



Centre Interuniversitaire sur le Risque,
les Politiques Économiques et l'Emploi

Cahier de recherche/Working Paper **08-33**

The Information Content of Implied Probabilities to Detect Structural Change

Alain Guay
Jean-François Lamarche

Octobre/October 2008
(First version : July 2005)

Guay: Département de sciences économiques, Université du Québec à Montréal, Québec, Canada, CIRPÉE and CIREQ

guay.alain@uqam.ca

Lamarche: Department of Economics, Brock University, St. Catharines, Ontario, Canada

jfl@brocku.ca

The first author gratefully acknowledges financial support of le Fonds québécois de la recherche sur la société et la culture (FQRSC). The first and second authors are thankful for the financial support of the Social Sciences and Humanities Research Council of Canada (SSHRC). We are grateful to Chuan Goh for his comments and also for the comments of participants at the CIREQ Conference on Generalized Method of Moments, Montréal, 2007, ESEM Budapest 2007 and CEA meeting 2007.

Abstract:

This paper proposes Pearson-type statistics based on implied probabilities to detect structural change. The class of generalized empirical likelihood estimators (see Smith (1997)) assigns a set of probabilities to each observation such that moment conditions are satisfied. These restricted probabilities are called implied probabilities. Implied probabilities may also be constructed for the standard GMM (see Back and Brown (1993)). The proposed test statistics for structural change are based on the information content in these implied probabilities. We consider cases of structural change with unknown breakpoint which can occur in the parameters of interest or in the overidentifying restrictions used to estimate these parameters. The test statistics considered here have good size and power properties.

Keywords: Generalized empirical likelihood, generalized method of moments, parameter instability, structural change

JEL Classification: C12, C32

1 Introduction

This paper proposes structural change tests based on implied probabilities resulting from estimation methods based on unconditional moment restrictions. The Generalized Method of Moments (GMM) is the standard method to estimate parameters of interest through moment restrictions. However, Monte Carlo results reveal that the GMM estimators may be seriously biased in small sample.¹ Recently, a number of alternative estimators to GMM have been proposed. Hansen, Heaton and Yaron (1996) suggested the continuous updated estimator (CUE) which shares same objective function that the GMM but with a weighting matrix depending on the parameters of interest. The empirical likelihood (EL) (see Qin and Lawless (1994)) and the exponential tilting (ET) estimators (see Kitamura and Stutzer (1997)) have also been proposed. Kitamura (2001) showed that tests for the validity of moment restrictions based on EL have optimal properties in terms of large deviations. In particular EL tests are shown to be more powerful than other standard tests. These alternative estimators are special cases of the generalized empirical likelihood (GEL) class considered by Smith (1997) and may be shown to be based on the Cressie and Read (1984) family of power divergence criteria. Newey and Smith (2004) showed (in an i.i.d. setting) that, although estimators based on the GMM, EL, ET or that are CUE have the same first order asymptotic distribution, they have different higher order asymptotic properties. Amongst their findings it is shown that the expression for the second order asymptotic bias of GEL has fewer components than the one of GMM (with EL having the fewest). Anatolyev (2005) extended the Newey and Smith setting to allow for weakly dependent data correlation and show that the asymptotic bias of smoothed GEL estimators is less than the GMM estimators especially with many moment conditions.

GEL estimators assign a probability to each observation such that the moment conditions are satisfied (see Smith (2004)). This resulting empirical measure is called implied probabilities. Implied probabilities may also be constructed for the standard GMM as shown by Back and Brown (1993). The interpretation of the implied probabilities is the following: less weight is assigned to an observation for which the moment restrictions are not satisfied at the estimated values of the parameters and more weight to an observation for which the moment restrictions are satisfied. As suggested by Back and Brown (1993), implied probabilities may then provide a useful diagnostic device for the model specification. In particular, implied probabilities may contain interesting information to detect instability in the sample. Consequently, we propose the use of these weights in detecting an unknown structural change in the model. Antoine, Bonnal and Renault (2007) use the weights given by implied probabilities to propose a three-step estimation procedure asymptotically higher equivalent to empirical likelihood. Schennach (2004) also discusses the use of these weights in the context of model misspecification. Ramalho and Smith (2005) considered Pearson-type test statistics (statistics based on the difference between restricted and unrestricted esti-

¹See in particular the special issue of *Journal of Business and Economic Statistics*, 1996, volume 14.

mators of the weights) for the validity of moment restrictions and parametric restrictions using implied probabilities.

The proposed test statistics to detect structural change are based on different measures of the discrepancy between these implied probabilities and the unconstrained empirical probabilities $\frac{1}{T}$. These test statistics fall in three categories: 1) general structural change tests to detect instability in the identifying restrictions and in the overidentifying restrictions; 2) structural change tests specially designed to detect instability in the identifying restrictions (see for example Andrews (1993)) and 3) structural change tests to detect instability in the overidentifying restrictions (see for example Hall and Sen (1999)). The asymptotic distribution of the statistic tests is derived for the case of unknown breakpoint under the null and under the alternative hypotheses. In a simulation study, we find that targeted tests based on implied probabilities performed quite well if the structural change corresponds to the target. That is, if instability is present in the identifying restrictions or in the overidentifying restrictions, then the targeted tests have size and power that are at least as good as those of the more standard tests. General tests for structural change have size and power properties that are between those of the targeted tests. Overall the test statistics based on implied probabilities considered in this paper have a nice intuitive appeal, are based on an estimation method that has been shown to have nice higher order asymptotic properties and are relatively easy to compute.

The paper is organized as follows. A discussion on the GMM and GEL estimators are presented in section 2. Section 3 presents formally the full-sample and partial-sample GMM and GEL estimators. Section 4 presents the test statistics proposed based on the implied probabilities. The simulation results are in Section 5 while the proofs are in the Appendix.

2 Discussion of GMM and GEL Estimators

In this section we present the estimators used in this paper. We start with an entropy-based formulation of the problem which puts emphasis on the informational content of the estimated weights. We then move to the more recent GEL formulation (see Newey and Smith (2004) and Smith (2004)).

We consider a random variable $\{x_t : 1 \leq t \leq T, T \geq 1\}$. Suppose we have $q \times 1$ vector function of the data $g(x_t, \theta)$ which depends on some unknown p -vector of parameters $\theta \in \Theta$ with $\Theta \subset R^p$ and that in the population their expected value is 0, namely

$$E[g(x_t, \theta_0)] = 0$$

Along this paper we consider the overidentifying case with $q > p$.

The standard GMM estimators (Hansen (1982)) are obtained as the solution of

$$\tilde{\theta}_T = \arg \min_{\theta \in \Theta} g_T(\theta)' W_T g_T(\theta)$$

where W_T is a random positive definite symmetric $q \times q$ matrix and $g_T(\theta) = \frac{1}{T} \sum_{t=1}^T g(x_t, \theta)$. The optimal weighting matrix is defined to be the inverse of the covariance matrix of the moment conditions, $W_T = \Omega_T^{-1}$ where Ω_T is a consistent estimator of

$$\Omega = \lim_{T \rightarrow \infty} \text{Var} \left(\frac{1}{\sqrt{T}} \sum_{t=1}^T g(x_t, \theta) \right).$$

The optimal weighting matrix can be estimated consistently using methods developed by Gallant (1987), Andrews and Monahan (1992) and Newey and West (1994), among several others.

We note that, in the GMM criterion function, the moment conditions receive equal weight ($1/T$) for each observations. Back and Brown (1993) derive a set of *implicit* weights using the GMM estimators given by

$$\pi_t(\theta) = \frac{1}{T} - \frac{1}{T-p} [J_{tT}(\theta) - J_T(\theta)]' \hat{\Omega}_T(\theta)^{-1} g_T(\theta) \quad (1)$$

with $J_T(\theta) = \frac{1}{T} \sum_{t=1}^T J_{tT}(\theta)$ and

$$J_{tT}(\theta) = \sum_{j=1}^T \kappa(|t-j|) g(x_{t-j}, \theta)$$

where $\kappa(|t-j|)$ is a real valued weighting function. The following estimator

$$\hat{\Omega}_T(\theta) = \frac{1}{T-p} \sum_{t=1}^T J_{tT}(\theta) g(x_t, \theta)'$$

is a consistent and positive definite estimator of the variance-covariance matrix and it has the usual form of a heteroskedasticity and autocorrelation consistent (HAC) weight matrix for standard kernels $\kappa(|t-s|)$ considered in the vast literature on consistent estimation of variance-covariance matrix (see e.g. Andrews (1991), Newey and West (1987) and Newey and West (1994)). The implied probabilities defined above are the empirical measure that ensures that moment conditions are satisfied in the sample.

Now letting the T -vector of *explicit* weights (explicit because the weights are used directly in estimation) be $\{\pi_t : 1 \leq t \leq T, T \geq 1\}$ we can recast the population moment conditions as

$$E_\pi[g(x_t, \theta_0)] = 0.$$

The vector π is determined by finding the most probable data distribution of the outcomes given the data. We can think of π as containing information on the content of the moment conditions. Therefore, $g(x, \theta)$ is viewed as a message. That is, when π is small, the message is informative and vice-versa. This relation is summarized by the function $f(\pi) = -\ln \pi$. The average information is then

$$S(\pi) \equiv E_\pi f(\pi) = -\sum_{t=1}^T \pi_t \ln \pi_t.$$

In this case, $S(\pi)$ can be interpreted as the entropy measure of Shannon (1948) and it captures the degree of uncertainty in the distribution π with respect to whether or not the distribution is concentrated or dispersed. The vector π is obtained by maximizing entropy

$$\max S(\pi) = - \sum_{t=1}^T \pi_t \ln \pi_t,$$

subject to $\sum_{t=1}^T \pi_t = 1$ and $\sum_{t=1}^T \pi_t g(x_t, \theta) = 0$.

With no constraint, we get $\pi_t = 1/T \quad \forall t$, the maximally uninformative uniform distribution, while with constraints, we want to choose π_t to be as maximally uninformative as the moment conditions will allow. We do not want to assert more about the distribution than is known via the moment conditions. In this sense, the probabilities make use of all the information that is available, and nothing more. In particular, we focus on detecting a structural change in the moment conditions. With no structural change, the weights will fluctuate around $1/T$, otherwise the entropy formulation will attempt to reduce the weight on the observation characterized by the change, and at the same time put more weights on the remaining observations so as to make $S(\pi)$ as large as possible.

We can transpose this formulation using the Kullback and Leibler (1951) information criterion which measures the discrepancy between two distributions p and π . If the subject distribution is π and the reference distribution is $p_t = 1/T, \forall t$ we have

$$KLIC(\pi, p = 1/T) = \sum_{t=1}^T \pi_t [\ln(\pi_t) - \ln(1/T)] = \sum_{t=1}^T \pi_t \ln(\pi_t) + \ln(T).$$

So that maximizing entropy is equivalent to minimizing the $KLIC(\pi, p = 1/T)$. Estimates of π , given θ , are obtained by maximizing $S(\pi)$ (or by minimizing $KLIC(\pi, p = 1/T)$) subject to the weighted zero functions and the probability constraint. The solution to the Lagrangian yields

$$\pi_t^{ET}(\theta) = \frac{\exp(\gamma' g(x_t, \theta))}{\sum_{t=1}^T \exp(\gamma' g(x_t, \theta))}$$

where the q -vector γ contains the Lagrange multipliers and as such measure the degree of departure from zero of the moment conditions and ET stands for exponential tilting (see Kitamura and Stutzer (1997)). Estimates of θ are obtained by substituting π in $S(\pi)$, maximizing it with respect to γ and then with respect to θ (see for example Kitamura and Stutzer (1997)).

If we interchange the subject/reference distributions we get

$$KLIC(p = 1/T, \pi) = \sum_{t=1}^T (1/T) [\ln(1/T) - \ln(\pi_t)]$$

and the solution to the optimization problem yields a different set of weights given by

$$\pi_t^{EL}(\theta) = \frac{1}{T [1 + \gamma' g(x_t, \theta)]}$$

where *EL* stands for empirical likelihood (see Qin and Lawless (1994) for example). When we evaluate the weights at some estimators we obtain $\pi_t^{ET}(\tilde{\theta}_T)$ and $\pi_t^{EL}(\tilde{\theta}_T)$. Recently, Schennach (2004) combined ET and EL into the ETEL estimator that combines the advantages of each approach.

We mentioned in the introduction that less weight is assigned to an observation for which the moment conditions are not satisfied. In this section we have seen that the vector π contains all the relevant information with respect to the moment conditions. We now provide a graphical intuition on the use of the weights in the detection of a structural change. We consider a small simulation study that contains three examples that have been studied in the structural change and entropy literature. The first example, which encompasses three cases, is similar to the one used by Imbens, Spady and Johnson (1998) and consists of estimating a single parameter, θ , with 2 moment conditions:

$$E[x_t - \theta] = 0 \text{ and } E[(x_t - \theta)^2 - 4] = 0$$

with a sample of 100 observations and $x_t = \theta_t + \epsilon_t$ where $\epsilon_t \sim N(0, 4)$. We consider a pulse, a break and a regime shift cases. In the pulse case we have $\theta_t = 5$ for $t = 50$ and $\theta = 10$ otherwise. For the break case we have $\theta_t = 10$ for $t \leq 20$ and $t > 80$ while $\theta_t = 15$ for $21 \leq t \leq 80$. Finally, the regime shift case has $\theta_t = 10$ for $t \leq 20$ and $\theta_t = 15$ otherwise. In these cases, structural change occurs via the parameters and can be tested using procedures proposed by Andrews (1993) and by Andrews and Ploberger (1994).

In contrast, the next two examples consist of structural change through the moment conditions. Following Hall and Horowitz (1996) and Gregory, Lamarche and Smith (2002) we study a simulated environment with CRRA preferences and making a distributional assumption on consumption growth, x_t , with i.i.d data and $T = 100$. In particular, we assume that consumption growth follows a $N(0, \sigma^2 = 0.16)$. There is a single parameter to be estimated, γ , the coefficient of CRRA and two moment conditions are used:

$$\begin{aligned} E_t \exp[-\gamma \ln x_{t+1} - 9\sigma^2/2 + (3 - \gamma)z_t] &= 1 \\ E_t z_t \exp[-\gamma \ln x_{t+1} - 9\sigma^2/2 + (3 - \gamma)z_t - 1] &= 0 \end{aligned}$$

with $z_t \sim N(0, \sigma^2)$. The moment conditions are satisfied when $\gamma = 3$. The structural break occurs in period 51 and is summarized by a shift in γ from 3 to 4.

Lastly, as in Ghysels, Guay and Hall (1997), we have the estimation of an autoregressive parameter using two moment conditions when the data generating process is an *AR*(1) ($x_t = \rho x_{t-1} + \epsilon_t$), for $t \leq 50$, and an *ARMA*(1,2) otherwise ($x_t = \rho x_{t-1} + \epsilon_t + 0.5\epsilon_{t-2}$). There are 100 observations and $\epsilon_t \sim N(0, 1)$. The two instruments used are the first and second lags of x_t . The two moment conditions are then:

$$\begin{aligned} E[x_{t-1}(x_t - \rho x_{t-1})] &= 0 \\ E[x_{t-2}(x_t - \rho x_{t-1})] &= 0 \end{aligned}$$

The instability occurs because the second moment condition is violated after $t > 50$.

Figure 1 shows the average of the vector of implied probabilities π over 10,000 replications. The key feature of these panels is that when there is no break, the weights fluctuate around $1/T = 1/100$ (upper right panel). With a structural break in the parameter or in the moment conditions, however, more weight is given to observations (and moment conditions) for which there is no break while less weight is assigned to an observation which violates the moment conditions. This simple simulation study clearly show that implied probabilities contain interesting information to detect structural change. In this paper, we examine the information contained in the estimated implied probabilities to detect structural change and propose test statistics based on some function of implied probabilities.

Now following the recent econometric literature (see Caner (2004), Newey and Smith (2004), Smith (2004), Caner (2005), Guggenberger and Smith (2007) and Ramalho and Smith (2005)) on GEL we let $\rho(\phi)$ be a function of a scalar ϕ that is concave on its domain, an open interval Φ that contains 0. Also, let $\tilde{\Gamma}_T(\theta) = \{\gamma : \gamma'g(x_t, \theta) \in \Phi, t = 1, \dots, T\}$. Then, the GEL estimator is a solution to the problem

$$\tilde{\theta}_T = \arg \min_{\theta \in \Theta} \sup_{\gamma \in \tilde{\Gamma}_T(\theta)} \sum_{t=1}^T \frac{[\rho(\gamma'g(x_t, \theta)) - \rho_0]}{T}$$

where $\rho_j(\cdot) = \partial^j \rho(\cdot) / \partial \phi^j$ and $\rho_j = \rho_j(0)$ for $j = 0, 1, 2, \dots$. Under this formulation a number of estimators can be obtained. First, the ET estimator of θ is found by setting $\rho(\phi) = -\exp(\phi)$. Second, the EL estimator of θ by setting $\rho(\phi) = \ln(1 - \phi)$. Third, the continuously updated estimator, as opposed to the two-step estimator presented above, of Hansen *et al.* (1996) can also be obtained from the GEL formulation by using a quadratic function for $\rho(\phi) = -(1 + \phi)^2/2$.

As in the GMM context an adjustment for the dynamic structure of $g(x_t, \theta)$ can also be made in the GEL context (see Kitamura and Stutzer (1997), Smith (2000), Smith (2004) and Guggenberger and Smith (2007)). The adjustment consists of smoothing the original moment conditions $g(x_t, \theta)$. Defining the smoothed moment conditions as

$$g_{tT}(\theta) = \frac{1}{S_T} \sum_{s=t-T}^{t-1} k\left(\frac{s}{S_T}\right) g(x_{t-s}, \theta)$$

for $t = 1, \dots, T$ and S_T is a bandwidth parameter, $k(\cdot)$ a kernel function and we define where $k_j = \int_{-\infty}^{\infty} k(a)^j da$. In the time series context, the criteria is then given by:

$$\sum_{t=1}^T \frac{[\rho(k\gamma'g_{tT}(\theta)) - \rho_0]}{T}$$

where $k = \frac{k_1}{k_2}$ (see Smith (2004)).

