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Innovation and Information Acquisition Under Time Inconsistency and Uncertainty

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Abstract:
We propose to analyse the hyperbolic discounting preferences effect on the innovator's research investment decision. Investing in research allows him to acquire information, and then to reduce the uncertainty of the risks of his project. We find that whatever the innovator's preferences, that is hyperbolic or time-consistent, there exists a research investment constraint that limits the information acquisition. However, even if the information is free, while a time-consistent agent always acquires information, a hyperbolic agent may prefer staying ignorant. We also emphasize that hyperbolic discounting preferences induce an information precision constraint that leads the hyperbolic innovator to ignore the information while the time-consistent innovator gets it. Moreover, the possibility that the agent has a commitment power in the future strengthens this ignorance strategy. Finally, we investigate the impact of existing liability rules on the innovator's decision to acquire information.

Keywords: Innovation, information acquisition, uncertainty, self-control, time inconsistency, liability rules

JEL Classification: D81, D83, D92

Introduction

A recent report of the European Commission (Aho, 2006) emphasizes the large gap between the scientific and technological knowledge of European countries and the relatively low level of innovation. Actually, the non-stability and/or the lack of information characterizing innovations constitute one of the main barriers to innovation. As an example of scientific innovation, one could point out genetically modified organisms (GMO)([29]). The general principle of producing GMO is to add genetic material into an organism's genome to generate new traits. Examples of GMO are highly diverse fish species, transgenic plants (e.g. tomatoes), medicines (e.g. gene therapy), or agricultural products (e.g. golden rice). As for several innovations, we do not perfectly know the effects that GMO may entail on people's health and/or on the environment in the long run. Under such a scientific uncertainty context, the ability of States to take adequate and proportionate decisions might be restricted. Indeed, in Europe the current debate on the potential risks that GMO-fertilizers use might pose to the environment (dissemination, insects resistance) illustrate such difficulties. Hence, which behaviour should we adopt in such circumstance? Should we limit technological and scientific innovation to prevent a possible risk as it is done regarding GMO production in California (United States) or in Prince Edward Island (Canada)? Or would not it be more relevant to encourage innovation's and research's efforts at the same time in order to increase the level of innovation and to reduce scientific uncertainties?

Aho (2006) concludes that there is a real need for action to implement what it defines as a *"pact for research and innovation"*. In this regard, the precautionary principle,[1] as a public decision criteria, proposes to combine innovation, security and information acquisition (Pouillard, 1999, Henry and Henry, 2004). This inspires many European directives and propositions of which the aim is to protect health and environment. Hence, the directive on environmental liability proposes to apply the 'pollutant-payer' principle, that is: if a damage happens, the pollutant has to pay for it. The investors are then liable for an eventual incident due to their activities. This principle's goal is to act directly on the investors' behaviours to increase their prevention efforts. Mechanisms based on liability rules, such as a strict liability rule or a negligence rule, are used to protect consumers as well as the environment by improving the control and the prevention of risks induced by firms' activities and products. However, uncertainty limits their efficiency but also may induce contradictory effects on both innovation and security. Moreover, regarding others liability rules that control firms activities, Sinclair-Desgagné and Vachon (1999) note that a limited liability rule leads to less prevention and the

[1] The precautionary principle states that the lack of certainty regarding the risk should not be used as an excuse to do nothing to avert that risk.

extension of the liability to all the firm's partners may limit innovating efforts. To go further in the current debate on precaution and economic growth, one should investigate what determines, under existing liability frameworks, the investor's behaviour regarding research allowing him to reduce his scientific knowledge on potential risky activities.

Actually, the investor's behaviour regarding risks perception, and then risks assessment is never exempt of subjectivity. For example, as stated Kahneman et al. (1982), people undervalue or overvalue small probabilities in proportion to the importance of potential damages, and according to their backgrounds. In a more general way, as emphasized by several empirical studies, risk perception and thus, individual preferences change over time (Frederick et al., 2002). If Strotz (1956) was the first to suggest an alternative to exponential discounting, Phelps and Pollack (1968) introduced the hyperbolic discounted utility function as a functional form of this kind of changing preferences.

Elster (1979) applies this formalization to a decision problem in characterizing time inconsistency by a decreasing discount rate between the present and the future, and a constant discount rate between two future periods. Laibson (1997, 1998) then uses this formulation to saving and consumption problems, while Bénabou and Tirole (2002, 2004), and Carrillo and Mariotti (2000) consider a problem of information acquisition. Indeed, Bénabou and Tirole (2002, 2004) shows that a 'comparative optimism', or a 'self-confidence' is at the origin of a time-inconsistent behaviour. Such behaviours inhibit all learning processes. Uncertainty may strengthen this effect. Carrillo and Mariotti (2000) study intertemporal consumption's decisions involving a potential risk in the long run. They show that hyperbolic discounting preference can favour *strategic ignorance*. Indeed, a person with time-inconsistent preferences might choose not to acquire free information in order to avoid over consumption or engagement in activities, which may require much more fundamental research on potential social costs or externalities they could involve in the long term. So, if ignorance is a self-disciplining device when an agent is confronted with uncertainty and hyperbolic discounting preferences, is it then useless for him to acquire information to develop a potential risky project?

We propose to analyse the impact of hyperbolic discounting preferences on the innovator's information acquisition decision in an uncertainty context. We choose to define information acquisition as an investment in research[2] to reduce the uncertainty on the potential risk of damage on health and the environment. We represent an agent with hyperbolic discounting preferences as a collection of risk-neutral incarnations with conflicting goals.

[2]Such investment in research could be considered as a resort to private experts like private laboratory, or to any other private party able to provide scientific knowledge on the dangerousness, or more generally on the characteristics of the innovator's activities.

We find that whatever the innovator preferences (hyperbolic or time-consistent), there exists a research investment constraint which limits the information acquisition. However, even if the information is free, while a time-consistent agent always gets information, a hyperbolic agent may prefer staying ignorant. In addition, we obtain that hyperbolic discounting preferences induce an information precision constraint that leads the hyperbolic innovator to ignore the information, while the time-consistent innovator acquires it. Hence, both the research investment constraint and the agent's hyperbolic discounting preferences restrict his ability to acquire information. Moreover, the possibility that the agent's future selves may commit in the long run, that is he may have a self-control in the future, strengthens this ignorance behaviour.

With regard to these results we investigate the impact of existing liability rules on the innovator's decision to acquire information. We find that the efficiency of a strict liability rule to push the innovator to get information is limited by both the existence of a research investment constraint and the hyperbolic discounting preferences. On the other hand, a negligence rule is efficient in improving the information acquisition. However, despite this information, an agent with hyperbolic discounting preferences may decide to always continue his project whatever the received signal. This information is then not reliable for him, and does not lead to a precautionary behaviour. Finally, we show that under a limited liability rule the innovator always chooses to neglect information on the dangerousness of his project. Hence, under uncertainty and with regard to the innovator's hyperbolic discounting preferences, none of these liability rules could provide the right incentives to make the innovator adopt a precautionary behaviour in terms of risk management.

The remainder of paper is organized as follows. Section 1 presents the model. Section 2 investigates the optimal decision-making. Section 3 studies the self-control effect. Finally, Section 4 analyses the efficiency of existing liability frameworks, such that the strict liability rule, the negligence rule, and the limited liability rule, to motivate the agent to acquire information. All proofs are in appendix.

1 The model

We consider a three period model. At period 0, the innovator invests a given amount of money I in a project that may create damage to people's health, or to the environment.

There are two possible states of the world H and L associated with different probabilities of damage θ^H and θ^L, respectively. We assume that state H is more dangerous than state L, so:

$$\theta^L < \theta^H.$$

At period 0, the prior beliefs of the innovator are p_0 on state H, and $1-p_0$ on state L. The expected probability of the damage is thus given by:

$$E(\theta) = p_0\theta^H + (1-p_0)\theta^L.$$

At period 0, the innovator invests $C \geq 0$ in research to obtain information at period 1 through a signal $\sigma \in \{h, l\}$ on the true state of the world. The probability to receive the signal corresponding to the state represents the precision of the signal. We define it as an increasing and concave function $f(C)$ such that:

$$P(h|H,C) = P(l|L,C) = f(C) \text{ and } P(h|L,C) = P(l|H,C) = 1 - f(C)$$

and

$$f(0) = \tfrac{1}{2};\ f'(0) = +\infty \text{ and } f'(+\infty) = 0.$$

The precision depends on the research investment. If the innovator does not invest in research ($C = 0$) then the signal is not informative.[3] However, a larger research investment implies a higher precision.
According to Bayes' rule, the probability to be in state H given the signal h and the research investment C, and the probability to be in state H given the signal l and the research investment C are respectively:

$$P(H|h,C) = \frac{p_0 f(C)}{p_0 f(C) + (1-p_0)(1-f(C))} \text{ and } P(H|l,C) = \frac{p_0(1-f(C))}{p_0(1-f(C)) + (1-p_0)f(C)}.$$

At period 1, according to signal $\sigma \in \{l, h\}$, let define $x_\sigma \in \{0,1\}$ as the innovator's decision to stop his project, or to continue it. If the innovator completes his project $x_\sigma = 0$, and he recovers a part of his investment D such that $0 < D < I$. On the other hand, if he maintains it, $x_\sigma = 1$.

At period 2, an accident may happen. If it does while the innovator has prematurely stopped his project, the innovator has to pay a financial cost K' related to the damages. On the other hand, if the agent has continued his project, he earns a positive return R_2, and if an accident occurs he has to pay a higher financial cost K. So, we have:

$$0 < K' < K.$$

In order to formalize the hyperbolic discounting preferences, we use the Phelps and Pollack (1968)'s functional form. Let $D(k)$ represent a discount function such that:

$$\begin{cases} D(k) = 1 & \text{if } k = 0; \\ D(k) = \beta\delta^k & \text{if } k > 0. \end{cases}$$

[3] We do not consider exogenous information, like the agent's background or public information. Actually, exogenous information is attainable by all freely. Instead we are interested in this paper by the own initiative of an agent to acquire information and his willingness to pay for it.

Finally, hyperbolic discounted utility function is defined as follows:

$$U_t = \sum_{k=0}^{T-t} D(k)u_k \text{ with } u_k \text{ the net instantaneous utility at period t.}$$

As Frederick et al. (2002) suggest it, the discount rate β gathers all the psychological motives of the agent's investment choice such that anxiety, confidence or impatience. Actually, Akerlof (1991) defines β as the *"salience of current payoffs relative to the future stream of returns"*, but it is also interpreted in the literature as a lack of willpower (Bénabou and Tirole, 2002), of foresight (Masson, 2002, O'Donoghue and Rabin, 1999) or as impulsiveness (Ainslie, 1992). Hence, if $\beta = 1$, the psychological motives have no influence on the agent's choice, and his preferences are time-consistent. On the other hand, if $\beta < 1$, the innovator is temporally inconsistent. Indeed, the agent's preferences change over time, indicating that what the agent decides today might be discordant with what he decides tomorrow. Moreover, to simplify the Phelps and Pollack (1968)'s formalization, we assume that $\delta = 1$.[4]

We consider that an agent with hyperbolic discounting preferences as being made up of many different risk neutral selves with conflicting goals.[5] Each self represents the agent at a different point in time. Hence, at each period t there is only one self called "self-t". Each self-t depreciates the following period with a discount rate β. In our model, we consider that β represents all the psychological motives of the agent's investment choice such that anxiety, confidence or impatience. For example, a farmer producing GMO may lose his confidence in GMO over time because of a better knowledge of their potential negative effects. This reduction of the scientific uncertainty could imply different period-to-period decisions.

