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ROBUST INFLATION-TARGETING RULES AND THE GAINS FROM INTERNATIONAL POLICY COORDINATION

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Robust Inflation-Targeting Rules and the Gains from International Policy Coordination*

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Abstract

This paper empirically assesses the performance of interest-rate monetary rules for interdependent economies characterized by model uncertainty. We set out a two-bloc dynamic stochastic general equilibrium model with habit persistence (that generates output persistence), Calvo pricing and wage-setting with indexing of non-optimized prices and wages (generating inflation persistence), incomplete financial markets and the incomplete pass-through of exchange rate changes. We estimate a linearized form of the model by Bayesian maximum-likelihood methods using US and Euro-zone data. From the estimates of the posterior distributions we then examine monetary policy conducted both independently and cooperatively by the Fed and the ECB in the form of robust inflation-targeting interest-rate rules. Comparing the utility outcome in a closed-loop Nash equilibrium with the outcome from a coordinated design of policy rules, we find a new result: the gains from monetary policy coordination rise significantly when CPI inflation targeting interest-rate rules are designed to account for model uncertainty.

JEL Classification: E52, E37, E58

Keywords: monetary policy coordination, robustness, inflation-targeting interest-rate rules.

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Contents

1 Introduction

The emergence of the new micro-founded open-economy models (NOEM) has led naturally for the literature to revisit the economics of monetary policy interdependence. Following the seminal contribution of Obstfeld and Rogoff (1996), a number of papers have studied spillover effects and the resulting gains from policy coordination for interdependent economies using a rudimentary NOEM (e.g., Betts and Devereux (2000), Corsetti and Pesenti (2001), Obstfeld and Rogoff (2002), Clarida *et al.* (2002a), Benigno and Benigno (2003)). A consensus emerging from this body of work is that the coordination gains are very small.

Our paper belongs to a more recent literature that reassesses the no gains result using a more developed NOEM. We examine inflation-targeting monetary rules that are robust in the face of model uncertainty and are operational in the sense that a zero-lower-bound (henceforth, ZLB) constraint on the nominal interest rate is observed. We develop a two-bloc NOEM with traded and non-traded sectors, habit persistence (that generates output persistence in the model), Calvo pricing with indexing of non-optimized prices (generating inflation persistence), imperfect financial markets and the incomplete pass-through of exchange rate changes. Wage stickiness is introduced using an analogous form of staggered wage setting. We estimate a linearized form of the model by Bayesian maximum-likelihood methods using US and Euro-zone data.[1]

We first assess the gains from coordination in the absence of model uncertainty. Both cooperative and non-cooperative optimized IFB targeting rules are computed. Comparisons between the outcomes under these two sets of rules provide an empirical assessment of the coordination gains. As in Batini *et al.* (2005), Batini *et al.* (2006) and Levine *et al.* (2008), we then proceed to use the estimated posterior densities of parameters to design IFB rules that are robust in two senses: they guarantee stable and unique equilibria for all parameter combinations and, in addition, use the posterior parameter density functions to minimize an expected loss function of the central bank subject to this estimated model uncertainty.[2]

The rest of the paper is set out as follows: section 2 describes our model, the steady

[1]Coenen *et al.* (2007) set out a similar model, but without non-traded sectors. In a calibrated model they find only small gains, of the order of a 0.05% permanent increase in steady-state consumption.

[2]We assume *model consistent expectations* throughout in the sense that two monetary authorities and the private sector all agree on the 'true' model drawn from the distribution. Frankel and Rockett (1988) and Holtham and Hughes Hallett (1992) study policy coordination where the central banks may believe in different models. Levine *et al.* (2008) examine policy rules in a single economy where the private sector and the central bank may believe in different models.

state and the linearization about the latter.[3] Section 3 describes the estimation methodology and results. Sections 4 and 5 describes our procedures for approximating the optimization problems in a linear-quadratic form and for imposing an approximate ZLB constraint. Section 6 provides results for optimized IFB rules where model parameters are known with certainty. Section 7 tackles the case where there is parameter uncertainty and inflation-targeting rules are designed to be robust. Section 8 summarizes our main results.

2 The Model

There are two asymmetric unequally-sized blocs with the different household preferences and technologies. In each bloc there are traded and non-traded sectors and the relative size of the sectors can differ. The model is set up so as to incorporate two large blocs and a small open economy embedded in a world economy within one framework. We first assume complete asset markets before modifying the model to incorporate imperfect markets. The exchange rate is perfectly flexible. The consumption index in each bloc is of Dixit-Stiglitz nested CES form with domestic and foreign components consisting of a basket of differentiated goods produced in each bloc. Goods producers and household suppliers of labor have monopolistic power. Wages and nominal domestic prices of both domestically produced and imported goods are sticky. Following Devereux and Engel (2002) we introduce heterogenity in the way goods are exported. Some firms are local currency pricers (LCPs) and market directly to the overseas markets which creates a departure from the law of one price. Other firms are producer currency pricers (PCPs,) setting prices in their own currency. To keep the model at manageable 'medium' size (and in common with much of the New Keynesian DSGE literature) labour is the only input. Apart from this feature, as each bloc tends to its closed economy limit and we shut down the open-economy aspects it resembles the single closed economy model of Smets and Wouters (2003a), but without capital.[4]

[3]Levine *et al.* (2007a) gives a fuller description of the model, its estimation and simulation properties.

[4]It follows that 'consumption' should in fact be interpreted as total private expenditure with the risk aversion parameter, σ interpreted accordingly (see Woodford (2003), page 352.)

2.1 Households

There are ν households in the 'home' bloc and ν^* households in the 'foreign' bloc. A representative household r in the home bloc maximizes

$$E_t \sum_{t=0}^{\infty} \beta^t U \left(C_t(r) - H_{C,t}, \frac{M_t(r)}{P_t}, L_t(r), G_t, U_{C,t}, U_{M,t}, U_{L,t} \right) \tag{1}$$

where E_t is the expectations operator indicating expectations formed at time t, β is the household's discount factor, $U_{C,t}$, $U_{M,t}$ and $U_{L,t}$ are preference shocks $C_t(r)$ is an index of consumption, $L_t(r)$ are hours worked, $H_{C,t}$ represents the habit in consumption, or desire not to differ too much from other households, and we choose $H_{C,t} = hC_{t-1}$, where $C_t = \frac{1}{\nu}\sum_{r=1}^{\nu} C_t(r)$ is the average consumption index, $h \in [0, 1)$. When $h = 0$, $\sigma > 1$ is the risk aversion parameter (or the inverse of the intertemporal elasticity of substitution)[5]. $M_t(r)$ are end-of-period nominal money balances and G_t is exogenous per capita real government spending assumed to be exclusively on non-traded domestic output. An analogous symmetric intertemporal utility is defined for the 'foreign' representative household and the corresponding variables (such as consumption) are denoted by $C_t^*(r)$, etc.

The representative household r must obey a budget constraint:

$$\begin{aligned} P_t C_t(r) + E_t[Q_{t,t+1} D_{t+1}(r)] + M_t(r) &= (1 - T_t) W_t(r) L_t(r) + D_t(r) + M_{t-1}(r) \\ &+ (1 - T_t)\Gamma_t(r) + TR_t \end{aligned} \tag{2}$$

where P_t is a Dixit-Stiglitz price index defined in (17) below, $D_{t+1}(r)$ is a random variable denoting the payoff of the portfolio purchased at time t and $Q_{t,t+1}$, the stochastic discount factor, is the period-t price of an asset that pays one unit of domestic currency in a particular state of period $t + 1$ divided by the probability of an occurrence of that state given information available in period t. $W_t(r)$ is the wage rate, T_t the income tax rate and $\Gamma_t(r)$ are dividends from ownership of firms.[6] Finally TR_t are lump-sum transfers to households by the government net of lump-sum taxes

Assume the existence of nominal one-period riskless bonds denominated in domestic currency with nominal interest rate I_t over the interval $[t, t + 1]$. Then arbitrage considerations imply that $E_t[Q_{t,t+1}] = \frac{1}{1+I_t}$. In addition, if we assume that households' labour supply is differentiated with elasticity of supply η, then (as we shall see below) the demand

[5]When $h \neq 0$, σ is merely an index of the curvature of the utility function.

[6]The tax rate T_t can be interpreted as a total tax wedge (see Levine *et al.* (2006)).

for each consumer's labor supplied by ν identical households is given by

$$L_t(r) = \left(\frac{W_t(r)}{W_t}\right)^{-\eta} L_t \tag{3}$$

where $W_t = \left[\frac{1}{\nu}\sum_{r=1}^{\nu} W_t(r)^{1-\eta}\right]^{\frac{1}{1-\eta}}$ and $L_t = \left[\frac{1}{\nu}\sum_{r=1}^{\nu} L_t(r)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$ are the average wage index and average employment respectively.[7]

Let the number of differentiated *traded* goods produced in the home and foreign blocs be n_H and n_F respectively and the number of differentiated *non-traded goods* be n_N and n_N^* respectively. Let $n = n_H + n_N$ and $n^* = n_F + n_N^*$ be the corresponding total numbers of goods in the two blocs. Each good is produced by a single firm and we assume that the the ratio of households to firms are the same in each bloc, i.e., $\frac{\nu}{n} = \frac{\nu^*}{n^*}$. It follows that n and n^* (or ν and ν^*) are measures of *size*. Then the per capita consumption index in the home bloc is given by

$$C_t(r) = \left[\mathrm{w}_N^{\frac{1}{\mu}} C_{N,t}(r)^{\frac{\mu-1}{\mu}} + (1-\mathrm{w}_N)^{\frac{1}{\mu}} C_{T,t}(r)^{\frac{\mu-1}{\mu}}\right]^{\frac{\mu}{\mu-1}} \tag{4}$$

where μ is the elasticity of substitution between non-traded and traded goods,

$$C_{N,t}(r) = \left[\left(\frac{1}{n_N}\right)^{\frac{1}{\zeta_N}} \sum_{f=1}^{n_N} C_{N,t}(f,r)^{(\zeta_N-1)/\zeta_N}\right]^{\zeta_N/(\zeta_N-1)} \tag{5}$$

and

$$C_{T,t}(r) = \left[\mathrm{w}^{\frac{1}{\mu_T}} C_{H,t}(r)^{\frac{\mu_T-1}{\mu_T}} + (1-\mathrm{w})^{\frac{1}{\mu_T}} C_{F,t}(r)^{\frac{\mu_T-1}{\mu_T}}\right]^{\frac{\mu_T}{\mu_T-1}} \tag{6}$$

where μ_T is the elasticity of substitution between home and foreign traded goods,

$$C_{H,t}(r) = \left[\left(\frac{1}{n_H}\right)^{\frac{1}{\zeta_T}} \sum_{f=1}^{n_H} C_{H,t}(f,r)^{(\zeta_T-1)/\zeta_T}\right]^{\zeta_T/(\zeta_T-1)} \tag{7}$$

$$C_{F,t}(r) = \left[\left(\frac{1}{n_F}\right)^{\frac{1}{\zeta_T}} \left(\sum_{f=1}^{\theta^* n_F} C_{F,t}(f,r)^{(\zeta_T-1)/\zeta)T} + \sum_{f=1}^{(1-\theta^*)n_F} C_{F,t}(f,r)^{(\zeta_T-1)/\zeta_T}\right)\right]^{\zeta_T/(\zeta_T-1)} \tag{8}$$

where $C_{N,t}(f,r)$ denotes the home consumption of the non-traded good of household r, $C_{H,t}(f,r)$ and $C_{F,t}(f,r)$ denote the home consumption of traded variety f produced in blocs H and F respectively, ζ_N and ζ_T are the elasticities of substitution between varieties

[7] Note that if we normalize $\nu = 1$ then as is more customary in the literature we can write $W_t \simeq \left[\int_0^1 W_t(r)^{1-\eta} dr\right]^{\frac{1}{1-\eta}}$. However here we need to impose different sized blocs with the foreign number of households $\nu^* \neq \nu$.

in each bloc (note that we impose equality between blocs for the traded elasticity, i.e., $\zeta_T^* = \zeta_T$), and

$$\mathrm{w} = \frac{n_H \omega}{n_H \omega + n_F(1-\omega)} \tag{9}$$

In (9) $\omega \in [\frac{1}{2}, 1]$ is a parameter that captures the degree of 'bias' in the home bloc. If $\omega = 1$ we have autarky, while the lower extreme of $\omega = \frac{1}{2}$ gives us the case of perfect integration. If blocs are of equal size then $n_H = n_F$, $\mathrm{w} = \omega$ and consumption only favours home consumption if there is home bias.[8] In the absence of home bias $\mathrm{w} = \frac{n_H}{n_H+n_F}$ and domestic/foreign consumption decisions depend only on relative size. As $\mu \to 1$ and $w_N \to 0$ we approach a one-sector model with a Cobb-Douglas utility function $C_t(r) = C_{T,t}(r) = \mathrm{w}^{-\mathrm{W}}(1-\mathrm{w})^{-(1-\mathrm{W})} C_{H,t}(r)^{\mathrm{W}} C_{F,t}(r)^{1-\mathrm{W}}$ as in Clarida *et al.* (2002b).