3 Full and Partial-Samples GMM and GEL Estimators

To establish the asymptotic distribution theory of tests for structural change in the parameters based on implied probabilities we need to elaborate on the specification of the parameter vector in our generic setup. We will consider parametric models indexed by parameters (β, δ) . With no structural change we define a vector of parameters $(\beta, \delta) \in B \times \Delta \in R^p$ with $p = r + \nu$. Following Andrews (1993) we make a distinction between pure structural change when no subvector δ appears and the entire parameter vector is subject to structural change under the alternative and partial structural change which corresponds to cases where only a subvector β is subject to structural change under the alternative. The generic null can be written as follows:

$$H_0 : \beta_t = \beta_0 \quad \forall t = 1, \dots, T. \quad (2)$$

The tests that we will consider assume as alternative that at some point in the sample there is a single structural break, like for instance:

$$\beta_{tT} = \begin{cases} \beta_1(s) & t = 1, \dots, [Ts] \\ \beta_2(s) & t = [Ts] + 1, \dots, T \end{cases}$$

where s determines the fraction of the sample before and after the assumed breakpoint and $[.]$ denotes the greatest integer function. The separation $[Ts]$ represents a possible breakpoint which is governed by an unknown parameter s . Hence, we will consider a setup with a parameter vector which encompasses any kind of partial or pure structural change involving a single breakpoint. In particular, we consider a $2r + \nu$ dimensional parameter vector $\theta = (\beta'_1, \beta'_2, \delta)'$ where β_1 and $\beta_2 \in B \subset R^r$ and $\theta \in \Theta = B \times B \times \Delta \subset R^{2r + \nu}$. The parameters β_1 and β_2 apply to the samples before and after the presumed breakpoint and the null implies that:

$$H_0 : \beta_1 = \beta_2 = \beta_0. \quad (3)$$

We will formulate all our models in terms of θ . Special cases could be considered whenever restrictions are imposed in the general parametric formulation. One such restriction would be that $\theta_0 = (\beta'_0, \beta'_0)'$, which would correspond to the null of a pure structural change hypothesis. Once we have defined the moment conditions we will also translate this into overidentifying restrictions and relate it to structural change tests, following the analysis of Sowell (1996a) and Hall and Sen (1999).

3.1 Definitions

We need to impose restrictions on the admissible class of functions and processes involved in estimation to guarantee well-behaved asymptotic properties of GMM and GEL estimators using the entire data sample or subsamples of observations. A set of regularity conditions is required to obtain weak convergence of partial-sample GMM and GEL estimators to a function of Brownian motions. To streamline the

presentation we provide a detailed description of them in Appendix 7.1. We now formally define the standard GMM estimator using the full sample.

Definition 3.1. *The full-sample General Method of Moments estimator $\{(\tilde{\beta}_T, \tilde{\delta}_T)\}$ is a sequence of random vectors such that:*

$$\left(\tilde{\beta}'_T, \tilde{\delta}'_T\right)' = \arg \min_{(\beta, \delta) \in B \times \Delta} g_T(\beta, \delta)' \hat{W}_T g_T(\beta, \delta)$$

where \hat{W}_T is a random positive definite symmetric $q \times q$ matrix.

The optimal weighting matrix W is defined to be the inverse of Ω which is defined as:

$$\Omega = \lim_{T \rightarrow \infty} \text{Var} \left(\frac{1}{\sqrt{T}} \sum_{t=1}^T g(x_t, \beta_0, \delta_0) \right).$$

The optimal weighting matrix can be estimated consistently using methods developed by Gallant (1987), Andrews and Monahan (1992) and Newey and West (1994), among several others.

To characterize the asymptotic distribution we define the following matrices:

$$G_\beta = \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T E \partial g(x_t, \beta_0, \delta_0) / \partial \beta' \in R^{q \times r},$$

$$G_\delta = \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T E \partial g(x_t, \beta_0, \delta_0) / \partial \delta' \in R^{q \times \nu},$$

$$G = [G_\beta \quad G_\delta] \in R^{q \times p}.$$

where $p = r + \nu$. Finally, let $\tilde{\theta}_T = \left(\tilde{\beta}'_T, \tilde{\beta}'_T, \tilde{\delta}'_T\right)'$ be a $2r + \nu$ -vector. Hereafter, the vector $\tilde{\theta}_T$ is called the full-sample estimator of θ . This restricted estimator is consistent only under the null that $\beta_1 = \beta_2$.

Several tests for structural change involve partial-sample GMM estimators defined by Andrews (1993). We consider again the two subsamples, the first based on observations $t = 1, \dots, [Ts]$ and the second covering $t = [Ts] + 1, \dots, T$ where $s \in S \subset (0, 1)$. The partial-sample GMM estimators for $s \in S$ based on the first and the second subsamples are formally defined as:

Definition 3.2. *A partial-sample General Method of Moments estimator $\{\hat{\theta}_T(s)\}$ is a sequence of random vectors such that:*

$$\hat{\theta}_T(s) = \arg \min_{\theta \in \Theta} g_T(\theta, s)' \hat{W}_T(s) g_T(\theta, s)$$

for all $s \in S$. Define $g_t(\theta, s) = (g(x_t, \beta_1, \delta)' , 0)'$ for $t = 1, \dots, [Ts]$ and $g_t(\theta, s) = (0', g(x_t, \beta_2, \delta))' \in R^{2q \times 1}$ for $t = [Ts] + 1, \dots, T$ such that

$$g_T(\theta, s) = \frac{1}{T} \sum_{t=1}^T g_t(\theta, s) = \frac{1}{T} \sum_{t=1}^{[Ts]} \begin{bmatrix} g(x_t, \beta_1, \delta) \\ 0 \end{bmatrix} + \frac{1}{T} \sum_{t=[Ts]+1}^T \begin{bmatrix} 0 \\ g(x_t, \beta_2, \delta) \end{bmatrix}$$

and $\hat{W}_T(s)$ is a random positive definite symmetric $2q \times 2q$ matrix.

According to the definition above $\hat{\theta}_T(s) = \left(\hat{\beta}_{1T}(s)', \hat{\beta}_{2T}(s)', \hat{\delta}_T(s)' \right)'$ is a $2r + \nu$ -vector with an estimator $\hat{\beta}_{1T}(s)$ that uses the first subsample $t = 1, \dots, [Ts]$, an estimator $\hat{\beta}_{2T}(s)$ that uses the second subsample $t = [Ts] + 1, \dots, T$ and an estimator $\hat{\delta}_T(s)$ that uses all the sample.

The partial-sample optimal weighting matrix is defined as the inverse of $\Omega(s)$ where

$$\Omega(s) = \lim_{T \rightarrow \infty} \text{Var} \left(\frac{1}{\sqrt{T}} \begin{bmatrix} \sum_{t=1}^{[Ts]} g(x_t, \beta_0, \delta_0) \\ \sum_{t=[Ts]+1}^T g(x_t, \beta_0, \delta_0) \end{bmatrix} \right)$$

which under the null (3) is asymptotically equal to

$$\Omega(s) = \begin{bmatrix} s\Omega & 0 \\ 0 & (1-s)\Omega \end{bmatrix}.$$

We also define

$$G(s) = \begin{bmatrix} sG_\beta & 0 & sG_\delta \\ 0 & (1-s)G_\beta & (1-s)G_\delta \end{bmatrix} \in R^{2q \times (2r+\nu)}.$$

In the GEL setting, the parameter vector is augmented by a vector of auxiliary parameters γ where each element of this vector is associated to an element of the smoothed moment conditions $g_{tT}(\theta)$. Under the null of no structural change relative to the specification of the model, the generic null hypothesis for this vector of auxiliary parameters can be written as follows:

$$H_0 : \gamma_t = \gamma_0 = 0 \quad \forall t = 1, \dots, T. \quad (4)$$

As for the parameter vector β , the tests that we will consider assume as alternative that at some point in the sample there is a single structural break, namely:

$$\gamma_t = \begin{cases} \gamma_1 & t = 1, \dots, [Ts] \\ \gamma_2 & t = [Ts] + 1, \dots, T. \end{cases}$$

Thus under the null of no structural change $H_0 : \gamma_1 = \gamma_2 = \gamma_0 = 0$. Guay and Lamarche (2007) show that a structural change in γ is associated with instability in the overidentifying restrictions.

We now formally define the GEL estimator using the full sample.

Definition 3.3. Let $\rho(\phi)$ be a function of a scalar ϕ that is concave on its domain, an open interval Φ that contains 0. Also, let $\tilde{\Gamma}_T(\beta, \delta) = \{\gamma : k\gamma'g_{tT}(\beta, \delta) \in \Phi, t = 1, \dots, T\}$ with $k = \frac{k_1}{k_2}$. Then, the full-sample GEL estimator $\{\tilde{\theta}_T\}$ is a sequence of random vectors such that:

$$\left(\tilde{\beta}'_T, \tilde{\delta}'_T \right)' = \arg \min_{(\beta, \delta) \in B \times \Delta} \sup_{\gamma \in \tilde{\Gamma}_T(\beta, \delta)} \sum_{t=1}^T \frac{[\rho(k\gamma'g_{tT}(\beta, \delta)) - \rho_0]}{T}$$

where $\rho_j(\cdot) = \partial^j \rho(\cdot) / \partial \phi^j$ and $\rho_j = \rho_j(0)$ for $j = 0, 1, 2, \dots$

The criteria is normalized so that $\rho_1 = \rho_2 = -1$ (see Smith (2004)). As mentioned earlier, the GEL estimator admits a number of special cases recently proposed in the econometrics literature. The CUE of Hansen, Heaton and Yaron (1996) corresponds to the following quadratic function $\rho(\phi) = -(1 + \phi)^2/2$. The EL estimator (Qin and Lawless, 1994) is a GEL estimator with $\rho(\phi) = \ln(1 - \phi)$. The ET estimator (Kitamura and Stutzer, 1997) is obtained with $\rho(\phi) = -\exp(\phi)$.

More precisely, the GEL estimator is obtained as the solution to a saddle point problem. Firstly, the criterion is maximized for given vector (β, δ) . Thus,

$$\tilde{\gamma}_T(\beta, \delta) = \arg \sup_{\gamma \in \tilde{\Gamma}(\beta, \delta)} \sum_{t=1}^T \frac{[\rho(k\gamma' g_{tT}(\beta, \delta)) - \rho_0]}{T}.$$

Secondly, the GEL estimator $(\tilde{\beta}'_T, \tilde{\delta}'_T)'$ is given by the following minimization problem:

$$(\tilde{\beta}'_T, \tilde{\delta}'_T)' = \arg \min_{(\beta, \delta) \in B \times \Delta} \sum_{t=1}^T \frac{[\rho(k\tilde{\gamma}_T(\beta, \delta)' g_{tT}(\beta, \delta)) - \rho_0]}{T}.$$

From now on, following Kitamura and Stutzer (1997) and Guggenberger and Smith (2007) we focus on the truncated kernel defined by

$$k(x) = 1 \text{ if } |x| \leq 1 \text{ and } k(x) = 0 \text{ otherwise}$$

to obtain the following smoothed moment conditions

$$g_{tT}(\beta, \delta) = \frac{1}{2K_T + 1} \sum_{j=-K_T}^{K_T} g(x_{t-j}, \beta, \delta).$$

To handle the endpoints in the smoothing we use the approach of Smith (2004) and Guggenberger and Smith (2007) which sets

$$g_{tT}(\beta, \delta) = \frac{1}{2K_T + 1} \sum_{j=\max\{t-T, -K_T\}}^{\min\{t-1, K_T\}} g(x_{t-j}, \beta, \delta).$$

We can easily show for this kernel that $k = \frac{k_1}{k_2} = 1$. A consistent estimator of the long run covariance matrix is then given by:

$$\tilde{\Omega}_T(\beta, \delta) = \frac{2K_T + 1}{T} \sum_{t=1}^T g_{tT}(\beta, \delta) g_{tT}(\beta, \delta)'$$

The weighting matrix thus obtained using this type of kernel is similar to the one obtained with the Bartlett kernel estimator of the long run covariance matrix of the moment conditions (see Smith (2004)). Define also the derivatives of the smoothed moment conditions as:

$$G_{tT}(\beta, \delta) = \frac{1}{2K_T + 1} \sum_{j=-K_T}^{K_T} \frac{\partial g(x_{t-j}, \beta, \delta)}{\partial (\beta', \delta')}.$$

Now consider a possible breakpoint $[Ts]$. Define the vector of auxiliary parameters $\gamma(s) = (\gamma_1(s)', \gamma_2(s)')$ where γ_1 is the vector of the auxiliary parameters for the first part of the sample e.g. $t = 1, \dots, [Ts]$ and γ_2 for the second part of the sample; $t = [Ts] + 1, \dots, T$. The partial-sample GEL estimators for $s \in S$ based on the first and the second subsamples are formally defined as:

Definition 3.4. Let $\rho(\phi)$ be a function of a scalar ϕ that is concave on its domain, an open interval Φ that contains 0. Also, let $\hat{\Gamma}_T(\theta, s) = \{\gamma(s) = (\gamma_1(s)', \gamma_2(s)')' : k\gamma(s)'g_{tT}(\theta, s)\}$. A partial-sample General Empirical Likelihood (PS-GEL) estimator $\{\hat{\theta}_T(s)\}$ is a sequence of random vectors such that:

$$\begin{aligned}\hat{\theta}_T(s) &= \arg \min_{\theta \in \Theta} \sup_{\gamma(s) \in \hat{\Gamma}_T(\theta, s)} \sum_{t=1}^T \frac{[\rho(k\gamma(s)'g_{tT}(\theta, s)) - \rho_0]}{T} \\ &= \arg \min_{\theta \in \Theta} \sup_{\gamma(s) \in \hat{\Gamma}_T(\theta, s)} \left(\sum_{t=1}^{[Ts]} \frac{[\rho(k\gamma_1'g_{tT}(\beta_1, \delta)) - \rho_0]}{T} + \sum_{t=[Ts]+1}^T \frac{[\rho(k\gamma_2'g_{tT}(\beta_2, \delta)) - \rho_0]}{T} \right)\end{aligned}$$

for all $s \in S$, where $g_{tT}(\theta, s) = (g_{tT}(\beta_1, \delta)', 0) \in \mathbb{R}^{2q \times 1}$ for $t = 1, \dots, [Ts]$ and $g_{tT}(\theta, s) = (0', g_{tT}(\beta_2, \delta)') \in \mathbb{R}^{2q \times 1}$ for $t = [Ts] + 1, \dots, T$ with $\gamma(s) = (\gamma_1(s)', \gamma_2(s)') \in \mathbb{R}^{2q \times 1}$.

To be more precise, the first order conditions corresponding to the Lagrange multiplier γ are obtained from the maximization of the partial-sample GEL criterion for a given β_1, β_2, δ . Thus,

$$\begin{aligned}\hat{\gamma}_{1T}(\beta_1, \delta) &= \arg \sup_{\gamma_1 \in \hat{\Gamma}_{1T}(\beta_1, \delta)} \sum_{t=1}^{[Ts]} \frac{[\rho(k\gamma_1(\beta_1, \delta)'g_{tT}(\beta_1, \delta)) - \rho_0]}{T}, \\ \hat{\gamma}_{2T}(\beta_2, \delta) &= \arg \sup_{\gamma_2 \in \hat{\Gamma}_{2T}(\beta_2, \delta)} \sum_{t=[Ts]+1}^T \frac{[\rho(k\gamma_2(\beta_2, \delta)'g_{tT}(\beta_2, \delta)) - \rho_0]}{T}.\end{aligned}$$

with $\hat{\Gamma}_{1T}(\beta_1, \delta) = \{\gamma_1 : k\gamma_1'g_{tT}(\beta_1, \delta) \in \Phi, t = 1, \dots, [Ts]\}$ and $\hat{\Gamma}_{2T}(\beta_2, \delta) = \{\gamma_2 : k\gamma_2'g_{tT}(\beta_2, \delta) \in \Phi, t = [Ts] + 1, \dots, T\}$.

We now present the corresponding implied probabilities defined by Back and Brown (1993) and Smith (2004) for the most commonly used full and partial-sample estimators. Following Back and Brown (1993), the full-sample GMM implied probabilities are defined as:

$$\pi_t(\tilde{\theta}_T) = \frac{1}{T} - \frac{1}{T} \left[J_{tT}(\tilde{\theta}_T) - J_T(\tilde{\theta}_T) \right]' \hat{\Omega}_T(\tilde{\theta}_T)^{-1} g_T(\tilde{\theta}_T)$$

with $J_T(\tilde{\theta}_T) = \frac{1}{T} \sum_{t=1}^T J_{tT}(\tilde{\theta}_T)$ and

$$J_{tT}(\tilde{\theta}_T) = \sum_{j=1}^T \kappa(|t-j|) g(x_{t-j}, \tilde{\theta}_T)$$

where $\kappa(|t-j|)$ is a real valued weighting function. In practice, however, some of the estimated probabilities may be negative in finite sample although these probabilities are asymptotically positive. Antoine,

Bonnal, and Renault (2007) proposed a shrinkage procedure defined as a weighted average of the standard 2S-GMM's implied probabilities ($1/T$) and the computed implied probabilities to guarantee the non-negativity of these implied probabilities in finite sample.

The general formula of the implied probabilities for the full-sample GEL estimator is defined by the following ratio (see Smith, 2004):

$$\pi_t^{GEL}(\tilde{\theta}_T) = \frac{\rho_1 \left(\tilde{\gamma}'_T g_{tT}(\tilde{\theta}_T) \right)}{\sum_{t=1}^T \rho_1 \left(\tilde{\gamma}'_T g_{tT}(\tilde{\theta}_T) \right)}.$$

Implied probabilities for the full-sample ET, EL and CUE estimators with the smoothed moment conditions are respectively given by:

$$\pi_t^{ET}(\tilde{\theta}_T) = \frac{\exp[\tilde{\gamma}'_T g_{tT}(\tilde{\theta}_T)]}{\sum_{t=1}^T \exp[\tilde{\gamma}'_T g_{tT}(\tilde{\theta}_T)]},$$

$$\pi_t^{EL}(\tilde{\theta}_T) = \frac{1}{T[1 + \tilde{\gamma}'_T g_{tT}(\tilde{\theta}_T)]},$$

and

$$\pi_t^{CUE}(\tilde{\theta}_T) = \frac{1}{T} - \frac{1}{T} g_{tT}(\tilde{\theta}_T) \left[\frac{1}{T} \sum_{t=1}^T g_{tT}(\tilde{\theta}_T) g_{tT}(\tilde{\theta}_T)' \right]^{-1} \frac{1}{T} \sum_{t=1}^T g_{tT}(\tilde{\theta}_T).$$

Note that $\frac{2K+1}{T} \sum_{t=1}^T g_{tT}(\tilde{\theta}_T) g_{tT}(\tilde{\theta}_T)'$ is a consistent estimator of Ω .