Hence, intertemporal expected payoffs of self-2, self-1 and self-0 may be expressed recursively. If signal σ has been perceived, self-2's intertemporal expected payoff is written as:

$$\begin{aligned} V_2(x_\sigma, \sigma, C) = \quad & x_\sigma[P(H|\sigma, C)(R_2 - \theta^H K) + (1 - P(H|\sigma, C))(R_2 - \theta^L K)] \\ & -(1 - x_\sigma)[P(H|\sigma, C)\theta^H K' + (1 - P(H|\sigma, C))\theta^L K']. \end{aligned}$$

Likewise, self-1's intertemporal expected payoff is:

$$V_1(x_\sigma, \sigma, C) = (1 - x_\sigma)D + \beta V_2(x_\sigma, \sigma, C).$$

Finally, self-0's intertemporal expected payoff is:

$$\begin{aligned} V_0(x_h, x_l, C) = \quad & -I - C + \beta[p_0 f(C) + (1 - p_0)(1 - f(C))][V_2(x_h, h, C) + (1 - x_h)D] \\ & +\beta[(1 - p_0)f(C) + p_0(1 - f(C))][V_2(x_l, l, C) + (1 - x_l)D]. \end{aligned}$$

[4]Actually, $\delta < 1$ only implies a lower discount rate. Taking $\delta = 1$ does not change the results.
[5]Following Strotz (1956), this conflict captures the innovator's time-inconsistency preferences.

For a given state of the world $S \in \{L, H\}$, let us define, from self-1's perspective, $B^S(\beta)$ as the difference between the intertemporal expected payoff when self-1 decides to carry on his project, and the intertemporal expected payoff when he decides to stop it. In other words, $B^S(\beta)$ is the discounted benefit to continue the project instead of stopping it at period 1. It is given by:

$$B^S(\beta) = \beta R_2 - D - \beta\theta^S(K - K').$$

We assume that we examine two opposite states of the world. Hence, we suppose that the discounted benefit is positive in state L, while it is negative in state H so:

$$B^H(\beta) < 0 < B^L(\beta).$$

For a given signal $\sigma \in \{l, h\}$, let us also define, from self-0's perspective, $B_0(\sigma, C)$ as the difference between the intertemporal expected pay-off of an informed agent who decides to completely achieve his project, and the intertemporal expected pay-off of an informed agents who decides to prematurely withdraw it. For each signal $\sigma \in \{l, h\}$, $B_0(\sigma, C)$ is given by:

$$B_0(h, C) = \beta[p_0 f(C) + (1 - p_0)(1 - f(C))][R_2 - D - E(\theta|h, C)(K - K')]$$

and

$$B_0(l, C) = \beta[p_0(1 - f(C)) + (1 - p_0)f(C)][R_2 - D - E(\theta|l, C)(K - K')].$$

Moreover, self-1's intertemporal expected payoff when self-1 decides to achieve the project, while self-0 has previously chosen not to be informed (No Learning) is defined by:

$$V_1^{NL}(1) = \beta[p_0(R_2 - \theta^H K) + (1 - p_0)(R_2 - \theta^L K)]$$

and self-1's intertemporal expected payoff when self-1 decides to stop the project, while self-0 has previously chosen not to be informed (No Learning) is given by:

$$V_1^{NL}(0) = D - \beta[p_0\theta^H K' + (1 - p_0)\theta^L K'].$$

We consider that an agent who begins a project without information always completes it. Indeed, we assume that without information, it is more profitable for the agent to continue the project than stopping it at period 1. Formally, this means that for $\beta \leq 1$:

$$V_1^{NL}(0) < V_1^{NL}(1) \text{ which is equivalent to } E(\theta) < \frac{\beta R_2 - D}{\beta(K - K')}.$$

For all $\beta \leq 1$, we define $\hat{\theta}(\beta)$ as the probability that a damage happens when the innovator is indifferent between continuing his project and stopping it at period 1. That is:

$$\beta(R_2 - \hat{\theta}(\beta)K) = D - \beta\hat{\theta}(\beta)K' \text{ which is equivalent to } \hat{\theta}(\beta) = \frac{\beta R_2 - D}{\beta(K - K')}.$$

So, for $\beta \leq 1$ and at period 1, it is more profitable for the agent to continue his project than stopping it when he does not acquire information if:

$$E(\theta) < \hat{\theta}(\beta). \tag{1}$$

Under such an assumption, we say that an uninformed agent adopts a non-precautionary behaviour. Indeed, in ignoring information, he does not make effort to reduce the uncertainty linked to his project.

However, this assumption can only hold for certain value of β. Indeed, we note that the derivative of $\hat{\theta}(\beta)$ is equal to $\hat{\theta}'(\beta) = \frac{D}{\beta^2(K-K')}$, which is positive. So, $\hat{\theta}(\beta)$ is increasing with β. We define the discount rate $\tilde{\beta}$ such that an uninformed agent is indifferent between maintaining his project and stopping it at period 1, that is:

$$E(\theta) = \hat{\theta}(\tilde{\beta}).$$

According to the intertemporal expected payoffs of self-1, an uninformed agent with a discount rate $\beta = 0$ does not perceive future payoffs. He then always prefers stopping his project in order to recover at least a part of his investment D. As when $\beta = \tilde{\beta}$, the innovator is indifferent between stopping and carrying on the project, then $\tilde{\beta}$ may not be equal to zero.

Overall, to ensure that an uninformed innovator always chooses to complete his project, we restrict our study to an agent with hyperbolic discounting preferences that satisfy condition (1). So, we analyse the hyperbolic agent's behaviour with a discount rate $\beta \in]\tilde{\beta}, 1[$.

2 The optimal decision-making

We now turn to the innovator's optimal decision-making. Subsection 2.1 studies self-1's optimal decision to continue the project or to stop it. Subsection 2.2 determines self-0's optimal decision to acquire information by investing in research.

2.1 Stopping or continuing the project

By assumption, if self-1 is uninformed he always continues the project. However, if he gets information and receives a signal $\sigma \in \{h, l\}$ on the probability of damage, he updates his beliefs and chooses whether to complete the project. Formally, for $\sigma \in \{h, l\}$ and for $C \geq 0$, self-1 continues the project if his expected payoff by continuing the project is higher than the expected payoff by stopping it. That is:

$$V_1(0, \sigma, C) < V_1(1, \sigma, C).$$

For $\sigma \in \{h, l\}$ and for $C \geq 0$, we define the revised expected probability of a damage by $E(\theta|\sigma, C) = P(H|\sigma, C)\theta^H + (1 - P(H|\sigma, C))\theta^L$, and the equilibrium strategy by x^*_σ. Conditions under which self-1 stops or continues his project are given by the following proposition:

Proposition 1 *For $\sigma \in \{h, l\}$, for $C \geq 0$ and for $\beta \in]\tilde{\beta}, 1]$: If $E(\theta|\sigma, C) < \hat{\theta}(\beta)$, then the innovator continues the project, that is $x^*_\sigma = 1$; If $\hat{\theta}(\beta) < E(\theta|\sigma, C)$, then the innovator stops the project, that is $x^*_\sigma = 0$; Finally, if $\hat{\theta}(\beta) = E(\theta|\sigma, C)$, then the innovator is indifferent between stopping and continuing his project, that is $x^*_\sigma \in \{0, 1\}$. Moreover, for all $C \geq 0$, we have $E(\theta|l, C) \leq E(\theta) \leq E(\theta|h, C)$, and $E(\theta|h, C)$ is increasing with C while $E(\theta|l, C)$ is decreasing with C.*

Proposition 1 shows that the conditions under which self-1 decides to partially or completely achieve his project is based on the comparison between the revised expected probability of a damage, $E(\theta|\sigma, C)$, and the probability of a damage when self-1 is indifferent between continuing and stopping his project, $\hat{\theta}(\beta)$. According to Proposition 1, three cases arise: If $E(\theta|\sigma, C)$ is lower than (resp. larger than) $\hat{\theta}(\beta)$, self-1's optimal decision is to carry on (resp. to stop) the project, and if they are equal, self-1 is indifferent between continuing and stopping his project. So, condition (1) implies that self-1 has two strategies: either whatever the signal he always continues the project, or if he receives the signal l he continues the project while he stops it if he receives the mboxsignal h. We may say that the latter strategy is more cautious than the first one. Indeed, the innovator decides to stop his project, and then to reduce the potential damages when he receives the signal h being in the most dangerous state of the world. On the other hand, in the first strategy, he always continues his project even if the received signal indicates he is in the most risky state.

In addition, Proposition 1 entails that the research investment chosen at period 0 leads to a certain information precision of the signal, and thus determines self-1's optimal decision.

Finally, since $\hat{\theta}(\beta)$ is increasing with β, in order to stop his project at period 1, a time-consistent agent ($\beta = 1$) needs a higher precision of the signal h, that is a larger research investment, than an agent with hyperbolic discounting preferences ($\beta \in]\tilde{\beta}, 1[$). So, when self-1 receives the signal h, for a given level of research investment, hyperbolic discounting preferences favour the decision to give up the project.

2.2 Information acquisition

We now determine self-0's optimal decision to acquire information by investing in research. In part 2.2.1, we study the time-consistent self-0's optimal research investment. In part 2.2.2, we analyse this decision in considering that the agent has hyperbolic discounting preferences. Finally, in part 2.2.3 we represent the research investment of both kinds of preferences.

2.2.1 Time-consistent agent

Time consistency supposes that an agent allows the same weight to the present and to the future, and thus his future selves act according to the preferences of his current self. So, the self-0 of a time-consistent agent ($\beta = 1$) chooses his optimal research investment to acquire information on the potential risk of accident knowing that self-1 either always continues the project whatever the signal (case 1), or stops it if he received the signal h and continues it if he receives the signal l (case 2).