If $P_{H,t}(f)$, $P_{F,t}(f)$ are the prices in domestic currency of the good produced by firm f in the relevant bloc, then the optimal intra-temporal decisions are given by standard results:

$$C_{H,t}(r,f) \;=\; \left(\frac{P_{H,t}(f)}{P_{H,t}}\right)^{-\zeta_T} C_{H,t}(r)\,;\;\; C_{F,t}(r,f) = \left(\frac{P_{F,t}(f)}{P_{F,t}}\right)^{-\zeta_T} C_{F,t}(r) \tag{10}$$

$$C_{H,t}(r) \;=\; \mathrm{w}\left(\frac{P_{H,t}}{P_{T,t}}\right)^{-\mu_T} C_{T,t}(r)\,;\;\; C_{F,t}(r) = (1-\mathrm{w})\left(\frac{P_{F,t}}{P_{T,t}}\right)^{-\mu_T} C_{T,t}(r) \tag{11}$$

$$C_{N,t}(r) \;=\; \mathrm{w}_N\left(\frac{P_{N,t}}{P_t}\right)^{-\mu} C_t(r)\,;\;\; C_{T,t}(r) = (1-\mathrm{w}_N)\left(\frac{P_{T,t}}{P_t}\right)^{-\mu} C_t(r) \tag{12}$$

where aggregate price indices for domestic and foreign consumption bundles of traded goods, and for consumption of non-traded goods are given by, respectively,

$$P_{H,t} \;=\; \left[\frac{1}{n_H}\sum_{f=1}^{n_H} P_{H,t}(f)^{1-\zeta_T}\right]^{\frac{1}{1-\zeta_T}} \tag{13}$$

$$P_{F,t} \;=\; \left[\frac{1}{n_F}\sum_{f=1}^{n_F} P_{F,t}(f)^{1-\zeta_T}\right]^{\frac{1}{1-\zeta_T}} \tag{14}$$

$$P_{N,t} \;=\; \left[\frac{1}{n_N}\sum_{f=1}^{n_N} P_{N,t}(f)^{1-\zeta_N}\right]^{\frac{1}{1-\zeta_N}} \tag{15}$$

[8] The effect of home bias in open economies is also studied in Corsetti *et al.* (2002) and De Fiore and Liu (2002).

and the aggregate price indices $P_{T,t}$ and P_t are given by

$$P_{T,t} = U_{ROW,t}\left[\mathrm{w}(P_{H,t})^{1-\mu_T} + (1-\mathrm{w})(P_{F,t})^{1-\mu_T}\right]^{\frac{1}{1-\mu_T}} \quad (16)$$

$$P_t = \left[\mathrm{w}_N(P_{N,t})^{1-\mu} + (1-\mathrm{w}_N)(P_{T,t})^{1-\mu}\right]^{\frac{1}{1-\mu}} \quad (17)$$

where $U_{ROW,t}$ is a price shock arising from trade with the rest of the world. Aggregate nominal consumption is then given by

$$P_tC_t = P_{T,t}C_{T,t} + P_{N,t}C_{N,t} = P_{H,t}C_{H,t} + P_{F,t}C_{F,t} + P_{N,t}C_{N,t} \quad (18)$$

We now need to distinguish between the pricing decisions of PCP and LCP firms. Let a proportion θ of home firms export their goods as PCPs and the remaining proportion $1-\theta$ as LCPs. Similarly a proportion of foreign firms θ^* are PCPs and $1-\theta$ are LCPs. Let the average prices of these categories of firms are given respectively by

$$P^p_{F,t} = \left[\frac{1}{\theta^* n_F}\sum_{f=1}^{\theta^* n_F} P_{F,t}(f)^{1-\zeta_T}\right]^{\frac{1}{1-\zeta_T}} \quad (19)$$

$$P^\ell_{F,t} = \left[\frac{1}{(1-\theta^*)n_F}\sum_{f=1}^{(1-\theta^*)n_F} P_{F,t}(f)^{1-\zeta_T}\right]^{\frac{1}{1-\zeta_T}} \quad (20)$$

Then we have that

$$P_{F,t} = \left[\theta^*(P^p_{F,t})^{1-\zeta_T} + (1-\theta^*)(P^\ell_{F,t})^{1-\zeta_T}\right]^{\frac{1}{1-\zeta_T}} \quad (21)$$

The existence of a distributors of imports and local currency pricing means that the law of one price does not hold i.e. aggregate prices of traded goods in home and foreign blocs are linked by $\Phi_{H,t} = \frac{S_tP^*_{H,t}}{P_{H,t}} \neq 1$ and $\Phi_{F,t} = \frac{S_tP^*_{F,t}}{P_{F,t}} \neq 1$ necessarily, where $P^*_{H,t}$ and $P^*_{F,t}$ are the foreign currency prices of the home and foreign-produced goods and S_t is the nominal exchange rate. Let

$$P^*_{T,t} = U_{ROW,t} = \left[\mathrm{w}^*(P^*_{F,t})^{1-\mu^*} + (1-\mathrm{w}^*)(P^*_{H,t})^{1-\mu^*}\right]^{\frac{1}{1-\mu^*}} \quad (22)$$

be the foreign aggregate traded price index corresponding to (16). Then it follows that aggregate relative traded prices $\frac{S_tP^*_{T,t}}{P_{T,t}}$, the 'real exchange rate' for traded goods, and the terms of trade, defined as the domestic currency relative price of imports to exports, $\mathcal{T}_t = \frac{P_{F,t}}{P_{H,t}}$, are related by the relationship

$$RER_{T,t} \equiv \frac{S_tP^*_{T,t}}{P_{T,t}} = \frac{\left[\mathrm{w}^*(\Phi_{F,t}\mathcal{T})^{1-\mu^*} + (1-\mathrm{w}^*)\Phi_{H,t}^{1-\mu^*}\right]^{\frac{1}{1-\mu^*}}}{\left[\mathrm{w} + (1-\mathrm{w})\mathcal{T}_t^{1-\mu}\right]^{\frac{1}{1-\mu}}} \quad (23)$$

Thus if the law of one price holds for differentiated goods; i.e., $\Phi_{H,t} = \Phi_{F,t} = 1$, and $\mu = \mu^*$, then the law of one price applies to the aggregate traded price indices iff $\mathrm{w}^* = 1 - \mathrm{w}$. The latter condition holds if there is no home bias. If there is home bias, the real exchange rate depreciates ($\frac{S_t P^*_{T,t}}{P_{T,t}}$ rises) as the terms of trade improves.

For later use we require the the CPI real exchange rate $RER_t \equiv \frac{S_t P^*_t}{P_t}$. The two real exchange rates are related as follows. Let $\mathcal{N}_t = \frac{P_{N,t}}{P_{T,t}}$ be the relative price of non-traded to traded goods in the home bloc with an analogous definition of $\mathcal{N}^*_t$ for the foreign bloc. Then

$$RER_{T,t} = \frac{RER_t \frac{P^*_{T,t}}{P^*_t}}{\frac{P_{T,t}}{P_t}} = \frac{RER_t \left[\mathrm{w}_N \mathcal{N}_t^{1-\mu} + 1 - \mathrm{w}_N\right]^{\frac{1}{1-\mu}}}{\left[\mathrm{w}^*_N (\mathcal{N}^*_t)^{1-\mu^*} + 1 - \mathrm{w}^*_N\right]^{\frac{1}{1-\mu^*}}} \tag{24}$$

Now consider the consumption, money demand and labour supply decisions of the representative household. We first consider the case of flexible wages. Then maximizing (58) subject to (2) and (3), treating habit as exogenous, and imposing symmetry on households (so that $C_t(r) = C_t$, etc) yields standard results:

$$Q_{t,t+1} = \beta \frac{MU^C_{t+1}}{MU^C_t} \frac{P_t}{P_{t+1}} \tag{25}$$

$$MU^M_t = MU^C_t \left[\frac{I_t}{1+I_t}\right] \tag{26}$$

$$\frac{W_t(1-T_t)}{P_t} = -\frac{1}{\left(1-\frac{1}{\eta}\right)} \frac{MU^L_t}{MU^C_t} \equiv \frac{1}{\left(1-\frac{1}{\eta}\right)} MRS_t \tag{27}$$

where MU^C_t, MU^M_t and $-MU^L_t$ are the marginal utility of consumption, money holdings and the marginal disutility of work respectively. Taking expectations of (25) we arrive at the following familiar Keynes-Ramsey rule:

$$1 = \beta(1+I_t)E_t \left[\frac{MU^C_{t+1}}{MU^C_t} \frac{P_t}{P_{t+1}}\right] \tag{28}$$

In (26), the demand for money balances depends positively on consumption relative to habit and negatively on the nominal interest rate. Given the central bank's setting of the latter and ignoring seignorage in the government budget constraint, (26) is completely recursive to the rest of the system describing our macro-model and will be ignored in the rest of the paper. In (27) the real disposable wage is proportional to the marginal rate of substitution between consumption and leisure, $-\frac{MU^N_t}{MU^C_t}$, this constant of proportionality reflecting the market power of households that arises from their monopolistic supply of a differentiated factor input with elasticity η.

2.2 Domestic Producers

In the domestic goods non-traded and trade sectors, each good differentiated good f is produced by a single firm f using only differentiated labour with another constant returns CES technology:

$$Y_{i,t}(f) = A_t^i \left[\left(\frac{1}{\nu}\right)^{\frac{1}{\eta}} \sum_{r=1}^{\nu} L_{i,t}(f,r)^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)} \equiv A_{i,t} L_{i,t}(f) \; ; \; i = N, T \tag{29}$$

where $L_{i,t}(f,r)$ is the labour input of type r by firm f in sector i and $A_{i,t}$ is an exogenous shock capturing shifts to trend total factor productivity in this sector. Minimizing costs $\sum_{f=1}^{\nu} W_t(r) L_t(f,r)$ gives the demand for each household's labour by firm f as

$$L_{i,t}(f,r) = \left(\frac{W_t(r)}{W_t}\right)^{-\eta} L_{i,t}(f) \; ; \; i = N, T \tag{30}$$

and aggregating over firms leads to the demand for labor as shown in (3).[9] Per capita aggregate outputs in the home bloc is given by

$$Y_{i,t} = A_{i,t} L_{i,t} \; ; \; i = N, T \tag{31}$$

where $Y_{i,t}$ and $L_{i,t}$ are aggregated as for consumption aggregates $C_{N,t}(r)$ and $C_{H,t}(r)$ in (5) and (7), respectively.

In a equilibrium of equal households, all wages adjust to the same level W_t. For later analysis it is useful to define the real marginal cost (MC) as the wage relative to domestic producer price. Using (27) and (31) this can be written as

$$\mathrm{MC}_{T,t} \equiv \frac{W_t}{A_{T,t} P_{H,t}} = \frac{\eta U_{L,t}}{(\eta-1)(1-T_t) A_{T,t}} L_t^{\phi} (C_t - H_{C,t})^{\sigma} \left(\frac{P_t}{P_{H,t}}\right) \tag{32}$$

$$\mathrm{MC}_{N,t} \equiv \frac{W_t}{A_{N,t} P_{N,t}} = \frac{\eta U_{L,t}}{(\eta-1)(1-T_t) A_{N,t}} L_t^{\phi} (C_t - H_{C,t})^{\sigma} \left(\frac{P_t}{P_{N,t}}\right) \tag{33}$$

for the traded and non-traded sectors respectively.

Turning to price-setting in the *traded sector*, we assume that there is a probability of $1 - \xi_H$ at each period that the price of each good f is set optimally to $\hat{P}_{H,t}(f)$. If the price is not re-optimized, then it is indexed to last period's aggregate producer price inflation.[10] With indexation parameter $\gamma_H \geq 0$, this implies that successive prices with no

[9] Note that in a *symmetric equilibrium* of identical firms and households, total demand for labour of type r by firms in the traded sector is $L_{T,t}(r) = \sum_{f=1}^{n_H} L_{T,t}(f,r)$. Hence $L_{T,t} = \sum_{f=1}^{n_H} L_{T,t}(f) = \sum_{r=1}^{n_H} L_{T,t}(r)$, $n_H L_{T,t}(f) = \nu L_{T,t}(r)$. Similarly $n_N L_{N,t}(f) = \nu L_{N,t}(r)$. Such a symmetric equilibrium applies to the flexi-price case of our model, but *not* to the sticky-price case where, at each point in time, some firms are locked into price and wage contracts, but others are re-optimizing these contracts.

[10] Thus we can interpret $\frac{1}{1-\xi_H}$ as the average duration for which prices are left unchanged.

re-optimization are given by $\hat{P}_{H,t}(f)$, $\hat{P}_{H,t}(f)\left(\frac{P_{H,t}}{P_{H,t-1}}\right)^{\gamma_H}$, $\hat{P}_{H,t}(f)\left(\frac{P_{H,t+1}}{P_{H,t-1}}\right)^{\gamma_H}$, ... For each producer f the objective is at time t to choose $\hat{P}_{H,t}(f)$ to maximize discounted profits

$$E_t \sum_{k=0}^{\infty} \xi_H^k Q_{t,t+k} Y_{T,t+k}(f) \left[\hat{P}_{H,t}(f)\left(\frac{P_{H,t+k-1}}{P_{H,t-1}}\right)^{\gamma_H} - P_{H,t+k}\mathrm{MC}_{T,t+k}\right] \tag{34}$$

where $Q_{t,t+k}$ is the discount factor over the interval $[t, t+k]$, subject to a common[11] downward sloping demand from domestic consumers and foreign importers of elasticity ζ_T as in (10). The solution to this is

$$E_t \sum_{k=0}^{\infty} \xi_H^k Q_{t,t+k} Y_{T,t+k}(f) \left[\hat{P}_{H,t}(f)\left(\frac{P_{H,t+k-1}}{P_{H,t-1}}\right)^{\gamma_H} - \frac{\zeta_T}{(\zeta_T - 1)} P_{H,t+k}\mathrm{MC}_{T,t+k}\right] = 0 \tag{35}$$

and by the law of large numbers the evolution of the price index is given by

$$P_{H,t+1}^{1-\zeta_T} = \xi_H \left(P_{H,t}\left(\frac{P_{H,t}}{P_{H,t-1}}\right)^{\gamma_H}\right)^{1-\zeta_T} + (1-\xi_H)(\hat{P}_{H,t+1}(f))^{1-\zeta_T} \tag{36}$$

Similarly for the *non-traded sector* we have

$$E_t \sum_{k=0}^{\infty} \xi_H^k Q_{t,t+k} Y_{T,t+k}(f) \left[\hat{P}_{N,t}(f)\left(\frac{P_{N,t+k-1}}{P_{H,t-1}}\right)^{\gamma_N} - \frac{\zeta_N}{(\zeta_N - 1)} P_{N,t+k}\mathrm{MC}_{N,t+k}\right] = 0 \tag{37}$$

and

$$P_{N,t+1}^{1-\zeta_N} = \xi_N \left(P_{N,t}\left(\frac{P_{N,t}}{P_{N,t-1}}\right)^{\gamma_N}\right)^{1-\zeta_N} + (1-\xi_N)(\hat{P}_{N,t+1}(f))^{1-\zeta_N} \tag{38}$$

2.3 Exchange Rate Pass-Through

The home bloc consumer purchases imported goods either via distributors who import foreign differentiated goods for which the law of one price holds, or directly from the producer. The first range of varieties produced by the PCP foreign firms have aggregate price $P_{F,t}^p$ given by (19) and the second range produced by LCP firms $P_{F,t}^\ell$ given by (20).

2.3.1 PCP Importers

For good f imported by the home bloc from PCP foreign firms the price $P_{F,t}^p(f)$, set by retailers, is given by $P_{F,t}^p(f) = S_t P_{F,t}^*(f)$. Similarly $P_{H,t}^{*\,p}(f) = \frac{P_{H,t}(f)}{S_t}$.