The corresponding unrestricted partial-sample GMM implied probabilities are defined for $s \in S$ as:

$$\pi_t(\hat{\theta}_T(s), s) = \frac{1}{T} - \frac{1}{T} \left[J_{tT}(\hat{\theta}_T(s), s) - J_T(\hat{\theta}_T(s), s) \right]' \hat{\Omega}_T(s)^{-1} g_T(\hat{\theta}_T(s), s)$$

with $J_T(\hat{\theta}_T(s), s) = \frac{1}{T} \sum_{t=1}^T J_{tT}(\hat{\theta}_T(s), s)$ and

$$J_{tT}(\hat{\theta}_T(s), s) = \sum_{j=1}^T \kappa(|t-j|) g_{t-j}(\hat{\theta}_T(s), s)$$

where $\kappa(|t-j|)$ is a real valued weighting function and $g_T(\hat{\theta}_T(s), s) = \frac{1}{T} \sum_{t=1}^T g_t(\hat{\theta}_T(s), s)$. The general formula for the unrestricted partial-sample implied probabilities for the GEL are defined for $s \in S$ as:

$$\pi_t^{GEL}(\hat{\theta}_T(s), s) = \frac{\rho_1 \left(\hat{\gamma}_T(s)' g_{tT}(\hat{\theta}_T(s), s) \right)}{\sum_{t=1}^T \rho_1 \left(\hat{\gamma}_T(s)' g_{tT}(\hat{\theta}_T(s), s) \right)}. \quad (5)$$

For example, in the case of t between observations 1 and $[Ts]$, we get for the unrestricted implied probabilities at t :

$$\pi_t^{GEL}(\hat{\beta}_{1T}(s), \hat{\delta}_T(s), s) = \frac{\rho_1 \left(\hat{\gamma}_{1T}(s)' g_{tT}(\hat{\beta}_{1T}(s), \hat{\delta}_T(s)) \right)}{\sum_{t=1}^T \rho_1 \left(\hat{\gamma}_{1T}(s)' g_{tT}(\hat{\beta}_{1T}(s), \hat{\beta}_{2T}(s), \hat{\delta}_T(s), s) \right)}.$$

For the commonly used GEL partial-sample estimators, we get

$$\begin{aligned}\pi_t^{ET}(\hat{\theta}_T(s), s) &= \frac{\exp[\hat{\gamma}_T(s)' g_{tT}(\hat{\theta}_T(s), s)]}{\sum_{t=1}^T \exp[\hat{\gamma}_T(s)' g_{tT}(\hat{\theta}_T(s), s)]}, \\ \pi_t^{EL}(\hat{\theta}_T(s), s) &= \frac{1}{T[1 + \hat{\gamma}_T(s)' g_{tT}(\hat{\theta}_T(s), s)]}, \\ \pi_t^{CUE}(\hat{\theta}_T(s), s) &= \frac{1}{T} - \frac{1}{T} g_{tT}(\hat{\theta}_T(s), s) \left[\frac{1}{T} \sum_{t=1}^T g_{tT}(\hat{\theta}_T(s), s) g_{tT}(\hat{\theta}_T(s), s)' \right]^{-1} \frac{1}{T} \sum_{t=1}^T g_{tT}(\hat{\theta}_T(s), s).\end{aligned}$$

The purpose of the next subsection is to refine the null hypothesis of no structural change. Such a refinement will enable us to construct various tests for structural change in the spirit of Sowell (1996a) and Hall and Sen (1999).

3.2 Refining the Null Hypothesis

The moment conditions for the full sample under the null can be written as:

$$Eg(x_t, \theta_0) = 0, \quad \forall t = 1, \dots, T. \quad (6)$$

Following Sowell (1996b), we can project the moment conditions on the subspace identifying the parameters and the subspace of overidentifying restrictions. In particular, considering the (standardized) moment conditions for the full-sample GMM estimator, such a decomposition corresponds to:

$$\Omega^{-1/2} Eg(x_t, \theta_0) = P_G \Omega^{-1/2} Eg(x_t, \theta_0) + (I_q - P_G) \Omega^{-1/2} Eg(x_t, \theta_0), \quad (7)$$

where $P_G = \Omega^{-1/2} G [G' \Omega - 1G]^{-1} G' \Omega^{-1/2}$. The first term is the projection identifying the parameter vector and the second term is the projection for the overidentifying restrictions. The projection argument enables us to refine the null hypothesis (3). For instance, following Hall and Sen (1999) we can consider the null, for the case of a single possible breakpoint s , which separates the identifying restrictions across the two subsamples:

$$H_0^I(s) = \begin{cases} P_G \Omega^{-1/2} E[g(x_t, \theta_0)] = 0 & \forall t = 1, \dots, [Ts] \\ P_G \Omega^{-1/2} E[g(x_t, \theta_0)] = 0 & \forall t = [Ts] + 1, \dots, T. \end{cases}$$

Moreover, the overidentifying restrictions are stable if they hold before and after the breakpoint. This is formally stated as $H_0^O(s) = H_0^{O1}(s) \cap H_0^{O2}(s)$ with:

$$\begin{aligned}H_0^{O1}(s) &: (I_q - P_G) \Omega^{-1/2} E[g(x_t, \theta_0)] = 0 \quad \forall t = 1, \dots, [Ts] \\ H_0^{O2}(s) &: (I_q - P_G) \Omega^{-1/2} E[g(x_t, \theta_0)] = 0 \quad \forall t = [Ts] + 1, \dots, T.\end{aligned}$$

The projection reveals that instability must be a result of a violation of at least one of the three hypotheses: $H_0^I(s)$, $H_0^{O1}(s)$ or $H_0^{O2}(s)$. Various tests can be constructed with local power properties

against any particular one of these three null hypotheses (and typically no power against the others). To elaborate further on this we consider a sequence of Pitman local alternatives based on the moment conditions:

Assumption 3.1. *A sequence of local alternatives is specified as:*

$$Eg(x_t, \theta_0) = \frac{h(\eta, \tau, \frac{t}{T})}{\sqrt{T}} \quad (8)$$

where $h(\eta, \tau, s)$, for $r \in [0, 1]$, is a q -dimensional function. The parameter τ locates structural changes as a fraction of the sample size and the vector η defines the local alternatives.² These local alternatives are chosen to show that the structural change tests presented in this paper have non trivial power against a large class of alternatives. Also, our asymptotic results can be compared with Sowell's results for the GMM framework.

Accordingly with the decomposition in equation (7), the sequence of alternatives can be rewritten as:

$$\Omega^{-1/2}Eg(x_t, \theta_0) = P_G \Omega^{-1/2} \frac{h(\eta, \tau, \frac{t}{T})}{\sqrt{T}} + (I_q - P_G) \Omega^{-1/2} \frac{h(\eta, \tau, \frac{t}{T})}{\sqrt{T}} \quad (9)$$

where the first component is the local alternative on the identifying moments and the second is the local alternative on the overidentifying restrictions.

For instability in the parameter vector, consider a general local alternative of the form (see Sowell (1996a))

$$\beta_{tT} = \beta_0 + \frac{f(\eta, \tau, \frac{t}{T})}{\sqrt{T}}$$

for $t = 1, \dots, T$. A Taylor expansion of $g_{tT}(x_t, \theta_{tT})$ yields

$$Eg(x_t, \beta_0) = -G_\beta \frac{f(\eta, \tau, \frac{t}{T})}{\sqrt{T}} + o_p(1)$$

and by substituting this expression into (9) this shows that the expression above is orthogonal to the second component of (9) and puts restrictions on the first component (the identifying restrictions). In the case of pure structural $P_G = P_{G_\beta} = \Omega^{-1/2} G_\beta \left[G'_\beta \Omega^{-1} G_\beta \right]^{-1} G'_\beta \Omega^{-1/2}$. The alternative that at some point there is a single structural break at τ , $H_A^I(\tau)$, is represented as:

$$\beta_{tT} = \begin{cases} \beta_0 & t = 1, \dots, [T\tau] \\ \beta_0 + \frac{\eta}{\sqrt{T}} & t = [T\tau] + 1, \dots, T \end{cases}$$

which corresponds to a specific form for $f(\eta, \tau, \frac{t}{T})$. Note that the true structural change breakpoint τ is allowed to differ than the possible breakpoint s chosen by the researcher.

²The function $h(\cdot)$ allows for a wide range of alternative hypotheses (see Sowell (1996b)). In its generic form it can be expressed as the uniform limit of step functions, $\eta \in R^i$, $\tau \in R^j$ such that $0 < \tau_1 < \tau_2 < \dots < \tau_j < 1$ and θ^* is in the interior of Θ . Therefore it can accommodate multiple breaks.

For instability of overidentifying restrictions at a single breakpoint τ occurring before and/or after the breakpoint, this is formally stated as $H_A^O(\tau) = H_A^{O1}(\tau) \cap H_A^{O2}(\tau)$ with:

$$\begin{aligned} H_A^{O1}(\tau) : (I_q - P_G)\Omega^{-1/2}E[g(x_t, \theta_0)] &= \frac{\eta_1}{\sqrt{T}} & \forall t = 1, \dots, [T\tau] \\ H_A^{O2}(\tau) : (I_q - P_G)\Omega^{-1/2}E[g(x_t, \theta_0)] &= \frac{\eta_2}{\sqrt{T}} & \forall t = [T\tau] + 1, \dots, T. \end{aligned}$$

and $\eta_1 \neq \eta_2$.³ This formulation of the alternative for a single breakpoint corresponds to a specific form of $h(\eta, \tau, \frac{t}{T})$ in (9).

4 Tests for a structural change based on implied probabilities

Ramalho and Smith (2005) introduced in i.i.d. setting a specification test for moment conditions based on implied probabilities similar in spirit to the classical Pearson Chi-Square goodness-of-fit test. The test is based on the following statistic:

$$\sum_{t=1}^T \left(T\pi_t^{GEL}(\tilde{\theta}_T) - 1 \right)^2.$$

They showed that such statistic is asymptotically equivalent to the overidentifying moment restrictions J-test proposed by Hansen (1982). Guay and Pelgrin (2007) and Guggenberger, Ramalho and Smith (2007) also used this statistic in the time series context and showed that:

$$\frac{1}{2K+1} \sum_{t=1}^T \left(T\pi_t(\tilde{\theta}_T) - 1 \right)^2. \quad (10)$$

is asymptotically first order equivalent to the overidentifying moment restrictions J-test. However, as shown by Ghysels and Hall (1990), the J-test has no power to detect structural change in parameter values, a property that is shared by the specification tests above proposed by those authors as we demonstrate below.

In the same spirit, we first consider a statistic test based on the partial-sample implied probabilities evaluated at the restricted estimator for GEL. The implied probabilities structural change (IPSC) test statistic proposed to detect instability is given by the following partial sum:

$$IPSC_T^{GEL}(s) = \frac{s}{2K_T+1} \sum_{t=1}^{[Ts]} \left(T\pi_t^{GEL}(\tilde{\theta}_T, s) - 1 \right)^2 \quad (11)$$

with

$$\pi_t^{GEL}(\tilde{\theta}_T, s) = \frac{\rho_1 \left(\tilde{\gamma}_{1T}(s)' g_{tT}(\tilde{\theta}_T, s) \right)}{\sum_{t=1}^T \rho_1 \left(\tilde{\gamma}_T(s)' g_{tT}(\tilde{\theta}_T, s) \right)}.$$

³This specification allows for the overidentifying restrictions to be violated just after the breakpoint ($\eta_1 = 0$ and $\eta_2 \neq 0$), just before the breakpoint ($\eta_1 \neq 0$ and $\eta_2 = 0$) or both ($\eta_1 \neq 0$, $\eta_2 \neq 0$ and $\eta_1 \neq \eta_2$).

where for the numerator $\tilde{\gamma}_{1T}(s)$ is the solution of the following maximization problem:

$$\gamma_{1T}(\beta, \delta) = \arg \sup_{\gamma_1 \in \hat{\Gamma}_T(\beta, \delta)} \sum_{t=1}^{[Ts]} \frac{[\rho(\gamma_1(\beta, \delta)' g_{tT}(\beta, \delta)) - \rho_0]}{T} \quad (12)$$

evaluated at the restricted estimator $\tilde{\theta}_T$ and for the denominator $\tilde{\gamma}_T(s) = (\tilde{\gamma}_{1T}(s)', \tilde{\gamma}_{2T}(s)')'$ with $\tilde{\gamma}_{1T}(s)$ defined as above and

$$\gamma_{2T}(\beta, \delta) = \arg \sup_{\gamma_2 \in \hat{\Gamma}_T(\beta, \delta)} \sum_{t=[Ts]+1}^T \frac{[\rho(\gamma_2(\beta, \delta)' g_{tT}(\beta, \delta)) - \rho_0]}{T}. \quad (13)$$

It is crucial to note that this statistic is based on the **unrestricted** implied probabilities evaluated at the **restricted** estimator of θ .

The next theorem establishes the asymptotic distribution for this general test of a structural change under the null and the sequence of alternatives defined in (8).

Theorem 4.1. *Under Assumptions (7.1) to (7.10), the following processes indexed by s for $s \in [0, 1]$ satisfy, under the null (6),*

$$IPSC_T^{GEL}(s) \Rightarrow BB_p(s)' BB_p(s) + B'_{(q-p)}(s) B_{(q-p)}(s)$$

and under the alternative (8)

$$\begin{aligned} IPSC_T^{GEL}(s) \Rightarrow & BB_p(s)' BB_p(s) + (H(s) - sH(1))' \Omega^{-1/2} P_G \Omega^{-1/2} (H(s) - sH(1)) \\ & + B_{q-p}(s)' B_{q-p}(s) + H(s)' \Omega^{-1/2} (I - P_G) \Omega^{-1/2} H(s), \end{aligned}$$

where $B_{(q-p)}(s)$ is a $(q-p)$ -vector of standard Brownian motion, $BB_p(s) = B_p(s) - sB_p(1)$ is a p -vector of Brownian bridge with $p = r + \nu$ and $H(s) = \int_0^s h(\eta, \tau, r) dr$.

Proof: See the Appendix.

The Theorem shows that the structural change test based on this quadratic form of the partial-sample sum of the implied probabilities evaluated at the full-sample estimator combines two components. The first component of the limiting distribution is a function of the Brownian bridges corresponds to a parameter stability tests for the whole set of parameters (β and δ) and the second component to a stability of overidentifying restrictions. This test statistic, based on implied probabilities, can be viewed as a more general form of misspecification due to instability than just a test for parameter variation. The predictive tests proposed by Ghysels, Guay and Hall (1997) shares the same properties. In the Appendix we show that the *IPSC* test statistic is asymptotically equivalent to the test statistic proposed by Sowell (1996b). He showed that his test statistic is optimal for a one time jump in all moment conditions where the location of the jump is unknown and consistent for arbitrary alternatives. These properties are then shared by our test. Note that the limiting distribution exists for $s = 0$ which is trivially equal to 0. For $s = 1$, the test statistic corresponds to the specification test for moment conditions developed by Guay

and Pelgrin (2007) and Guggenberger *et al.* (2007) and by the above theorem the limiting distribution is given by:

$$B_{q-p}(1)'B_{q-p}(1) + H(1)'\Omega^{-1/2}(I - P_G)\Omega^{-1/2}H(1).$$

This limiting distribution shows that this test statistic has a chi-square distribution with $q - p$ degrees of freedom under the null and that has local power equal to the size to detect instability in parameter values as the J-test proposed by Hansen (1982). Moreover, the test statistic (10) can not detect asymptotically instability in the overidentifying restrictions for which $(I - P_G)\Omega^{-1/2}H(1) = 0$.

When the breakpoint is unknown, one can construct statistics across $s \in S$. In the context of maximum likelihood estimation, Andrews and Ploberger (1994) derive asymptotic optimal tests for a gaussian a priori of the amplitude of the structural change based on the Neyman-Pearson approach which are characterized by an average exponential form. The Sowell (1996a) optimal tests are a generalization of the Andrews and Ploberger approach to the case of two measures that do not admit densities. The most powerful test is given by the Radon-Nikodym derivative of the probability measure implied by the local alternative with respect to the probability measure implied by the null hypothesis.

The optimal average exponential form applied to a statistic $Q_T(s)$ for $s \in S$ has the following form:

$$Exp - Q_T = (1 + c)^{-q/2} \int \exp\left(\frac{1}{2} \frac{c}{1 + c} Q_T(s)\right) dJ(s),$$

where various choices of c determine power against close or more distant alternatives and $J(\cdot)$ is the weight function over the value of $s \in S$. In the case of close alternatives ($c = 0$), the optimal test statistic takes the average form, $aveQ_T = \int_S Q_T(s)dJ(s)$. For a distant alternative ($c = \infty$), the optimal test statistics takes the exponential form, $expQ_T = \log\left(\int_S \exp[\frac{1}{2}Q_T(s)]dJ(s)\right)$. The supremum form often used in the literature corresponds to the case where $\frac{c}{(1+c)} \rightarrow \infty$. The sup test is given by $\sup Q_T = \sup_{s \in S} Q_T(s)$.

The following Theorem gives the asymptotic distribution for the average exponential mapping for $Q_T^{IPSC}(s)$ where $Q_T^{IPSC}(s)$ corresponds to the statistic presented above based on the implied probabilities.

Theorem 4.2. *Under the null hypothesis H_0 in (6) and Assumptions 7.1 to 7.10, the following processes indexed by s for a given set S whose closure lies in $[0,1]$ satisfy:*

$$\sup Q_T^{IPSC} \Rightarrow \sup_{s \in S} Q_{p,q-p}(s), \quad aveQ_T^{IPSC} \Rightarrow \int_S Q_{p,q-p}(s)dJ(s), \quad expQ_T^{IPSC} \Rightarrow \log\left(\int_S \exp[\frac{1}{2}Q_{p,q-p}(s)]dJ(s)\right),$$

with

$$Q_{p,q-p}(s) = BB_p(s)'BB_p(s) + B_{(q-p)}(s)'B_{(q-p)}(s)$$

and $J(s)$ is the weighting distribution function for the location of the instability s .

This result is obtained through the application of the continuous mapping theorem (see Pollard (1984)). The asymptotic critical values were obtained using simulated Brownian motions and Brownian

bridges over 10,000 replications for maximum values of p and $q - p$ set at 10. The critical values appear in Tables 1 to 3 for symmetric intervals $S = [s_0, 1 - s_0]$. Critical values for the entire sample appear at $s_0 = 0$.

An asymptotically equivalent modified statistic to (11) in the spirit of the Neyman-modified chi-square is given by:

$$IPSCM_T^{GEL}(s) = \frac{s}{2K_T + 1} \sum_{t=1}^{[Ts]} \frac{\left(T\pi_t^{GEL}(\tilde{\theta}_T, s) - 1\right)^2}{T\pi_t^{GEL}(\tilde{\theta}_T, s)}$$

since $T\pi_t^{GEL}(\tilde{\theta}_T, s) = 1 + o_p(1)$ under the null.

In the sequel, we propose structural change tests based on implied probabilities specially design to detect instability in the parameters of interest or in the overidentifying restrictions.

4.1 Tests for a structural change in the parameters based on implied probabilities

The test statistics proposed to specifically detect parameter instability are based on the difference between the partial sum of unrestricted implied probabilities evaluated at the unrestricted estimator $\hat{\theta}_T(s)$ with the corresponding partial sum of unrestricted implied probability but at the restricted estimator $\tilde{\theta}_T$. More precisely the test statistic is defined as:

$$IPSC_T^{I,GEL}(s) = \frac{1}{2K_T + 1} \sum_{t=1}^T \left(T\pi_t^{GEL}(\hat{\theta}_T(s), s) - T\pi_t^{GEL}(\tilde{\theta}_T, s)\right)^2$$

with

$$\pi_t^{GEL}(\tilde{\theta}_T, s) = \frac{\rho_1 \left(\tilde{\gamma}_T(s)' g_{tT}(\tilde{\theta}_T, s)\right)}{\sum_{t=1}^T \rho_1 \left(\tilde{\gamma}_T(s)' g_{tT}(\tilde{\theta}_T, s)\right)}$$

where $\tilde{\gamma}_T(s) = (\tilde{\gamma}_{1T}(s)' \tilde{\gamma}_{2T}(s)')'$ is the solution of the respective maximization problems defined in equations (12) and (13) and $\pi_t^{GEL}(\hat{\theta}_T(s), s)$ is defined in equation (5).

