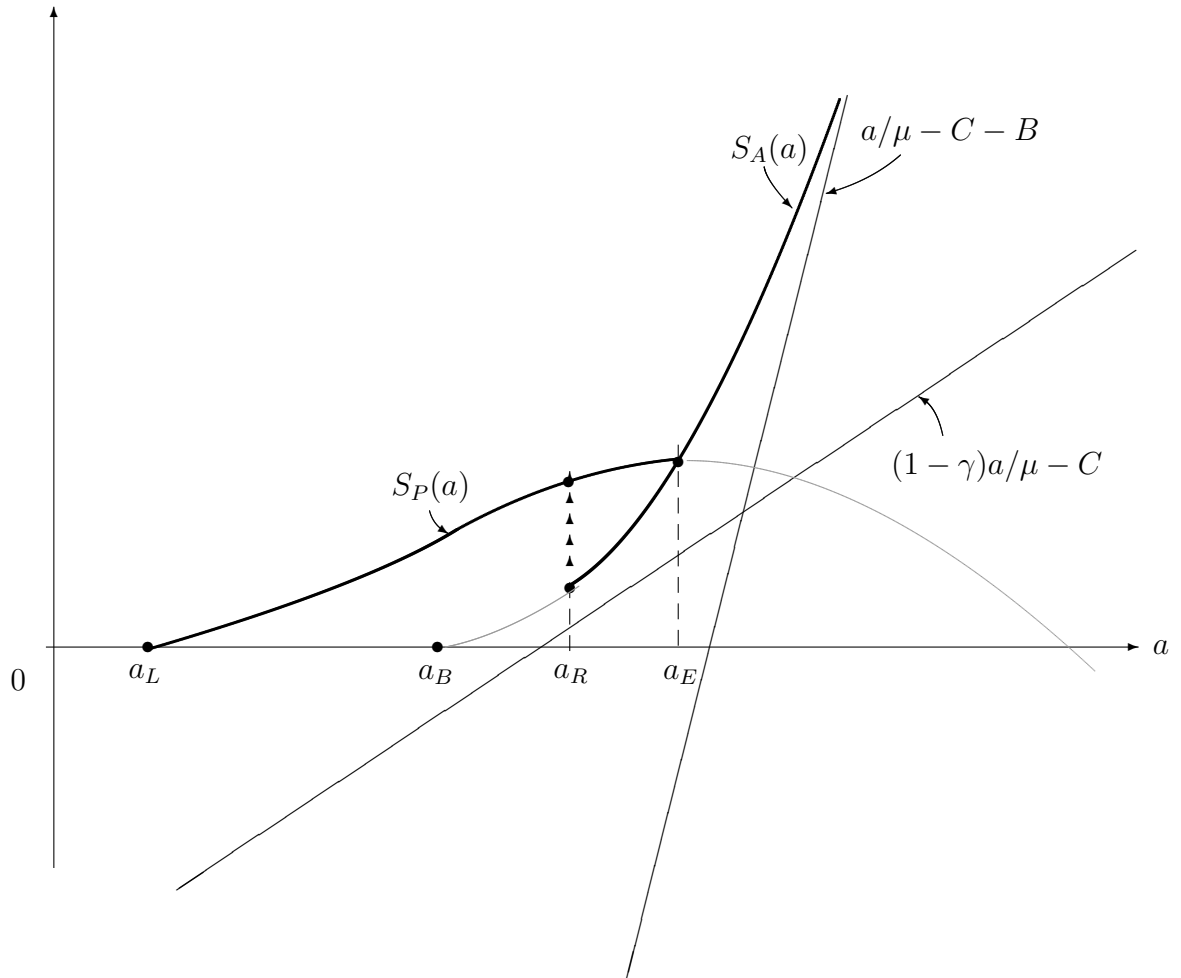


Figure 2: *Ex ante* and *ex post* equity values

If the bank accepts the renegotiation offer, the equity value jumps at a_R from $S_A(a_R)$ to $S_P(a_R)$. Then, it follows $S_P(a_t)$ until a_t reaches a_E or falls to a_L , in which case the firm is liquidated.

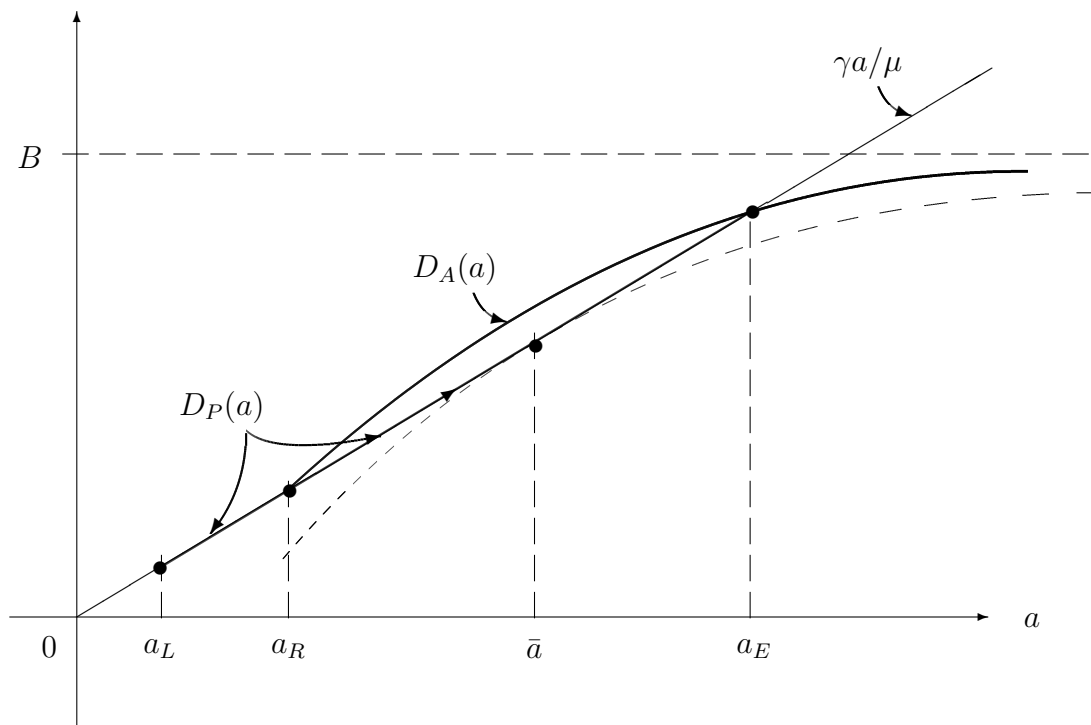


Figure 3: Bank debt value under renegotiation

Observe that the debt value crosses the liquidation value $\gamma a/\mu$ twice. Renegotiation is triggered when a_t falls to a_R . Then the debt value equals the liquidation value until a_t climbs to a_E or falls to a_L , in which case the firm is liquidated. The dashed curve corresponds to the debt value when the bank always accepts to renegotiate the debt service (relevant values are for $a > \bar{a}$; for $a \leq \bar{a}$, it follows the firm's liquidation value). It meets the liquidation value tangentially at \bar{a} , the corresponding renegotiation threshold.