[11]Note that we impose a symmetry condition $\zeta_T = \zeta_T^*$; i.e., the elasticity of substitution between differentiated goods produced in any one bloc is the same for consumers in both blocs.

2.3.2 LCP Exporters

Price setting in export markets by domestic LCP exporters follows is a very similar fashion to domestic pricing. Note that non-optimized prices are indexed to last period's aggregate imported price inflation in the LCP distribution sector. The optimal price in units of domestic currency is $\hat{P}^{\ell}_{H,t}S_t$, costs are as for domestically marketed goods so (35) and (36) become

$$E_t \sum_{k=0}^{\infty} \xi_H^k Q_{t,t+k} Y^*_{T,t+k}(f) \left[\hat{P}_{H,t}(f)^{*\,\ell} S_{t+k} \left(\frac{P^{*\,\ell}_{H,t+k-1}}{P^{*\,\ell}_{H,t-1}} \right)^{\gamma^{*\,\ell}_H} - \frac{\zeta_T}{(\zeta_T - 1)} P_{H,t+k} \mathrm{MC}_{T,t+k} \right] = 0 \tag{39}$$

and by the law of large numbers the evolution of the price index is given by

$$(P^{*\,\ell}_{H,t+1})^{1-\zeta_T} = \xi_H \left(P^{*\,\ell}_{H,t} \left(\frac{P^{*\,\ell}_{H,t}}{P^{*\,\ell}_{H,t-1}} \right)^{\gamma^{*\,\ell}_H} \right)^{1-\zeta_T} + (1-\xi_H)(\hat{P}^{*\,\ell}_{H,t+1}(f))^{1-\zeta_T} \tag{40}$$

Price setting of P^{ℓ}_F by foreign LCP exporters follows in an analogous way. Table 1 summarizes the notation used.

Origin of Good	Domestic Market	Export Market (PCP)	Export Market(LCP)
Home	P_H	$P^{*\,p}_H = \frac{P_H}{S_t}$	$P^{*\,\ell}_H \neq \frac{P_H}{S_t}$
Foreign	P^*_F	$P^p_F = S_t P^*_F$	$P^{\ell}_F \neq S_t P^*_F$

Table 1. Notation for Prices

2.4 Staggered Wage-Setting

We introduce wage stickiness in an analogous way. There is a probability $1 - \xi_W$ that the wage rate of a household of type r is set optimally at $\hat{W}_t(r)$. If the wage is not re-optimized then it is indexed to last period's CPI inflation. With a wage indexation parameter γ_W the wage rate trajectory with no re-optimization is given by $\hat{W}_t(r)$, $\hat{W}_t(r)\left(\frac{P_t}{P_{t-1}}\right)^{\gamma_W}$, $\hat{W}_t(r)\left(\frac{P_{t+1}}{P_{t-1}}\right)^{\gamma_W}, \cdots$. The household of type r at time t then chooses $W^0_t(r)$ to maximize

$$E_t \sum_{k=0}^{\infty} (\xi_w \beta)^k \left[\hat{W}_t(r)(1 - T_{t+k}) \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{\gamma_w} L_{t+k}(r)\Lambda_{t+k}(r) + L_{t+k}(r) MU^L_{t+k}(r) \right] \tag{41}$$

where $\Lambda_t(r) = \frac{MU_t^C(r)}{P_t}$ is the real marginal utility of consumption income and $L_t(r)$ is given by (3). The first-order condition for this problem is

$$E_t \sum_{k=0}^{\infty} (\xi_w \beta)^k \; W_{t+k}^{\eta} \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{-\gamma_w \eta} L_{t+k} \Lambda_{t+k}(r) \Big[\hat{W}_t(r)(1 - T_{t+k}) \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{\gamma_w} - \frac{1}{(1-\frac{1}{\eta})} P_{t+k} MRS_{t+k}(r) \Big] = 0 \tag{42}$$

Note that as $\xi_w \to 0$ and wages become perfectly flexible, only the first term in the summation in (41) counts and we then have the result (27) obtained previously. By analogy with (36), by the law of large numbers the evolution of the wage index is given by

$$W_{t+1}^{1-\eta} = \xi_w \left(W_t \left(\frac{P_t}{P_{t-1}} \right)^{\gamma_w} \right)^{1-\eta} + (1 - \xi_w)(\hat{W}_{t+1}(r))^{1-\eta} \tag{43}$$

2.5 The Equilibrium

In equilibrium, goods markets, money markets and the bond market all clear. Equating the supply and demand of the home consumer good and assuming that exogenous government expenditure goes exclusively on non-traded goods we obtain

$$Y_{T,t} = C_{H,t} + \frac{\nu^*}{\nu} C_{H,t}^* \tag{44}$$

$$Y_{N,t} = C_{N,t} + G_t \tag{45}$$

$$L_t = L_{T,t} + L_{N,t} \tag{46}$$

Fiscal policy is rudimentary: a balanced government budget constraint[12]

$$P_{N,t} G_t + TR_t = T_t (P_{H,t} C_{H,t} + \frac{\nu^*}{\nu} S_t P_{H,t}^* C_{H,t}^* + P_{N,t} Y_{N,t}) \equiv T_t \, GDP_t \tag{47}$$

where GDP_t is nominal GDP, completes the model. As in Coenen *et al.* (2005) we further assume that changes in government spending are financed exclusively by changes in lump-sum taxes with the tax rates T_t, held constant at its steady-state value.

Given nominal interest rates I_t, I_t^* the money supply is fixed by the central banks to accommodate money demand. By Walras' Law we can dispense with the bond market equilibrium condition. Then the equilibrium is defined at $t = 0$ as stochastic sequences C_t, C_{Ht}, C_{Ft}, $C_{N,t}$, P_{Ht}, $P_{N,t}$, P_{Ft}, P_t, M_t, W_t, $Y_{H,t}$, $Y_{N,t}$, L_t, $L_{N,t}$, $L_{T,t}$, P_{Ht}^0, 16 foreign

[12]In this cashless economy, we ignore seignorage and consistent with this we later ignore the utility from money balances in the household welfare function.

counterparts C_t^*, etc, RER_t, $RER_{T,t}$, $\mathcal{N}_t$ and $\mathcal{T}_t$, given past price indices and exogenous processes $U_{C,t}$, $U_{M,t}$, $U_{L,t}$, $A_{N,t}$, $A_{T,t}$, TR_t, G_t and foreign counterparts.

From (25) and its foreign counterpart we have

$$Q_{t,t+1} = \beta \frac{MU_{t+1}^C}{MU_t^C} \frac{P_t}{P_{t+1}} = \beta \frac{MU_{t+1}^{C*}}{MU_t^{C*}} \frac{P_t^* S_t}{P_{t+1}^* S_{t+1}} \tag{48}$$

Let $z_t = \frac{S_t P_t^*}{P_t} \frac{MU_t^C}{MU_t^{C*}}$. Then assuming identical holdings of initial wealth in the two blocs, (48) implies that $z_{t+1} = z_t = z_0$ where initial relative consumption in prices denominated in the home currency reflects different initial wealth in the two blocs. Therefore[13]

$$\frac{MU_t^C}{MU_t^{C*}} = \frac{z_0 P_t}{S_t P_t^*} \tag{49}$$

2.6 Financial Market Incompleteness

We now modify our model to allow for incomplete financial markets, to incorporate foreign debt dynamics without inducing non-stationarity. We assume there is no inter-bloc trade in state-contingent bonds so that risk-sharing between blocs no longer applies. There is however a full set of state-contingent bonds within each bloc so the marginal utilities of consumption are equated across households at all dates and states of nature in each bloc. Therefore we can assume a representative household for each bloc. Following Benigno (2001) there are two risk-free one-period bonds denominated in the currencies of each bloc, $B_{H,t}$ and $B_{F,t}$ respectively in aggregate. The prices of these bonds are given by

$$P_{B,t} = \frac{1}{1+I_t} \tag{50}$$

$$P_{B,t}^* = \frac{1}{(1+I_t^*)\phi(\frac{S_t B_{F,t}}{P_t})} \tag{51}$$

where $\phi(\cdot)$ captures the cost in the form of a risk premium for home households to hold foreign bonds. We assume $\phi(0) = 0$ and $\phi' < 0$.

For analytical convenience only the home households can hold foreign bonds. Then net foreign assets in the home bloc equals holdings of foreign assets, $B_{F,t}$. Assuming a

[13](49) is the risk-sharing condition for consumption, because it equates marginal rate of substitution to relative price, as would be obtained if utility were being jointly maximized by a social planner (see Sutherland (2002)). Note that (28) and (49) together imply the stochastic UIP condition (see Benigno and Benigno (2001)).

cashless economy, for the home bloc the household budget constraint for household r now becomes

$$\begin{aligned} P_t C_t(r) + P_{B,t} B_{H,t}(r) + P^*_{B,t} S_t B_{F,t}(r) &= (1 - T_t) W_t(r) L_t(r) + B_{H,t-1}(r) + S_{t-1} B_{F,t-1}(r) \\ &+ (1 - T_t)\Gamma_t(r) + TR_t \end{aligned} \tag{52}$$

Maximizing (58) subject to (52) and (3), treating habit as exogenous, and imposing symmetry on households as before gives the following first-order conditions for holdings of home and foreign bonds

$$P_{B,t} = \frac{\beta E_t \left[MU^C_{t+1} \frac{P_t}{P_{t+1}} \right]}{MU^C_t} = E_t[Q_{t,t+1}] \tag{53}$$

$$P^*_{B,t} = \frac{\beta E_t \left[MU^C_{t+1} \frac{S_{t+1} P_t}{S_t P_{t+1}} \right]}{MU^C_t} = E_t[Q_{t,t+1}] \tag{54}$$

which replaces (25). Dividing (53) by (54) gives the *modified risk-sharing condition*

$$\frac{P_{B,t}}{P^*_{B,t}} = \frac{E_t \left[MU^C_{t+1} \frac{P_t}{P_{t+1}} \right]}{E_t \left[MU^C_{t+1} \frac{S_{t+1} P_t}{S_t P_{t+1}} \right]} \tag{55}$$

Let $\sum_{r=1}^{\nu} B_{F,t}(r) = \nu B_{F,t}$ are the net holdings by the household sector of foreign bonds. Summing over the household budget constraints and subtracting (47), we arrive at the national resource identity describing the accumulation of net foreign assets

$$P^*_{B,t} S_t B_{F,t} = S_{t-1} B_{F,t-1} + W_t L_t + \Gamma_t - P_t C_t - P_{H,t} G_t \equiv S_{t-1} B_{F,t-1} + TB_t \tag{56}$$

where, noting that national income $W_t L_t + \Gamma_t = GDP_t$, $TB_t = GDP_t - P_t C_t - P_{N,t} G_t$ is the trade balance. For later use we can write the trade balance, nominal exports minus nominal imports, as

$$\nu TB_t = \nu^* S_t P^*_{H,t} C^*_{H,t} - \nu P_{F,t} C_{F,t} \tag{57}$$

2.7 Specialization of the Utility Function

In this paper we adopt a standard form of the utility function of the form

$$E_0 \sum_{t=0}^{\infty} \beta^t U_{C,t} \left[\frac{(C_t(r) - H_{C,t})^{1-\sigma}}{1 - \sigma} + U_{M,t} \frac{\left(\frac{M_t(r)}{P_t} \right)^{1-\varphi}}{1 - \varphi} - U_{L,t} \frac{L_t(r)^{1+\phi}}{1 + \phi} + u(G_t) \right] \tag{58}$$

where $u(G_t)$ is the utility from exogenous per capita government spending G_t.

Before proceeding it is informative at this point to discuss an alternative choice of utility function is non-separable in consumption, labour effort and money balances. The former feature allows the model to be consistent with the balanced growth path (henceforth BGP) set out in previous sections. As pointed out in Barro and Sala-i-Martin (2004), chapter 9, section 9.3, this requires a careful choice of the form of the utility as a function of consumption and labour effort. Again it is achieved by a utility function which is non-separable in these two arguments. A utility function that satisfies these requirements takes the form:

$$U \equiv \frac{U_{C,t}\left[\Phi(r)^{1-\varrho_t}(1-L_t(r))^{\varrho_t}\right]^{1-\sigma}}{1-\sigma} \tag{59}$$

where effort is measured as a proportion of a day, normalized at unity, and

$$\varrho_t \equiv \varrho + \varepsilon_{L,t} \tag{60}$$

$$\Phi_t(r) \equiv \left[aZ_t(r)^{\frac{\theta-1}{\theta}} + (1-a)U_{M,t}\left(\frac{M_t(r)}{P_t}\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{61}$$

$$Z_t(r) \equiv \left[b(C_t(r)-H_{C,t})^{\frac{\chi-1}{\chi}} + (1-b)G_t^{\frac{\chi-1}{\chi}}\right]^{\frac{\chi}{\chi-1}} \tag{62}$$

The utility function (59) has a number of notable features. First, $U_{C,M} > 0$ iff $\sigma\theta > 1$ in which case money holdings and consumption are *complements*. Second, $U_{\Phi L} > 0$ so that private and public consumption, and money holdings together, and leisure (equal to $(1 - L_t(r))$ are substitutes. Third, it leads to a non-zero Ramsey steady-state inflation rate. Finally, a BGP requires that the real wage and consumption grow at the same rate at the steady state. From (27) this requires that $\frac{MU_t^L}{C_t MU_t^C}$ is constant at the BGP growth steady state. The implications of this alternative form of utility function is left to further research.