Define by $C^*_{x_h x_l}(1)$ self-0's optimal research investment under the strategy $\{x_h, x_l\}$. Under time-consistent preferences, self-0's optimal research investment $C^*_{x_h x_l}(1)$ solves the following problem:

$$\max_{C \geq 0} V_0(x_h, x_l, C).$$

Let us first study case 1 in which self-1 always continues the project, that is $\{x_h = 1, x_l = 1\}$. Self-0's expected payoff under this strategy is:

$$V_0(1, 1, C) = -I - C + [p_0(R_2 - \theta^H K) + (1 - p_0)(R_2 - \theta^L K)].$$

Since $V_0(1, 1, C)$ is decreasing with C, it is obvious that self-0's optimal investment in research under this strategy is: $C^*_{11}(1) = 0$. In such a case, the signal does not have any influence on self-1's behaviour, it is then not reliable for him to acquire it.

Let us turn to case 2 in which self-1 only gives up the project if he receives the signal h, that is $\{x_h = 0, x_l = 1\}$. In order to choose his optimal research investment $C^*_{01}(1)$, self-0's maximizes the following expected payoff with respect to C:

$$\begin{aligned} V_0(0, 1, C) = \quad & -I - C + [p_0(1 - f(C))(R_2 - \theta^H K) + (1 - p_0)f(C)(R_2 - \theta^L K)] \\ & +[p_0 f(C)(D - \theta^H K') + (1 - p_0)(1 - f(C))(D - \theta^L K')]. \end{aligned}$$

Lemma 1 $C^*_{01}(1)$ *is characterized by:*

$$f'(C^*_{01}(1)) = \frac{1}{[(1 - p_0)B^L(1) - p_0 B^H(1)]}.$$

$C^*_{01}(1)$ *is strictly positive.*

Self-0 anticipates that self-1 will continue the project if he receives the signal l and will stop it with the signal h. The information is trustworthy to him, and it is thus optimal to choose a positive research investment.

Finally, define $C^*(1)$ as the optimal research investment over all the strategies. To determine $C^*(1)$, we compare self-0's expected payoffs of both strategies and select the level of research investment that leads, from self-0's perspective, to the highest expected payoff.

Proposition 2 *If*

$$C^*_{01}(1) < -B_0(h, C^*_{01}(1)) \tag{2}$$

then $C^*(1) = C^*_{01}(1)$*; Otherwise* $C^*(1) = 0$.

We note that according to Proposition 1, $-B_0(h, C^*_{01}(1))$ is always positive. So, Proposition 2 entails that if for the same precision, the research investment is free then condition (2) always holds and the innovator always acquires information. Condition (2) is a research investment constraint: if the research investment is too costly, the agent does not acquire any information.

Overall, this research investment constraint restrains the ability of a time-consistent agent to acquire information.

2.2.2 Agent with hyperbolic discounting preferences

The future selves of an agent with hyperbolic discounting preferences choose strategies which are optimal for them, even if these strategies are suboptimal from the current self's point of view.[6] So, the self-0 of an agent with hyperbolic discounting preferences ($\beta \in]\tilde{\beta}, 1[$) selects his optimal research investment knowing that the future selves may deviate from his optimal choice. As previously, two cases arise: self-1 may either always maintain the project whatever the signal (case 1), or may stop it if he receives the signal h and continue it if he receives the signal l (case 2).

In both cases, define by $C^*_{x_h x_l}(\beta)$ self-0's optimal research investment under the strategy $\{x_h, x_l\}$. As self-0 first anticipates that self-1 will implement the strategy $\{x_h, x_l\}$, he chooses the research investment that maximizes his expected payoff under the anticipated strategy $\{x_h, x_l\}$. If this research investment is rather precise to insure that self-1 will actually choose the anticipated strategy, then this research investment is optimal. Otherwise, if it is not that precise, self-0 has to invest the lowest research investment that leads self-1 to implement the strategy $\{x_h, x_l\}$.

[6]According to the literature on hyperbolic discounting preferences, we consider that our agent is sophisticated.

So, if self-1 always continues the project (case 1), self-0's expected payoff under the strategy $\{x_h = 1, x_l = 1\}$ is given by:

$$V_0(1,1,C) = -I - C + \beta[p_0(R_2 - \theta^H K) + (1-p_0)(R_2 - \theta^L K)].$$

With hyperbolic discounting preferences, self-0's optimal research investment $C^*_{11}(\beta)$ solves the following problem:

$$\max_{C \geq 0 / E(\theta|l,C) < E(\theta|h,C) < \hat{\theta}(\beta)} V_0(1,1,C).$$

As already said in part 2.2.1, $V_0(1,1,C)$ is decreasing with C which implies that $C^*_{11}(\beta) = 0$. According to condition (1), when self-0 decides not to invest in research, then self-1 always chooses to continue his project.

If self-1 only gives up the project if he receives the signal h (case 2), self-0's expected payoff under the strategy $\{x_h = 0, x_l = 1\}$ is given by:

$$\begin{aligned} V_0(0,1,C) = \; & -I - C + \beta[p_0(1-f(C))(R_2 - \theta^H K) + (1-p_0) f(C)(R_2 - \theta^L K)] \\ & + \beta[p_0 f(C)(D - \theta^H K') + (1-p_0)(1-f(C))(D - \theta^L K')]. \end{aligned}$$

Let $C^*_{01}(\beta)$ be the optimal investment in research under the assumption that the strategy $\{x_h = 0, x_l = 1\}$ is optimal for self-1. In words this means that $C^*_{01}(\beta)$ provides an enough precise signal to be taken into account by self-1. In this case, the strategy $\{x_h = 0, x_l = 1\}$ is ex-post optimal. Thus, with hyperbolic discounting preferences, self-0's optimal research investment $C^*_{01}(\beta)$ solves the following problem:

$$\max_{C \geq 0 / E(\theta|l,C) < \hat{\theta}(\beta) < E(\theta|h,C)} V_0(0,1,C).$$

To completely define $C^*_{01}(\beta)$, let us first consider $C_{01}(\beta)$, the optimal research investment when self-0 anticipates that self-1 implements the strategy $\{x_h = 0, x_l = 1\}$. For all $\beta \in]\tilde{\beta}, 1[$, $C_{01}(\beta)$ solves the following problem:

$$\max_{C \geq 0} V_0(0,1,C). \tag{3}$$

Lemma 2 For all $\beta \in]\tilde{\beta}, 1[$, $C_{01}(\beta)$ *is characterized by*

$$f'(C_{01}(\beta)) = \frac{1}{\beta[(1-p_0)B^L(1) - p_0 B^H(1)]}.$$

$C_{01}(\beta)$ *is increasing with* β, *and for all* $\beta \in]\tilde{\beta}, 1[$ $C_{01}(\beta)$ *is strictly positive.*

We immediately note that $f'(C_{01}(1)) = f'(C^*_{01}(1))$. Indeed, $C_{01}(\beta)$ is the optimal research investment of a hyperbolic agent who behaves as a time-consistent agent, that is his future selves act according to the preferences of his current self.[7]

[7] In the literature, this agent is called a myopic agent.

However, to select $C^*_{01}(\beta)$, self-0 has to check that self-1 does not deviate to the strategy $\{x_h = 0, x_l = 1\}$. Let us define by $\hat{C}(\beta)$ the smallest $C \geq 0$ which satisfies that $E(\theta|l, C) \leq \hat{\theta}(\beta) \leq E(\theta|h, C)$. In other words, $\hat{C}(\beta)$ is the smallest investment that ensure that the strategy $\{x_h = 0, x_l = 1\}$ is optimal for self-1.

Lemma 3 *For $\beta \in]\tilde{\beta}, 1[$, $\hat{C}(\beta)$ is characterized by:*

$$f(\hat{C}(\beta)) = \frac{(1-p_0)B^L(\beta)}{-p_0 B^H(\beta) + (1-p_0)B^L(\beta)}.$$

So, if $C_{01}(\beta)$ is such that $E(\theta|l, C_{01}(\beta)) < \hat{\theta}(\beta) < E(\theta|h, C_{01}(\beta))$ then $C^*_{01}(\beta) = C_{01}(\beta)$. Otherwise, if from self-0's perspective, the signal given by the research investment $C_{01}(\beta)$ is not enough informative, then self-0 selects the smallest research investment which leads self-1 to implement the strategy $\{x_h = 0, x_l = 1\}$, that is $C^*_{01}(\beta) = \hat{C}(\beta)$.

Lemma 4 *For $\beta \in]\tilde{\beta}, 1[$, if*

$$f(\hat{C}(\beta)) < f(C_{01}(\beta)) \tag{4}$$

then $C^*_{01}(\beta) = C_{01}(\beta)$. Otherwise, $C^*_{01}(\beta) = \hat{C}(\beta)$. Moreover, $\hat{C}(\beta)$ is increasing with β, and $f(\hat{C}(\tilde{\beta})) = f(0) = \frac{1}{2}$.

Overall, as for a time-consistent agent, we determine the optimal research investment $C^*(\beta)$ in comparing self-0's expected payoff of both strategies and select the level of research investment that leads, from self-0's perspective, to the highest expected payoff.

Proposition 3 *For $\beta \in]\tilde{\beta}, 1[$ if*

$$C_{01}(\beta) < -B_0(h, C_{01}(\beta)) \tag{5}$$

and (4) hold then $C^*(\beta) = C_{01}(\beta)$, otherwise if one of these both conditions is not satisfied then $C^*(\beta) = 0$.

We note, that contrary to the time-consistent case, $-B_0(h, C_{01}(\beta))$ may be negative. Hence, if for the same information precision the research investment is free, then condition (5) cannot hold. Acquiring information on the potential risks is not an optimal strategy for the innovator. Actually, when he is uninformed the possibility that self-1 gives up the project is so high that self-0 may choose not to acquire information, even if it is free, in order to avoid such a risk of withdrawing. It is thus noteworthy that the discount factor β has an impact on the information acquisition. Indeed, while a time-consistent agent always chooses to get information when it is free, the agent with hyperbolic discounting preferences may prefer staying ignorant.

However, when $-B_0(h, C_{01}(\beta))$ is positive, condition (5) may not hold either. Indeed, if the research investment cost is too high, the innovator never gets any information. In this case, condition (5) is a research investment constraint that restrains the information acquisition.

In addition, self-0 is aware that his research investment determines the precision of the signal received by self-1. When self-1 receives the signal h, being in the most dangerous state, the information precision may lead him either to stop his project, or to continue it. For this latter option, self-1 does not pay any attention to the signal h because self-0's investment in research is not high enough. To avoid such an useless investment, self-0 prefers staying uninformed because the cost of research that would provide an enough precise signal h to self-1, would also yield a lower expected payoff, from self-0's perspective, than the one without information. So, condition (4) is an information precision constraint.

2.2.3 The research investment

In this part, we propose to represent the research investment according to the discount rate $\beta \in]\tilde{\beta}, 1]$.[8] We first characterize conditions (2) and (5). We then describe condition (4).