The next theorem establishes the asymptotic distribution for this test of a structural change in the parameter values under the null that the vector β is constant throughout the sample.

Theorem 4.3. *Under the null hypothesis H_0 in (3) and Assumptions (7.1) to (7.10), the following processes indexed by s for a given set S whose closure lies in $(0, 1)$ satisfy:*

$$IPSC_T^{I,GEL}(s) \Rightarrow Q_r(s) = \frac{BB_r(s)' BB_r(s)}{s(1-s)}$$

and under the alternative (8)

$$Q_r(s) = \frac{BB_r(s)' BB_r(s)}{s(1-s)} + \frac{(H(s) - sH(1))' \Omega^{-1/2} P_{G_\beta} \Omega^{-1/2} (H(s) - sH(1))}{s(1-s)},$$

where $BB_r(s) = B_r(s) - sB_r(1)$ is a Brownian bridge, B_r is r -vector of independent Brownian motions and $P_{G_\beta} = \Omega^{-1/2}G_\beta [(G_\beta)'\Omega^{-1}G_\beta]^{-1} (G_\beta)' \Omega^{-1/2}$. Moreover,

$$\begin{aligned} \sup IPSC_T^{I,GEL} &\Rightarrow \sup_{s \in S} Q_r(s), & aveIPSC_T^{I,GEL} &\Rightarrow \int_S Q_r(s) dJ(s), \\ expIPSC_T^{I,GEL} &\Rightarrow \log \left(\int_S \exp\left[\frac{1}{2} Q_r(s)\right] dJ(s) \right). \end{aligned}$$

Proof: See the Appendix.

The Theorem shows that the asymptotic distribution of the test based on implied probabilities is asymptotically equivalent under the null and the alternative to the *Wald*, *LM* and *LR* tests for parameter instability (see Andrews 1993). More precisely, the limiting distribution is function of a r -vector of Brownian bridge with the same dimension than the parameter vector β . However, the small sample properties can differ compared to those more standard tests. Note that, in contrast to the preceding Theorem, the limiting distribution in Theorem 4.3 is valid only for S in the open interval $(0, 1)$.

The two following modified statistics are asymptotically equivalent to the one defined above:

$$IPSCM1_T^{I,GEL}(s) = \frac{1}{2K_T + 1} \sum_{t=1}^T \frac{\left(T\pi_t^{GEL}(\hat{\theta}_T(s), s) - T\pi_t^{GEL}(\tilde{\theta}_T, s) \right)^2}{T\pi_t^{GEL}(\hat{\theta}_T(s), s)}$$

and

$$IPSCM2_T^{I,GEL}(s) = \frac{1}{2K_T + 1} \sum_{t=1}^T \frac{\left(T\pi_t^{GEL}(\hat{\theta}_T(s), s) - T\pi_t^{GEL}(\tilde{\theta}_T, s) \right)^2}{T\pi_t^{GEL}(\tilde{\theta}_T, s)}.$$

4.2 Tests for a structural change for overidentifying restrictions based on implied probabilities

Now, we propose a test statistic designed specially to detect instability in the overidentifying restrictions based on implied probabilities. The statistic is powerful against violation of $H_0^{O1}(s)$ and $H_0^{O2}(s)$. The statistic is asymptotically equivalent to the ones proposed by Hall and Sen (1999) and thus shares its asymptotic properties. As previously, the sample is split in two subsamples with a single breakpoint at $[Ts]$. An estimator of the parameter vector is obtained with the first subsample (for $t = 1, \dots, [Ts]$) and with the second subsample (for $t = [Ts] + 1, \dots, T$). The entire parameter vector is allowed to vary for both subsamples. The proposed statistic specifically designed to detect instability for overidentifying restrictions is based on the specification test statistic for moment conditions given in equation (10) for the first and the second subsamples.

More precisely the statistic is defined as:

$$IPSC_T^{O,GEL}(s) = \frac{1}{2K_T + 1} \sum_{t=1}^{[Ts]} \left([Ts] \pi_t^{GEL}(\hat{\theta}_{1T}(s), s) - 1 \right)^2 \quad (14)$$

$$+ \frac{1}{2K_T + 1} \sum_{t=[Ts]+1}^T \left((T - [Ts]) \pi_t^{GEL}(\hat{\theta}_{2T}(s), s) - 1 \right)^2 \quad (15)$$

where $\hat{\theta}_{1T}(s) = \hat{\beta}_{1T}(s)$ and $\hat{\theta}_{2T}(s) = \hat{\beta}_{2T}(s)$. The statistic is the sum of the overidentifying restrictions statistics (10) but for the first and the second parts of the sample evaluated at the unrestricted estimator.

The next Theorem establishes the asymptotic distribution of this statistic and the corresponding average mappings.

Theorem 4.4. *Under Assumptions (7.1) to (7.10), the following processes indexed by s for a given set S whose closure lies in $(0, 1)$ satisfy*

$$IPSC_T^{O,GEL}(s) \Rightarrow Q_{q-r}(s)$$

with under the null of no structural change

$$Q_{q-r}(s) \Rightarrow \frac{B_{q-r}(s)' B_{q-r}(s)}{s} + \frac{[B_{q-r}(1) - B_{q-r}(s)]' [B_{q-r}(1) - B_{q-r}(s)]}{1-s}$$

and under the alternative (8)

$$Q_{q-r}(s) \Rightarrow \frac{B_{q-r}(s)' B_{q-r}(s)}{s} + \frac{H(s)' \Omega^{-1/2} (I - P_G) \Omega^{-1/2} H(s)}{s} \\ + \frac{[B_{q-r}(1) - B_{q-r}(s)]' [B_{q-r}(1) - B_{q-r}(s)]}{1-s} + \frac{[H(1) - H(s)]' \Omega^{-1/2} (I - P_G) \Omega^{-1/2} [H(1) - H(s)]}{(1-s)},$$

where $B_{q-r}(s)$ is a $q-r$ -vector of standard Brownian motion and $P_G = P_{G_\beta}$ with $P_{G_\beta} = \Omega^{-1/2} G_\beta \left(G_\beta' \Omega^{-1} G_\beta \right)^{-1} G_\beta' \Omega^{-1/2}$. Moreover,

$$\sup IPSC_T^{O,GEL} \Rightarrow \sup_{s \in S} Q_{q-r}(s), \quad ave IPSC_T^{O,GEL} \Rightarrow \int_S Q_{q-r}(s) dJ(s), \\ exp IPSC_T^{O,GEL} \Rightarrow \log \left(\int_S \exp \left[\frac{1}{2} Q_{q-r}(s) \right] dJ(s) \right).$$

Proof: See the Appendix.

The Theorem shows that the proposed test statistics can detect instability occurring in overidentifying restrictions before and after the breakpoint. Indeed, the term $\frac{H(s)' \Omega^{-1/2} (I - P_G) \Omega^{-1/2} H(s)}{s}$ corresponds to a structural change in the moment conditions before the breakpoint s while the term $\frac{[H(1) - H(s)]' \Omega^{-1/2} (I - P_G) \Omega^{-1/2} [H(1) - H(s)]}{(1-s)}$ to a structural change after the breakpoint s .

The asymptotic critical values for the interval $S = [.15, .85]$ can be found in Hall and Sen (1999). For other symmetric interval $[s_0, 1 - s_0]$, critical values can be obtained in Guay (2003), Tables 1 to 3 for a number of overidentifying restrictions divided by 2 (in those Tables). To see this, note that the critical

values for the supremum, the average and the log exponential mappings applied to $\frac{B_{2q-2r}(s)'B_{2q-2r}(s)}{s}$ are equivalent to ones corresponding to $\frac{B_{q-r}(s)'B_{q-r}(s)}{s} + \frac{(B_{q-r}(1)-B_{q-r}(s))'(B_{q-r}(1)-B_{q-r}(s))}{1-s}$ for a symmetric interval S .⁴

An asymptotic equivalent statistic to (14) in the spirit of the Neyman-modified chi-square is given by:

$$IPSCM_T^{O,GEL}(s) = \frac{1}{2K_T + 1} \sum_{t=1}^{[Ts]} \frac{\left([Ts]\pi_t^{GEL}(\hat{\theta}_{1T}(s), s) - 1\right)^2}{[Ts]\pi_t^{GEL}(\hat{\theta}_{1T}(s), s)} + \frac{1}{2K_T + 1} \sum_{t=[Ts]+1}^T \frac{\left((T - [Ts])\pi_t^{GEL}(\hat{\theta}_{2T}(s), s) - 1\right)^2}{(T - [Ts])\pi_t^{GEL}(\hat{\theta}_{2T}(s), s)}.$$

5 Simulation Evidence

To evaluate the performance of the test statistics we use the data generating process in Ghysels *et al.* (1997) and in Hall and Sen (1999). The time series model used is an AR(1) process for the variable x_t . One parameter is estimated, the autoregressive parameter (denoted by θ in the expression below), using two moment conditions formed with the lagged values of x_t .

The data generating process is given by

$$x_t = \theta_i x_{t-1} + u_t \quad (16)$$

for $t = 1, \dots, T$. Structural change in the identifying restrictions (in the parameter) is studied by considering different values of θ_i where the index $i = 1, 2$ denotes the first or second subsamples. Structural stability in the overidentifying restrictions is studied by allowing for an ARMA(1,2) model

$$x_t = \theta_i x_{t-1} + u_t + \alpha u_{t-2} \quad (17)$$

and considering nonzero values of α in the second subsample. The change is set at $T/2$. In the above, $u_t \sim N(0, 1)$. The sample size was set to 200 observations and the number of Monte Carlo replications was limited to 500 since the time required to estimate by exponential tilting, the method used in these simulations experiments.

Table 7 summarizes the different parametrization and is taken from Hall and Sen (1999). The null hypothesis of structural stability is denoted by H_0^{SS} (DGP 1 to 3). For those DGPs we vary the magnitude of the autoregressive parameter θ . The alternatives of instability in the parameters or in the overidentifying restrictions are denoted by H_A^I (DGP 4 to 6), where we vary the magnitude of the change in the autoregressive parameter, and H_A^O (DGP 7 to 10) where we consider various values of the moving average

⁴This is verified by comparing the critical values in Hall and Sen (1999) and Guay (2003). The critical values in Table 1 in Hall and Sen for $q - r$ in our notation are the same than the critical values in Guay (2003) but for $2q - 2r$.

parameter, respectively. In this situation only one parameter is estimated using two moment conditions created with the first two lags of x_t . Under H_0^{SS} , where $\alpha = 0$, the instruments are appropriate. Under the first class of alternative hypothesis (H_A^I) the two instruments are also valid while they no longer are for the second part of the sample with the second class of alternative hypothesis (H_A^O).

Smoothing the moment conditions is done via an appropriate choice of K_T . In a GMM setting this is equivalent to using some form of estimate of the long run covariance matrix of the moment conditions (for example using the Newey-West estimator as the weight matrix in quadratic form). Most of the previous simulation work considered a fixed degree of smoothing (see for example Gregory *et al.* (2002) and Guggenberger and Smith (2007)). Otsu (2006) did also look at fixed smoothing but also looked at applying the automatic bandwidth selection rule of Newey and West (1994).

Our Monte Carlo study is not specially designed to investigate smoothing because under the null hypothesis the optimal value of K_T is 0 while it is not 0 under the alternative hypothesis H_A^O . For this reason we set $K_T = 0$ for all DGPs except DGPs 7, 8, 9 and 10. For these DGPs we select a bandwidth parameter chosen via the automatic, data-driven, procedure which chooses a bandwidth m_T which is then transformed as $K_T = [(m_T - 1)/2]$. The average, taken over Monte Carlo replications, K_T was found to vary between 1.6 and 2.3, increasing with the moving average component. A complete analysis of the effects of smoothing is left for future work. Lastly, a trimming rule of 0.15 was used, namely $S = [.15, .85]$.

Table 5 contains the results for the general specification tests *IPSC* and *IPSCM* (the supremum, exponential and average version of the test statistics are presented). Table 6 contains the rejection frequencies for the test statistics designed to have power against a structural change in the parameters while Table 7 presents the results for test statistics which are designed to have power against a structural change in the overidentifying restrictions. All the test statistics were all computed in the GEL setting. The tests used for comparison appear in Guay and Lamarche (2008). For completeness we report them also here:

$$Wald_T(s) = T \left(\hat{\beta}_{1T}(s) - \hat{\beta}_{2T}(s) \right)' \left(\hat{V}_\Omega(s) \right)^{-1} \left(\hat{\beta}_{1T}(s) - \hat{\beta}_{2T}(s) \right),$$

where $\hat{V}_\Omega(s) = \left(\hat{V}_1(s)/s + \hat{V}_2(s)/(1-s) \right)$ and $\hat{V}_i(s) = \left(\hat{G}_{i,tT}^\beta(s)' \hat{\Omega}_{i,T}^{-1}(s) \hat{G}_i^\beta(s) \right)^{-1}$ for $i = 1, 2$ corresponding to the first and the second part of the sample. For the first part of the sample:

$$\begin{aligned} \hat{G}_{1,tT}^\beta(s) &= \frac{1}{[Ts]} \sum_{t=1}^{[Ts]} \frac{\partial g_{tT}(\hat{\beta}_1(s), \hat{\delta}(s))}{\partial \beta_1'} \\ \hat{\Omega}_{1T}(s) &= \frac{2K_T + 1}{[Ts]} \sum_{t=1}^{[Ts]} g_{tT}(\beta_1(s), \delta) g_{tT}(\hat{\beta}_1(s), \hat{\delta}(s))' \end{aligned}$$

and for the second part of the sample:

$$\begin{aligned}\widehat{G}_{2,tT}^\beta &= \frac{1}{T - [Ts]} \sum_{t=[Ts]+1}^T \frac{\partial g_{tT}(\widehat{\beta}_2(s), \widehat{\delta}(s))}{\partial \beta_2'} \\ \widehat{\Omega}_{2T}(s) &= \frac{2K_T + 1}{T - [Ts]} \sum_{t=[Ts]+1}^T g_{tT}(\widehat{\beta}_2(s), \widehat{\delta}(s)) g_{tT}(\widehat{\beta}_2(s), \widehat{\delta}(s))'\end{aligned}$$

The Lagrange Multiplier statistic is given by:

$$LM_T(s) = \frac{T}{s(1-s)} g_{1T}(\tilde{\theta}_T, s)' \widehat{\Omega}_T^{-1} \widehat{G}_{tT}^\beta \left[(\widehat{G}_{tT}^\beta)' \widehat{\Omega}_T^{-1} \widehat{G}_{tT}^\beta \right]^{-1} (\widehat{G}_{tT}^\beta)' \widehat{\Omega}_T^{-1} g_{1T}(\tilde{\theta}_T, s).$$

where

$$\begin{aligned}g_{1T}(\tilde{\theta}_T, s) &= \frac{1}{T} \sum_{t=1}^{[Ts]} g_{tT}(\tilde{\theta}_T), \\ \widehat{G}_{tT}^\beta &= \frac{1}{T} \sum_{t=1}^T \frac{\partial g_{tT}(\tilde{\beta}, \tilde{\delta})}{\partial \beta'}, \\ \widehat{\Omega}_T &= \frac{2K_T + 1}{T} \sum_{t=1}^T g_{tT}(\tilde{\beta}, \tilde{\delta}) g_{tT}(\tilde{\beta}, \tilde{\delta})'\end{aligned}$$

The LR-like statistic is defined as:

$$LR_T(s) = \frac{2T}{2K + 1} \left[\sum_{t=1}^T \frac{[\rho(\tilde{\gamma}(s))' g_{tT}(\tilde{\theta}, s) - \rho_0]}{T} - \sum_{t=1}^T \frac{[\rho(\hat{\gamma}(s))' g_{tT}(\hat{\theta}(s), s) - \rho_0]}{T} \right]$$

An finally, the test statistic proposed by Hall and Sen (1999) is

$$O_T(s) = O_{1T}(s) + O_{2T}(s)$$

where

$$O_{1T}(s) = \left[\frac{1}{\sqrt{[Ts]}} \sum_{t=1}^{[Ts]} g_{tT}(\widehat{\beta}_1(s), \widehat{\delta}(s)) \right]' \widehat{\Omega}_{1T}^{-1}(s) \left[\frac{1}{\sqrt{[Ts]}} \sum_{t=1}^{[Ts]} g_{tT}(\widehat{\beta}_1(s), \widehat{\delta}(s)) \right]$$

and

$$O_{2T}(s) = \left[\frac{1}{\sqrt{(T - [Ts])}} \sum_{t=[Ts]+1}^T g_{tT}(\widehat{\beta}_2(s), \widehat{\delta}(s)) \right]' \widehat{\Omega}_{2T}^{-1}(s) \left[\frac{1}{\sqrt{(T - [Ts])}} \sum_{t=[Ts]+1}^T g_{tT}(\widehat{\beta}_2(s), \widehat{\delta}(s)) \right]$$

Focusing first on size we find that the modified tests ($IPSCM1^I$, $IPSCM2^I$ and $IPSCM^O$) based on implied probabilities have rejection frequencies that are much too large. The intuition for this is that the modified test statistics contain a more volatile term in the denominator which can inflate the value of the tests and hence increase the rejection frequencies. So for brevity, we don't report those results here.⁵

⁵Obviously, the results can be obtained upon request.

On the other hand, all other tests have rejection frequencies that are quite good, with some overrejection except for the O test statistics which slightly underreject. The $\text{sup}IPSC^I$ has a nominal size close to the true one while the average and exponential forms for this statistic slightly overreject, a property shared by the *Wald* and *LR* tests. The *LM* test statistics (supremum, average and exponential mappings) significantly underreject for all DGPs (1 to 3) under the null and have poor power for other DGPs. For these reasons, the rejection frequencies for the *LM* test statistics are not reported in the tables here.

The study of power is divided into two cases. In case 1 structural change occurs in the parameter values while in case 2 structural change occurs in the overidentifying restrictions. Under the alternative of instability in the parameter, H_A^I (DGP 4 to 6), we see that the newly proposed test statistics based on implied probabilities have good rejection frequencies. The power of these test statistics is equal or larger than the power of the standard *Wald* and *LR* tests. The $IPSC^O$ and O test statistics, geared toward instability in the overidentifying restrictions, have not useful power while the more general specification test, $IPSC$, has some reasonable power.

Under the alternative of instability in the overidentifying restrictions, e.g. H_A^O , (DGP 7 to 10), we see that the test statistics specially designed to detect a change in the parameter $IPSC^I$, and the standard *Wald* and *LR* tests have little power while the targeted $IPSC^O$ and O tests have very good power. Importantly the tests based on implied probabilities are seen to have significantly higher power than O tests for all cases. In some cases, the gain in power can be twice as important. As expected, the general specification tests based on implied probabilities have rejection frequencies that fall between those of $IPSC^O$ and $IPSC^I$.