2.8 The Steady State

A deterministic zero-inflation steady state, denoted by variables without the time subscripts, with $E_{t-1}(U_{C,t}) = 1$, $E_{t-1}(U_{L,t}) = \kappa$, a zero trade balance $TB = 0$ and zero net foreign assets is given by

$$C_H = \mathrm{w}\left(\frac{P_H}{P_T}\right)^{-\mu_T} C_T \tag{63}$$

$$C_F = (1-\mathrm{w})\left(\frac{P_F}{P}\right)^{-\mu_T} C_T \tag{64}$$

$$C_N = \mathrm{w}_N \left(\frac{P_N}{P}\right)^{-\mu} C \tag{65}$$

$$C_T = (1-\mathrm{w}_N)\left(\frac{P_T}{P}\right)^{-\mu} C \tag{66}$$

$$PC = P_T C_T + P_N C_N = P_H C_H + P_F C_F + P_N C_N \tag{67}$$

$$P = \left[\mathrm{w}_N P_H^{1-\mu} + (1-\mathrm{w}_N) P_T^{1-\mu}\right]^{\frac{1}{1-\mu}} \tag{68}$$

$$P_T = \left[\mathrm{w} P_H^{1-\mu_T} + (1-\mathrm{w}) P_F^{1-\mu_T}\right]^{\frac{1}{1-\mu_T}} \tag{69}$$

$$P_F = \left[\theta^* (P_F^p)^{1-\zeta_T} + (1-\theta^*)(P_F^\ell)^{1-\zeta_T}\right]^{\frac{1}{1-\zeta_T}} \tag{70}$$

$$\frac{W(1-T)}{P} = \frac{\kappa L^\phi ((1-h)C)^\sigma}{1-\frac{1}{\eta}} \tag{71}$$

$$1 = \beta(1+I) \tag{72}$$

$$Y_T = A_T L_T \tag{73}$$

$$Y_N = A_N L_N \tag{74}$$

$$L = L_T + L_N \tag{75}$$

$$P_N = \hat{P}_N = \frac{W}{A_N\left(1-\frac{1}{\zeta_N}\right)} \tag{76}$$

$$P_H = \hat{P}_H = \frac{W}{A_T\left(1-\frac{1}{\zeta_T}\right)} \tag{77}$$

$$P_F^p = \hat{P}_F^p = S P_F^* \tag{78}$$

$$P_F^\ell = S P_F^* \tag{79}$$

$$\Phi_F = \frac{S P_F^*}{P_F} = 1 \tag{80}$$

$$Y_T = C_H + \frac{\nu^*}{\nu} C_H^* \tag{81}$$

$$Y_N = C_N + G \tag{82}$$

$$T = \frac{P_H G + TR}{GDP} = \frac{P_H G + TR}{PC + P_N G + TB} \tag{83}$$

plus the 19 foreign counterparts and

$$\mathcal{N} = \frac{P_N}{P_T} \tag{84}$$

$$\mathcal{T} = \frac{P_F}{P_T} \tag{85}$$

$$RER = \frac{SP^*}{P} \tag{86}$$

$$
\begin{aligned}
RER_T &= \frac{\left[w^* \mathcal{T}^{1-\mu^*} + 1 - w^*\right]^{\frac{1}{1-\mu^*}}}{\left[w + (1-w)\mathcal{T}^{1-\mu}\right]^{\frac{1}{1-\mu}}} \qquad (87)\\
\nu TB &= \nu^* S P_H^* C_H^* - \nu P_F C_F = 0 \qquad (88)
\end{aligned}
$$

It should be noted that in the steady state the law of one price holds for each differentiated good and for aggregate traded prices. We now have gives 47 equations to determine the steady state of 49 endogenous variables: C, C_H, C_F, C_N, C_T, P, P_T, P_N, W, L_T, L_N, L, R, Y_T, Y_N, $P_H = \hat{P}_H$, $P_F = \hat{P}_F$, Φ_F, P_F^p, P_F^ℓ, T, 21 foreign counterparts C^* etc, $\mathcal{T}$, $\mathcal{N}$, S, RER_T and RER given G and TR.

To pin down price levels we need to re-introduce money equate money demand and its foreign counterpart with exogenously set money supplies in the two blocs, which then gives us a determinate steady state of the model. It is convenient to assume that money supplies in our steady state are set so as to result in $S = 1$ and dispense with the money demand equations. Furthermore, as is standard in general equilibrium models, we choose units of output appropriately so that prices of the two non-traded goods, and those of the traded goods in their own currencies are unity; i.e, $P_N = P_N^* = P_H = P_F^* = 1$. With this normalization and the fact that the law of one price holds in the steady state, we have that $P = P_F^p = P_F^\ell = P_F = P_T = \mathcal{N} = \mathcal{T} = RER_T = 1$. Similarly for the foreign bloc $P^* = P_H^{*\,p} = P_H^{*\,\ell} = P_H^* = P_T^* = 1$ and therefore $RER = 1$. Thus in *the steady state we can normalize all prices at unity*, an extremely convenient property when it comes to the linearization.[14]

2.9 Linearization and State-Space Representation

We now linearize around a baseline and, in general, asymmetric, steady state in which consumption, output, employment and prices in the two blocs are constant. Then inflation is zero. Output is then at its inefficient natural rate studied in the previous section and the nominal rate of interest is given by (72). Define all lower case variables as proportional deviations from this baseline steady state except for rates of change which are absolute deviations.[15]

[14] Note that with a retail sector introducing an extra LCP mark-up, as in Monacelli (2003), PPP and this convenient normalization of all prices no longer holds in the steady state (see Batini *et al.* (2005)).

[15] That is, for a typical variable X_t, $x_t = \frac{X_t - X}{X} \simeq \log\left(\frac{X_t}{X}\right)$ where X is the baseline steady state. For variables expressing a rate of change over time such as r_t and π_t, $x_t = X_t - X$. Levine *et al.* (2007a)

The whole model can now be written in state space form as

$$\begin{bmatrix} \mathsf{z}_{t+1} \\ E_t \mathsf{x}_{t+1} \end{bmatrix} = A \begin{bmatrix} \mathsf{z}_t \\ \mathsf{x}_t \end{bmatrix} + B\mathsf{o}_t + C \begin{bmatrix} i_t \\ i_t^* \end{bmatrix} + D\epsilon_{t+1} \tag{89}$$

$$F\mathsf{o}_\mathsf{t} = H \begin{bmatrix} \mathsf{z}_t \\ \mathsf{x}_t \end{bmatrix} \tag{90}$$

where z_t is a vector of predetermined variables and x_t is a vector of non-predetermined or 'jump' variables.

3 Calibration and Estimation

3.1 Econometric Strategy

Traditionally, DSGE models are calibrated such that certain theoretical moments given by the model match as closely as possible their empirical counterparts.[16] However, this method lacks formal statistical foundations (Kim and Pagan, 1994)) and makes testing the results difficult.[17]

Following Sargent (1989), and preceding the Bayesian literature, the common praxis was to estimate DSGE models with maximum-likelihood (ML). For instance Kim and Pagan (1994) analyzes the effects of taxation in an estimated business cycle model and Leeper and Sims (1994) and Kim (2000) estimated DSGE models for the analysis of monetary policy. Well known problems arising with this method are that parameters take on corner solutions or implausible values, and that the likelihood function may be flat in some dimensions. GMM estimation is a popular alternative for estimating intertemporal models (see (Galí and Gertler, 1999)) . However , Christiano and Haan (1996) has shown by estimating a business cycle model on U.S. data that GMM estimators often do not have the distributions implied by asymptotic theory. In addition, Lindé (2005) finds that parameters in a simple New Keynesian model are likely to be estimated imprecisely and with bias.

provide full details of this linearization.

[16]For an overview see Favero (2001).

[17]See, however, Canova and Ortega (2000) for a discussion on how testing in calibrated DSGE models could be conducted.

The Bayesian approach taken in this paper follows work by DeJong *et al.* (2000b,a), Otrok (2001), and Smets and Wouters (2003b). There are by now numerous applications of the approach, for example Adolfson *et al.* (2005), Adolfson *et al.* (2007), Justiniano and Preston (2004), Lubik and Schorfheide (2004) and Rabanal and Rubio-Ramírez (2005), and can be seen as a combination of likelihood methods and the calibration methodology. Bayesian analysis allows formally incorporating uncertainty and prior information regarding the parametrization of the model by combining the likelihood with a prior density for the parameters of interest. The moments of the prior density can be based on results from earlier microeconometric or macroeconometric studies, that is appropriate values could be employed as the means or modes of the prior density, while a priori uncertainty can be expressed by choosing the appropriate prior variance. For example, the restriction that AR(1)-coefficients lie within the unit interval can be implemented by choosing a prior density that covers only that interval, such as a truncated normal or a beta density. This strategy may help to mitigate numerical problems stemming e.g. from a flat likelihood function as estimates of the maximum likelihood are pulled towards values that the researcher would consider sensible a priori. This effect will be stronger when the data carry little information about a certain parameter, that is the likelihood is relatively flat whereas the effect will only be moderate when the likelihood is very peaked.

By Bayes' theorem, the posterior density $\varphi(\xi \mid Y)$ is related to prior and likelihood as follows

$$\varphi(\xi \mid Y) = \frac{f(Y \mid \xi)\pi(\xi)}{f(Y)} \propto f(Y \mid \xi)\pi(\xi) = L(\xi \mid Y)\pi(\xi),$$

where $\pi(\xi)$ denotes the prior density of the parameter vector ξ, $L(\xi \mid Y) \equiv f(Y \mid \xi)$ is the likelihood of the sample Y and $f(Y) = \int f(Y \mid \xi)\pi(\xi)d\xi$ is the unconditional sample density. The unconditional sample density does not depend on the unknown parameters and consequently serves only as a proportionality factor that can be neglected for estimation purposes. In this context it becomes clear that the main difference between 'classical' and Bayesian statistics is a matter of conditioning. Likelihood-based non-Bayesian methods condition on the unknown parameters ξ and compare $f(Y \mid \xi)$ with the observed data. Bayesian methods condition on the observed data and use the full distribution $f(\xi, Y) = f(Y \mid \xi)\pi(\xi)$ and require specification of a prior density $\pi(\xi)$.

Computation of the posterior distribution $\varphi(\xi \mid Y)$ requires calculating the likelihood and then multiplying by the prior density. The likelihood function can be computed with the Kalman filter using the state-space representation of the solution to the rational expectations model.

We proceed in two steps: Parameters that are not identified or difficult to estimate are calibrated based on earlier studies, evidence from micro data or where applicable on sample averages. The other model parameters are estimated using the Bayesian approach. following, by now, well-known work of DeJong *et al.* (2000a,b), Otrok (2001), Smets and Wouters (2003c, 2004), and in particular for the open economy by Adolfson *et al.* (2004).[18] This allows us to express our subjective believes about these parameters in a statistical coherent way and update them with the data used. Further, due to the complexity of the model we first estimate a version of the model where all goods are traded thus eliciting a prior for the full model with traded and non-traded goods.

We estimate these models on a set of 15 time series: real GDP, real consumption expenditure, hours worked, the GDP-deflator, consumer prices, nominal wages, nominal interest rates and the euro-dollar exchange rate, where we take the euro area as the home country and express the exchange rate in euros per dollar, so that a rising exchange rate implies a depreciation of the euro. The real variables are expressed in per capita terms. We use data from 1980q1 to 2005q4, where the first four years are used to initialize the state of the Kalman filter. For the US the data stems from the Federal Reserve Bank of St. Louis database (FRED), worked hours and hourly compensation have been retrieved from the Bureau of Labour Statistics. For the euro area the data is taken from the ECB database first compiled by Fagan *et al.* (2001). The exchange rate is obtained from EcoWinPro.

For the euro area there is no long time series on worked hours available. We use employment instead and add the following measurement equation to the system

$$empl_t = empl_{t-1} + E_t empl_{t+1} - empl_t + \frac{(1-\beta\xi_e)(1-\xi_e)}{\xi_e}(n_t - empl_t),$$

where *empl* denotes employment. The idea is that employment reacts more sluggishly in response to macroeconomic shocks than hours worked

The data is pre-filtered such that we remove means and linear trends in order to obtain stationary series. However, the exchange rate is measured in first differences.

[18]See also Justiniano and Preston (2004) and Rabanal and Tuesta (2006).

3.2 Calibrated Parameters

The set of parameters is split into a set of calibrated parameters that are difficult to estimate because they are linked to steady state conditions or because we think that we have very good a priori information about them, e.g. the Calvo contract lengths. The other set contains estimated parameters mainly pertaining to model dynamics and stochastic properties.

We discuss the set of calibrated parameters in turn. The discount factor is assumed to be equal across blocs and set to 0.99. We calibrate the relative size of the blocs according to average relative population size in the euro area and U.S over the years 2001 to 2006. This implies that EMU makes up about 51% of the population in both blocs. The substitution elasticity between different kinds of labour is set to 3 for both blocs. The import shares of traded consumption goods are set to 0.10 in the euro area, taken from the new area wide model and 0.09 for the US, taken from the Federal Reserve Board model, SIGMA . The share of labour traded used for production of non-traded goods is set to 0.34 in the euro area and 0.28 in the U.S.-bloc. Further we assume that 80% of all goods, be they traded or non-traded, are going to consumption.

As regards price setting we choose to set all Calvo contract lengths to four quarters but estimate the degree of indexation. However due to possible identification problems we assume that all firms, i.e. PCP- and LCP-firms, apply the same degree of indexation in each bloc. We also assume that price setters in each bloc are hit by the same markup shocks.

Consumption and labour supply elasticities are calibrated as well and set $\sigma = \sigma^* = \varphi = \varphi^* = 2$. The substitution elasticities between traded and non-traded goods are calibrated to 1.5 and the substitution elasticities between home and foreign traded goods are set to 2 in each bloc respectively. Finally, as the first difference of the real exchange rate turned out to be very persistent we calibrated this value to 0.99.

3.3 Priors

We have 50 remaining parameters to estimate and assume a priori that the parameters are independent of each other and apply the following general convention. For all parameters that should lie in the unit interval a beta density is chosen as a prior. For the standard deviations of the innovations we choose fairly uninformative inverted gamma densities and for the remaining parameters we formulate our subjective beliefs about location and uncertainty in terms of normal densities. For a full overview see the column ´prior' in Tables 1 to 3 below.

3.4 Results

The results in Tables 1 to 3 are obtained from maximizing the posterior mode as well as calculating the mean from 500 000 Metropolis-Hastings simulations to approximate the posterior density. Overall differences between the two blocs are not big, though there are notable differences. We note that price indexation for traded goods does not play an important role whereas it is more important for price indexation for non-traded goods with a much higher value in the euro area. This is also the case for wage indexation in the euro areas vs the U.S. Across both blocs it turns out that the fraction of PCP-price setters is very small, only about 1% of the firms set prices in the producer currency.