Lemma 5 *If condition (2) holds then there exists a $\bar{\beta} \in]\tilde{\beta}, 1]$ such that for all $\beta \in]\tilde{\beta}, \bar{\beta}]$ condition (5) does not hold and for all $\beta \in]\bar{\beta}, 1]$ condition (5) holds. Otherwise, for all $\beta \in]\tilde{\beta}, 1[$, condition (5) does not hold.*[9]

Hyperbolic discounting preferences limit the fulfilment of the research investment constraint. Indeed, a lower discount factor implies a higher preference for the present. The cost of the research investment may then be more valuable to the agent than the future payoffs, implying a non respect of the research investment constraint.

Regarding the information precision constraint, that is condition (4), the conditions under which this constraint is satisfied are not straightforward. According to Lemmas 2, 3 and 4, $f(\hat{C}(\tilde{\beta})) = \frac{1}{2} < f(C_{01}(\tilde{\beta}))$. However, due to the general form of our precision function, we are not able to define when $f(\hat{C}(\beta))$ is higher than, equal to, or lower than $f(C_{01}(\beta))$. So, three possibilities arise:

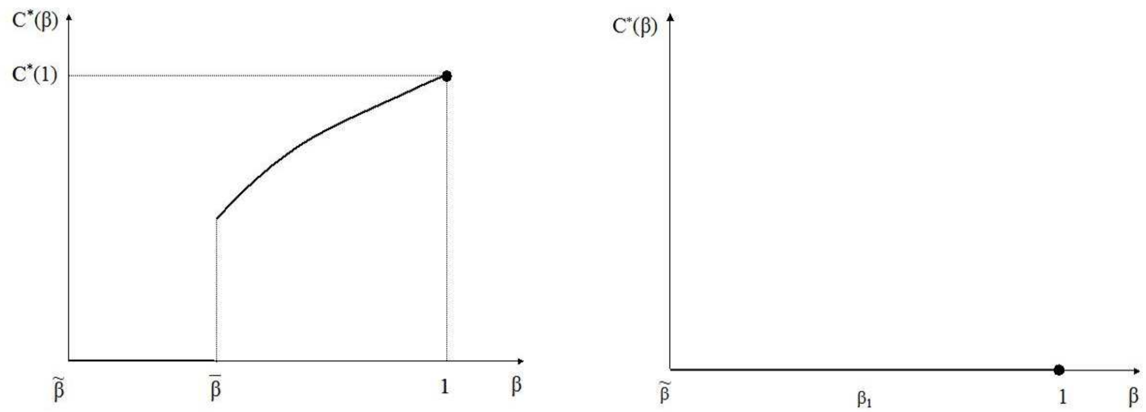
[8] We only give insight to the reader of the agent's information acquisition. To simplify, we represent this acquisition by a concave research investment function. However, we are aware that the research investment's increase may be slowed down or accelerated according to the discount rate values. In addition, we do not lose the hyperbolic preferences effect, which implies that a higher discount rate value leads to an increase of the level of research investment (when the agent acquires information).

[9] For $\beta = 1$, conditions (2) and (5) are equivalent.

- $(P1)$: There is no intersection between $f(\hat{C}(\beta))$ and $f(C_{01}(\beta))$. That is for all $\beta \in]\tilde{\beta}, 1[$, $f(\hat{C}(\beta)) < f(C_{01}(\beta))$;
- $(P2)$: There exists at least one intersection between $f(\hat{C}(\beta))$ and $f(C_{01}(\beta))$. Define $\beta_1 \in]\tilde{\beta}, 1[$ such that $f(\hat{C}(\beta_1)) = f(C_{01}(\beta_1))$. Then, for all $\beta \in]\tilde{\beta}, \beta_1[$, $f(\hat{C}(\beta)) < f(C_{01}(\beta))$ while for all $\beta \in [\beta_1, 1[$, $f(C_{01}(\beta)) \leq f(\hat{C}(\beta))$;
- $(P3)$: There exist several intersections between $f(\hat{C}(\beta))$ and $f(C_{01}(\beta))$. So, for certain $\beta \in]\tilde{\beta}, 1[$, $f(\hat{C}(\beta)) < f(C_{01}(\beta))$, and for the others $f(C_{01}(\beta)) \leq f(\hat{C}(\beta))$.

The first possibility $(P1)$ is the simplest case. It supposes that the precision constraint (condition (4)) is always satisfied and thus allows basic and traditional comments on the impact of hyperbolic discounting preferences on the innovator's ability to get information (Brocas and Carrillo, 2000, Carrillo and Mariotti, 2000). For $\beta \in]\tilde{\beta}, 1[$, the optimal research investment chosen by self-0 always gives a rather precise information to lead self-1 to choose the strategy $\{x_h = 0, x_l = 1\}$. However, the innovator's research investment is limited by the cost of the research or by the agent's preferences for the present. Indeed, the innovator may not be willing to pay it, or his preferences for the present may be so high that he prefers ignoring information in order to avoid a premature stop of his project.

Figure 1 represents the optimal research investment for possibility (P1).



(a) $\bar{\beta}$ exists. (b) $\bar{\beta}$ does not exist.

Figure 1: Optimal research investment for possibility $(P1)$.

As the investment in research only depends on the research investment constraint, according to Lemma 5, two cases arise: either below $\bar{\beta}$ condition (5) does not hold while above $\bar{\beta}$ condition (5) holds (Figure 1-a), or for all β condition (5) is never satisfied (Figure 1-b).

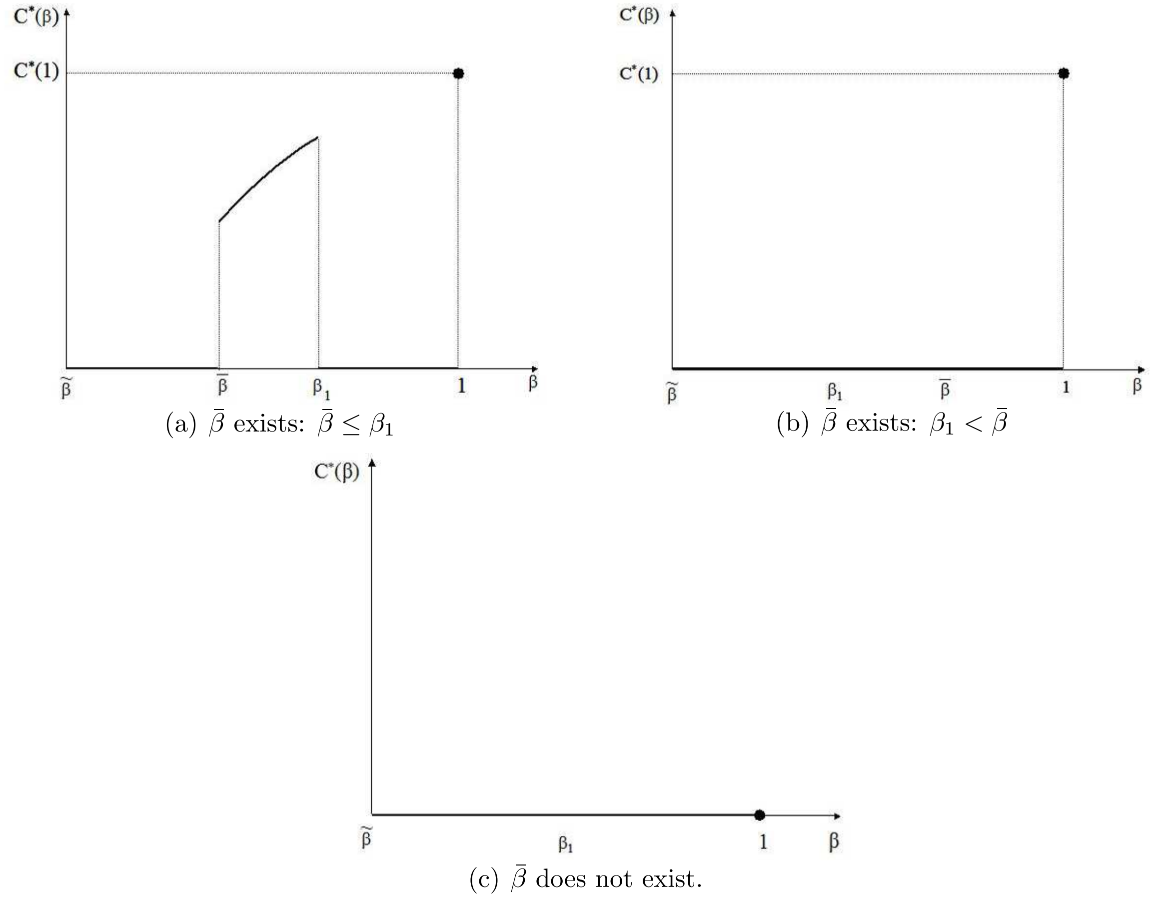
On Figure 1-a, below $\bar{\beta}$ the innovator may have so strong preferences for the present that his research expenditure may be greater than the expected payoff. As he cannot respect his research investment constraint to get a useful signal, it is not optimal for him to acquire information. Moreover, due to his preferences for the present, he is also more willing to withdraw the project and to earn money now, than to wait for a future payoff and to potentially pay a financial cost. To avoid such a behaviour self-0 may choose to stay uninformed rather than reducing the uncertainty linked to the dangerousness of his activity. This strategic ignorance could be considered as non-precautionary with regard to people's health and the environment. This kind of behaviour was already discussed by Carrillo and Mariotti (2000) who consider that hyperbolic discounting preferences influence the agent's choice to get information on the consumption of a product that may be harmful. They predict that an uninformed agent consumes less than an informed one, and contrary to our approach, this ignorance is defined as a strategic tool to prevent the consumption of a potentially dangerous product. In addition, this result also fits into the literature on hyperbolic discounting. Akerlof (1991) points out that a hyperbolic agent always postpones a costly activity. In our model since the innovator has only the choice to invest in research at period 0, postponing this investment is equivalent to not doing it.

On the other hand, in Figure 1-a, above $\bar{\beta}$, the research investment constraint always holds. The innovator is willing to pay to be informed and follows the strategy $\{x_h = 0, x_l = 1\}$. By acquiring information in order to reduce the uncertainty on the potential risks, the agent chooses to adopt a precautionary behaviour. We remark that the optimal investment in information is increasing with β. It tends towards $C^*(1)$ the optimal investment reached when the agent has time-consistent preferences. Actually, weaker preferences for the present incite the agent to continue his project and to wait for a future payoff. He then needs a greater information precision to make him stop this project when he receives the signal h of the most dangerous state. Overall, both the existence of a research investment constraint and the innovator's hyperbolic discounting preferences limit the information acquisition.

Figure 1-b represents the possibility in which whatever the agent's preferences (time-consistent or hyperbolic) the cost of research in order to get an enough precise signal is so high that the innovator always gets a higher expected payoff without information. In such a case, it is optimal for him not to invest in research and to acquire information. The research investment constraint limits the information acquisition.

The two remaining possibilities are more sophisticated cases in so far as conditions (2), (5) and (4) interact with the innovator's decision to get information.