The increase in the autoregressive coefficient from 0 to 0.8 does not impact greatly on the rejection frequencies under the null hypothesis but under the alternative hypotheses the magnitude of the change is important. Under H_A^I , for example, we see that power is close to unity when the change in the autoregressive parameter is quite extreme (0 to 0.8). Under H_A^O , which captures a change in the overidentifying restrictions, an increase (in absolute terms) in the moving average coefficient increases power.

6 Conclusion

As noted by Back and Brown (1993) implied probabilities obtained from estimation of models using estimating equations could be used as an additional tool for model specification in the researcher's tool kit. An important specification test that has received considerable attention in the econometric literature has been a test for structural stability of either the underlying key parameters composing the estimation equations and or the stability of the additional equations (the overidentifying restrictions) that are often used in estimation. In this paper we have focused on the class of estimators based on the Generalized Empirical Likelihood approach. This approach is appealing intuitively because it requires the search for a

vector of weights, one for each observation, that yields the most probable data distribution. We view these weights as potentially containing information on the content of the moment conditions (the estimating equations). In a pure entropy setting with no estimating equations the vector of weights is found to be maximally uninformative. That is we obtain weights fluctuating around $1/T$ where T is the sample size. With constraints, the weights are chosen to be as maximally uninformative as the constraints will allow.

In this sense the weights make use of the available information contained in the sample. In particular we use the weights to detect an unknown structural change. We suggest three types of testing procedures for the detection of a structural change each based on different measures of the discrepancy between the estimated weights and the unconstrained weights $\frac{1}{T}$. Specifically, we propose general structural change tests to detect instability both in the identifying restrictions and in the overidentifying restrictions, instability in the identifying restrictions and finally instability in the overidentifying restrictions. This type of classification is similar to the one found in classical papers on structural change in time series as proposed by Andrews (1993) and Hall and Sen (1999) for example. We found that tests based on these implied probabilities have good finite sample size and power properties. An issue that was not investigated in length in this paper was the impact of smoothing (to take into account serial dependence) on the performance of the tests. This interesting avenue of research is left for future work. The important question of structural change tests for GEL robust to weak identification is also left for future investigation.

7 Appendix

7.1 Assumptions

Assumption 7.1. The process $\{x_t\}_{t=1}^{\infty}$ is a finite dimensional stationary and strong mixing coefficients $\sum_{j=1}^{\infty} \alpha(j)^{(\nu-1)/\nu} < \infty$ for some $\nu > 1$.

Assumption 7.2. (a) $(\beta_0, \delta_0) \in B \times \Delta$ is the unique solution to $E[g(x_t, \beta, \delta)] = 0$ where $B \times \Delta$ is compact, $g(x_t, \beta, \delta)$ is continuous at each $(\beta, \delta) \in B \times \Delta$ with probability one; (b) $E\left[\sup_{(\beta, \delta) \in B \times \Delta} \|g(x_t, \beta, \delta)\|^\alpha\right] < \infty$ for some $\alpha > \max\left(4\nu, \frac{1}{\eta}\right)$; (c) $\Omega(\beta, \delta)$ is finite and positive definite for all $(\beta, \delta) \in B \times \Delta$.

Define the smoothed moment conditions as:

$$g_{tT}(\beta, \delta) = \frac{1}{S} \sum_{s=t-T}^{t-1} k\left(\frac{s}{S_T}\right) g_{t-s}(x_t, \beta, \delta)$$

for an appropriate kernel. From now on, we consider the uniform kernel proposed by Kitamura and Stutzer (1997):

$$g_{tT}(\beta, \delta) = \frac{1}{2K_T + 1} \sum_{s=-K_T}^{K_T} g_{t-s}(x_t, \beta, \delta)$$

Assumption 7.3. $K_T^2/T \rightarrow 0$ and $K_T \rightarrow \infty$ as $T \rightarrow \infty$.

Assumption 7.4. (a) $\rho(\cdot)$ is twice continuously differential and concave on its domain, an open interval Φ containing 0, $\rho_1 = \rho_2 = -1$; (b) $\gamma \in \Gamma$ where $\Gamma = \{\gamma : \|\gamma\| \leq D(T/K_T^2)^{-\zeta}\}$ for some $D > 0$ with $\frac{1}{2} > \zeta > \frac{1}{2\alpha\eta}$.

Under Assumptions 7.1 to 7.4, Smith (2004) shows that for the full-sample estimators $\tilde{\theta}_T \xrightarrow{p} \theta_0$, $\tilde{\gamma}_T \xrightarrow{p} 0$, $\|\tilde{\gamma}_T\| = O_p((T/2K_T + 1)^2)^{-1/2}$ and $\|\frac{1}{T} \sum_{t=1}^T g_{tT}(\tilde{\theta}_T)\| = O_p(T^{-1/2})$. Now, Let $G(\beta, \delta) = E[\partial g(x_t, \beta, \delta)/\partial(\beta', \delta)']$ and $G = G(\beta_0, \delta_0)$.

Assumption 7.5. $g(\cdot, \beta, \delta)$ is differentiable in $(\beta_0, \delta_0) \in B_0 \times \Delta_0$ where $B_0 \times \Delta_0$ is a neighborhood of (β_0, δ_0) , $\sup_{1 \leq t \leq T} E\left[\sup_{(\beta, \delta) \in B_0 \times \Delta_0} \|\partial g(x_t, \beta, \delta)/\partial(\beta', \delta)'\|^\alpha\right] < \infty$ and $\text{rank}(G) = p + \nu$.

Assumptions 7.1 to 7.5 yield the asymptotic distribution of $T^{1/2}(\tilde{\theta}_T - \theta_0)$ and $(T/(2K_T + 1)^2)^{1/2} \tilde{\gamma}_T$ (see Smith, 2004). Also, under these assumptions:

$$\frac{1}{T} \sum_{t=1}^T G_{tT}(\tilde{\beta}_T, \tilde{\delta}_T) \xrightarrow{p} G$$

and

$$\tilde{\Omega}_T(\tilde{\theta}_T) = \frac{2K + 1}{T} \sum_{t=1}^T g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T) g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T)' \xrightarrow{p} \Omega.$$

The following high level assumptions are sufficient to derive the weak convergence under the null of the PS-GEL estimators $\hat{\theta}_T(s)$ and $\hat{\gamma}_T(s)$ (see Guay and Lamarche, 2008). These assumptions are similar to the ones in Andrews (1993).

Assumption 7.6. $\text{Var}\left(\frac{1}{\sqrt{T}} \sum_{t=1}^{Ts} g(x_t, \theta)\right) \rightarrow s\Omega \forall s \in [0, 1]$ for some positive definite matrix Ω where $\Omega = \lim_{T \rightarrow \infty} \text{Var}\left(\frac{1}{\sqrt{T}} \sum_{t=1}^T g(x_t, \theta)\right)$.

Assumption 7.7. $\sup_{s \in S} \|\hat{\Omega}_T(s) - \Omega(s)\| \xrightarrow{P} 0$ where $\Omega(s)$ is defined in Section 3.1 and S whose closure lies in $(0, 1)$.

Assumption 7.8. $\sup_{s \in S} \|\hat{\theta}_T(s) - \theta_0\| \xrightarrow{P} 0$ for some θ_0 in the interior of Θ and $\sup_{s \in S} \|\hat{\gamma}_T(s) - 0\| \xrightarrow{P} 0$ and S whose closure lies in $(0, 1)$.

Assumption 7.9. $\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^{Ts} E \partial g(x_t, \beta_0, \delta_0) / \partial (\beta', \delta')$ exists uniformly over $s \in S$ and equals sG $\forall s \in S$ and S whose closure lies in $(0, 1)$.

Assumption 7.10. $G(s)' \Omega(s)^{-1} G(s)$ is nonsingular $\forall s \in S$ and has eigenvalues bounded away from zero $\forall s \in S$ and S whose closure lies in $(0, 1)$.

7.2 Lemmas

The following Lemmas are necessary to establish the proofs of the Theorems:

Lemma 7.1. We denote $\{B(s) : s \in [0, 1]\}$ as q -dimensional vectors of mutually independent Brownian motion on $[0, 1]$ and define

$$J(s) = \begin{bmatrix} \Omega^{1/2} B(s) \\ \Omega^{1/2} (B(1) - B(s)) \end{bmatrix}$$

where $B(\pi)$ is a q -dimensional vector of standard Brownian motion. Under Assumptions 7.1 to 7.10, every sequence of PS-GEL estimators $\{\hat{\theta}(\cdot), \hat{\gamma}(\cdot), T \geq 1\}$ satisfies

$$\begin{aligned} \sqrt{T} \left(\hat{\theta}_T(\cdot) - \theta_0 \right) &\Rightarrow - (G(\cdot)' \Omega(\cdot)^{-1} G(\cdot))^{-1} G(\cdot)' \Omega(\cdot)^{-1} J(\cdot), \\ \frac{\sqrt{T}}{2K_T + 1} \hat{\gamma}_T(\cdot) &\Rightarrow - \left(\Omega(\cdot)^{-1} - \Omega(\cdot)^{-1} (G(\cdot)' \Omega(\cdot)^{-1} G(\cdot))^{-1} G(\cdot)' \Omega(\cdot)^{-1} \right) J(\cdot) \end{aligned}$$

as a process indexed by $s \in S$, where S has closure in $(0, 1)$. Further, the sequence GEL estimators $\hat{\theta}_T(\cdot)$ and the sequence of auxiliary estimators $\hat{\gamma}_T(\cdot)$ are asymptotically uncorrelated. Under the alternative (8), the same results hold except that:

$$J(s) = \begin{bmatrix} \Omega^{1/2} B(s) + H(s) \\ \Omega^{1/2} (B(1) - B(s)) + H(1) - H(s) \end{bmatrix}$$

where $H(s) = \int_0^s h(r) dr$ with $h(r) = h(\eta, \tau, r)$ to simplify the notation.

Proof of Lemma 7.1: see Guay and Lamarche (2008).

Lemma 7.2. Under Assumptions 7.1 to 7.10, for the unrestricted implied probabilities evaluated at the restricted estimator, we get

$$\pi_t^{GEL}(\tilde{\theta}_T, s) = \frac{1}{T} + \frac{1}{T} g_{tT}(\tilde{\theta}_T, s)' \tilde{\gamma}_T(s) (1 + o_p(1)) + O_p(K_T/T^2)$$

and $\pi_t^{GEL}(\tilde{\theta}_T) = \frac{1}{T} + o_p(1)$ uniformly in $t = 1, \dots, T$. According to notation in Definition 3.4, for $t = 1, \dots, [Ts]$,

$$T \pi_t^{GEL}(\tilde{\theta}_T, s) - 1 = g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T)' \tilde{\gamma}_{1T} (1 + o_p(1)) + O_p(K_T/T)$$

and for $t = [Ts] + 1, \dots, T$,

$$T \pi_t^{GEL}(\tilde{\theta}_T, s) - 1 = g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T)' \tilde{\gamma}_{2T} (1 + o_p(1)) + O_p(K_T/T).$$

Proof of Lemma 7.2

We need to derive the asymptotic distributions of the partial-sample implied probabilities evaluated at the full-sample estimator, namely:

$$\pi_t^{GEL}(\tilde{\theta}_T, s) = \frac{\rho_1 \left(\tilde{\gamma}_T(s)' g_{tT}(\tilde{\theta}_T, s) \right)}{\sum_{t=1}^T \rho_1 \left(\tilde{\gamma}_T(s)' g_{tT}(\tilde{\theta}_T, s) \right)}.$$

A mean value expansion for $\pi_t^{GEL}(\tilde{\theta}_T, s)$ around $(\tilde{\theta}_T, \tilde{\gamma}_T) = (\tilde{\theta}_T, 0)$ yields:

$$\begin{aligned} \pi_t^{GEL}(\tilde{\theta}_T, s) &= \frac{1}{T} + \frac{\rho_2 \left(\tilde{\gamma}_T(s)' g_{tT}(\tilde{\theta}_T, s) \right)}{\sum_{t=1}^T \rho_1 \left(\tilde{\gamma}_T(s)' g_{tT}(\tilde{\theta}_T, s) \right)} g_{tT}(\tilde{\theta}_T, s)' \tilde{\gamma}_T(s) - \\ &\quad \frac{\rho_1 \left(\tilde{\gamma}_T(s)' g_{tT}(\tilde{\theta}_T, s) \right)}{\left[\sum_{t=1}^T \rho_1 \left(\tilde{\gamma}_T(s)' g_{tT}(\tilde{\theta}_T, s) \right) \right]^2} \sum_{t=1}^T \rho_2 \left(\tilde{\gamma}_T(s)' g_{tT}(\tilde{\theta}_T, s) \right) g_{tT}(\tilde{\theta}_T, s)' \tilde{\gamma}_T(s) \end{aligned}$$

where $\tilde{\gamma}_T(s)$ lies on the line segment joining $\tilde{\gamma}_T(s)$ and 0 and may differ from row to row.

Since $\tilde{\gamma}_T(s)$ converges in probability to 0 and using Lemma A.4 in Smith (2004), this yields that $\rho_j \left(\tilde{\gamma}_T(s)' g_{tT}(\tilde{\theta}_T, s) \right) = \rho_j(0) + o_p(1) = -1 + o_p(1)$, for $j = 1, 2, \forall t$. Thus, we get:

$$\pi_t^{GEL}(\tilde{\theta}_T, s) = \frac{1}{T} + \frac{1}{T} \left[g_{tT}(\tilde{\theta}_T, s)' \tilde{\gamma}_T(s) (1 + o_p(1)) \right] - \frac{1}{T^2} \left[\sum_{t=1}^T g_{tT}(\tilde{\theta}_T, s)' \tilde{\gamma}_T(s) (1 + o_p(1)) \right].$$

As $\sum_{t=1}^T g(\tilde{\theta}_T) = \sum_{t=1}^T g_{tT}(\tilde{\theta}_T) + O_p(K^2/(2K+1)) = O_p(T^{1/2})$ which implies $\sum_{t=1}^T g_{tT}(\tilde{\theta}_T, s) = O_p(T^{1/2})$, $g_{tT}(\tilde{\theta}_T, s) = O_p((2K_T+1)^{-1/2})$ and $\tilde{\gamma}_T(s) = O_p(2K_T+1/\sqrt{T})$ ⁶yields:

$$\pi_t^{GEL}(\tilde{\theta}_T, s) = \frac{1}{T} + \frac{1}{T} \left[g_{tT}(\tilde{\theta}_T, s)' \tilde{\gamma}_T(s) (1 + o_p(1)) \right] + O_p(K_T/T^2)$$

and

$$\pi_t^{GEL}(\tilde{\theta}_T, s) = \frac{1}{T} (1 + o_p(1)).$$

uniformly in $t = 1, \dots, T$. Thus, we get:

$$T\pi_t^{GEL}(\tilde{\theta}_T, s) - 1 = g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T)' \tilde{\gamma}_{1T} (1 + o_p(1)) + O_p(K_T/T). \quad (18)$$

uniformly in $t = 1, \dots, [Ts]$ and

$$T\pi_t^{GEL}(\tilde{\theta}_T, s) - 1 = g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T)' \tilde{\gamma}_{2T} (1 + o_p(1)) + O_p(K_T/T). \quad (19)$$

uniformly in $t = [Ts] + 1, \dots, T$.

⁶See Kitamura and Stutzer (1997) and Guay and Lamarche (2008).

7.3 Proofs of Theorems

Proof of Theorem 4.1

We expand the FOC of the Lagrange multiplier vector for the partial-sample GEL evaluated at the restricted estimator in a Taylor series about 0 for the first part of the sample $t = 1, \dots, [Ts]$. Thus

$$\begin{aligned} \frac{1}{T} \sum_{t=1}^{[Ts]} \rho_1 \left(\tilde{\gamma}'_{1T} g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T) \right) g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T) &= -\frac{1}{T} \sum_{t=1}^{[Ts]} g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T) - \frac{1}{T} \sum_{t=1}^{[Ts]} g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T) g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T)' \tilde{\gamma}_{1T} \\ &+ \frac{1}{T} \sum_{t=1}^{[Ts]} g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T) \sum_{j=2}^{\infty} \frac{1}{j!} \rho_{j+1}(0) \left(g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T)' \tilde{\gamma}_{1T} \right)^j \end{aligned}$$

where $\tilde{\gamma}_{1T}$ is the partial-sample Lagrange multiplier evaluated for the restricted estimator for $t = 1, \dots, [Ts]$ and $\rho_1(0) = \rho_2(0) = -1$. By the fact that $g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T) = O_p\left((2K_T + 1)^{-1/2}\right)$, this yields

$$\begin{aligned} \frac{1}{T} \sum_{t=1}^{[Ts]} \rho_1 \left(\tilde{\gamma}'_{1T} g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T) \right) g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T) &= -\frac{1}{T} \sum_{t=1}^{[Ts]} g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T) - \frac{1}{T} \sum_{t=1}^{[Ts]} g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T) g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T)' \tilde{\gamma}_{1T} \\ &+ O_p\left((2K_T + 1)^{-3/2} \|\tilde{\gamma}_{1T}\|^2\right). \end{aligned}$$

By using a consistent estimator $\tilde{\Omega}_T = \frac{2K_T+1}{[Ts]} \sum_{t=1}^{[Ts]} g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T) g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T)'$ and $\tilde{\gamma}_{1T} = O_p\left((2K_T + 1)/\sqrt{T}\right)$, we get under the null:

$$0 = \frac{1}{T} \sum_{t=1}^{[Ts]} g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T) + \frac{s}{2K_T + 1} \tilde{\Omega}_T \tilde{\gamma}_{1T} + O_p\left((2K_T + 1)^{-3/2} \|\tilde{\gamma}_{1T}\|^2\right)$$

which yields

$$\frac{s}{2K_T + 1} \tilde{\gamma}_{1T} = -\tilde{\Omega}_T^{-1} \frac{1}{T} \sum_{t=1}^{[Ts]} g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T) + O_p\left(K_T^{1/2}/T\right). \quad (20)$$

Now, expanding this expression around β_0 and δ_0 gives:

$$\frac{s\sqrt{T}}{2K_T + 1} \tilde{\gamma}_{1T} = -\tilde{\Omega}_T^{-1} \frac{1}{\sqrt{T}} \sum_{t=1}^{[Ts]} g_{tT}(\beta_0, \delta_0) - s\tilde{\Omega}_T^{-1} G\sqrt{T} \begin{pmatrix} \tilde{\beta}_T - \beta_0 \\ \tilde{\delta}_T - \delta_0 \end{pmatrix} + o_p(1).$$

We can easily show for the restricted estimators under the null that (see also Smith, 2004):

$$\sqrt{T} \begin{pmatrix} \tilde{\beta}_T - \beta_0 \\ \tilde{\delta}_T - \delta_0 \end{pmatrix} = - (G'\Omega^{-1}G)^{-1} G'\Omega^{-1} \frac{1}{\sqrt{T}} \sum_{t=1}^T g_{tT}(\beta_0, \delta_0) + o_p(1). \quad (21)$$