The notable differences between the two blocs are as follows: consumption habit in the US is far higher than for EMU pointing to a higher degree of output persistence in that bloc. By contrast indexation in the non-traded sector and for wages is far higher EMU indicating a higher degree of non-traded goods inflation and wages in that bloc. All shocks are very persistent with small differences between blocs. The most important supply-side shocks in terms of the standard deviation of the white noise components are for labour supply, non-traded goods technology, and the mark-up in the non-traded sector. For all these shocks EMU is more volatile than the US. Most of the other parameters estimates are relatively similar across blocs. *Taken as a whole the differences between the two blocs suggest a stronger role for stabilization in EMU than the US.*

Structural Parameters		Prior			Posterior			
Parameter		Type	Mean	Std	Mode	Mean	5/95% Interval	
Calvo employment EMU	ξ_e	beta	0.750	0.100	0.727	0.735	0.697	0.770
Indexation PCP/LCP EMU	γ_H	beta	0.500	0.150	0.047	0.058	0.023	0.107
Indexation PCP/LCP US	$\gamma_H^{\ell *}$	beta	0.500	0.150	0.095	0.113	0.047	0.204
Indexation wage setters EMU	γ_W	beta	0.500	0.150	0.529	0.402	0.208	0.600
Indexation wage setters US	γ_W^*	beta	0.500	0.150	0.267	0.373	0.181	0.583
Indexation nontraded goods EMU	γ_N	beta	0.500	0.150	0.739	0.615	0.361	0.821
Indexation nontraded goods US	γ_N^*	beta	0.500	0.150	0.305	0.360	0.164	0.586
Habit EMU	h	beta	0.500	0.150	0.455	0.475	0.372	0.579
Habit US	h^*	beta	0.500	0.150	0.670	0.657	0.551	0.766
PCP-fraction EMU	θ	beta	0.500	0.200	0.012	0.018	0.004	0.039
PCP-fraction US	θ^*	beta	0.500	0.200	0.014	0.024	0.006	0.050
Taylor rule inflation EMU	f_π	norm	1.500	0.200	1.682	1.517	1.235	1.839
Taylor rule output EMU	f_y	norm	0.500	0.200	0.518	0.229	0.043	0.560
Taylor rule interest rate EMU	f_r	beta	0.700	0.200	0.842	0.871	0.836	0.900
Taylor rule inflation US	f_π^*	norm	1.500	0.200	1.201	1.495	1.225	1.772
Taylor rule output US	f_y^*	norm	0.500	0.200	0.488	0.525	0.359	0.731
Taylor rule lagged interest rate US	f_r^*	beta	0.700	0.200	0.819	0.814	0.756	0.866

Table 1. Structural Parameters

Shock processes - Persistence		Prior			Posterior			
Parameter		Type	Mean	Std	Mode	Mean	5/95% Interval	
Cons. preference EMU	ρ_c	beta	0.850	0.100	0.880	0.900	0.849	0.938
Cons. preference US	ρ_c^*	beta	0.850	0.100	0.843	0.842	0.734	0.915
Labour supply EMU	ρ_L	beta	0.850	0.100	0.952	0.950	0.923	0.976
Labour supply US	ρ_L^*	beta	0.850	0.050	0.928	0.857	0.691	0.956
Techn. traded goods EMU	ρ_{a_T}	beta	0.850	0.100	0.909	0.914	0.854	0.992
Techn. traded goods US	$\rho_{a_T}^*$	beta	0.850	0.100	0.977	0.976	0.955	0.993
Techn. nontraded goods EMU	ρ_{a_N}	beta	0.850	0.100	0.997	0.989	0.973	0.998
Techn. nontraded goods US	$\rho_{a_N}^*$	beta	0.850	0.100	0.937	0.935	0.892	0.974
Government expenditure EMU	ρ_g	beta	0.850	0.100	0.940	0.945	0.910	0.974
Government expenditure US	ρ_g^*	beta	0.850	0.100	0.980	0.978	0.957	0.994
ROW oil	ρ_{ROW}	beta	0.850	0.100	0.837	0.870	0.768	0.949

Table 2. Persistence of Shocks

Parameter		Prior Type	Prior Mean	Prior Dof	Posterior Mode	Posterior Mean	Posterior Interval	
Cons pref EMU	ϵ_C	invg	0.670	4.000	0.421	0.434	0.354	0.527
Cons pref US	ϵ_C^*	invg	0.670	4.000	0.457	0.475	0.384	0.586
Labour pref EMU	ϵ_L	invg	3.000	4.000	2.325	2.530	1.848	3.341
Labour pref US	ϵ_L^*	invg	3.000	4.000	1.425	1.756	1.035	2.765
Tech. traded goods EMU	ϵ_{a_T}	invg	0.600	4.000	0.256	0.317	0.175	0.552
Tech. traded goods US	$\epsilon_{a_T}^*$	invg	0.600	4.000	0.329	0.486	0.207	0.914
Tech. nontraded goods EMU	ϵ_{a_N}	invg	0.600	4.000	0.840	0.928	0.749	1.133
Tech. nontraded goods US	$\epsilon_{a_N}^*$	invg	0.600	4.000	0.660	0.653	0.557	0.750
Gov exp EMU	ϵ_G	invg	2.000	4.000	2.388	2.511	2.007	3.014
Gov exp US	ϵ_G^*	invg	2.000	4.000	4.095	4.180	3.664	4.790
Oil shock ROW	ϵ_{ROW}	invg	0.300	4.000	0.278	0.268	0.201	0.334
Wage markup EMU	ϵ_W	invg	0.500	4.000	0.122	0.141	0.119	0.164
Wage markup US	ϵ_W^*	invg	0.500	4.000	0.232	0.235	0.207	0.267
Risk premium	ϵ_E	invg	1.400	4.000	0.368	0.381	0.332	0.434
Monetary policy EMU	ϵ_R	invg	0.200	4.000	0.092	0.166	0.064	0.432
Monetary policy US	ϵ_R^*	invg	0.200	4.000	0.092	0.178	0.066	0.459
Markup traded goods EMU	ϵ_{π_H}	invg	0.500	4.000	0.180	0.178	0.139	0.220
Markup traded goods US	$\epsilon_{\pi_F}^*$	invg	2.000	4.000	0.321	0.306	0.263	0.354
Markup nontraded goods EMU	ϵ_{π_N}	invg	0.500	4.000	0.398	0.416	0.360	0.482
Markup nontraded goods US	$\epsilon_{\pi_N}^*$	invg	0.500	4.000	0.264	0.276	0.160	0.414
Correlation between techn.	$\sigma_{a_T a_T^*}$	norm	0.740	0.200	0.707	0.691	0.440	0.935

Table 3. Standard Deviations of Shocks

4 LQ Approximation and Equilibrium Concepts

We focus exclusively on monetary policy where the monetary authorities can commit.[19] We consider one or two Ramsey planners, for the cooperative and non-cooperative problems respectively, choosing monetary instruments to maximize household welfare in an environment consisting of a decentralized economy with possibly large distortions in the

[19]Levine and Pearlman (2007) consider further discretionary equilibria where commitment to the private sector is not possible.

zero-inflation steady state. As shown in Levine *et al.* (2007b), the procedure for achieving an accurate LQ approximation for each optimization problem is as follows[20]:

1. Define the optimization problem for the Ramsey planner. For the cooperation this is a standard problem. For non-cooperative games we need to define the appropriate equilibrium concept. Our ultimate aim is to obtain an accurate quadratic approximation of welfare for the state-space representation of the game, (89) and (90). Since interest-rates are given in this representation, we choose an open-loop Nash equilibrium in interest-rate paths for the purposes of the approximation.

2. Set out the deterministic non-linear form of each Ramsey problem, to maximize the representative agents utility subject to non-linear dynamic constraints.

3. Write down the single Lagrangian for the cooperative problem, and the Lagrangians for the two blocs for the non-cooperative problem. For the cooperative problem it is assumed that the single Ramsey planner maximizes a weighted sum of the expected utilities of the representative households in the two blocs using population ratios as the weights. Associated with each Lagrangian is a Hamiltonian consisting of the utility and a sum of *all* appropriately expressed constraints for the decentralized economy time multipliers.

4. Calculate the first order conditions. We do not require the initial conditions for an optimum since we ultimately only need the steady-state about which we are approximating.

5. Calculate the steady state of the first-order conditions. The terminal condition implied by this procedure is that the system converges to this steady state.

6. Calculate a second-order Taylor series approximation, about the steady state, of the Hamiltonian associated with the Lagrangian or Lagrangians in 3.

7. Calculate a first-order Taylor series approximation, about the steady state, of the first-order conditions and the original constraints.

[20]MATLAB software to implement this procedure is in preparation and will be available on request from the authors.

8. Use 5. to eliminate the steady-state Lagrangian multipliers in 6. By appropriate elimination both the Hamiltonian and the constraints can be expressed in minimal form.

To be more specific, let us consider the general deterministic dynamic programming problem for an individual policymaker:

$$\max \sum_{t=0}^{\infty} \beta^t [U(X_{t-1}, W_t) \; s.t. \; X_t = f(X_{t-1}, W_t) \tag{91}$$

where X_t and W_t are vectors of state vector and instrument respectively, which has a Lagrangian $\mathcal{L}$ given by

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t [U(X_{t-1}, W_t) - \lambda_t^T (X_t - f(X_{t-1}, W_t))] \tag{92}$$

If the solution to this problem tends to a steady state, it is easy to show that this steady state $\{\bar{X}, \bar{W}, \bar{\lambda}\}$ satisfies the steady-state first-order conditions:

$$U_W + \lambda_t^T f_W = 0 \qquad U_X - \frac{1}{\beta}\bar{\lambda}^T + \bar{\lambda}^T f_X = 0 \tag{93}$$

If we now expand (92) about the steady state, then all first-order terms are zero, while the second order terms are given by

$$\Delta\mathcal{L} = \sum_{t=0}^{\infty} \beta^t [\nabla^2 H - \Delta\lambda_t^T (\Delta X_t - \Delta f(X_{t-1}, W_t))] \tag{94}$$

where $\nabla^2 H$ is the second-order expansion of $H = U + \bar{\lambda}^T f$. This corresponds to the problem

$$max \sum_{t=0}^{\infty} \beta^t \nabla^2 H \; s.t. \; \Delta X_t = \Delta f(X_{t-1}, W_t) \tag{95}$$

which is what we think of as the LQ approximation.

In fact, the set of nonlinear constraints are slightly more complicated than this, in that the lead terms may be a function of several lead terms, and there are also static relationship. Thus for example, the nonlinear equations that define the domestic inflation rate $\pi_{H,t}$ are given by

$$H_t = \beta\xi_{H,t}\Pi_{H,t+1,t}^{\zeta-1}\Pi_{H,t}^{\gamma_H(1-\zeta)} H_{t+1,t} + (C_{H,t} + C_{H,t}^{p*})e^{UC_t}(C_t - h_C C_{t-1})^{-\sigma}(P_{H,t}/P_t) \tag{96}$$

$$\Lambda_t = \beta\xi_{H,t}\Pi_{H,t+1,t}^{\zeta}\Pi_{H,t}^{-\gamma_H\zeta}\Lambda_{t+1,t} + (C_{H,t} + C_{H,t}^{p*})e^{UC_t}(C_t - h_C C_{t-1})^{-\sigma}\frac{WR_t e^{-A_t}}{(1-1/\zeta)} \tag{97}$$

$$Q_t H_t = \Lambda_t \qquad 1 = \xi_H \Pi_{H,t}^{\zeta-1} \Pi_{H,t-1}^{\gamma_H(1-\zeta)} + (1-\xi_H) Q_t^{1-\zeta} \tag{98}$$

where $\Pi_{H,t}$ is the gross inflation rate, and H_t, Λ_t, Q_t are defining variables that are eliminated from the linear approximation. However, the basic approach outlined above is essentially unchanged other than to include both the static equations and terms like $\beta\xi_{H,t}\Pi_{H,t+1,t}^{\zeta-1}\Pi_{H,t}^{\gamma_H(1-\zeta)}H_{t+1,t}$ in the Hamiltonian.

This then gives us the accurate LQ approximation of the original non-linear optimization problem in the form of a minimal linear state-space representation of the constraints and a quadratic form of the utility expressed in terms of the states. The quadratic form of the utility function obtained for the cooperative Ramsey planners is then appropriate for cooperative LQ games irrespective of the monetary instrument, although we use interest rates as the instrument. For the non-cooperative problem, where there is more than one policymaker, we obtain an analogous LQ approximation; in this case, each policymaker has its own set of steady state Lagrange multipliers, so the the quadratic approximations to the utility functions differ not merely due to the differing objectives of the two policymakers but also because of the different weights on the constraints. These quadratic approximations obtained for the non-cooperative Ramsey planners are then appropriate for for non-cooperative LQ games.

For the non-cooperative problem, for the home and foreign blocs we then arrive at the approximations

$$\Omega_0 = \frac{1}{2} E_t \sum_{t=0}^{\infty} \beta^t [\mathsf{y}_t' Q \mathsf{y}_t] \tag{99}$$

$$\Omega_0^* = \frac{1}{2} E_t \sum_{t=0}^{\infty} \beta^t [\mathsf{y}_t' Q^* \mathsf{y}_t] \tag{100}$$

whilst for the cooperative problem we have

$$\Omega_0^C = \frac{1}{2} E_t \sum_{t=0}^{\infty} \beta^t [\mathsf{y}_t' Q^C \mathsf{y}_t] \tag{101}$$

Letting the population weights be $[\omega, 1-\omega]$, it is important to note that in general the cooperative loss function is *not* a simple linear sum of the non-cooperative ones as is commonly assumed in the literature; i.e.,

$$\Omega_0^C \neq \omega\Omega_0 + (1-\omega)\Omega_0^* \tag{102}$$

To get some idea about the objectives of the policymakers from the perspective of the LQ approximation, we can examine the elements of the square matrix that multiplies percentage deviations of the variables from their steady-state values. Given that this is a 49×49 matrix, it is more useful to gain some idea of the policy tradeoffs by listing the largest diagonal elements; we present these as positive numbers so as to represent welfare losses. Since EMU and the US have approximately equal populations, we put $\omega = 0.5$.

Weight on	Coop	Non-Coop Euro	Non-Coop US
c	4	8	-1
c^*	14	-3	30
π	156	249	24
π^*	184	36	306
π_H	11	21	2
π_F^*	16	3	28
π_N	33	74	-4
π_N^*	61	5	112
$wr - wr_{-1}$	145	256	34
$wr^* - wr_{-1}^*$	184	43	326
π_F	24	29	98
π_H^*	-63	26	16
π_F^p	17	22	23
π_H^{p*}	-47	19	12

Table 4. Largest weights on squares of variables under cooperation, and for each bloc under non-cooperation

As can seen from this table, the weights on consumption deviations are easily outweighed by those on inflation, and particularly by CPI inflation and real wage inflation. Also note in this table that the weights under cooperation are not equal to the average of those under non-cooperation as noted in (102). Given the heavy weighting of inflation and real wage inflation, one would expect that an interest-rate rule based on one or both of these is likely to be very effective at stabilizing welfare.