Let us turn to the possibility ($P2$). In this case, the precision constraint is not always satisfied: below β_1 condition (4) holds while above β_1 condition (4) does not hold. Indeed, below β_1 the optimal research investment chosen by self-0 provides a rather precise information to lead self-1 to give up the project when he receives the signal h. While above β_1, the information is not rather precise, self-1 always prefers continuing it. For fear of such a behaviour, self-0 chooses not to invest in research. Indeed a higher research investment yields a lower expected payoff than the one he could get without investing in research. Moreover, as noticed in $(P1)$, the innovator's expectations in terms of signal precision are increasing with β. In words, in period 1, the more the innovator pays attention to the future, the more precise the signal h has to be to make him stop his activity. According to Proposition 2 and 3, three cases arise. We represent them in Figure 2.



(a) $\bar{\beta}$ exists: $\bar{\beta} \leq \beta_1$

(b) $\bar{\beta}$ exists: $\beta_1 < \bar{\beta}$

(c) $\bar{\beta}$ does not exist.

Figure 2: Optimal research investment for possibility $(P2)$.

In Figure 2-a, an innovator with hyperbolic discounting preferences such that the discount rate is below $\bar{\beta}$ has either too strong preferences for the present and prefers being uniformed, or he is not willing to pay the cost of research. In both cases, he optimally chooses not to invest in research. If his discount rate is within $\bar{\beta}$ and β_1, the innovator invests in research to get information. Indeed, his optimal research investment is rather precise to make him stop his project when he receives the signal h: his preferences for the present are not too strong and the research investment constraint is satisfied. Above β_1, the innovator prefers not investing in research because of a lack of information precision while a time-consistent innovator invests because his research investment constraint holds.

In Figure 2-b, an innovator with hyperbolic discounting preferences never acquires information while a time-consistent innovator does. Indeed, there is no cost of research that could both provide useful information and satisfy the research investment constraint. Then, it is never optimal for such an innovator to invest in research. On the other hand, the time-consistent agent is always willing to be informed.

In the last figure 2-c, whatever the agent's preferences, it is not optimal to invest in research. Indeed, in this case the research investment constraint is never satisfied as the cost of research is too high.

The third possibility brings more sophistication to $(P2)$. Instead of allowing only one intersection between the two precision functions, it allows several ones and thus for a replication of the two cases underlined by $(P2)$: either condition (5) is satisfied or not. Moreover, as previously, for each of these cases, it is necessary to analyse the optimal research investment when the conditions (2) and (5) hold or not. Thus the analysis of $(P3)$ allows the same comments than in the previous possibility $(P2)$. An innovator with hyperbolic discounting preferences such that the discount rate is higher than $\bar{\beta}$ invests in research only if the information precision he could get is rather precise. In addition, whatever his preferences if the cost of research is too high, it is not optimal for him to acquire information.

Finally, we sum up our result:

Result 1: *Both the research investment constraint and the hyperbolic discounting preferences of the innovator limit information acquisition.*

3 Self-control

According to Salanié and Treich (2006), hyperbolic discounting preferences induce a self-control problem. The self-control may be defined as the agent's ability to commit in the future. Hence, a lack of self-control means that there is no possibility of commitment between the future selves of the innovator, that is the agent has hyperbolic preferences ($\beta \in]\tilde{\beta}, 1[$ between period 1 and period 2). On the other hand, self-control means that we allow the agent's future selves to have a commitment power. Indeed, self-1 allows the same weight to his current period and to the future one, and thus he behaves as a time consistent agent. Formally, self-control is thus characterized in the following by $\beta = 1$ between the period 1 and the period 2.

As an example, we consider a farmer who undertakes a GMO project. The parameter β represents the farmer's confidence in the project. When the farmer invests in the project, he may decide to invest in research, or not to acquire information on the effects of GMO. He is not confident with the future because consequences of GMO on health and the environment, in particular the contamination of other crops, are not well known. He then depreciates his future payoff ($\beta < 1$). However, in the following period the information on the potential risks received may increase his confidence in the project until making him entirely trusting of his project in the future ($\beta = 1$). In addition, without information, the farmer's confidence may also increase. Indeed, the farmer may be so motivated by his project that after a period of doubt at the beginning, his optimism leads him to trust perfectly in his project in the long term ($\beta = 1$).

This section is organized as follows: In subsection 3.1, we examine the agent with self-control's optimal decision-making; And in subsection 3.2 we determine the self-control effect on the information acquisition.

3.1 Optimal decision-making of an agent with self-control

In previous sections we analyse the behaviour of an agent with a lack of self-control. We now determine the behaviour of an innovator with self-control. Both kinds of innovators follows the same timing of the intra-personal game. However, we have to replace $\beta < 1$ by $\beta = 1$ between periods 1 and 2 in the intertemporal payoffs of the innovator with a lack of self-control to get the new intertemporal expected payoffs of an innovator with self-control.

Let us define the equilibrium strategy by x_σ^{**}. The following proposition establishes the conditions under which self-1 stops or continues his project:

Proposition 4 *For $\sigma \in \{h, l\}$, for $C \geq 0$ and for $\beta \in]\tilde{\beta}, 1]$: If $E(\theta|\sigma, C) < \hat{\theta}(1)$, then the innovator continues the project, that is $x_\sigma^{**} = 1$; If $\hat{\theta}(1) < E(\theta|\sigma, C)$, then the innovator stops the project, that is $x_\sigma^{**} = 0$; Finally, if $\hat{\theta}(1) = E(\theta|\sigma, C)$, then the innovator is indifferent between stopping and continuing his project, that is $x_\sigma^{**} \in \{0, 1\}$.*

Although the self-1's strategies do not change, the conditions to apply to them shift. In Proposition 1 the equilibrium strategy x_σ^* is replaced by x_σ^{**}, and $\hat{\theta}(\beta)$ by $\hat{\theta}(1)$.

Let us define $C^{**}(\beta)$ as the optimal research investment of an innovator with self-control. The following proposition represents the optimal research investment of an agent with self-control:

Proposition 5 *For $\beta \in]\tilde{\beta}, 1[$ if condition (5) and*

$$f(\hat{C}(1)) = \frac{(1 - p_0)B^L(1)}{-p_0 B^H(1) + (1 - p_0)B^L(1)} < f(C_{01}(\beta)). \tag{6}$$

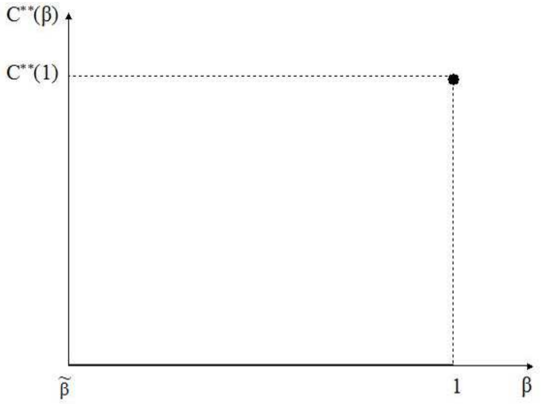
hold then $C^{**}(\beta) = C_{01}(\beta)$; Otherwise if one of these both conditions is not satisfied then $C^{**}(\beta) = 0$.

Self-0's conditions to acquire information are also modified. In Proposition 3, $C^*(\beta)$ is replaced by $C^{**}(\beta)$, and condition (4) by (6).

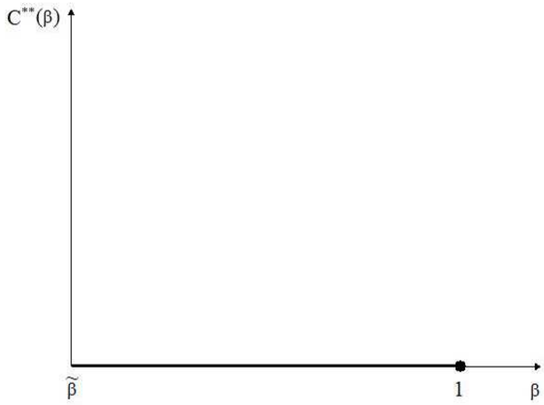
As previously, three possibilities arise when considering the optimal research investment:

- $(\tilde{P}1)$: There is no intersection between $f(\hat{C}(1))$ and $f(C_{01}(\beta))$. That is for all $\beta \in]\tilde{\beta}, 1[$, $f(C_{01}(\beta)) \leq f(\hat{C}(1))$;
- $(\tilde{P}2)$: There exists one intersection between $f(\hat{C}(1))$ and $f(C_{01}(\beta))$. Define $\beta_2 \in]\tilde{\beta}, 1[$ such that $f(\hat{C}(1)) = f(C_{01}(\beta_2))$. Then, for all $\beta \in]\tilde{\beta}, \beta_2]$, $f(C_{01}(\beta)) \leq f(\hat{C}(1))$ while for all $\beta \in [\beta_2, 1[$, $f(C_{01}(\beta)) < f(\hat{C}(\beta))$;
- $(\tilde{P}3)$: There is no intersection between $f(\hat{C}(1))$ and $f(C_{01}(\beta))$. That is for all $\beta \in]\tilde{\beta}, 1[$, $f(\hat{C}(1)) < f(C_{01}(\beta))$.

Let us turn to the first possibility $(\tilde{P}1)$. In this case, the precision constraint (condition (6)) is never satisfied, and thus the hyperbolic agent, even if his futures selves have a commitment power, never invests in research. Indeed, he prefers staying ignorant in order to avoid a lower expected payoff with a research investment. Nevertheless, the decision to get information of a time-consistent agent, who can completely commit in the long run, only depends on the fulfilment of his research investment constraint (condition (2)), and as previously, on the existence of $\bar{\beta}$. Figure 3 summarizes these comments.



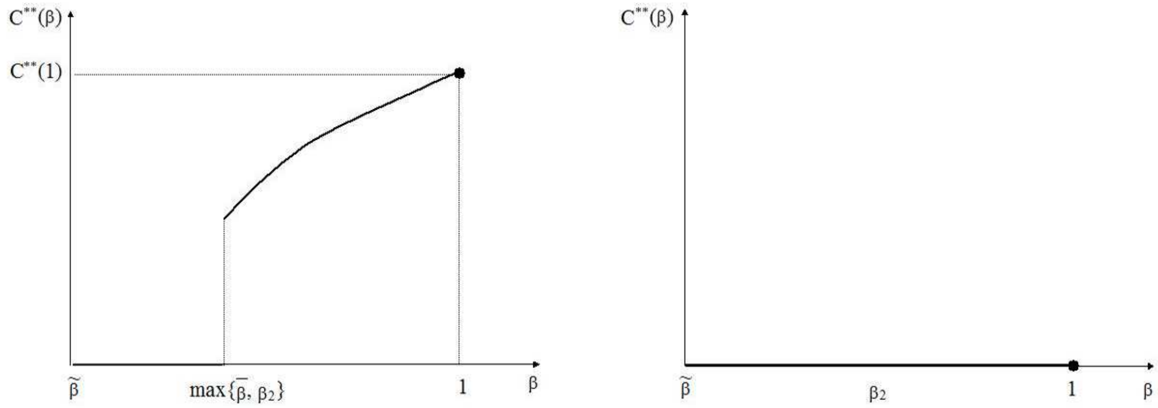
(a) $\bar{\beta}$ exists.