Combining the two preceding results, we obtain

$$\frac{s\sqrt{T}}{2K_T + 1} \tilde{\gamma}_{1T} = -\Omega^{-1} \frac{1}{\sqrt{T}} \sum_{t=1}^{[Ts]} g_{tT}(\beta_0, \delta_0) + s\Omega^{-1} G(G'\Omega^{-1}G)^{-1} G'\Omega^{-1} \frac{1}{\sqrt{T}} \sum_{t=1}^T g_{tT}(\beta_0, \delta_0) + o_p(1).$$

By Lemma 5.1 in Guay and Lamarche (2008), under the null of no structural change, we have

$$\Omega^{-1/2} \frac{1}{\sqrt{T}} \sum_{t=1}^{[Ts]} g_{tT}(\beta_0, \delta_0) \Rightarrow B(s) \quad (22)$$

and under the generic alternative (8),

$$\Omega^{-1/2} \frac{1}{\sqrt{T}} \sum_{t=1}^{[Ts]} g_{tT}(\beta_0, \delta_0) \Rightarrow B(s) + \Omega^{-1/2} H(s) \quad (23)$$

where $B(s)$ is a q -vector of standard Brownian motions and $H(s) = \int_0^s h(r) dr$. Note also that:

$$\frac{1}{T} \sum_{t=1}^{[Ts]} g_{tT}(\beta_0, \delta_0) = \frac{1}{T} \sum_{t=1}^{[Ts]} g_t(\beta_0, \delta_0) + o_p(1).$$

Using Lemma 7.2 and the derivation above,

$$T\pi_t^{GEL}(\tilde{\theta}_T, s) - 1 = - \left(\frac{2K+1}{s} \tilde{\Omega}_T^{-1} \frac{1}{T} \sum_{t=1}^{[Ts]} g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T) + O_p(K_T^{3/2}/T) \right)' \times \quad (24)$$

$$g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T)(1 + o_p(1)) + O_p(K_T/T). \quad (25)$$

Let us now examine the asymptotic distribution of the expression on the right-hand side. First, we show the following asymptotic result for the partial sums of the moments conditions evaluated at $\tilde{\theta}_T$:

$$\Omega^{-1/2} \left[\frac{1}{\sqrt{T}} \sum_{t=1}^{[Ts]} g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T) \right] \Rightarrow B(s) - s\Omega^{-1/2} G[G'\Omega^{-1}G]^{-1} G'\Omega^{-1/2} B(1).$$

By a mean value expansion:

$$\frac{1}{\sqrt{T}} \sum_{t=1}^{[Ts]} g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T) = \frac{1}{\sqrt{T}} \sum_{t=1}^{[Ts]} g_{tT}(\beta_0, \delta_0) + \frac{1}{T} \sum_{t=1}^{[Ts]} \frac{\partial g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T)}{\partial (\beta', \delta')} \begin{pmatrix} \tilde{\beta}_T - \beta_0 \\ \tilde{\delta}_T - \delta_0 \end{pmatrix}$$

where $\tilde{\beta}_T$ lies on the line segment joining $\tilde{\beta}_T$ and β_0 and may differ from row to row and respectively for $\tilde{\delta}_T$. By applying eq. (21), this gives

$$\frac{1}{\sqrt{T}} \sum_{t=1}^{[Ts]} g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T) = \frac{1}{\sqrt{T}} \sum_{t=1}^{[Ts]} g_{tT}(\beta_0, \delta_0) - sG[G'\Omega^{-1}G]^{-1} G'\Omega^{-1} \frac{1}{T} \sum_{t=1}^T g_{tT}(\beta_0, \delta_0) + o_p(1). \quad (26)$$

It follows by (22),

$$\Omega^{-1/2} \frac{1}{\sqrt{T}} \sum_{t=1}^{[Ts]} g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T) \Rightarrow B(s) - s\Omega^{-1/2} G[G'\Omega^{-1}G]^{-1} G'\Omega^{-1/2} B(1). \quad (27)$$

By (24), the following partial sum can be shown to be:

$$\begin{aligned} \sum_{t=1}^{[Ts]} [T\pi_t^{GEL}(\tilde{\theta}_T, s) - 1]^2 &= \left(\frac{2K+1}{s} \tilde{\Omega}_T^{-1} \frac{1}{T} \sum_{t=1}^{[Ts]} g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T) \right)' \sum_{t=1}^{[Ts]} g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T) g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T)' \\ &\quad \times \left(\frac{2K+1}{s} \tilde{\Omega}_T^{-1} \frac{1}{T} \sum_{t=1}^{[Ts]} g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T) \right) + o_p(1). \end{aligned}$$

Considering that $\tilde{\Omega}_T = \frac{2K+1}{[Ts]} \sum_{t=1}^{[Ts]} g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T) g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T)'$, the expression above gives:

$$\sum_{t=1}^{[Ts]} [T \pi_t^{GEL}(\tilde{\theta}_T, s) - 1]^2 = \frac{2K+1}{s} \left(\frac{1}{\sqrt{T}} \sum_{t=1}^{[Ts]} g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T) \right)' \tilde{\Omega}_T^{-1} \left(\frac{1}{\sqrt{T}} \sum_{t=1}^{[Ts]} g_{tT}(\tilde{\beta}_T, \tilde{\delta}_T) \right) + o_p(1).$$

By the result above, the partial sum of interest yields by equations (27) and the consistency of $\tilde{\Omega}_T$:

$$\frac{s}{2K+1} \sum_{t=1}^{[Ts]} [T \pi_t^{GEL}(\tilde{\theta}_T, s) - 1]^2 \Rightarrow \left[B(s) - s\Omega^{-1/2} G[G'\Omega^{-1}G]^{-1} G'\Omega^{-1/2} B(1) \right]' \times \quad (28)$$

$$\left[B(s) - s\Omega^{-1/2} G[G'\Omega^{-1}G]^{-1} G'\Omega^{-1/2} B(1) \right]. \quad (29)$$

The expression $[B(s) - s\Omega^{-1/2} G[G'\Omega^{-1}G]^{-1} G'\Omega^{-1/2} B(1)]$ can be rewritten as:

$$[I - P_G] B(s) + P_G [B(s) - sB(1)]$$

with $P_G = \Omega^{-1/2} G[G'\Omega^{-1}G]^{-1} G'\Omega^{-1/2}$. We can now decompose:

$$\Omega^{-1/2} G(G'\Omega^{-1}G)^{-1} G'\Omega^{-1/2} = C'\Lambda C \quad (30)$$

where $CC' = I$ and

$$\Lambda = \begin{bmatrix} I_p & 0 \\ 0 & 0 \end{bmatrix}$$

and I_p is an identity matrix with dimension $p \times p$ where $p = r + \nu$ with r and ν the respective dimension the vectors β and δ . Note also that $CB(s)$ has the same asymptotic distribution as $B(s)$. The RHS of expression (29) can then be rewritten as:

$$B(s)' [I - C'\Lambda C] B(s) + [B(s) - sB(1)]' C'\Lambda C [B(s) - sB(1)]$$

which is equal in distribution to

$$B_{(q-p)}(s)' B_{(q-p)}(s) + BB_p(s)' BB_p(s)$$

where $B_{(q-p)}(s)$ is a $(q-p)$ -vector of standard Brownian motion and $BB_p(s) = B_p(s) - sB_p(1)$ is a p -vector of Brownian bridge. The result follows. The asymptotic distribution under the alternative (8) can be easily obtained similarly by using (23).

Proof of Theorem 4.3

By Lemma 7.2 applied to the difference between the partial-sample implied probabilities evaluated at the unrestricted and at the restricted estimators we get:

$$T \left[\hat{\pi}_t(\hat{\theta}_T(s), s) - \tilde{\pi}_t(\tilde{\theta}_T, s) \right] = g_{tT}(\hat{\theta}_T(s), s)' \hat{\gamma}_T(s) - g_{tT}(\tilde{\theta}_T, s)' \tilde{\gamma}_T(s) + o_p(T^{-1/2}).$$

Let us define the following selection matrices:

$$H_1 = \begin{bmatrix} I_{r \times r} & 0_{r \times r} & 0_{r \times \nu} \\ 0_{\nu \times r} & 0_{\nu \times r} & I_{\nu \times \nu} \end{bmatrix}$$

and

$$H_2 = \begin{bmatrix} 0_{r \times r} & I_{r \times r} & 0_{r \times \nu} \\ 0_{\nu \times r} & 0_{\nu \times r} & I_{\nu \times \nu} \end{bmatrix}.$$

and the corresponding estimator: $\hat{\theta}_{1T}(s) = H_1 \hat{\theta}_T(s) = \left(\hat{\beta}_{1T}(s)', \hat{\delta}_T(s)' \right)'$ and $\hat{\theta}_{2T}(s) = H_2 \hat{\theta}_T(s) = \left(\hat{\beta}_{2T}(s)', \hat{\delta}_T(s)' \right)'$. Similarly, for the restricted estimator $\tilde{\theta}_{1T}(s) = H_1 \tilde{\theta}_T(s) = \left(\tilde{\beta}_T(s)', \tilde{\delta}_T(s)' \right)'$ and $\tilde{\theta}_{2T}(s) = H_2 \tilde{\theta}_T(s) = \left(\tilde{\beta}_T(s)', \tilde{\delta}_T(s)' \right)'$ where $\tilde{\theta}_T(s) = \left(\tilde{\beta}_T(s)', \tilde{\beta}_T(s)', \tilde{\delta}_T(s)' \right)'$. Accordingly, we define $\theta_{1,0} = H_1 \theta_0 = (\beta'_0, \delta'_0)'$ and $\theta_{2,0} = H_2 \theta_0 = (\beta'_0, \delta'_0)'$.

Let us examine the expression for the first part of the sample, namely $t = 1, \dots, [Ts]$. Replacing the unrestricted and the restricted estimators of γ by the corresponding expression (20), we get:

$$\begin{aligned} T \left[\pi_t(\hat{\theta}_T(s), s) - \pi_t(\tilde{\theta}_T, s) \right] &= - \left[(2K_T + 1)\Omega(s)^{-1} \frac{1}{[Ts]} \sum_{t=1}^{[Ts]} g_{tT}(\hat{\theta}_T(s), s) + o_p(1) \right]' g_{tT}(\hat{\theta}_T(s), s)(1 + o_p(1)) \\ &+ \left[(2K_T + 1)\Omega(s)^{-1} \frac{1}{[Ts]} \sum_{t=1}^{[Ts]} g_{tT}(\tilde{\theta}_T, s) + o_p(1) \right]' g_{tT}(\tilde{\theta}_T, s)(1 + o_p(1)) \\ &+ O_p(K_T/T) \end{aligned}$$

which equals

$$T \left[\pi_t(\hat{\theta}_T(s), s) - \pi_t(\tilde{\theta}_T, s) \right] = - \left[\frac{1}{[Ts]} \sum_{t=1}^{[Ts]} g_{tT}(\hat{\theta}_{1T}(s)) - \frac{1}{[Ts]} \sum_{t=1}^{[Ts]} g_{tT}(\tilde{\theta}_{1T}) + o_p(1) \right]' \quad (31)$$

$$\times \Omega^{-1}(2K_T + 1)g_{tT}(\hat{\theta}_{1T}(s))(1 + o_p(1)) + O_p(K_T/T) \quad (32)$$

since $G_{tT}(\cdot) = O_p((2K_T + 1)^{-1/2})$ and $\hat{\theta}_{1T}(s) - \tilde{\theta}_{1T} = O_p(T^{-1/2})$.

Now, consider the following mean value expansion:

$$\frac{1}{[Ts]} \sum_{t=1}^{[Ts]} g_{tT}(\tilde{\theta}_{1T}) = \frac{1}{[Ts]} \sum_{t=1}^{[Ts]} g_{tT}(\hat{\theta}_{1T}(s)) + \frac{1}{[Ts]} \sum_{t=1}^{[Ts]} \frac{\partial g_{tT}(\bar{\theta}_{1T}(s))}{\partial \theta'_1} (\tilde{\theta}_{1T} - \hat{\theta}_{1T}(s)). \quad (33)$$

where $\bar{\theta}_{1T}(s)$ lies on the line segment joining $\hat{\theta}_{1T}(s)$ and $\tilde{\theta}_{1T}$ and may differ from row to row. Thus,

$$\begin{aligned} T \left[\pi_t(\hat{\theta}_T(s), s) - \pi_t(\tilde{\theta}_T, s) \right] &= \left[\frac{1}{[Ts]} \sum_{t=1}^{[Ts]} \frac{\partial g_{tT}(\bar{\theta}_{1T}(s))}{\partial \theta'_1} (\tilde{\theta}_{1T} - \hat{\theta}_{1T}(s)) + o_p(1) \right]' \\ &\times \Omega^{-1}(2K_T + 1)g_{tT}(\hat{\theta}_{1T}(s))(1 + o_p(1)) + O_p(K_T/T). \end{aligned}$$

The partial sum over the first subsample for the square of the LHS expression above yields:

$$\frac{1}{2K + 1} \sum_{t=1}^{[Ts]} \left[T \pi_t(\hat{\theta}_T(s), s) - T \pi_t(\tilde{\theta}_T, s) \right]^2 = [Ts] \left(\tilde{\theta}_{1T} - \hat{\theta}_{1T}(s) \right)' G' \Omega^{-1} G \left(\tilde{\theta}_{1T} - \hat{\theta}_{1T}(s) \right) + o_p(1)$$

as $(2K_T + 1) \frac{1}{[Ts]} \sum_{t=1}^{[Ts]} g_{tT}(\hat{\theta}_{1T}(s), s) g_{tT}(\hat{\theta}_{1T}(s), s)'$ is a consistent estimator of Ω .

Similarly for the second subsample $t = [Ts] + 1, \dots, T$, we obtain

$$\frac{1}{2K+1} \sum_{t=[Ts]+1}^T \left[T\pi_t(\hat{\theta}_T(s), s) - T\pi_t(\tilde{\theta}_T, s) \right]^2 = (T - [Ts]) \left(\tilde{\theta}_{2T} - \hat{\theta}_{2T}(s) \right)' G' \Omega^{-1} G \left(\tilde{\theta}_{2T} - \hat{\theta}_{2T}(s) \right) + o_p(1)$$

By results derived above and some calculations, this yields

$$\frac{1}{2K+1} \sum_{t=1}^T \left[T\pi_t(\hat{\theta}_T(s), s) - T\pi_t(\tilde{\theta}_T, s) \right]^2 = T \left(\tilde{\theta}_T - \hat{\theta}_T(s) \right)' G(\theta_0, s)' \Omega^{-1}(s) G(\theta_0, s) \left(\tilde{\theta}_T - \hat{\theta}_T(s) \right) + o_p(1).$$

Andrews (1993, p. 851-852) shows that this expression is asymptotically equivalent to the $LM_T(s)$ statistic for parameters instability. This gives the result follows under the null. Under the alternative, Guay and Lamarche (2008) show that the $LM_T(s)$ statistic for the restricted GEL estimator has the same asymptotic distribution as in Theorem 4.3. Since the asymptotic equivalence between the expression above and the $LM_T(s)$ statistic, the result follows.

Proof of Theorem 4.4

The two subsamples are now evaluated separately. A direct application of Lemma 7.2 but for the estimation of the subsample with the first $[Ts]$ observations gives

$$[Ts]\pi_t^{GEL}(\hat{\theta}_{1T}(s), s) - 1 = g_{tT}(\hat{\theta}_{1T}(s))' \hat{\gamma}_{1T}(s) + o_p([Ts]^{-1/2})$$

uniformly in $t = 1, \dots, [Ts]$. This is obtained with a proof similar to the one of Lemma 7.2 but only for the first part of the sample. This yields

$$\left([Ts]\pi_t^{GEL}(\hat{\theta}_{1T}(s), s) - 1 \right)^2 = \hat{\gamma}_{1T}(s)' g_{tT}(\hat{\theta}_{1T}(s)) g_{tT}(\hat{\theta}_{1T}(s))' \hat{\gamma}_{1T}(s) + o_p([Ts]^{-1}).$$

By summing the expression above to $t = \dots, [Ts]$, and by (20) for the unrestricted estimator applied to the sample $t = 1, \dots, [Ts]$, this yields

$$\begin{aligned} \sum_{t=1}^{[Ts]} \left([Ts]\pi_t^{GEL}(\hat{\theta}_{1T}(s), s) - 1 \right)^2 &= (2K_T + 1)^2 \frac{1}{\sqrt{[Ts]}} \sum_{t=1}^{[Ts]} g_{tT}(\hat{\theta}_{1T}(s))' \Omega^{-1} \frac{1}{[Ts]} \sum_{t=1}^{[Ts]} g_{tT}(\hat{\theta}_{1T}(s)) \\ &\quad \times \left(g_{tT}(\hat{\theta}_{1T}(s)) \right)' \Omega^{-1} \frac{1}{\sqrt{[Ts]}} \sum_{t=1}^{[Ts]} g_{tT}(\hat{\theta}_{1T}(s)) + o_p(1). \end{aligned}$$

As $(2K_T + 1) \frac{1}{[Ts]} \sum_{t=1}^{[Ts]} g_{tT}(\hat{\theta}_{1T}(s)) g_{tT}(\hat{\theta}_{1T}(s))'$ is a consistent estimator of Ω , this gives

$$\frac{1}{2K_T + 1} \sum_{t=1}^{[Ts]} \left([Ts]\pi_t^{GEL}(\hat{\theta}_{1T}(s), s) - 1 \right)^2 = \frac{1}{\sqrt{[Ts]}} \sum_{t=1}^{[Ts]} g_{tT}(\hat{\theta}_{1T}(s))' \Omega^{-1} \frac{1}{\sqrt{[Ts]}} \sum_{t=1}^{[Ts]} g_{tT}(\hat{\theta}_{1T}(s)) + o_p(1). \quad (34)$$

Similarly, we can easily obtain that

$$\frac{1}{2K_T + 1} \sum_{t=[Ts]+1}^T \left((T - [Ts]) \pi_t^{GEL}(\hat{\theta}_{2T}(s), s) - 1 \right)^2 =$$

$$\frac{1}{\sqrt{T - [Ts]}} \sum_{t=[Ts]+1}^T g_{tT}(\hat{\theta}_{2T}(s))' \Omega^{-1} \frac{1}{\sqrt{T - [Ts]}} \sum_{t=[Ts]+1}^T g_{tT}(\hat{\theta}_{2T}(s)) + o_p(1). \quad (35)$$

Now, by a mean value expansion and the consistency of $\hat{\theta}_T(s)$,

$$\Omega(s)^{-1/2} \frac{1}{\sqrt{T}} \sum_{t=1}^T g_{tT}(\hat{\theta}_T(s), s) = \Omega(s)^{-1/2} \left[\begin{array}{c} \frac{1}{\sqrt{T}} \sum_{t=1}^{[Ts]} g_{tT}(\beta_0, \delta_0) \\ \frac{1}{\sqrt{T}} \sum_{t=[Ts]+1}^T g_{tT}(\beta_0, \delta_0) \end{array} \right] + \Omega(s)^{-1/2} G(s) \sqrt{T} (\hat{\theta}_T(s) - \theta_0) + o_p(1).$$