We can now define the three LQ games and equilibria used in the rest of the paper:

Optimal Policy: Cooperation with Commitment: OPTCC

A single Ramsey planner maximizes Ω_0^C given by (101) with respect to $\{i_t\}, \{i_t^*\}$ subject to the state-space representation (89) and (90).

Simple Rule: Cooperation with Commitment: SIMCC

A single Ramsey planner maximizes Ω_0^C given by (101) subject to simple feedback constraints $i_t = Dy_t$ and $i_t^* = D^* y_t$ and to the state-space representation (89) and (90), with respect to D, D^*.

Simple Rule: Non-Cooperation with Commitment: SIMNC

The home Ramsey planner maximizes Ω_0 given by (99) subject to simple feedback constraints $i_t = Dy_t$ and to the state-space representation (89) and (90), with respect to D, given $i_t^* = D^* y_t$. In a closed-loop Nash equilibrium the foreign Ramsey planner chooses D^* in an analogous fashion.

Details of these equilibria are provided in Appendix A.

5 The Zero Lower Bound Constraint

Following Woodford (2003), chapter 6, Levine *et al.* (2007c) and Levine *et al.* (2008), we can impose an effect that approximates the interest-rate ZLB constraint by modifying the LQ optimization problems. For the non-cooperative game, this is implemented as a constraint on the *variance* of the interest rate by modifying the home and foreign blocs welfare loss functions to, respectively

$$\Omega_0 = \frac{1}{2} E_t \sum_{t=0}^{\infty} \beta^t [\mathrm{y}_t' Q \mathrm{y}_t + w_i i_t^2] \tag{103}$$

$$\Omega_t^* = \frac{1}{2} E_t \sum_{t=0}^{\infty} \beta^t [\mathrm{y}_t' Q^* \mathrm{y}_t + w_i^* {i_t^*}^2] \tag{104}$$

For the cooperative game the loss function is modified to

$$\Omega_0^C = \frac{1}{2} E_t \sum_{t=0}^{\infty} \beta^t [\mathrm{y}_t' Q^C \mathrm{y}_t + w_i i_t^2 + w_i^* {i_t^*}^2] \tag{105}$$

As explained in Levine *et al.* (2008), for the non-cooperative game, the home optimization problem is to choose an unconditional distribution for i_t (i.e., at the steady-state, such that the probability, p, of the interest rate hitting the lower bound is very low. This is implemented by calibrating the weight w_i so that $z_0(p)\sigma_i < I$ where $z_0(p)$ is the critical value of a standard normally distributed variable Z such that prob $(Z \leq z_0) = p$, $I = \frac{1}{\beta} - 1 + \bar{\pi}$ is the zero-inflation, steady-state nominal interest rate and σ_i is the unconditional variance. An analogous choice of w_i^* applies to the foreign bloc. For the cooperative game the single Ramsey planner chooses (w_i, w_i^*) so that the ZLB constraint is satisfied in both blocs, though it may only *bind* in one bloc.[21]

The stages of the interest-rate rule cooperative and non-cooperative games with a ZLB constraint are as follows:

1. For the cooperative rules (w_i, w_i^*) are chosen jointly so that the the probability of hitting the ZLB is p or less in both blocs. For the non-cooperative rules, EMU chooses w_i given w_i^* such that the probability of hitting the ZLB is p or less. EU similarly chooses w_i^* given w_i. The intersection of the reaction functions $w_i = f(w_i^*)$ and $w_i^* = f(w_i)$ is the Nash equilibrium in interest-rate penalties at stage 1 of the game.

2. Now given (w_i, w_i^*), for the cooperative rules the two blocs jointly choose rules to minimize a joint welfare loss function (that incorporates the ZLB constraints). For non-cooperative rules, EMU choose an welfare-optimum feedback interest-rate rule to minimize its welfare loss (that again incorporates its own ZLB constraint) given the rule in the US. The US acts in an analogous way resulting in a closed-loop Nash equilibrium at stage 2 of the game.

3. Given the rules designed at stage 2 both countries responds to shocks in accordance with these rules.

[21] The ZLB constraint can be further eased by shifting the interest rate distribution to the right. Then steady state inflation rate in the optimal policy is positive. Let $\bar{\pi} > 0$ be this rate. Then $I = \frac{1}{\beta} - 1 + \pi^*$ is the steady state nominal interest rate. Given σ_r the steady state positive inflation rate that will ensure $r_t \geq 0$ with probability $1 - p$ is given by $\bar{\pi} = \max[z_0(p)\sigma_i - \left(\frac{1}{\beta} - 1\right) \times 100, 0]$. Furthermore if π^* is chosen in a optimal fashion, it is a credible new steady state inflation rate. (See Levine *et al.* (2007c)). In this paper however we retain zero inflation as a steady state feature of the policy rules.

6 Coordination Gains without Model Uncertainty

6.1 Results with no ZLB Constraint

First let us ignore the ZLB constraint. Table 5 presents results for this case. We consider three forms of simple commitment rules. The first feed back from current CPI inflation in each bloc and take the form

$$i_t = \rho i_{t-1} + \theta_\pi \pi_t \tag{106}$$

$$i_t^* = \rho^* i_{t-1}^* + \theta_\pi^* \pi_t^* \tag{107}$$

and the second includes a feedback from wage inflation:

$$i_t = \rho i_{t-1} + \theta_\pi \pi_t + \theta_{\Delta w} \Delta w_t \tag{108}$$

$$i_t^* = \rho^* i_{t-1}^* + \theta_\pi^* \pi_t^* + \theta_{\Delta w}^* \Delta w_t^* \tag{109}$$

Finally we consider a rule close to the standard inflation - output gap rule. Since there are two sectors, traded and non-traded goods, and two outputs we use an employment gap, the difference between employment with sticky prices and imperfect financial markets and that with flexible prices and perfect financial markets, as the target variable. Denoting this by egap_t the rule takes the form

$$i_t = \rho i_{t-1} + \theta_\pi \pi_t + \theta_{\mathrm{egap}} \mathrm{egap}_t \tag{110}$$

$$i_t^* = \rho^* i_{t-1}^* + \theta_\pi^* \pi_t^* + \theta_{egap}^* \mathrm{egap}_t^* \tag{111}$$

As before we denote by SIMCC the coordinated optimized simple rule of this type whilst SIMNC denotes the corresponding closed-loop Nash game in interest-rate rules between the countries. We compare the outcomes of these equilibria with the optimal coordination and commitment rule, OPTCC. c_e is the percentage consumption permanent equivalent loss in the US from sub-optimal rules compared with OPT. Throughout the paper, we adopt a *conditional* welfare loss measure, starting at the zero-inflation steady state (see Appendix A). Let $(\Omega_0 + \Omega_0^*)^i$, i = SIMCC, SIMNC, OPT be the expected welfare loss under these two optimized and optimal rule respectively. Then we have $c_e = \frac{\Omega_0 + \Omega_0^* - (\Omega_0 + \Omega_0^*)^{CC}}{k} \times 10^{-2}$ where $k = (C(1-h))^{1-\sigma} \simeq 2$ for central parameter values.

In table 5 we first consider the interest volatilities across the various equilibria which are measures of the degree of monetary policy activism required to minimize the expected welfare loss function. We denote by $\text{var}(i_t)$ the steady-state conditional variance of the nominal interest rate. For OPTCC these are very high and higher in EMU than in the US. This is as one would expect from the estimated parameter differences noted in section 3. For such values there is a large probability per period of violating the ZLB indicated in the final column of 0.31 and 0.25 in EMU and the US respectively.[22]

For equilibrium SIMCC there is very little difference in outcomes across our three forms of simple feedback rule. Moreover the interest-rate variances are far lower indicating that being restricted to such rules considerably reduces the welfare gains from stabilization by the use of the nominal interest rate. The cost of simplicity is significant, around 0.6% in consumption equivalent terms. *Violations of the ZLB are not serious for cooperative simple rules, but the rules are severely sub-optimal.* Once we turn to non-cooperative simple rules however this feature changes remarkably. Now countries have a incentive to manipulate the terms of trade in their favour in a direction dependent on the shocks hitting their economies. Each country then designs a more active rule that constitutes a closed-loop Nash equilibrium in feedback rules but has a beggar-thy-neighbour character. As a consequence *the gains from cooperation are high* varying from 0.53% for the CPI infaltion rule to 0.78 for the rule that also responds to wage inflation.

6.2 Imposing the ZLB Constraint

The results obtained without ZLB considerations follow most of the coordination gains literature. In our set-up there are two sources of gains from coordinating monetary policy. First, as we have seen in section 4, the approximate quadratic loss function are different for the cooperative and non-cooperative games and the former is not a simple linear combination of the latter. Second there is the familiar terms of trade externality that encourages beggar-thy-neighbour policies. From the work of Canzoneri *et al.* (2005), Liu and Pappa (2005) and others we know that a non-traded sector add a relative price of trade to non-traded goods effect that can magnify the unilateral benefits of terms of trade changes. We then appear to confirm the literature that suggests the coordination gains can be quite large in richer models with these features.

[22]Note that we needed to choose $w_i = w_i^* = 1.6$ in OPTCC to obtain a solution. For simple rules there are no computational problems with putting $w_i = w_i^* = 0$.

However this conclusion is premature because our non-cooperative rules involve a severe violation of the ZLB constraint and are therefore not operational. As described in section 5, we now modify the policy rules to incorporate a ZLB constrain with a low probability per period of $p = 0.02$ or less of hitting the ZLB. Table 6 shows the results for same policy rules as in Table 5. Three features of the results are particularly notable. First, the costs of simplicity are much smaller with the ZLB constraint imposed. Second, owing to the asymmetries in the estimated model in a number of cases we have equilibria where the ZLB constraint only binds in the EMU. Third, and most importantly, the gains from policy coordination are much smaller with ZLB considerations, down to consumption equivalent gains of 0.03, 0.01 and 0.07 for the CPI, wage inflation and employment gap rules respectively. The conclusion we draw from these results is: *rather than cooperate in the details of the rule, countries can simply agree to adopt a rule that responds to CPI and wage inflation.*[23]

In order to understand the workings of the model under cooperative rules, we now examine the responses under the optimal rule OPTCC and SIMCC in Table 6 to common 1% shocks to total factor productivity in the traded sector ($A_N(0) = A_N(0)^* = 1$) and to government spending ($G(0) = G^*(0) = 1$). Figures 1 and 2 show the simulations.

Consider first the supply-side shocks in the non-traded sectors. The features that OPTCC and the SIMCC rules have in common are a rise in output in the non-traded sectors, a corresponding fall in the traded sectors a switch facilitated on the demand side by a fall in the relative prices of non-traded and traded goods in the two blocs (n_t, n_t^*), a rise in consumption and a fall in CPI inflation. The rise in non-traded output is sluggish and never achieves the productivity increase of 1% because some of the benefit of this benign supply-side shock results in households taking more leisure. The relevant asymmetry between the two blocs for these shocks is the greater persistence of the TFP non-traded shock in EMU resulting in a more persistent output effect for that bloc.

To understand movements in the real exchange rate (and the related terms of trade) consider the following linearization of the modified UIP condition (55):

$$rer_t \quad = \quad E_t rer_{t+1} + E_t(r_t^* - r_t) - \delta_r b_{F,t} \qquad (112)$$

[23]Interestingly such a rule is implicitly advocated by a current member of the monetary policy committee of the Bank of England, David Blanchflower (see Blanchflower and Shadforth (2007)).

Solving (112) forward in time we see that the real exchange rate is a sum of future expected real interest-rate US-EMU differentials plus a term proportional to the sum of future expected net liabilities of EMU. The EMU real exchange will depreciate (a rise in rer_t) if the sum of expected future US-EMU interest-rate differentials is positive and/or the sum of expected future net liabilities are positive. The second asset effect, a deviation from risk-sharing, is shown in Figures 1 and 2 as $rerd$ and is negative indicating an accumulation of assets in EMU. However this effect is offset by a long-run interest-rate differential in favour of the US causing the EMU real and nominal exchange rate to eventually depreciate.

After around 10 quarters, SIMCC closely mimics OPTCC for the non-traded goods technology shocks but prior to that the nominal interest paths differ in both blocs and indeed the interest-rate differentials are of opposite sign. Turning to the government spending shock, in Figure 2 we now see more prolonged differences between SIMCC and OPTCC. Under the latter optimal regimes there is very nominal response in terms of the interest rate and CPI inflation rate. Under SIMCC however, the interest rate falls in both blocs with a differential in favour of EMU for 20 quarters. Again EMU accumulates assets and its real exchange rate first appreciates then depreciates as the interest-rate differential moves in favour of the US. SIMCC does not closely mimic OPTCC even after 20 quarters for this shock. This serves to highlight the sense in which simple rules are optimal: they are designed to minimize the expected welfare loss over the full range of shocks. They are optimal only for the particular estimated persistence parameters and standard deviations of white noise disturbances and are non-certainty equivalence: different estimates for standard deviations result in different optimized rules. Responses to some shocks can be severely sub-optimal as is clearly the case for this common government spending shock.

7 Coordination Gains with Model Uncertainty

In this section we consider model uncertainty in the form of uncertain estimates of the non-policy parameters of the model, Γ. Suppose the state of the world s is described by a model with $\Gamma = \Gamma^s$ expressed in state-space form as

$$\begin{bmatrix} \mathsf{z}_{t+1}^s \\ E_t \mathsf{x}_{t+1}^s \end{bmatrix} = A^s \begin{bmatrix} \mathsf{z}_t^s \\ \mathsf{x}_t^s \end{bmatrix} + B^s \begin{bmatrix} i_t \\ i_t^* \end{bmatrix} + C^s \epsilon_t \tag{113}$$

where z_t^s is a vector of predetermined variables at time t and x_t are non-predetermined variables in state s of the world. For parameter-robust rules, (103) is replaced with the average expected utility loss across a large number of draws, n, from all models constructed using both the posterior model probabilities and the posterior parameter distributions for each model.

$$\Omega_0 = \frac{1}{2}\sum_{s=1}^{n} E_t \sum_{t=0}^{\infty} \beta^t [\mathsf{y}_t^{s\prime} Q^s \mathsf{y}_t^r + w_i i_t^2] \tag{114}$$

A similar reformulation of the average expected utility applies to Ω_0^* and Ω_0^C.