(b) $\bar{\beta}$ does not exist.

Figure 3: Optimal research investment for possibility ($\tilde{P}1$).

Regarding the second possibility ($\tilde{P}2$), the information precision (condition (6)), conditions (2) and (5) have to be characterized to determine the optimal research investment. Let us first consider the information precision constraint. The level of information precision from which self-1 decides to implement strategy $\{x_h = 0, x_l = 1\}$ is independent of β because self-1 and self-2 have a commitment power. Since a higher β implies a higher research investment $C_{01}(\beta)$ and since f is increasing, then below β_2 the precision constraint is not satisfied while it is above β_2. When taking the research investment constraint into account, two cases arise: If the investor's preferences are such that the cost of research that provides a precise enough signal to implement the strategy $\{x_h = 0, x_l = 1\}$, respects the research investment constraint, that is $\beta \in]\max\{\bar{\beta}, \beta_2\}, 1]$, then the investor always chooses to pay this cost and thus to acquire information; On the other hand, if either the cost of research that needs to be paid to get a useful signal does not satisfy the research investment constraint, or if the agent's preferences for the present are too strong, the hyperbolic agent prefers staying ignorant. Moreover, according to Lemma 5, if the time-consistent agent does not satisfies his research investment constraint, the hyperbolic agent with self-control does not either. Then, both kinds of agent do not acquire information. Figure 4 summarizes such remarks.



(a) $\bar{\beta}$ exists. (b) $\bar{\beta}$ does not exist.

Figure 4: Optimal research investment for possibility ($\tilde{P}2$).

Let us turn to the last possibility ($\tilde{P}3$). The precision constraint is always satisfied, and thus we get the same results as for possibility ($P1$) described in the previous section.

Overall, we remark that even if its futures selves have the possibility to commit, the innovator may still prefer staying ignorant while an innovator with a complete commitment power, that is a time-consistent agent, optimally chooses to be informed.

3.2 Self-control effect

According to Propositions 1 and 4, since $\hat{\theta}(\beta)$ is increasing with β, at period 1, an innovator with self-control is more likely to carry on his project than an innovator with a lack of self-control. Moreover, according to Propositions 3 and 5, since f and $\hat{C}$ are increasing, when the information precision constraint that characterizes an innovator with self-control (condition (6)) holds, then the information precision constraint that characterizes an innovator with a lack of self-control (condition (4)) is also satisfied. However, the reverse is not always true.

Actually, in period 1, an innovator with a lack of self-control gives more weight to the present than to the future. On the contrary, an innovator with self-control, in period 1, does not make any difference between the present and the future, since he has a commitment power. Thus, when both kinds of agent receive signal h, the one with a lack self-control requires a lower information precision to stop the project than the innovator with self-control. Indeed, with a lack of self-control the innovator is more concerned by current rewards and thus is more willing to give up in order to recover a part of his investment. The following result summarizes the self-control effect:

Result 2: *The self-control effect limits information acquisition.*

Overall, the possibility to commit in the future does not solve the problem of information acquisition and in some cases it may strengthen it. Actually, hyperbolic discounting also implies preferences for the present and the existence of a research investment constraint that imposes conditions on the precision of the information received and thus particularly influences the decision of both an innovator with and with a lack of self-control to acquire information.

4 Are liability rules enough efficient to influence the innovator's decision to get information?

All firms are constrained by a legal framework, in which liability rules specify how to allocate financial damages from an accident. Regarding technological innovations as well as other risky activities, it is important that firms receive the right incentives in order to not neglect risk and uncertainty learning. In this regard, this section proposes to analyse if, at the innovator level, the existing regulatory frameworks of risk, such as strict liability rule, negligence rule and limited liability rule, are efficient to incentive the innovator to acquire information, or at least does not limit its acquisition.

We study in subsections 4.1, 4.2 and 4.3 the efficiency of the strict liability rule, the negligence rule, and the limited liability rule to encourage the information acquisition respectively.

4.1 The strict liability rule

Under a strict liability rule, it is said that *if the victims can demonstrate a causality link between the damages and the innovator's activity or the product sold, the innovator is fully liable and thus he must pay for the damages caused by his activities.* Shavell (1980) and Miceli (1997) show that under time-consistency, such a rule is an incentive for innovators to consider the effect of both their level of care and their level of activity on accident losses. Hence, this rule allows one to prevent risks and to reduce the potential damages by leading the agent to exercise an optimal level of prevention. So, in an uncertainty and time inconsistency context, is this rule efficient enough to encourage the information acquisition in order to reduce the possible risks?

In the model we assume that, if an accident happens, the innovator is liable for damages and must pay for them. So, in defining the level of care as the level of research investment that the innovator needs to get information, we implicitly suppose that a strict liability rule is enforced.[10] Acquiring information reflects the innovator's level of

[10]However, we do not take into account of the just way in which the victims have to demonstrate a causality link between the damages and the activity or the product sold.

interest for the potential losses that his activity may cause. Indeed, if the innovator is informed he may stop his activity and then limit the cost of damages to K'. On the other hand, if he does not acquire information, he never stops his project and therefore exposes people and environment to a more severe risk.

Let us consider that the efficiency of the strict liability rule is linked to the innovator's ability to acquire information through the research investment that he is optimally willing to undertake. As previous sections suggest the model emphasizes that whatever the innovator's preferences (time-consistent or hyperbolic), if the optimal research investment related to the strategy $\{x_h = 0, x_l = 1\}$ does not satisfy the research investment constraint, the innovator never chooses to implement this strategy and always prefers staying uninformed. In others words, even if the investor is fully liable, if developing research activities or resorting to experts is too costly with regard to the expected returns of his project, the innovator has to choose to stay ignorant about the dangerousness of this project in order to be able to realize it. In such a case, the liability framework enforced does not lead the innovator to behave in a precautionary way and thus could be viewed as inefficient.

However, even if the optimal research investment related to the strategy $\{x_h = 0, x_l = 1\}$ satisfies the research investment constraint, it does not mean that the innovator always chooses to undertake this investment. Indeed, as shown by the model, an innovator with hyperbolic discounting preferences may prefer staying uninformed, while in such a case a time-consistent innovator always chooses to acquire information. Even if he is fully liable, when an investor has strong preferences for the present, he then prefers not getting information, even if it is free, to be sure to complete his project. The strict liability rule enforced does not lead him to adopt a precautionary behaviour, it is thus inefficient.

The optimal decisions taken by the agent also show that even if he has a weaker preference for the present, such a behaviour may still appear. Hyperbolic discounting preferences induce a precision constraint on the information that the innovator could get. If the signal provided by the research investment is not viewed as reliable, that is the signal received has no influence on the agent's behaviour, who always continues the project whatever its dangerousness, the innovator chooses again to stay ignorant. In such a case, a strict liability rule does not allow to avoid the impact of the agent's bias for the present on his decision to get information.

Overall, a strict liability rule does not seem to be a useful tool to encourage the innovator to acquire information.

4.2 The negligence rule

Under a negligence rule, it is said that *the injurer is liable for the victims' damages only if he fails to take a minimum level of care.* In other words, after an accident the Court of Law does not consider that an innovator is liable and must pay a financial cost for the damages if he has exercised the minimum level of care specified by the legal framework. In such a case, the victims or States have to assume the financial costs.

However, how to define the minimum level of care? According to Shavell (1980, 1992) and Miceli (1997), under a negligence rule the minimum level of care is defined as the optimal level of care that an injurer chooses if he has to pay for the damages. In our model, the minimum level of care is then the optimal research investment $C^*(\beta)$. Propositions 2 and 3 imply that when the research investment constraint does not hold for the innovator with time-consistent preferences, and when both condition (5) and the precision constraint do not hold for the innovator with hyperbolic discounting preferences, then it is not optimal for him to invest in research and to acquire information ($C^*(\beta) = 0$). On the other hand, when these conditions are satisfied, acquiring information is optimal ($C^*(\beta) > 0$) for the innovator whatever his preferences. Hence, this definition of the minimum level of care may lead the innovator to neglect information without being liable for the damages. This does not encourage the innovator to acquire information and then to decide whether he continues or stops his project in order to limit the cost of damages.

So, how to make the innovator get information? We propose to define the minimum level of care as the minimum of research investment $C^{min}(\beta)$ that an innovator exercises so as not to be liable if an accident occurs, and which leads him to acquire information whatever his preferences. In this respect, the innovator has an incentive to be informed.

Self-0's intertemporal expected payoff when the innovator is not responsible for the financial damages in case of an accident occurs is thus given by:

$$\begin{aligned} V_0^{NR}(x_h, x_l, C) = \; & -I - C + \beta[p_0 f(C) + (1-p_0)(1-f(C))](x_h R_2 + (1-x_h)D) \\ & +\beta[(1-p_0)f(C) + p_0(1-f(C))](x_l R_2 + (1-x_l)D). \end{aligned}$$

Actually, $V_0^{NR}(x_h, x_l, C)$ equals self-0's intertemporal expected payoff $V_0(x_h, x_l, C)$ with $K = K' = 0$.

Propositions 2 and 3 show that whatever his preferences, the innovator may optimally prefer not to acquire information and to be responsible for the damages in case of an accident. To avoid such an effect, the legal framework should impose that the minimum level of care $C^{min}(\beta)$ verifies that regarding the strategy $\{x_h = 0, x_l = 1\}$, self-0's intertemporal expected payoff when self-0 invests $C^{min}(\beta)$ and is not liable for the damages, that is $V_0^{NR}(0, 1, C^{min}(\beta))$, is at least equal to self-0's intertemporal expected payoff when self-0 does not invest in research and is then liable for the damages. According to condition (1), we know that when an innovator decides not to invest in research he always continues his project. Then, self-0's intertemporal expected payoff when self-0 decides not to invest in research and is liable for the damages is $V_0(1,1,0)$. The minimum level of care $C^{min}(\beta)$ is then characterized by:

$$V_0^{NR}(0,1,C^{min}(\beta)) = V_0(1,1,0)$$

which is equivalent to

$$C^{min}(\beta) + \beta[p_0 f(C^{min}(\beta)) + (1-p_0)(1-f(C^{min}(\beta)))](R_2 - D) = \beta E(\theta)K.$$

We can easily check that $C^{min}(\beta)$ exists and it is strictly positive. So, for whatever preferences an innovator has an incentive to invest at least $C^{min}(\beta)$. After this investment in research, a time-consistent agent follows the strategy $\{x_h = 0, x_l = 1\}$, while since the agent is not responsible of the damages ($K = K' = 0$), it is optimal for the hyperbolic agent's self-1 to always continues his project.