By Lemma 7.1 and (22),

$$\begin{aligned} \Omega(s)^{-1/2} \frac{1}{\sqrt{T}} \sum_{t=1}^T g_{tT}(\hat{\theta}_T(s), s) &\Rightarrow \left[\frac{\frac{1}{\sqrt{s}} B(s)}{\sqrt{(1-s)} (B(1) - B(s))} \right] - \Omega(s)^{-1/2} G(s) (G(s)' \Omega(s)^{-1} G(s))^{-1} \\ &\quad \times G(s)' \Omega(s)^{-1/2} \left[\begin{array}{c} B(s) \\ (B(1) - B(s)) \end{array} \right] \end{aligned}$$

The RHS can be rewritten as

$$\left(I - \Omega(s)^{-1/2} G(s) (G(s)' \Omega(s)^{-1} G(s))^{-1} G(s)' \Omega(s)^{-1/2} \right) \left[\begin{array}{c} \frac{1}{\sqrt{s}} B(s) \\ \frac{1}{\sqrt{(1-s)} (B(1) - B(s))} \end{array} \right]$$

Since the entire parameter vector is estimated for both subsample, e.g. $\hat{\theta}_{1T}(s) = \hat{\beta}_{1T}(s)$ and $\hat{\theta}_{2T}(s) = \hat{\beta}_{2T}(s)$, the matrices $\Omega(s)$ and $G(s)$ are block-diagonal and by the definition of $g_{tT}(\hat{\theta}_T(s), s)$, we get:

$$\Omega^{-1/2} \frac{1}{\sqrt{T}} \sum_{t=1}^{[Ts]} g_{tT}(\hat{\beta}_{1T}(s)) \Rightarrow \left(I_q - \Omega^{-1/2} G_\beta (G'_\beta \Omega^{-1} G_\beta)^{-1} G'_\beta \Omega^{-1/2} \right) B(s) \quad (36)$$

and

$$\Omega^{-1/2} \frac{1}{\sqrt{T}} \sum_{t=[Ts]+1}^T g_{tT}(\hat{\beta}_{2T}(s)) \Rightarrow \left(I_q - \Omega^{-1/2} G_\beta (G'_\beta \Omega^{-1} G_\beta)^{-1} G'_\beta \Omega^{-1/2} \right) (B(1) - B(s)).$$

We can now decompose:

$$\left(I - \Omega^{-1/2} G_\beta (G'_\beta \Omega^{-1} G_\beta)^{-1} G'_\beta \Omega^{-1/2} \right) = (I - C' \Lambda C)$$

where $CC' = I$ and

$$\Lambda = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$$

and I_r is an identity matrix with dimension $r \times r$ with r is the dimension of the vectors β . By noting that $C(I - C' \Lambda C)B(s) = (I - \Lambda)CB(s)$ and $CB(s)$ is also a r -vector of Brownian motion. Now consider the multiplication of the RHS term of equation (36) by the matrix C , this yields

$$C \Omega^{-1/2} \frac{1}{\sqrt{T}} \sum_{t=1}^{[Ts]} g_{tT}(\hat{\beta}_{1T}(s)) \Rightarrow (I_q - \Lambda) B(s)$$

By using this result, we obtain directly the asymptotic distribution of the statistic for the first subsample by equation (34) considering that $C'C = I_q$. The proof is similar for the second subsample (eq. 35), the asymptotic distribution under the null is then derived.

Under the alternative (8) by using Lemma 7.1 it is straightforward to show the asymptotic distribution by proceeding in the same manner as above.

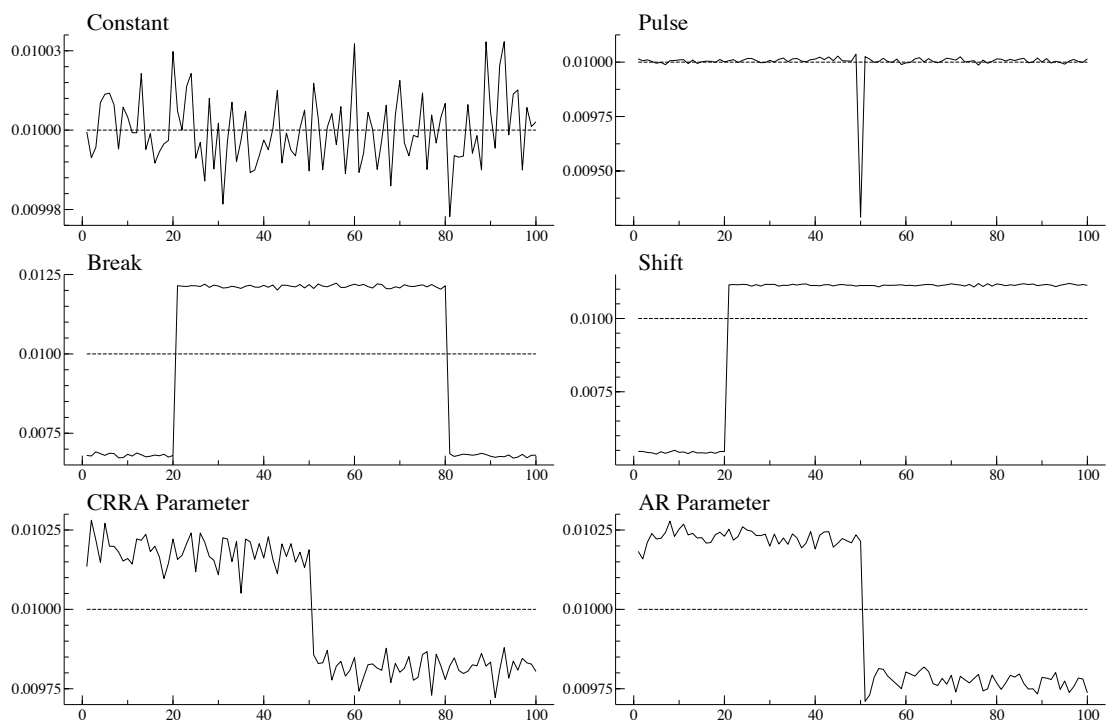
References

- Anatolyev, S. (2005) ‘GMM, GEL, serial correlation, and asymptotic bias.’ *Econometrica* 73(3), 983–1002
- Andrews, D.W.K. (1991) ‘Heteroskedasticity and autocorrelation consistent covariance matrix estimation.’ *Econometrica* 59, 817–858
- (1993) ‘Tests for parameter instability and structural change with unknown change point.’ *Econometrica* 61, 821–856
- Andrews, D.W.K., and J.C. Monahan (1992) ‘An improved heteroskedasticity and autocorrelation consistent covariance matrix estimator.’ *Econometrica* 60, 953–966
- Andrews, D.W.K., and W. Ploberger (1994) ‘Optimal tests when a nuisance parameter is present only under the alternative.’ *Econometrica* 62, 1383–1414
- Antoine, B., H. Bonnal, and E. Renault (2007) ‘On the efficient use of the informational content of estimating equations: implied probabilities and euclidean empirical likelihood.’ *Journal of Econometrics* 138, 461–487
- Back, K., and D.P. Brown (1993) ‘Implied probabilities in GMM estimators.’ *Econometrica* 61, 971–975
- Caner, M. (2005) ‘Exponential tilting and weak instruments: estimation and testing.’ Manuscript, University of Pittsburg
- Caner, Mehmet (2004) ‘Nearly singular design in GMM and generalized empirical likelihood estimators.’ Manuscript, University of Pittsburg
- Cressie, N., and T. Read (1984) ‘Multinomial goodness-of-fit tests.’ *Journal of the Royal Statistical Society B* 46, 440–464
- Gallant, A.R. (1987) *Nonlinear statistical models* (New York: Wiley)
- Ghysels, E., A. Guay, and A. Hall (1997) ‘Predictive tests for structural change with unknown breakpoint.’ *Journal of Econometrics* 82, 209–233
- Ghysels, E., and A. Hall (1990) ‘Are consumption-based intertemporal capital asset pricing models structural.’ *Journal of Econometrics* 45, 121–139
- Gregory, A.W., Jean-Francois Lamarche, and Gregor W. Smith (2002) ‘Information-theoretic estimation of preference parameters: macroeconomic applications and simulation evidence.’ *Journal of Econometrics* 107, 213–233
- Guay, A., and F. Pelgrin (2007) ‘Using implied probabilities to improve estimation in unconditional moment models: Re-assessing the new keynesian phillips curve.’ Manuscript

- Guay, A., and J-F Lamarche (2007) ‘Structural change tests for GEL criteria.’ Manuscript
- Guggenberger, P., and R. J. Smith (2007) ‘Generalized empirical likelihood tests in time series models with potential identification failure.’ Technical Report
- Guggenberger, P., J.J.S. Ramalho, and R. J. Smith (2007) ‘Gel pearson-type statistics under weak identification.’ Manuscript
- Hall, A.R., and A. Sen (1999) ‘Structural stability testing in models estimated by generalized methods of moments.’ *Journal of Business and Economic Statistics* 17, 335–348
- Hall, P., and J.L. Horowitz (1996) ‘Bootstrap critical values for tests based on generalized-method-of-moments estimators.’ *Econometrica* 64, 891–916
- Hansen, L.P. (1982) ‘Large sample properties of generalized method of moments estimators.’ *Econometrica* 50, 1029–1054
- Hansen, L.P., J. Heaton, and A. Yaron (1996) ‘Finite-sample properties of some alternative GMM estimators.’ *Journal of Business and Economic Statistics* 14, 262–280
- Imbens, G.W., R.H. Spady, and P. Johnson (1998) ‘Information theoretic approaches to inference in moment condition models.’ *Econometrica* 66, 333–357
- Kitamura, Y. (2001) ‘Asymptotic optimality of empirical likelihood for testing moment restrictions.’ *Econometrica* 69, 1661–1672
- Kitamura, Y., and M. Stutzer (1997) ‘An information-theoretic alternative to generalized method of moments estimation.’ *Econometrica* 65, 861–874
- Kullback, S., and R.A. Leibler (1951) ‘On information and sufficiency.’ *Annals of Mathematical Statistics* 22, 79–86
- Newey, W.K., and K.D. West (1987) ‘A simple positive semidefinite, heteroskedastic and autocorrelation consistent covariance matrix.’ *Econometrica* 55, 703–708
- (1994) ‘Automatic lag selection in covariance matrix estimation.’ *Review of Economic Studies* 61, 631–654
- Newey, W.K., and R.J. Smith (2004) ‘Higher order proprieties of GMM and generalized empirical likelihood estimators.’ *Econometrica* 72, 219–256
- Otsu, T. (2006) ‘Generalized empirical likelihood inference for nonlinear and time series models under weak identification.’ *Econometric Theory* 22, 513–527
- Pollard, D. (1984) *Convergence of Stochastic Processes* (New York: Springer-Verlag)

- Qin, J., and J. Lawless (1994) ‘Empirical likelihood and general estimating equations.’ *Annals of Statistics* 22, 300–325
- Ramalho, J.J.S., and R. J. Smith (2005) ‘Goodness of fit tests for moment condition models.’ Documento de trabalho 20005/05
- Schennach, S.M. (2004) ‘Point estimation with exponentially tilted empirical likelihood.’ Manuscript (accepted for publication at *Annals of Statistics*), University of Chicago
- Shannon, C.E. (1948) ‘Axiomatic derivation of the principle of maximum entropy and the principle of minimum cross-entropy.’ *IEEE Transactions on information theory* 26, 26–37
- Smith, R. J. (2004) ‘GEL criteria for moment condition models.’ CEMMAP Working Paper CWP19/04
- Smith, R.J. (1997) ‘Alternative semi-parametric likelihood approaches to generalised method of moments.’ *The Economic Journal* 107, 503–519
- Smith, R.J. (2000) ‘Empirical likelihood estimation and inference.’ In *Applications of Differential Geometry to Econometrics*, ed. M. Salmon and P. Marriott (Cambridge: Cambridge University Press) pp. 119–150
- Sowell, F. (1996a) ‘Optimal tests of parameter variation in the generalized method of moments framework.’ *Econometrica* 64, 1085–1108
- (1996b) ‘Test for violations of moment conditions.’ Manuscript, Carnegie Mellon University

Figure 1: Simulated Implied Probabilities



Notes: The *constant* case refers to no break, the *pulse* case to a one-time temporary jump at observation 50, the *break* case to a change in the mean for 60 time periods and the *shift* to a permanent change in the mean at observation 20. For the CRRA case, a preference parameter is estimated using two moments. The moment conditions are violated at observation 51. For the AR case a autoregressive parameter is estimated. The data generating process is represented by an $AR(1)$ process for $t \leq 50$ and by an $ARMA(1, 2)$ otherwise. The sample is of size 100 and in all cases the horizontal lines correspond to the empirical weights of $1/100$.