We use the draws from the Markov Chain Monte Carlo (MCMC) Bayesian estimation as a representation of the ex post probability distribution of the parameters of the system. The results that follow are based on $n = 100$ such draws. For each draw we use the variance of the interest rate to calculate the probability of hitting the zero lower bound; once again the average of these appears as Prob ZLB in the tables and the average variance of these is included in the table as var(i_t). Thus with an equilibrium interest rate of 1% per quarter (4% per annum), the latter are given by

$$\mathrm{var}(i_t) \equiv \sigma_i^2 = \frac{1}{n}\sum_{j=1}^{n} \sigma_i^2(j) \tag{115}$$

$$\text{Prob } ZLB = \frac{1}{n}\sum_{j=1}^{n} Z\left(-\frac{1}{\sigma_i(j)}\right) \tag{116}$$

where $Z(x)$ is the probability that a standard normal random variable has a value less than x.

As we have found little improvement in rules that target wage inflation and the output gap in addition to CPI inflation, in Table 7 we present the results for robust CPI inflation targeting rule only. Consumption equivalent losses are measured relative to SIMCC without the ZLB. How do our robust rules compare with their non-robust counterparts of Table 6? With or without a ZLB we see that SIMCC calls for *far less activism* when there is model uncertainty, a result resembling that of Brainard (1967). With or without model uncertainty, SIMNC is far more activist than SIMCC as each bloc seeks to use the exchange rate to its advantage. Cooperation prevents such beggar-thy-neighbour behaviour, and when blocs cooperate to account for model uncertainty the benefits from cooperation grow substantially. With a ZLB, the consumption equivalent gain grows from 0.03% without model uncertainty to to 0.41% with model uncertainty. We have then a new result:

the gains from monetary policy coordination rise significantly when CPI interest-rate rules are designed to account for model uncertainty.

8 Conclusions

This paper has examined the the gains from monetary policy coordination in the design of CPI inflation interest-rate rules using a developed NOEM fitted to EMU-US data by Bayesian-ML methods. We incorporate two novel features not found in coordination literature to date: the incorporation of a ZLB interest-rate constraint and model uncertainty. Both these aspects have interesting consequences for the size of the gains from coordination summarized in Table 8.

First we recall two sources of gains from coordinating monetary policy: approximate quadratic loss functions that are different for the cooperative and non-cooperative games with the former not a simple linear combination of the latter, and the familiar terms of trade externality that encourages beggar-thy-neighbour policies. From the existing literature we know that a non-traded sector adds a relative price of traded to non-traded goods effect that can magnify the unilateral benefits of terms of trade changes. From the no-ZLB, no-model uncertainty cell of Table 8 we then appear to confirm the literature that suggests the coordination gains can be quite large in richer models with this feature. But this result is misleading because interest-rate rules that ignore the ZLB constraint are not operational. When the ZLB constraint is introduced, from the ZLB, no-model uncertainty cell of Table 8 we see that the the scope for beggar-thy-neighbour exchange rate policy under SIMNC is severely curtailed and the coordination gains become very small. However adding the second aspect: the need to design robust rules in the face of model uncertainty creates new incentives for exploiting the exchange rate channel under SIMNC that increase the inefficiency of the Nash equilibrium compared with SIMCC. The consequence is that even with a ZLB constraint the coordination gains become significant. This suggests a new result that may have general applicability to both monetary and fiscal stabilization policy: the gains from coordination can rise significantly when rules are designed to account for model uncertainty. We have established such a result for CPI inflation targeting interest-rate rules and a particular two-bloc model. Future research is required to establish the more general proposition.

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A The Policy Rules

Consider first the deterministic problem. Substituting out for outputs, the state-space representation (89) and (90) becomes:

$$\begin{bmatrix} z_{t+1} \\ x^e_{t+1,t} \end{bmatrix} = A \begin{bmatrix} z_t \\ x_t \end{bmatrix} + Bw_t \tag{A.1}$$

where z_t is an $(n-m)\times 1$ vector of predetermined variables including non-stationary processes, z_0 is given, $w_t = [i_t, i^*_t]^T$ is a vector of policy variables, x_t is an $m \times 1$ vector of non-predetermined variables and $x^e_{t+1,t}$ denotes rational (model consistent) expectations of x_{t+1} formed at time t. Then $x^e_{t+1,t} = x_{t+1}$ and letting $y^T_t = [z_t, x_t]^T$, (A.1) becomes

$$y_{t+1} = Ay_t + Bw_t \tag{A.2}$$

The policymakers' loss function under cooperation at time t with a ZLB is given by

$$\Omega^C_t = \frac{1}{2}\sum_{i=0}^{\infty} \lambda^t [y^T_{t+i} Q^C y_{t+i} + w^T_{t+i} R w_{t+i}] \tag{A.3}$$

The procedures for evaluating the three policy rules are outlined in the rest of this appendix (or Currie and Levine (1993) for a more detailed treatment).

A.1 The Optimal Policy: Cooperation with Commitment (CC)

Consider the policy-maker's *ex-ante* optimal policy at $t = 0$. This is found by minimizing Ω^C_0 given by (A.3) subject to (A.2) and given z_0. We proceed by defining the Hamiltonian

$$H_t(y_t, y_{t+1}, \mu_{t+1}) = \frac{1}{2}\beta^t (y^T_t Q^C y_t + w^T_t R w_t) + \mu_{t+1}(Ay_t + Bw_t - y_{t+1}) \tag{A.4}$$

where μ_t is a row vector of costate variables. By standard Lagrange multiplier theory we minimize

$$L_0(y_0, y_1, \ldots, w_0, w_1, \ldots, \mu_1, \mu_2, \ldots) = \sum_{t=0}^{\infty} H_t \tag{A.5}$$

with respect to the arguments of L_0 (except z_0 which is given). Then at the optimum, $L_0 = \Omega_0^C$.

Redefining a new costate vector $p_t = \beta^{-1}\mu_t^T$, the first-order conditions lead to

$$w_t = -R^{-1}\beta B^T p_{t+1} \tag{A.6}$$

$$\beta A^T p_{t+1} - p_t = -Q^C y_t \tag{A.7}$$

Substituting (A.6) into (A.2)) we arrive at the following system under control

$$\begin{bmatrix} I & \beta B R^{-1} B^T \\ 0 & \beta A^T \end{bmatrix} \begin{bmatrix} y_{t+1} \\ p_{t+1} \end{bmatrix} = \begin{bmatrix} A & 0 \\ -Q^C & I \end{bmatrix} \begin{bmatrix} y_t \\ p_t \end{bmatrix} \tag{A.8}$$

To complete the solution we require $2n$ boundary conditions for (A.8). Specifying z_0 gives us $n-m$ of these conditions. The remaining condition is the 'transversality condition'

$$\lim_{t\to\infty} \mu_t^T = \lim_{t\to\infty} \beta^t p_t = 0 \tag{A.9}$$

and the initial condition

$$p_{20} = 0 \tag{A.10}$$

where $p_t^T = \begin{bmatrix} p_{1t}^T & p_{2t}^T \end{bmatrix}$ is partitioned so that p_{1t} is of dimension $(n-m)\times 1$. Equation (**??**), (A.6), (A.8) together with the $2n$ boundary conditions constitute the system under optimal control.

Solving the system under control leads to the following rule

$$w_t = -F \begin{bmatrix} I & 0 \\ -N_{21} & -N_{22} \end{bmatrix} \begin{bmatrix} z_t \\ p_{2t} \end{bmatrix} \tag{A.11}$$

$$\begin{bmatrix} z_{t+1} \\ p_{2t+1} \end{bmatrix} = \begin{bmatrix} I & 0 \\ S_{21} & S_{22} \end{bmatrix} G \begin{bmatrix} I & 0 \\ -N_{21} & -N_{22} \end{bmatrix} \begin{bmatrix} z_t \\ p_{2t} \end{bmatrix} \tag{A.12}$$

$$N = \begin{bmatrix} S_{11} - S_{12}S_{22}^{-1}S_{21} & S_{12}S_{22}^{-1} \\ -S_{22}^{-1}S_{21} & S_{22}^{-1} \end{bmatrix} = \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} \tag{A.13}$$

$$x_t = -\begin{bmatrix} N_{21} & N_{22} \end{bmatrix} \begin{bmatrix} z_t \\ p_{2t} \end{bmatrix} \tag{A.14}$$

where $F = -(R + B^T SB)^{-1}(B^T SA + U^T), G = A - BF$ and

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \tag{A.15}$$

partitioned so that S_{11} is $(n-m)\times(n-m)$ and S_{22} is $m \times m$ is the solution to the steady-state Ricatti equation

$$S = Q^C + F^T RF + \beta(A - BF)^T S(A - BF) \tag{A.16}$$

The welfare loss at time t is

$$\Omega_t^{CCOPT} = -\frac{1}{2}(\mathrm{tr}(N_{11}Z_t) + \mathrm{tr}(N_{22}p_{2t}p_{2t}^T)) \tag{A.17}$$

where $Z_t = z_t z_t^T$. To achieve optimality the policy-maker sets $p_{20} = 0$ at time $t = 0$. At time $t > 0$ there exists a gain from reneging by resetting $p_{2t} = 0$. It can be shown that $N_{22} < 0$, so the incentive to renege exists at all points along the trajectory of the optimal policy. This is the time-inconsistency problem.

A.2 Optimized Simple Commitment Rules (SIMCC and SIMNC)

We now consider simple sub-optimal rules of the form

$$w_t = Dy_t = D\begin{bmatrix} z_t \\ x_t \end{bmatrix} \tag{A.18}$$

where D is constrained to be sparse in some specified way. Rule (A.18) can be quite general. By augmenting the state vector in an appropriate way it can represent a PID (proportional-integral-derivative)controller (though the paper is restricted to a simple proportional controller only).

First consider the design of cooperative simple rules. Substituting (A.18) into (A.3) gives

$$\Omega_t = \frac{1}{2}\sum_{i=0}^{\infty}\beta_t y_{t+i}^T P_{t+i} y_{t+i} \tag{A.19}$$

where $P = Q^C + D^T RD$. The system under control (A.1), with w_t given by (A.18), has a rational expectations solution with $x_t = -Nz_t$ where $N = N(D)$. Hence

$$y_t^T P y_t = z_t^T T z_t \tag{A.20}$$

where $T = P_{11} - N^T P_{21} - P_{12}N + N^T P_{22}N$, P is partitioned as for S in (A.15) onwards and

$$z_{t+1} = (G_{11} - G_{12}N)z_t \tag{A.21}$$

where $G = A + BD$ is partitioned as for P. Solving (A.21) we have

$$z_t = (G_{11} - G_{12}N)^t z_0 \tag{A.22}$$

Hence from (A.23), (A.20) and (A.22) we may write at time t

$$\Omega_t^{SIMCC} = \frac{1}{2}z_t^T V z_t = \frac{1}{2}\mathrm{tr}(VZ_t) \tag{A.23}$$

where $Z_t = z_t z_t^T$ and V satisfies the *Lyapunov* equation

$$V = T + H^T V H \tag{A.24}$$

where $H = G_{11} - G_{12}N$. At time $t = 0$ the optimized simple rule is then found by minimizing Ω_0 given by (A.23) with respect to the non-zero elements of D given z_0 using a standard numerical technique. An important feature of the result is that unlike the previous solution the optimal value of D is not independent of z_0. That is to say

$$D = D(z_0)$$

For the non-cooperative case, in a closed-loop Nash equilibrium we assume each policymaker chooses rules $w_t = Dy_t$ and $w_t^* = D^* y_t$ independently taking the rule of the other bloc as given. The equilibrium is then computed by iterating between the two countries until the solutions converge.

A.3 The Stochastic Case

Consider the stochastic generalization of (A.1)

$$\begin{bmatrix} z_{t+1} \\ x^e_{t+1,t} \end{bmatrix} = A \begin{bmatrix} z_t \\ x_t \end{bmatrix} + Bw_t + \begin{bmatrix} u_t \\ 0 \end{bmatrix} \tag{A.25}$$

where u_t is an $n \times 1$ vector of white noise disturbances independently distributed with $\text{cov}(u_t) = \Sigma$. Then, it can be shown that certainty equivalence applies to all the policy rules apart from the simple rules (see Currie and Levine (1993)). The expected loss at time t is as before with quadratic terms of the form $z_t^T X z_t = \text{tr}(X z_t, Z_t^T)$ replaced with

$$E_t\left(\text{tr}\left[X\left(z_t z_t^T + \sum_{i=1}^{\infty}\beta^t u_{t+i}u_{t+i}^T\right)\right]\right) = \text{tr}\left[X\left(z_t^T z_t + \frac{\beta}{1-\beta}\Sigma\right)\right] \tag{A.26}$$

where E_t is the expectations operator with expectations formed at time t.

Thus for the optimal policy with commitment (A.17) becomes in the stochastic case

$$\Omega_t^{OPTCC} = -\frac{1}{2}\text{tr}\left(N_{11}\left(Z_t + \frac{\beta}{1-\beta}\Sigma\right) + N_{22}p_{2t}p_{2t}^T\right) \tag{A.27}$$

For the simple rule, generalizing (A.23)

$$\Omega_t^{SIMCC} = -\frac{1}{2}\text{tr}\left(V\left(Z_t + \frac{\beta}{1-\beta}\Sigma\right)\right) \tag{A.28}$$

(A.27) and (A.28) are *conditional* welfare loss measures at time t given z_t. The paper reports conditional welfare losses at the steady state ($z_t = Z_t = 0$). An *unconditional* welfare loss measure averages over all possible initial states using the distribution of states calculated under the optimal commitment policy. These are obtained from (A.27) and (A.28) by replacing Z_t with the variance-covariance matrix $\text{cov}(z_t)$. However, for a discount factor close to unity, the stochastic terms dominate so the difference between these two measures is small.

For the conditional welfare loss measures at time $t = 0$, the optimized cooperative simple rule is found by minimizing Ω_0^{SIMCC} given by (A.28). Now we find that

$$D^* = D^* \left(z_0 + \frac{\beta}{1-\beta} \Sigma \right) \tag{A.29}$$

or, in other words, the optimized rule depends both on the initial displacement z_0 and on the covariance matrix of disturbances Σ. The non-cooperative rule for the stochastic case follows in a similar way.