Hence, the negligence rule is perfectly efficient for a time-consistent innovator because it leads him to acquire information and to choose the cautious strategy $\{x_h = 0, x_l = 1\}$. Note that such a result is consistent with the findings of the literature on liability rules (Shavell, 1980, 1992). On the other hand, this rule is partially efficient for an innovator with hyperbolic discounting preferences. Indeed, this rule leads him to acquire information but this information is not useful for him: he always chooses to continue his project whatever the information he receives. Such a level of research investment does not influence him to behave in a precautionary way and does not limit the exposure of people or the environment to a potential danger.

4.3 The limited liability rule

Under a limited liability rule, *a catastrophic accident, the damages of which are higher than the financial capacities of the firm, is considered as a bankruptcy. Then, the total cost of accident is limited to the returns of the firms.*

In our model, if a limited liability rule is enforced then $R_2 = K$ and $D = \beta K'$. In this case, we get that the probability that a damage happens when the innovator is indifferent between continuing his project and stopping it at period 1 is:

$$\hat{\theta}(\beta) = \frac{\beta R_2 - D}{\beta(K - K')} = 1.$$

According to Propositions 1, 2, and 3 the innovator always decides to continue his project. It is then not optimal for him to invest in research and acquire information. Whatever the agent's preferences, this rule limits his ability to acquire information.

Actually, this result is not surprising. The limited liability rule is already discussed because it limits the responsibility of the innovator and leads him to be less concerned by potential catastrophes. In addition, from the innovation's point of view, such a rule may also have perverse effects. Indeed, if an accident happens and the innovator goes bankrupt, he may not reimburse his debt to the bank. This behaviour may lead the bank to restrict his credit and then to reduce the financing of current or future innovating projects.

5 Conclusion

In this paper we study the hyperbolic discounting preferences effect on the innovator's research investment behaviour. Investing in research allows him to acquire information, and then to reduce the uncertainty linked to his project. Possible examples of application of this model include innovations in new technologies (e.g. nanotechnologies, mobile phones), pharmaceutical firms (e.g. development and production of new drugs), or chemical firms (e.g. production of new fertilizers). As for the GMO plants farm's example, in all those preceding examples, innovators may produce while they may have an incomplete knowledge on the dangerousness of their activities in the long run. Acquiring information allows them to learn about the potential risks of their activities.

We find that whatever the innovator preferences (hyperbolic or time-consistent), there exists a research investment constraint which limits the information acquisition. Indeed, if the cost of research investment is too high the agent never acquires information. However, as in Carrillo and Mariotti (2000) if the information is free, while a time-consistent agent always gets information, a hyperbolic agent may prefer staying ignorant. In addition, we obtain that hyperbolic discounting preferences induce an information precision constraint that leads the hyperbolic innovator to ignore the information while the time-consistent innovator gets it. Hence, both the existence of a research investment constraint and the hyperbolic discounting preferences restrict the agent's ability to acquire information. Indeed, this constraint and such preferences lead him to undertake an investment whose risks are imperfectly known, while if he gets information he could stop prematurely his activity and limit the damages in case of accident.

Moreover, we show that the possibility that the agent's futures selves may commit in the long run (he may have a self-control in the future) strengthens this ignorant behaviour. Actually, in the long run the agent does not discount his future payoff. He is more concerned by the information precision because he is more concerned by the potential loses. In addition, the research investment required may be too high, and leads to a lower payoff than the one without investment in research. This may imply a non-precautionary behaviour regarding the potential risks of the project.

We then analyse the efficiency of liability rules to improve the information acquisition. In the model, we assume that the agent is fully liable and must pay for the damages caused by his activities. He is then under a strict liability rule. In terms of information acquisition, the efficiency of this rule is limited by the research investment constraint and the innovator's hyperbolic discounting preferences. When we apply a negligence rule, we underline that even if this rule is efficient enough to improve the information acquisition, an agent with hyperbolic discounting preferences may decide to always continue his project whatever the received signal. This information is then not reliable for him and does not lead to a precautionary behaviour. Finally, we investigate the impact of another liability rule that control firm's activities, that is the limited liability rule, on the innovator's decision to get information. We find that this rule totally restricts the information acquisition whatever the agent's preferences. Overall, such liability rules do not seem to encourage the information acquisition and the reduction of damages.

Thus, if the information acquisition is not spontaneous, other responsibility forms or rules might need to be considered. Strotz (1956) emphasizes the necessity to define pre-commitment strategies in a context of hyperbolic preferences in order to reduce the impact of the hyperbolic discounting on the agents' decision. In an innovation context, pre-commitment could be realized with contracts establishing the innovation's agenda in the long run. From this perspective, the negligence rule combined with a pre-commitment controlled by the State, and saying that the firm takes into account the information provided by research, could be an interesting alternative solution. Agents who are familiar with risk and uncertainty, such as the insurers, could also define such a pre-commitment strategy. One could then imagine insurance contracts with a deductible to allow better control of the precautionary efforts undertaken by innovators in order to reduce the financial risk they are exposed to.

However, regarding the current application of a strict liability rule, experience also underlines the persuasive role that such a rule can play on producers' behaviour. Weill (2005) notes that when the 'burden of the proof' is on the potential injurer, and not on the victims, as it is the case under a negligence rule, producers are more likely to withdraw potentially dangerous products from the market. The recent European legislation on chemicals (REACH directive)[11] tackles the challenging issue related to the application of the precautionary principle to both enhance innovation as well as people and environmental protection. It is based on a strict liability rule, under which the 'burden of the proof' is on the industry, but it also requires manufacturers and importers to take the responsibility "to gather information on the properties and risks of all substances

[11]REACH stands for Registration, Evaluation, Authorization and Restriction of Chemicals

produced or imported".[12] This legislation proposes an interesting way to implement the precautionary principle to deal with chemicals, by combining the positive effects of a strict liability rule to a research obligation for firms that should avoid the negative ones. This approach should provide relevant elements in the current debate on the regulation of other kinds of scientific and/or technological innovation.

Appendix

Proof of Proposition 1

At period 1, the innovator receives the signal $\sigma \in \{h, l\}$. For all $C \geq 0$ and for all $\beta \in]\tilde{\beta}, 1]$:

He chooses to continue, that is $x_\sigma = 1$ if:

$$V_1(0, \sigma, C) < V_1(1, \sigma, C) \quad \text{i.e.} \quad E(\theta|\sigma, C) < \hat{\theta}(\beta) \equiv \frac{\beta R_2 - D}{\beta(K - K')};$$

He chooses to stop, that is $x_\sigma = 0$ if:

$$V_1(1, \sigma, C) < V_1(0, \sigma, C) \quad \text{i.e.} \quad \hat{\theta}(\beta) \equiv \frac{\beta R_2 - D}{\beta(K - K')} < E(\theta|\sigma, C);$$

He is indifferent between stopping and continuing the project, that is $x_\sigma \in \{0, 1\}$ if:

$$V_1(1, \sigma, C) = V_1(0, \sigma, C) \quad \text{i.e.} \quad \hat{\theta}(\beta) \equiv \frac{\beta R_2 - D}{\beta(K - K')} = E(\theta|\sigma, C).$$

Since $\theta^H > \theta^L$, and for all $C \geq 0$ we have $f(C) \geq \frac{1}{2}$, we obtain that:

$$E(\theta|l, C) - E(\theta) = \frac{(1 - p_0)p_0(\theta^H - \theta^L)(1 - 2f(C))}{(1 - p_0)f(C) + p_0(1 - f(C))} \leq 0$$

and

$$E(\theta) - E(\theta|h, C) = \frac{(1 - p_0)p_0(\theta^L - \theta^H)(2f(C) - 1)}{p_0 f(C) + (1 - p_0)(1 - f(C))} \leq 0.$$

Thus, for all $C \geq 0$, $E(\theta|l, C) \leq E(\theta) \leq E(\theta|h, C)$.

We differentiate $E(\theta|h, C)$ with respect to C, we obtain:

$$\frac{\partial E(\theta|h, C)}{\partial C} = \frac{(1 - p_0)p_0 f'(C)(\theta^H - \theta^L)}{[(1 - p_0)(1 - f(C)) + p_0 f(C)]^2}$$

[12] For more details on REACH, see European Commission [28].

which is positive because f is increasing and $\theta^H > \theta^L$. Thus, $E(\theta|h, C)$ is increasing with C.

We differentiate $E(\theta|l, C)$ with respect to C, we obtain:

$$\frac{\partial E(\theta|l, C)}{\partial C} = \frac{(1-p_0)p_0 f'(C)((\theta^L - \theta^H)}{[p_0(1-f(C)) + (1-p_0)f(C)]^2}$$

which is negative because f is increasing and $\theta^H > \theta^L$. Thus, $E(\theta|l, C)$ is decreasing with C.

■

Proof of Lemma 1

For $\beta = 1$, we study the concavity of $V_0(0, 1, C)$: We differentiate $V_0(0, 1, C)$ with respect to C, we obtain:

$$\frac{\partial V_0(0, 1, C)}{\partial C} = -1 + [(1-p_0)B^L(1) - p_0 B^H(1)]f'(C). \tag{7}$$

We differentiate equation (7) with respect to C, we obtain:

$$\frac{\partial^2 V_0(0, 1, C)}{\partial C^2} = [(1-p_0)B^L(1) - p_0 B^H(1)]f''(C)$$

which is negative because f is concave, $B^L(1)$ is positive and $B^H(1)$ is negative. Thus, for $\beta = 1$, $V_0(0, 1, C)$ is concave.

There exists a solution $C_{01}^*(1)$ to the maximization of $V_0(0, 1, C)$ with $\beta = 1$ and with respect to C. $C_{01}^*(1)$ is characterized by the first order condition:

$$\frac{\partial V_0(0, 1, C)}{\partial C} = 0 \Leftrightarrow f'(C_{01}^*(1)) = \frac{1}{[(1-p_0)B^L(1) - p_0 B^H(1)]}.$$

Since $B^L(1)$ is positive and $B^H(1)$ is negative, we get $f'(C_{01}^*(1)) > 0$.

We suppose that $C_{01}^*(1) = 0$. We then get $f'(C_{01}^*(1)) = +\infty$. However, since $0 < (1-p_0)B^L(1) - p_0 B^H(1)$ we cannot have $f'(C_{01}^*(1)) = +\infty$. So, there is a contradiction that implies that $C_{01}^*(1) > 0$.

■

Proof of Proposition 2

For $\beta = 1$, the optimal research investment $C^*(1)$ is such that:

- if $V_0(1, 1, 0) < V_0(0, 1, C^*_{01}(1))$ then $C^*(\beta) = C^*_{01}(1)$;
- otherwise $C^*(1) = 0$.