Table 2: Asymptotic critical values for the exponential mapping

p	s_0	$q - p = 1$					$q - p = 2$					$q - p = 3$					$q - p = 4$					$q - p = 5$																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																			
		10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
1	0	0.8342	1.1957	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	2	0	0.8778	1.1839	2.1343	1.4994	1.9330	3.1166	2.1388	2.7414	4.1997	2.7060	3.3159	4.8419	3.2466	3.9516	5.5217	3	0	1.0553	1.3942	2.2999	1.6339	2.0587	3.1116	2.1948	2.7097	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696	4	0	1.1518	1.4518	2.2657	1.7297	2.2657	3.2484	2.2657	2.4284	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696	5	0	1.2498	1.5596	2.3774	1.8510	2.3774	3.3774	2.3774	2.5596	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696	0.15	0.8385	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	6	0	1.3481	1.6449	2.2876	1.7584	2.2497	3.0756	2.3635	2.7608	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696	0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	7	0	1.4518	1.7479	2.3414	1.8510	2.3414	3.3774	2.3774	2.5596	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696	0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	8	0	1.5596	1.8510	2.4445	1.9501	2.4445	3.4774	2.4774	2.7408	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696	0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	9	0	1.6637	1.9501	2.5596	2.0350	2.5596	3.5001	2.5001	2.7635	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696	0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	10	0	1.7714	2.0350	2.6637	2.1116	2.6637	3.6066	2.6066	2.8700	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696	0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374
	0.15	0.8385	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374		6	0	1.3481	1.6449	2.2876	1.7584	2.2497	3.0756	2.3635	2.7608	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696	0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525		5.5374	7	0	1.4518	1.7479	2.3414	1.8510	2.3414	3.3774	2.3774	2.5596	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696	0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523		3.9525	5.5374	8	0	1.5596	1.8510	2.4445	1.9501	2.4445	3.4774	2.4774	2.7408	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696	0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664		3.2523	3.9525	5.5374	9	0	1.6637	1.9501	2.5596	2.0350	2.5596	3.5001	2.5001	2.7635	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696	0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449		4.8664	3.2523	3.9525	5.5374	10	0	1.7714	2.0350	2.6637	2.1116	2.6637	3.6066	2.6066	2.8700	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696	0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771		3.3449	4.8664	3.2523	3.9525	5.5374																																																																											
	0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374		6	0	1.3481	1.6449	2.2876	1.7584	2.2497	3.0756	2.3635	2.7608	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066			5.0696	0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	7	0	1.4518	1.7479	2.3414	1.8510	2.3414	3.3774	2.3774	2.5596	4.0457	2.8531	3.2733	4.9643		3.3735		3.6066	5.0696	0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	8	0	1.5596	1.8510	2.4445	1.9501	2.4445	3.4774	2.4774	2.7408	4.0457	2.8531		3.2733	4.9643		3.3735	3.6066	5.0696	0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	9	0	1.6637	1.9501	2.5596	2.0350	2.5596	3.5001	2.5001	2.7635		4.0457	2.8531	3.2733		4.9643	3.3735	3.6066	5.0696	0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	10	0	1.7714	2.0350	2.6637	2.1116	2.6637	3.6066		2.6066	2.8700	4.0457	2.8531		3.2733	4.9643	3.3735	3.6066	5.0696	0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374																																																																																							
	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374		6	0	1.3481	1.6449	2.2876	1.7584	2.2497	3.0756	2.3635	2.7608	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066			5.0696	0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294			3.9290	5.5184	0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	7	0	1.4518	1.7479	2.3414	1.8510	2.3414	3.3774	2.3774	2.5596	4.0457	2.8531	3.2733	4.9643	3.3735		3.6066	5.0696	0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542		3.3224		4.8436	3.2294	3.9290	5.5184	0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	8	0	1.5596	1.8510	2.4445	1.9501	2.4445	3.4774	2.4774	2.7408	4.0457	2.8531	3.2733	4.9643		3.3735	3.6066	5.0696	0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166		2.5697	3.9513		2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	9	0	1.6637	1.9501	2.5596	2.0350	2.5596	3.5001	2.5001	2.7635	4.0457	2.8531	3.2733		4.9643	3.3735	3.6066	5.0696	0.15	0.8379	1.1546	2.1342	1.4567		1.9452	3.2459	2.0166		2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	10	0	1.7714	2.0350	2.6637	2.1116	2.6637	3.6066	2.6066	2.8700	4.0457	2.8531		3.2733	4.9643	3.3735	3.6066	5.0696	0.15	0.8379		1.1546	2.1342	1.4567	1.9452		3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374																																																																																																		
	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374		6	0	1.3481	1.6449	2.2876	1.7584	2.2497	3.0756	2.3635	2.7608	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066			5.0696	0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294			3.9290	5.5184	0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436			3.2294	3.9290	5.5184	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	7	0	1.4518	1.7479	2.3414	1.8510	2.3414	3.3774	2.3774	2.5596	4.0457	2.8531	3.2733	4.9643	3.3735		3.6066	5.0696	0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224		4.8436	3.2294	3.9290	5.5184	0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697		3.9513		2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	8	0	1.5596	1.8510	2.4445	1.9501	2.4445	3.4774	2.4774	2.7408	4.0457	2.8531	3.2733	4.9643		3.3735	3.6066	5.0696	0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513		2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.20	0.8379	1.1546	2.1342	1.4567		1.9452	3.2459		2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	9	0	1.6637	1.9501	2.5596	2.0350	2.5596	3.5001	2.5001	2.7635	4.0457	2.8531	3.2733		4.9643	3.3735	3.6066	5.0696	0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166		2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.20		0.8379	1.1546	2.1342		1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	10	0	1.7714	2.0350	2.6637	2.1116	2.6637	3.6066	2.6066	2.8700	4.0457	2.8531		3.2733	4.9643	3.3735	3.6066	5.0696	0.15	0.8379	1.1546	2.1342	1.4567	1.9452		3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436		3.2294	3.9290	5.5184	0.20		0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374																																																																																																													
6	0	1.3481	1.6449	2.2876	1.7584	2.2497	3.0756	2.3635	2.7608	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696	0.15		0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.20		0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.25		0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30		0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	7	0	1.4518	1.7479	2.3414	1.8510	2.3414	3.3774	2.3774	2.5596	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696	0.15	0.8379	1.1546		2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.20		0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290		5.5184	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387		3.9381	5.5279	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	8	0	1.5596	1.8510	2.4445	1.9501	2.4445	3.4774	2.4774	2.7408	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696	0.15	0.8379	1.1546		2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436		3.2294	3.9290	5.5184	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635		3.3315	4.8529	3.2387	3.9381	5.5279	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	9	0	1.6637	1.9501	2.5596	2.0350	2.5596	3.5001	2.5001	2.7635	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696	0.15	0.8379	1.1546		2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290		5.5184	0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542		3.3224	4.8436	3.2294	3.9290	5.5184	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214		2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	10	0	1.7714	2.0350	2.6637	2.1116	2.6637	3.6066	2.6066	2.8700	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696	0.15	0.8379	1.1546		2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294		3.9290	5.5184	0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697		3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.25	0.8491	1.1587	2.1387	1.4616		1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374																																																																																																																																	
	0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.20		0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.25		0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30		0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	7	0	1.4518	1.7479	2.3414	1.8510	2.3414	3.3774	2.3774	2.5596	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.20	0.8379	1.1546		2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.25		0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381		5.5279	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	8	0	1.5596	1.8510	2.4445	1.9501	2.4445	3.4774	2.4774	2.7408	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.20	0.8379	1.1546		2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529		3.2387	3.9381	5.5279	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	9	0	1.6637	1.9501	2.5596	2.0350	2.5596	3.5001	2.5001	2.7635	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.20	0.8379	1.1546		2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290		5.5184	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635		3.3315	4.8529	3.2387	3.9381	5.5279	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	10	0	1.7714	2.0350	2.6637	2.1116	2.6637	3.6066	2.6066	2.8700	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.20	0.8379	1.1546		2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294		3.9290	5.5184	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747		3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374																																																																																																																																																		
	0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.25		0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30		0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	7	0	1.4518	1.7479	2.3414	1.8510	2.3414	3.3774	2.3774	2.5596	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.25	0.8491	1.1587		2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30		0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	8	0	1.5596	1.8510	2.4445	1.9501	2.4445	3.4774	2.4774	2.7408	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.25	0.8491	1.1587		2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	9	0	1.6637	1.9501	2.5596	2.0350	2.5596	3.5001	2.5001	2.7635	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.25	0.8491	1.1587		2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381		5.5279	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	10	0	1.7714	2.0350	2.6637	2.1116	2.6637	3.6066	2.6066	2.8700	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184	0.25	0.8491	1.1587		2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387		3.9381	5.5279	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374																																																																																																																																																																			
	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30		0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	7	0	1.4518	1.7479	2.3414	1.8510	2.3414	3.3774	2.3774	2.5596	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30	0.8585	1.1723		2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	8	0	1.5596	1.8510	2.4445	1.9501	2.4445	3.4774	2.4774	2.7408	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30	0.8585	1.1723		2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	9	0	1.6637	1.9501	2.5596	2.0350	2.5596	3.5001	2.5001	2.7635	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30	0.8585	1.1723		2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	10	0	1.7714	2.0350	2.6637	2.1116	2.6637	3.6066	2.6066	2.8700	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279	0.30	0.8585	1.1723		2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374																																																																																																																																																																																				
	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	7	0	1.4518	1.7479	2.3414	1.8510	2.3414	3.3774	2.3774	2.5596	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	8	0	1.5596	1.8510	2.4445	1.9501	2.4445	3.4774	2.4774	2.7408	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	9	0	1.6637	1.9501	2.5596	2.0350	2.5596	3.5001	2.5001	2.7635	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	10	0	1.7714	2.0350	2.6637	2.1116	2.6637	3.6066	2.6066	2.8700	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374																																																																																																																																																																																																					
7	0	1.4518	1.7479	2.3414	1.8510	2.3414	3.3774	2.3774	2.5596	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	8	0	1.5596	1.8510	2.4445	1.9501	2.4445	3.4774	2.4774	2.7408	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	9	0	1.6637	1.9501	2.5596	2.0350	2.5596	3.5001	2.5001	2.7635	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	10	0	1.7714	2.0350	2.6637	2.1116	2.6637	3.6066	2.6066	2.8700	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374																																																																																																																																																																																																																						
	0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	8	0	1.5596	1.8510	2.4445	1.9501	2.4445	3.4774	2.4774	2.7408	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	9	0	1.6637	1.9501	2.5596	2.0350	2.5596	3.5001	2.5001	2.7635	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	10	0	1.7714	2.0350	2.6637	2.1116	2.6637	3.6066	2.6066	2.8700	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374																																																																																																																																																																																																																																							
	0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	8	0	1.5596	1.8510	2.4445	1.9501	2.4445	3.4774	2.4774	2.7408	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	9	0	1.6637	1.9501	2.5596	2.0350	2.5596	3.5001	2.5001	2.7635	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	10	0	1.7714	2.0350	2.6637	2.1116	2.6637	3.6066	2.6066	2.8700	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374																																																																																																																																																																																																																																																								
	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	8	0	1.5596	1.8510	2.4445	1.9501	2.4445	3.4774	2.4774	2.7408	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	9	0	1.6637	1.9501	2.5596	2.0350	2.5596	3.5001	2.5001	2.7635	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	10	0	1.7714	2.0350	2.6637	2.1116	2.6637	3.6066	2.6066	2.8700	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374																																																																																																																																																																																																																																																																									
	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	8	0	1.5596	1.8510	2.4445	1.9501	2.4445	3.4774	2.4774	2.7408	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	9	0	1.6637	1.9501	2.5596	2.0350	2.5596	3.5001	2.5001	2.7635	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	10	0	1.7714	2.0350	2.6637	2.1116	2.6637	3.6066	2.6066	2.8700	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374																																																																																																																																																																																																																																																																																										
8	0	1.5596	1.8510	2.4445	1.9501	2.4445	3.4774	2.4774	2.7408	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	9	0	1.6637	1.9501	2.5596	2.0350	2.5596	3.5001	2.5001	2.7635	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	10	0	1.7714	2.0350	2.6637	2.1116	2.6637	3.6066	2.6066	2.8700	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374																																																																																																																																																																																																																																																																																																											
	0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	9	0	1.6637	1.9501	2.5596	2.0350	2.5596	3.5001	2.5001	2.7635	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	10	0	1.7714	2.0350	2.6637	2.1116	2.6637	3.6066	2.6066	2.8700	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374																																																																																																																																																																																																																																																																																																																												
	0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	9	0	1.6637	1.9501	2.5596	2.0350	2.5596	3.5001	2.5001	2.7635	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	10	0	1.7714	2.0350	2.6637	2.1116	2.6637	3.6066	2.6066	2.8700	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374																																																																																																																																																																																																																																																																																																																																													
	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	9	0	1.6637	1.9501	2.5596	2.0350	2.5596	3.5001	2.5001	2.7635	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	10	0	1.7714	2.0350	2.6637	2.1116	2.6637	3.6066	2.6066	2.8700	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374																																																																																																																																																																																																																																																																																																																																																														
	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	9	0	1.6637	1.9501	2.5596	2.0350	2.5596	3.5001	2.5001	2.7635	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	10	0	1.7714	2.0350	2.6637	2.1116	2.6637	3.6066	2.6066	2.8700	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374																																																																																																																																																																																																																																																																																																																																																																															
9	0	1.6637	1.9501	2.5596	2.0350	2.5596	3.5001	2.5001	2.7635	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	10	0	1.7714	2.0350	2.6637	2.1116	2.6637	3.6066	2.6066	2.8700	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374																																																																																																																																																																																																																																																																																																																																																																																																
	0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	10	0	1.7714	2.0350	2.6637	2.1116	2.6637	3.6066	2.6066	2.8700	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374																																																																																																																																																																																																																																																																																																																																																																																																																	
	0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	10	0	1.7714	2.0350	2.6637	2.1116	2.6637	3.6066	2.6066	2.8700	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374																																																																																																																																																																																																																																																																																																																																																																																																																																		
	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	10	0	1.7714	2.0350	2.6637	2.1116	2.6637	3.6066	2.6066	2.8700	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374																																																																																																																																																																																																																																																																																																																																																																																																																																																			
	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374	10	0	1.7714	2.0350	2.6637	2.1116	2.6637	3.6066	2.6066	2.8700	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374																																																																																																																																																																																																																																																																																																																																																																																																																																																																				
10	0	1.7714	2.0350	2.6637	2.1116	2.6637	3.6066	2.6066	2.8700	4.0457	2.8531	3.2733	4.9643	3.3735	3.6066	5.0696		0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374																																																																																																																																																																																																																																																																																																																																																																																																																																																																																					
	0.15	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																						
	0.20	0.8379	1.1546	2.1342	1.4567	1.9452	3.2459	2.0166	2.5697	3.9513	2.6542	3.3224	4.8436	3.2294	3.9290	5.5184		0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																							
	0.25	0.8491	1.1587	2.1387	1.4616	1.9501	3.2501	2.0214	2.5747	3.9604	2.6635	3.3315	4.8529	3.2387	3.9381	5.5279		0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																								
	0.30	0.8585	1.1723	2.1523	1.4752	1.9637	3.2637	2.0350	2.5883	3.9737	2.6771	3.3449	4.8664	3.2523	3.9525	5.5374																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																									

Table 4: Data Generating Processes

	H_0^{SS}	H_A^I	H_A^O
DGP1	$\theta_1 = \theta_2 = 0$		$\alpha = 0$
DGP2	$\theta_1 = \theta_2 = 0.4$		$\alpha = 0$
DGP3	$\theta_1 = \theta_2 = 0.8$		$\alpha = 0$
DGP4		$\theta_1 = 0, \theta_2 = 0.4$	$\alpha = 0$
DGP5		$\theta_1 = 0, \theta_2 = 0.8$	$\alpha = 0$
DGP6		$\theta_1 = 0.4, \theta_2 = 0.8$	$\alpha = 0$
DGP7	$\theta_1 = \theta_2 = 0.4$		$\alpha = 0.5$
DGP8	$\theta_1 = \theta_2 = 0.4$		$\alpha = 0.9$
DGP9	$\theta_1 = \theta_2 = 0.4$		$\alpha = -0.5$
DGP10	$\theta_1 = \theta_2 = 0.4$		$\alpha = -0.9$

Table 5: Rejection Frequencies for General Tests of Structural Change

DGP	Size (%)	<i>supIPSC</i>	<i>aveIPSC</i>	<i>expIPSC</i>	<i>supIPSCM</i>	<i>aveIPSCM</i>	<i>expIPSCM</i>
DGP1	1	0.0200	0.0200	0.0180	0.0720	0.0420	0.0560
	5	0.0620	0.0660	0.0720	0.1380	0.0960	0.1180
	10	0.1160	0.1140	0.1140	0.2020	0.1620	0.1640
DGP2	1	0.0140	0.0140	0.0140	0.0680	0.0340	0.0520
	5	0.0660	0.0420	0.0480	0.1520	0.0900	0.1100
	10	0.1300	0.1080	0.1240	0.2160	0.1660	0.1840
DGP3	1	0.0340	0.0160	0.0220	0.0860	0.0540	0.0660
	5	0.0760	0.0720	0.0740	0.1560	0.1020	0.1140
	10	0.1120	0.1240	0.1120	0.2140	0.1680	0.1820
DGP4	1	0.1420	0.1680	0.1680	0.3620	0.2820	0.3280
	5	0.3720	0.3780	0.3760	0.5820	0.5260	0.5460
	10	0.5640	0.5820	0.5820	0.6920	0.6700	0.6720
DGP5	1	0.9820	0.9640	0.9780	1.0000	0.9940	0.9980
	5	0.9980	1.0000	1.0000	1.0000	1.0000	1.0000
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
DGP6	1	0.4000	0.3960	0.4180	0.6800	0.5860	0.6440
	5	0.7080	0.6640	0.6840	0.8460	0.7840	0.8100
	10	0.8260	0.7860	0.8020	0.9000	0.8620	0.8820
DGP7	1	0.2920	0.1780	0.2480	0.5340	0.3520	0.4700
	5	0.5360	0.3780	0.4560	0.6920	0.5240	0.5960
	10	0.6680	0.5480	0.5980	0.7840	0.6260	0.7000
DGP8	1	0.6000	0.4420	0.5460	0.7980	0.6460	0.7480
	5	0.8140	0.6920	0.7540	0.9100	0.7720	0.8420
	10	0.8980	0.8220	0.8620	0.9500	0.8560	0.8980
DGP9	1	0.4060	0.2420	0.3400	0.5620	0.3240	0.4640
	5	0.6220	0.5040	0.5920	0.7180	0.5140	0.6400
	10	0.7540	0.6480	0.7020	0.8040	0.6840	0.7360
DGP10	1	0.7860	0.6640	0.7400	0.8340	0.6780	0.8060
	5	0.9340	0.8680	0.9080	0.9280	0.8520	0.8960
	10	0.9660	0.9520	0.9540	0.9580	0.9240	0.9520

Table 6: Rejection Frequencies for Tests of Structural Change in the Parameters

DGP	Size (%)	<i>supIPSC^I</i>	<i>aveIPSC^I</i>	<i>expIPSC^I</i>	<i>supW</i>	<i>aveW</i>	<i>expW</i>	<i>supLR</i>	<i>aveLR</i>	<i>expLR</i>
DGP1	1	0.0140	0.0120	0.0200	0.0160	0.0120	0.0200	0.0180	0.0140	0.0240
	5	0.0540	0.0660	0.0700	0.0640	0.0620	0.0680	0.0600	0.0600	0.0680
	10	0.1300	0.1260	0.1280	0.1200	0.1140	0.1240	0.1300	0.1260	0.1360
DGP2	1	0.0160	0.0120	0.0140	0.0180	0.0140	0.0180	0.0160	0.0140	0.0140
	5	0.0600	0.0720	0.0800	0.0720	0.0660	0.0780	0.0660	0.0680	0.0760
	10	0.1260	0.1400	0.1480	0.1220	0.1160	0.1400	0.1280	0.1340	0.1420
DGP3	1	0.0060	0.0120	0.0120	0.0300	0.0140	0.0180	0.0100	0.0120	0.0120
	5	0.0600	0.0680	0.0760	0.0860	0.0640	0.0800	0.0700	0.0720	0.0800
	10	0.1400	0.1260	0.1360	0.1380	0.1300	0.1380	0.1460	0.1280	0.1360
DGP4	1	0.4720	0.5100	0.5360	0.4740	0.4900	0.5240	0.4620	0.4980	0.5340
	5	0.6960	0.7540	0.7600	0.6920	0.7380	0.7460	0.6840	0.7400	0.7540
	10	0.7880	0.8440	0.8400	0.7880	0.8360	0.8340	0.7820	0.8440	0.8360
DGP5	1	1.0000	1.0000	1.0000	1.0000	0.9980	1.0000	1.0000	1.0000	1.0000
	5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
DGP6	1	0.6840	0.7460	0.7620	0.6960	0.7460	0.7600	0.6820	0.7320	0.7580
	5	0.8680	0.8940	0.9000	0.8660	0.8920	0.9020	0.8700	0.8980	0.9060
	10	0.9120	0.9340	0.9360	0.9080	0.9300	0.9360	0.9060	0.9300	0.9320
DGP7	1	0.3100	0.2720	0.3220	0.2800	0.2220	0.2920	0.1720	0.1820	0.1980
	5	0.4560	0.4580	0.4860	0.4220	0.3820	0.4340	0.3400	0.3620	0.3800
	10	0.5660	0.5480	0.5860	0.5100	0.4860	0.5300	0.4460	0.4700	0.4780
DGP8	1	0.5100	0.4820	0.5340	0.4200	0.3700	0.4480	0.3460	0.3560	0.3740
	5	0.6500	0.6320	0.6640	0.5640	0.5480	0.6000	0.5220	0.5420	0.5640
	10	0.7400	0.7280	0.7700	0.6720	0.6560	0.6900	0.6180	0.6320	0.6460
DGP9	1	0.3060	0.1640	0.2640	0.2280	0.1300	0.2140	0.1240	0.1040	0.1280
	5	0.4800	0.3620	0.4500	0.3740	0.2840	0.3660	0.2660	0.2400	0.2740
	10	0.5840	0.4700	0.5560	0.4760	0.4200	0.4720	0.3760	0.3880	0.4060
DGP10	1	0.5400	0.2740	0.5040	0.4240	0.2560	0.4080	0.1860	0.1180	0.1680
	5	0.7200	0.5340	0.6740	0.5840	0.4540	0.5500	0.3820	0.3060	0.4020
	10	0.8080	0.6720	0.7600	0.6520	0.5820	0.6540	0.5420	0.4660	0.5200

Table 7: Rejection Frequencies for Tests of Structural Change in the Overidentifying Restrictions

DGP	Size (%)	$supIPSC^O$	$aveIPSC^O$	$expIPSC^O$	$supO$	$aveO$	$expO$
DGP1	1	0.0200	0.0200	0.0160	0.0040	0.0100	0.0100
	5	0.0720	0.0720	0.0620	0.0200	0.0500	0.0380
	10	0.1200	0.1280	0.1400	0.0600	0.1040	0.0960
DGP2	1	0.0380	0.0160	0.0300	0.0060	0.0120	0.0120
	5	0.0840	0.0620	0.0860	0.0420	0.0480	0.0600
	10	0.1300	0.1260	0.1340	0.0680	0.0980	0.0860
DGP3	1	0.0240	0.0140	0.0220	0.0020	0.0080	0.0040
	5	0.0780	0.0760	0.0880	0.0240	0.0580	0.0540
	10	0.1320	0.1220	0.1360	0.0700	0.1120	0.1000
DGP4	1	0.0340	0.0240	0.0320	0.0060	0.0100	0.0120
	5	0.1100	0.0740	0.1040	0.0380	0.0520	0.0520
	10	0.1740	0.1520	0.1780	0.0960	0.1060	0.1200
DGP5	1	0.3060	0.1400	0.3000	0.1560	0.0940	0.1800
	5	0.5120	0.3500	0.4860	0.3660	0.2660	0.4000
	10	0.6420	0.5060	0.6220	0.5260	0.4400	0.5360
DGP6	1	0.0480	0.0240	0.0520	0.0100	0.0120	0.0140
	5	0.1120	0.0960	0.1120	0.0580	0.0720	0.0800
	10	0.1900	0.1540	0.1740	0.1040	0.1300	0.1300
DGP7	1	0.6180	0.5840	0.6440	0.0440	0.1920	0.1580
	5	0.8060	0.8020	0.8200	0.2860	0.5660	0.5100
	10	0.8540	0.8740	0.8760	0.4780	0.7380	0.7220
DGP8	1	0.8120	0.8140	0.8340	0.0840	0.3560	0.2620
	5	0.8960	0.9140	0.9120	0.4060	0.7800	0.6940
	10	0.9260	0.9600	0.9560	0.6240	0.8840	0.8580
DGP9	1	0.8660	0.8900	0.9060	0.0940	0.4120	0.2880
	5	0.9580	0.9560	0.9660	0.4340	0.8200	0.7800
	10	0.9840	0.9880	0.9840	0.7080	0.9340	0.9220
DGP10	1	0.9920	0.9900	0.9980	0.1000	0.6620	0.4900
	5	0.9960	1.0000	0.9980	0.6160	0.9660	0.9400
	10	1.0000	1.0000	1.0000	0.8660	0.9960	0.9880