Equilibrium	**Form of Rule**	(w_i, w_i^*)	Loss	$c_e(\%)$	$(\mathrm{var}(i_t), \mathrm{var}(i_t^*))$	Pr ZLB
OPTCC	complex	(1.6, 1.6)	59.8	0	$(3.96, 2.11)$	$(0.31, 0.25)$
SIMCC CPI Inflation	$(\rho, \theta_\pi) = (0.83, 0.31)$ $(\rho^*, \theta_\pi^*) = (0.6, 1.19)$	(0,0)	180.6	0.60	(0.18.0.34)	$(0.01, 0.06)$
SIMNC CPI Inflation	$(\rho, \theta_\pi) = (0.51, 10.0)$ $(\rho^*, \theta_\pi^*) = (0.64, 10.0)$	(0,0)	286	1.13	(2.67, 2.56)	$(0.27, 0.26)$
SIMCC Wage Inflation	$(\rho, \theta_\pi, \theta_{\Delta w}) = (0.98, 0.08, 0.15)$ $(\rho^*, \theta_\pi^*, \theta_{\Delta w}^*) = (0.82, 0.4, 0.78)$	(0,0)	178.7	0.60	$(0.04, 0.34)$	$(0.00, 0.04)$
SIMNC Wage Inflation	$(\rho, \theta_\pi, \theta_{\Delta w}) = (0.63, 10.0, 0.0)$ $(\rho^*, \theta_\pi^*, \theta_{\Delta w}^*) = (1.0, 0.0, 10.0)$	(0,0)	335.9	1.38	$(2.54, 9.46)$	$(0.27, 0.46)$
SIMCC Employment Gap	$(\rho, \theta_\pi, \theta_{egap}) = (0.9, 0.17, 0.01)$ $(\rho^*, \theta_\pi^*, \theta_{egap}^*) = (0.65, 1.0, 0.0)$	(0,0)	180.3	0.60	$(0.1, 0.3)$	$(0.00, 0.03)$
SIMNC Employment Gap	$(\rho, \theta_\pi, \theta_{egap}) = (0.56, 10.0, 0.0)$ $(\rho^*, \theta_\pi^*, \theta_{egap}^*) = (0.94, 10.0, 0.55)$	(0,0)	283.5	1.12	$(2.64, 2.17)$	$(0.27, 0.25)$

Table 5. Gains from Coordination: No Model Uncertainty; no ZLB Constraint.

Equilibrium	**Form of Rule**	(w_i, w_i^*)	Loss	$c_e(\%)$	$(\mathrm{var}(i_t), \mathrm{var}(i_t^*))$	Pr ZLB
OPTCC	complex	(4.6, 2.8)	111.8	0	$(0.25, 0.25)$	$(0.02, 0.02)$
SIMCC CPI Inflation	(ρ, θ_π) =(0.83,0.31) $(\rho^*, \theta_\pi^*) = (0.72, 0.84)$	(0.0, 0.17)	181.7	0.35	(0.19.0.25)	$(0.02, 0.02)$
SIMNC CPI Inflation	$(\rho, \theta_\pi) = (1.0, 0.13)$ $(\rho^*, \theta_\pi^*) = (0.93, 1.06)$	(0.59, 1.2)	189.1	0.38	$(0.04, 0.25)$	$(0.00, 0.02)$
SIMCC Wage Inflation	$(\rho, \theta_\pi, \theta_{\Delta w}) = (0.97, 0.09, 0.13)$ $(\rho^*, \theta_\pi^*, \theta_{\Delta w}^*) = (0.88, 0.0.28, 0.62)$	(0.09, 0.09)	179.1	0.34	$(0.05, 0.24)$	$(0.00, 0.02)$
SIMNC Wage Inflation	$(\rho, \theta_\pi, \theta_{\Delta w}) = (1.0, 0.48, 0.52)$ $(\rho^*, \theta_\pi^*, \theta_{\Delta w}^*) = (1.0, 0.3, 0.64)$	(0.55, 1.1)	181.4	0.35	$(0.15, 0.22)$	$(0.03, 0.02)$
SIMCC Employment Gap	$(\rho, \theta_\pi, \theta_{egap}) = (0.9, 0.17, 0.01)$ $(\rho^*, \theta_\pi^*, \theta_{egap}^*) = (0.65, 1.0, 0.0)$	(0.0, 0.11)	180.9	0.25	$(0.12, 0.25)$	$(0.03, 0.02)$
SIMNC Employment Gap	$(\rho, \theta_\pi, \theta_{egap}) = (1.0, 0.48, 0.52)$ $(\rho^*, \theta_\pi^*, \theta_{egap}^*) = (0.94, 10.0, 0.55)$	(0.56, 1.07)	196.5	0.42	$(0.25, 0.23)$	$(0.02, 0.02)$

Table 6. Gains from Coordination: No Model Uncertainty; ZLB Constraint Imposed.

Equilibrium	**Form of Rule**	(w_i, w_i^*)	Loss	$c_e(\%)$	$(\text{var}(i_t), \text{var}(i_t^*))$	Pr ZLB
SIMCC (No ZLB) CPI Inflation	(ρ, θ_π) =(0.81,0.2) $(\rho^*, \theta_\pi^*) = (0.433, 10)$	(0.0, 0.0)	27.7	0	(0.74.2.53)	$(0.12, 0.264)$
SIMNC (No ZLB) CPI Inflation	$(\rho, \theta_\pi) = (0.94, 0.12)$ $(\rho^*, \theta_\pi^*) = (0.97, 10)$	(0.0, 0.0)	114.3	0.43	$(0.06, 2.22)$	$(0.00, 0.25)$
SIMCC (ZLB) CPI Inflation	$(\rho, \theta_\pi, \theta_\theta) = (0.81, 0.2)$ $(\rho^*, \theta_\pi^*, \theta_\theta^*) = (0.639, 0.656)$	(6, 0.5)	42.8	0.08.	$(0.25, 0.25)$	$(0.024, 0.024)$
SIMNC (ZLB) CPI Inflation	$(\rho, \theta_\pi, \theta_\theta) = (0.95, 0.1)$ $(\rho^*, \theta_\pi^*, \theta_\theta^*) = (1.0, 1.12)$	(1.0, 1.12)	126.5	0.49	$(0.06, 0.25)$	$(0.00, 0.023)$

Table 7. Gains from Coordination: Robust Rules with Model Uncertainty; with and without ZLB

	No ZLB Constraint	ZLB Constraint
No Model Uncertainty	0.53	0.03
Model Uncertainty	0.43	0.41

Table 8. Summary of Gains from Coordination in Consumption Equivalent Terms (%)

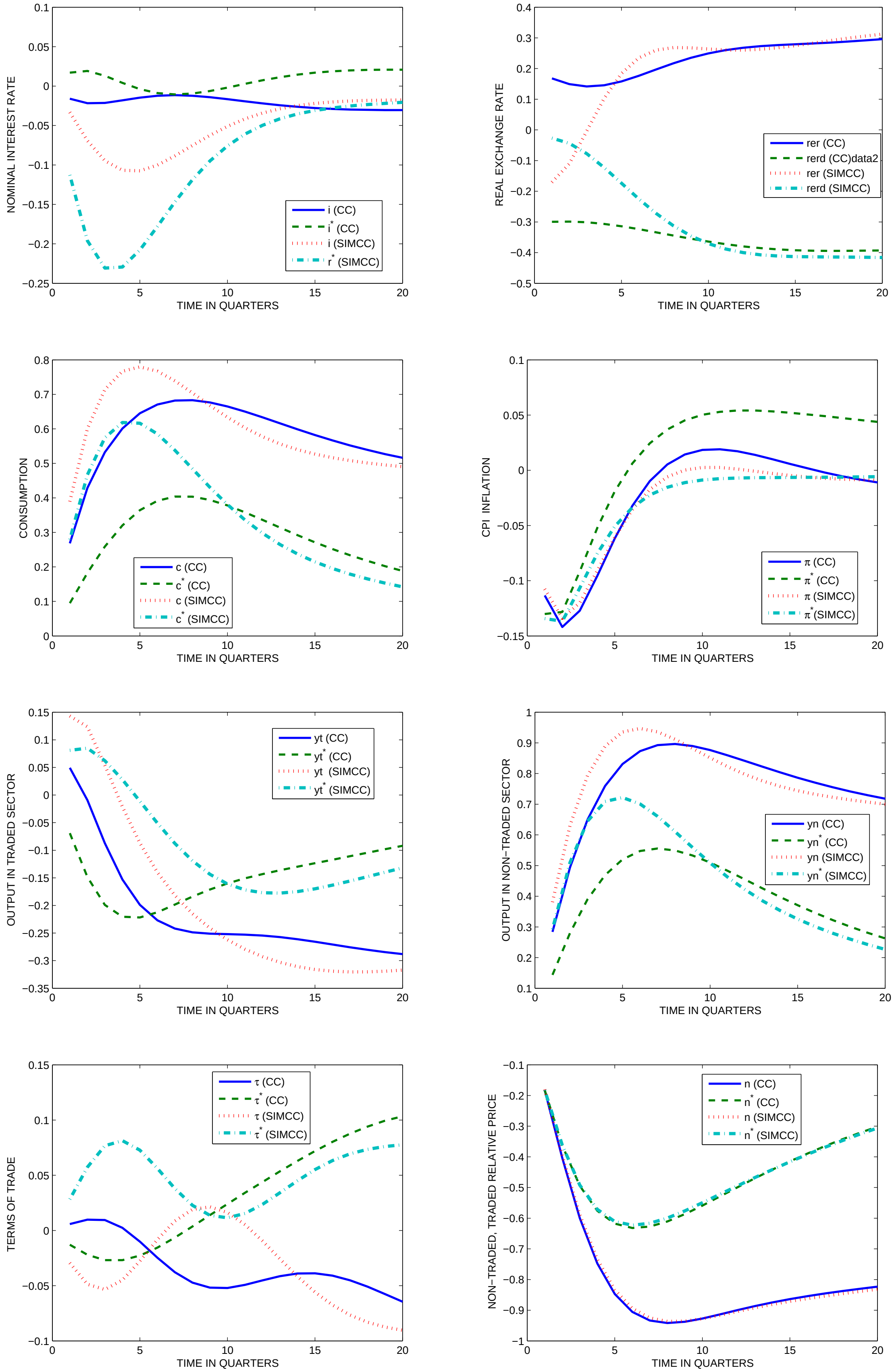

Figure 1: **Impulse Responses to a 1% Non-Traded Good Technology Shock**

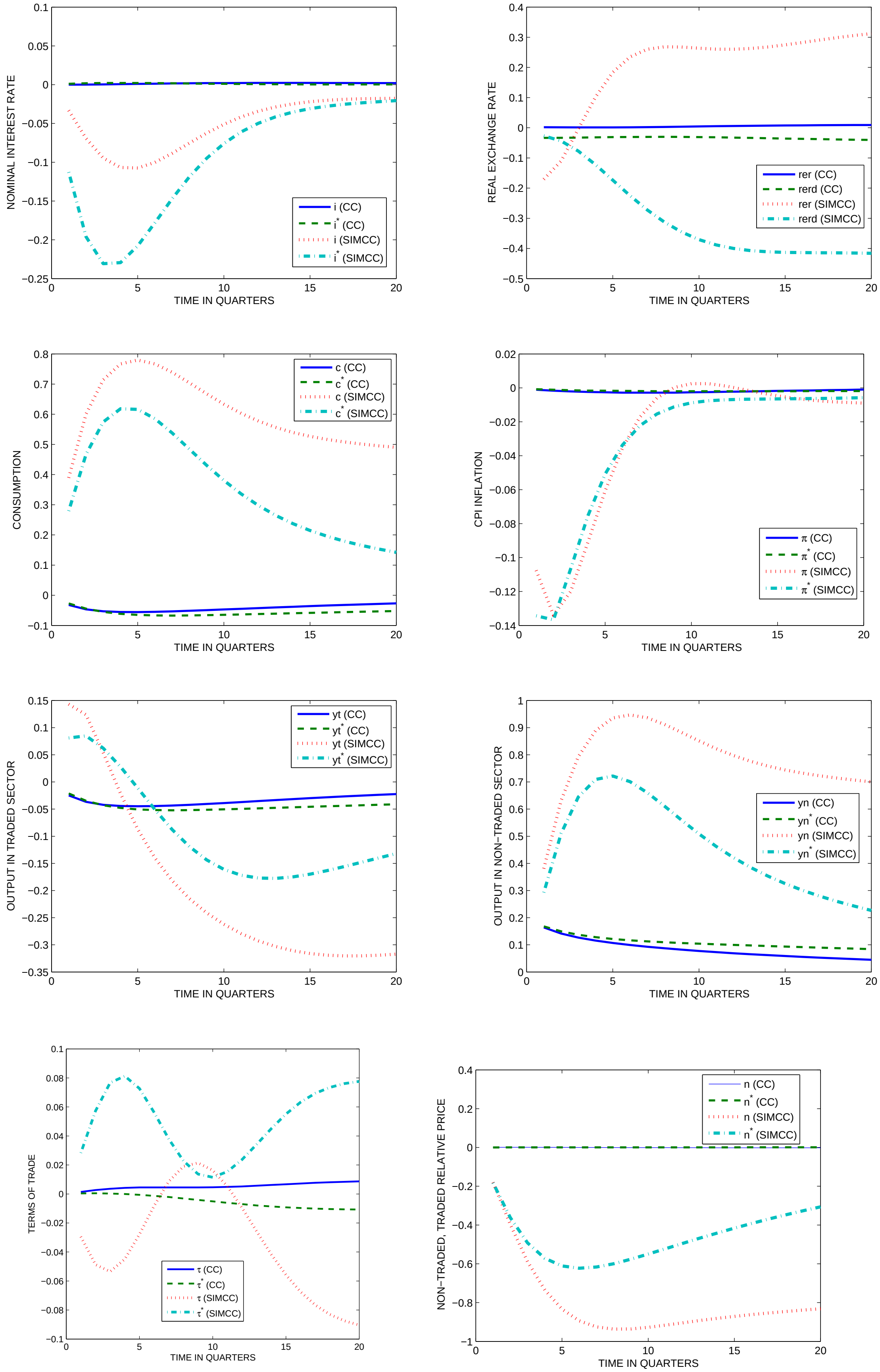

Figure 2: **Impulse Responses to a 1% Government Spending Shock**