We first compare $V_0(0, 1, C^*_{01}(1))$ and $V_0(1, 1, 0)$. We obtain:

$$\begin{aligned} V_0(0,1,C^*_{01}(1)) - V_0(1,1,0) &= -C^*_{01}(1) - [p_0 f(C^*_{01}(1))B^H(1) + (1-p_0)(1-f(C^*_{01}(1)))B^L(1)]. \\ &= -C^*_{01}(1) - [p_0 f(C^*_{01}(1)) + (1-p_0)(1-f(C^*_{01}(1)))] \\ & [R_2 - D - E(\theta|h,C)(K-K')]. \end{aligned}$$

Hence, $V_0(1, 1, 0) < V_0(0, 1, C^*_{01}(1))$ if and only if:

$$C^*_{01}(1) < -[p_0 f(C^*_{01}(1)) + (1-p_0)(1-f(C^*_{01}(1)))][R_2 - D - E(\theta|h,C)(K-K')]$$

which is equivalent to

$$C^*_{01}(1) < -B_0(h, C^*_{01}(1)).$$

Overall, we have the following result: for $\beta = 1$ if $C^*_{01}(1) < -B_0(h, C^*_{01}(1))$ holds then $C^*(1) = C_{01}(1)$, otherwise $C^*(1) = 0$.

■

Proof of Lemma 2

Similar to the proof of Lemma 1, thus omitted.

We differentiate $f'(C_{01}(\beta))$ with respect to β. We obtain:

$$f''(C_{01}(\beta))C'_{01}(\beta) = \frac{-1}{\beta^2[(1-p_0)B^L - p_0 B^H]}$$

which is negative. Since f is concave then C_{01} is increasing with β.

We suppose that there exists a $\beta_0 \in]\tilde{\beta}, 1]$ such that $C_{01}(\beta_0) = 0$. We then get $f'(C_{01}(\beta_0)) = +\infty$. Since $0 < (1-p_0)B^L(1) - p_0 B^H(1)$ then $f'(C_{01}(\beta_0)) = +\infty$ if and only if $\beta_0 = 0$. However, $\beta_0 \neq 0$ because $\tilde{\beta} > 0$. So, there is a contradiction that implies for all $\beta \in]\tilde{\beta}, 1[$ we obtain $C_{01}(\beta) > 0$.

■

Proof of Lemma 3

$\hat{C}(\beta)$ is the smallest $C \geq 0$ which satisfies that $E(\theta|l, C) \leq \hat{\theta}(\beta) \leq E(\theta|h, C)$.

Define $\hat{C}_1(\beta)$ the smallest $C \geq 0$ which satisfies $E(\theta|l, \hat{C}(\beta)) \leq \hat{\theta}(\beta)$, $\hat{C}_2(\beta)$ the smallest $C \geq 0$ which satisfies $\hat{\theta}(\beta) \leq E(\theta|h, \hat{C}(\beta))$, and $\hat{C}(\beta)$ the smallest $C \geq 0$ which satisfies $E(\theta|l, C) \leq \hat{\theta}(\beta) \leq E(\theta|h, C)$. Since $E(\theta|h, C)$ is increasing with C and $E(\theta|l, C)$ is decreasing with C then $\hat{C}(\beta) = \max\{\hat{C}_1(\beta), \hat{C}_2(\beta)\}$.

We first look for $\hat{C}_1(\beta)$: According to condition (1) and Proposition 1, for all $C \geq 0$ we have $E(\theta|l, C) \leq \hat{\theta}(\beta)$. We then get $\hat{C}_1(\beta) = 0$.

Now, we turn to $\hat{C}_2(\beta)$: Since $E(\theta|h, C)$ is increasing then $\hat{C}_2(\beta)$ is such that:

$$\hat{\theta}(\beta) = E(\theta|h, \hat{C}_2(\beta)) \text{ which is equivalent to } f(\hat{C}_2(\beta)) = \frac{(1-p_0)B^L(\beta)}{-p_0 B^H(\beta) + (1-p_0)B^L(\beta)}.$$

According to condition (1) we easily verify that $f(\hat{C}_2(\beta)) \geq \frac{1}{2}$. Since $B^H(\beta)$ is negative, we immediately get $f(\hat{C}_2(\beta)) \leq 1$. Moreover, since $\frac{1}{2} = f(\hat{C}_1(\beta)) \leq f(\hat{C}_2(\beta))$ and f is increasing, we get that $\hat{C}(\beta) = \max\{\hat{C}_1(\beta), \hat{C}_2(\beta)\} = \hat{C}_2(\beta)$.

Hence, for $\beta \in]\tilde{\beta}, 1[$:

$$f(\hat{C}(\beta)) = \frac{(1-p_0)B^L(\beta)}{-p_0 B^H(\beta) + (1-p_0)B^L(\beta)}.$$

■

Proof of Lemma 4

By definition of $\hat{C}(\beta)$, $C_{01}(\beta)$ and $C^*_{01}(\beta)$ and since f is increasing, if $f(\hat{C}(\beta)) < f(C_{01}(\beta))$ then $C^*_{01}(\beta) = C_{01}(\beta)$ otherwise $C^*_{01}(\beta) = \hat{C}(\beta)$. That is $C^*_{01}(\beta) = \max\{\hat{C}(\beta), C_{01}(\beta)\}$.

Moreover, we differentiate $f(\hat{C}(\beta))$ with respect to β, we obtain:

$$f'(\hat{C}(\beta))\hat{C}'(\beta) = \frac{p_0(1-p_0)D(\theta^H - \theta^L)(K - K')}{[-p_0 B^H(\beta) + (1-p_0)B^L(\beta)]^2}$$

which is positive. Since f is increasing then $\hat{C}(\beta)$ is increasing with β.

By definition of $\tilde{\beta}$ and $\hat{C}(\beta)$, we get $E(\theta) = \hat{\theta}(\tilde{\beta}) = E(\theta|h, \hat{C}(\tilde{\beta}))$. According to Proposition 1: $E(\theta) = E(\theta|h, 0) = E(\theta|l, 0)$. Hence, since $\hat{C}(\beta)$ is increasing with β and

$E(\theta|h, C)$ is increasing with C then $\hat{C}(\tilde{\beta}) = 0$. So, since f is increasing we obtain that $f(\hat{C}(\tilde{\beta})) = f(0) = \frac{1}{2}$.

■

Proof of Proposition 3

For $\beta \in]\tilde{\beta}, 1[$, the optimal research investment $C^*(\beta)$ is such that

- if $V_0(0, 1, C^*_{01}(\beta)) > V_0(1, 1, 0)$ then $C^*(\beta) = C^*_{01}(\beta)$;
- otherwise $C^*(\beta) = 0$.

According to Lemma 4, for $\beta \in]\tilde{\beta}, 1[$ we have two cases : if condition (4) holds then $C^*_{01}(\beta) = C_{01}(\beta)$ but if condition (4) does not hold we get $C^*_{01}(\beta) = \hat{C}(\beta)$.

We first assume that condition (4) does not hold. We then compare $V_0(0, 1, \hat{C}(\beta))$ and $V_0(1, 1, 0)$. We obtain:

$V_0(0, 1, \hat{C}(\beta)) - V_0(1, 1, 0) = -\hat{C}(\beta) - \beta[p_0 f(\hat{C}(\beta))B^H(1) + (1 - p_0)(1 - f(\hat{C}(\beta)))B^L(1)]$.

Since $f(\hat{C}(\beta)) = \frac{(1-p_0)B^L(\beta)}{-p_0 B^H(\beta)+(1-p_0)B^L(\beta)}$. We get:

$$V_0(0, 1, \hat{C}(\beta)) - V_0(1, 1, 0) = -\hat{C}(\beta) - \beta p_0(1 - p_0)\left[\frac{(1 - \beta)(\theta^H - \theta^L)D(K - K')}{-p_0 B^H(\beta) + (1 - p_0)B^L(\beta)}\right]$$

which is negative.
So, if condition (4) does not hold: $V_0(0, 1, \hat{C}(\beta)) < V_0(1, 1, 0)$ and $C^*_{01}(\beta) = 0$.

Now, we assume that condition (4) holds. We then compare $V_0(0, 1, C_{01}(\beta))$ and $V_0(1, 1, 0)$. We obtain:

$$\begin{aligned}V_0(0, 1, C_{01}(\beta)) - V_0(1, 1, 0) &= -C_{01}(\beta) - \beta[p_0 f(C_{01}(\beta))B^H(1) + (1 - p_0)(1 - f(C_{01}(\beta)))B^L(1)].\\ &= -C_{01}(\beta) - \beta[p_0 f(C_{01}(\beta)) + (1 - p_0)(1 - f(C_{01}(\beta)))]\\ &[R_2 - D - E(\theta|h, C_{01}(\beta))(K - K')].\\ &= -C_{01}(\beta) - B_0(h, C_{01}(\beta))\end{aligned}$$

So, if condition (4) holds: If $C_{01}(\beta) < -B_0(h, C_{01}(\beta))$ then $V_0(1, 1, 0) < V_0(0, 1, C_{01}(\beta))$ and $C^*_{01}(\beta) = C_{01}(\beta)$; Otherwise $V_0(0, 1, C_{01}(\beta)) \leq V_0(1, 1, 0)$ and $C^*_{01}(\beta) = 0$.

Overall, we have the following result: For all $\beta \in]\tilde{\beta}, 1[$ if $C_{01}(\beta) < -B_0(h, C_{01}(\beta))$ and (4) hold then $C^*(\beta) = C_{01}(\beta)$; Otherwise if one of these both conditions is not satisfied then $C^*(\beta) = 0$.

■

Proof of Lemma 5

For $\beta \in]\tilde{\beta}, 1]$, we define:

$$g(\beta) = -C_{01}(\beta) - B_0(h, C_{01}(\beta)).$$

We differentiate $g(\beta)$ with respect to β, we obtain:

$$g'(\beta) = -B_0(h, C_{01}(\beta)). \tag{8}$$

We differentiate $g'(\beta)$ with respect to β, we obtain:

$$g''(\beta) = \frac{C_{01}(\beta)}{\beta}$$

which is positive and so g is convex.

If we assume that equation (8) is positive then g is increasing. Hence, since g is increasing and convex, condition (2) holds then there exists a $\bar{\beta} \in]\tilde{\beta}, 1]$ such that for all $\beta \in]\tilde{\beta}, \bar{\beta}]$ condition (5) does not hold and for all $\beta \in]\bar{\beta}, 1]$ condition (5) holds. However, if condition (2) does not hold then for all $\beta \in]\tilde{\beta}, 1[$, condition (5) does not hold either.

If we assume that equation (8) is negative or equal to zero then for $\beta \in]\tilde{\beta}, 1]$, $g(\beta) < 0$. This implies that for all $\beta \in]\tilde{\beta}, 1[$ condition (5), and condition (2) do not hold.
■

Proof of Proposition 4

Similar to the proof of Proposition 1, thus omitted.
■

Proof of Proposition 5

Similar to the proof of Proposition 3, thus omitted.
■

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